Solar sail trajectory optimization for an Earth-Saturn Mission

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I. Abstract

This report aims to generate time-optimal trajectories for an Earth-Saturn solar sail mission. Solar sails are being increasingly investigated for interplanetary missions and other spaceflight applications due to their characteristic of requiring little to no thrust. Trajectory optimization is carried out on MATLAB with the inclusion of solar radiation pressure terms as applied on an ideal solar sail, using the solar sail pitch angle as the control variable. Preliminary results with literature, and the problem is then built on to increase fidelity and achieve the goal of reaching Saturn. This mission aims to be a precursor for interplanetary missions to the Jovian planets and outer Solar System, and to provide a deeper understanding of the heliosphere.

II. Introduction

Saturn missions can be important due to its largest moon Titan being an important candidate for the exploration of potential for life, due to the presence of liquid water, ethane, and methane as found by earlier missions like the Cassini-Huygens probe. Solar sail propulsion was chosen due to its advantages of less fuel requirements over long distances. With a goal of minimizing the time of flight, this project can serve as a precursor for further missions to the Jovian planets and beyond, as well as help understand the structure of the heliosphere for future interplanetary missions.

Low thrust optimization is increasingly desired for the future of spaceflight and especially long-distance interplanetary missions due to the high expense, mass, and inconvenience of using liquid fuel as propellant. An upcoming solution being investigated is the use of solar sails, which use solar radiation pressure as the source of thrust. Solar sails have been conceptually studied since the 1970s, with mentions in literature as early as the 1600s. The first spacecraft using solar sail technology was IKAROS in 2010, launched by JAXA using a solar-photon propulsion method to reach Venus. The use of solar sails for the trajectory in this report was inspired by Stephen Hawking's upcoming mission Breakthrough Starshot, the first interstellar mission aiming to travel to the Alpha Centauri star system with a flyby mission to Promixa Centauri b. The mission envisions launching thousands of tiny spacecrafts equipped to employ solar sail propulsion mounted on one large spaceship.

This report aims to design a trajectory to Saturn, as Saturn missions are of significance due to its largest moon Titan as it is an important candidate for the exploration of the potential for life, due to the presence of liquid water, ethane, and methane as found by earlier missions like the Cassini-Huygens probe. The trajectory is formulated with a goal to minimize time for multiple scenarios – with free and fixed final angular position, as well as considering the effect of different area sizes of the solar sail. With a goal of minimizing the time of flight, this project can serve as a precursor for further missions to the Jovian planets and beyond, as well as help understand the structure of the heliosphere for future interplanetary missions.

This project mainly references Zhang et al.'s [1] and Kim et al.'s [2] work on optimizing a solar sail trajectory for an Earth-Mars mission. These papers describe the equations of motions for a spacecraft using solar sail propulsion along with the initial and final boundary conditions, using the pitch angle as the primary control variable and minimizing the performance index chosen as the time of flight. Pitch angle is the angle between the normal vector to the sail surface and the radial velocity vector [1]. They also formulate the necessary conditions, i.e., costate equations, stationary condition, and terminal conditions. Similar work was conducted by T.S. Jayaraman [3] for ISRO, where he investigates a time-optimal solution from Earth to Mars with the pitch angle of the solar sails as the unbounded control variable solved using the penalty function approach. The paper uses the conjugate gradient algorithm for free final time to minimize the final time of flight. The solutions are then compared with those achieved by ionic propulsion. A paper by Colasurdo and Casalino [4] gives a broader approach to interplanetary missions using non-ideal solar sails that include their properties of emission and absorption on top of reflection as considered for an ideal sail. The paper makes use of Pontryagin's maximum principle as well applied to an indirect optimization problem to minimize the time of flight for missions to Mars and Mercury. This study also takes inspiration from the conceptual study carried out by JPL with a goal to Rendezvous with Halley's Comet [6].

