Poker Theory and Analytics

Prerak Srivastava
Ruthvik Kodati

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Introduction

- **Poker** refers to a collection of card games in which players compare ranked **hands** in competition for a **pot** of money.
- The size of the pot increase through **betting**, where the players bet on the hand that they have or could potentially get. In order to win at poker, one must use an optimal **Bet**
- What defines "winning" in Poker?
    - **Return on Investment**
    - Deciding when to bet and when to not bet to minimize risk and maximize profits.

Poker Variations

Poker is a generic name for hundreds of game variations, but we will primarily be focusing on the **Texas Hold'em** variation.

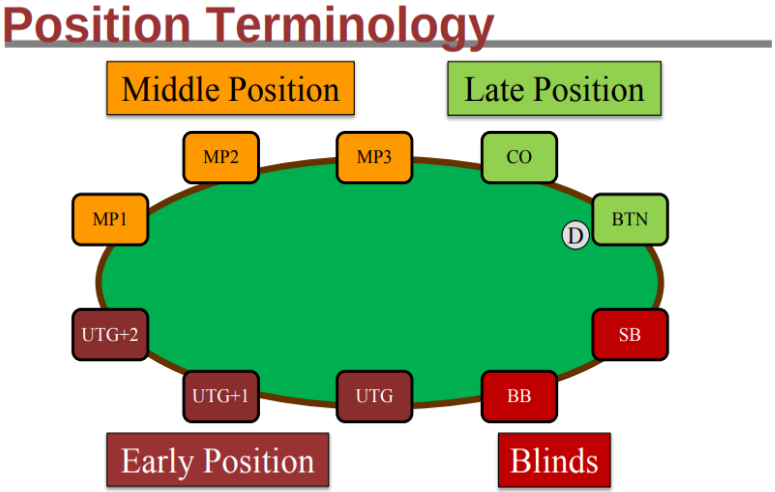
- The game starts off with each **player** being dealt 2 cards called **hole cards** face down by **the dealer**.
- 3 **community cards** are then dealt face up after a fourth and a fifth card is dealt separately. These are called the **flop**, **turn**, and **river**.
- Bets are placed after each step as players call, raise or fold after checking their cards.
- The aim is to make the best five-card poker hand out of the seven cards available.

Texas Hold’em Poker Variations

There are mainly 2 variations of Hold’em Poker, out of which No Limit is the more popular variation.

- **Pot Limit Texas Hold’em**:
  - The betting range for each player is capped by the size of the pot.
  - Therefore it’s not possible for a player to go all-in if they have more chips than the size of the pot. So the concept of over betting is typically not applicable in this variant.
  - Pre-flop action is much less aggressive as compared to No-limit Texas Hold’em merely because of the betting cap.
- **No Limit Texas Hold’em**:
  - The betting range for each player is uncapped.
  - This variant allows player to go **all in** whenever they see fit.
  - This variant is more risk prone, making it more fun to watch and play the game.

Texas Hold’em Poker Player Positions



Texas Hold'em Poker Player Positions

- **Blinds**:
  - In the image, **SB** and **BB** refer to the Blinds position at the poker table.
  - The blinds introduce a regular cost to take part in the game, thus inducing a player to enter the pot in an attempt to compensate for that expense.
  - The blinds are paid each hand by the players who are occupying the small blind and big blind seats at the table.
  - An **x/y poker game** has two blinds x and y, where x is **small blind** and y is **big blind**.
  - The bets are called blinds because they are made blind, that is, before you know what your cards are.

There are 4 **Streets of Texas Hold'em**:

- Preflop
- Flop
- Turn
- River

First Betting Round: Preflop

- The first round of betting takes place right after each player has been dealt two hole cards.
- In this round, the blinds are posted and each player is dealt two hole cards.
- The first player to act is the player to the left of the big blind (under the gun).
- UTG has 3 options:
  - **Call**: Match the amount of the big blind.
  - **Raise**: Increase the bet within the specific limits of the game.
  - **Fold**: Throw the hand away.
- If the player chooses to fold, he or she is out of the game and no longer eligible to win the current hand.
- Once the last bet is called and the action is 'closed,' the preflop round is over and play moves on to the "flop."

