INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

Independent Study Final Report

Poker Theory and Analytics

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Contents

1 Introduction

Poker refers to a collection of card games in which players compare ranked **hands** in competition for a **pot** of money that typically grows through betting as the hand proceeds. The size of the pot increases through **betting**, where the players bet on the hand that they have or could potentially get. All serious versions of poker have the following property in common: while it is quite easy to learn the basic rules of the game, it is extremely difficult to develop a strategy that might be described as optimal. The game involves a combination of strategy, skill, and luck, and requires players to make bets based on the strength of their hand and their ability to read their opponents. The game typically involves a standard deck of 52 cards and can be played with as few as two players or as many as ten or more players. Each player is dealt a certain number of cards, depending on the specific variation of poker being played, and the goal of the game is to win the pot, which is the total amount of money or chips that have been wagered by all players. Poker is a game of **incomplete information**, meaning that players do not have access to all of the cards in play or the other players' hands [6]. This makes it challenging and exciting, as players must use their knowledge of probabilities and their ability to read their opponents to make informed decisions and maximize their chances of winning. Poker is a game of incomplete information, meaning that players do not have access to all of the cards in play or the other players' hands. This makes it challenging and exciting, as players must use their knowledge of probabilities and their ability to read their opponents to make informed decisions and maximize their chances of winning.

The main aspect we are interested in is deciding the criteria for "winning" in poker. To win at the game, it is pivotal to decide when to bet and when to not bet to minimize risk and maximize profits. As a result, this ensures a **return on investment** and such a strategy should be employed. In this report, our goal will be to understand the theory as well as some of the ways in which mathematics can be applied to the analysis of poker. Moreover, we also implement important *hand assessment* functions which have been mentioned in this paper.

1.1 No Limit Texas Hold'em

Poker is a generic name for hundreds of game variations, but we will be focusing on a variant of Texas Hold'em.

- **Texas Hold'em:**
  The game starts off with each **player** being dealt 2 cards called **hole cards** face down by **the dealer**. After that, 3 community cards are then dealt face up which is called the **flop**. The fourth and fifth cards are dealt separately in later betting rounds and are called the **turn**, and **river** respectively. Bets are placed after each step as players call, raise or fold after checking their cards. Overall, the aim is to make the best five-card poker hand out of the seven cards available.

There are mainly 2 variations of Hold'em Poker, out of which No Limit is the more popular variation:

- **Pot Limit Texas Hold'em**:
  In this variant, the betting range for each player is capped by the size of the pot. Therefore it's not possible for a player to go all-in if they have more chips than the size of the pot. As a result, the concept of overbetting is not possible in this variant. Ultimately, pre-flop action is much less aggressive as compared to No-limit Texas Hold'em merely because of the betting cap.

- **No Limit Texas Hold'em**:
  As opposed to Pot Limit Texas Hold'em, the betting range for each player is uncapped. Therefore, this variant allows players to go all in whenever they see fit. Overall, No Limit Texas Hold'em is more risk-prone, making it more fun to view and play the game [6].

For our report, we will be focusing on **No Limit Texas Hold'em**.

1.2 Position Terminology

In this section, we will be explaining the terminology of the positions in No Limit Texas Hold'em.

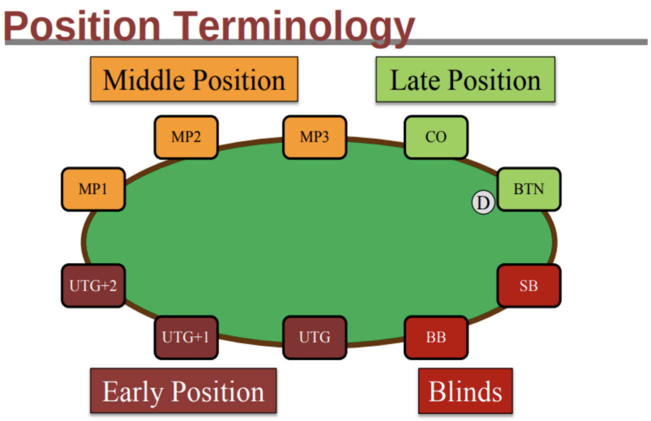


Figure 1: Position Terminology

As seen in Figure 1, we can see that there are multiple positions that are of different value [3]. Therefore, the chances of a player earning money are greatly affected by where they are seated at the table.

- **The Button**:
  The play moves clockwise around the table, starting with action to the left of the dealer button. The first two players sitting to the immediate left of the button are required to post a **'small blind'** and a **'big blind'** to initiate the betting. The hand is played starting from the small blind position and ending at **the button** position making this position highly advantageous since they get to play their hand after every other player has made their decision.

- **Other Terminology**:
  - **UTG**: Under the Gun - position right after the big blind.
  - **MP**: Middle Position - no particular advantage at this position.
  - **CO**: Cut Off - player that plays just before the button, another highly advantageous position but not as much as the button.

1.3 Blinds

In figure 1, **SB** and **BB** refer to the Blinds position at the poker table. In the game of poker, the blinds are mandatory bets that two players sitting to the left of the dealer must place before any cards are dealt. The bets are called “blinds” because they are made “blind”, that is, before you know what your cards are. The player sitting immediately to the left of the dealer must post the *small blind*, while the player sitting to their left must post the *big blind*. An **x/y poker game** has two blinds x and y, where x is **small blind** and y is **big blind**.

The purpose of the blinds is to create a pot for players to compete for, even when no one has voluntarily put any money into the pot yet [3]. In other words, the blinds introduce a regular cost to take part in the game, thus inducing a player to enter the pot in an attempt to compensate for that expense. Blind strategy is an essential part of poker, as players must decide when to defend their blinds, steal the blinds from other players, or adjust their betting based on their position relative to the blinds.

1.4 Streets of No Limit Texas Hold’em Poker:

There are 4 **Streets of Texas Hold’em**: Preflop, Flop, Turn, and River. Streets in poker refer to betting rounds that subsequently occur until the players must show their cards to decide the winner [3].

1. **First Betting Round: Preflop -**
   In the first betting round (**preflop**), the blinds are posted and each player is dealt two hole cards. The first player to act is the player to the left of the big blind (under the gun). The UTG has 3 options:

   - **Call**: Match the amount of the big blind.
   - **Raise**: Increase the bet within the specific limits of the game.
   - **Fold**: Throw the hand away.

   If the player chooses to fold, he or she is out of the game and no longer eligible to win the current hand. Once the last bet is called and the action is ’closed,’ the preflop round is over and play moves on to the "flop."

2. **Second Betting Round: Flop -**
   After the first preflop betting round has been completed, the **first three community cards are dealt** and a second betting round (**flop**) follows which involves only the players who have not folded already. From here onwards, the action starts with the player to the left of the button (BTN), which is the SB (small blind). Along with the options to bet, call, fold, or raise, a player now has the option to **check** if no betting action has occurred beforehand.**A check** simply means to pass the action to the next player in the hand.

3. **Third Betting Round: Turn -**
   The fourth community card, called **the turn**, is dealt face-up following all betting action on the flop. Once this has been completed, another round of betting occurs, similar to that on the previous street of play.

4. **Final Betting Round: River -**
   The fifth community card, called **the river**, is dealt face-up following all betting action on the turn and another

round of betting occurs. After all betting action has been completed, the remaining players in the hand with hole cards now expose their holdings to determine a winner. This is called **the showdown**.

At **showdown**, the remaining players show their hole cards, and with the assistance of the dealer, a winning hand is determined. This is done by choosing the player with the best combination of five cards. After that, the winner is then rewarded with all the money from the pot.

1.5 Hand Hierarchy in Texas Hold'em

The hand hierarchy [6] that is used during the showdown is based on the probability of the hand occurring in the game:

1. **Royal Flush** (Ace High Straight Flush): Five cards of the same suit, ranked ace through ten. Eg. A♥ K♥ Q♥ J♥ 10♥
2. **Straight Flush**: Five cards of the same suit and consecutively ranked. Eg. 9♠ 8♠ 7♠ 6♠ 5♠
3. **Four of a Kind**: Four cards of the same rank. Eg. 5♥ 5♠ 5♣ 5♦ 10♥
4. **Full House**: Three cards of the same rank and two more cards of the same rank. Eg. 3♥ 3♦ 3♣ J♥ J♦
5. **Flush**: Any five cards of the same suit. Eg. A♥ 8♥ 6♥ 4♥ 2♥
6. **Straight***: Any five cards consecutively ranked. Eg. A♥ 2♣ 3♥ 4♠ 5♦
7. **Three of a Kind**: Three cards of the same rank. Eg. K♦ K♥ K♦ J♥ 10♥
8. **Two Pair**: Two cards of the same rank and two more cards of the same rank. Eg. J♥ J♦ 3♥ 3♠ 10♥
9. **One Pair**: Two cards of the same rank. Eg. A♥ A♦ Q♥ J♥ 10♣
10. **High Card**: Five unmatched cards. Eg. J♥ 9♦ 6♣ 4♥ 3♥

Hand	Probability	Odds Against	Frequency
Royal Flush	0.000154%	649,739 : 1	$\binom{4}{1} = 4$
Straight Flush	0.00139%	72,192.33 : 1	$\binom{10}{1}\binom{4}{1} - \binom{4}{1} = 36$
Four of a Kind	0.02401%	4,165 : 1	$\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1} = 624$
Full House	0.1441%	693.1667 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3744$
Flush	0.1965%	508.8019 : 1	$\binom{13}{5}\binom{4}{1} - \binom{10}{1}\binom{4}{1} = 5108$
Straight	0.3925%	253.8 : 1	$\binom{10}{1}\binom{4}{1}^2 - \binom{10}{1}\binom{4}{1} = 10200$
Three of a Kind	2.1128%	46.32955 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2 = 54912$
Two Pair	4.7539%	20.03535 : 1	$\binom{13}{1}\binom{4}{2}^2\binom{11}{1}\binom{4}{1} = 123552$
One Pair	42.2569%	1.366477 : 1	$\binom{13}{1}\binom{4}{2}^2\binom{12}{3}\binom{4}{1}^3 = 123552$
High Card	50.1177%	0.9953015 : 1	$[\binom{13}{5} - \binom{10}{1}][\binom{4}{1}^5 - \binom{4}{1}] = 1302540$

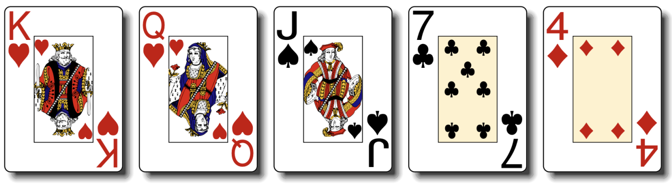
Figure 2: Hand Hierarchy With Probabilities

Figure 2 displays the probabilities of getting a certain hand. These are the number of ways of making 5 card hands (assigned to a particular hand) from a deck of 52 cards (this is equal to $\binom{52}{5}$). In Figure 2, frequency refers to the number of 5 card hands that match a hand from the hierarchy. After determining frequency, the probability is calculated by dividing the frequency of the hand by the total number of combinations. For example, take the hand *Royal Flush*. Its frequency is 4 since there are only 4 possibilities: A♥ K♥ Q♥ J♥ 10♥, A♣ K♣ Q♣ J♣ 10♣, A♠ K♠ Q♠ J♠ 10♠, and A♦ K♦ Q♦ J♦ 10♦. Therefore, the probability of getting a *Royal Flush* is: $\frac{\binom{4}{1}}{\binom{52}{5}} = 0.000154\%$.

1.6 Constructing the Best Possible Hand

Players construct their hands by choosing the five best cards from the seven available (their two hole cards and the five community cards). When comparing two hands, the winner is chosen using the **First Point of Difference** which follows the rank order of the cards.