III. Methodology

A. Equations of Motion

The equations of motion [2] are modified from standard coplanar equations to include the solar radiation pressure term for an ideal solar sail with inclusion of $\beta = a_c r_e b_2$, where $a_c = \frac{2P_0A}{m}$ is the characteristic acceleration, P_0 is the solar radiation pressure at r_e and b_2 is the thermo-optical property of the solar sail and is equal to 1 for an ideal solar sail. The equations of motion are:

$$\dot{r} = u$$

$$\dot{\theta} = {}^{v}/_{r}$$

$$\dot{u} = \frac{\beta \cos^3 \alpha}{r^2} + \frac{v^2}{r} - \frac{\mu}{r^2}$$

$$\dot{v} = \frac{\beta \sin\alpha \cos^2 \alpha}{r^2} - \frac{uv}{r}$$

Typical boundary values are applied. The final angular position can be either fixed or free. This project investigated both cases. The boundary conditions are:

$$r(t_0) = r_e \mid \theta(t_0) = 0 \mid u(t_0) = 0 \mid v(t_0) = \sqrt{\frac{\mu}{r_e}}$$

$$r(t_f) = r_s \mid \theta(t_f) = \theta_s \; or \; free \mid u(t_f) = 0 \mid v(t_f) = \sqrt{\frac{\mu}{r_f}}$$

The optimal control problem is the minimal time of flight problem; therefore, the following cost function and Hamiltonian were used:

$$\boldsymbol{J = t_f}$$

$$H = L + \lambda^T f$$

$$H = \lambda_r(u) + \lambda_\theta\left(\frac{v}{r}\right) + \lambda_u\left(\frac{\beta\, cos^3\,\alpha}{r^2} + \frac{v^2}{r} - \frac{\mu}{r^2}\right) + \lambda_v\left(\frac{\beta\, sin\alpha\, cos^2\,\alpha}{r^2} - \frac{uv}{r}\right)$$

Typical necessary conditions were applied. They are:

$$\dot{\lambda}_r = -\frac{\partial H}{\partial r} = \lambda_\theta\frac{v_\theta}{r^2} + \lambda_u\left(\frac{2\beta\, cos^3\,\alpha}{r^3} + \frac{v^2}{r^2} - \frac{2\mu}{r^3}\right) + \lambda_v\left(\frac{2\beta\, sin\alpha\, cos^2\,\alpha}{r^3} - \frac{uv}{r^2}\right)$$

$$\dot{\lambda}_\theta = -\frac{\partial H}{\partial \theta} = c = 0 \text{ iff } \theta_f = 0 \mid \lambda_\theta = c \neq 0 \text{ iff } \theta_f = \theta_s$$

$$\dot{\lambda}_u = -\frac{\partial H}{\partial u} = -\lambda_r + \lambda_v\left(\frac{v}{r}\right)$$

$$\dot{\lambda}_v = -\frac{\partial H}{\partial v} = -\frac{\lambda_\theta}{r} - \frac{2\lambda_u v}{r}\frac{+\lambda_v u}{r}$$

The terminal constraint was also applied:

$$\psi\big(x(t_f), t_f\big) = 0$$

For the case of a spacecraft using solar sail propulsion, there is only 1 control parameter. That parameter is the pitch angle of the sail:

$$u = \alpha$$

The stationary conditions are also needed. Additional constraints are needed because this problem has free final time of flight.

- Stationary conditions

$$\frac{\partial H}{\partial u} = \frac{\partial H}{\partial \alpha} = 0$$

$$\frac{\partial H}{\partial \alpha} = \frac{-3\lambda_u\beta\sin\alpha\cos^2\alpha}{r^2} + \frac{\lambda_v\beta(\cos^3\alpha\ -\ 2\, sin^2\,\alpha\, cos\,\alpha)}{r^2} = 0$$

$$\alpha^* = tan^{-1}\left[\frac{-3\lambda_u - \sqrt{9\lambda_u^2 + 8\lambda_v^2}}{4\lambda_v}\right]\ if\ \lambda_v \neq 0$$

- Free final time problem

$$\phi = t_f \qquad | \qquad \frac{\partial \Phi}{\partial t_f} + H = 0$$

$$H + 1 = 0$$

The following units were used:

- Time (TU = 58.13 days) | Distance (AU) | Mass (kg)

B. Models and Simulations

The code created for the Earth-Saturn free θ_f problem was modified to fit the Earth-Mars problem. By doing so, the generated results could then be compared to literature [5]. For the 2 different values of β as shown in the literature, the code was run.

