PERFORMANCE

Term	Remarks	Staff Member's Signature
I	Completed	17/10/19
II	Completed	27/01/2020

THAKUR COLLEGE OF SCIENCE & COMMERCE

NAAC Accredited with Grade "A" (3rd Cycle)

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ISO 9001 : 2015 Certified

Degree College

Computer Journal

CERTIFICATE

SEMESTER II **UID No.** ____________

Class Fy BSc CS **Roll No.** 1709 **Year** 2019-20

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27/01/2020

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PRACTICAL - 1

Topic : Limit and Continuity

Q. 1

i. $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3}\,x}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

ii. $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

iii. $$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

iv. $$\lim_{x \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$$

Q. Examine the continuity from the following function

i. $$f(x) = \begin{cases} \dfrac{\sin 2x}{1-\cos 2x}, & 0 < x \le \pi/2 \\[2ex] \dfrac{\cos x}{\pi - 2x}, & \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

ii. $$f(x) = \frac{x^2-9}{x-3}, \quad 0 < x < 3$$

$$= x+3 \qquad 3 < x < 6$$

$$= \frac{x^2-9}{x+3}$$

Q. Find value of k, so that the function f(x) is continuous at the indicated point.

i. $f(x) = \dfrac{1-\cos 4x}{x^2}$, $x < 0$
$= k$, $x = 0$ } at $x = 0$

ii. $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$
$= k$, $x = 0$ } at $x = 0$

iii. $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \pi/3$
$= k$, $x = \pi/3$

Q. Discuss the continuity of the following function have removable discontinuity? Redefine function so as to remove the discontinuity

i. $f(x) = \dfrac{1-\cos 3x}{x \tan x}$, $x \neq 0$
$= 9$, $x = 0$ } at $x = 0$

ii. $f(x) = \dfrac{(\cancel{e^{3x}}-1)\sin x^\circ}{x^2}$, $x \neq 0$
$= \pi/60$, $x = 0$

Q. If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$
for $x \neq 0$ is continuous at $x = 0$ find $f(0)$

Q. If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$
is continuous at $x = \pi/2$
find $f(\pi/2)$.

Solutions :

Q.1

i. $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$$= \lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{\cancel{(a-x)}}{\cancel{(a-x)}} \frac{(\sqrt{3a+x} + 3\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$= \frac{1}{3} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \frac{2\sqrt{a} + 2\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{1}{3} \cdot \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2\sqrt{a}}{3\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

i. $$\lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}$$

$$= \lim_{y \to 0} \frac{\sqrt{a+x} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+x} + \sqrt{a}}{\sqrt{a+x} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{(a+y-a)}{(y\sqrt{a+y})(\sqrt{a+x} + \sqrt{a})}$$

$$= \lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}(\sqrt{a+y} \cdot \sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0}\,(\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})} = \frac{1}{\sqrt{a}\,(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

iii. $$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

$x - \pi/6 = h$

$x = h + \pi/6 \qquad h \to 0$

$$= \lim_{h \to 0} \frac{\cos(h+\pi/6) - \sqrt{3}(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

$$= \lim_{h \to 0} \frac{(\cos h \cos \pi/6 - \sin h \cdot \sin \pi/6) - \sqrt{3}(\sin h \cdot \cos \pi/6 + \cos h \cdot \sin \pi/6)}{-6h}$$

$$= \lim_{h \to 0} \frac{\left[\left(\cos h \frac{\sqrt{3}}{2}\right) - \left(\sin h \frac{1}{2}\right)\right] - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{2}\right)}{-6h}$$

$$= \lim_{h \to 0} \frac{\left(\cos \frac{\sqrt{3}}{2} h - \sin \frac{h}{2}\right) - \left(\sin \frac{3h}{2} + \cos \frac{\sqrt{3}}{2} h\right)}{-6h}$$

$$\lim_{h \to 0} \frac{\frac{-\sin 4h}{2}}{-6h}$$

$$= \frac{1}{3} \times \frac{\sin h}{h}$$

$$= \frac{1}{3} \times 1 = \frac{1}{3}$$

iv. $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}}{\sqrt{x^2+3}} \quad \dfrac{-\sqrt{x^2-3}}{\sqrt{x^2+1}}\right]$

$= \lim_{x\to\infty} \dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}$

$= \lim_{x\to\infty} \dfrac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$= \lim_{x\to\infty} \dfrac{8}{2} \dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$= \lim_{x\to 0} 4 \dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{\sqrt{x^2+5}+\sqrt{x^2-3}}$

$= \lim_{x\to\infty} 4 \dfrac{(x^2(1+3/x^2)) + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$

$= 4 \dfrac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{x^2(1+5/x^2) + x^2(1-3x^2)}$

$= 4$

Q.

i. $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \leq \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$ $\Big\}$ at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1 - \cos 2(\pi/2)}}$

1. $f(\pi/2) = 0$

$\therefore$ f at $x = \pi/2$ define

$\lim\limits_{x \to \pi/2^+} f(x) = \lim\limits_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

put $x - \pi/2 = h$

$\therefore x = \pi/2 + h$

as $x \to \pi/2$ $h \to 0$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - \cancel{2}\left(\dfrac{2h + \pi}{\cancel{2}}\right)}$

$\cancel{\pi} - 2h - \cancel{\pi}$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$$

$$\lim_{h \to 0^+} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$$

$$\lim_{h \to 0^+} \cos h \cdot \frac{0 - \sin h}{-2h}$$

$$\lim_{x \to 0^+} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2}$$

b. $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$

$$\lim_{x \to \pi/2} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$$

$$= \lim_{x \to \pi/2} \frac{2 \sin x \cdot \cos x}{\sqrt{2} \sin x}$$

$$= \frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cos x$$

$$= 0$$

LHL $\neq$ RHL

$\therefore$ f is not continuous at $x = \pi/2$

ii. $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x < 6$ $\Big\}$ at $x=3$, $x=6$

