

1. Given an array of  $(4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, 11, -9)$  integers find the maximum and minimum product that can be obtained by multiplying two integers from the array.

Sol. Array is  $(4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11)$

We need to consider the largest and smallest products that can be formed by selecting two numbers from the array.

1. Sort the array.

Sorted array  $[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

2. Identify possible candidates for maximum product.

3. Identify possible candidates for minimum product.

calculating maximum product

\* The two largest positive numbers are 10 and 11  $\therefore 10 \times 11 = 110$

\* The two smallest negative numbers are -9 and -8  $\therefore 72$

The maximum product is 110

calculating minimum products

The largest positive and negative numbers is 11, -9  $\therefore -99$

The smallest positive and negative numbers -9 & -8  $\therefore 72$

2. Demonstrate the primary search method to search for the key = 23 from the array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

Sol. Given key = 23 & array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

\* Initialise pointers

low = 0 and high = 9

$$\text{calculate mid} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4$$

compare arr[mid] with key:

$$\text{arr}[4] = 16$$

since  $16 < 23$  update low = mid + 1 = 5

$$\text{calculate mid} = \frac{\text{low} + \text{high}}{2} = \frac{5 + 9}{2} = 7$$

arr[7] = 56  $56 > 23$  update high = mid - 1 = 6

$$\text{mid} = \left(\frac{5 + 6}{2}\right) = 5 \Rightarrow \text{arr}[\text{mid}] = \text{arr}[5] = 23; \quad 23 = 23$$

The key is found at index 5 The key = 23 is found at index 5

3. Apply merge sort and other list of 8 elements, Data  $d = (45, 67, -12, 5, 22, 30, 50, 20)$ . Set up a recurrence relation for the no. of key comparisons made by merge sort.

Sol Merge sort:-

|    |    |     |   |    |    |    |    |
|----|----|-----|---|----|----|----|----|
| 45 | 67 | -12 | 5 | 22 | 30 | 50 | 20 |
|----|----|-----|---|----|----|----|----|

|    |    |     |   |
|----|----|-----|---|
| 45 | 67 | -12 | 5 |
|----|----|-----|---|

|    |    |    |    |
|----|----|----|----|
| 22 | 30 | 50 | 20 |
|----|----|----|----|

|    |    |     |   |
|----|----|-----|---|
| 45 | 67 | -12 | 5 |
|----|----|-----|---|

|    |    |    |    |
|----|----|----|----|
| 22 | 30 | 50 | 20 |
|----|----|----|----|

|     |   |    |    |
|-----|---|----|----|
| -12 | 5 | 45 | 67 |
|-----|---|----|----|

|    |    |    |    |
|----|----|----|----|
| 20 | 22 | 30 | 50 |
|----|----|----|----|

|     |   |    |    |    |    |    |    |
|-----|---|----|----|----|----|----|----|
| -12 | 5 | 20 | 22 | 30 | 45 | 50 | 67 |
|-----|---|----|----|----|----|----|----|

$\therefore$  The sorted list =  $(-12, 5, 20, 22, 30, 45, 50, 67)$

$$T(n) = 2 + (n/2) + O(n)$$

if  $n \geq 1$ ,  $T(1) = 0$ , Best case

$\rightarrow$  At each level of recursion we make at most  $n/2$  comparisons merge two halves of size  $n/2$  so it becomes  $T(n) = 2 + (n/2) + (n-1)$

solving recurrence relation we get

$$T(n) = n \log_2 n - (n+1)$$

$$\therefore T(n) = O(n \log n)$$

$$T(n) = n \log_2 n - n + 1$$

4. Find the no. of times to perform solving swapping for sel. sort also estimate the time complexity for others of notations sets  $\{12, 7, 5, -2, 18, 6, 13, -7\}$

sol The selection sort algorithm always makes exactly  $n-1$  swaps in the worst case, where  $n$  is the no. of elements in the list.

$$S \rightarrow \{12, 7, 5, -2, 18, 6, 13, 44\}$$

$$\text{No of elements } n = 8$$

$$\text{no of swaps } n-1 = 8-1 = 7$$

Time complexity:- The time complexity of selection sort in Big O-Notation is  $O(n^2)$ . So the no. of swap and time complexity is  $O(n^2)$ .

5. Find the index of the target value to visiting binary search team the following list of elements  $\{2, 4, 6, 8, 10, 12, 14, 16, 20\}$

sol. Given, List  $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$  and  $\text{value} = 10$

$$\text{low} = 0 \text{ and } \text{high} = 9$$

$$\text{mid} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4$$

$$\text{mid} = 10, \text{mid} == \text{value}$$

since  $= 10$  the target is found at index 4

∴ The target value  $= 10$  is found at index 4