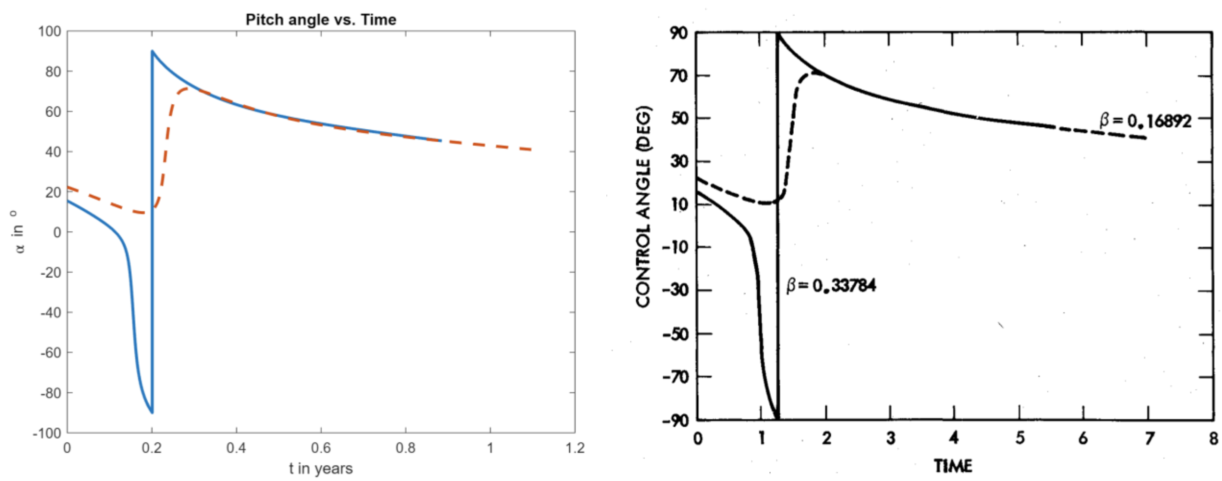

Figure 1: Comparative study for validation of code

From inspection of the above graphs, the code works for the Earth-Mars case. And the results from Earth-Saturn case can be presented with confidence.

The optimal control problem was solved using shooting technique. An error function was created, and then shooting technique was used to find the solution that results in minimal error. This was done in MATLAB using fsolve() to do the shooting and ode45() to propagate the differential equations.

The final values of solar sail mass and area were chosen via trial-and-error method. Even such a large value of area is at par with the area value used in a JPL's conceptual study [6].

The following solar sail properties were used:

- Area = $1.4 \times 10^5\ m^2$

- Mass = $300\ kg$

Fixed θ_f

The relative final angular position of Saturn was calculated to be around $\theta_f = 216^0$ using the minimized time of flight from the free final angular position case as reference and a live planet positioning online tool (theplanetstoday.com/). This is an approximate calculation.