$= \frac{x^2-9}{x+9}$ $\quad 6 \leq x < 9$

at $x=3$

$f(3) = \frac{x^2-9}{x-3} = 0$

$\therefore$ f at $x=3$ define

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}$

$f(3) = x + 3 = 3+3 = 6$

f is defined at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

LHL = RHS

$\therefore$ f is continuous at $x=3$.

for $x=6$

$$f(6)=\frac{x^2-9}{x+3}=\frac{36-9}{6+3}=\frac{27}{9}=\frac{3}{1}$$

$$\lim_{x\to 6^+}\frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$\lim_{x\to 6^+}(x-3)=6-3=3$$

$$\lim_{x\to 6^-} x+3=3+6=9$$

LHL ≠ RHS

f is not continuoas at $x=6$

Q. Find k

i. $f(x)=\dfrac{1-\cos 4x}{2}$ $\quad x<0$ $\Big\}$ at $x=0$

$=12$ $\quad x=0$

f is cts at $x=0$

$$\lim_{x\to 0} f(x)=f(0)$$

$$\lim_{x\to 0}\frac{1-\cos 4x}{x^2}=12$$

$$\lim_{x\to 0}\frac{2\sin^2 2x}{x^2}=12$$

$$2\lim_{x\to 0}\left(\frac{\sin^2 2x}{x}\right)^2=k$$

$2(2)^2=k$ $\quad\therefore k=8$

ii. $f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that

$\lim_{x \to 0} (1+px)^{\frac{1}{px}} = e$

$\therefore k = e$

iii.

$f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$

$= 12$

$x = \pi/3 + h$ $\qquad x \to \pi/3$

$x = \pi/3 + h$ $\qquad x \to \pi/3$

$h \to 0$ $\qquad h = x - \pi/3$

$f(\pi/3+h) = \frac{\sqrt{3} - \tan(\pi/3+h)}{\pi - 3(\pi/3+h)}$

$= \frac{\sqrt{3} - \tan(\pi/3+h)}{\pi - 3(\pi/3+h)}$

$= \frac{\sqrt{3} - \frac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$

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$$= \frac{\sqrt{3} - \frac{\tan\pi}{3} \cdot \tan h \cdot \tan\sqrt{3}/3 + \tan h}{\frac{1 - \tan\pi/3 \cdot \tan h}{-3h}}$$

$$= \frac{\sqrt{3} - 3 \cdot \tan h - \sqrt{3} \cdot \tan h}{\frac{1 - \sqrt{3} \cdot \tan h}{-3h}}$$

$$= \frac{\frac{-4 \cdot \tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \frac{-4 \tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{1 - \sqrt{3}\tan x}$$

$$= \frac{4}{3} \frac{1}{1 - \sqrt{3}(0)}$$

$$= \frac{4}{3}$$

Q. $f(x) = \dfrac{1 - \cos 3x}{x \tan x}$

$$= \frac{2\sin^2 3/2\, x}{x + \tan x}$$

Mult x^2 in N and D

$$= \frac{\frac{2\sin^2 3x/2}{x^2} \cdot x^2}{x \cdot \frac{\tan x}{x^2} \cdot x^2}$$

$$= 2 \lim_{x \to 0} \left(\frac{3}{3}\right)^2$$

$$\cancel{2} \times \left(\frac{9}{4}\right) = \frac{9}{2}$$

$$\lim_{x \to 0} f(x) = \frac{9}{2} \quad 9 = f(0)$$

$\therefore$ f is not ctis at $x=0$

Redefine function

$$f(x) = \frac{1-\cos 3x}{x\tan x} \qquad x \neq 0$$

$$9/2 \qquad x = 0$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuous as $x=0$

11. $f(x) = \frac{(e^{3x}-1)\sin x^0}{x^2} \qquad x \neq 0$

$$= \frac{\pi}{60} \qquad x = 0$$

$$= \lim_{x \to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$= 3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is cts at $x = 0$

Q. if $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x \neq 0$

is cts at $x = 0$

$\therefore f$ is cts at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$= \frac{e^{x^2}-1}{x^2} + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

SA

$= \log e + 2\left(\frac{\sin x/2}{x}\right)^2$

Mult with 2 on Num and Den

$= 1 + 2 \times \frac{1}{4}$

$= \frac{3}{2}$

$f(0) = 3/2$

Q. $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

f is cts at $x = \pi/2$

$= \frac{\sqrt{2} - 1 + \sin x}{\cos^2(\sqrt{2} + \sqrt{1+\sin x})} \times 2$

$= \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} - \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \frac{2 - 1 + \sin x}{\cos^2(\sqrt{2} + \sqrt{1-\sin x})}$

$= \frac{1 + \sin x}{1 - \sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{(1-\sin)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2} + \sqrt{2})} = \frac{1}{2 \times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$

AV 02/12/19

Q.1

Q.2

Q.3

Q.4

PRACTICAL - 2

Topic: Derivative

Q.1 Show that the following function defined from IR to IR are differentiable.

