1. Big Omega Notation prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

$g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

For finding constants c and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + \frac{2}{n} + \frac{4}{n^2} \simeq 1$

Example $c = \frac{1}{2}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1 \quad (1 \geq \frac{1}{2}, n \geq 1)$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2} \quad (n \geq 1, n_0 = 1)$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

2. Big Theta Notation determine where $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

$C_1 n^2 \leq h(n) \leq C_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(10(n^2))$

$h(n) = 4n^2 + 3n$

$h(n) \leq C_2 n^2$

$4n^2 + 3n \leq C_2 n^2 \Rightarrow 4n^2 + 3n \leq Cn^2$

Let $C_2 = 5$

divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $((C_2 = \cancel{5}4, n_0 = 1))$

$1 + \frac{n}{n \log n} \leq 2$ simplify

$1 + \frac{1}{\log n} \leq C_2$ $C_2 = 2$

$((C_2 = 2, n_0 = 2))$

$1 + \frac{1}{\log n} \leq 2$

Then $h(n)$ is $O(n \log n)$

lower bound.

$h(n) \geq C_1(n \log n)$

$h(n) = n \log n + n$

$n \log n + n \geq C_1 . n \log n$

divide both sides by $n \log n$

$1 + \frac{n}{n \log n} \geq C_1$ $C_1 = 1$

$1 + \frac{1}{\log n} \geq S_1$

$\frac{1}{\log n} \geq 0$ for all $n > 1$

$h(n) = n \log n + n$ is $\Theta(n \log n)$

3 Solve the following recurrence relations and find the growth of solution $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$f(n) \geq C_1 g(n)$

substituting $f(n)$ & $g(n)$ into this inequality we get $n^3 - 2n^2 + n \geq C(-n^2)$

A and C and n_0 holds $n \geq n_0$

$n^3 - 2n^2 + n \geq Cn^2$

$n^3 + (C(-2))n^2 + n \geq 0$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$

$T(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

∴ The statement $f(n) = \Omega\, g(n)$ is true

4 Determine whether $h(n) = n\log n + n$ is $\Theta(n \log n)$ prove a vigorous proof for your conclusion.

$C_1 n \log n \leq h(n) \leq C_2 n \log n$

upper bound:-

$h(n) \leq C_2 . n\log n$

$h(n) = n \log n + n$

$n\log n + n \leq C_2 n \log n$

divide both sides with $n\log n$

$1 + \frac{n}{n\log n} \leq C$

$1 + \frac{1}{\log n} \leq C_2$

$$1+\frac{1}{\log n} \leq 2$$

Then $h(n)$ is $O(n \log n)$

lower bound:-

$$h(n) \geq C_1 n \log n$$

$$h(n) = n \log n + n$$

$$n \log n + n \geq C_1 n \log n$$

divide bothsides with $n \log n$

$$1+\frac{n}{n \log n} \geq C_1$$

$$1+\frac{1}{\log n} \geq C_1$$

$$1+\frac{1}{\log n} \geq 1$$

$$\frac{1}{\log n} \geq 0$$

$h(n)$ is $\Omega(n \log n)$ $(C_1 = 1, n_0 = 1)$

$h(n) = n \log n + n$ is $\Theta(n \log n)$

5 Solve the following recurrence relations and find the order of growth of solutions.

$$T(n) = 4T(n/2) + n^2, T(1) = 1$$

$T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4 \quad b = 2 \quad f(n) = n^2$

Applying masters thereom

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b^a - \epsilon})$

$f(n) = \Theta(n^{\log_b^a})$, then $T(n) = \Theta(n^{\log_b^a} \log n)$

$f(n) = \Omega(n^{\log_b^a + \epsilon})$, Then $T(n) = f(n)$

calculating $\frac{a}{b}$

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = O(n^2)$ (comparing $f(n)$ with $n^{\log_b^a}$)

$f(n) = \Theta(n^2) = \Theta(n^{\log_b^a})$

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b^a} \log n) = \Theta(n^2 \log n)$

Order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$

$i > O(n^2 \log n)$