6) $L(\sin 2t \cos t \cosh 2t)$

$$= L\left[\sin 2t \cos t \left(\frac{e^{2t}+e^{-2t}}{2}\right)\right]$$

$$= L\left\{\frac{1}{2}\left[e^{2t}\sin 2t \cos t + e^{-2t}\sin 2t \cos t\right]\right\}$$

$$= L\left\{\frac{1}{4}\left[e^{2t}\sin 3t + e^{2t}\sin t + e^{-2t}\sin 3t + e^{-2t}\sin t\right]\right\} \text{ — } (1)$$

$L[\sin 3t] = 3/s^2+9$

$L[e^{2t}\sin 3t] = 3/(s-2)^2+9$ — (2)

$L[e^{-2t}\sin 3t] = 3/(s+2)^2+9$ — (3)

$L[\sin t] = 1/s^2+1$

$L[e^{2t}\sin t] = 1/(s-2)^2+1$ — (4)

$L[e^{-2t}\sin t] = 1/(s+2)^2+1$ — (5)

Put in (1)

$$= \frac{1}{4}\left[\frac{3}{(s-2)^2+9} + \frac{3}{(s+2)^2+9} + \frac{1}{(s-2)^2+1} + \frac{1}{(s+2)^2+1}\right]$$

4) If $L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{\sqrt{s}}$ then find $\left[\sqrt{\frac{\pi}{t}}\right]$

$f(t) = 1/\sqrt{\pi t}$ $\quad f(at) = \sqrt{\pi/t}$

$\therefore 1/\sqrt{\pi} = \sqrt{\pi}/\sqrt{a}$

$\sqrt{a} = \sqrt{\pi}\cdot\sqrt{\pi}$

$\sqrt{a} = \pi$

$a = \pi^2$

Now,

$L\left[\sqrt{\pi/t}\right] = 1/a\,(s/a)$.

$1/\pi^2 = \pi/s$

$\therefore L\left[\sqrt{\frac{\pi}{t}}\right] = \frac{1}{\pi\sqrt{s}}$

5) $L\left(\frac{\cos^3 t}{e^{3t}}\right)$

$\cos^3 t/e^{3t} = e^{-3t}\cdot\cos^3 t$

$\cos^3 t = \cos 3t + 3\cos t\ /4$

$L[\cos^3 t] = 1/4\left[s/s^2+9 + 3s/s^2+1\right] = \phi s$

By 1st simplifying theorem,

$L[f(t)] = \phi s$ then

$L[e^{at}\cdot f(t)] = \phi(s-a)$

Here $a = -3$

$L[e^{-3t}\cdot\cos^3 t] = \phi(s+3) = 1/4\left[\frac{(s+3)}{(s+3)^2+9} + \frac{3(s+3)}{(s+3)^2+1}\right]$

$= 1/4\left[\frac{s+3}{s^2+6s+18} + \frac{3}{s^2+6s+10}\right]$

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1) $L[\cos t \cdot \cos 2t \cdot \cos 3t]$

$= L[\frac{1}{2}(\cos 3t + \cos t)\cos 3t]$

$= \frac{1}{2} L[\cos^2 3t + \cos t \cdot \cos 3t]$

$= \frac{1}{2} L[\frac{1}{2}(\cos 6t + 1) + \frac{1}{2}(\cos 4t + \cos 2t)]$

$= \frac{1}{4} L[1 + \cos 2t + \cos 4t + \cos 6t]$

$= \frac{1}{4}\left[\frac{1}{s} + \frac{s}{s^2+4} + \frac{s}{s^2+16} + \frac{s}{s^2+36}\right]$

2) $L(\cosh^3 2t)$

$\cosh^3 2t = \left(\frac{e^{2t} - e^{-2t}}{2}\right)^3 = \left(\frac{e^{4t}-1}{2e^{2t}}\right)^3$

$= e^{12t} - 1 - 3e^{8t} + 3e^{4t} / 8e^{6t}$

$= \frac{1}{8}[e^{6t} - e^{-6t} - 3e^{+2t} + 3e^{-2t}]$

$= \frac{1}{8}\left[\frac{1}{(s-6)} - \frac{1}{(s+6)} - \frac{1}{(s-2)} + \frac{1}{(s+2)}\right]$

3) $L(3t^2 + e^{-t} + \sin^3 t)$

$= L\left[3t^2 + e^{-t} + \left(\frac{3\sin t - \sin 3t}{4}\right)\right]$

$= L\left[3t^2 + e^{-t} + \frac{3}{4}\sin t - \frac{1}{4}\sin 3t\right]$

$= 3\left(\frac{2!}{s^{2+1}}\right) + \frac{1}{s+1} + \frac{3}{4}\left(\frac{1}{s^2+1}\right) - \frac{1}{4}\left(\frac{3}{s^2+9}\right)$

$= 6/s^3 + 1/s+1 + 3/4(1/s^2+1) - 1/4(9/s^2+9)$