i. $\cot x$

ii. $\operatorname{cosec} x$

iii. $\sec x$

Q.2 If $f(x) = 4x+1$, $x \leq 2$

$= x^2+5$ $x > 0$ at $x=2$, then find function is differentiable or not

Q.3 If $f(x) = 4x+7$, $x < 3$

$= x^2+3x+1$, $x \geq 3$ at $x=3$ then find function is differentiable or not

Q.4 If $f(x) = 8x-5$, $x \leq 2$

$= 3x^2-4x+7$, $x > 2$ at $x=2$ then find f is differentiable or not

Solution

Q.1

i. $f(x) = \cot x$

$$f(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$$

$$= \lim_{x\to 0} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$$

put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$

$$f(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\cdot \tan a}$$

$\because$ $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\cdot\tan B}$

$\therefore$ $\tan A - \tan B = \tan(A-B)(1+\tan A\cdot\tan B)$

$$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan a\cdot\tan(a+h))}{h\cdot\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \cdot \frac{1+\tan a\cdot\tan(a+h)}{\tan(a+h)\tan a}$$

$= -1 \cdot \dfrac{1+\tan^2 a}{\tan^2 a}$

$= \dfrac{-\sec^2 a}{\tan^2 a}$

$= \dfrac{-1}{\cos^2 a} \cdot \dfrac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore f(a) = -\cos^2 a$

$\therefore$ function is differentiable $\forall a \in R$

ii. $f(x) = \operatorname{cosec} x$

$f(a) = \lim\limits_{x\to a} \dfrac{f(x) - f(a)}{x-a}$

$= \lim\limits_{x\to a} \dfrac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim\limits_{x\to a} \dfrac{1/\sin x - 1/\sin a}{x-a}$

$= \lim\limits_{x\to a} \dfrac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$

put $x - a = h$

$\therefore x = a+h$

as $x \to a$, $h \to 0$

$f(h) = \lim\limits_{h\to 0} \dfrac{\sin a - \sin(a+h)}{(a+h-a)\cdot \sin a \cdot \sin(a+h)}$

$\because \sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right) \cdot \cancel{to} \sin\left(\dfrac{C-D}{2}\right)$

$= \lim\limits_{h\to 0} \dfrac{2\cos\left(\dfrac{a+h+a}{2}\right) \cdot \sin\left(\dfrac{a-a-h}{2}\right)}{h \cdot \sin a \cdot \sin(a+h)}$

$= \lim\limits_{h\to 0} \dfrac{-\sin h/2}{h/2} \cdot \dfrac{1}{2} \cdot \dfrac{2\cos\left(\dfrac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$

$$= \frac{-1}{2} \cdot \frac{2\cos\left(\frac{2a+0}{2}\right)}{\cancel{2}\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a}$$

$$= -\cot a \cdot \operatorname{cosec} a$$

iii. $f(x) = \sec x$

$$f(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\cos x - 1/\cos a}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cdot \cos x}$$

put $x - a = h$

$\therefore x = a+h$

as $x \to a$, $h \to 0$

$$= \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \cdot \cos a \cdot \cos(a+h)}$$

$\because -2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\frac{(a+a+h)}{2} \sin\left(\frac{a-a-h}{2}\right)}{h \cdot \cos a \cdot \cos(a+h}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \sin\frac{-h}{2}}{\cos a \cdot \cos(a+h) \times \frac{-h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \cdot \frac{-2 \sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \frac{-1}{2} * -2 \cdot \frac{\sin a}{\cos a + \cos a}$$

$$= \tan a \cdot \sec a$$

Q.2

LHD :

$$f(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{x-2}$$

$$= 4$$

RHD :

$$f(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2+2 = 4$$

$\therefore$ RHD = LHD $\therefore$ f is differentiable at $x=2$

Q.3. RHD:

$$f(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3(3) + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (19)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{x - 3}$$

$$= 3 + 6$$

$$= 9$$

LHD

$$f(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4(x-3)}{x - 3}$$

$$= 4$$

$\therefore$ LHD $\neq$ RHD

$\therefore$ f is not differentiable at $x = 3$

Q.4.

RHD :

$$f(2^+) = \lim_{x \to 2^+} \cancel{3x}\,\frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - (3(2)^2 - 4(2) + 7)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - (12 - 8 + 7)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - (11)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x - 2) + 2(x - 2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x + 2)(x - 2)}{x - 2}$$

$$= \lim_{x \to 2^+} 3x + 2$$

$$= 3(2) + 2$$

$$= 6 + 2$$

$$= 8$$

$$\therefore f(2^+) = 8$$

LHD :

$$f(2^-) = \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{x-2}$$

$$= 8$$

$\therefore$ LHD = RHD

$\therefore$ f is differentiable at $x = 2$

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PRACTICAL - 3

Topic : Application of Derivative

Q.1 Find the intervals in which function is increasing or decreasing

i. $f(x) = x^3 - 5x - 11$
ii $f(x) = x^2 - 4x$
iii. $f(x) = 2x^3 + x^2 - 20x + 4$
iv. $f(x) = x^3 - 27x + 5$
v. $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q.2 Find the intervals in which function is concave upwards

i. $y = 3x^2 - 2x^3$
ii $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
iii $y = x^3 - 27x + 5$
iv $y = 69 - 24x - 9x^2 + 2x^3$
v. $y = 2x^3 + x^2 - 20x + 4$

Solution

Q.1

i. $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$\sqrt{5/3}$ $\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

- f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$-\sqrt{5/3}$ $\sqrt{5/3}$

ii. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

iii. $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

f is increasing iff $f'(x) > 0$

$\therefore 6x^2+2x-20 > 0$

$2(3x^2+x-10) > 0$

$3x^2+x-10 > 0$

$3x^2+6x-5x-10 > 0$

$(x+2)(3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore 6x^2+2x-20 < 0$

$2(3x^2+x-10) < 0$

$3x^2+x-10 < 0$

$3x^2+6x-5x-10 < 0$

$(x+2)(3x-5) < 0$

$x \in (-2, 5/3)$

iv. $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2-9) > 0$

$(x-3)(x+3) > 0$

$x \in (-\infty, -3) \cup (3, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore 3x^2-27 < 0$

