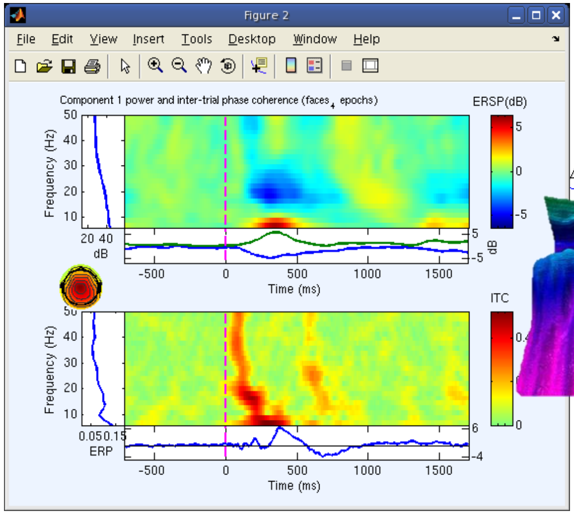
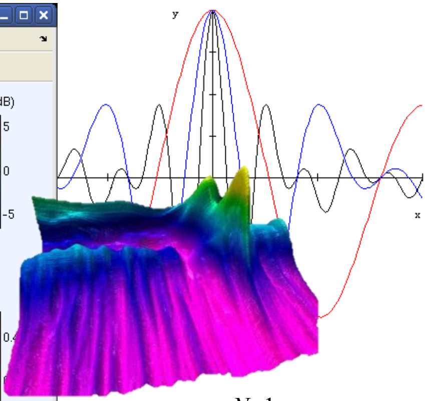
Time-Frequency Analysis of Biophysical Time series

Arnaud Delorme

(with contributions from Tim Mullen and John Iversen)

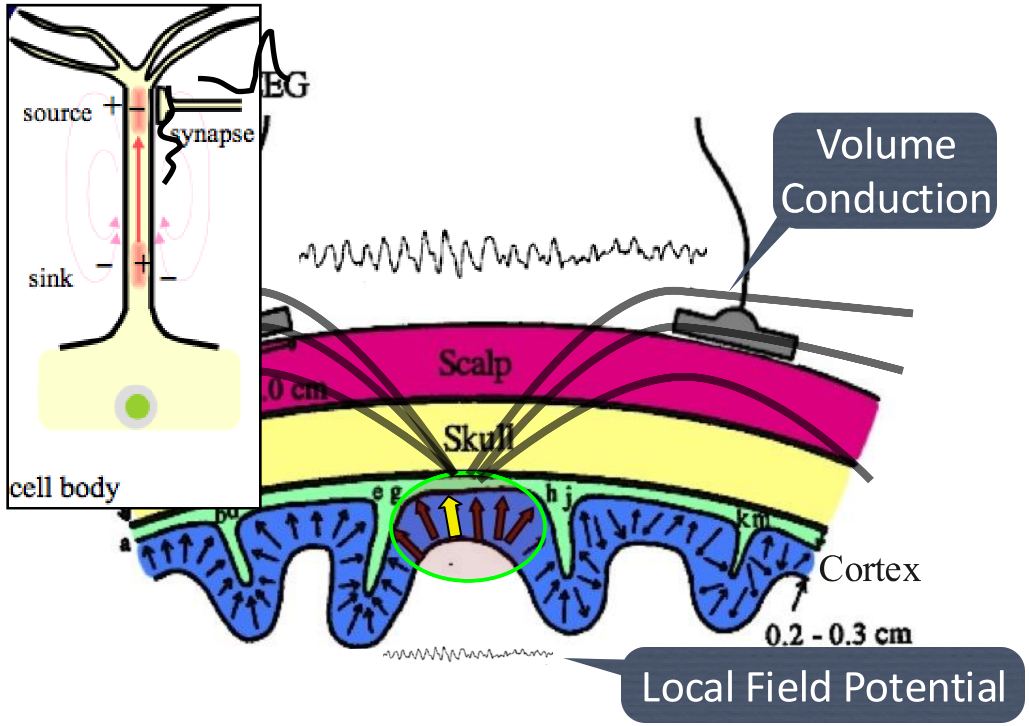




$$S(f) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)e^{-2\pi ift/N}$$

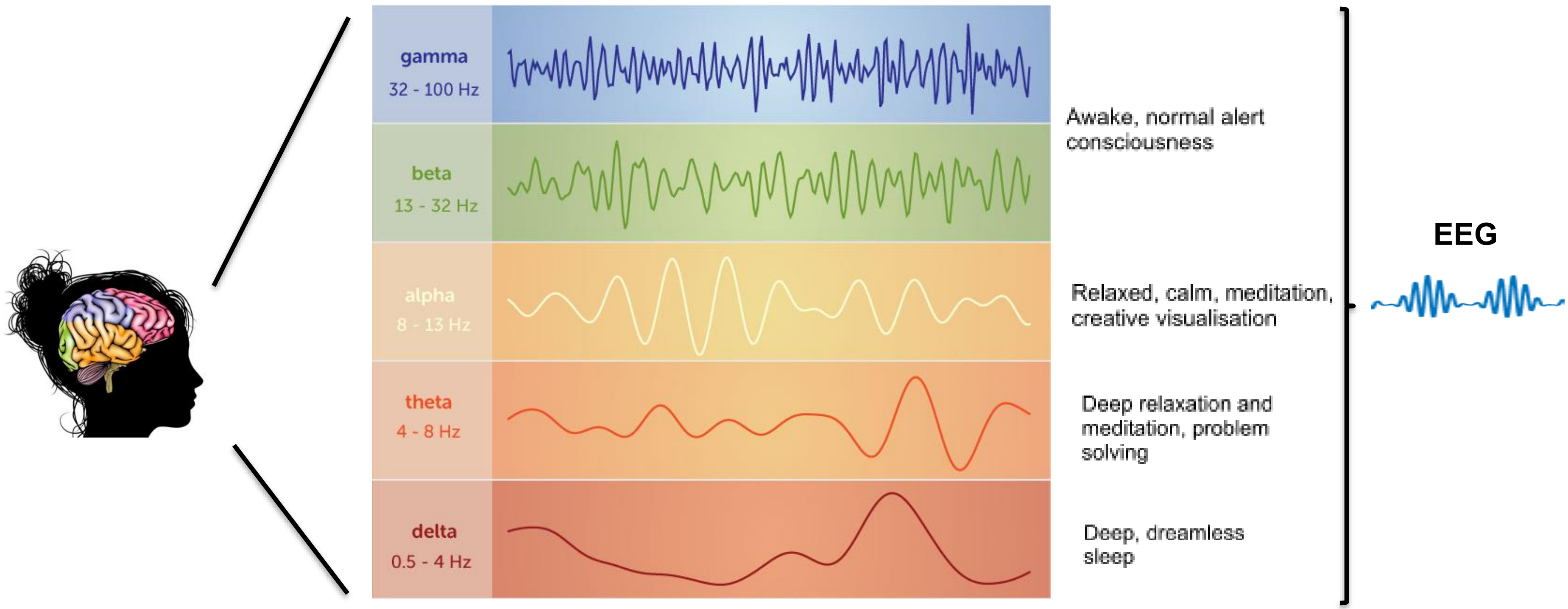
Biophysics of EEG

Hans Berger (1873-
1971)



Synchronicity of cell excitation (due to recurrent cortico-cortical and cortico-thalamo-cortical projections) determines amplitude and rhythm of the EEG signal

Brain oscillations



Sinusoids

phase shift

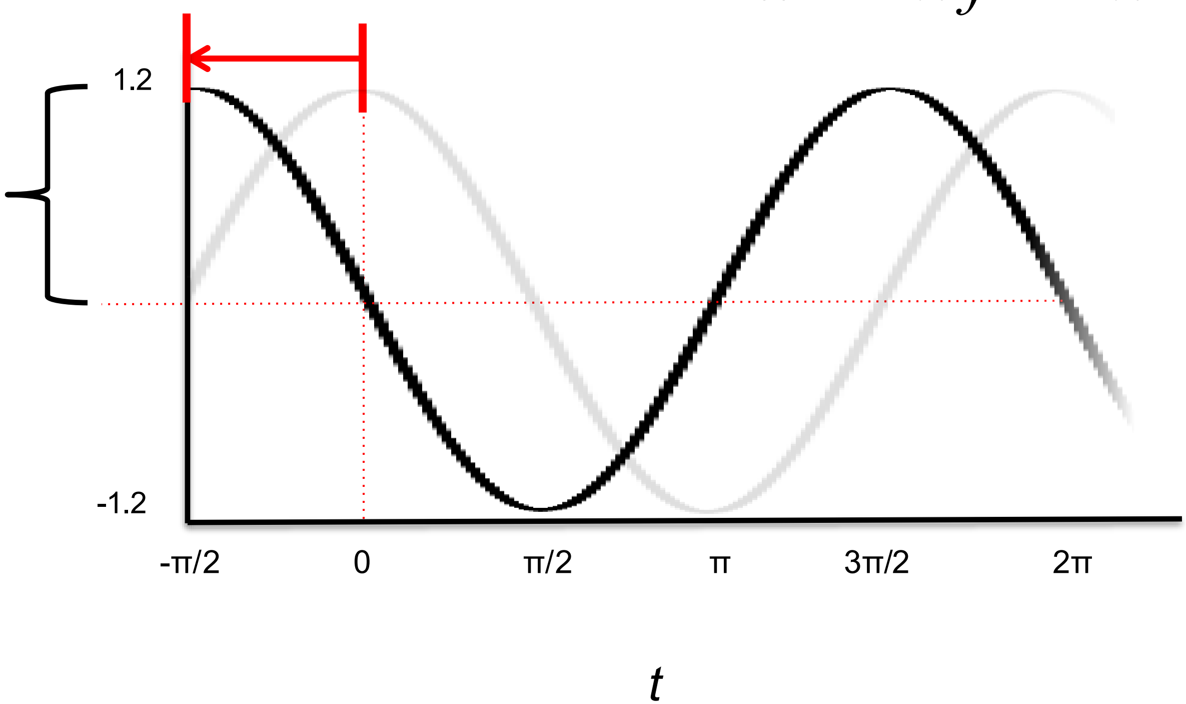
$$\theta = \pi / 2$$

angular frequency

$$\omega = 2\pi f = 2\pi \text{ rad/sec}$$

Amplitude

A=*1.2*

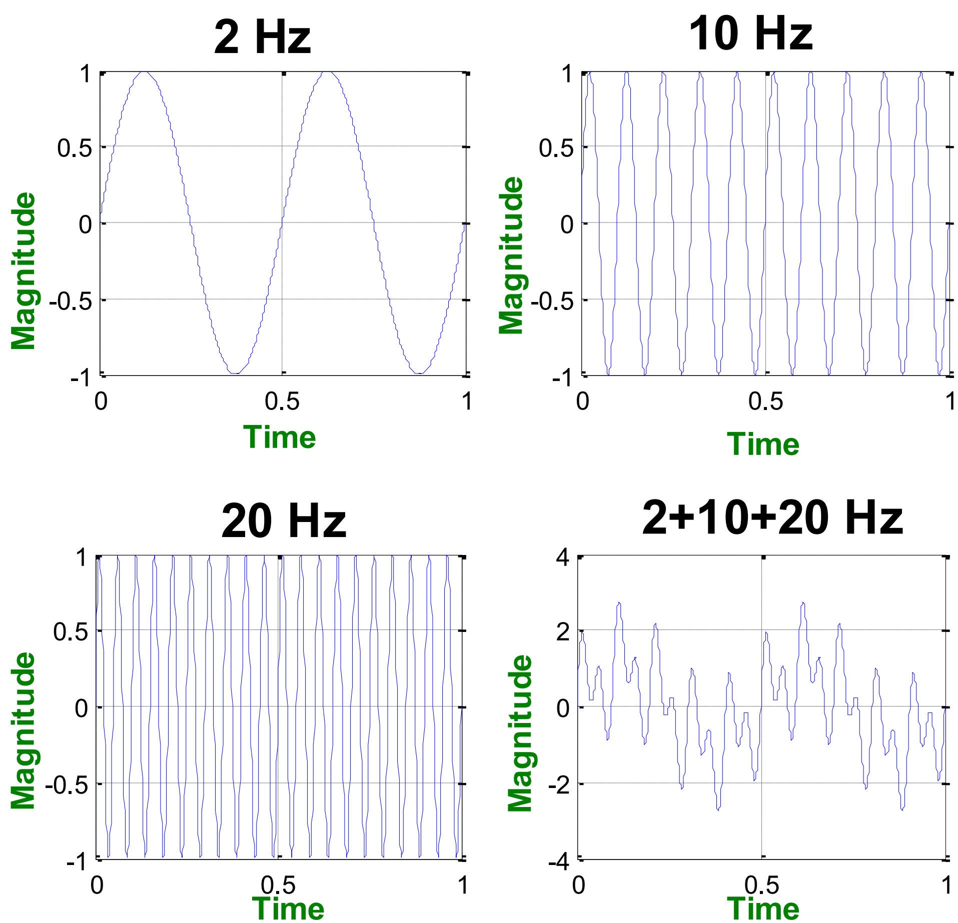


Wide-sense stationary signals

The first and second moments (mean and variance) of the data distribution do not depend on time.

Wide-sense stationary signals

Cyclostationary signals



Wide-sense

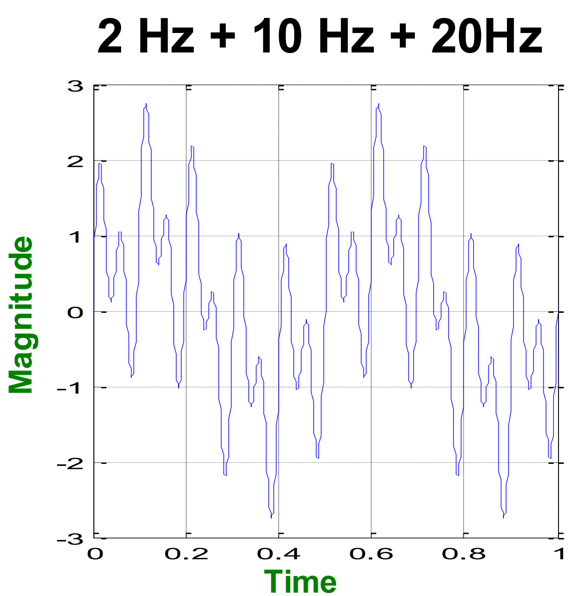
Stationary

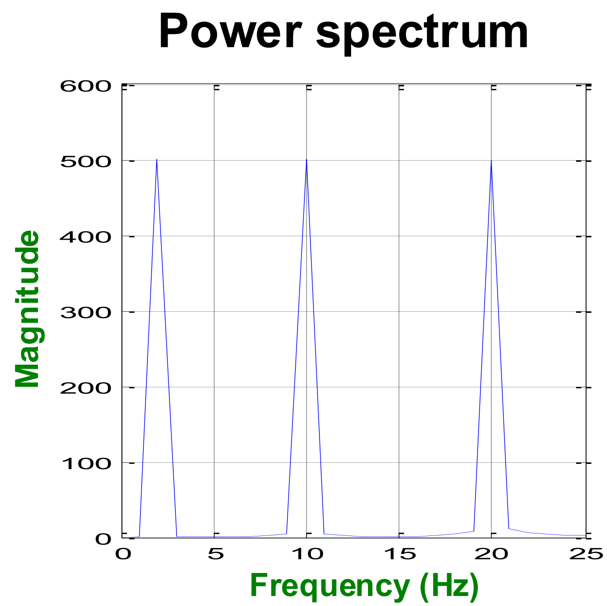
Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Wide-sense stationary signals

Wide-sense

Stationary

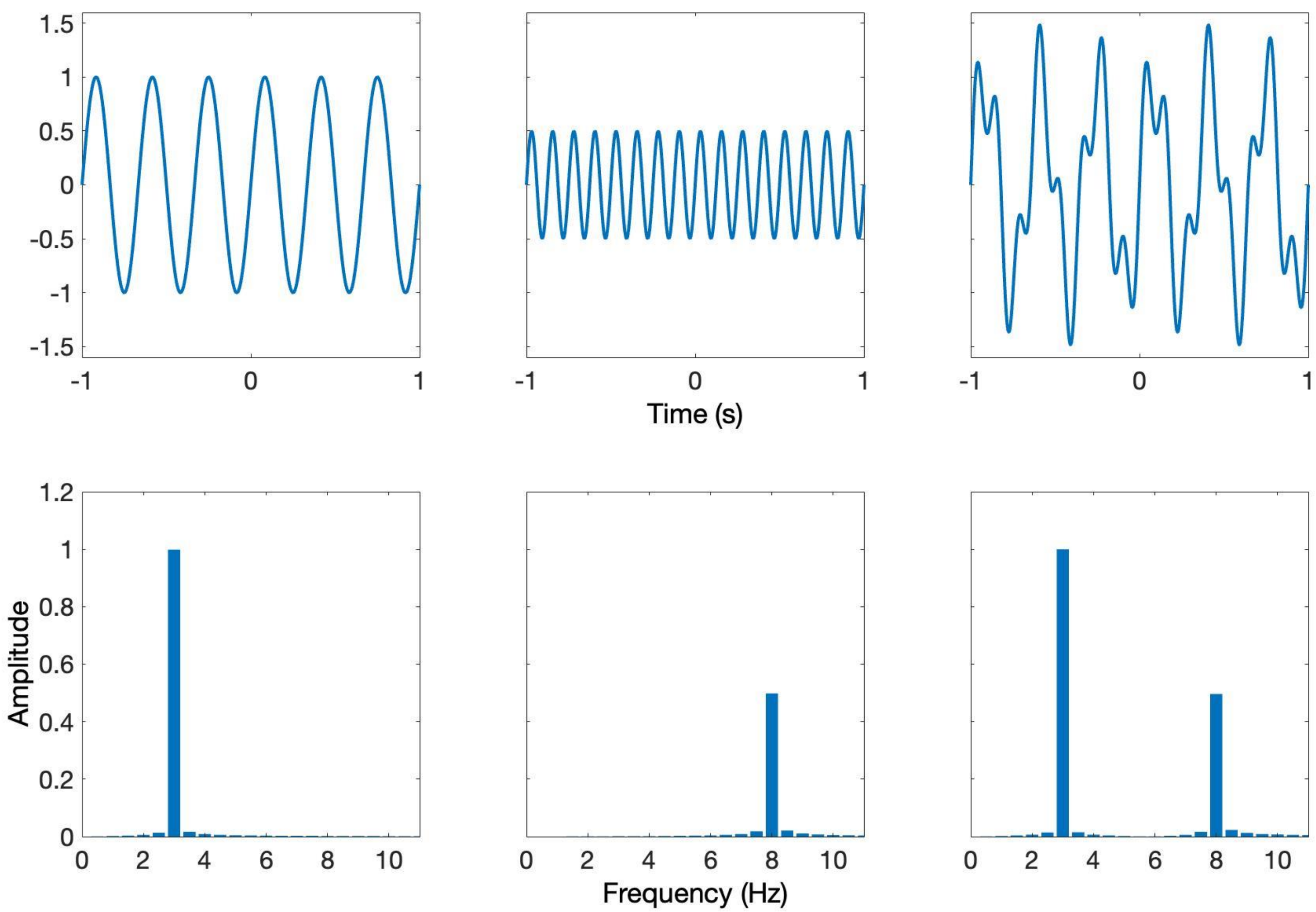




By looking at the Power spectrum of the signal we can recognize three frequency Components (at 2, 10, 20Hz respectively).