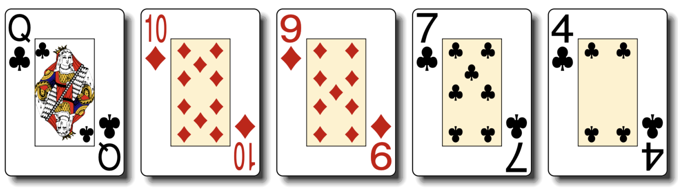
For example, let the 5 community cards for a given hand be as follows:



Let's say **The Hero** (the player that we are concerned about) is dealt the cards K♣ 8♥, and **the Villain** (the player the hero is going against in the showdown) is dealt K♦ 5♥. The best possible hand that the Hero can construct will be K♥ K♣ Q♥ J♠ 8♥. Firstly, the Hero identifies a hand (in this case it's One Pair), followed by choosing the remaining cards in their hand according to the cards' ranks. Therefore, the two Kings are selected and from the remaining 5 cards (Q, J, 8, 7, 4) the top 3 are selected.

Similarly, the best possible hand that the Villain can construct will be K♥ K♦ Q♥ J♠ 7♣. We can see that both the hands have a King Pair, and therefore we can see that the first point of difference between the two hands from the remaining cards occurs at the 5th card. Since the rank of the Hero's last card is higher than that of the Villain's (8♥ > 7♣), therefore in this case, the Hero would win the showdown. Alternatively, if the Hero had K♣ 6♥, the Hero and the Villain both will have the same hand K♥ K♦ Q♥ J♠ 7♣. Hence, there is no point of difference between the hands here and the pot would be split as the showdown has resulted in a draw.

Let's consider another example. Let the 5 community cards for a given hand be as follows:



Let's say **The Hero** is dealt the cards A♣ 2♣ , and **the Villain** is dealt K♣ 8♣ . The best possible hand that the Hero can construct will be A♣ Q♣ 7♣ 4♣ 2♣ , whereas the Villain's will be K♣ Q♣ 8♣ 7♣ 4♣ . Here, the Hero's hand is called a **Ace-High Flush** or **Nut Flush** since it's the best possible flush with an Ace as the high card. The Villain's hand is a King-High Flush which is the second best possible flush. Therefore, in this case the Hero wins, because the first point of difference is found at the first position (A♣ > K♣).

2 Expected Value Calculation Criteria For Betting and Calling

The main goal of developing all tools for poker theory is to answer the following question: what is the play the Hero can make to ensure positive expectation? To quantify this, we define **Expected Value**. EV is considered as the most important and fundamental concept in poker. It is the average result of a given play if it were played hundreds or even thousands of times. It is deduced by finding the probability-weighted average of possible results. We will refer to it as the **expected value of winning**(denoted by $E[W_c]$). In general, we seek to make **Positive EV plays** to ensure profits over a long period of playtime. Before jumping into expected value directly, let's first take a look at **winning percentage** and **number of outs**, which are terms involved in its calculation.

2.1 Winning Percentage

Winning Percentage is defined as the **equity** of a hand in Poker. It denotes the probability of the hand winning in a particular scenario. This probability differs after every street in Poker.

2.2 Number of Outs

In Poker, postflop, there are hands that are **established** and hands that are **drawing**. Established hands are hands that possess some sort of strength at present. In other words, they are words that have decent or great equity at a particular betting round. On the other hand, drawing hands currently have low equity, but have the potential to be a very strong hand on later streets. In calculation for number of outs, we consider only drawing hands. An **Out** is defined as a card that can appear in subsequent streets, which on appearing would complete the hand that the drawer is drawing to.

Consider the following example: suppose the flop is Q♥ 8♣ 7♠ and the Hero has 9♦ 6♦ . In this scenario, the Hero has an **Open-ended Straight Draw**. The draw is called a **Straight Draw** because the Hero currently has 4 consecutive cards and only requires one more card to obtain the straight. On top of that, the draw is called an **Open Ended Straight Draw** since the draw can be completed from both ends, on getting a Ten or Five of any suit in the subsequent streets.

Here we can say that the Hero has **8 outs**:

- Any 10: 10♣ 10♥ 10♦ 10♠
- Any 5: 5♣ 5♥ 5♦ 5♠

2.3 Gordon's Rule of 2 or 4

After observing the probabilities of various hands, Phil Gordon established a basic rule of thumb to assign equity to outs [4]. Each out is worth approximately 2% equity per card. If you get to see the card on both the turn and the river without any additional betting, then each out is worth approximately 4% equity per card. The probability gets doubled since you get to see 2 cards rather than 1.

Consider the following example: suppose the flop is Q♦ 8♦ 7♠ and the Hero has 9♦ 2♦ . In this scenario, the Hero has an **Flush Draw**. Also, here the Villain has not gone all-in, which means that subsequent betting rounds will take place. The draw is called a **Flush Draw** because the Hero currently has 4 cards of the same suit and only requires one more card to obtain the flush. Here we can say that the Hero has **9 outs** since there are 2 diamond card on the table and the Hero also has 2 diamond cards. Therefore the winning percentage will be

$$q = \frac{9}{47} = 0.191 \approx 18\% = 9 \cdot 2\%$$

However, suppose the flop is Q♦ 8♦ 7♠ and the Hero has 9♦ 2♦ and the villain has gone **all-in** (if the Hero calls, the next two cards will be seen) on the flop itself. Therefore, the winning percentage will be

$$q = \frac{9 \cdot 2}{47} = 0.382 \approx 36\% = 9 \cdot 4\%$$

Suppose the flop is Q♦ 8♦ 7♠ and the Hero has 9♦ 6♦ . In this scenario, the Hero has a **Straight/Flush Draw**. Again, the Villain has not gone all-in on the flop. Here we can say that the Hero has **15 outs** :

- 9 Flush Outs: The remaining diamond cards.
- 8 Straight Outs: All 5's and 10's.
- Here 5♦ and 10♦ has been counted twice so they'll be subtracted from the total.

Therefore, the winning percentage will be

$$q = \frac{15}{47} = 0.319 \approx 30\% = 15 \cdot 2\%$$

2.4 Expected Value Formula

The Expected Value of Winning $E[W_c]$ is defined as:

$$E[W_c] = Pq - B(1 - q)$$

Here, W_c refers to the winnings if the Hero calls, P refers to the size of the pot, q refers to the **Win Percentage**, and B refers to the calling amount (current bet size) [2].

In general, decision rules will be made based on Expected Value. In order to calculate the EV in a given scenario the following method can be followed: The first step would be to **Deduce whether Hand is Established or Drawing**.

2.5 Expected Value Calculation For Established Hand

If the hand is **established**, then we have to make a bet substantial enough to reduce the effective EV of the players that are drawing to a hand.

Let us consider the scenario where the Hero and the Villain are playing with a pot of size P. The Villain makes a bet of \$V, making the pot of size $P + V$. Let the win percentage of the Hero be q_H.
Therefore,

$$E[W_c] = (P + 2V)q_H - V(1 - q_H) = Pq_H - V(1 - 3q_H)$$

Here, we can see that EV of the Hero is **directly proportional** to $-V$.

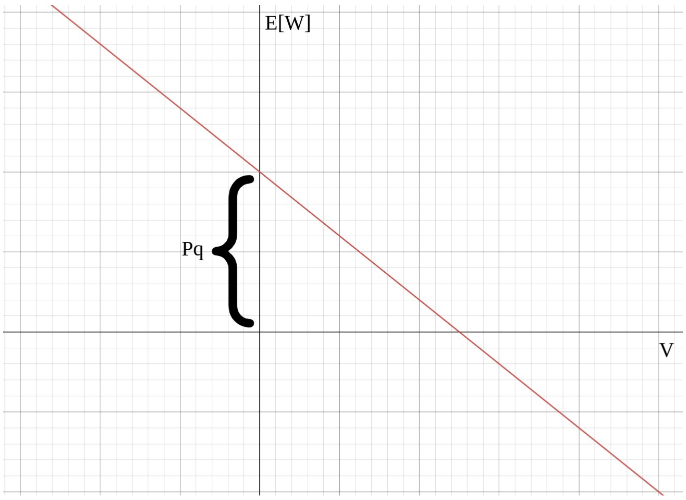


Figure 3: Expected Value for Established Hands

Since the Hero has an unestablished hand, $q < \frac{1}{3}$. Therefore, as the Villain increases the size of his bet (bloats the pot), the EV of the Hero during their turn decreases. In order for the Villain to force the Hero into making a $0EV$ decision, the size of the bet that would need to be made would be

$$Pq_H - V(1 - 3q_H) = 0$$

$$\therefore\ V = \frac{Pq_H}{1 - 3q_H}$$

Any bet that is made higher than this amount will force the Hero into making a $-EV$ decision. Obviously, the following calculation is done only given that the Villain know the approximate winning percentage of the Hero.

Example: Let us consider the general case where the Hero is drawing to either a straight or a flush. Typically, the Hero's hand will have either 8 or 9 outs in this case, which translates to a winning percentage to around $16\% - 18\%$. Let us assume $q = 20\%$, just to be on the safe side. Using the equation above, we get

$$V = \frac{P \cdot 0.2}{0.4} = \frac{P}{2}$$

Therefore, in the event that we have an established hand, it is advisable to bet more than **half** of the current pot to force the opponent into making a $-EV$ bet. [7]

2.6 Expected Value Calculation For Drawing Hand

On the other hand, if the hand is **drawing**, we must first understand the **pot odds** for the hand.

2.6.1 Pot Odds

Pot Odds: Pot odds represent the ratio between the size of the total pot and the size of the bet facing you. Keep in mind that the size of the total pot includes the bet(s) made in the current round [2]. It is defined as

$$PO = \frac{B}{P + B}$$

For example, let the pot be \$100 and the Villain bets \$50. The Pot Odds for the Hero would be equal to

$$\frac{50}{100 + 50 + 50} = \frac{1}{4} = 0.25$$

In addition, the Hero will want to convert their pot odds into a percentage so they know exactly how much equity their hand needs to profitably call the bet.

Referring back to the expected value formula,

$$E[W_c] = Pq - B(1 - q)$$

one should call only if $E[W_c] > 0$ which gives us

$$q > \frac{B}{P + B}$$

$$\therefore q > PO$$

Ultimately, the **positive expectation** play for the Hero would be when their **winning percentage** exceeds their **pot odds** (valid for a situation in which they are looking to call a bet from the Villain).

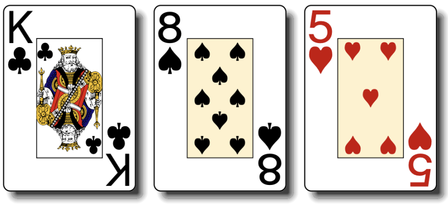
Applications: There are many situations where a player has an **established hand** such as One-pair, Two-pair etc. and the player they are facing has a **drawing hand** (currently possesses nothing but has potential to become stronger hand in the subsequent streets). For example, a One-pair on flop v/s Straight/Flush Draw is a very common scenario. The drawer has to take into account the chance of hitting his draw (**whether or not this is greater than pot odds**) and the cost of seeing further streets. Through this analysis, the Hero makes a $+EV$ or $-EV$ play.

2.6.2 Steps for Calculation

If the hand is **drawing**, we do the following:

1. **Deduce Drawing Hand**: Finding the hand that the Hero can draw to.
2. **Calculating Outs**: Calculating the number of outs through which the Hero can complete their draw.
3. **Calculating Win Percentage**(q): Through the number of outs and the Rule of 2 or 4, the Hero can calculate the equity of their hand.
4. **Calculating Pot Odds**: Using the amount of money that the Hero needs to call in their turn and the money that is already in the pot, we can calculate the pot odds.
5. **Calculating EV**: Calculate the EV using q which will be used by the Hero to make the decision.