IV. Results

<u>Plots</u>

a) Earth-Mars

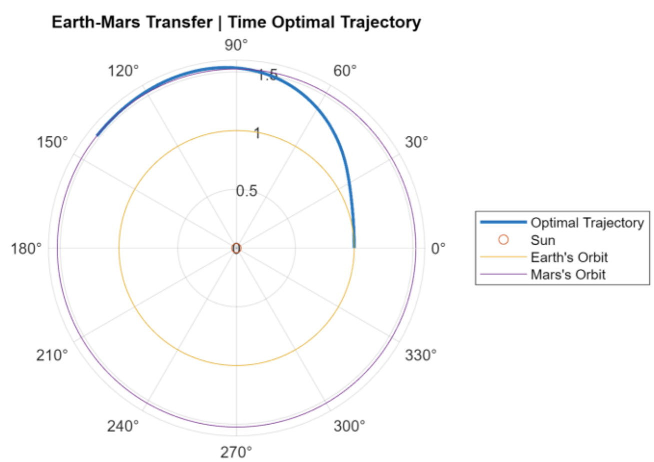

Figure 2: Time optimal trajectory for an Earth-Mars transfer

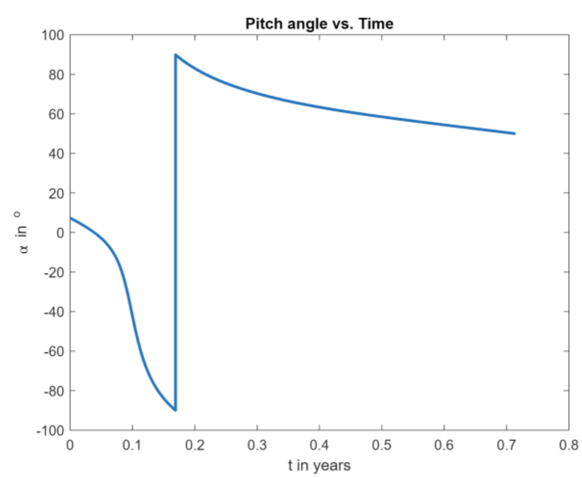

Figure 3: Pitch angle variation with time

These results show that the optimal trajectory and the pitch angle control policy. There is one point of concern, the vertical line in the control policy indicates that the solar sail must quickly change pitch to obtain the most optimal trajectory. This may not be feasible in real life, as a sudden 180-degree turn may not be possible.

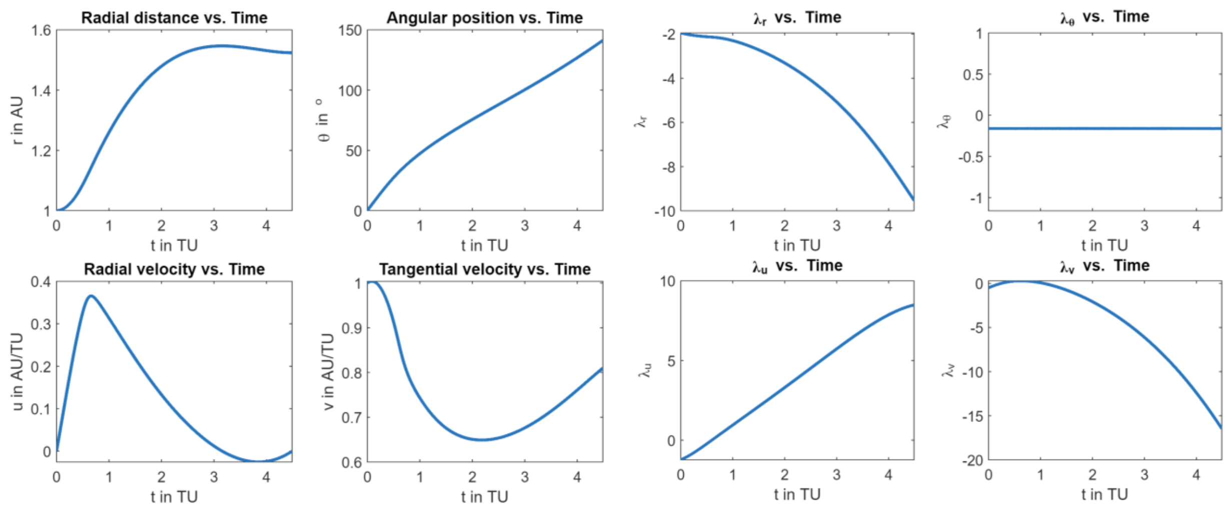

Figure 4: States and costates plot

b) Earth-Saturn | free θ_f

These results include the optimal trajectory, the control policy, graphs for the states and costates, graphs of the acceleration due to the solar sail's solar radiation pressure, and graphs related to the Hamiltonian over time.