$\therefore 3(x^2-9) < 0$

$(x-3)(x+3) < 0$

$x \in (-3, 3$

v. $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

f is increasing iff $f'(x) > 0$

$6x^2 - 18x - 24 > 0$

$6(x^2 - 2x - 4) > 0$

$$x^2 - 3x - 4 > 0$$
$$x^2 - 4x + x - 4 > 0$$
$$x(x-4) + 1(x-4) > 0$$
$$(x-4)(x+1) > 0$$
$$x \in (-\infty, -1) \cup (4, \infty)$$

f is decreasing iff $f'(x) < 0$

$$\therefore 6x^2 - 18x - 24 < 0$$
$$6(x^2 - 3x - 4) < 0$$
$$x^2 - 3x - 4 < 0$$
$$(x+1)(x-4) < 0$$
$$x \in (-1, 4)$$

Q.2.

i. $Y = 3x^2 - 2x^3$

$$f(x) = 3x^2 - 2x^3$$
$$f'(x) = 6x - 6x^2$$
$$f''(x) = 6 - 12x$$

f is concave upward if $f''(x) > 0$

$$\therefore (6 - 12x) > 0$$
$$12(6/12 - x) > 0$$
$$x - 1/2 > 0$$
$$x > 1/2$$
$$\therefore f''(x) > 0$$
$$x \in (1/2, \infty)$$

ii. $Y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x^2 - 2x - x + 2 > 0$

$(x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

iii. $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$x \in (0, \infty)$

iv. $f(x) = \cancel{69 - 24x}\ 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$x - 3/2 > 0 \qquad x > 3/2$

$x \in (3/2, \infty)$

Q.A:

v. $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$$12x + 2 > 0$$

$$12(x + 2/12) > 0$$

$$x + 1/6 > 0$$

$$x > -1/6$$

$$f''(x) \not> 0$$

$\therefore$ No interval exists.

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PRACTICAL - 4

Topic: Application of Derivative and Newton's Method

Q1 Find maximum and minimum value of following function

i. $f(x) = x^2 + \frac{16}{x^2}$ ii. $f(x) = 3 - 5x^3 + 3x^5$

iii. $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

iv. $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2. Find root of following equation by Newton's method

i. $f(x) = x^3 - 3x^2 - 55x + 9.5$

ii. $f(x) = x^3 - 4x - 9$, in $[2, 3]$

iii. $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Answers

Q.1

i. $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = \frac{2+96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2+6$

$= 8 > 0$

$\therefore f$ has min. value at $x=2$.

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4+4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2+6$

$= 8 > 0$

$\therefore f$ has min. value at $x = -2$

$\therefore$ function reaches min value at $x = 2, -2$

ii. $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider,

$f'(x) = 0$

$-16x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has min value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + (60)(-1)^3$
$= 30 - 60$
$= -30 < 0$

$\therefore f$ has max value at $x = -1$

$\therefore f(1) = 3 - 5(-1)^3 + (3)(-1)^5$
$= 3 + 5 - 3$
$= 5$

$\therefore f$ has max value 5 at $x = -1, 1$

iii) $f(x) = x^3 - 3x^2 + 1$
$f'(x) = 3x^2 - 6x$
Consider,
$f'(x) = 0$
$3x^2 - 6x = 0$
$3x(x-2) = 0$
$x = 0$ or $x = 2$

$f''(x) = 6x - 6$
$f''(0) = 6(0) - 6$
$= -6 < 0$
$\therefore f$ has max value at $x = 0$
$\therefore f(0) = (0)^3 - 3(0)^2 + 1$
$= 1$
$f''(2) = 6(2) - 6$
$= 6 > 0$

$\therefore f$ has min value at $x = 2$
$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12$
$= -3$
$\therefore f$ has max value at $x = 0, 2$

iv. $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 18 > 0$

$\therefore f$ has min value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

f has max value at $x = -1$

$\therefore f(-1) = (2)(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has max value at $x = -1, 2$

Q2.

i. $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 0 + \frac{9.5}{55}$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0829}{55.9467}$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= -55.9393$

$x_3 = 0.1712 + \frac{0.0011}{55.9393}$

$= 0.1712$

The root is 0.1712

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ii. $f(x) = x^3 - 4(x) - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be initial approximation.

By Newton's Method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{6}{23}$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 18.5096$

$x_2 = 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 17.9851$

$x_3 = 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7065$

iii. $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= -2.2$

$x_0 = 2$

By Newton's Method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$

$= 2 - \frac{2.2}{5.2}$

$= 2 - 0.4230$
$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= -8.2164$

$\therefore x_2 = 1.577 + \frac{0.6755}{8.2164}$

$= 1.6592$

$f(x_2) = (1.659)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.659) - 10$
$= -7.7143$

$x_3 = 1.6592 + \frac{0.204}{7.7143}$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= -7.6977$

$x_4 = 1.6618 + \frac{0.0004}{7.6977}$
$= 1.6618$

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$\therefore$ The root is 1.6618