For this **drawing hand** example, the flop is



The Hero has 6♦ 7♦ and the Villain has gone **all-in** by putting \$150 into the pot, making the pot a total of \$470.

The calculation is as follows:

1. **Deduce Drawing Hand**: The Hero has an **Open-ended Straight Draw**.
2. **Calculating Outs**: Any 9 or Any 4 would complete the draw. Therefore there are **8 Outs** in total.
3. **Calculating Win Percentage**:
$$q \approx 8 \cdot 4 = 32\%$$
4. **Calculating Pot Odds**:
$$PO = \frac{B}{P+B} = \frac{150}{620} \approx 24\% = 0.24$$
5. **Calculating EV of decision**:
$$E[W_c] = Pq - B(1-q) = 32 \cdot 470 - 68 \cdot 150 = 48.4$$

Since $q > PO$, in this scenario **calling will be +EV play** and the Hero should call.

Let us alter the scenario, where the Villain **does not go all-in**, and bets the same \$150, making the pot a total of \$470. The procedure will be the same as the previous example except for the win percentage and the EV.

1. **Calculating Win Percentage**:
$$q \approx 8 \cdot 2 = 16\%$$

2. **Calculating Pot Odds**:

$$PO = \frac{B}{P + B} = \frac{150}{620} \approx 24\% = 0.24$$

3. **Calculating EV of decision**:

$$E[W_c] = Pq - B(1 - q) = 16 \cdot 470 - 84 \cdot 150 = -50.8$$

Since $q < PO$, in this scenario **calling will be -EV play** and the Hero should fold.

2.7 Implied Odds

Implied odds are the amount of money that you **expect to win** on later streets if you hit one of your outs. Typically the Hero is expected to fold whenever they have to make a $-EV$ play (according to pot odds), but using implied odds, the Hero can gauge taking a risk by making a $-EV$ play, which could eventually turn into a $+EV$ play by the end of the river. If you expect to win more money from your opponent after you hit your draw, then you have **good implied odds**. But if you anticipate not being able to get any more money from your opponent on future streets, then you have **little or no implied odds** [2].

2.7.1 Calculating Implied Odds

Implied Odds are calculated by figuring out what the pot would have to be after our call to make the x% chance of winning equal to the x% of the pot for the call. For example, if we have a flush draw (18% to hit), and we are facing a bet of $180 into a pot of $300, then our call represents $\frac{180}{660}$ = 27% of the pot (i.e. too expensive to call). This would be a good call if we contributed 18% of the pot. So we need to find $1000 – $660 = $340 in **dead money**. The additional $340 after the draw makes our $180 bet worth 18% of a $1000 pot.

Given the scenario where the Hero is facing a bet of $B with a pot of $P. Let the win percentage of the Hero be q and the pot odds be PO. The Hero calling would be a $-EV$ play.

$$PO = \frac{B}{P + B}$$

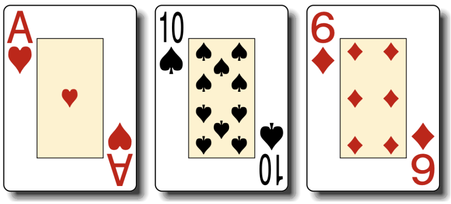
Since this is a $-EV$ play, $q < PO$. Therefore, for this play to become $+EV$, we need to calculate the amount of dead money that needs to be added to the pot in order for this play to become $+EV$. Let the amount of extra dead money for the hero to break even ($0EV$) be $X. At $0EV$, $q = PO$

$$q = \frac{B}{P + B + X}$$

$$\therefore X = \frac{B}{q} - P - B$$

2.7.2 Implied Odds Examples

Consider the flop: The Hero has K♦ 10♥ and faces a bet of \$100 with a pot of \$375. The Hero knows that they are currently



losing to the Villain and their hand needs to improve in order to win. Therefore, they are drawing to a better hand in the hierarchy such as Two-pair or Three of a Kind.

1. **Deduce Drawing Hand**: Two-Pair or Three of a Kind
2. **Calculating Outs**: 3 Kings and 2 Tens, therefore **5 outs**.
3. **Calculating Win Percentage for next street**:

$$q \approx 5 \cdot 2 = 10\%$$

4. **Calculating Pot Odds**:

$$PO = \frac{100}{575} \approx 17\%$$

which is too expensive since $PO > q$.

5. **Calculating EV of decision**:

$$E[W_c] = 575 \cdot 10\% - 100 \cdot 83\% = -25.5$$

Therefore, according to pot odds, it would be a **negative expectation** play for the Hero to call. However, let's consider implied odds. **Calculating Implied Odds:**

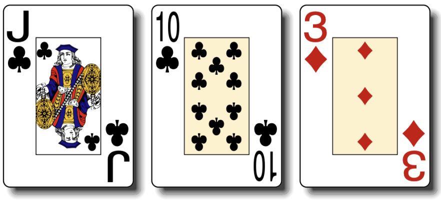
$$X = \frac{B}{q} - P - B = 1000 - 575 = \$425 \text{ more}$$

Therefore, if we're making a call of \$100 on this street, our call is $+EV$ iff \$425 or more is extracted from other players on the next street. In this scenario, the Villain made a $\frac{100}{375} \cdot P = 0.26P$ bet on the flop. According to our implied odds the Villain would have to bet at least \$425 into a \$575 pot ($\frac{425}{575} \cdot P = 0.73P > 0.26P$) to make our call on the flop $+EV$ (assuming our draw hit on the turn). Since there is such a big discrepancy in the betting sizes between the street, it is quite unlikely that the Villain would cover our implied odds and therefore the $+EV$ play would be to fold on the flop itself.

For the next example, the flop is as follows:

The Hero has K♣ Q♣ and the faces a bet of \$800 and the pot now becomes \$1000.

1. **Deduce Drawing Hand**: Straight or Flush



2. **Calculating Outs**: Any A, Any 9, Any club (7) therefore **15 outs**.

3. **Calculating Win Percentage for next street**: $q \approx 15 \cdot 2\% = 30\%$

4. **Calculating Pot Odds**:
$$PO = \frac{800}{1000} \approx 80\%$$
which is too expensive.

5. **Calculating EV of decision**:
$$E[W_c] = 1000 \cdot 30\% - 800 \cdot 70\% = -26$$

Therefore, according to pot odds, it would be a **negative expectation** play for the Hero to call. **Calculating Implied Odds:**
$$X = \frac{B}{q} - P - B = 2666.6 - 1800 = \$866.6 \text{ more}$$
Hence, if we're making a call of \$800 on this street, our call is $+EV$ iff \$866.6 or more is extracted from other players on the next street. In this scenario, the Villain made a $\frac{800}{200} \cdot P = 4P$ bet on the flop. According to our implied odds the Villain would have to bet at least \$866.6 into a \$1000 pot ($\frac{866}{1000} \cdot P = 0.86P < 4P$) to make our call on the flop $+EV$ (assuming our draw hit on the turn). Since the betting size to cover the Hero's implied odds is far lesser than the bet that the Villain made on the flop, we can say that the Villain is quite likely to cover the Hero's implied odds in this case. Hence, it is a $+EV$ play for the Hero to call on the flop.

3 Expected Value Calculation Criteria For Bluffing and Semi-Bluffing

Bluffing in Poker is the act of making the Villain fold a better hand what the Hero holds by making them think the Hero has a good hand. We can say that bluffing is the exact opposite of a value bet. This is because when betting for value, the Hero hopes to be called by a worse hand. On the other hand, when betting as a bluff, the Hero hopes the Villain folds the better hand.

Types of Bluffing:

- **Continuation Bet Bluff**: This is a bet where the Hero has the initiative and is relying on the Villain not being able to make a good hand from board.

- **Semi-Bluff**: This is a bet when the (drawing) hand is currently weak, but not totally devoid of showdown value. It has a decent number of outs to make it a strong hand. It is a bet with $+EV$ even though it has $E_f < 0$ because of sufficiently high q
- **0EV/Stone-Cold Bluff**: This is a bluff where you have almost no chance of improving to the best hand and it relies entirely on **fold equity** (explained in next section). It is a $+EV$ bet only because $E_f > 0$

3.1 Fold Equity

Fold equity refers to the probability that the Hero can win the pot by making the Villain fold their hand, assuming that a call will result in a loss. The most important use of this concept is for determining **profitable bluffs**. Fold equity helps shape the optimal strategy on all streets, considering you won't always have a good hand, it should be taken into account to make the highest EV decisions.
Fold Percentage: refers to the percentage of times a player folds over the course of multiple hands. It is a property of the player and is calculated over multiple hands.

Let us represent **Showdown Win Percentage** as q. This represents the probability of a hand winning in a showdown. For $q = 0$,

$$E_f = P_i f - B(1 - f)$$

For $q > 0$,

$$E_f = P_i f + E[W_c](1 - f) = P_i f + EV_{SD}$$

Here, E_f refers to Fold Equity, P_i refers to **Current Pot** before the Hero's bet. $P_i + B = P$, B refers to the size of the **Bet**, $E[W_c]$ refers to EV if called, and f represents the **Fold Percentage** of the Villain [2].

- **Bluffing**: A bet which has $+EV$ because $E_f > 0$

$$f > \frac{B}{P_i + B} = \frac{B}{P}$$

- **Semi-Bluffing**: A bet where $E_f < 0$ but has $+EV$ because of the high enough q

$$Pf < (1 - f)(B(1 - q) - (P_i + B)q)$$

$$f < \frac{E[W_c]}{E[W_c] - P_i}$$

$$f < \frac{(P + B)q - B}{(P + B)q - P}$$

Given that we know the Villain's fold percentage, we can calculate the maximum possible amount that the Hero can bet in bluff/semi-bluff.

- **Stone-cold Bluff**:

$$B_{\max} = fP$$

- **Semi-Bluff**:

$$fq(P + B_{\max}) - Pf = (P + B)q - B_{\max}$$

$$B_{\max} = \frac{P(q + f(1 - q))}{1 - q(1 - f)}$$

Let us look at the formula,

$$f = \frac{(P + B)q - B}{(P + B)q - P}$$

Here f would represent fold percentage at $E_f = 0$

- The curve between f and q will be **hyperbolic**. The part of the curve that we're interested in is $0 \leq q \leq 1$ and $0 \leq f \leq 1$
- At $q = 0$, $f = \frac{B}{P} \implies 0EV$ bluff
- At $f = 0$, $q = \frac{B}{B+P} = PO \implies$ No bluff

3.2 Semi-Bluffing Formula Analysis

In figure 4 below, we can see that every bet that is made lies on a spectrum where one end is a complete stone-cold bluff, and the other end is playing at exact pot odds where no bluffing is involved. The rest of the points on the group represent **semi-bluffing**, where for each point (q,f) that lies on the graph, the EV for every pair of win and fold percentage will be 0.