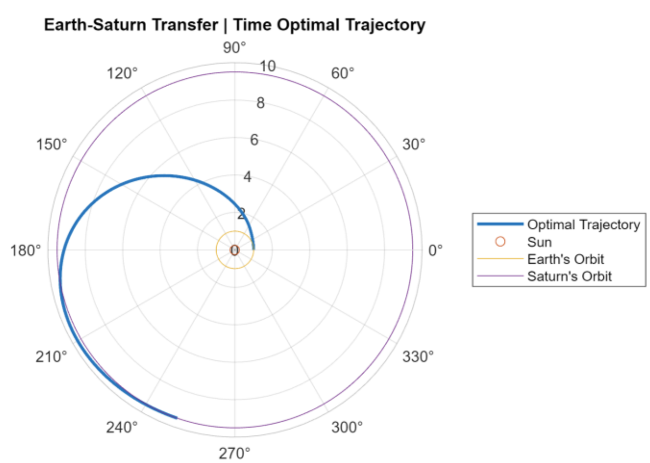

Figure 5: Time optimal trajectory for an Earth-Saturn transfer

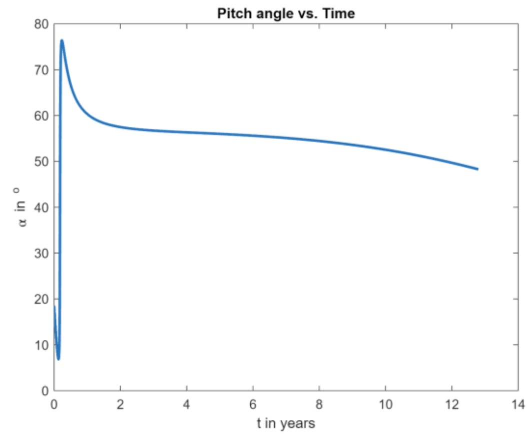

Figure 6: Pitch angle variation with time

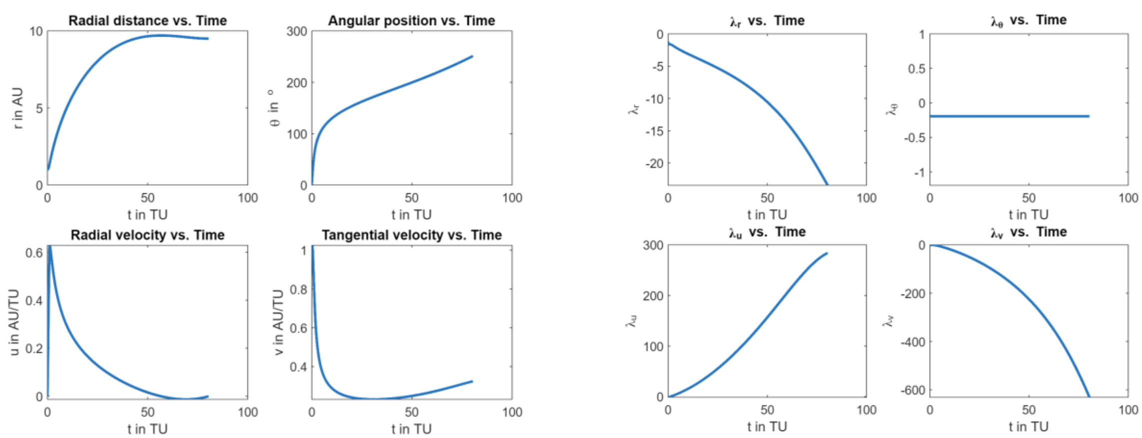

Figure 7: States and costates plot

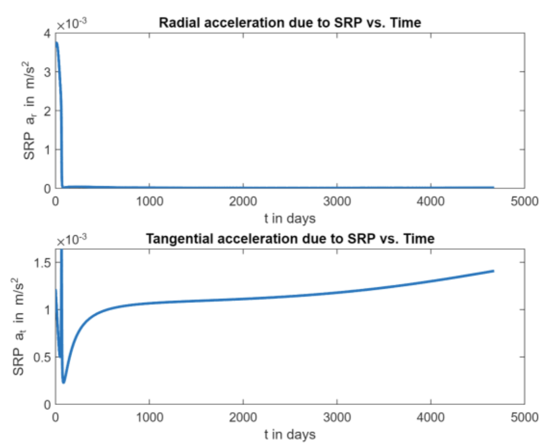

Figure 8: Variation of radial and tangential acceleration due to SRP

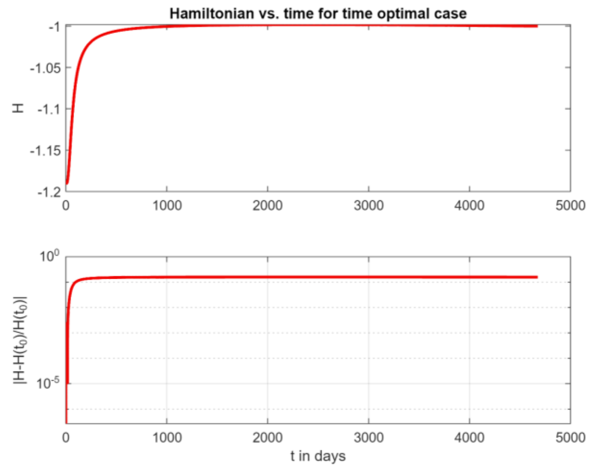

Figure 9: Variation of Hamiltonian with time