PRACTICAL-5

Topic: Integration

Q.1 Solve the following Integration

i. $\int \frac{dx}{\sqrt{x^2+2x-3}}$ ii. $\int (4e^{3x}+1)\,dx$ iii. $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ v. $\int t^7 \sin(2t^4)\,dt$

vi. $\int \sqrt{x}(x^2-1)\,dx$ vii. $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii. $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ ix. $\int e^{\cos^2 x} \sin 2x\,dx$

x. $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

Answers

Q.1

i. $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$=$

$= \int \frac{1}{x^2+2x(1)-3}$

$= \int \frac{1}{\sqrt{x^2+2x(1)+(1)^2-(1)^2-3}}$

$= \int \frac{1}{\sqrt{(x+1)^2-(1)^2-3}}$

$= \frac{1}{\sqrt{(x+1)^2-4}}$

$= \int \frac{1}{(x+1)^2-(2)^2}$

$= \log|x+\sqrt{x^2+a^2}|+c$

$= \log|x+1+\sqrt{(x+1)^2-(2)^2}|+c$

$= \log|x+1+\sqrt{x^2+2x+1}-4|+c$

$= \log|x+1+\sqrt{x^2+2x-3}|+c$

2. $\int (4e^{3x}+1)\, dx$

$= \int (4e^{3x}+x^0)\, dx$

$= 4\int e^{3x} dx + \int x^0 dx$

$= \frac{4e^{3x}}{3} + \frac{x^{0+1}}{0+1} + c$

$= \frac{4e^{3x}}{3} + x + c$

3. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2\int x^2 dx - 3\int \sin x \, dx + 5\int x^{1/2} dx$

$= 2\frac{x^3}{3} - 3(\cos x) + \frac{5x^{1/2+1}}{1/2+1} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{5x^{3/2}}{3/2} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x^{3/2}}{3} + c$

$= \frac{2x^3+10x\sqrt{x}}{3} + 3\cos x + c$

4. $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\, dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}}\, dx$

$= \dfrac{x^{5/2+1}}{5/2+1}$

$= \dfrac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5. $\int t^7 \cdot \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \cdot \sin(2t^4) \times \dfrac{1}{2 \cdot 4t^3}\,du$

$= \int t^4 \sin(2t^4) \cdot \dfrac{1}{2 \cdot 4}\,du$

$= \int t^4 \sin(2t^4) \cdot \dfrac{1}{8}\,du = \dfrac{t^4 \cdot \sin(2t^4)}{8}\,du$

Subst. t^4 with $\dfrac{u}{2}$

$= \int \dfrac{u/2 \cdot \sin}{8}\,du$

$= \int \dfrac{\frac{u \cdot \sin(u)}{2}}{8}\,du$

$= \int \dfrac{u \cdot \sin(u)}{16}\,du$

$= \dfrac{1}{16} \int u \cdot \sin(u)\,du$

$\int u\,dv = uv - \int v\,du$

$= \dfrac{1}{16}\left(u \times (-\cos(u)) - \int \cos(u)\,du\right)$

$= \dfrac{1}{16}\left(u \times (-\cos(u)) + \int \cos(u)\,du\right)$

$= \dfrac{1}{16} \times \left(u \times (-\cos(u)) + \sin(u)\right)$

$= \dfrac{1}{16} \times \left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4)\right)$

$= -\dfrac{t^4 \times \cos(2t^4)}{8} + \dfrac{\sin(2t^4)}{16} + C$

$\int \sqrt{x}(x^2 - 1)\,dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \cdot x^2 - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= I_1 = \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7} = \dfrac{2\sqrt{x^7}}{7} = \dfrac{2x^3\sqrt{x}}{7}$

$I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3/2} = \dfrac{2\sqrt{x^3}}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x^3}}{3} + C$

$\int \dfrac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \int \dfrac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin x$ $\quad t = \cos x$

$= \int \dfrac{\cos(x)}{\sin(x)^{3/2}} \cdot \dfrac{1}{\cos(x)}\,dx$

$$= \frac{1}{\sin x^{3/2}} \, dt$$

$$= \frac{1}{t^{2/3}} \, dt$$

$$I = \int \frac{1}{t^{2/3}} \, dt = \frac{-1}{(2/3-1)\, t^{2/3-1}} = \frac{-1}{(2/3-1)\, t^{2/3-1}}$$

$$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

$$= 3\sqrt[3]{\sin x} + c$$

$$\int \frac{x^2-2x}{x^3-3x^2+1} \, dx$$

put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2-2x}{x^3-3x^2+1} \cdot \frac{1}{3x^2 - 3x \cdot 2x} \, dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x} \, dt$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)} \, dt$$

$$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} \, dt$$

$$= \int \frac{1}{3(x^3-3x^2+1)} \cancel{dt} = \int \frac{1}{3t} \, dt$$

$$= 1/3 \int \frac{1}{t} \, \cancel{dt}$$

$$= 1/3 \times \log|t| + c$$

$$= 1/3 \times \log|(x^3-3x^2+1)| + c$$

$$\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$$

$$I = \int x^3 \sin\left(\frac{1}{x^2}\right) dx$$

let $\frac{1}{x^2} = t$

$$x^{-2} = t$$

$$\frac{-2}{x^3} \, dx = dt$$

$$I = \frac{-1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$$

$$= \frac{-1}{2} \int \sin t$$

$$= \frac{-1}{2} (-\cos t) + c$$

$$= \frac{1}{2} \cos t + c$$

$$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$$

$$\int e^{\cos^2 x} \cdot \sin^2 x \, dx$$

$$= \int e^{\cos^2 x} \cdot \sin 2x \, dx$$

$$\cos^2 x = t$$

$$-2\cos x \cdot \sin x \, dx = dt$$

$$-2\sin 2x \, dx = dt$$

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$$I = \int -\sin 2x \, e^{\cos 2x} \, dx$$

$$= -\int e^t dt$$

$$= e^t + c$$

$$\therefore I = -e^{\cos 2x} + c$$

PRACTICAL - 6

Topic: Application of integration and Numerical integration

Q.1 Find the length of the following curve

1. $x = t - \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$
2. $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$
3. $y = x^{3/2}$ in $[0, 4]$
4. $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$
5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2 Using simpson's Rule solve the following