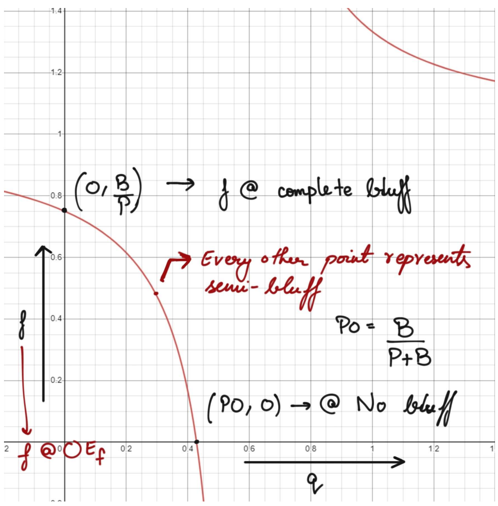
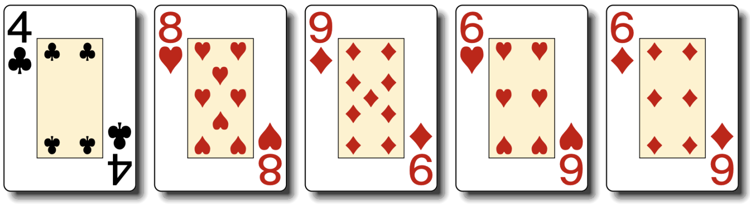


Figure 4: Semi-Bluffing Formula Analysis

3.3 Bluffing and Semi-Bluffing Examples

For this example, the **board** is as follows:



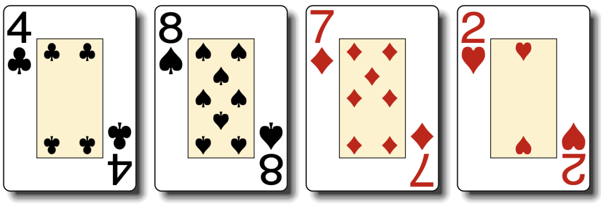
The Hero has 10♦ 5♠. The pot is currently \$375 and the Villain has checked on their turn. We also know that the Villain calls 25% of the time. **Steps:**

- **Deduce Drawing Hand:** High Card
- **Number of Outs:** 0 $\implies q = 0\%$
- **Type of Bluff:** Stone-cold Bluff
- **Calculate Breakeven Bet:**

$$B_{\max} = fP = \$468.75.$$

Therefore, the maximum amount that the Hero can safely bet in this bluff would \$468.75.

For this example, the **turn** is as follows:



The Hero has 9♦ 10♠. The pot is currently \$775 and the Villain has checked on their turn. We also know that the Villain calls 80% of the time. **Steps:**

- **Deduce Drawing Hand:** Open-ended Straight Draw
- **Number of Outs: 8** $\implies q = 16\%$
- **Type of Bluff** Semi-Bluff
- **Calculate Breakeven Bet:**

$$B_{\max} = \frac{P(q + f(1-q))}{1 - q(1-f)} = \frac{775(0.16 + 0.2(1-0.16))}{1 - 0.16(1-0.2)} = \$305.5$$

Therefore, the maximum amount that the Hero can safely bet in this bluff would \$305.5.

4 Preflop Analysis

Preflop understanding in poker is an important aspect of the game that can greatly influence a player's long-term success. The following are some key concepts that must be analysed during preflop:

- **Starting hands:** The strength of your starting hand is one of the most important factors to consider preflop. It's generally a good idea to play tight and only enter pots with strong hands, especially in early position.
- **Position:** Your position at the table is also crucial in determining which hands to play. Being in later position allows you to see what your opponents do before you have to act, giving you more information and more control over the pot.
- **Bet sizing:** The size of your bets preflop can also have an impact on the game. Generally, a larger bet size will deter players with weaker hands from calling, while a smaller bet size may entice more players to enter the pot.
- **Player tendencies:** Paying attention to your opponents' tendencies can also help you make better preflop decisions. If a player has been playing aggressively, for example, you may want to tighten up your range and only play strong hands against them.

In the following sections, we will touch upon **preflop betting**, **preflop hand rankings**, and **ranges**.

4.1 Preflop Betting

Preflop Betting is an important aspect of NLH poker which should be played by the Hero as well as possible in order to be in good position during the game. Making the decision of calling/raising-reraising/folding after the Hero recieves their starting hand depends on the Hero's position at the table, the Playing Style of the Villain (Call Range), the Hero's Stack (M value), and the Hero's Starting Hand Ranking.
Preflop Bet Sizing: If the Hero decides that they are going to enter the pot, they should be looking to make a raise of about **3BB to 4BB**. By making a minimum raise, the Hero is letting the opponents with marginal hands come in cheaply, which defeats the purpose of making a preflop raise. The idea of a preflop raise is to reduce the amount of players who follow you to see a flop, as it is easier to make profitable decisions when there are fewer players in the pot. **Table Position**: It is advised to play **tighter at earlier positions** and **looser at later positions** at the table. This is because at the early positions, the Hero cannot see what decisions the other players will make, making it difficult to anticipate how many players will raise eventually. At late positions such as the dealer, the Hero is the last to act and can see how many players are raising, making it an advantageous position allowing them to play loose.

4.2 Preflop Hand Rankings

Preflop Hand Rankings is a method of assigning a relative value or strength to different starting hands. Its purpose is to help the player make informed decisions about which hands to play and which hands to fold in different scenarios. There are several different hand rankings in poker: Sklansky-Karlson Hand Rankings, Chen Formula, and Harrington Staring Hand Guide. In our paper, we will be highlighting the **Sklansky-Karlson Hand Rankings** as it is the most used ranking system.

4.2.1 Sklansky-Karlson Hand Rankings

- The **Sklansky-Karlson Hand Rankings** are based on extensive analysis of hand histories and statistics by poker expert **David Sklansky**, and takes into account multiple factors such as:
  - Likelihood of making a strong hand
  - Ability to bluff effectively
  - Which starting hands are profitable in the long run

The Sklansky-Karlson Hand Rankings group together certain hands based on their strength. The hands are depicted as XXb, where X represents any card from 2 to A and $b = \{s, o\}$ where s represents suited and o represents off-suit.

A **range strand** is represented as $XXba$ where $a = \{+, -\}$, which represents the set of hands better or worse than the given hand. For example, “JJ+”, is a range strand that says "select pocket Jacks and all pocket pairs above it" so JJ, QQ, KK, and AA. The plus sign after a starting hands tells you to include all similar hands that are higher than it.

Examples:

- **22+** means you should include all pocket pairs (22, 33, 44, ..., QQ, KK, AA)
- **98s+** means you should include suited connectors 98s and higher (so 98s, T9s, etc.)
- **AQ+** doesn’t have an "s" (suited) or "o" (offsuit) qualifier, so you would include **all versions** of AQ and also AK.

The Sklansky-Karlson Rankings are as follows:

- **Top 10 hands**: AA, KK, AKs, QQ, AKo, JJ, AQs, TT, AQo, 99
- **Top 2.5%**: QQ+, AK
- **Top 5%**: TT+, AQs+, AQo+
- **Top 10%**: 44+, AJ+, KQ, KJs
- **Top 20%**: 22+, ATB, 54s+
- **Top 33%**: 22+, ATB, A2s+, A7o+, T9+, 43s+, 53s+, J8s+, K8s

4.3 Ranges

A **range** is a collection of all possible hands a player can have at the given moment. During preflop, it is important to consider the range of cards the Villain can have before considering betting. The Hero can start building the Villain’s range preflop and refine it at subsequent streets. Overall, ranges are used for developing rules to determine plays (call/raise/fold).

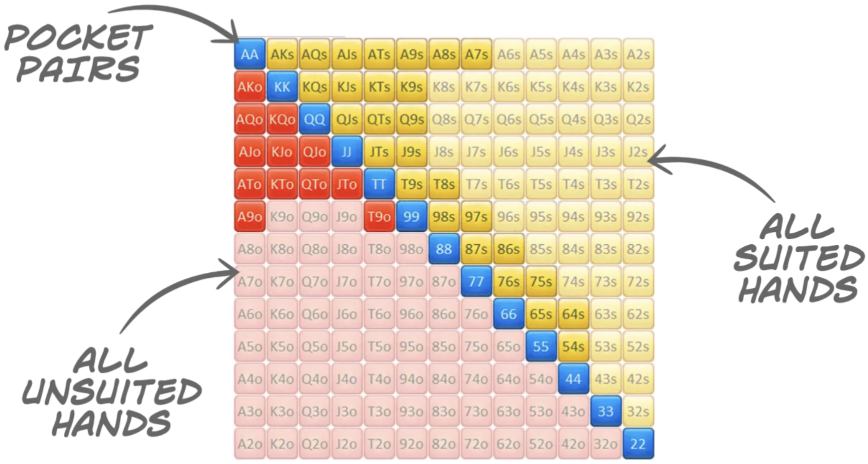


Figure 5: Range Matrix

4.3.1 Range Matrix

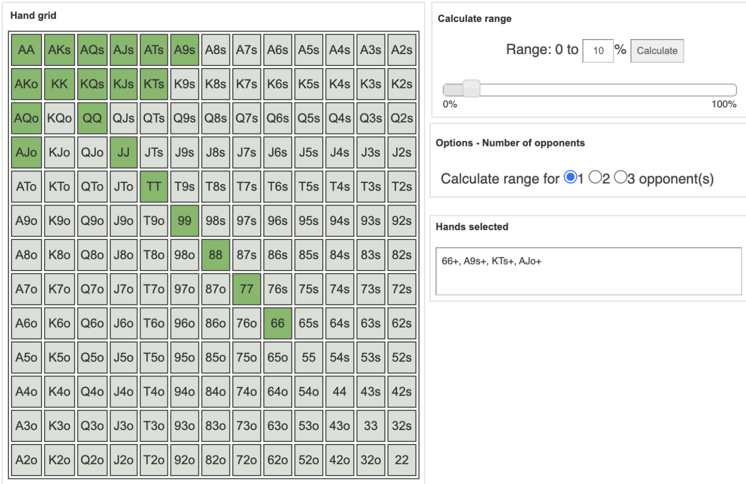
When analysing ranges, we look at the **range matrix** which is the possible hands in poker and determine the percentage of all possible starting hands which are within that range.
The matrix lists out all 169 possible starting hands in poker ($13 \cdot 13 = 169$) and is the means through which poker players visualize ranges. As you can see, the diagonal consists of all pocket pairs and every suited combo is above the diagonal whereas the unsuited combos can be found below the diagonal.

Before looking at the percentage form, let's first understand the combos possible for the hands in poker. The 169 possible starting hands become $\binom{52}{2} = 1326$ unique combos which represent the spectrum of possible hands (suits included) a player can have preflop. To find the percentage form of the poker hand matrix, we count the number of combinations for a particular hand.

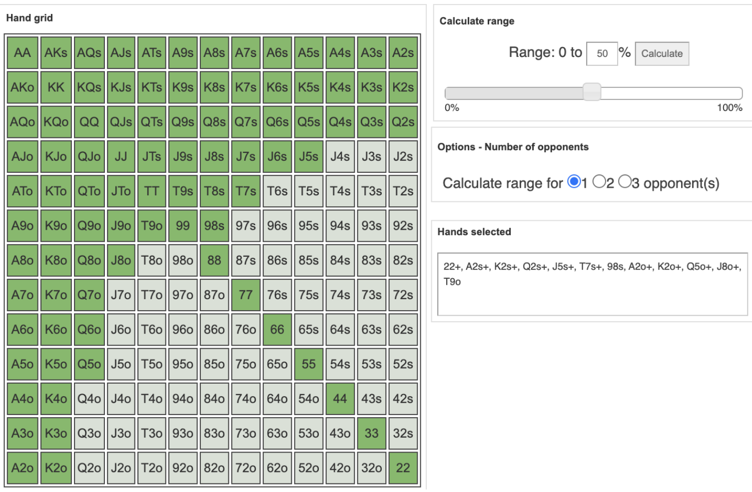
- For any pair of cards that aren't pairs, there are 16 ($\frac{\binom{8}{2}-\binom{4}{2}-\binom{4}{2}}{\binom{52}{2}}$) different combinations of them (excluding suits), out of which 4 are suited and 12 are unsuited.
- We calculate the **probability of occurence** for 3 different kinds of hands:
  - **Pocket Hands**: $P(\text{pocket}) = \frac{\binom{4}{2}}{\binom{52}{2}} = 0.45\%$
  - **Suited Hands**: $P(\text{suited}) = \frac{4}{\binom{52}{2}} = 0.3\%$
  - **OffSuit Hands**: $P(\text{unsuited}) = \frac{12}{\binom{52}{2}} = 0.9\%$

Examples:

If we consider 133 combos ($\frac{1326}{10}$), the percentage form would be 10% which basically means the opponent's range can be classified into the following hands (that range can be selected to represent his highest probable starting two cards):



50% of the hands in the range matrix would equate to 663 combos ($\frac{1326}{2}$) and we can compute the range strand of the Villain accordingly:



4.3.2 Range Calculation

- Using the parameters player position, stack size, calling and raising percentages the Hero can make a good approximation of the range that the Villain has. The steps for range calculation are as follows:
  1. Define the action the opponent is making (calling/raising)
  2. Estimate the frequency (percentage of hands they play in that position and the action that they are proceeding with)
  3. Remove the possibilities of hands that are not included in the range.