Slide courtesy of Petros Xanthopoulos, Univ. of Florida

Time and Frequency Domain

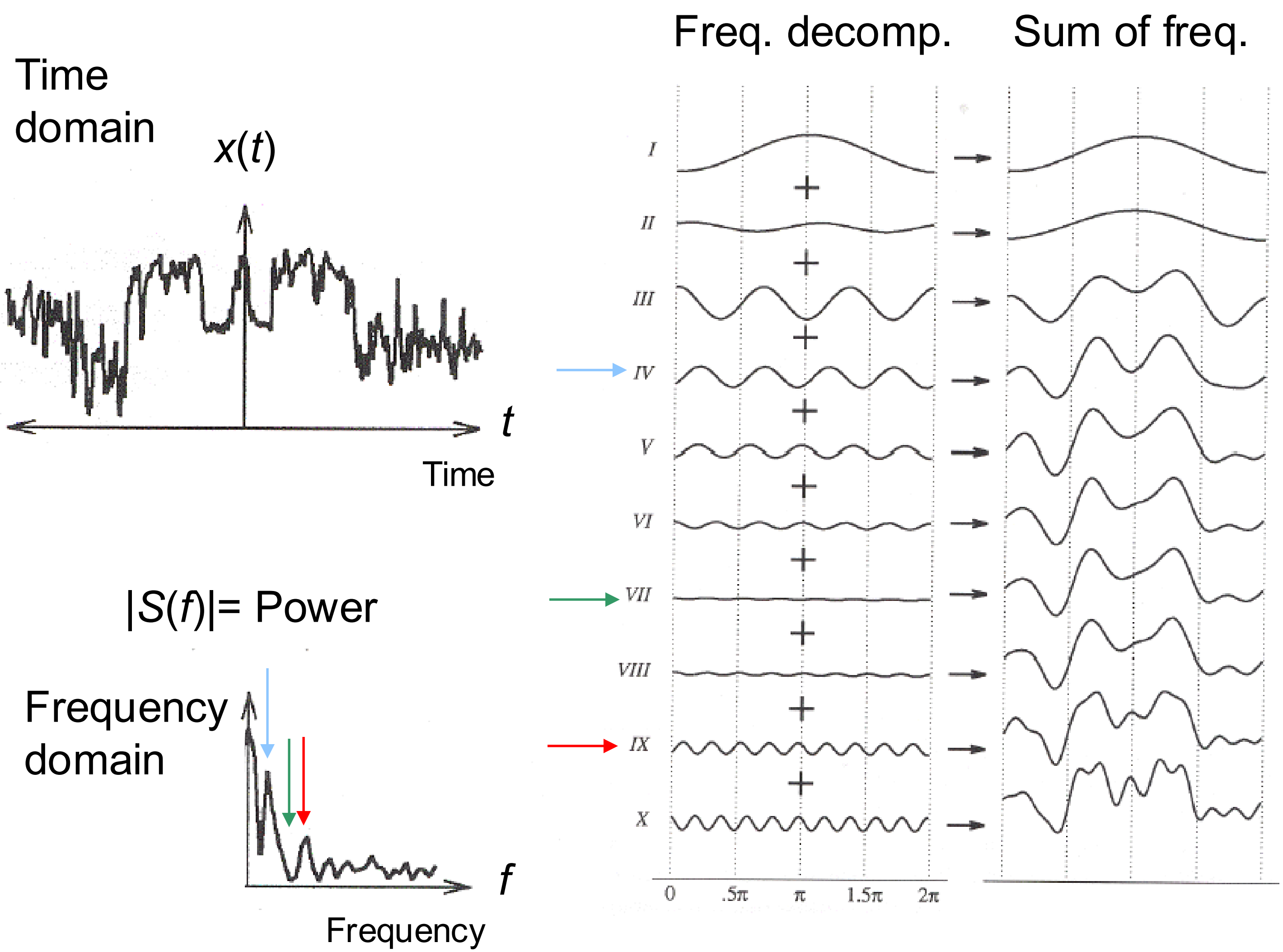


Fourier’s Theorem

Any stationary, continuous process can be exactly described by an infinite sum of sinusoids of different amplitudes and phases.

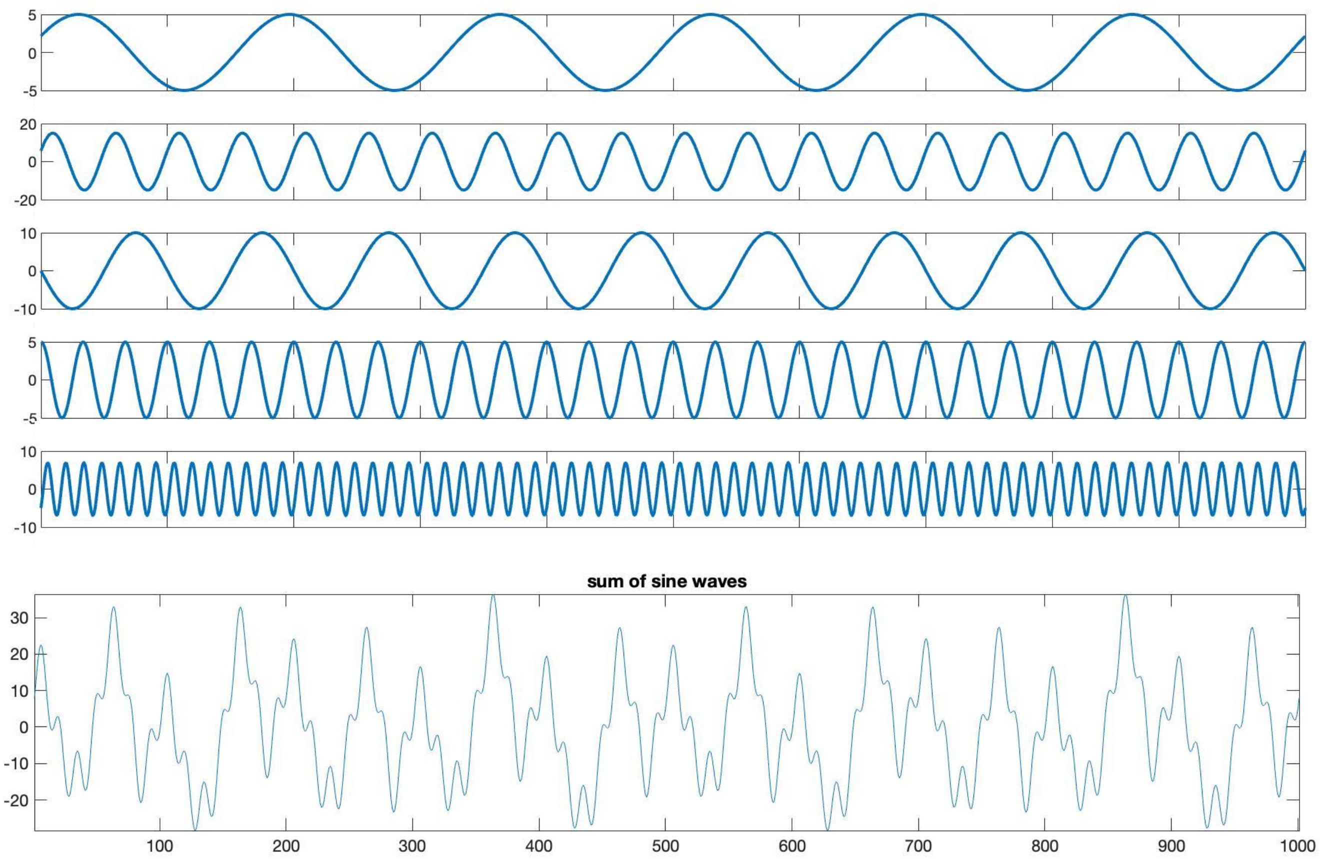
Jean Baptiste Joseph Fourier
(1768 –1830)

Fourier Analysis

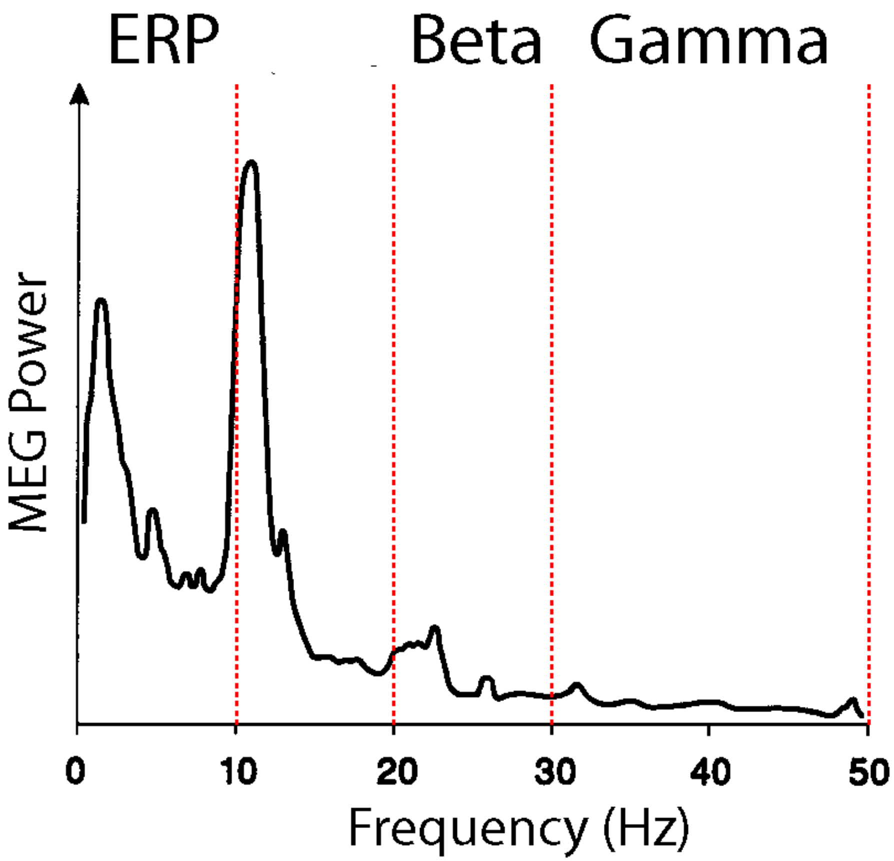


Figure, courtesy of Ravi Ramamoorthi & Wolberg

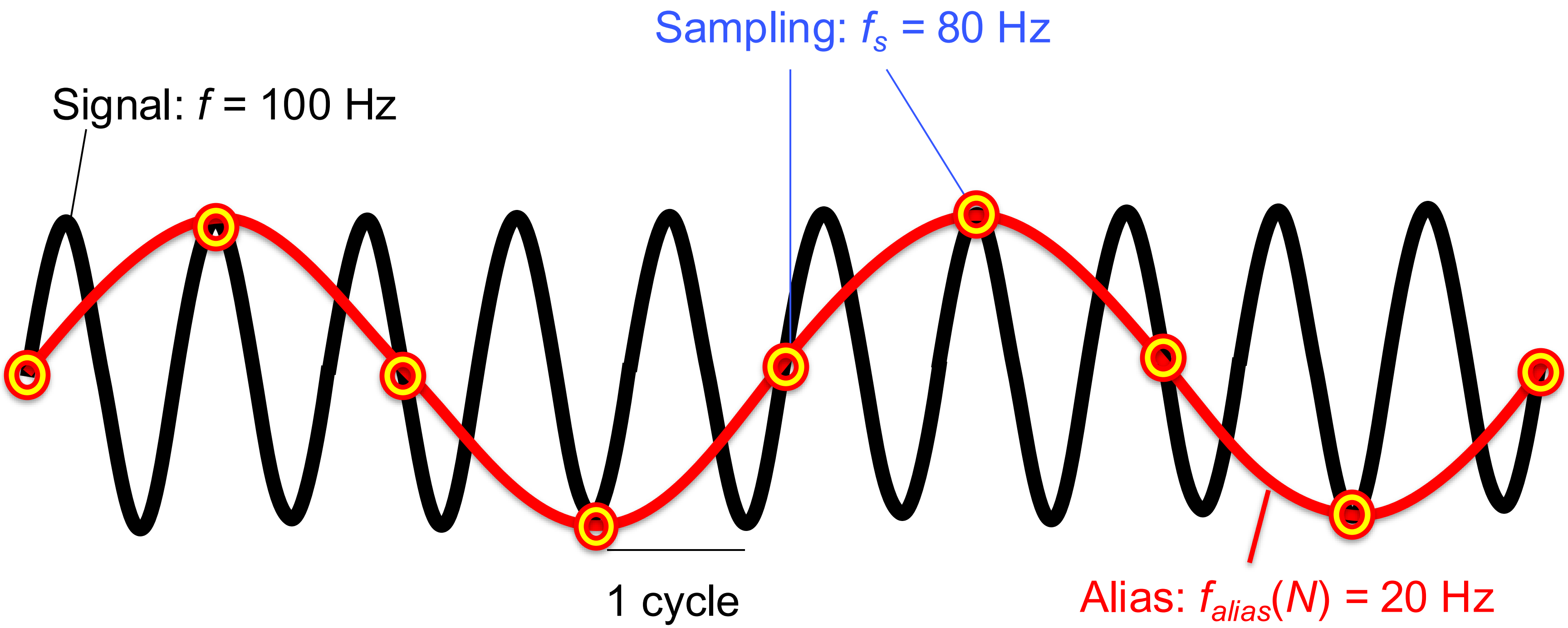
Adding up sine waves



MEEG Spectrum



Aliasing and the Nyquist Frequency



Nyquist Frequency:

The maximum frequency that can be uniquely recovered at a sampling rate of f_s

$$f_N = f_s / 2$$

$$f_{alias}(N) = \left| f - N f_s \right|$$

$$f_s = \text{sampling rate}$$

Quiz: What is N in the example above?

Euler's Formula

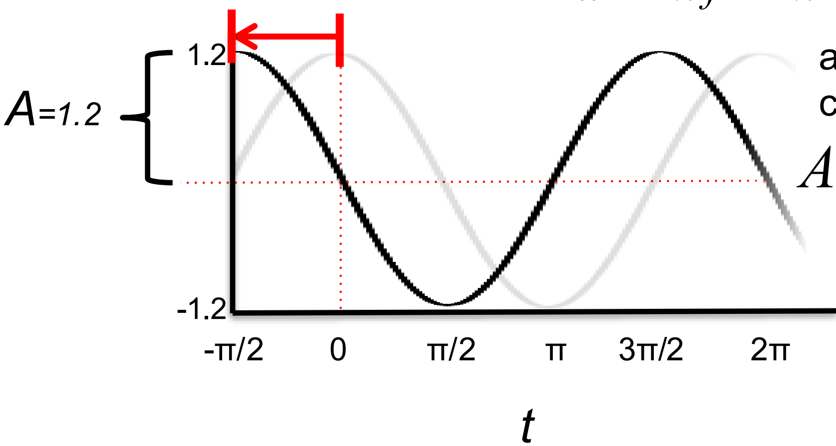
phase shift

$\theta = \pi / 2$

angular frequency

$\omega = 2\pi f = 2\pi$ rad/sec

instantaneous complex amplitude and phase



any sinusoid can be expressed as the sum of two complex numbers...

$$A \cdot \cos(\omega t + \theta) = \frac{A}{2} e^{i(\omega t + \theta)} + \frac{A}{2} e^{-i(\omega t + \theta)}$$

$$= \mathrm{Re}\{A e^{i(\omega t + \theta)}\} = \mathrm{Re}\{S(\omega, t)\}$$

Another version:

$$e^{i(\omega t + \theta)} = \cos(\omega t + \theta) + i \sin(\omega t + \theta)$$

Real part
Cosine component

Imaginary part
Sine component

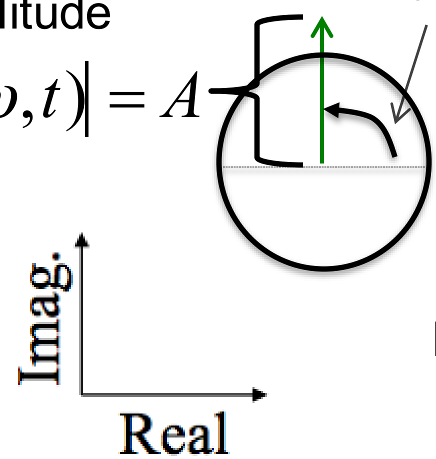
Phase

$\theta = \angle S(\omega, t)$

$= \pi / 2$

Amplitude

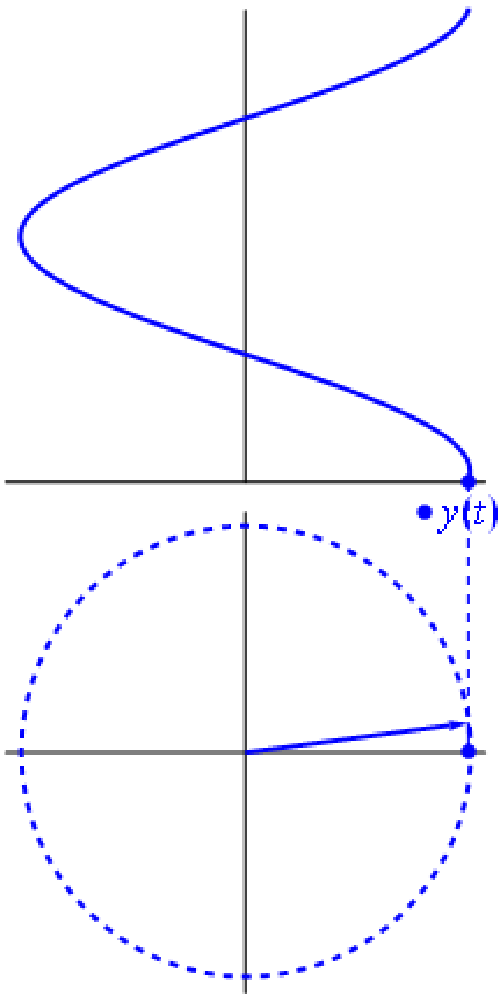
$|S(\omega, t)| = A$

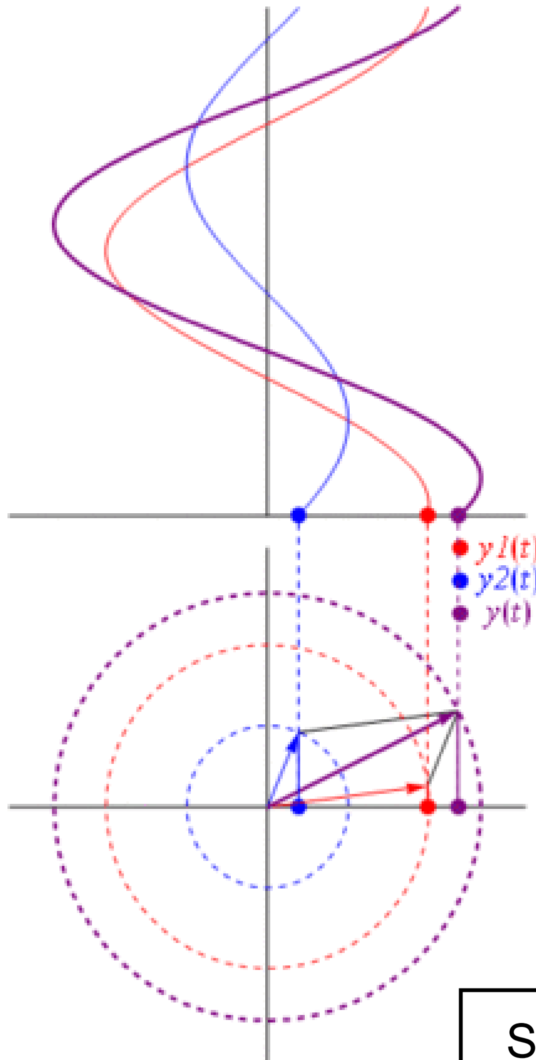


Phasor

Phasors

Rotation velocity (Rad/S; Hz)
= (angular) frequency (w; f)





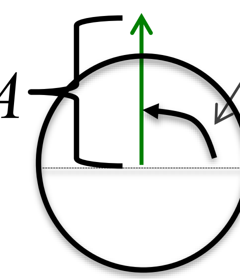
$$A \cos(\omega t+\theta) = \mathrm{Re}\{Ae^{i(\omega t+\theta)}\}$$
$$= \mathrm{Re}\{S(\omega,t)\}$$