These results, like the Earth-Mars results, require a sudden change in pitch angle. This may not be feasible in real life, depending on the construction of the solar sail. Notably, the time of flight for this case is **12.8 years**.

c) Effects of solar sail area on the minimum time

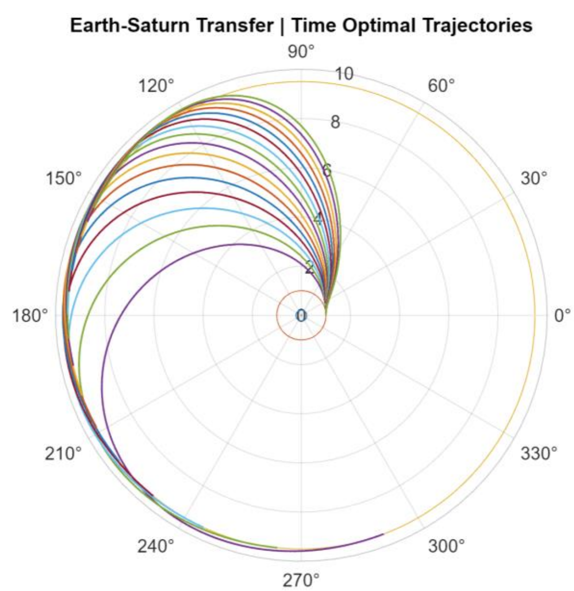

Figure 10: Time optimal trajectory plots for different areas

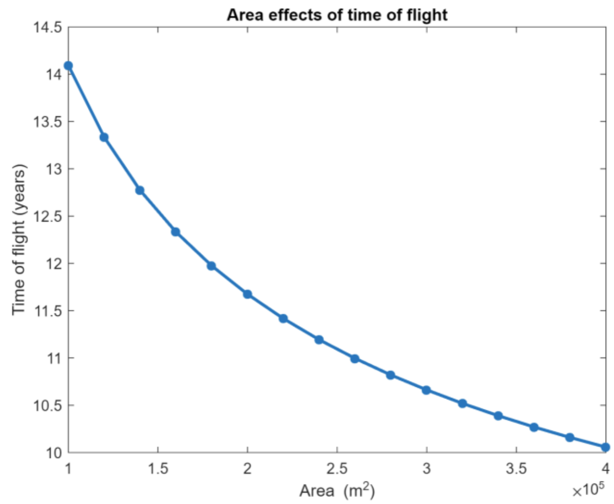

Figure 11: Optimized final time vs. Sail area

The results match the intuition that the larger solar sails cause the spacecraft to go faster.