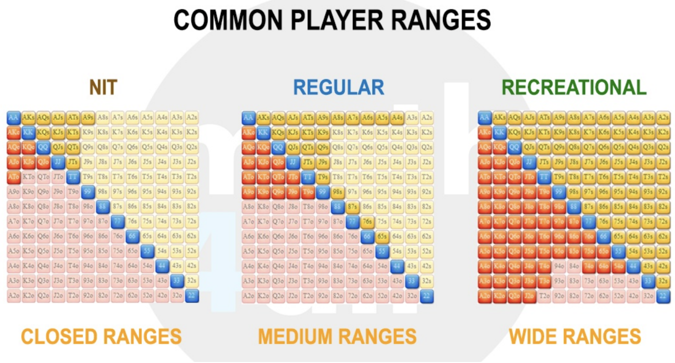
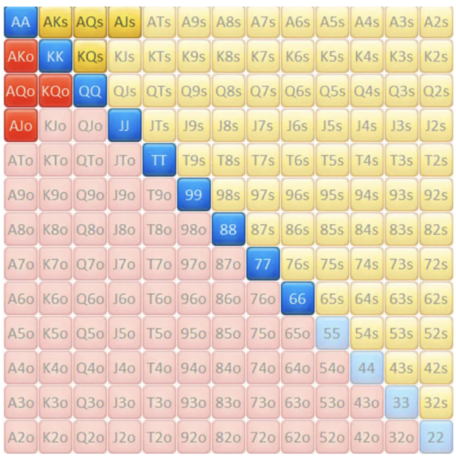


Figure 6: Common Player Ranges

For example, a tight player (low calling and raising percentages) is going to play tight from an early position and open-raise few hands in which he/she is first to act. This means that the player's range does not have that many hands in it and they are likely to have a strong hand. On the other hand, a player that open-raises on the button, is typically going to raise with many hands and overall has a higher calling and raising percentage leading to a wider range of hands. Given a tight player with a raising percentage of 15% in early position (UTG) that is open-raising in a game, one can say that the player most likely has a range of 15% (top 15% of hands) that they are playing with.

Examples:

Consider a player in early position that raises 9% of their preflop hands. Their range can be shown as:

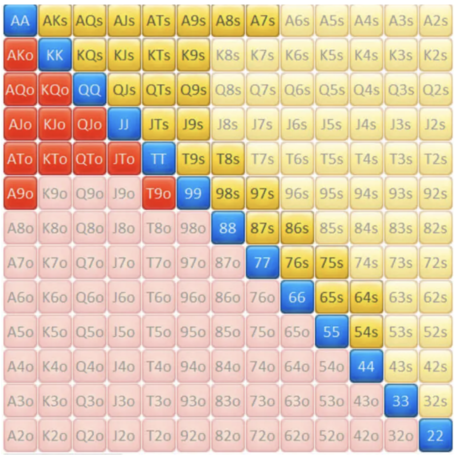


Range Strand: 66+, AJs+, KQs, AJo+, KQo

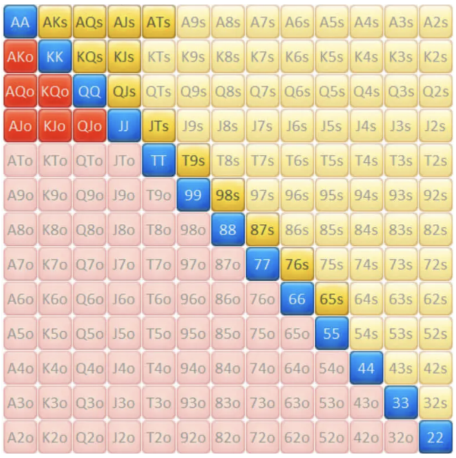
Now, consider a player in the later position (CO) who raises 25% of their preflop hands. Their range would be much wider and is shown below:

Range Strand: 22+, A7s+, K9s+, Q9s+, J9s+, T8s+, 97s+, 86s+, 75s+, 64s+, 54s, A9o+, KTo+, QTo+, JTo, T9o

The 9% range has fewer but stronger hands. As the percentage increases in the poker range matrix, it shows us the



extra possible hands that a player could raise for preflop. This ensures that the Hero can remove the cards they would most likely not have in a particular scenario and therefore, deduce an easy decision making process postflop. Finally, consider the scenario where a player raises 15% of their hands preflop. Their range would look something like this:



From this, the Hero can starting building the Villain's range and remove the unlikely starting hands from it. For example, if the flop were to come Q♥ 6♣ 2♦ , then with high liklihood the Hero would know that the Villain does not have a Q♦ 6♥ , therefore they need not worry about the Villain having a two-pair on the flop. Moreover, it is highly unlikely that they have QQ, 66 or 22 since a card from each of those ranks is already on the board. Hence, we can discount certain hands they have based on their range.

4.3.3 Range Application

Given the range of a player, if the player is loose then they would tend of have a lot of hands in their range that they are unlikely to have. If these extra hands are likely to fold if they face a raise or re-raise, the Hero is heavily incentivized to **bluff** against the opponents to apply pressure. If the Villain refuses to fold those extra hands, the Hero's focus should be on getting thinner value since they will continue onward with weak-marginal hand and will most likely have a higher win percentage when the flop arrives.

Ranges help the Hero deduce the possible hands the Villain can have based on their betting behavior. This comprises of the frequency at which they are calling/raising a bet, i.e., their **call percentage** and **raise percentage**. "The range you assign is a bi-product of *who* your opponent is, *what* action they are taking, *where* they are making that action, and *how* they might craft that specific range." Knowing what's included in the top-X% of hands can help the Hero remove hands from the Villain's range that could prove to be problematic later. Ex. A player that open-raises 15% of hands almost certainly is not going to have Q6s in their preflop range.

5 Hand Assessment

5.1 Introduction

Hand assessment tools are useful since they provide us with the means to **quantify** the strength of a hand during the game. Essentially, **Hand strength** assesses how strong your hand is in relation to the other hands. The stronger our cards are in a game, the better our position is and the more incentivized we are to take risks and call/make bets.

This strength can be defined for an established hand as well as a drawing hand. The strength of an established hand comes from being in good position **currently**, i.e, early into the game (on the flop or on preflop), whereas the strength of a drawing hand comes from the **potential** of converting into a strong hand on the subsequent streets. Therefore, it is important to bet optimally in both cases in order to avoid folding a good hand.

For our analysis, we have implemented 4 hand evaluation criteria (mentioned below) to estimate hand strength using two mainly two techniques - Monte Carlo Simulation and Enumeration Techniques. Enumeration techniques consider and evaluate a given evaluation function for **all possible cases**, giving a deterministic output for a given input. Monte Carlo Simulation relies on **repeated random sampling** in order to calculate an approximate value of an evaluation method and is used when it is practically not feasible to evaluate over all possible cases due to the number of cases being too large. Therefore, there is some margin of error in the output of evaluation carried out using monte carlo simulation.

HSE allows us to evaluate the current strength of the hand and Hand Potential calculates the probability of our current hand improving to a better hand or worsening to a worse hand. Moreover, EHS combines these two parameters to calculate the overall strength of a hand. We conclude that Preflop Analysis and Effective Hand Strength are necessary but not sufficient criteria for hand evaluation and subsequently, coming up with an optimal betting strategy. A more advanced evaluation would take into account factors such as the position at the table, history of betting for that hand, unpredictability, opponent modeling etc.

5.2 Library Used and Utility Functions

For our hand assessment methods, we developed utility functions using the *deuces* python library in order to allow us to perform hand comparisons. The `evaluator` class from *deuces* evaluates a hand of 5 cards by evaluating the given cards in integer form, mapping them to a **rank** in the range from 1 being the most powerful (Royal Flush, Eg. A♥ K♥ Q♥ J♥ 10♥) to 7462 being the worst (7 High, Eg. 7♦ 5♣ 4♥ 3♣ 2♦). Using this, the following utility functions were developed for our analysis: Percentage Rank and Vs Odds.

5.2.1 Percentage Rank

The percentage rank function tells us the percentage of hands the player's hand is beating. Given a hand of 5 cards, the percentage rank is defined as follows:

```
function percentage_rank(boarcards, hand) {
    handrank = evaluator.evaluate(boardcard, hand)
    percentage = 1.0 - handrank/7462.0

    return percentage
}
```

5.2.2 Versus Odds (vs Odds)

The Versus Odds function takes two starting hands as input and compares their strength against each other. The evaluation is as follows: given a starting Hero and Villain hand, over all possible boards that can appear, the function keeps track of how many times each hand wins and outputs each hand's winning percentage.
The pseudocode is as follows:

```
function odds_calculator(hero_hand, villain_hand) {
    hero_wins = villain_wins = 0
    for each case(boardcards) {
        hero_rank = evaluator.evaluate(boardcards, hero_hand)
        villain_rank = evaluator.evaluate(boardcards, villain_hand)

        if (hero_rank < villain_rank) hero_wins += 1
        else if (hero_rank > villain_rank) villain_score += 1
    }

    hero_wp = hero_wins / (hero_wins + villain_wins)
    villain_wp = villain_wins / (hero_wins + villain_wins)

    return hero_wp, villain_wp
}
```

We use a **Monte Carlo approach** for this evaluation since the enumeration method involves evaluating over $\binom{48}{5}$ boards given two hands, which is practically not feasible for calculation in a reasonable amount of time.

The Monte Carlo method is as follows: given a starting Hero and Villain hand, a random board is selected 100000 times and both hands are evaluated, keeping track of which hand wins in each iteration. The pseudocode is as follows:

```
function mc_odds_calculator(hero_hand, villain_hand, iterations=100000) {
    hero_wins = villain_wins = 0
    for i in range(iterations) {
        boardcards = deck.draw(5)
        hero_rank = evaluator.evaluate(boardcards, hero_hand)
        villain_rank = evaluator.evaluate(boardcards, villain_hand)

        if (hero_rank < villain_rank) hero_wins += 1
        else if (villain_rank < hero_rank) villain_wins += 1
    }

    hero_wp = hero_wins / (hero_wins + villain_wins)
    villain_wp = villain_wins / (hero_wins + villain_wins)

    return hero_wp, villain_wp
}
```

In order to validate the output of our Monte Carlo method, we calculate the margin of error for 10 randomly selected set of hands (taken from Testcase 1 mentioned later in the report).

ID	Cards	MC Odds	Absolute Odds	Error Percentage
1	9♥ 7♥	0.6895	0.68702	0.00362
	3♣ 2♠	0.3105	0.31298	0.00795
2	9♣ 7♠	0.67039	0.66916	0.00183
	3♠ 2♦	0.32961	0.33084	0.00371
3	J♣ 9♣	0.42251	0.42029	0.00529
	A♠ 10♦	0.57749	0.57971	0.00384
4	K♠ 8♣	0.40203	0.40102	0.00253
	A♦ 6♣	0.59797	0.59898	0.00169
5	A♦ 3♣	0.61425	0.61107	0.00519
	K♠ 4♣	0.38575	0.38893	0.00816
8	10♣ 8♠	0.47338	0.47171	0.00354
	6♠ 6♦	0.52662	0.52829	0.00316
9	J♣ 9♣	0.62947	0.62944	4e-05
	7♦ 5♦	0.37053	0.37056	7e-05
10	K♣ J♥	0.42338	0.42361	0.00054
	A♥ 3♣	0.57662	0.57639	0.0004
12	8♣ 3♠	0.22227	0.22168	0.00267
	K♦ 8♦	0.77773	0.77832	0.00076
13	Q♣ 10♦	0.64838	0.65152	0.00482
	8♣ 6♠	0.35162	0.34848	0.00901

From the table, we can see that the error percentage for every testcase is less than 1%, indicating that the Monte Carlo method is an **extremely good** approximation for the Absolute method.