Second Betting Round: The Flop

- After the first preflop betting round has been completed, the **first three community cards are dealt** and a second betting round follows involving only the players who have not folded already.
- From here onwards, the action starts with the player to the left of the button (BTN) , which is the SB (small blind).
- Along with the options to bet, call, fold, or raise, a player now has the option to **check** if no betting action has occurred beforehand.
- **A check** simply means to pass the action to the next player in the hand.

Third Betting Round: The Turn

- The fourth community card, called **the turn**, is dealt face-up following all betting action on the flop.
- Once this has been completed, another round of betting occurs, similar to that on the previous street of play.

Final Betting Round: The River

- The fifth community card, called **the river**, is dealt face-up following all betting action on the turn and another round of betting occurs.
- After all betting action has been completed, the remaining players in the hand with hole cards now expose their holdings to determine a winner. This is called **the showdown**.

The Showdown

- The remaining players show their hole cards, and with the assistance of the dealer, a winning hand is determined.
- The player with the best combination of five cards wins the pot according to the official poker hand rankings.
- The winner is then rewarded with all the money from the pot.

Hand Hierarchy in Texas Hold'em

The hand hierarchy that is used during the showdown is based on the probability of the hand occurring in game. These hand rankings aren't specifically part of Texas hold'em rules, but apply to many different poker games like Omaha and Seven-star stud.

1. **Royal Flush**(Ace High Straight Flush): Five cards of the same suit, ranked ace through ten. Eg. A♥ K♥ Q♥ J♥ 10♥
2. **Straight Flush**: Five cards of the same suit and consecutively ranked. Eg. 9♠ 8♠ 7♠ 6♠ 5♠
3. **Four of a Kind**: Four cards of the same rank. Eg. 5♥ 5♠ 5♣ 5♦ 10♥
4. **Full House**: Three cards of the same rank and two more cards of the same rank. Eg. 3♥ 3♦ 3♣ J♥ J♦

Hand Hierarchy in Texas Hold'em

5. **Flush**: Any five cards of the same suit. Eg. A♥ 8♥ 6♥ 4♥ 2♥
6. **Straight***: Any five cards consecutively ranked. Eg. A♥ 2♣ 3♥ 4♠ 5♦
7. **Three of a Kind**: Three cards of the same rank. Eg. K♦ K♥ K♦ J♥ 10♥
8. **Two Pair**: Two cards of the same rank and two more cards of the same rank. Eg. J♥ J♦ 3♥ 3♠ 10♥
9. **One Pair**: Two cards of the same rank. Eg. A♥ A♦ Q♥ J♥ 10♣
10. **High Card**: Five unmatched cards. Eg. J♥ 9♦ 6♣ 4♥ 3♥

* **NOTE**: The Ace card can be chosen to be either at the lowest rank or the highest rank in the hand.

Hand Hierarchy

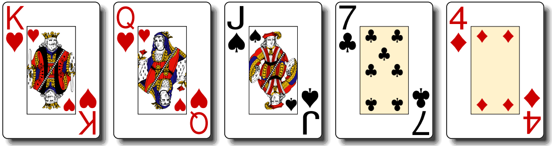
Hand	Probability	Odds Against	Frequency
Royal Flush	0.000154%	649,739 : 1	$\binom{4}{1} = 4$
Straight Flush	0.00139%	72,192.33 : 1	$\binom{10}{1}\binom{4}{1} - \binom{4}{1} = 36$
Four of a Kind	0.02401%	4,165 : 1	$\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1} = 624$
Full House	0.1441%	693.1667 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3744$
Flush	0.1965%	508.8019 : 1	$\binom{13}{5}\binom{4}{1} - \binom{10}{1}\binom{4}{1} = 5108$
Straight	0.3925%	253.8 : 1	$\binom{10}{1}\binom{4}{1}^2 - \binom{10}{1}\binom{4}{1} = 10200$
Three of a Kind	2.1128%	46.32955 : 1	$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2 = 54912$
Two Pair	4.7539%	20.03535 : 1	$\binom{13}{1}\binom{4}{2}^2\binom{11}{1}\binom{4}{1} = 123552$
One Pair	42.2569%	1.366477 : 1	$\binom{13}{1}\binom{4}{2}^2\binom{12}{3}\binom{4}{1}^3 = 123552$
High Card	50.1177%	0.9953015 : 1	$[\binom{13}{5} - \binom{10}{1}][\binom{4}{1}^5 - \binom{4}{1}] = 1302540$

Constructing the Best Possible Hand

- Players construct their hands by choosing the five best cards from the seven available (their two hole cards and the five community cards).
- When comparing two hands, the winner is chosen using the **First Point of Difference** which follows the rank order of the cards.

