Math 120 - Problem Set - Chapter 1

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1.The midpoint of a line segment is the point, C, halfway between endpoints A and B on line segment $\overline{AB}$ s.t. the length of $\overline{AC}$ is equivalent to the length of $\overline{CB}$.

2.
a. If $t > 3$, then $t^2 > 9$.
b. If x is positive, then $x + 1$ is also positive.

3.

A	B	A $\rightarrow B$	B $\rightarrow A$	$\neg A \rightarrow \neg B$	$\neg B \rightarrow \neg A$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

4. Let a, b, d, x, and y be integers. If d|a and d|b, then d|(ax+by). If b is divisible by d, and a is divisible by d by definition this means that there exists h, $i \in \mathbb{Z}$ s.t. $dh = b$ and $di = a$. Thus, for any $x, y \in \mathbb{Z}$ we have $ax+by = (dh)x+(di)y = d(hx+iy)$ which can be simplified to d|(ax + by).

5. Suppose we have consecutive integers x and $x + 1$. Adding x and x+1 together yields $x + (x + 1)$ which simplifies to $2x$+1 which, by definition, is odd.

6.
a. If x and y are distinct, non-consecutive perfect squares, then $y - x$ is composite. Since x and y are perfect squares, by definition there exists a, $b \in \mathbb{Z}$ s.t. $a^2 = x$ and $b^2 = y$. $y - x = b^2 - a^2 = (b-a)(b+a) \in \mathbb{Z}$. Since x and y are non-consecutive then $(b - a) \geq 2$. This can be simplified to $(b-a) \geq 2$ which $= b \geq a+2 = b+a \geq 2a+2 \geq 4$, therefore $b+a \geq 2$. Since $y-x$ can be written as two distinct factors, this by definition is composite.

b. Let $p = 2$ and $q = 3$. Adding these two integers together $= p + q$ which can be simplified to 5, which by definition is prime.

c. If $n = 41$, then $n^2 + n + 41 = 1763$, which by definition is not prime.

7.
a.

x	y	z	x $\leftrightarrow y$	y $\leftrightarrow z$	z $\leftrightarrow x$	x $\leftrightarrow y \wedge y \leftrightarrow z \wedge z \leftrightarrow x$
T	T	T	T	T	T	T
T	F	T	F	F	T	F
T	T	F	T	F	F	F
T	F	F	F	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	F	F
F	T	F	F	F	T	F
F	F	F	T	T	T	T

x	y	z	x$\rightarrow y$	y $\rightarrow z$	z $\rightarrow x$	(x $\rightarrow y) \wedge (y \rightarrow z \wedge (z \rightarrow x)$
T	T	T	T	T	T	T
T	F	T	F	T	T	F
T	T	F	T	F	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	F
F	F	T	T	T	F	F
F	T	F	T	F	T	T
F	F	F	T	T	T	T

b.

x	y	$\neg x$	$\neg y$	$\neg x \vee \neg y$	x $\wedge y$	$(\neg x \vee \neg y) \rightarrow (x \wedge y)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

x	y	x $\wedge y$
T	T	T
T	F	F
F	T	F
F	F	F