5.3 Preflop Analysis using Monte Carlo Simulation (MCP)

Preflop Evaluation is a fundamental aspect of poker that elucidates the strength of the starting hand which a player possesses before the flop comes. It is usually more advantageous to raise/re-raise if the player already knows that their starting hand is extremely strong since this forces the players with weaker starting hands to fold so they avoid making $-EV$ bets. For the initial two cards, there are $\binom{52}{2} = 1326$ possible combinations, out of which $13 \cdot 13 = 169$ hands are unique.

Let us consider the enumeration method of preflop evaluation: we are to calculate how many times a given hand will win against a random hand. Given a starting hand, for all possible starting hands the opponent can have and all possible boards that can appear, we keep track of the number of times the player's starting hand wins, loses and ties. This lets us produce a statistical measure of the approximate **Income Rate**(IR), i.e, profit expectation of the starting hand, which is defined as:

$$IR = \frac{\text{wins} + \text{ties}/2}{\text{wins} + \text{ties} + \text{losses}}$$

The preflop IR values for each starting hand that is calculated using enumeration can be found here [5]. The pseudocode for the enumeration method of preflop evaluation is as follows:

```
function enumeration_preflop(hero_hand) {
    wins = ties = losses = 0

    for each case(villain_hand) {
        for each case(boardcards) {
            hero_rank = evaluator.evaluate(boardcards, hero_hand)
            villain_rank = evaluator.evaluate(boardcards, villain_hand)

            if (hero_rank < villain_rank) wins += 1
            else if (villain_rank < hero_rank) losses += 1
            else ties +=1
        }
    }
    IR = (wins + ties/2)/(wins + ties + lesses)

    return IR
}
```

We use a **Monte Carlo Simulation** for this evaluation. Let us consider the case where we are calculating IR against one opponent. After giving the Hero their starting hand, out of 50 cards in the deck, there are $\binom{50}{2} = 1225$ possible hands the Villain can have. For each possible hand that the Villain can have, there are $\binom{48}{5} = 1712304$ possible cards that can appear on the board. Therefore, the enumeration method would have to evaluate $1225 \cdot 1712304 = 2097572400$ cases, which is practically not feasible. Therefore, we will use the Monte Carlo method and compare its output with the values from the online source to calculate the margin of error.

The Monte Carlo method is as follows: given a starting hand, over 100000 iterations, a random starting Villain hand and a random board are selected, and we keep track of number of wins, losses and ties. Since the number of cases evaluated are much smaller, we can increase the number of opponents and calculate Preflop IR against multiple opponents. The only change in this case would be that in every iteration, all opponents are given a random starting hand, and the rest of the calculation remains the same. The pseudocode for this method is as follows

```
function mc_preflop(hero_hand, num_players, iterations=100000) {
    wins = ties = losses = 0
    for i in range( iterations ) {
        boardcards = deck.draw(5)
        hero_rank = evaluator.evaluate(boardcards, hero_hand)
        oppRanks = []
```

```
        for i in range(num_players)  {
            oppCards = deck.draw(2)
            oppRanks.append(evaluator.evaluate(boardcards,oppCards))
        }
        if (hero_rank < min(oppRanks)) wins += 1
        else if (hero_rank == min(oppRanks)) ties += 1
        else losses += 1
    }
    IR = ( wins + ties/2 ) / (wins + ties + losses )

    return(IR)
}
```

In order to validate the output of our Monte Carlo method, we calculate the margin of error for 10 hands from the Sklansky-Karlson hand rankings, at an interval of 17 each.

Cards	MC Preflop	Absolute Preflop	Error(Difference)
AAo	85.3	84.93	0.37
66o	63.3	62.7	0.6
66o	63.3	62.7	0.6
K8o	56.3	54.43	1.87
Q8o	53.8	51.93	1.87
J6s	50.8	48.57	2.23
J6o	47.9	45.71	2.19
85s	44.8	41.99	2.81
54s	41.1	38.53	2.57
54o	37.9	35.07	2.83
32o	31.2	29.23	1.97

From the table, we can see that the error is always less than 3%, indicating that the Monte Carlo method is a **great** approximation for the Absolute method.

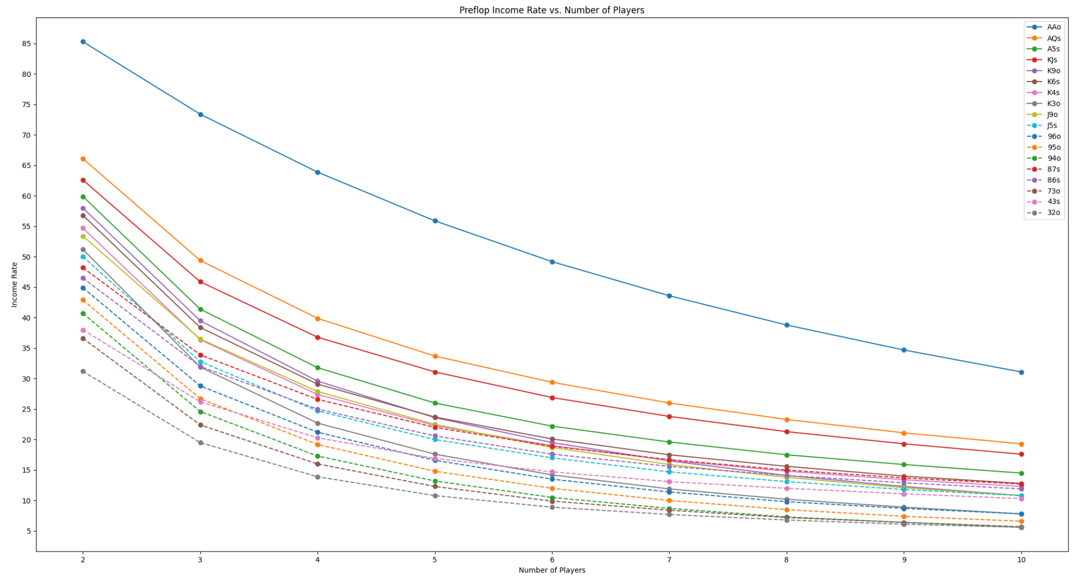


Figure 7: Preflop Evaluation Using Monte Carlo Simulation

In figure 7, 17 hands are selected from the Sklansky-Karlson hand rankings at an interval of 10 each. The essence of the graph is to highlight the decrease in *Income Rate (IR)* for a particular hand as the number of players at the table increases. It is very conspicuous that pocket Aces remain dominant regardless of the number of players at the table. The graph justifies why Aces is the best hand in the game and why it is common for professionals to go all-in at any position and with any chip stack when they possess this hand.

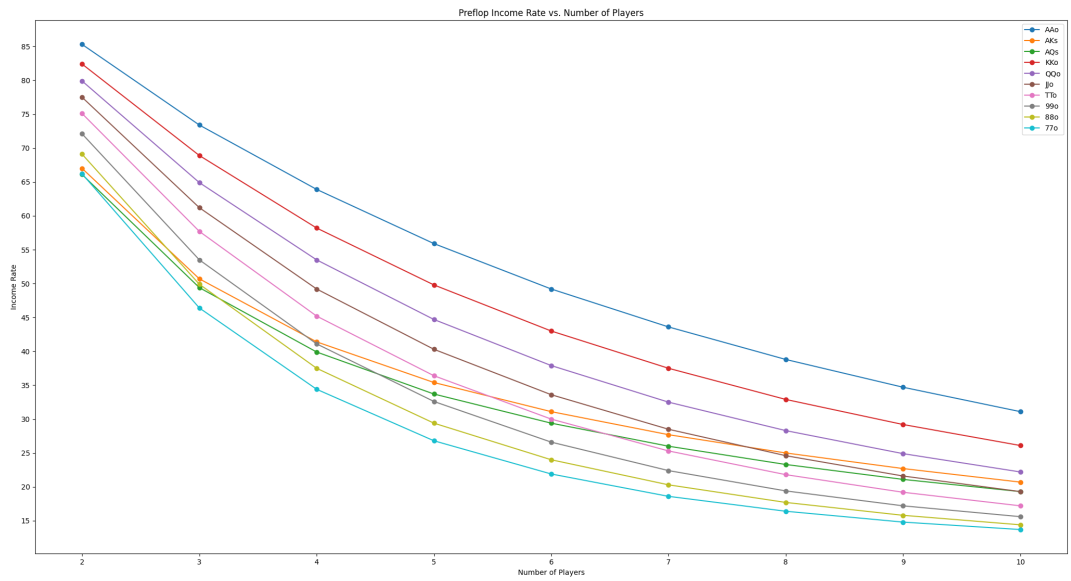


Figure 8: Preflop Evaluation Using Monte Carlo Simulation

In figure 8, the top 10 hands from the Sklansky-Karlson hand rankings are selected. This graph indicates the decrease in *Income Rates (IR)* for these hands as the number of players at the table increases. There are some interesting trends we can notice from the outcome. As mentioned earlier, pocket Aces remains the strongest holding with any number of players at the table. Therefore, as shown in the graph, no line ever surpasses the blue line which alludes to AA. Moreover, if we follow AKs as the number of players increases, we can notice that it surpasses several hands later on. In other words, it is in the 8th rank when 2 players are present but when the number of players is increased to 10, it jumps up to the 4th rank. This is a very informative trend since it provides a strong justification as to why professional players do not mind going all-in with AKs when there are at least 6 players at the table. Finally, it is known in the poker world that JJ vs AKs is basically a coin flip when there are at least 6 players at the table. This graph provides proof for that statement.

5.4 Hand Strength Estimation (HSE)

Hand Strength Estimation is a hand assessment criterion that is used to estimate the strength of a player's hand by looking at all possible cases that could possibly play out with that hand. At a minimum, it is a function of the player's cards and the current community cards.

Given that the player has a starting hand against one opponent, we will consider all possible starting cards $\binom{47}{2}$ that the opponent can have provided that the flop has come. Furthermore, we consider all possible cards that can appear on the turn and river $\binom{45}{2}$. Therefore, the total number of cases that would be evaluated would be $\binom{47}{2} \cdot \binom{45}{2} = 1116720$, which can easily be evaluated using enumeration methods.
The pseudocode for the enumeration method is as follows

```
function hand_strength(hero_hand, boardcards) {
    wins = ties = losses = 0
    hero_rank = evaluator.evaluate(boardcards, hero_hand)
    for each case(villain_hand) {
        villain_rank = evaluator.evaluate(boardcards, villain_hand)
        if (hero_rank < villain_rank) wins += 1
        else if (hero_rank == villain_rank) ties += 1
        else losses += 1
    }
    handstrength = (wins + ties/2) / wins + ties + losses
}
```

The hand strength calculation is with respect to one opponent but can be extrapolated to multiple opponents by raising it to the power of the number of active opponents. Let's say the hand strength of a particular hand is equal to 0.585 for 2 players total. If the number of players is increased to 5, the adjusted hand strength becomes $0.585^5 = .069$. Hence, the presence of additional opponents has reduced the likelihood of having the best hand to only 6.9 percent.

In practice, just by being able to tell our current position in the game isn't enough to make the most optimal decision. For example, if we have a straight flush draw, then by intuition we can say we have a very strong hand and should definitely make a continuation bet, but HSE would indicate that we have a very bad hand since it is currently only a high card. Therefore, we can conclude that HSE on its own is an **incomplete criteria for assessing the strength of a hand**.