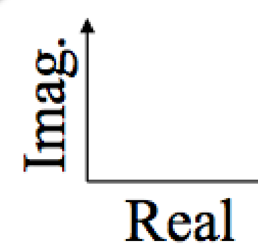
Amplitude

$$|S(\omega,t)| = A$$

Phase

$$\theta = \frac{\pi}{2}$$

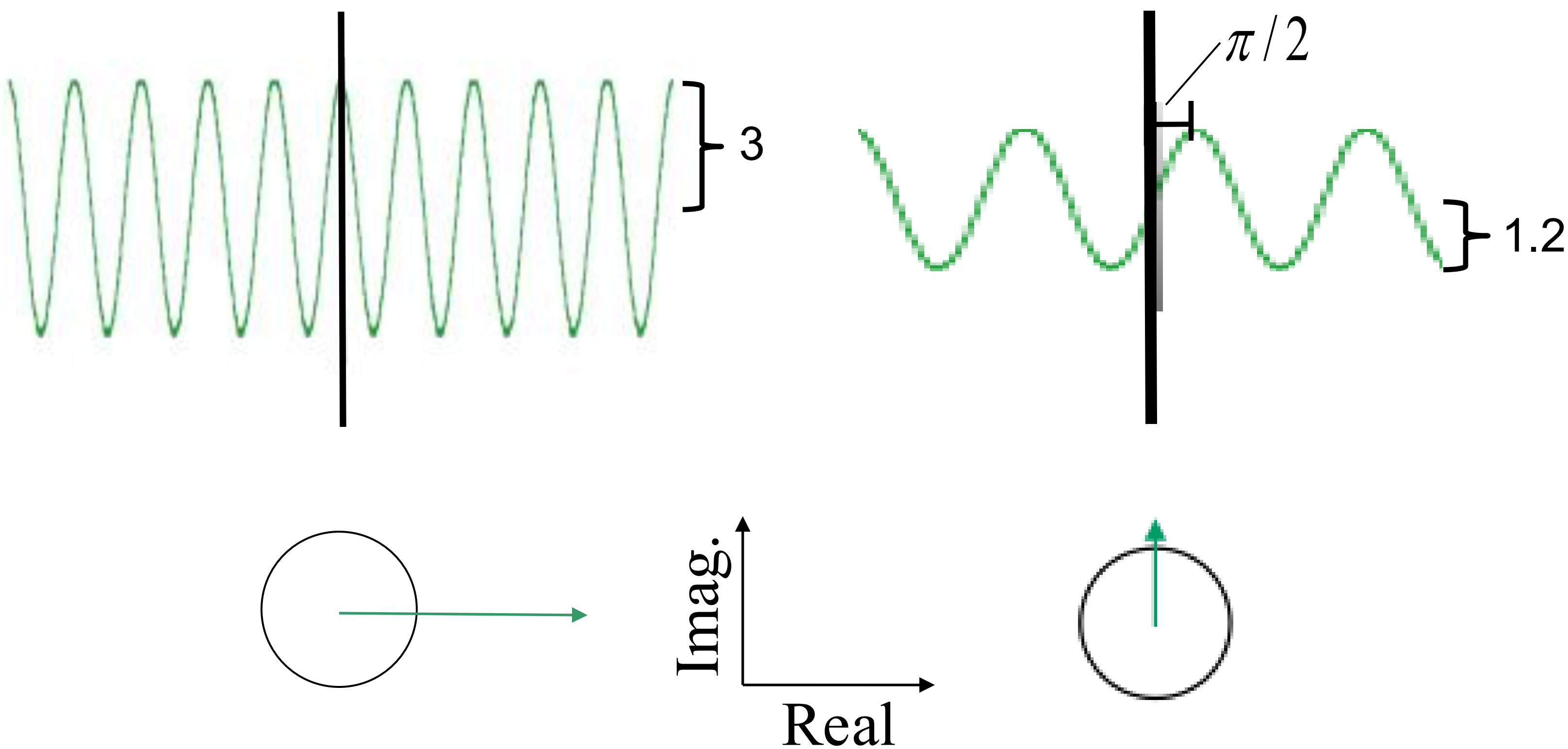




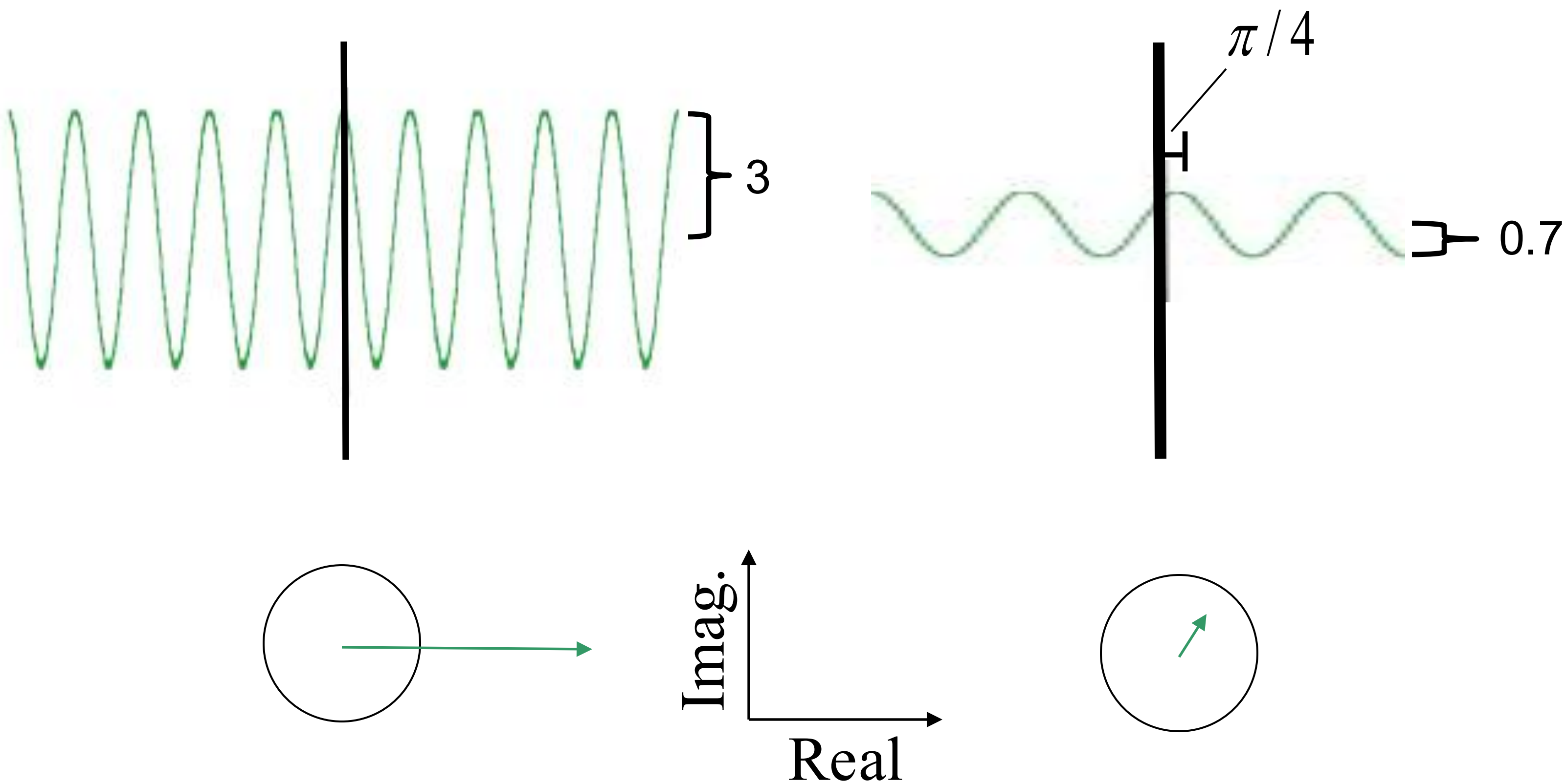
Shorthand phasor notation: $Ae^{i\phi}$

Polar animations courtesy Wikipedia

Phasors: Example



Phasors: Example



Discrete Fourier Transform

Time → Frequency

Forward transform

$$S(f) = \frac{1}{N} \sum_{t=0}^{N-1} x(t) e^{-2\pi i f t / N}$$

Frequency → Time

Inverse transform

$$x(t) = \frac{1}{N} \sum_{f=0}^{N-1} S(f) e^{2\pi i f t / N}$$

N = number of samples

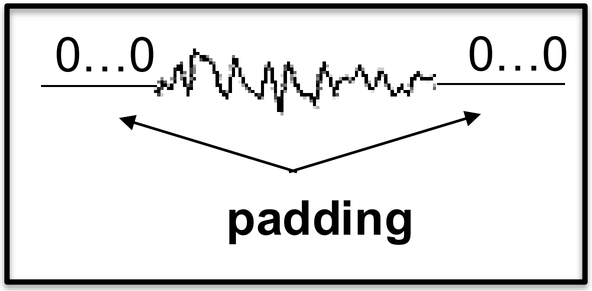
Fast Fourier Transform (FFT)

$$e^{\pm(2\pi i f t + \theta)} = \cos(2\pi f t + \theta) \pm i \sin(2\pi f t + \theta)$$

Power reflects the **covariance** between the original signal and a complex sinusoid at frequency *f*. Or you can think of it as the proportion of the signal variance explained by a sinusoid at frequency *f*

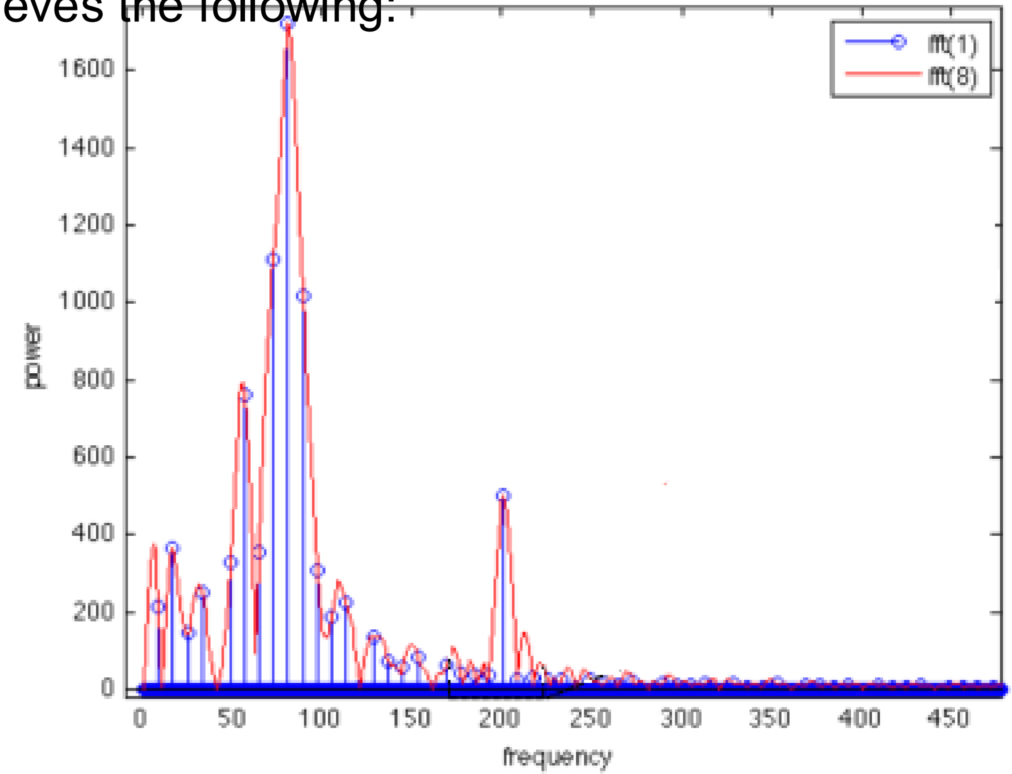
Zero-padding

The DFT/FFT of a sequence of length N produces power estimates at N frequencies evenly distributed between 0 and the sampling rate (Fs), or `floor`(N/2+1) frequencies between 0 and the Nyquist rate, Fn=Fs/2.



Padding the signal with Q zeros achieves the following:

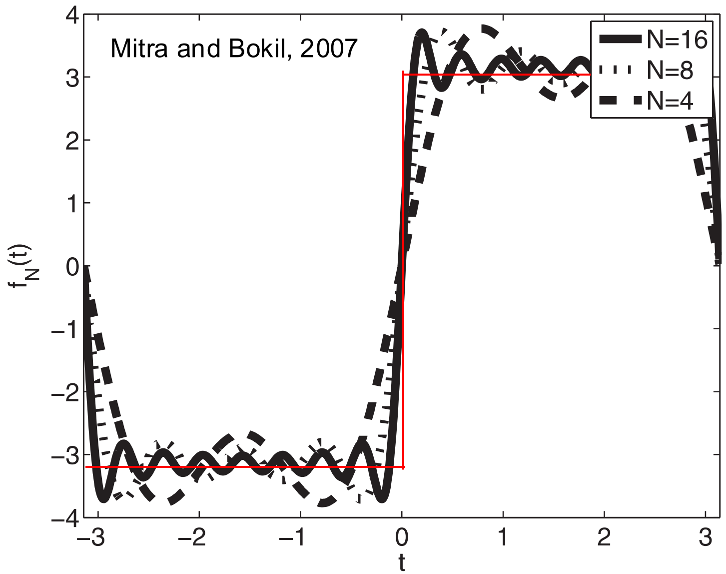
1) Allows enforcement of signal length as a power of two enabling FFT
2) Increases the number of frequency bins between 0 and Fs from N to N+Q (intermediate points are sinc interpolates)

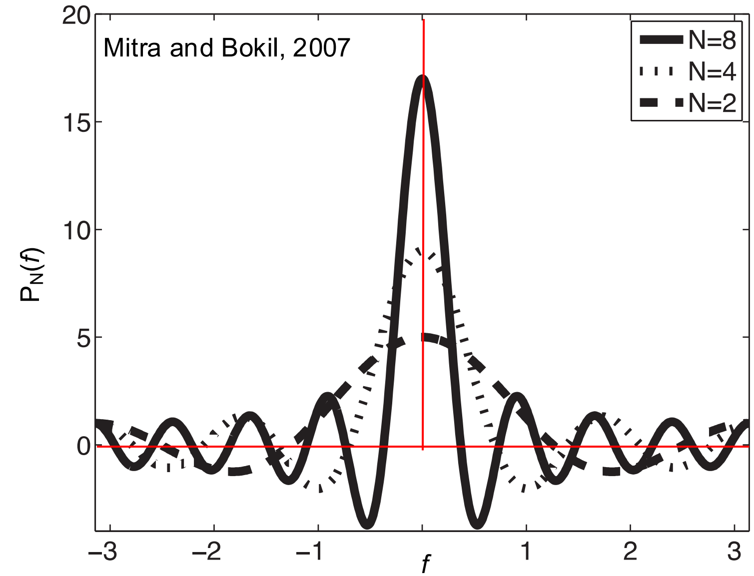


Zero-padding does not increase frequency resolution (number of independent degrees of freedom)

Tapering

Fourier's Theorem lets us exactly represent any length *N*, continuous, stationary signal using a weighted sum of *N* sinusoids. Discontinuous functions must be approximated.





Gibbs Phenomenon
"Rippling" effect due to discontinuities in signal (e.g. edges of the truncated signal)

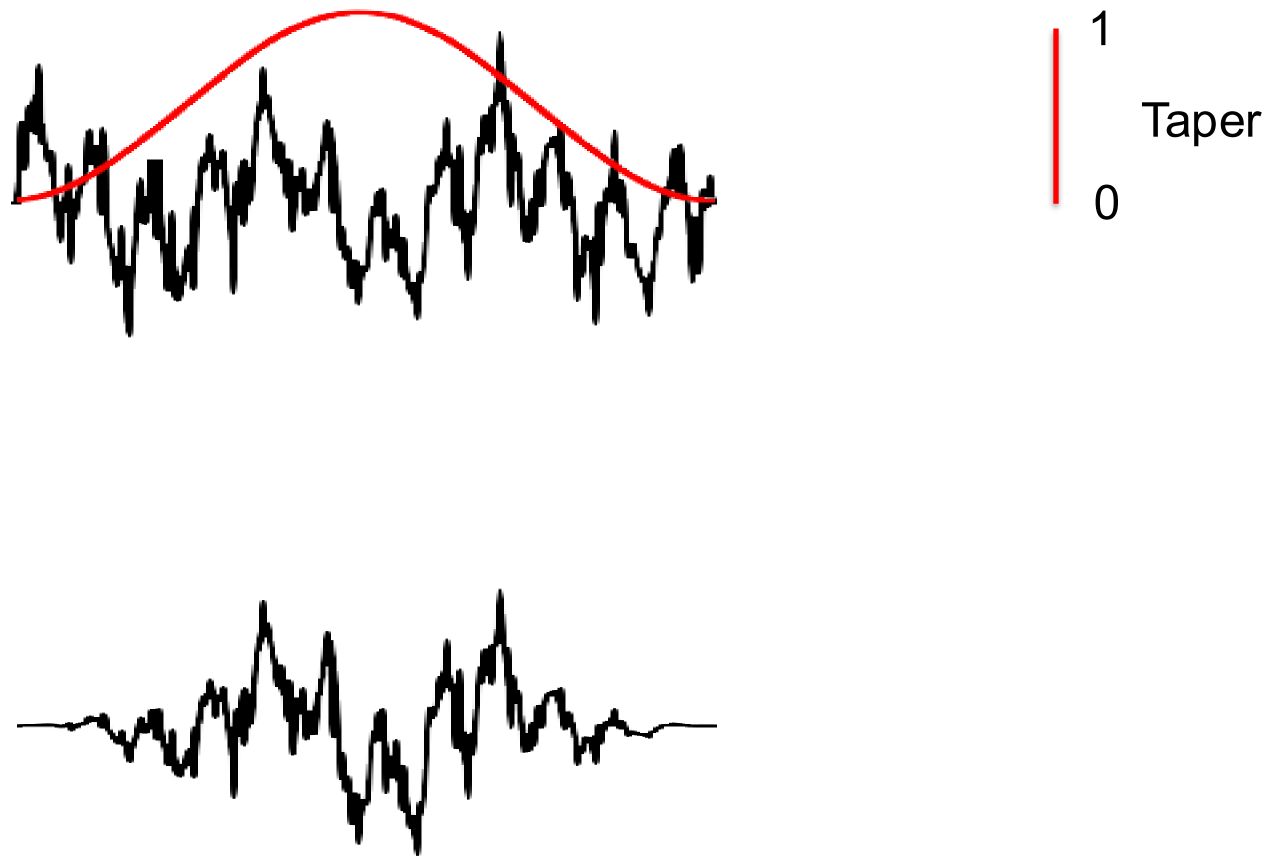
- Infinite number of frequencies required to approximate discontinuities
- This means infinite (or very large) number of samples required (not possible)

What can we do?

Tapering

Smoothly decay signal to zero at endpoints to smooth discontinuity

EEG



X

Tapered EEG

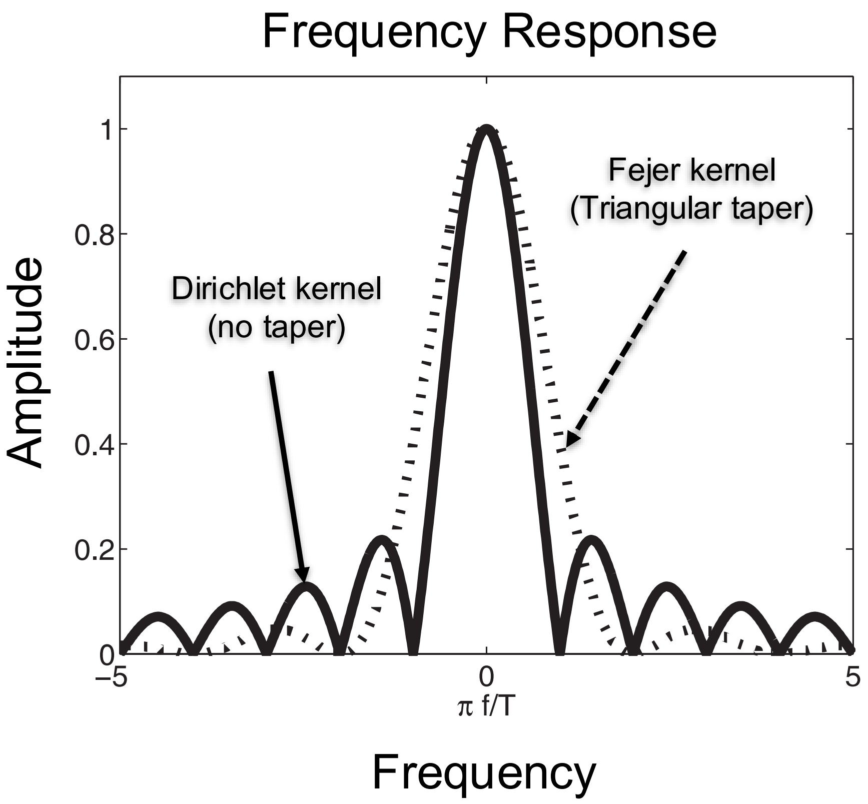
Windowing

- When we pick a short segment of signal, we typically window it with a smooth function (taper).
- Windowing in time = convolving (filtering) the spectrum with the Fourier transform of the window
- No window (=rectangular window) results in the most smearing of the spectrum
- There are many other windows optimized for different purposes: Hamming, Gaussian…

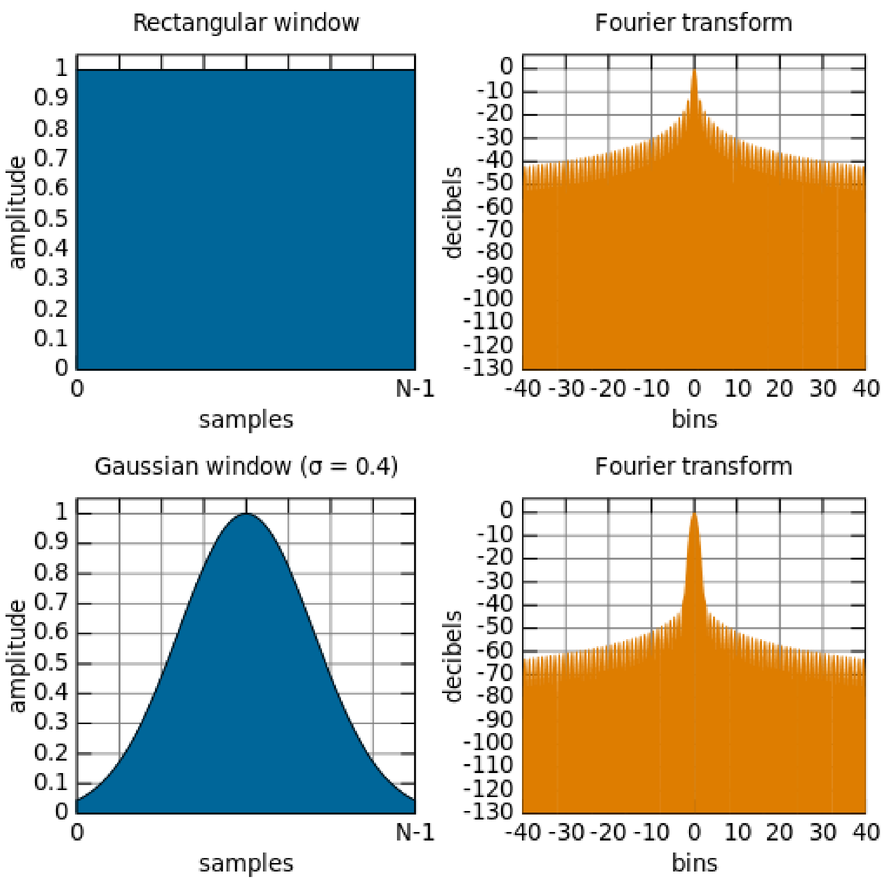
Tapering

Tapering reduces the effect of the Gibbs phenomenon making it easier to identify "true" peaks in the spectrum from spurious ripple peaks (minimized broadband bias or "spectral leakage")

The cost is increased width of central peak (narrowband bias).