Constructing the Best Possible Hand

For example, let the 5 community cards for a given hand be as follows:



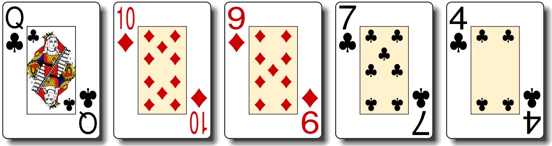
- And let's say **The Hero** (the player that we are concerned about) is dealt the cards K♣ 8♥, and **the Villain** (the player the hero is going against in the showdown) is dealt K♦ 5♥.
- The best possible hand that the Hero can construct will be K♥ K♣ Q♥ J♠ 8♥. Firstly, the Hero identifies a hand (in this case it's One Pair), followed by choosing the remaining cards in their hand according to the cards' ranks.
- Therefore, the two Kings are selected and from the remaining 5 cards (Q,J,8,7,4) the top 3 are selected.

Constructing the Best Possible Hand

- Similarly, the best possible hand that the Villain can construct will be K♥ K♦ Q♥ J♠ 7♣.
- We can see that both the hands have a King Pair, and therefore we can see that the first point of difference between the two hands from the remaining cards occurs at the 5th card.
- Since the rank of the Hero's last card is higher than that of the Villain's, therefore in this case, the Hero would win the showdown.
- Alternatively, if the Hero had K♣ 6♥, the Hero and the Villain both will have the same hand K♥ K♦ Q♥ J♠ 7♣. Hence, there is no point of difference between the hands here and the pot would be split as the showdown has resulted in a draw.

Constructing the Best Possible Hand

Another example, let the 5 community cards for a given hand be as follows:



- And let's say **The Hero** is dealt the cards A♣2♣, and **the Villain** is dealt K♣8♣. The best possible hand that the Hero can construct will be A♣Q♣7♣4♣2♣and the Villain's will be K♣Q♣8♣7♣4♣
- Here, the Hero's hand is called a **Ace-High Flush** or **Nut Flush** since it's the best possible flush with an Ace as the high card.
- The Villain's hand is a King-High Flush which is the second best possible flush. Therefore, in this case the Hero wins, because the first point of difference is found at the first position.

Pot Odds: Introduction

The main goal of developing all tools for poker theory is to answer the following question:

What is the maximum bet the Hero should call for a given hand?

To quantify this, we define **Expected Value**:

- *EV* is considered as the most important and fundamental concepts in Poker
- *EV* is the average result of a given play if it were played hundreds or even thousands of times.
- Expected Value is the probability-weighted average of possible results.
- Generally refers to the expected value of **winning** (denoted by $E[W_c]$)

Pot Odds: Terminology

In general, we seek to make **Positive EV plays** to ensure profits over a long period of playtime.

To deduce $E[W_c]$, we take into account a combination of the following factors:

- Money in the pot
- Bet that the Hero is facing
- Winning Percentage
- Fold Percentage

Pot Odds

Pot odds represent the ratio between the size of the total pot and the size of the bet facing you. Keep in mind that the size of the total pot includes the bet(s) made in the current round.

It is defined as

$$PO = \frac{B}{P + B}$$

For example, let the pot be \$100 and the Villain bets \$50. The Pot Odds for the Hero would be equal to

$$\frac{50}{100 + 50 + 50} = \frac{1}{4} = 0.25$$

From here, the Hero will want to convert your pot odds into a percentage so they know exactly how much equity their hand needs to profitably call the bet.

Pot Odds: Winning Percentage

- Winning Percentage is defined as the **equity** of a hand in Poker.
- It denotes the probability of the hand winning in a particular scenario.
- This probability differs after every street in Poker.
- For a play to be $+EV$, by definition, the winning percentage **q** must be greater than the pot odds.