d) Earth-Saturn | fixed θ_f

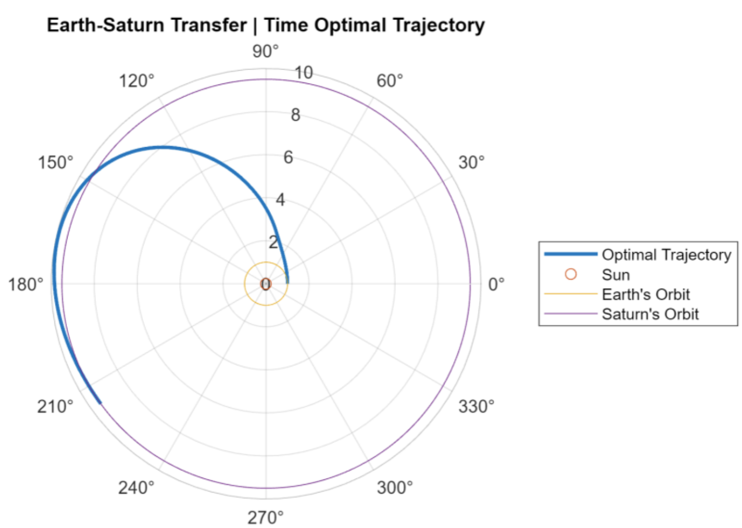

Figure 12: Time optimal trajectory for an Earth-Saturn transfer

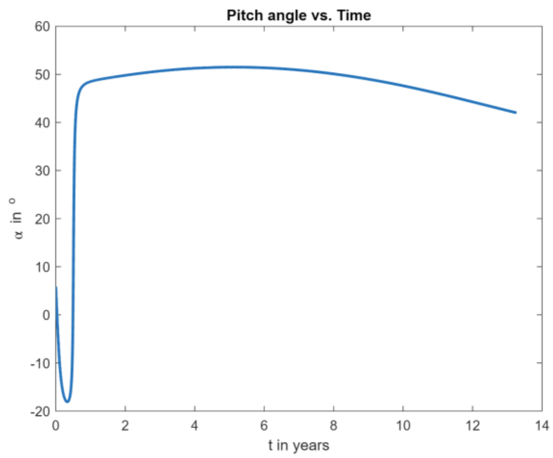

Figure 13: Pitch angle variation with time

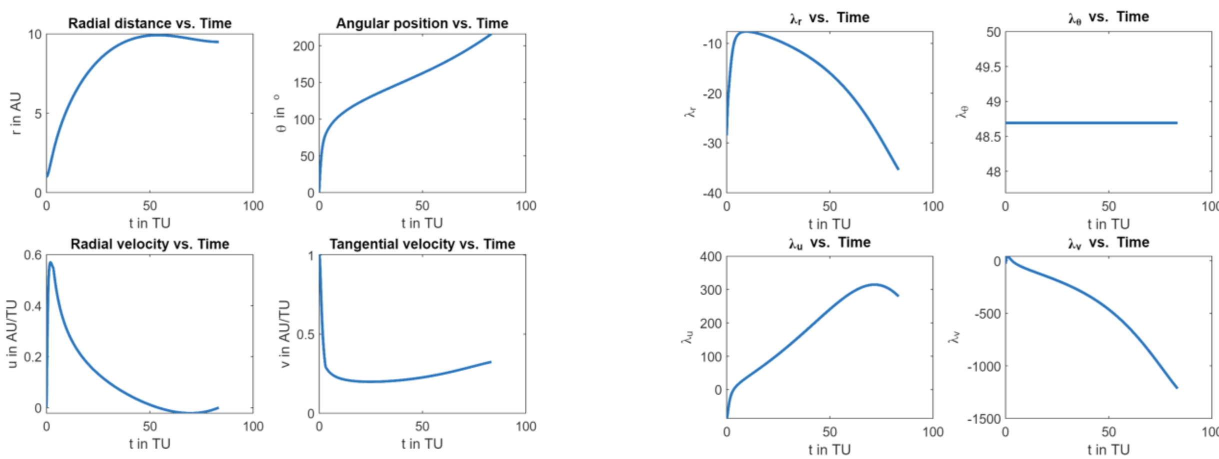

Figure 14: State and costate plots

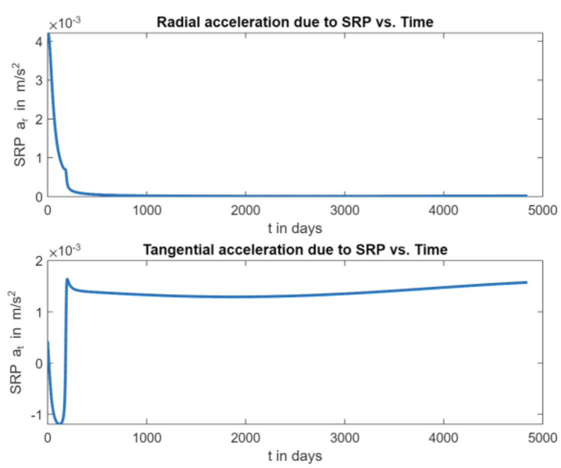

Figure 15: Variation of radial and tangential acceleration due to SRP

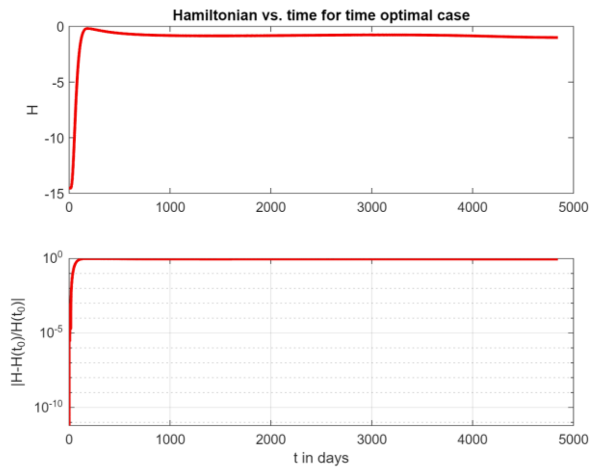

Figure 16: Variation of Hamiltonian with time

The minimum time for this fixed θ_f is **13.3 years**.

V. Conclusion

The project discusses multiple variations for optimizing a trajectory for a solar sail mission from Earth to Saturn, with the aim of improving fidelity with every step. This project discussed plots for free and fixed θ_f and observed the effect of solar sail area sizes on the trajectory.

- We confirm that larger sails would cause the spacecraft to travel faster.
- Due to the unbelievably large areas considered for the efficacy of the mission which would cause numerous structural constraints, this project falls in the category of a conceptual study (along the lines of JPL 1977 study of solar sail rendezvous with Halley's comet) [6].
- This project was conducted with the aim of developing time-optimal missions to explore the outer Solar System. We designed a trajectory to Saturn as a precursor for potential future missions to the Jovian planets.
- Since the calculation of the final position of Saturn (needed for the fixed θ_f case) by using the minimized time obtained from the free θ_f case was approximate, the mission would miss Saturn by about 6 months.