Windows and their Fourier Transforms

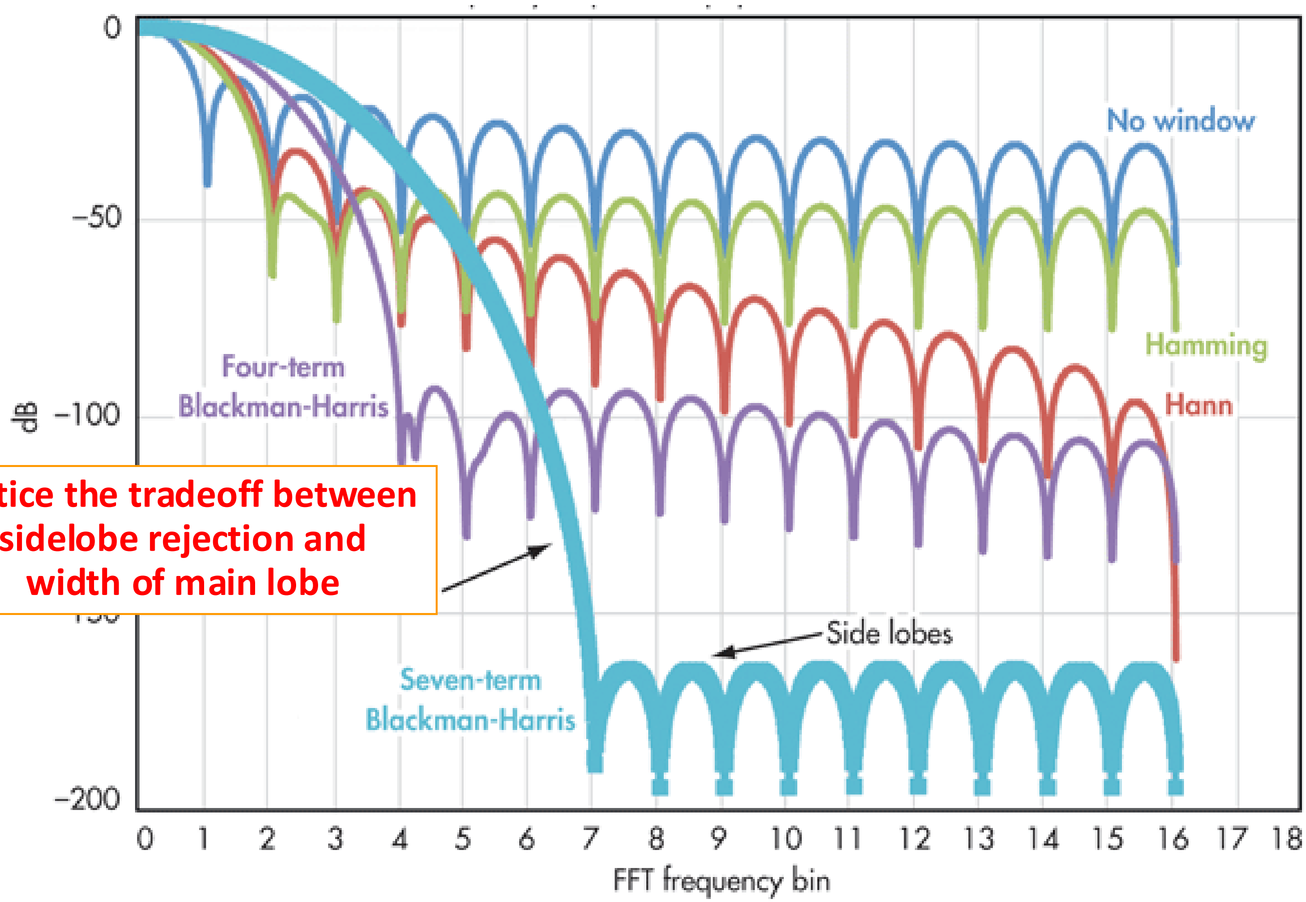


Narrowest main peak, but
Highest side-lobes
Most spectral ‘smearing’

Wider main peak, but
much lower side-lobes

Courtesy of John Iversen

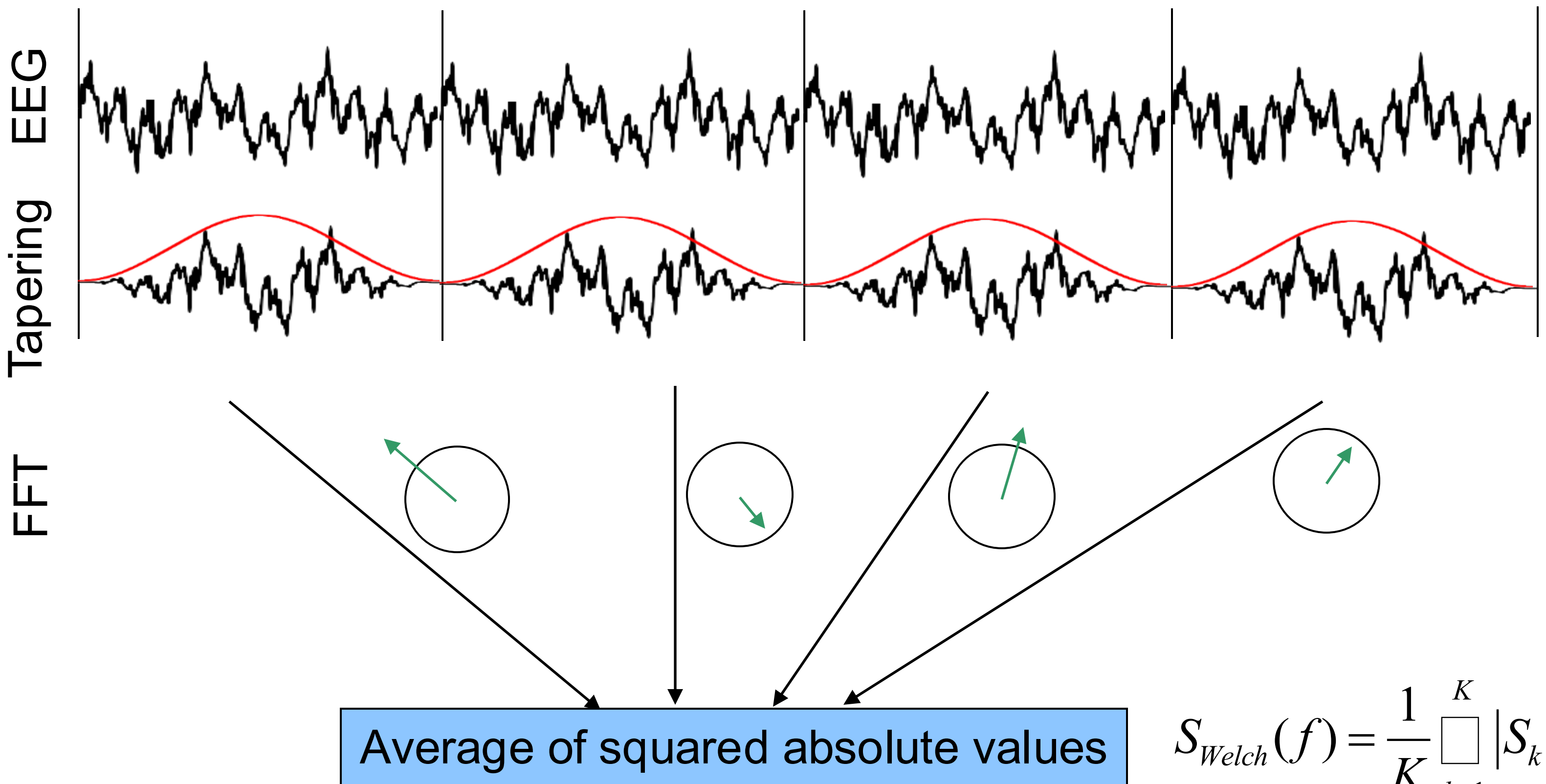
Frequency Responses of Windows



Notice the tradeoff between sidelobe rejection and width of main lobe

Courtesy of John Iversen

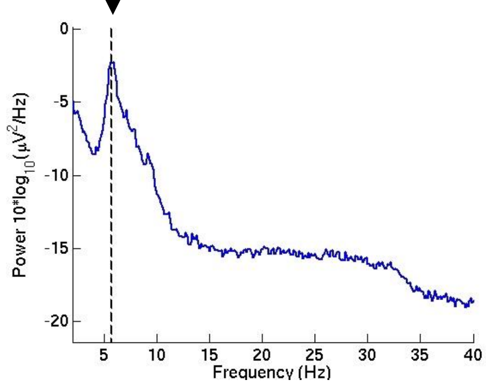
Spectral Estimation via Welch's Method



$$S_{Welch}(f) = \frac{1}{K}\sum_{k=1}^{K}\left|S_k(f)\right|$$

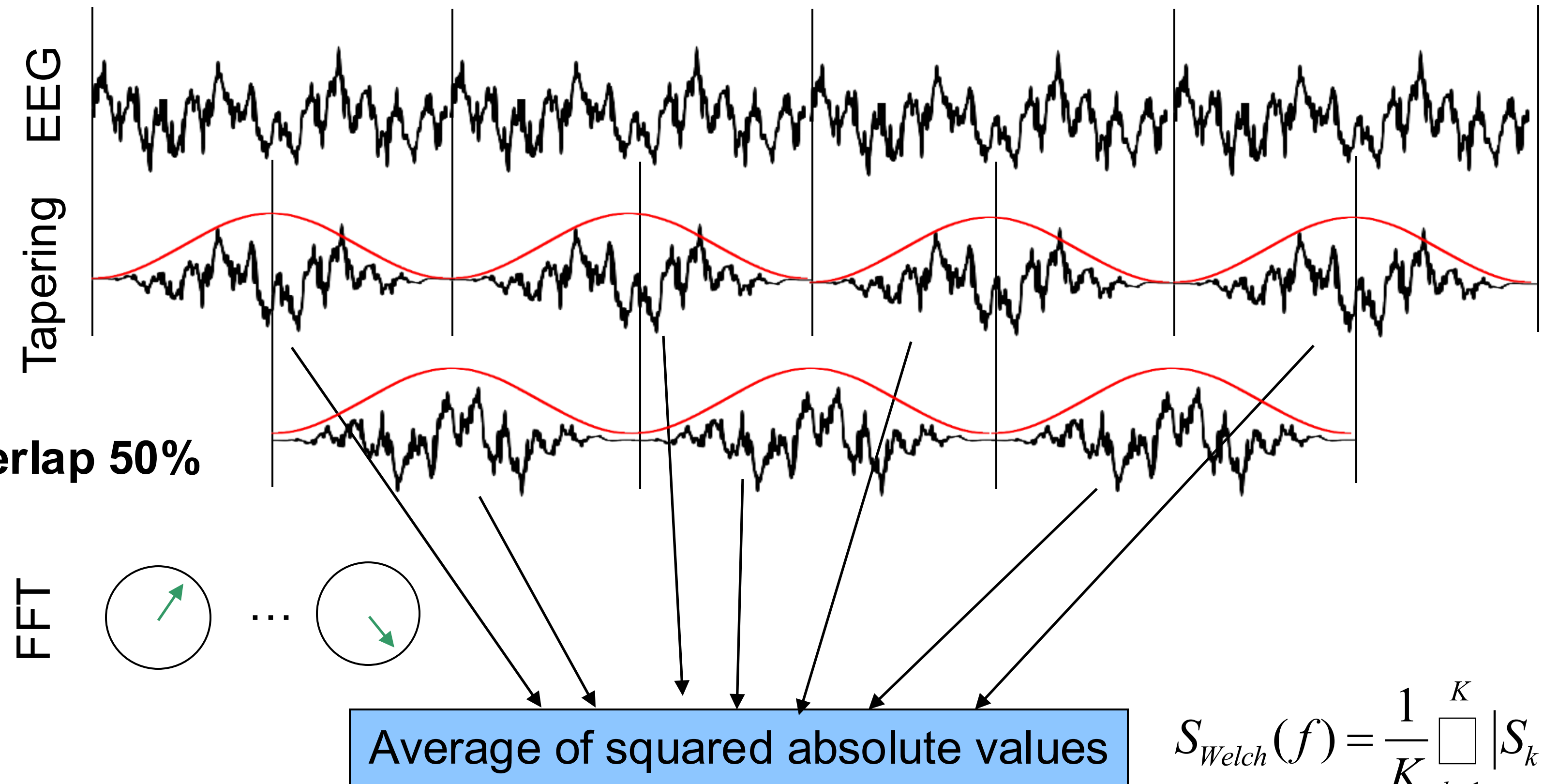
Given K windows:

- Variance is reduced by a factor of K
- Frequency resolution also reduced by a factor of K

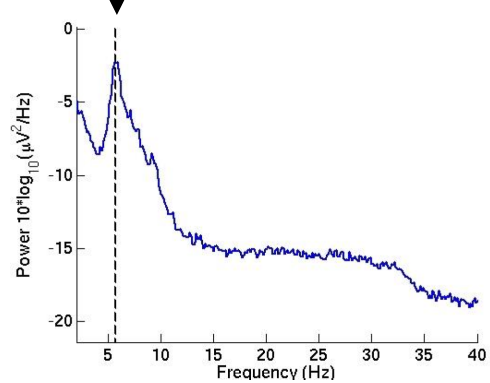


- Tapering also results in data loss → decreased frequency resolution (increased narrowband bias)
- Can we mitigate data loss?

Spectral Estimation via Welch's Method



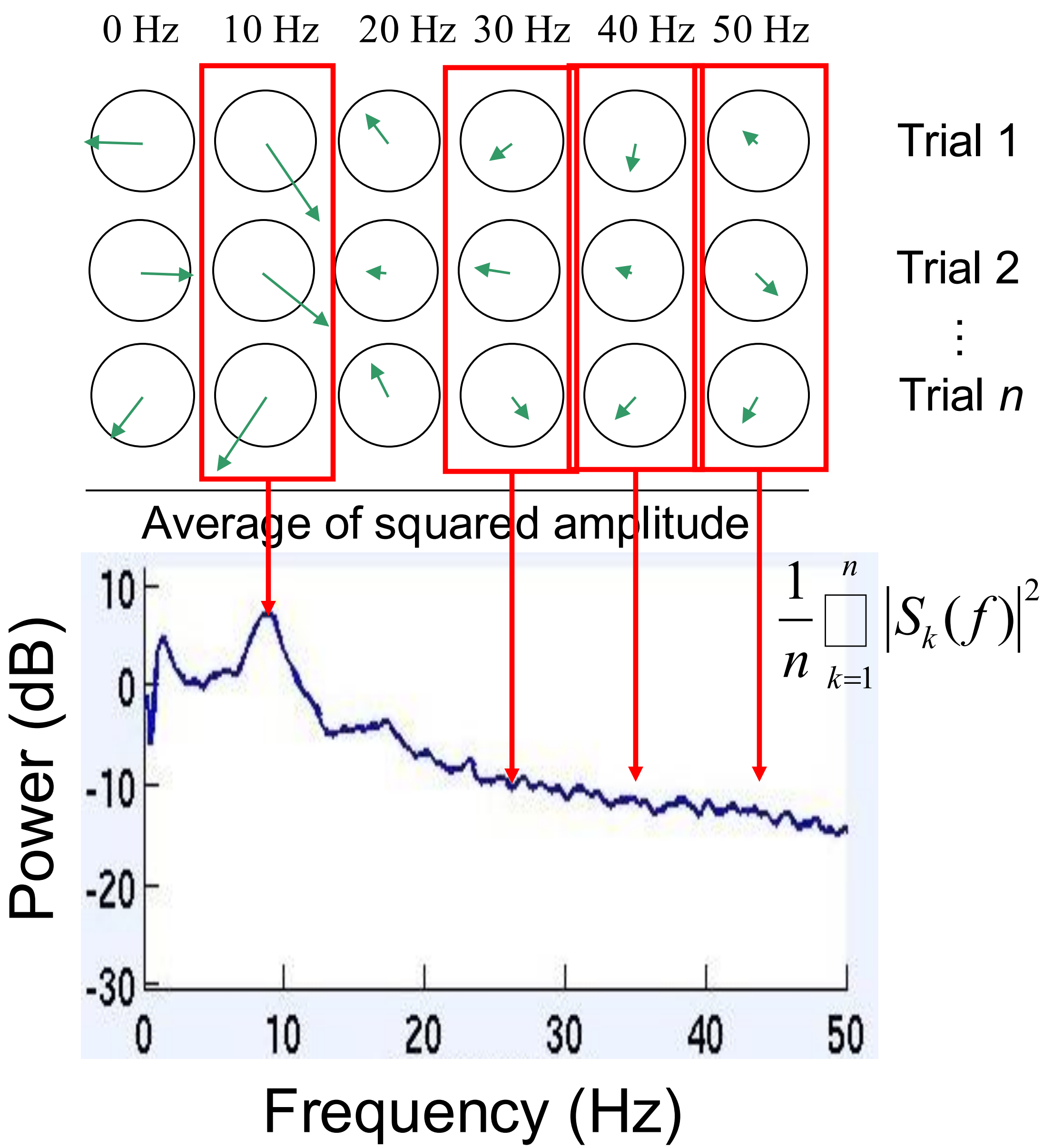
$$S_{Welch}(f) = \frac{1}{K}\sum_{k=1}^{K}\left|S_k(f)\right|$$



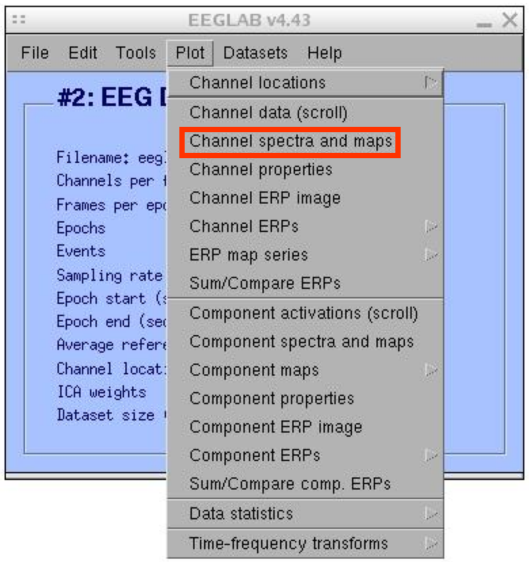
Overlap-add method results in reduced data loss due to tapering, while preserving spectral variance reduction ☺

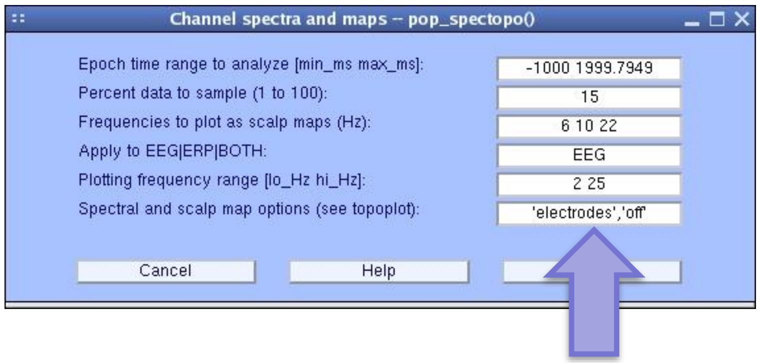
Trial Averaging

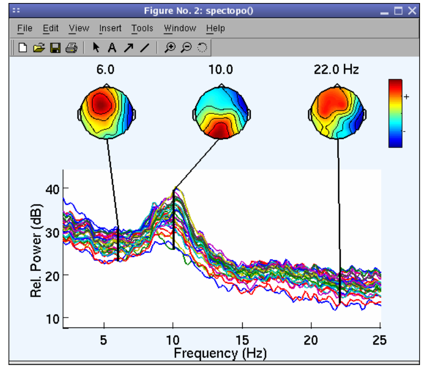
Averaging spectra over *n* independent trials leads to further reduction of variance by a factor of *n*



Hands on: Plot periodogram (spectrum) using the Welch's method







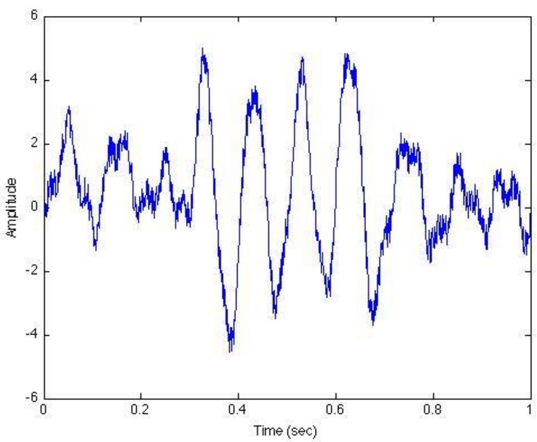
'winsize' , 256, (in samples; change FFT window length)
'nfft' , 256, (in samples; change FFT padding)
'overlap' , 128, (in samples; change window overlap)

1. Use menu item **File > Load existing dataset** to import the continuous data ds000117_pruned/derivatives/meg_derivatives/sub-01/ses-meg/meg/wh_S01_run_01_preprocessing_data_session_1_out.set

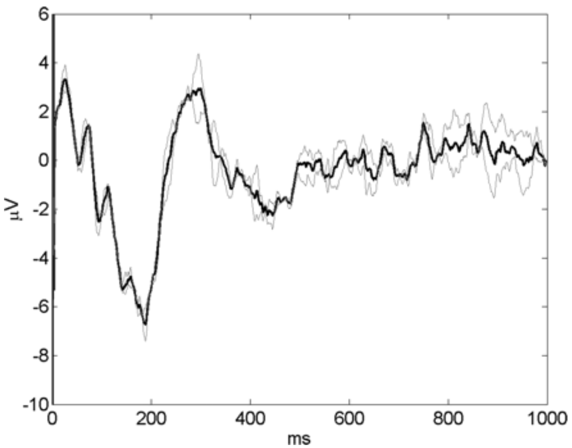
2. Use menu item **Plot > Plot spectra and maps**. Plot spectral decomposition with different overlap ('overlap'), window length ('winsize') and FFT length ('nfft'). The MATLAB command line shows you the values currently used.

