

# Quadratic Equations

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- **Identification of quadratic equations**

**Example:** Check whether the following are quadratic equations or not.

(i)  $(2x + 3)^2 = 12x + 3$

(ii)  $x(x + 3) = (x + 1)(x - 5)$

**Solution:**

(i)  $(2x + 3)^2 = 12x + 3$

$$\Rightarrow 4x^2 + 12x + 9 = 12x + 3$$

$$\Rightarrow 4x^2 + 6 = 0$$

It is of the form  $ax^2 + bx + c = 0$ , where  $a = 4$ ,  $b = 0$  and  $c = 6$

Therefore, the given equation is a quadratic equation

(ii)  $x(x + 3) = (x + 1)(x - 5)$

$$\Rightarrow x^2 + 3x = x^2 + x - 5x - 5$$

$$\Rightarrow 7x + 5 = 0$$

It is not of the form  $ax^2 + bx + c = 0$ , since the maximum power (or degree) of equation is 1.

Therefore, the given equation is not a quadratic equation.

- **Express given situation mathematically**

**Example 1:**

An express train takes 2 hour less than a passenger train to travel a distance of 240 km. If the average speed of the express train is 20 km/h more than that of a passenger train, then form a quadratic equation to find the average speed of the express train?

**Solution:**

Let the average speed of the express train be  $x$  km/h.

Since it is given that the speed of the express train is 20 km/h more than that of a passenger train,

Therefore, the speed of the passenger train will be  $x - 20$  km/h.

Also we know that  $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$$\text{Time taken by the express train to cover 240 km} = \frac{240}{x}$$

$$\text{Time taken by the passenger train to cover 240 km} = \frac{240}{x - 20}$$

And the express train takes 2 hour less than the passenger train. Therefore,

$$\frac{240}{x-20} - \frac{240}{x} = 2$$

$$\Rightarrow 240 \left[ \frac{x - (x-20)}{x(x-20)} \right] = 2$$

$$\Rightarrow 120 \left( \frac{20}{x^2 - 20x} \right) = 1$$

$$\Rightarrow 2400 = x^2 - 20x$$

$$\Rightarrow x^2 - 20x - 2400 = 0$$

This is the required quadratic equation.

- **Solution of Quadratic Equation by Factorization Method**

If we can factorize  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , into a product of two linear factors, then the roots of this quadratic equation can be calculated by equating each factor to zero.

**Example:**

Find the roots of the equation,  $2x^2 - 7\sqrt{3}x + 15 = 0$ , by factorisation.

**Solution:**

$$2x^2 - 7\sqrt{3}x + 15 = 0$$

$$\Rightarrow 2x^2 - 2\sqrt{3}x - 5\sqrt{3}x + 15 = 0$$

$$\Rightarrow 2x(x - \sqrt{3}) - 5\sqrt{3}(x - \sqrt{3}) = 0$$

$$\Rightarrow (x - \sqrt{3})(2x - 5\sqrt{3}) = 0$$

$$\Rightarrow (x - \sqrt{3}) = 0 \text{ or } (2x - 5\sqrt{3}) = 0$$

$$\Rightarrow x = \sqrt{3} \text{ or } x = \frac{5\sqrt{3}}{2}$$

Therefore,  $\sqrt{3}$  and  $\frac{5\sqrt{3}}{2}$  are the roots of the given quadratic equation.

- **Solution of Quadratic Equation by completing the square**

A quadratic equation can also be solved by the method of completing the square.

**Example:**

Find the roots of the quadratic equation,  $5x^2 + 7x - 6 = 0$ , by the method of completing the square.

**Solution:**

$$\begin{aligned}
5x^2 + 7x - 6 &= 0 \\
\Rightarrow 5 \left[ x^2 + \frac{7}{5}x - \frac{6}{5} \right] &= 0 \\
\Rightarrow x^2 + 2 \times x \times \frac{7}{10} + \left( \frac{7}{10} \right)^2 - \left( \frac{7}{10} \right)^2 - \frac{6}{5} &= 0 \\
\Rightarrow \left( x + \frac{7}{10} \right)^2 - \frac{49}{100} - \frac{6}{5} &= 0 \\
\Rightarrow \left( x + \frac{7}{10} \right)^2 &= \frac{169}{100} \\
\Rightarrow \left( x + \frac{7}{10} \right) &= \pm \sqrt{\frac{169}{100}} = \pm \frac{13}{10} \\
\Rightarrow x + \frac{7}{10} = \frac{13}{10} \text{ or } x + \frac{7}{10} &= -\frac{13}{10} \\
\Rightarrow x = \frac{13}{10} - \frac{7}{10} = \frac{3}{5} \text{ or } x = -\frac{13}{10} - \frac{7}{10} &= -2
\end{aligned}$$

Therefore,  $-2$  and  $\frac{3}{5}$  are the roots of the given quadratic equation.

• **Quadratic Formula to find solution of quadratic equation:**

The roots of the quadratic equation,  $ax^2 + bx + c = 0$ , are given

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } b^2 - 4ac \geq 0$$

by,

**Example:**

Find the roots of the equation,  $2x^2 - 3x - 44 = 0$ , if they exist, using the quadratic formula.

**Solution:**

$$2x^2 - 3x - 44 = 0$$

Here,  $a = 2$ ,  $b = -3$ ,  $c = -44$

$$\therefore b^2 - 4ac = (-3)^2 - 4 \times 2 \times (-44) = 9 + 352 = 361 > 0$$

The roots of the given equation are given by  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

$$\begin{aligned}
\Rightarrow x &= \frac{-(-3) \pm \sqrt{361}}{2 \times 2} = \frac{3 \pm 19}{4} \\
\Rightarrow x &= \frac{3 + 19}{4} = \frac{11}{2} \text{ or } x = \frac{3 - 19}{4} = -4
\end{aligned}$$

The roots are  $-4$  and  $\frac{11}{2}$ .

- **Nature of roots of Quadratic Equation**

For the quadratic equation,  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , the discriminant 'D' is defined as  $D = b^2 - 4ac$

The quadratic equation,  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , has

- 1.
- 1.
1. two distinct real roots, if  $D = b^2 - 4ac > 0$
2. two equal real roots, if  $D = b^2 - 4ac = 0$
3. has no real roots, if  $D = b^2 - 4ac < 0$

**Example:** Determine the nature of the roots of the following equations

(a)  $2x^2 + 5x - 117 = 0$

(b)  $3x^2 + 5x + 6 = 0$

**Solution:**

(a) Here,  $a = 2$ ,  $b = 5$ ,  $c = -117$

$$\therefore D = b^2 - 4ac = 5^2 - 4 \times 2 \times (-117) = 25 + 936 = 961 > 0$$

Therefore, the roots of the given equation are real and distinct.

(b) Here,  $a = 3$ ,  $b = 5$ ,  $c = 6$

$$\therefore D = b^2 - 4ac = 5^2 - 4 \times 3 \times 6 = 25 - 72 = -47 < 0$$

Therefore, the roots of the given equation are not real.