5.5 Hand Potential

We have established that in general, it is significant to be wary of how the potential of our current hand improving to a better hand can affect our hand's effective strength.

Hand Potential is a hand assessment criterion that allows us to calculate the potential of our drawing hands by calculating the following quantities:

1. **Positive Potential** (Ppot): The probability of a hand improving to a better hand in subsequent streets given that the hand is currently behind.

2. **Negative Potential** (Npot): The probability of a hand falling behind in the subsequent streets, i.e., the opponent's hand improves and the Hero falls behind.

These are calculated using enumeration techniques. Given the Hero's starting hand and the flop, the function assigns a hand to the opponent and for each possible opponent hand, calculates if we are ahead, tied or behind their hand. Then for all possible 2 cards that can appear on the boards, we again check if we are ahead, tied or behind the opponent with the updated board. These values are then used to calculate $Ppot$ and $Npot$.

The pseudocode for the following function is as follows:

```
function hand_potential(hero_hand, boardcards) {
    integer array HP[3][3]
    integer array HPTotal[3]
    hero_rank = evaluator.evaluate(boardcards,ourcards)
    for each case(oppcards) {
        opprank = evaluator.evaluate(oppcards,boardcards)
        if(hero_rank < opprank) index = ahead
        else if(hero_rank = opprank) index = tied
        else index = behind
        for each case(turn,river) {
            HPTotal[index] += 1
            board = [boardcards,turn,river]
            hero_best = evaluator.evaluate(ourcards,board)
            opponent_best = evaluator.evaluate(oppcards,board)
            if (hero_best < opponent_best) HP[index][ahead]+=1
            else if (hero_best == opponent_best) HP[index][tied]+=1
            else HP[index][behind]+=1
        }
    }
    Ppot = (HP[behind][ahead]+HP[behind][tied]/2 +HP[tied][ahead]/2)
            / (HPTotal[behind]+HPTotal[tied])

    Npot = (HP[ahead][behind]+HP[tied][behind]/2 +HP[ahead][tied]/2)
```

```
        / (HPTotal[ahead]+HPTotal[tied])

    return(Ppot,Npot)
}
```

The pseudocode provided refers to a **two-card lookahead** since we are considering all possibilities of the next two cards coming on the board. Therefore, the output of this function is referred to as $Ppot_2$ and $Npot_2$, the 2 indicating a two-card lookahead. If we *only* evaluate over all possibilities of the next card coming on the board (turn or river), then the values obtained here would be $Ppot_1$ and $Npot_1$, since we are only performing a **one-card lookahead**.

5.6 Effective Hand Strength (EHS)

HSE gives us information about the strength of our hand at a given moment and Hand Potential gives us information about the probability of the strength of our hand increasing. By combining these two criteria, we get **Effective Hand Strength** which is defined as:

$$EHS = HS + (1 - HS) \cdot Ppot$$

Here, we can see that the larger the value of HS, the smaller the value of (1-HS). This indicates that for a higher HSE, it will contribute more towards EHS. Similarly, the smaller the value of HSE, the greater the value of (1-HS). In this scenario, this indicates that for smaller a HSE, Ppot will contribute more towards EHS.

5.7 Testcases

We have created a multitude of testcases to display the results we get from the functions mentioned in the previous sections. Moreover, the testcases help to serve us in verifying whether or not the functions are accurate. To test the hand assessment criteria that we have implemented, we are considering the most common possible testcases that span the entire range of combination of hands. Furthermore, in our testcases, we follow a Hero vs Villain format, to assess the parameters of a specific hand compared to another hand, given a fixed flop. Initially, preflop analysis is done and then the flop is fixed to do further analysis. Ultimately, after verification through our testcases, we can confidently say that we have developed an accurate metric for calculating the **Effective Hand Strength (EHS)** for any hand. This is an important outcome as decision-making tools revolve around the EHS of a particular hand as different betting rounds ensue. The following are the testcases we used:

1. Nut Flush vs Nut Low
2. Nut Straight vs Nut Low
3. Open-ended Straight Draw vs Top Pair
4. 2nd Pair vs 3rd Pair
5. High Card vs High Card
6. Full House vs Full House
7. Full House vs High Card
8. Three of a Kind vs Two Pair
9. Lower Straight Draw vs Higher Straight Draw
10. Flush Draw vs Flush Draw
11. Quad vs Full House
12. Middle Pair Cards with different kicker

13. Bottom 2 pair vs Top pair
14. Overpair vs Three of a Kind
15. Overpair vs Top pair Top kicker
16. Overpair vs Nut Flush Draw
17. Overpair vs Bottom Two pair

On top of that, we added the testcase Open-ended Straight/Flush Draw vs Underpair to highlight the significance of **EHS** as compared to **HSE**. In the tables below, P corresponds to $Ppot_2$, N corresponds to $Npot_2$, ↓ implies *low*, ↑ implies *high*, and ~ implies *mediocre*. Some descriptions are given for several testcases to justify our predictions and to provide an explanation for the outcomes of the functions.

5.7.1 Testcase 1

Flop: 10♥ 8♥ 6♥

Table 1: Testcase 1 and their outputs for the fixed flop as shown above

ID	Hand	vsOdds(MC)	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
1	9♥ 7♥	0.686	0.49	1	0	≈0	1	$P = 0, N \approx 0$
	3♣ 2♠	0.313	0.32	0.003	0.122	0.08	0.125	$P \downarrow, N \approx 0$
2	9♣ 7♠	0.669	0.46	0.95	0.001	0.1	0.954	$P \approx 0, N \downarrow$
	3♠ 2♦	0.330	0.32	0.003	0.122	0.08	0.125	$P \downarrow, N \approx 0$
3	J♣ 9♣	0.419	0.55	0.234	0.356	0.309	0.506	$P \sim, N \uparrow$
	A♠ 10♦	0.58	0.62	0.896	0.009	0.3	0.906	$P \approx 0, N \uparrow$
4	K♠ 8♣	0.400	0.559	0.775	0.139	0.314	0.806	$P \downarrow, N \uparrow$
	A♦ 6♣	0.599	0.575	0.668	0.154	0.324	0.720	$P \downarrow, N \uparrow$
5	A♦ 3♣	0.609	0.558	0.47	0.113	0.418	0.53	$P \downarrow, N \approx max$
	K♠ 4♣	0.390	0.525	0.373	0.129	0.421	0.453	$P \downarrow, N \approx max$
8	10♣ 8♠	0.471	0.499	0.938	0.163	0.237	0.948	$P \downarrow, N \sim$
	6♠ 6♦	0.528	0.634	0.939	0.315	0.179	0.958	$P \sim, N \sim$
9	J♣ 9♣	0.638	0.557	0.234	0.356	0.309	0.507	$P \sim, N \uparrow$
	7♦ 5♦	0.362	0.437	0.106	0.307	0.357	0.381	$P \sim, N \uparrow$
10	K♣ J♥	0.438	0.604	0.42	0.428	0.199	0.669	$P \uparrow, N \sim$
	A♥ 3♣	0.562	0.559	0.471	0.427	0.193	0.697	$P \uparrow, N \sim$
12	8♣ 3♠	0.220	0.376	0.721	0.146	0.334	0.761	$P \downarrow, N \uparrow$
	K♦ 8♦	0.779	0.585	0.775	0.14	0.314	0.807	$P \downarrow, N \uparrow$
13	Q♣ 10♦	0.652	0.572	0.882	0.106	0.299	0.894	$P \downarrow, N \uparrow$
	8♣ 6♠	0.347	0.432	0.927	0.15	0.262	0.938	$P \downarrow, N \uparrow$

ID 1: In this testcase, we can predict that the **vs Odds** and **MCP** will strongly lean towards 9♥ 7♥ . This is because this hand already possesses a much better high card before the flop (suited cards add value too). After the flop, 9♥ 7♥ should have 0 potential to progress into a better hand since it is the best possible hand that can be constructed even if 2 more cards come (already a straight flush). On top of that, it has zero potential to become a losing hand later on since it is the *nuts*. Hence, $PPot_2$ and $Npot_2$ values should both be nearly zero which is verified by our results.

On the other hand, 3♣ 2♠ should have a very low positive potential since it possesses minimal situations in which it could improve to the best hand. Furthermore, its negative potential should be nearly zero since there are barely any hands it is beating on the flop. Therefore, there is a very low probability of 3♣ 2♠ becoming the worst hand since the subset of hands it could beat are too low. Our results confirm these predictions and hence, for **ID 1**, our **EHS** values should be correct.

ID 3: In this testcase, we can predict that the **vs Odds** and **MCP** will lean towards A♠ 10♦ . This is because this hand already possesses a much better high card before the flop. After the flop, J♣ 9♣ should have a decent potential to progress into a better hand since it is an *open-ended straight draw*. Therefore, any 7 or any Queen would complete its straight. On top of that, jacks can also be considered as cards that would improve this hand since it would produce a *top pair*. However, J♣ 9♣ also has a high potential to become a losing hand later on. This is because even though it is currently beating the set of hands that possess a heart, there is a good chance a heart or a pair that beats jack high would arise after the river. Hence, $PPot_2$ should be mediocre and $Npot_2$ must be high which is verified by our results.

On the other hand, A♠ 10♦ should have nearly zero positive potential since the only cases where it may improve are when either an Ace or a Ten comes in the further cards, but such improvements are nullified once a heart comes. Therefore, the probability of A♠ 10♦ ending up as the best hand is far too low. Furthermore, its negative potential should be high since it has a high potential of becoming a losing hand on further streets. Even though it is currently beating the set of hands with a heart as well as the majority of high cards, there is a strong chance a heart of a pair that beats ace high would arise after the showdown. Our results confirm these predictions and hence, for **ID 3**, our **EHS** values should be correct.

ID 5: In this testcase, we can predict that the **vs Odds** and **MCP** will lean towards A♦ 3♣ . This is because Ace high card strongly dominates King high card before the flop. After the flop, A♦ 3♣ should have a very low potential to progress into the better hand. This is because its possible outs that may project it into the winning hand will majorly be another Ace coming on the turn or river. A three coming on later streets would not be so impactful since it would result in a bottom pair. Therefore, its low (non-zero) positive potential would mostly arrive in the scenarios where another Ace comes and no further hearts (since flush beats pair or high card). However, A♦ 3♣ should have one of the highest negative potentials because even though it is currently beating the set of hands with a heart and also the set of hands with a high card below Ace, there is an unbelievably high chance that this hand will lose by showdown. This is because a heart or a pair that would beat Ace high will probably come in the next two cards. Hence, the results $PPot_2$ should be very low and $Npot_2$ must be nearly maximum which is verified by our results.

On the other hand, K♠ 4♣ should have low positive potential for the same reasons as A♦ 3♣ . Its possible improvement cards are basically the remaining kings (only three) and on top of that, only in scenarios where another heart doesn't come. Therefore, there are barely any scenarios where K♠ 4♣ becomes the better hand. However, K♠ 4♣ should have one of the highest negative potentials (similar to A♦ 3♣ because even though it is currently beating the set of hands with a heart and also the set of hands with a high card below King, there is a strong chance that this hand will lose by showdown. This is because a heart or a pair that would beat King high will probably come in the next two cards. Hence, $PPot_2$ should be very low and $Npot_2$ must be nearly maximum which is verified by our results. Our results confirm these predictions and hence, for **ID 5**, our **EHS** values should be correct.