Non-Stationary Signals

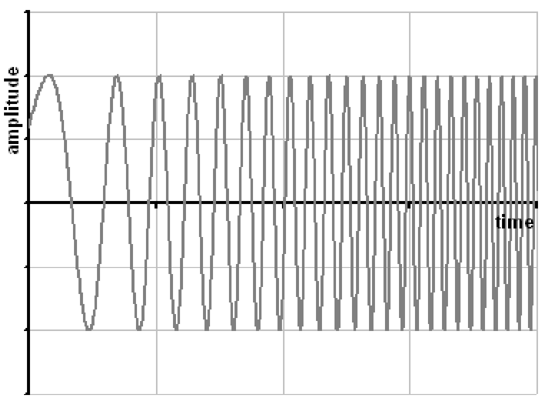
Non-stationary signals include bursts, chirps, evoked potentials, …



Burst

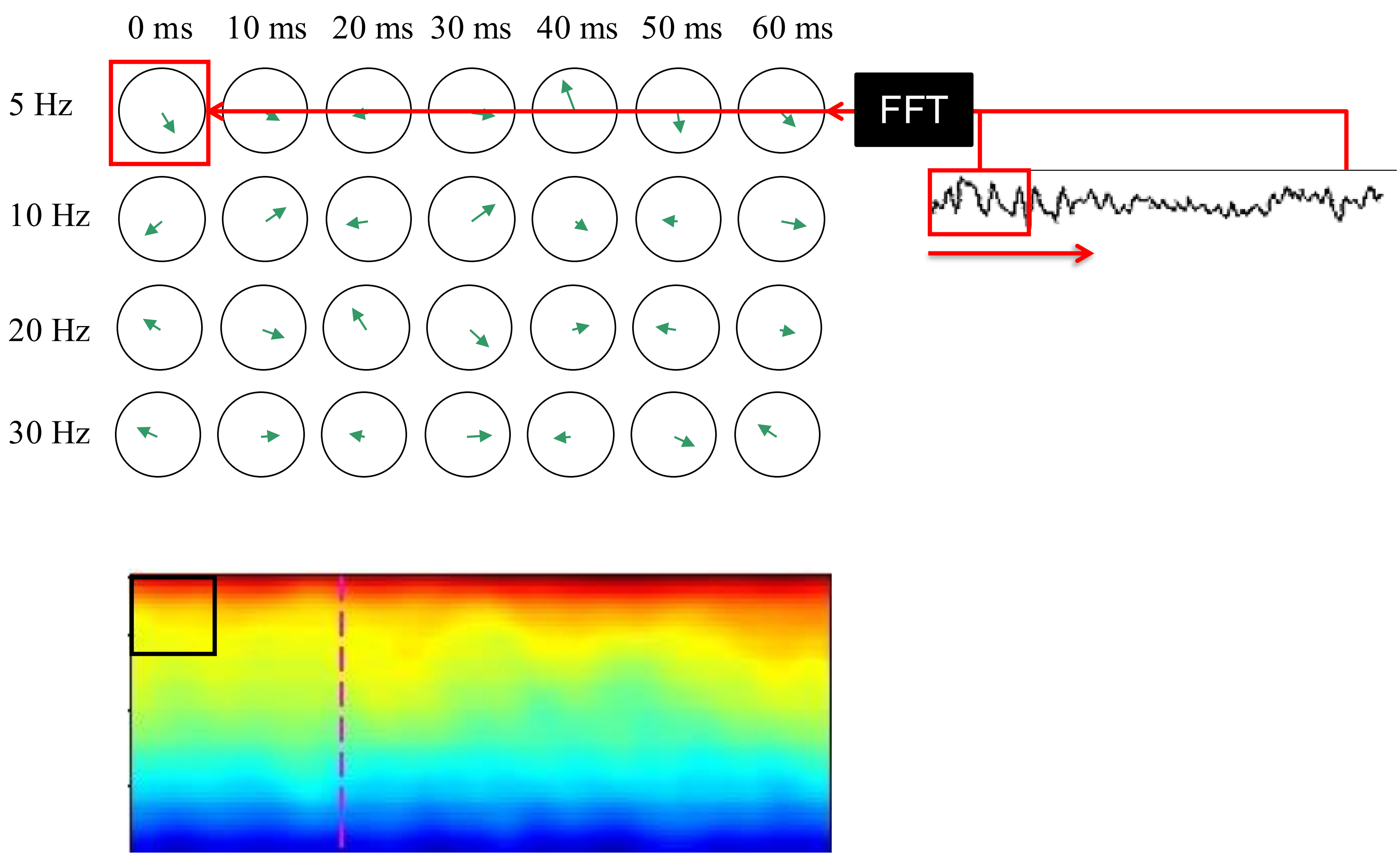


Evoked potential

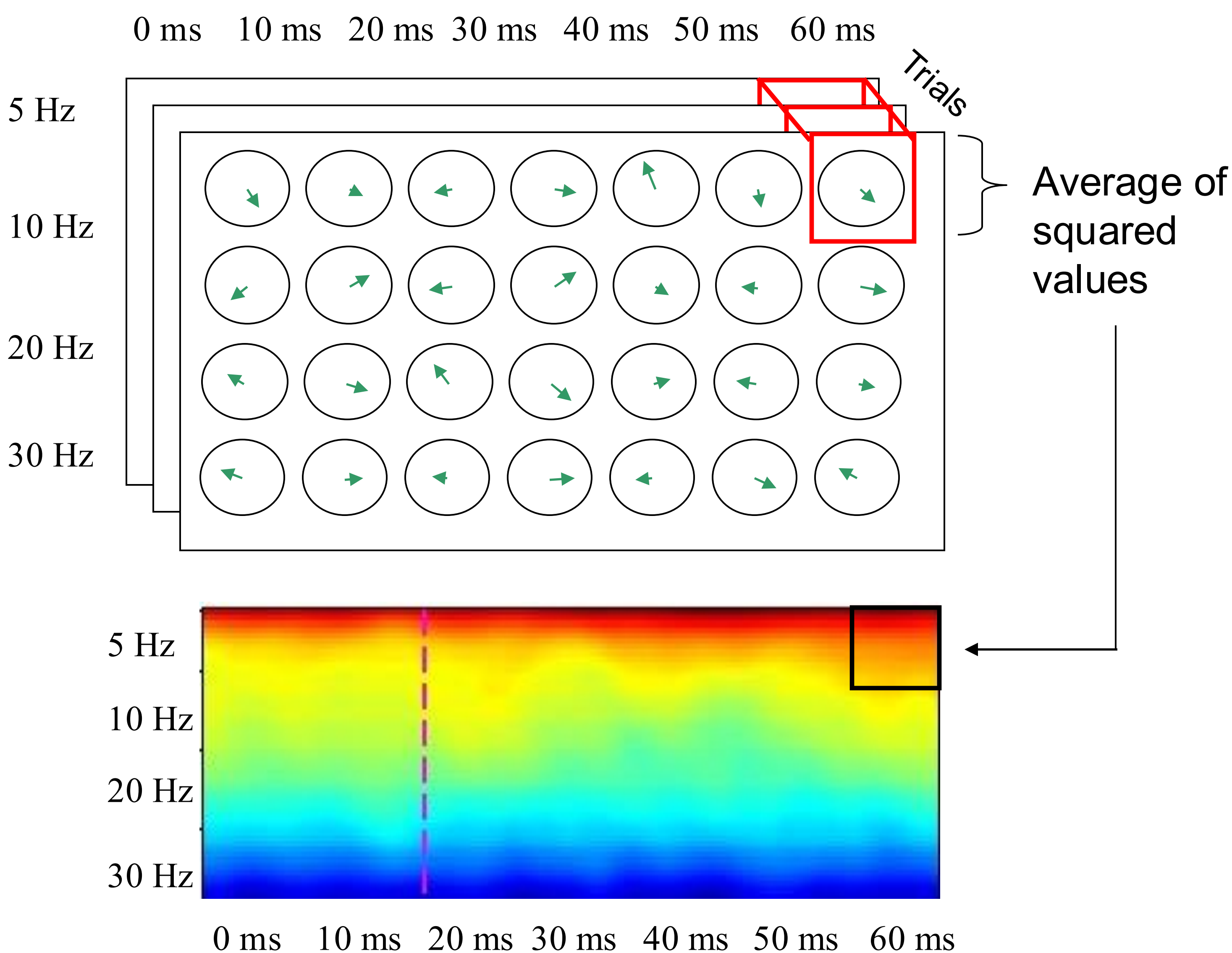


Chirp

Spectrogram or ERSP



Spectrogram or ERSP



Power spectrum and event-related spectral (perturbation)

$$ERS(f,t) = \frac{1}{n}\sum_{k=1}^{n}\left|S_k(f,t)\right|^2$$

Ensemble average

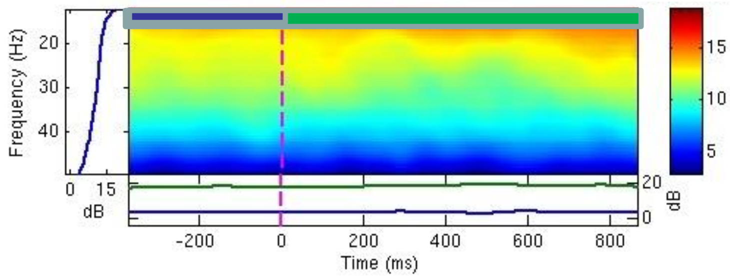
Complex number

Scaled to dB 10Log_{10}

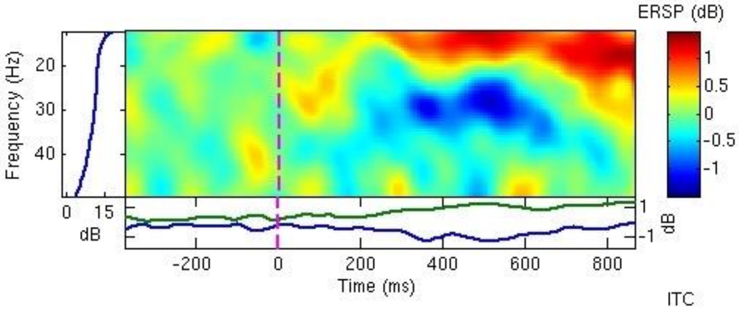
Here, there are *n* trials
Each trial is time-locked to the same *event* (hence "event-related" spectrum)
The ERS is the average power across event-locked trials

Absolute versus relative power

Absolute ERSP



Baselined ERSP (dB or %)



To compute the ERSP, we just subtract the pre-stimulus ERS from the whole trial

Grandchamp R, Delorme A. Single-trial normalization for event-related spectral decomposition reduces sensitivity to noisy trials. *Front Psychol*. 2011;2:236. Published 2011 Sep 30. doi:10.3389/fpsyg.2011.00236

The Uncertainty Principle

A signal <u>cannot</u> be localized arbitrarily well <u>both</u> in time/position <u>and</u> in frequency/momentum.

There exists a lower bound to the *<u>Heisenberg product</u>*:

$$\Delta t \, \Delta f \geq 1/(4\pi)$$

or $$\Delta f \geq 1/(4\pi\Delta t)$$

Werner Karl Heisenberg
(1901 – 1976)

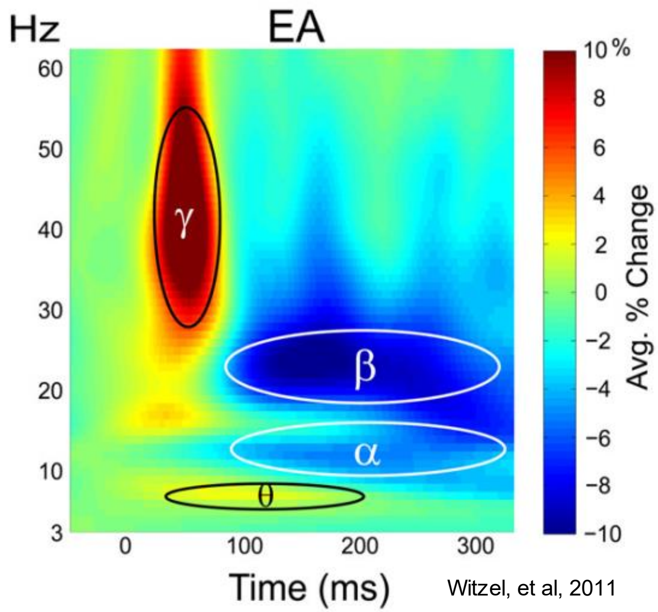
e.g. here are two possible (Δf, Δt) pairs:

Δf = 1Hz, Δt = 80 msec or
Δf = 2Hz, Δt = 40 msec

Note: Δf means “difference between successive frequencies” or the inverse of the frequency resolution. Ditto for Δt.

Time-Frequency Tradeoff

Natural biophysical processes may exhibit sustained changes in narrowband low-frequency oscillations along with rapidly-changing (e.g. "burst") high-frequency oscillations.



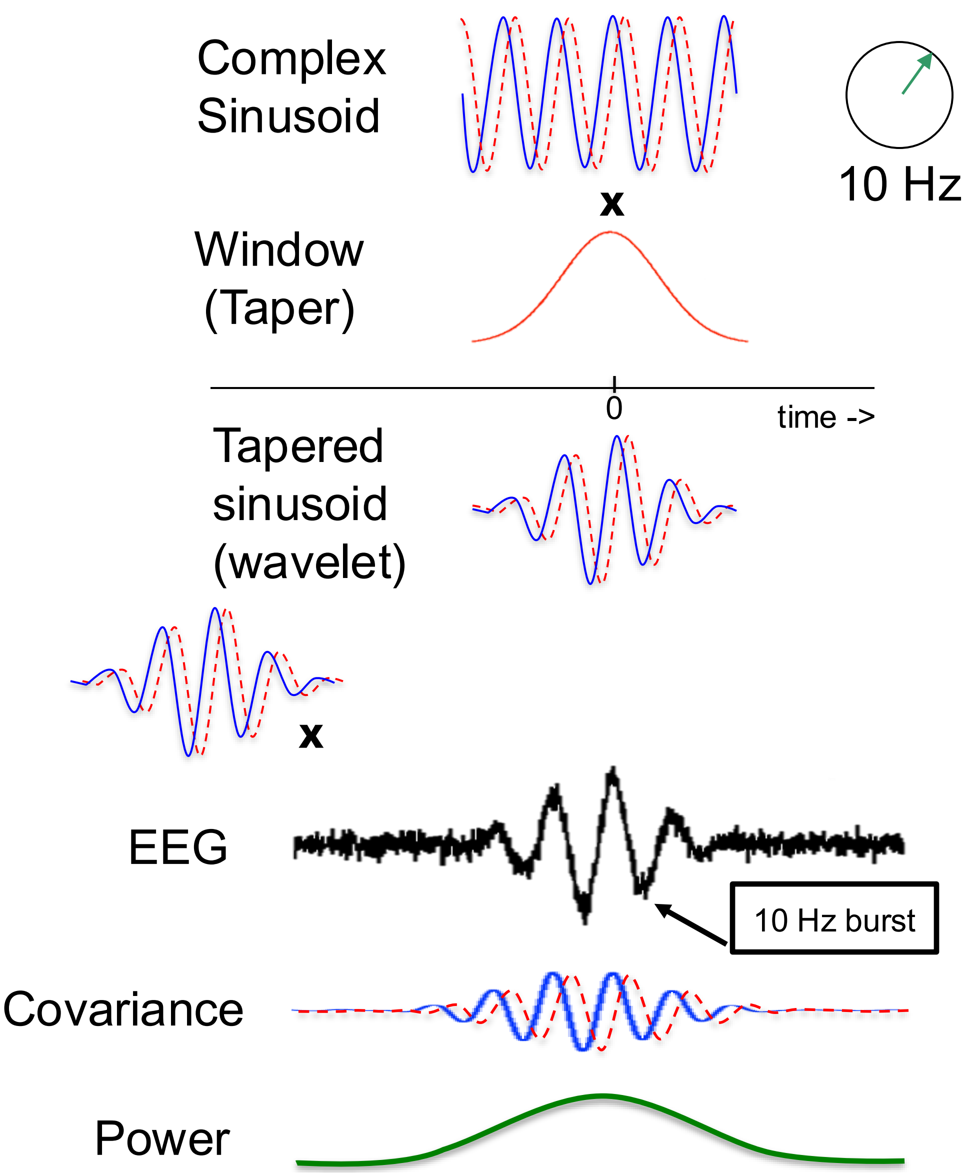
Witzel, et al, 2011

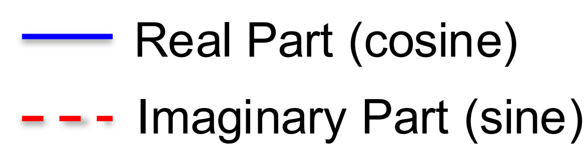
The Short-Time Fourier Transform has a constant temporal resolution for all frequencies.

Can we adapt the time-frequency resolution tradeoff for individual frequencies to improve spectral estimation?

Yes, we can!