Pot Odds: Expected Value

$$E[W_c] = Pq - B(1 - q)$$

Here,

- W_c refers to the winnings if the Hero calls
- P refers to the size of the pot
- q refers to the **Win Percentage**
- B refers to the calling amount (current bet size).

Generally speaking, one should call only if $E[W_c] > 0$ which gives us

$$q > \frac{B}{P + B}$$

$$\therefore q > PO$$

Established Hand VS Drawing Hand

- During the game, the player can have one of 2 kinds of hands:
  - **Established Hand**: A player with a hand that is formed on the flop, but could get weaker on subsequent streets because of having fewer outs to convert it to a stronger hand. Examples are One-pair, Two-pair etc.
  - **Drawing Hand**: A player could currently have nothing or a very weak hand, but it has more number of outs which increases its **potential** to become a stronger hand in the subsequent streets.

Established Hand VS Drawing Hand

- For example, the player could have a One-pair on the flop, but can turn into a flush on the turn or the river, and plays **based on the odds** of those cards appearing on further street which could make their hand better.
- The drawer has to take into account the chance of hitting his draw and the cost of seeing further streets. Based on this, the **betting strategy** is designed.
- Through this analysis, the Hero makes a $+EV$ or $-EV$ play.

Outs and Winning Percentage Definition

- Typically, bad players assume they have good odds in scenarios where they should ideally fold.
- The concept of **Outs** is introduced in order to calculate winning odds and determine winning percentage at any given point in the game.
- An **Out** is defined as a card that can appear in subsequent streets, which on appearing would complete the hand that the drawer is drawing to.
- **Winning Percentage** is used by the player with a **drawing hand** and is defined as the probability of one of our outs appearing in the subsequent streets. It can be written as

$$q = \frac{\text{Number of outs}}{\text{Number of Cards Remaining in the deck}}$$

Calculating Outs Examples

Suppose the flop is Q♥ 8♣ 7♠ and the Hero has 9♦ 6♦.
In this scenario, the Hero has an **Open-ended Straight Draw**.

- The draw is called a **Straight Draw** because the Hero currently has 4 consecutive cards and only requires one more card to obtain the straight.
- The draw is called an **Open Ended Straight Draw** since the draw can be completed from both ends, on getting a Ten or Five of any suit in the subsequent streets.

Here we can say that the Hero has **8 outs**:

- Any 10: 10♣ 10♥ 10♦ 10♠
- Any 5: 5♣ 5♥ 5♦ 5♠

Pot Odds: Gordon's Rule of 2 or 4

- Winning percentage is calculated as the probability of the player's outs appearing on subsequent streets. But calculating this value during a game for each possible hand in a reasonable amount of time is extremely difficult.
- After observing the probabilities of various hands, professional poker player **Phil Gordon** established a basic rule of thumb to **assign equity to outs**.

Pot Odds: Gordon's Rule of 2 or 4

- The Rule of 2 or 4 is used to calculate the approximate **Winning Percentage** of a hand.
- The rule is defined as follows:
    1. Each out is worth approximately 2% equity per card.
    2. If you get to see the card on both the turn and the river without any additional betting, then each out is worth approximately 4% equity per card. The probability gets doubled since you get to see 2 cards rather than 1.

Pot Odds: Gordon's Rule of 2 or 4 Calculation

Suppose the flop is Q♦ 8♦ 7♠ and the Hero has 9♦ 2♦. The Villain has not gone all in, which means subsequent betting rounds will take place.

- In this scenario, the Hero has an **Flush Draw**, because the Hero currently has 4 cards of the same suit and only requires one more card to obtain the flush. Any diamond card on the turn/river would complete the hand.
- Here we can say that the Hero has **9 outs** since there are 2 diamond card on the table and the Hero also has 2 diamond cards, so 9 diamond cards remaining in the deck.
- Therefore the winning percentage will be

$$q = \frac{9}{47} = 0.191 \approx 18\% = 9 \cdot 2\%$$

Pot Odds: Gordon's Rule of 2 or 4 Calculation

Suppose the flop is Q♦ 8♦ 7♠ and the Hero has 9♦ 2♦ and the villain has gone **all-in** on the flop itself.
Therefore the winning percentage will be

$$q = \frac{9 \cdot 2}{47} = 0.382 \approx 36\% = 9 \cdot 4\%$$

Pot Odds: Gordon’s Rule of 2 or 4 Calculation

Suppose the flop is Q♦ 8♦ 7♠ and the Hero has 9♦ 6♦.

