Algebraic Expressions and Identities

- The multiplication of a monomial by a monomial gives a monomial. While performing multiplication, the coefficients of the two monomials are multiplied and the powers of different variables in the two monomials are multiplied by using the rules of exponents and powers.

  $(-2ab^2c) \times (3abc^2) = (-2 \times 3) \times (a \times a \times b^2 \times b \times c \times c^2) = -6a^2b^3c^3$

  The multiplication of three or more monomials is also performed similarly.

  $(xy) \times (3yz) \times (3x^2z^2)$

  $= (3 \times 3) \times (x \times x^2) \times (y \times y) \times (z \times z^2)$

  $= 9x^3y^2z^3$

- There are two ways of arrangement of multiplication while multiplying a monomial by a binomial or trinomial or polynomial. These are horizontal arrangement and vertical arrangement.

  Multiplication in **horizontal arrangement** can be performed as follows:

  Here, we arrange monomial and polynomial both horizontally and multiply every term in the polynomial by the monomial by making use of distributive law.

  $5a \times (2b + a - 3b + c)$

  $= (5a \times 2b) + (5a \times a) + (5a \times (-3b)) + (5a \times c)$

  $= 10ab + 5a^2 - 15ab + 5ac$

  $= 5a^2 - 5ab + 5ac$

  Multiplication in **vertical arrangement** can be performed as follows:

  $$\begin{array}{r} 4x^2 + 2x \\ \underline{\times \quad\quad 3x} \\ \underline{12x^3 + 6x} \end{array}$$

  Here, we have first multiplied $3x$ with $2x$ and wrote the product with sign at the bottom. After doing this, we have multiplied $3x$ with $4x^2$ and wrote the product with sign at the bottom.
  Similarly, we can multiply a trinomial with monomial as follows:

$$
\begin{array}{r}
2y^3 - 5y + 1 \\
\times \qquad\qquad 2y \\
\hline
4y^4 - 10y^2 + 2y \\
\hline
\end{array}
$$

- While multiplying a polynomial by a binomial (or trinomial) in horizontal arrangement, we multiply it term by term. That is, every term of the polynomial is multiplied by every term of the binomial (or trinomial).

  Example: Simplify $(x + 2y)(x + 3) - (2x + 1)(y + x + 1)$.

  Solution: $(x + 2y)(x + 3) = x(x + 3) + 2y(x + 3)$

  $= x^2 + 3x + 2xy + 6y$

  $(2x + 1)(y + x + 1) = 2x(y + x + 1) + 1(y + x + 1)$

  $= 2xy + 2x^2 + 2x + y + x + 1$

  $= 2xy + 2x^2 + 3x + y + 1$

  $\therefore (x + 2y)(x + 3) - (2x + 1)(y + x + 1) = x^2 + 3x + 2xy + 6y - 2xy - 2x^2 - 3x - y - 1$

  $= - x^2 + 5y - 1$

- We can also perform multiplication of two polynomials using vertical arrangement.

  For example,

$$
\begin{array}{l}
\qquad\qquad l + 6m + 7n \\
\times \qquad\qquad\quad l + 3m \\
\hline
\qquad 3lm + 18m^2 + 21mn \\
+l^2 + 6lm + 7nl \\
\hline
l^2 + 9lm + 18m^2 + 21mn + 7nl \\
\hline
\end{array}
$$

- An identity is an equality which is true for all values of the variables in it. It helps us in shortening our calculations.

- Identities for "Square of Sum or Difference of Two Terms" are:

  - $(a + b)^2 = a^2 + 2ab + b^2$
  - $(a - b)^2 = a^2 - 2ab + b^2$

  Example: Evaluate $(5x + 2y)^2 - (3x - y)^2$.

Solution: Using identities (i) and (ii), we obtain
$(5x + 2y)^2 = (5x)^2 + 2\,(5x)\,(2y) + (2y)^2$
$= 25x^2 + 20xy + 4y^2$
$(3x - y)^2 = (3x)^2 - 2\,(3x)\,(y) + (y)^2$
$= 9x^2 - 6xy + y^2$
$\therefore (5x + 2y)^2 - (3x - y)^2 = 25x^2 + 20xy + 4y^2 - 9x^2 + 6xy - y^2 = 16x^2 + 26xy + 3y^2$
$(x + a)\,(x + b) = x^2 + (a + b)\,x + ab$

Example: Find 206 × 198.

Solution: We have,

$206 \times 198 = (200 + 6)\,(200 - 2)$

$= (200)^2 + (6 + (-2)) \times 200 + (6)\,(-2)$ [Using identity $(x + a)\,(x + b) = x^2 + (a + b)\,x + ab$]

$= 40000 + 800 - 12$

$= 40800 - 12$

$= 40788$

- $(a + b)\,(a - b) = a^2 - b^2$

Example: Evaluate 95 × 105.

Solution: We have, $95 \times 105 = (100 - 5) \times (100 + 5)$

$= (100)^2 - (5)^2$ [Using identity $(a + b)\,(a - b) = a^2 - b^2$]

$= 10000 - 25$

$= 9975$