1. $\int_0^2 e^{x} \, dx$ with $n = 4$
2. $\int_0^4 x^2 \, dx$ with $n = 4$
3. $\int_0^{\pi/3} \sqrt{\sin x} \, dx$ with $n = 6$

Answers

Q.1 i. $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

$x = t - \sin t$ $\therefore \frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$ $\therefore \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2} \, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \, dt = \int_0^{2\pi} \sqrt{2(1 - \cos t)} = \int_0^{2\pi} \sqrt{2 \times 2\sin^2 t/2} = \sqrt{4\sin^2 t/2}$

$= \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt$ $\therefore \sin^2 \frac{t}{2} = \frac{1 - \cos t}{2}$

$= \int_0^{2\pi} 2\sin \frac{t}{2} \, dt$

$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8$

ii. $y=\sqrt{4-x^2}$ $\quad x \in [-2,2]$

$$L=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$y=\sqrt{4-x^2}$ $\quad \therefore \frac{dy}{dx} = 2\int_0^2 1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$

$= 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\,dx$

$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

iii. $y = x^{3/2}$ in $[0,4]$

$f'(x) = \frac{3}{2}x^{1/2}$

$[f'(x)]^2 = \frac{9}{4}x$

$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

$= \int_0^4 \sqrt{1+\frac{9}{4}x}\,dx$

~~put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$~~

$= \int_0^4 \sqrt{\frac{4+9x}{4}}\,dx$

$= 1/2 \int_0^4 \sqrt{4+9x}\,dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$= \frac{1}{27}\left[(4+0)^{3/2}-(4+36)^{3/2}\right]$

$= \frac{1}{27}\left[(4)^{3/2}-(40)^{3/2}\right]$

iv. $x = 3\sin t$ $\quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$ $\quad \frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{(3\cos t)^2+(-3\sin)^2\,dt}$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$

$= \int_0^{2\pi} \sqrt{9(\sin^2 t+\cos^2 t)\,dt}$

$= \int_0^{2\pi} \sqrt{9(1)}\,dt$

$= \int_0^{2\pi} 3\,dt \quad = 3\int_0^{2\pi} dt \quad = 3[x]_0^{2\pi}$

$= 3(2\pi - 0)$

$= 6\pi$

5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ $\quad y = [1,2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$$

$$= \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2} dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$$

~~ii~~ i. $\int_0^2 e^{x^2} dx$ with $n=4$

$a=0$, $b=2$, $n=4$

$$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

x	0	0.5	~~2~~ 1	1.5	~~2~~ 2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By simpson's Rule

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(\cancel{y_0+}1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\right]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.6326\right]$$

$$= \cancel{0.1667[}$$

$$= 1.1779$$

ii $\int_0^4 x^2\, dx$

$$L = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= 64/3$$

$$\int_0^4 x^2 dx = 21.533$$

iii. $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ $n = 6$

$L = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$.	$8\pi/$
y	0	0.4166	0.58	0.70	0.80687	0.8727	0.9

$\int_0^{\pi/3} \sqrt{\sin x} = \cancel{1/3}\ \pi/54 \times 12.1163$

$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$

AK
09/01/2020

PRACTICAL - 7

Topic : Differential Equation

Q.1 Solve the following differential Equation

1. $x \frac{dy}{dx} + y = e^x$
2. $e^x \frac{dy}{dx} + 2e^x y = 1$
3. $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$
4. $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$
5. $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$
6. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$
7. $\frac{dy}{dx} = \sin^2(x - y + 1)$
8. $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

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Answers

1. $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$.

$IF = e^{\int 1/x}\, dx$

$= e^{\int 1/x\, dx}$

$IF = x$

$y(IF) = \int Q(x)(IF)\, dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x dx + 1$

$xy = e^x + c$

2. $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$\int P(x)dx$

$IF = e^{\int 2dx} = e^{2x}$

$y(IF) = \int Q(x)(IF)dx + c$

$y \cdot e^{2x} \int e^{-x} + 2x\, dx + c$

$= \int e^x dx + c$

$y \cdot e^{2x} = e^x + c$

3. $x\frac{dy}{dx} = \frac{\cos x}{x} \cdot 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$IF = e^{\int P(x)\, dx}$

$= e^{\int 2/x\, dx}$

$= e^{2/x}\, dx$

$= x^2$

$y(IF) = \int Q(x)(IF)dx + c$

$= \int \frac{\cos x}{x^2} - x^2 dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

4. $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$IF = e^{\int P(x)dx}$

$= e^{\int 3/x\, dx}$

$= e^{3/x}\, dx$

$= x^3$

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$y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + C$

$= \int \sin x \, dx + C$

$x^3 y = -\cos x + C$

5. $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$IF = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int 2xe^{-2x} e^{2x} dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

6. $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$

$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

7. $\frac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

$\therefore 1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = 1 - \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \sin^2(v)$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\therefore \int \sec^2 dv = \int dx$

$\therefore \tan v = x + c$

but $v = x - y + 1$

$\therefore \tan(x - y + 1) = x + c$

8. $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

$\frac{dy}{dx} = \frac{2x + 3y - 1}{3(2x + 3y + 2)}$

put $2x + 3y = v$

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$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$3\frac{dy}{dx} = \frac{dv}{dx} - 2$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = 2 + \frac{v-1}{v+2}$$

$$= \frac{2v+4+v-1}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2}{3v+3}\, dv = dx$$

$$\frac{1}{3}\left(\frac{v+2}{v+1}\right) dv = dx$$

$$\frac{1}{3}\left(\frac{v+1+1}{v+1}\right) dv = dx$$

$$\frac{1}{3}\int\left(1 + \frac{1}{v+1}\right) dv = \int dx$$

$$\frac{1}{3}\left(v + \log(v+1)\right) = x + C$$

$$\frac{1}{3}\left[(2x+3y) + \log(2x+3y+1)\right] = x + C$$

PRACTICAL-8

Topic: Euler's Method

Q. Using Euler's method find the following

1. $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$
2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$
3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$
4. $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$ for $h = 0.5$, $h = 0.25$
5. $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h = 0.2$