Wavelet Analysis

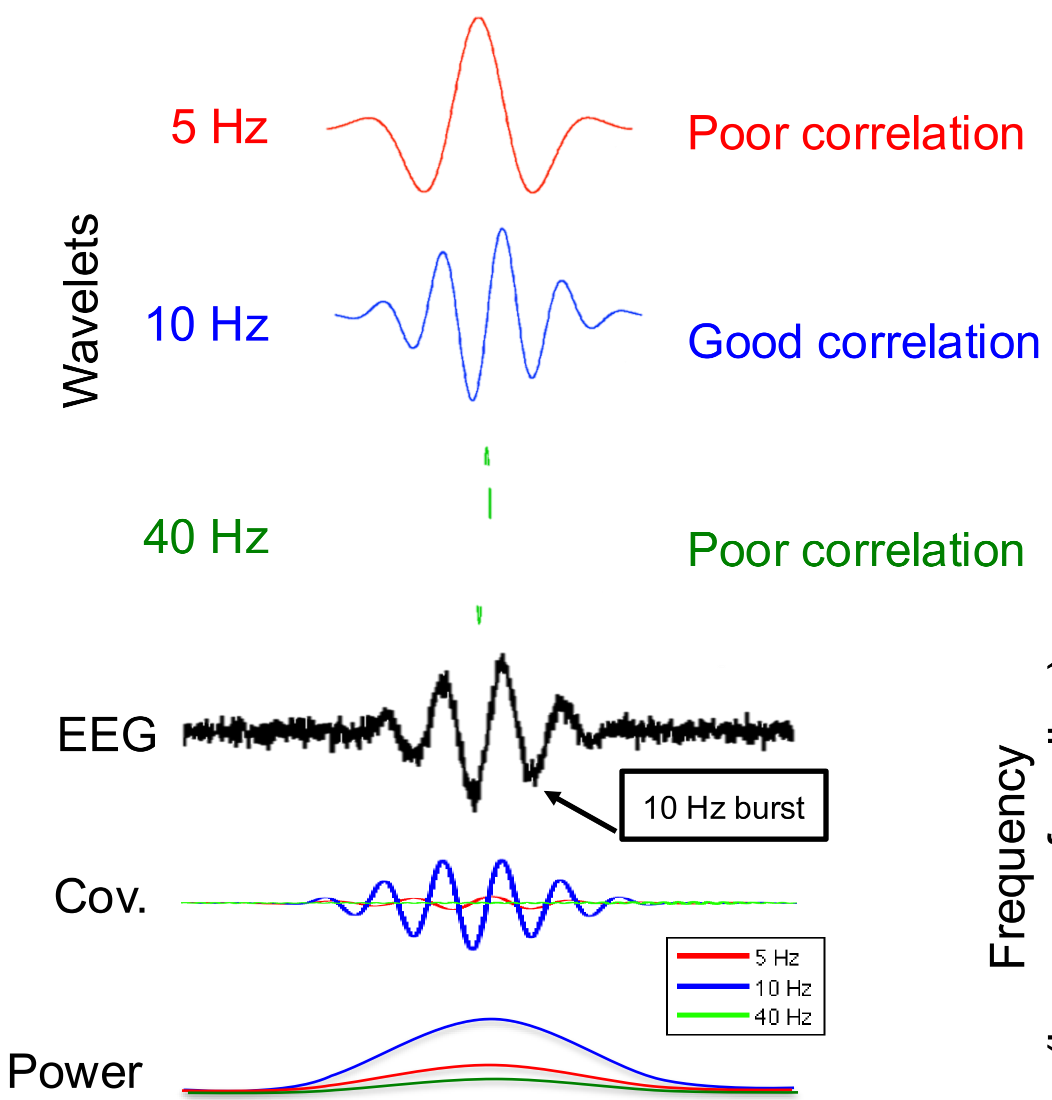




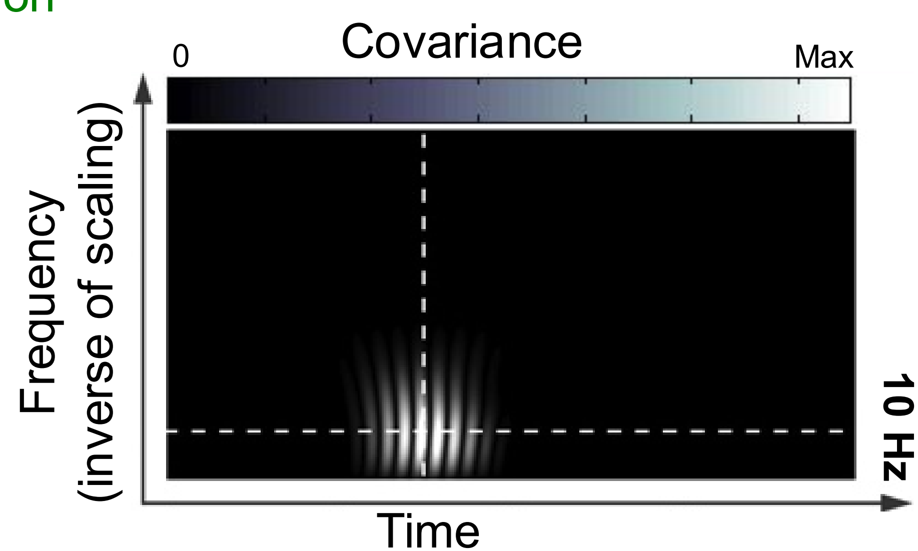
We estimate the time-varying power at 10 Hz by convolving EEG signal with a tapered 10 Hz complex sinusoid (Morlet wavelet)

$|S(f,t)| = |Re + Im|$

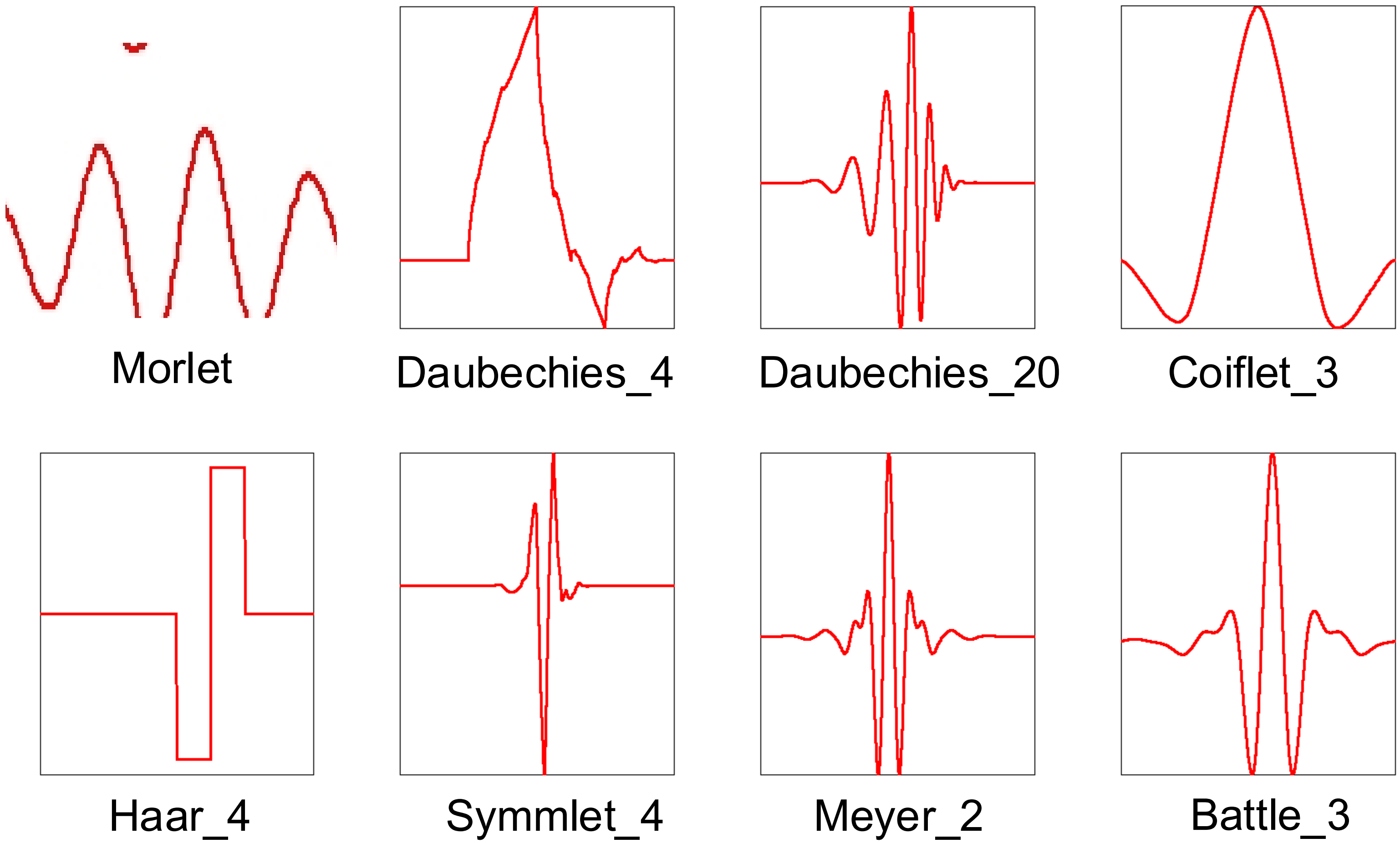
Wavelet Time-Frequency Image



By convolving stretched and scaled versions of the “mother” wavelet with the EEG signal, we determine the time-frequency distribution of power

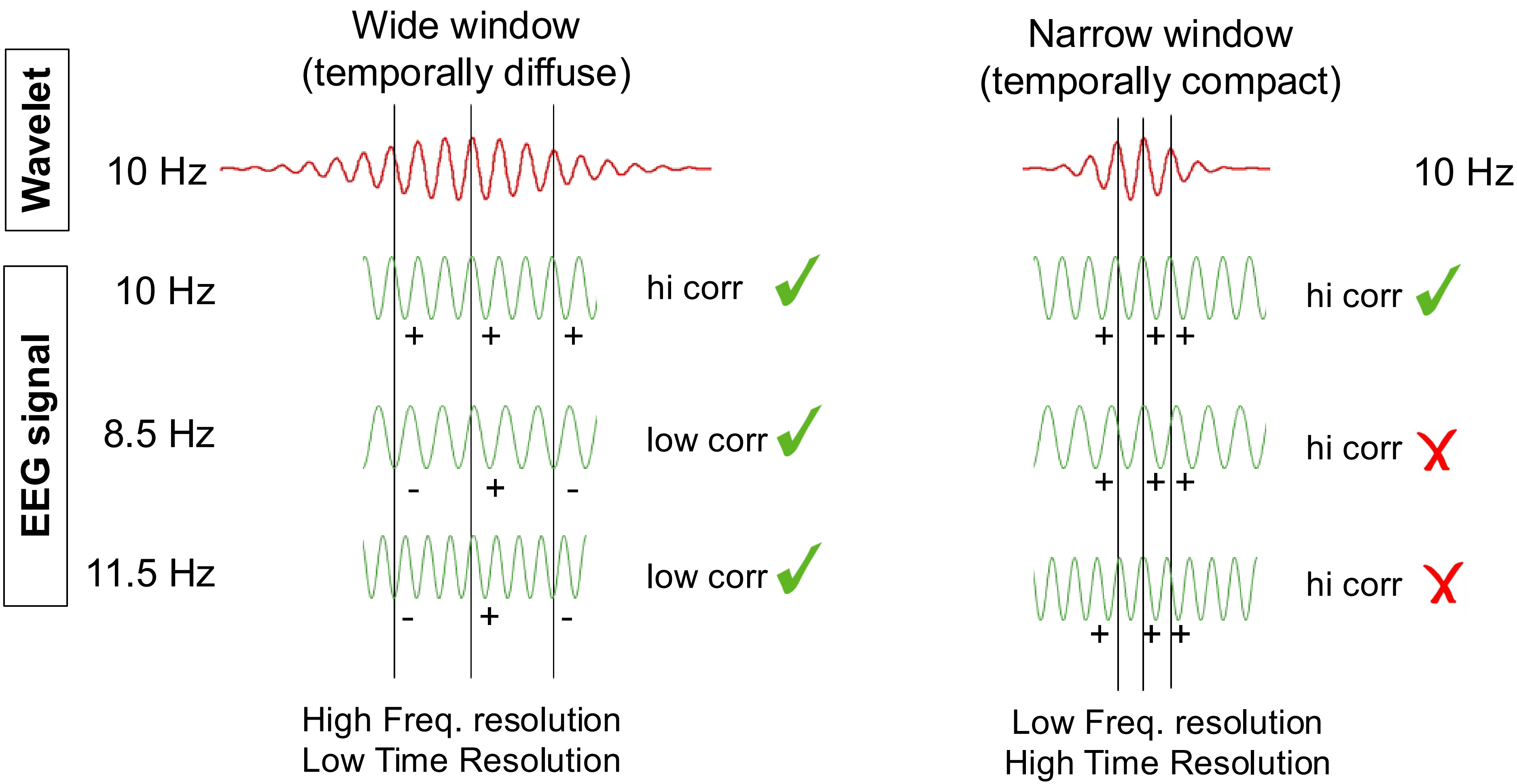


Some Wavelet Families



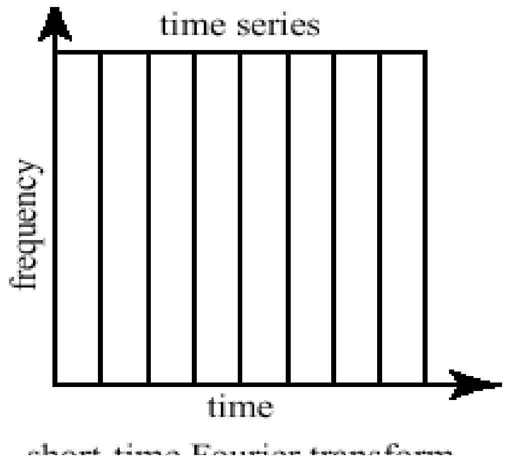
Trading Frequency for Time

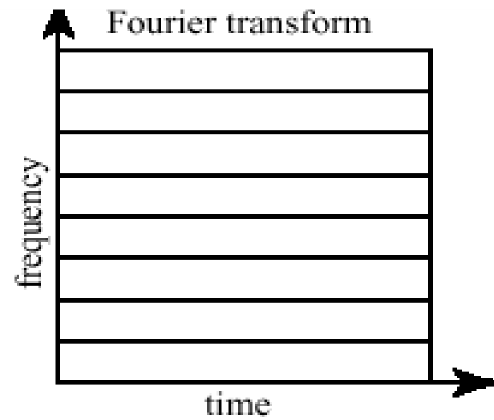
(and vice versa)



FFT versus Wavelets

Time series:
fixed time res
no freq res

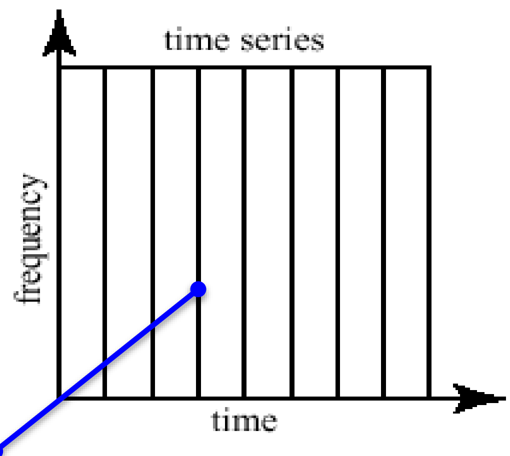


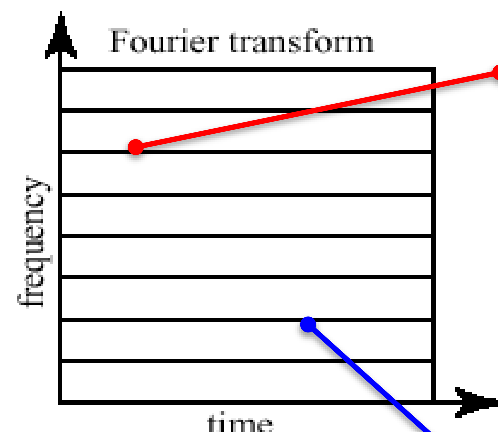


Fourier transform:
fixed freq res
no time res

ST Fourier:
fixed time res
fixed freq res

equal time
and freq
resolution





high time res
low freq res

Wavelet:
variable time res
variable freq res

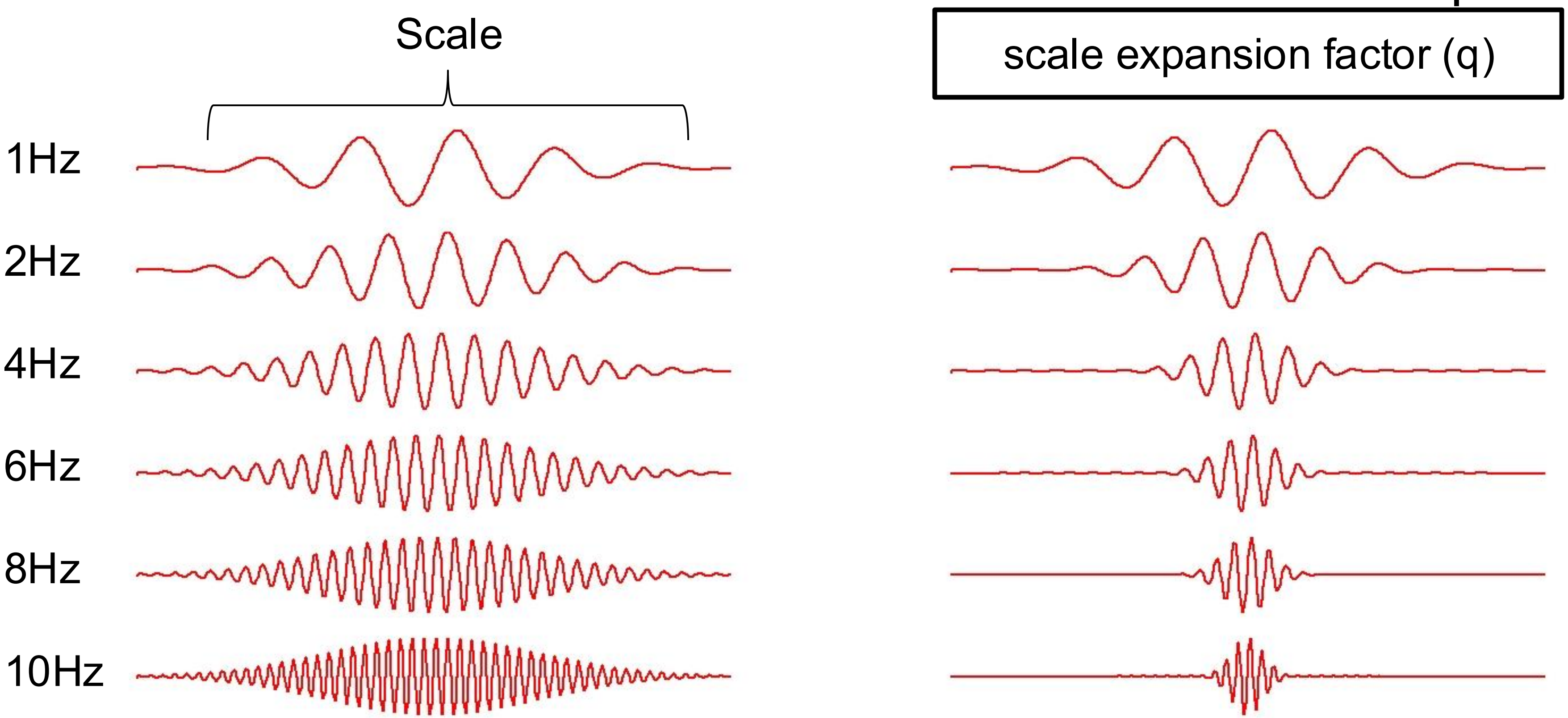
equal time
and freq
resolution

Adapted from http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10

Wavelet scale expansion factor

Wavelet (0)

Wavelet (1)

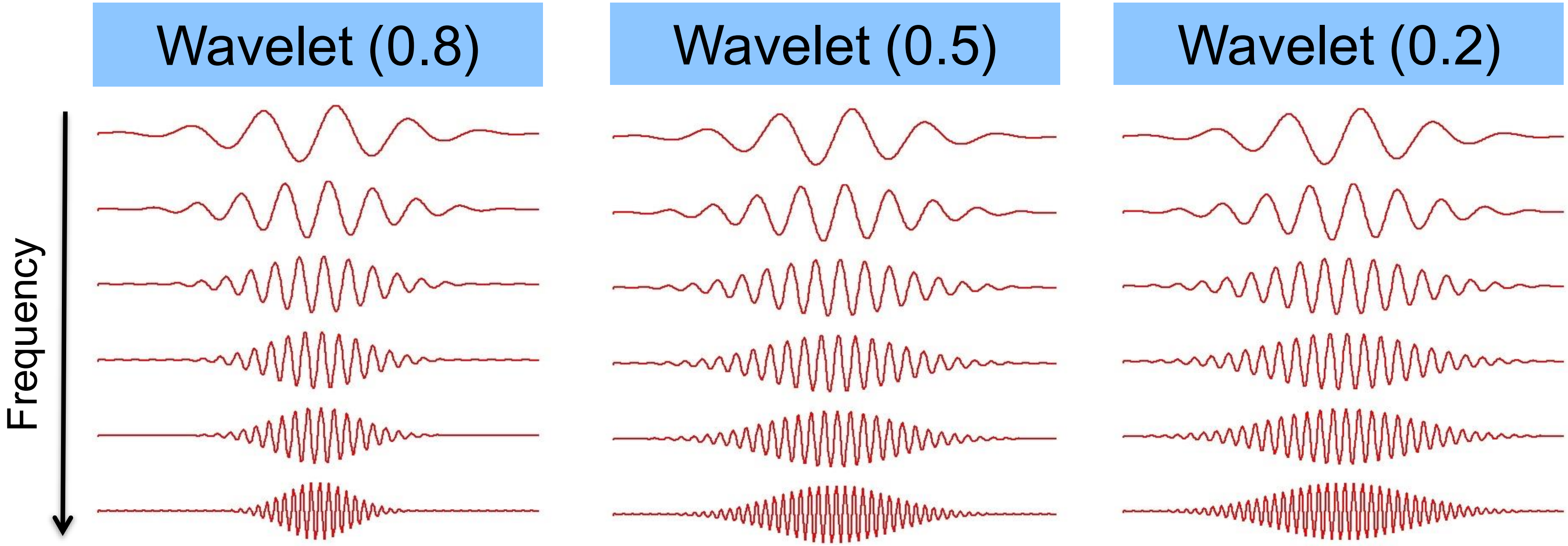


constant window size (time resolution) for increasing frequency → increasing # cycles with frequency.