ID 10: In this testcase, we can predict that the **vs Odds** will lean towards A♥ 3♣. This is because Ace high dominates King high before the flop. However, the **MCP** will lean towards K♣ J♥ since this would beat any two random cards more times than A♥ 3♣. Both King and Jack are strong cards that contribute to a pair whereas only the Ace is a good card to consider for a strong pair (Three does not hold much merit as practically any pair beats it). On top of that, there are more straight possibilities for K♣ J♥ compared to A♥ 3♣. From this reasoning, it explains the higher **MCP** for K♣ J♥. After the flop, K♣ J♥ should have quite a high potential to progress into the better hand. This is because we can consider Kings (except K♥ as this would produce a *one card flush*), the remaining hearts, and maybe even Jacks as the outs that would project it into the winning hand. Hence, since it has such a high potential of winning at showdown (because it has so many outs), we should expect a strong positive potential from it. However, this hand should have a decent negative potential because even though it is currently beating the set of hands without a pair (below King high card), there is a mediocre chance that a pair (which would beat King high card) could come on the turn or river. Hence, the results $PPot_2$ should be very high and $Npot_2$ must be decent which is verified by our results.

On the other hand, A♥ 3♣ should have high positive potential because it is the *nut flush draw*. This means that if another heart comes, A♥ 3♣ would be the best possible flush and most likely the best hand that can be constructed. Therefore, for this flop, possible outs that may progress this hand to the better hand are majorly the remaining Aces (to produce Ace top pair) and the remaining hearts. Since there are so many outs, this would justify the high $Ppot_2$ value that would arise. However, this hand should have a decent negative potential for the same reasons as K♣ J♥. Even though it is currently beating the set of hands without a pair (below Ace high card), there is a mediocre chance that a pair (which would beat King high card) could come on the turn or river. Our results confirm these predictions and hence, for **ID 10**, our **EHS** values should be correct.

5.7.2 Testcase 2

Flop: Q♠ Q♦ 10♣

Table 2: Testcase 2 and their outputs for the fixed flop as shown above

ID	Hand	vsOdds(MC)	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
6	Q♣ 10♦	0.334	0.572	0.999	0.0	0.005	0.999	$P = 0, N = 0$
	10♠ 10♥	0.665	0.753	0.997	0.0	0.028	0.997	$P = 0, N = 0$
7	Q♣ 10♦	0.427	0.569	0.999	0.0	0.005	0.999	$P = 0, N = 0$
	A♣ 8♠	0.572	0.598	0.704	0.153	0.244	0.749	$P \downarrow, N \sim$
11	Q♣ Q♥	0.817	0.797	1.0	0	0.0	1.0	$P = 0, N = 0$
	10♥ 10♠	0.182	0.748	0.997	0.0	0.028	0.997	$P = 0, N = 0$

ID 11: In this testcase, we can predict that the **vs Odds** will strongly leans towards Q♣ Q♥ . This is because it dominates the 10♥ 10♠ pair preflop and the only way for that hand to improve would be if another Ten or a Broadway (10 to Ace straight) were to arise by the river, which is very unlikely. However, the **MCP** values for both hands will be nearly similar since Q♣ Q♥ beats all the same hands as 10♥ 10♠ except for Jacks and Ace-King Suited (in some scenarios). After the flop, Q♣ Q♥ has literally zero potential of progressing into a better hand. This is because *Quads* have been flopped and that is the best possible situation. In addition, this hand has practically zero chance of becoming the worse hand. This is because that possibility would only arise if a Straight Flush or Royal Flush were possible, but since the board is rainbow (all different suits) and the hand is not suited, *Quads* is the best possible hand. Hence, the results $PPot_2$ must be zero and $Npot_2$ must be zero is verified by our results.

On the other hand, 10♥ 10♠ should have zero positive potential since it has already flopped the lower full house. Therefore, if it was currently the worse hand, there are no scenarios where it can improve and become the better hand. For example, at present, the only hands that make 10♥ 10♠ the worse hand are all QQ holdings and all Q10 holdings. In the QQ case, 10♥ 10♠ can never win since QQ has already formed the best cards. In the Q10 case, 10♥ 10♠ can only improve if another 10 comes but since there is already a 10 on the flop, all the 10's have been exhausted from the deck and hence, 10♥ 10♠ has zero outs to become the better hand. However, if we consider the situation where it is currently the better hand, there is nearly zero negative potential it becomes the worst hand. This is because it has already flopped a full house and there are minimal scenarios a better full house would arise from a losing position. Our results confirm these predictions and hence, for **ID 11**, our **EHS** values should be correct.

5.7.3 Testcase 3

Flop: 9♣ 5♣ 2♠

Table 3: Testcase 3 and their outputs for the fixed flop as shown above

ID	Hand	vsOdds(MC)	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
14	A♠ A♦	0.824	0.853	0.966	0.221	0.144	0.974	$P\sim, N\sim$
	2♣ 2♦	0.175	0.503	0.994	0.089	0.074	0.995	$P\approx 0, N\approx 0$
15	A♠ A♦	0.826	0.852	0.966	0.221	0.144	0.974	$P\sim, N\sim$
	K♥ 9♥	0.173	0.601	0.939	0.159	0.2	0.949	$P\downarrow, N\sim$
16	K♠ K♦	0.678	0.824	0.961	0.188	0.166	0.968	$P\sim, N\sim$
	A♣ Q♣	0.321	0.662	0.585	0.492	0.148	0.789	$P\uparrow, N\sim$
17	A♠ A♦	0.864	0.854	0.966	0.221	0.144	0.974	$P\sim, N\sim$
	5♠ 2♣	0.135	0.343	0.982	0.093	0.153	0.984	$P\approx 0, N\sim$

ID 16: In this testcase, we can predict that the **vs Odds** and **MCP** will strongly lean towards K♠ K♦ . This is the second best hand in the game (second best pair before the flop itself) which justifies that it would beat A♣ Q♣ (only Ace high card) more times in a 1v1 scenario as well as beat more random hands (as confirmed by greater MCP value). After the flop, the **HSE** value for K♠ K♦ is ≈ 0.38 greater than A♣ Q♣ . However, the latter hand is the *nut flush draw* and therefore, it will have a high positive potential to become the best hand. This is because any club or Ace that comes on the turn/river

would basically ensure that A♣ Q♣ is the winning hand. Hence, such a discrepancy should not be there between these hands' strength estimations which highlights the inaccuracy of **HSE** when it comes to hand strength estimation. On the other hand, **EHS** takes into account $PPot_2$ and as a result, the difference between their hand strength dwindles down to ≈ 0.18 (according to EHS).

5.7.4 Testcase 4

Flop: 7♦ 10♦ A♣

Table 4: Testcase 4 and their outputs for the fixed flop as shown above

ID	Hand	vsOdds(MC)	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
14	8♦ 9♦	0.532	0.508	0.263	0.6	0.12	0.705	$P = max, N \downarrow$
	3♠ 3♣	0.467	0.537	0.605	0.081	0.314	0.637	$P \approx 0, N \uparrow$

ID 14: This testcase was made to determine the (one of) highest possible positive potentials in the game. Furthermore, it highlights the importance of **EHS** over **HSE** for hand strength estimation. In this testcase, 8♦ 9♦ is an *open-ended straight/flush draw*. Hence, if we only consider that, it possesses 15 outs to become the better hand. On top of that, there are marginal cases where an 8 pair or a 9 pair could also win (against holdings that currently possess a 7 pair or holdings which are underpairs such as 66, 55, 44, 33, 22). Therefore, it has the maximum potential of becoming the better hand. On the other hand, if it is somehow currently the better hand, the subset of hands that 8♦ 9♦ beats is substantially low. Hence, the negative potential of becoming the worse hand would be quite low since the probability of that small subset hitting a pair is minimal. Hence, the results $PPot_2$ must be maximum and $Npot_2$ must be low is verified by our results.

On the other hand, 3♠ 3♣ has nearly zero positive potential since it requires another three (2 outs) to become the better hand. However, even in these cases, the hand must avoid flushes or better sets and such occurrences are improbable. If we consider its negative potential, we can confidently say that that will be high because there is a strong probability that a flush or maybe even a better set occurs in this situation. Therefore, the results $PPot_2$ must be around zero and $Npot_2$ must be high are verified by our results. Now, let's analyse the **HSE** and **EHS** for this testcase. After the flop, the **HSE** value for 8♦ 9♦ is ≈ 0.34 **lower** than 3♠ 3♣ . However, the latter hand is an *open-ended straight/flush draw* and therefore, it will have one of the highest positive potentials to become the best hand. **EHS** takes into account $PPot_2$ and as a result, the **EHS** value for 8♦ 9♦ comes out to be ≈ 0.07 **greater** than 3♠ 3♣ .

5.7.5 Inferences

As we can see in the tables aboves, our predictions for the testcases match with the values outputted by the evaluation functions. Hence, we can confidently claim that our **EHS** values are correct for any set of cards. Here, we mention some inferences that we notice from the testcases. Through our analysis, we found that the highest $Ppot_2$ came out to be ≈ 0.6 and the lowest being 0. On the other hand, the highest $Npot_2$ came out to be around 0.44 and the lowest being 0. ID 5 corresponds to the case with the maximum $Npot_2$ whereas ID 14 corresponds to the scenario with the maximum $Ppot_2$. These testcases were important as they helped in quantifying a range for $Ppot_2$ and $Npot_2$ values. Overall, we quantified our ranges for $Ppot_2$ as: 0-0.17 (**low**), 0.17-0.39 (**mediocre**), and 0.39-0.60 (**high**). Our ranges for $Npot_2$ were as follows: 0-0.15 (**low**), 0.15-0.26 (**mediocre**), and 0.26-0.44 (**high**).

We noticed a pattern that when both $Ppot_2$ and $Npot_2$ values are both equal to 0, the **EHS** of that hand is extremely high. This result makes sense since the hand must be extremely strong as there is no potential to get worse or better. In other words, the hand represents the **nuts** which corresponds to the highest **EHS** possible. Furthermore, when $NPot$ is high and $Ppot$ is low, the hands are quite strong (high EHS). This goes to show that established hands with low positive potential to get more established are strong hands even when they are prone to a decent chance of becoming the losing hand later. Hence, $Npot_2$ being a high value does not necessitate that a hand is bad if its establishment is strong enough to compensate the negative potential.

Overall, before even making the testcases, we had a strong feeling that **EHS** served as a strong hand strength tool than **HSE** since it also considered potential of becoming an established hand. In the HSE strength estimation, the establishment of the hand serves as a strong and only factor in determining its strength. However, in cases where a hand has a lot of outs to become established, these situations must be handled and EHS serves that purpose. Therefore, testcase 4 ultimately validated our inference since the **EHS** result is completely different from the **HSE** result.

6 Links to Codebase

Link to all of the code used to calculate the implemented hand assessment functions can be found here.

7 Conclusion

This final report was made for our Independent Study: **Poker Theory and Analytics**. Initially, our main priority was introducing ourselves to the rules and intricacies in the game of Poker. After that, we picked upon analytical techniques that would aid us in applying mathematical concepts within the game. Through such techniques, we developed a stronger understanding of important aspects of the game such as *pot odds*, *implied odds*, *fold equity*, *bluffing*, *semi-bluffing*, etc. In addition, we acquainted ourselves with preflop-analysis strategies which involve utilizing range matrices and interpreting the insight behind the Sklansky-Karlson Rankings. Instead of diving into more poker theoretical domains such as Poker Economics, Game Theory, and Game Theory, we wanted to focus more on the implementation of some aspect of Poker. This would aid us in developing a more keen knowledge of the game instead of incessantly pouring ourselves with knowledge. After doing some searching, we came upon the following paper [1] and decided to implement **Hand Assessment**. This is because a significant chunk of it involves Preflop Evaluation which was a topic we had just touched upon. Ultimately, we were able to successfully implement the **Preflop Evaluation**, **Hand Strength**, and **Hand Potential** sections from the paper. As a result, we were able to create an accurate implementation of the **EHS** tool which is a strong hand estimation approximator. We are content with the results so far and hope to further implement other functions that broach other aspects of Poker.

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