Answers

1. $\frac{dy}{dx} = y + e^x - 2 \quad y(0) = 2 \quad h = 0.5$

$f(x) = y + e^x - 2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore Y(2) = 9.8215$

2. $\frac{dy}{dx} = 1 + y^2; \quad y(0) = 0 \quad h(0.2)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$\therefore y(1) = 1.2939$

3. $\frac{dy}{dx} = \sqrt{\frac{x}{y}} \quad y(0) = 1, \quad h = 0.2.$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$\therefore Y(1) = 1.5051$

4. $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$\therefore Y(2) = 7.875$

$h = (0.25)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.996
4	2	299.996		

$\therefore Y(2) = 299.9966$

5. $\frac{dy}{dx} = \sqrt{xy}+2$ $y(1)=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$\therefore Y(1.2) = 3.6$

20/01/2020

PRACTICAL - 9

Topic: Limits and Partial Order derivative

1. Evaluate the following limits

i. $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

ii. $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii. $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

2. Find f_x, f_y for each of the following f

i. $f(x,y) = xye^{x^2+y^2}$ ii. $f(x,y) = e^x \cos y$

iii. $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

3. Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \dfrac{2x}{1+y^2}$

4. Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i. $f(x,y) = \dfrac{y^2-xy}{x^2}$ ii. $f(x,y) = x^3+3x^2y^2-\log(x^2$

iii. $f(x,y) = \sin(xy) + e^{x+y}$

5. Find linearization of $f(x,y)$ at given point

i. $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

ii. $f(x,y) = 1-\cancel{y}+y\sin x$ at $(\pi/2, 0)$

iii. $f(x,y) = \log x + \log y$ at $(1,1)$

Answers

Q1

1. $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1)+5}$

$= \dfrac{-64+3+1-1}{4+5}$

$= \dfrac{-61}{9}$

ii.

At $(2,0)$, Deno $\neq 0$

$\therefore \dfrac{(0+1)((2)^2+0-4(2))}{2+0}$

$= \dfrac{1(4+0-8)}{2}$

$= \dfrac{-4}{2}$

$= -2$

iii. At $(1,1,1)$, Denominator $\neq 0$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2-z^2}{x^3-x^2-yz}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

$$= \frac{1+1(1)}{(1)^2}$$

$$= 2$$

Q.2.

i. $f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$ $\therefore f_y = 2yxe^{x^2+y^2}$

ii. $f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$= e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$= -e^x \sin y$

iii. ~~f(x,y)~~ $f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2-3x^2y+y^3+1)$

$= 3x^2y^2-6xy$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2-3x^2y+y^3+1)$

$= 2x^3y-3x^2+3y^2$

Q.3. $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0} = 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial y}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

Q4.

i. $$f_x = \frac{x^2 \frac{d}{dx}(y^2-xy) - (y^2-xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$$

$$f_y = \frac{2y-x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(-x^2y - 2x(y^2-xy)\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^4}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial y} \frac{-x^2y - 2xy^2 + 2x^2y}{x^4}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{x^2\frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

Q.

$f_x = \frac{\partial}{\partial x}(x^3+3x^2y^2-\log(x^2+1))$

$= 3x^2+6xy^2-\frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y}(x^3+3x^2y^2-\log(x^2+1))$

$= 0+6x^2y-0$

$= 6x^2y$

$f_{xx} = 6x+6y^2-\left(\frac{x^2+\frac{\partial(2x)}{\partial x}-2x\frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2}\right)$

$= 6x+6y^2-\left(\frac{2(x^2+1)-4x^2}{(x^2+1)^2}\right)$

$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2$

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$

$= 0+12xy-0$

$= 12xy$

$f_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$= 12xy$

$\therefore f_{xy} = f_{yx}$

Q.5.

i. $f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{2}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

$f(x)$ at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$

$= \sqrt{2}+\frac{1}{\sqrt{2}}(x-1)+\frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2}+\frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2}+\frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2}+\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}y-\frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii. $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1-\pi/2+0 = 1-\pi/2$

$f_x = 0-1+y\cos x$

f_x at $(\pi/2, 0) = -1+0$

$= -1$

$f_y = 0-0+\sin x$

f_y at $(\pi/2, 0)$

$= \sin \pi/2 = 1$

$L(x,y) = f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$

$= 1-\pi/2+(-1)(x-\pi/2)+1(y-0)$

$= 1-\pi/2-x+\pi/2+y$

$= 1-x+y$

3. $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\qquad$ f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

PRACTICAL-10

Topic- Directional Derivative, Gradient vector & maxima, minima tangent and normal vector

Q.1 Find the directional derivative of the following function at given points & in the direction of given vector

i. $f(x,y) = x + 2y - 3$ $\quad a = (1,-1),\ u = 3i - j$

ii. $f(x,y) = y^2 - 4x + 1$ $\quad a = (3,4),\ u = i + 5j$

iii. $f(x,y) = 2x + 3y$ $\quad a = (1,2),\ u = 3i + 4j$

Q.2 Find gradient vector for the following function ~~of~~ at given point

i. $f(x,y) = x^y + y^x$ $\quad a = (1,1)$

ii. $f(x,y) = (\tan^{-1}x) \cdot y^2$ $\quad a = (1,-1)$

iii. $f(x,y) = xyz - e^{x+y+z}$ $\quad a = (1,-1,0)$

Q.3 Find the equation of tangent & normal to each of the following using curves at given points