window size decreases by a factor of 2 for each octave (power of 2) → constant # of cycles at each frequency

Wavelet scale expansion factor

Larger expansion factor produces larger scale decrements (increased time resolution, lower frequency resolution) for increasing frequency

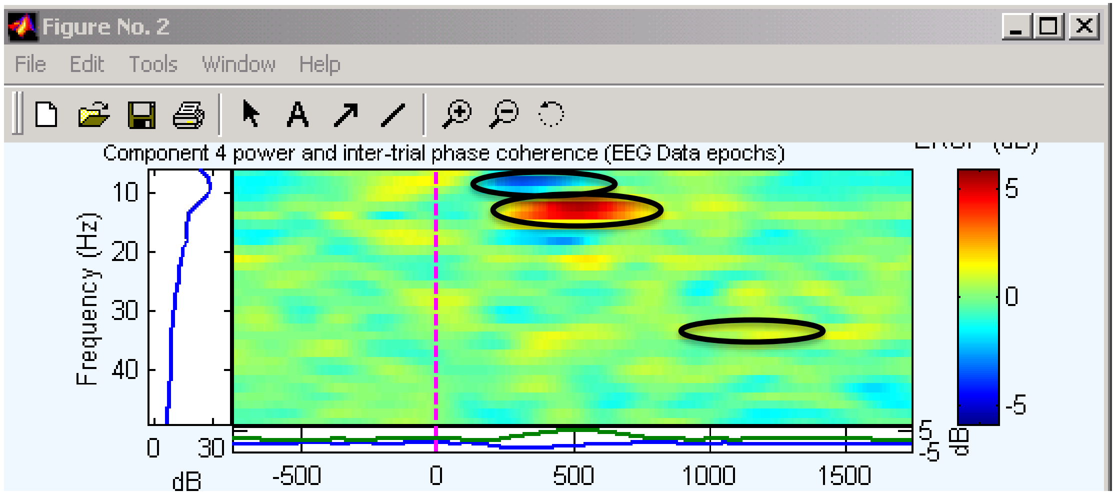


Number of cycles at highest frequency for an expansion factor of *q:*

$$C_{\mathrm{fmax}} = \frac{f_{max}}{f_{min}} C_{\mathrm{fmin}} (1-q)$$

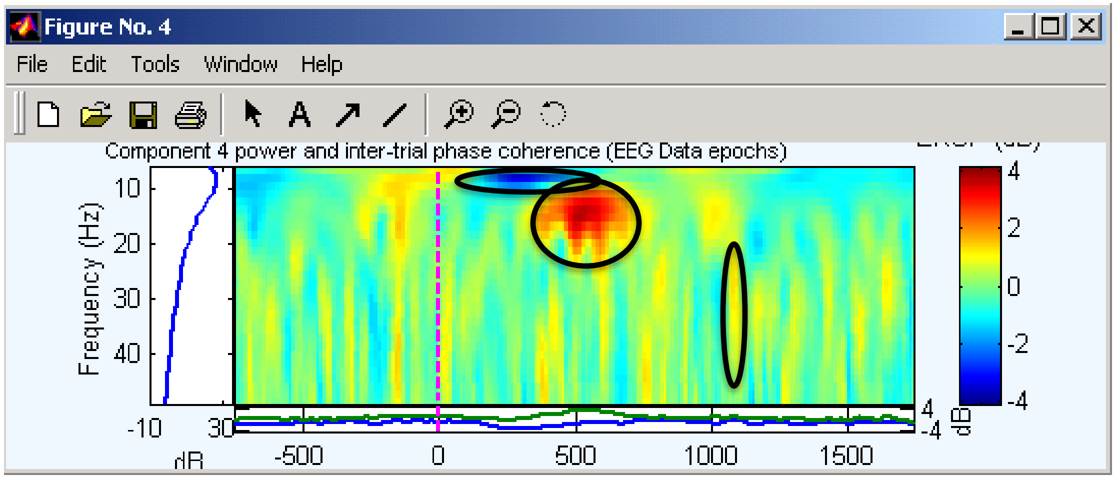
Wavelet scale expansion factor

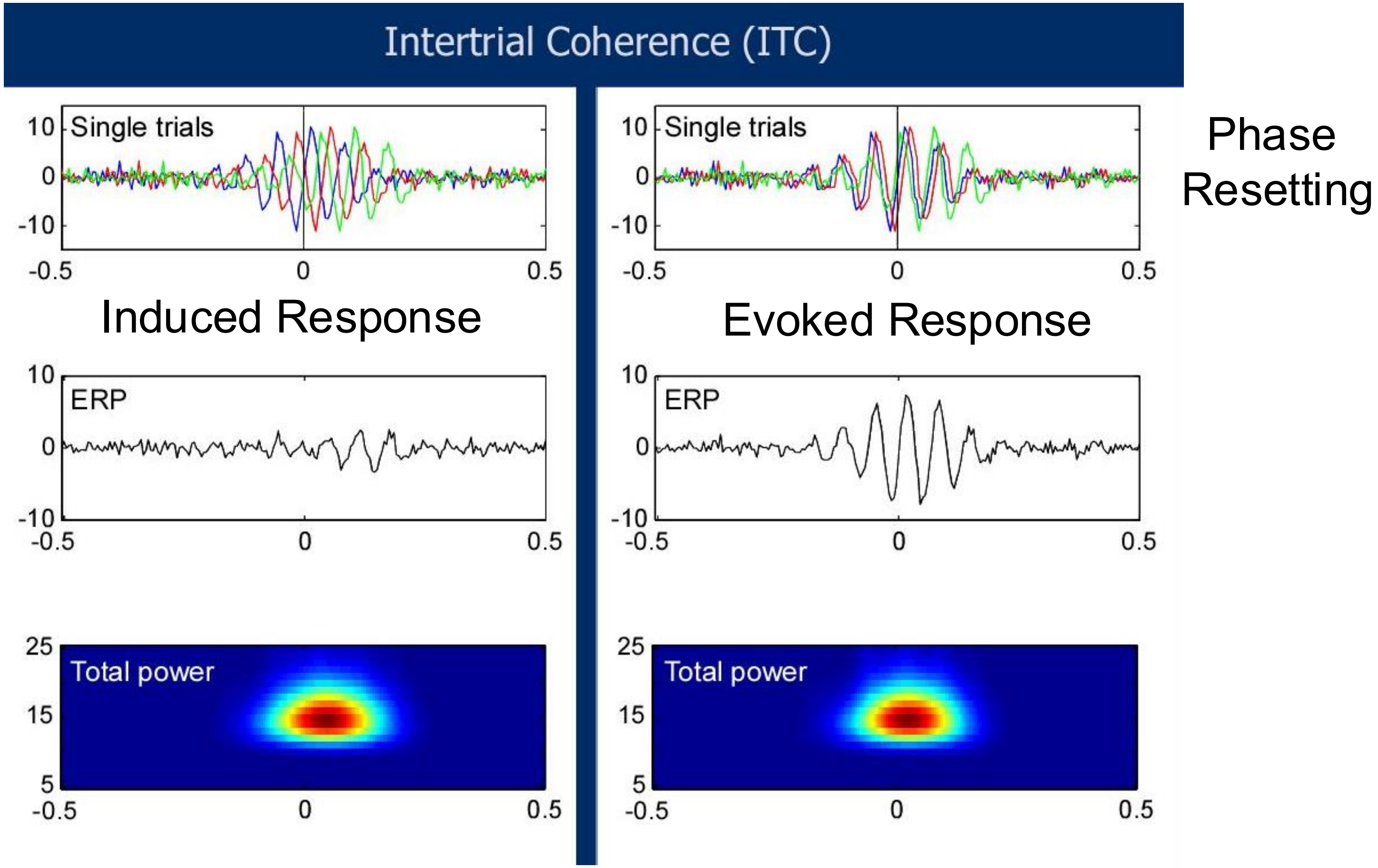
Wavelet(0)
(STFT)



Wavelet (1)

scale expansion factor (q)

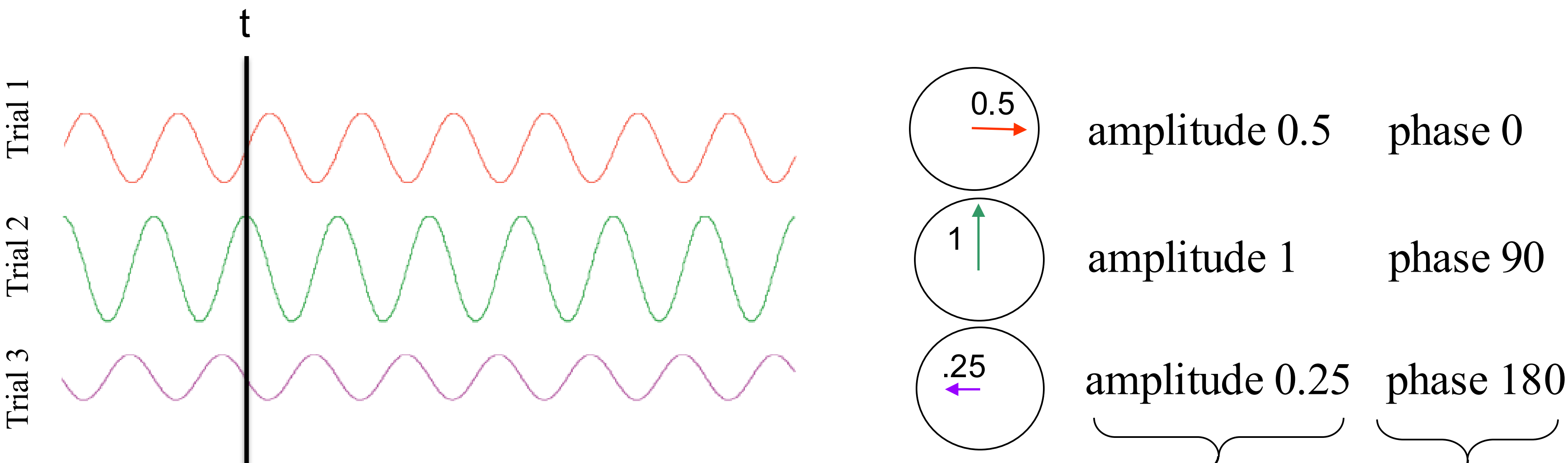




Slide courtesy of Stefan Debener

Inter-Trial Coherence (ITC)

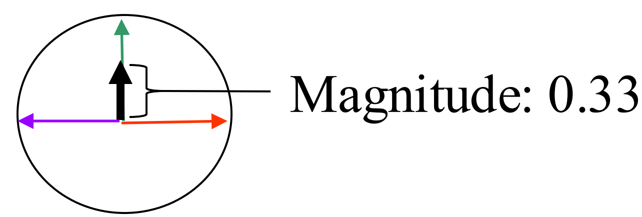
Tallon-Baudry, et al, 1996



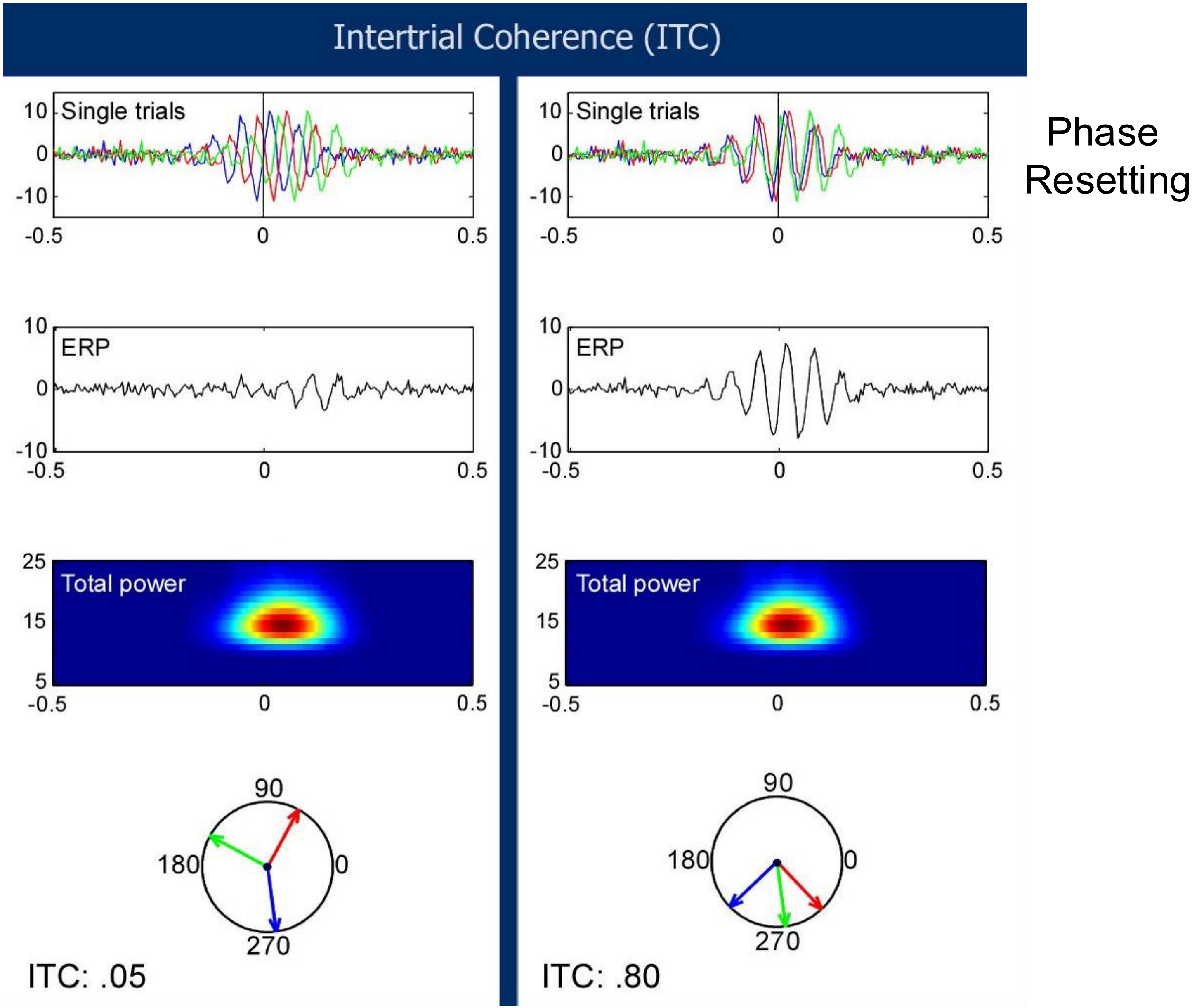
POWER = mean(amplitudes^2)

0.44 or –8.3 dB

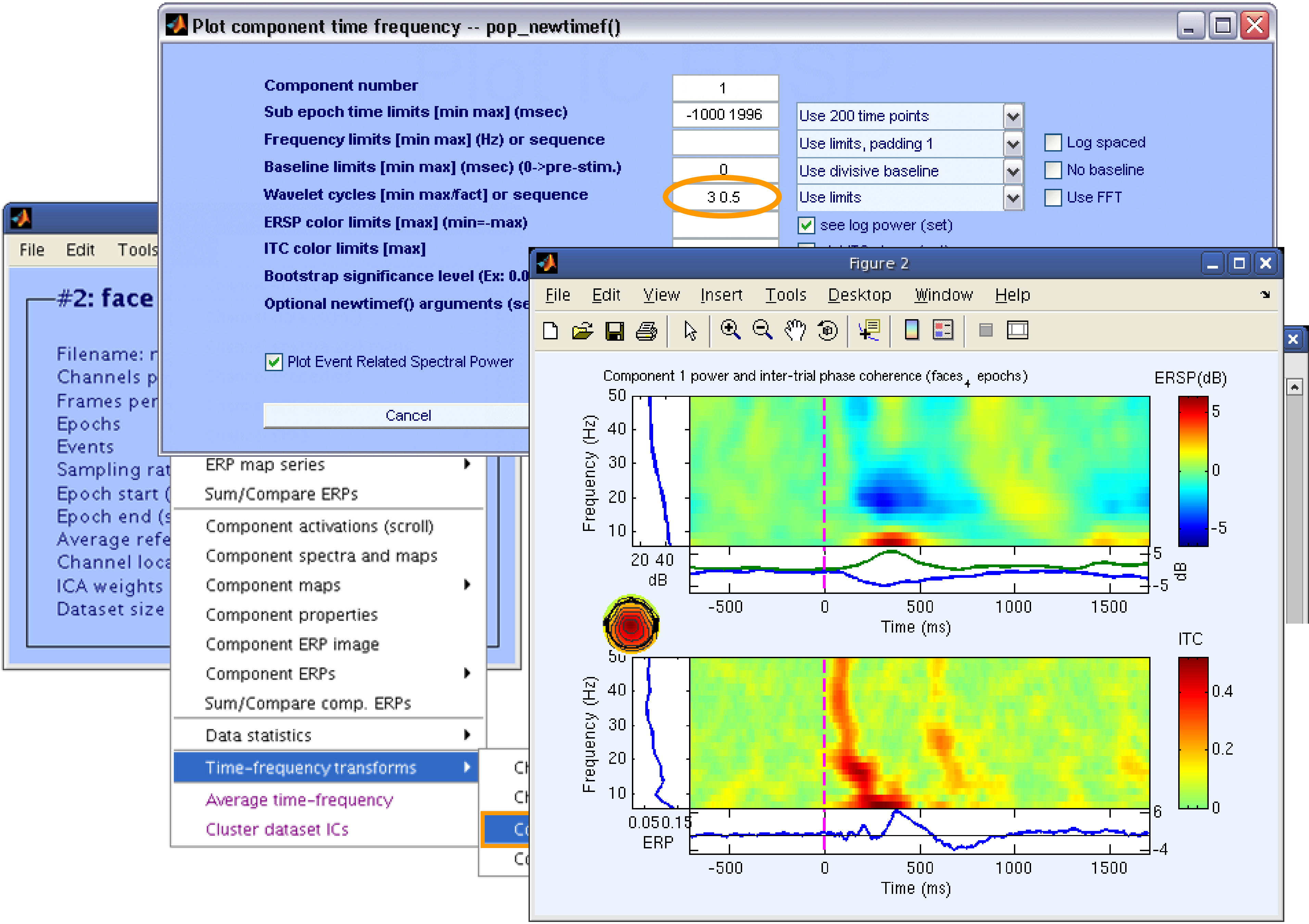
ITC = |mean(norm'd phase vectors)|




```
>> EEG = pop_newtimef(EEG,..., 'plotitc','on');
```



Slide courtesy of Stefan Debener



Plot component time frequency -- pop_newtimef()
Component number
Sub epoch time limits [min max] (msec)
Frequency limits [min max] (Hz) or sequence
Baseline limits [min max] (msec) (0->pre-stim.)
Wavelet cycles [min max/fact] or sequence
ERSP color limits [max] (min=-max)
ITC color limits [max]
1
-1000 1996
0
3 0.5
Use 200 time points
Use limits, padding 1
Use divisive baseline
Use limits
see log power (set)
Log spaced
No baseline
Use FFT
Plot Event Related Spectral Power
Cancel
File Edit Tools
#2: face
Filename:
Channels
Frames per
Epochs
Events
Sampling rat
Epoch start
Epoch end
Average refe
Channel loca
ICA weights
Dataset size
ERP map series
Sum/Compare ERPs
Component activations (scroll)
Component spectra and maps
Component maps
Component properties
Component ERP image
Component ERPs
Sum/Compare comp. ERPs
Data statistics
Time-frequency transforms
Average time-frequency
Cluster dataset ICs
Figure 2
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
ERSP(dB)
Frequency (Hz)
Time (ms)
dB
ITC
ERP