- Finally, this project also aims to provide a basic understanding of solar sails and their potential uses for the future specifically due to their low fuel consumption characteristics.

VI. Future Scope

- This project's overall fidelity can be improved by considering multiple perturbation effects on the preliminary trajectory. We have two specific perturbations in mind to refine this optimal trajectory.

- First, the problem can include third-body perturbations (Jupiter's gravity) – both in a simplified mathematical sense, and more specifically based on Jupiter's position in orbit at specific times, to be carried out with the help of JPL Spice. This may also be presented in either cartesian or polar coordinates. [7]
- Secondly, our project uses an ideal solar sail (i.e., a perfect reflector). For a more practical outcome, we aim to consider all three thermo-optical parameters, i.e., absorption, reflection, and emission specific to the material of the solar sail to achieve results for a non-ideal solar sail. This may be carried out by including any one extra parameter or all three for the highest fidelity. [4]

VII. Acknowledgement

This project was carried out in the partial fulfillment for the requirements of the AE508 Optimal Spacecraft Trajectories course under the guidance of Dr. Robyn Woollands.

Contributions: Raman Singh – Base code for the minimum time problem and its validation| Sanjana Srivastava – Theoretical formulation of the problem, its necessary conditions for all cases | Robert Cazeau – Implementation of the code for the different cases | Everyone – Literature and report

VIII. References

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