1. $x^2\cos y + e^{xy} = 2$ at $(1,0)$
2. $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

Q.4 Find the equation of tangent & normal line to each of the following surfaces

1. $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$ $\qquad$ 2. $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q.5. Find the local maxima & minima for the following ~~four~~ functions

i. $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

ii. $f(x,y) = 2x^4 + 3x^2y - y^2$

iii. $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

Answers

i. Here $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector $= \frac{1}{\sqrt{10}}(3, -1) = \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(0) = f(1,-1) = 1 + 2(-1) - 3) = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$\cancel{= f\, 2\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)}$

$= \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$= -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{4 + h/\sqrt{10} + 4}{h}$

$= \frac{1}{\sqrt{10}}$

ii. Here $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{1^2 + 5^2} = \sqrt{26}$

Unit vector $= \frac{1}{\sqrt{26}}(1, 5) = \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = 4^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$

$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii. Here $u = 3i + 4j$ is not a unit vector

$|u| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

Unit vector $= \frac{1}{5}(3,4) = \frac{3}{5}, \frac{4}{5}$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= 18h + 8$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$

$= \frac{18}{5}$

Q.2.

i. $\cancel{f_x = y \cdot x^{y-1} + y^x \log y}$

$\cancel{f_y = x^y \log x + xy^{x-1}}$

$\nabla f(x,y) = f_x, f_y$

$= (yx^{y-1} + y^x \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1, 1)$

ii. $f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = f_x, f_y$

$= \frac{y^2}{1+x^2}, 2y \tan^{-1} x$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1(-2))\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

iii. $f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)0 - e^{(1+(-1)+0)}, (1)(0) - e^{(1+(-1)+0)}, (1)(-1) - e^{(1+(-1)+0)})$

$= \cancel{0 - e^0}, 0 - e^0, -1, -e^0$

$= (-1, -1, -2)$

Q.3

i. $f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0) = \quad \therefore x_0 = 1, y_0 = 0$

eqn of tangent

$f_x(x-x_0)+f_y(y-y_0)=0$

$f_x(x_0,y_0) = \cos 0 \; 2(1) + e^{\circ} 0$

$= 1(2)+0$

$= 2$

$f_y(x_0,y_0) = 1^2(\sin 0) + e^{\circ}.1$

$= 0+1.1$

$= 1$

$2(x-1)+1(y-0)=0$

$2x-2+y=0$

$2x+y-2=0$

eqn of Normal $= ax+by+c=0$

$= bx+ay+d=0$

$1(1)+2y+d=0$

$1+2y+d=0$ at $(1,0)$

$= 1+2(0)+d=0$

$d+1=0$

$d=-1$

Q. $f_x = 2x+0-2+0+0.$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0,y_0)=(2,-2)$ $\therefore x_0=2, y_0=-2$

$f_x(x_0,y_0) = 2(2)-2=2$

$f_y(x_0,y_0) = 2(-2)+3=-1$

eqn of tangent

$f_x(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+1(-1(y+2))=0$

$2x-2-y-2=0$

$2x-y-4=0$

eqn of Normal $= -1(x)+2y+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore d=6$

Q.4.

i. $f_x = 2x-0+0+z$

$= 2x+z$

$f_y = 0-2z+3+0$

$= 2z+3$

$f_z = -2y+x$

$f_x(x_0,y_0,z_0) = 2(2)+0=4$

$f_y(x_0,y_0,z_0) = 2(0)+3=3$

$f_z(x_0,y_0,z_0) = -2(1)+2=0$

eqn of tangent $= 4(x-2)+3(y-1)+0(z-0)=2$

$= 4x-8+3y-3=0$

$4x+3y-11=0$

eqn of normal $= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$

ii. $f_x = 3yz - 1$
$f_y = 3xz - 1$
$f_z = 3xy + 1$
$(x_0, y_0, z_0) = (1, -1, 2)$
$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$
$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$
$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqn. of tangent
$-7(x-1) + 5(y+1) - 2(z-2) = 0$
$-7x + 7 + 5y + 5 - 2z + 4 = 0$
$-7x + 5y - 2z + 16 = 0$

Eqn of normal
$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$

Q.5

i. $f_x = 6x - 3y + 6$
$f_y = 2y - 3x - 4$
$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$

$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Substitute
$2(0) - y = -2$
$-y = -2 \quad y = 2$
∴ Critical points are $(0, 2)$

Here $r > 0$
$= rt - s^2$
$= 6(2) - (3)^2$
$= 12 - 9$
$= 3 > 0$

f is maximum at $(0,2)$
$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$0 + 4 - 0 + 0 - 8 = -4$

ii. $f(x,y) = 2x^4 + 3x^2y - y^2$
$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$
$f_x = 0$
$8x^3 + 6xy = 0$
$2x(4x^2 + 3y) = 0$
$4x^2 + 3y = 0$

$f_y = 0$

$3x^2 - 2y = 0$

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$11y = 0$

$\therefore y = 0$

Substitute

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - s^2$

$= 0 - 0$

$= 0$

$r = 0$ & $rt - s^2 = 0$

$\therefore$ nothing to say.

iii. $f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$2x + 2 = 0$

$x = -1$

$f_y = 0$

$-2y + 8 = 0$

$y = \frac{-8}{-2}$

$\therefore y = 4$

critical point is $(-1, 4)$

$r = 2$

$t = -2$

$s = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - 4^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$

27/01/2020