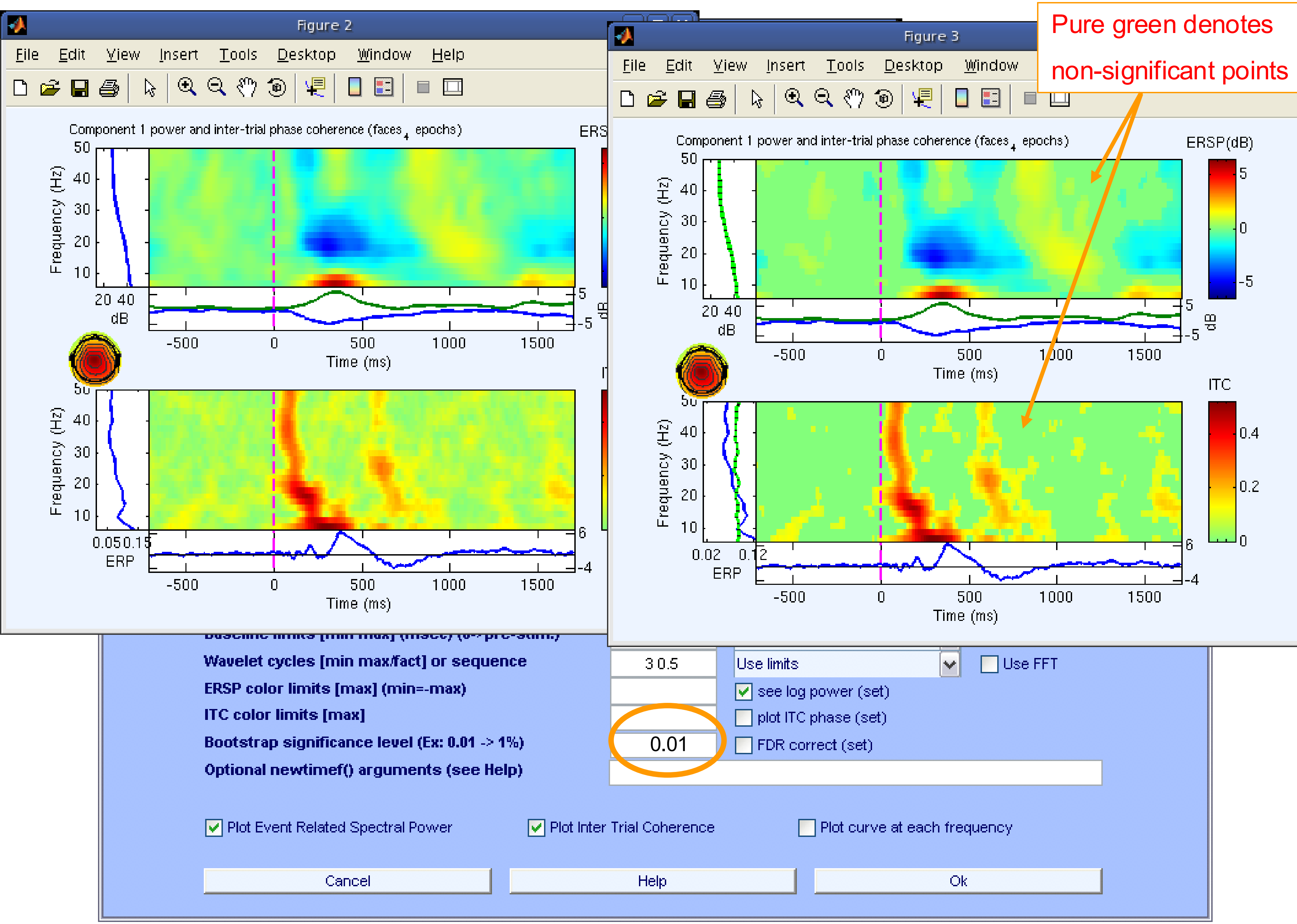


Figure 2
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
Frequency (Hz)
dB
ERP
Time (ms)
Figure 3
Pure green denotes
non-significant points
ERSP(dB)
ITC
Wavelet cycles [min max/fact] or sequence
ERSP color limits [max] (min=-max)
ITC color limits [max]
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional newtimef() arguments (see Help)
3 0.5
Use limits
Use FFT
see log power (set)
plot ITC phase (set)
0.01
FDR correct (set)
Plot Event Related Spectral Power
Plot Inter Trial Coherence
Plot curve at each frequency
Cancel
Help
Ok

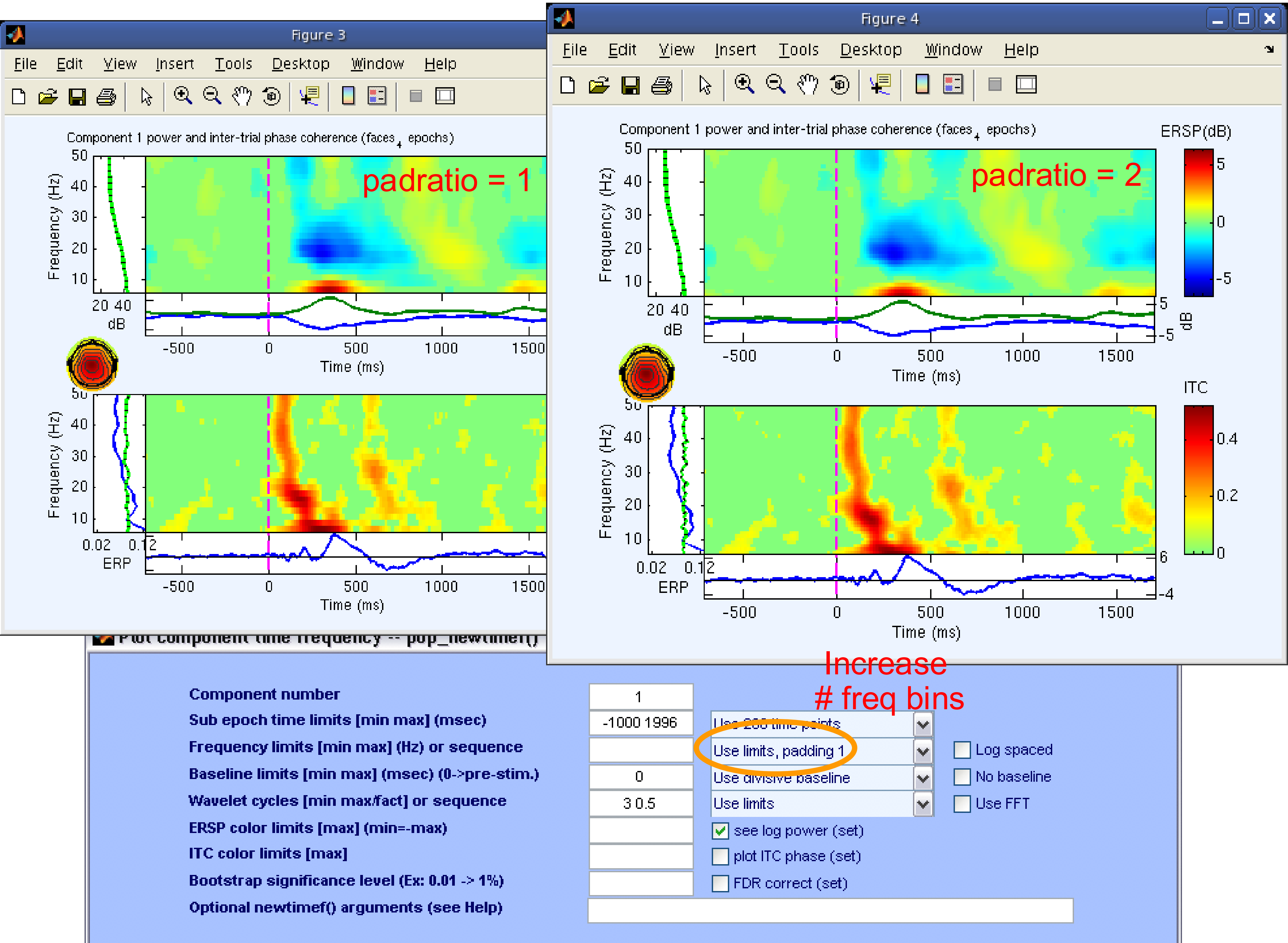
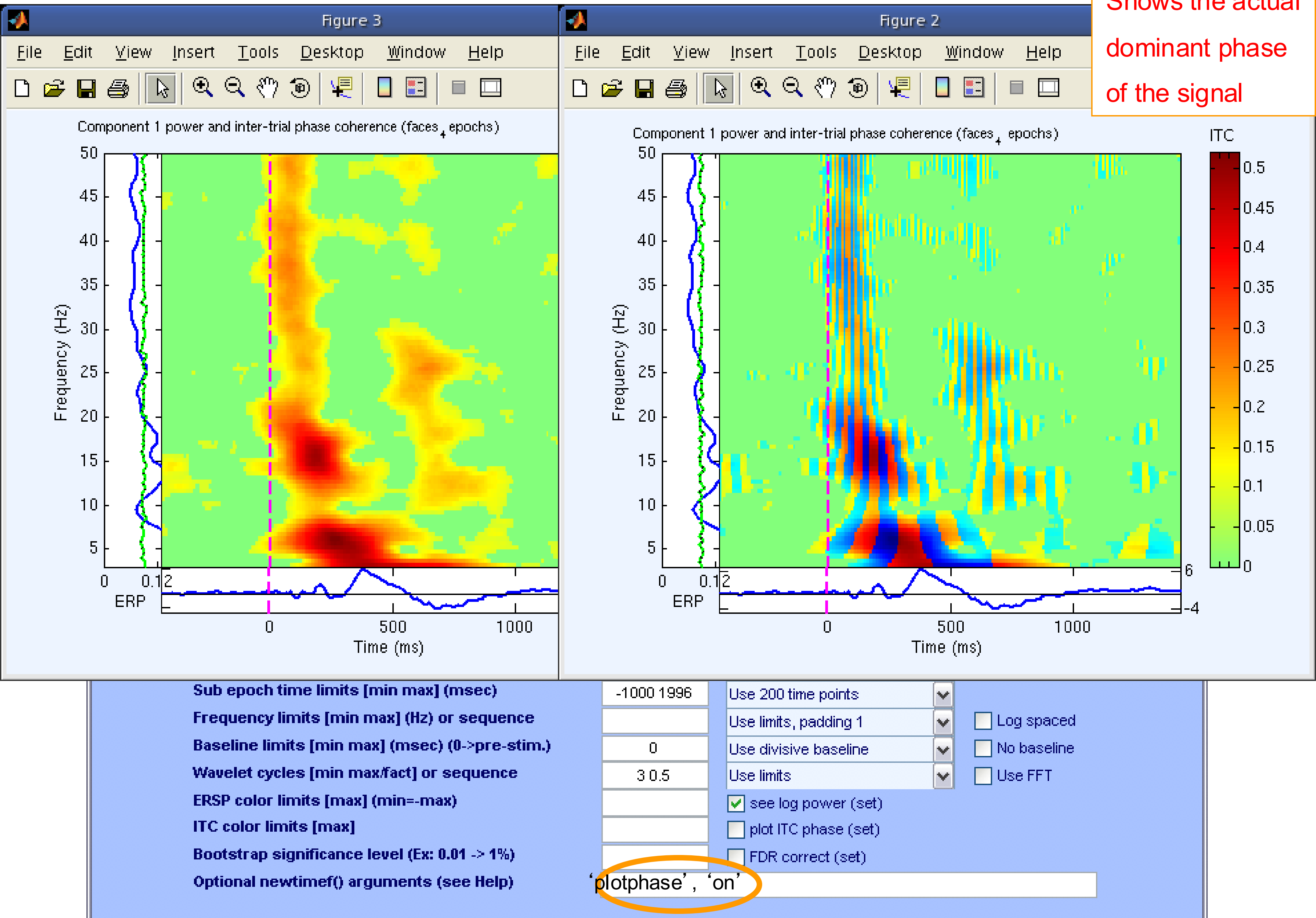


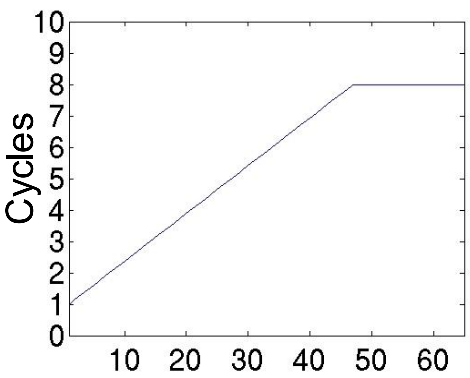
Figure 3
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
padratio = 1
Frequency (Hz)
20 40
dB
Time (ms)
0.02 0.12
ERP
Figure 4
File Edit View Insert Tools Desktop Window Help
Component 1 power and inter-trial phase coherence (faces epochs)
padratio = 2
ERSP(dB)
ITC
Frequency (Hz)
Time (ms)
ERP
Increase
freq bins
Plot component time frequency -- pop_newtimef()
Component number
Sub epoch time limits [min max] (msec)
Frequency limits [min max] (Hz) or sequence
Baseline limits [min max] (msec) (0->pre-stim.)
Wavelet cycles [min max/fact] or sequence
ERSP color limits [max] (min=-max)
ITC color limits [max]
Bootstrap significance level (Ex: 0.01 -> 1%)
Optional newtimef() arguments (see Help)
1
-1000 1996
0
3 0.5
Use limits, padding 1
Use divisive baseline
Use limits
Log spaced
No baseline
Use FFT
see log power (set)
plot ITC phase (set)
FDR correct (set)

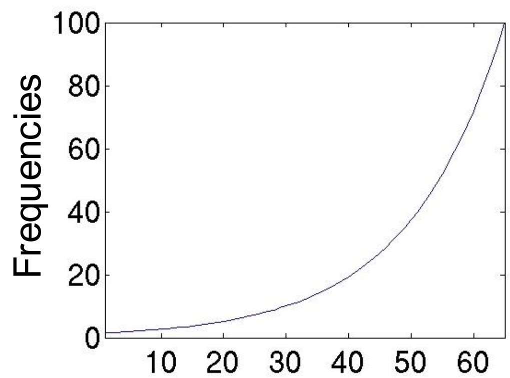
Shows the actual dominant phase of the signal

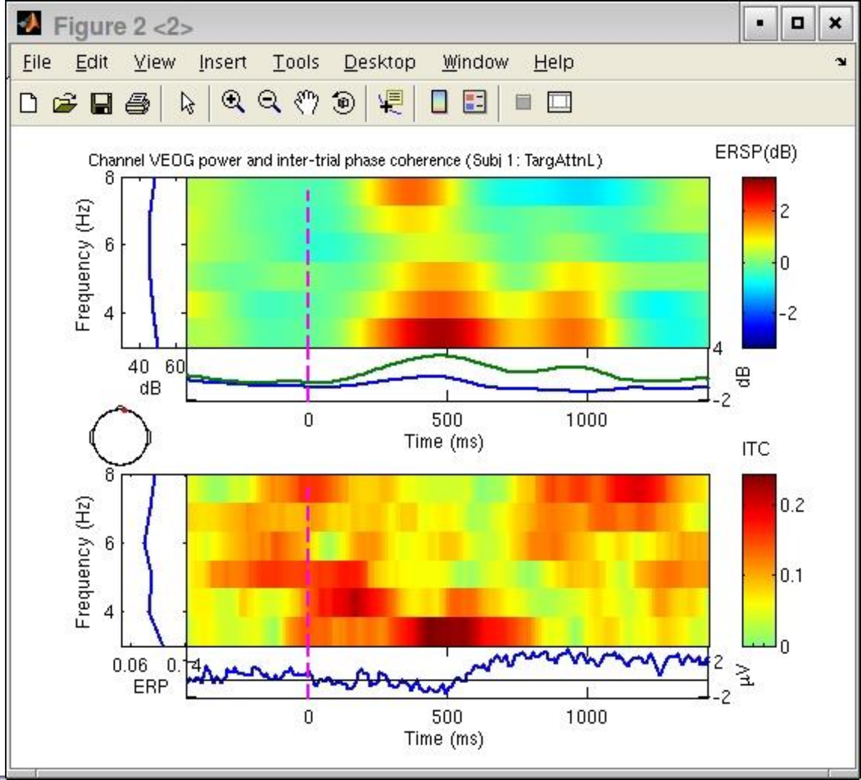


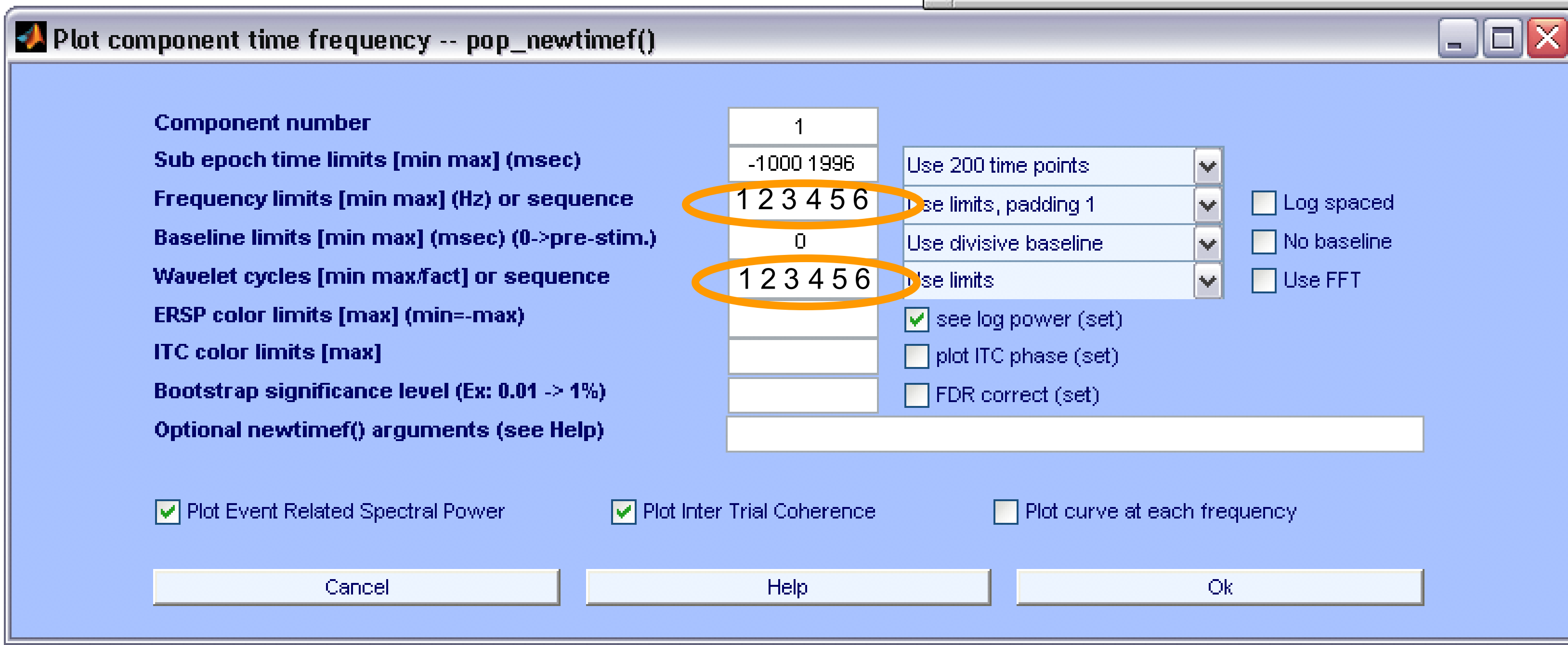
To visualize both low and high frequencies

```
freqs  = exp(linspace(log(1.5), log(100), 65));
cycles = [ linspace(1, 8, 47) ones(1,18)*8 ];
```





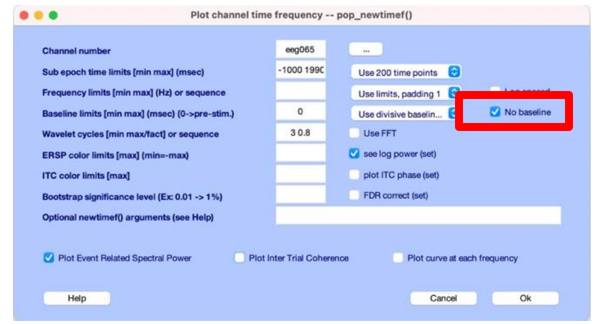




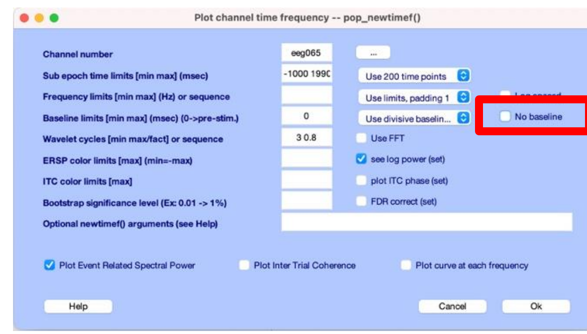
Hands-on: Display ERS vs ERSP, change parameters

1. Use menu item **File > Load existing dataset** to import epoched file for famous stimuli
ds000117_pruned/derivatives/meg_derivatives/sub-01/ses-meg/meg/wh_S01_run_01_ERP_Analysis_Session_2_famous_out.set

Use menu item **Plot > Time-frequency transforms > Channel time frequency**

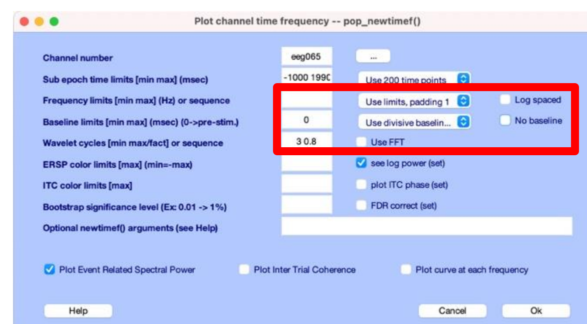


2. Plot event-related Spectrogram (no baseline)



3. Plot event-related Spectral Perturbation

4. Try changing parameters window size, number of wavelet cycles, padratio,



The end