- In this scenario, the Hero has an **Straight/Flush Draw**.
- Again, the Villain has not gone all-in on the flop.
- Here we can say that the Hero has **15 outs** :
  - 9 Flush Outs: The remaining diamond cards.
  - 8 Straight Outs: All 5’s and 10’s.
  - Here 5♦ and 10♦ has been counted twice so they’ll be subtracted from the total.
- Therefore the winning percentage by the turn will be

$$q = \frac{15}{47} = 0.319 \approx 30\% = 15 \cdot 2\%$$

Pot Odds: Expected Value Calculation

- In general, decision rules will be made based on Expected Value.
- In order to calculate the *EV* in a given scenario the first step would be to **Deduce whether Hand is Established or Drawing**
- Based on the type of hand, *EV* is calculated differently and the decision is taken based on that.

Pot Odds: Expected Value Calculation For Established Hand

If the hand is **established**, then we have to make a bet substantial enough to reduce the effective EV of the players that are drawing to a hand.

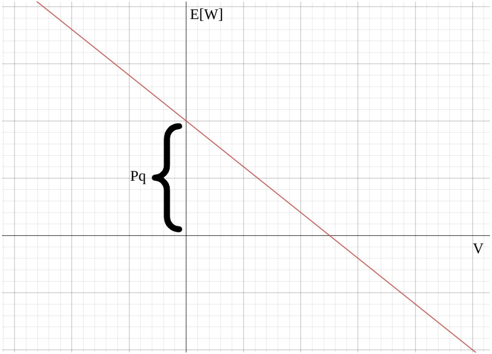
Let us consider the scenario where the Hero and the Villain are playing with a pot of size P. The Villain makes a bet of \$V, making the pot of size $P + V$. Let the win percentage of the Hero be q_H. Therefore,

$$E[W_c] = (P + 2V)q_H - V(1 - q_H) = Pq_H - V(1 - 3q_H)$$

Here, we can see that EV of the hero is **directly proportional** to $-V$.

Pot Odds: Expected Value Calculation For Established Hand

$$E[W_c] = Pq_H - V(1 - 3q_H)$$



Since the Hero has an unestablished hand, $q < \frac{1}{3}$. Therefore, as the Villain increases the size of his bet (bloats the pot), the *EV* of the Hero during their turn decreases.

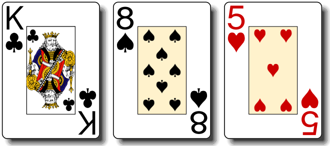
Pot Odds: Expected Value Calculation For Drawing Hand

On the other hand, if the hand is **drawing**, we do the following:

- **Deduce Drawing Hand**: Finding the hand that the Hero can draw to.
- **Calculating Outs**: Calculating the number of outs through which the Hero can complete their draw.
- **Calculating Win Percentage**(q): Through the number of outs and the Rule of 2 or 4, the Hero can calculate the equity of their hand.
- **Calculating Pot Odds**: Using the amount of money that the Hero needs to call in their turn and the money that is already in the pot, we can calculate the pot odds.
- **Calculating EV**: Calculate the EV using q which will be used by the Hero to make the decision.

Pot Odds: Drawing Hand Example

For this **drawing hand** example, the flop is



and the Hero has 6♦ 7♦ and the Villain has gone **all-in** by putting $150 into the pot, making the pot a total of $470

The calculation is as follows:

- **Deduce Drawing Hand**: The Hero has a **Open-ended Straight Draw**.
- **Calculating Outs**: Any 9 or Any 4 would complete the draw. Therefore there are **8 Outs** in total.

Pot Odds: Drawing Hand Example

- **Calculating Win Percentage**:

$$q \approx 8 \cdot 4 = 32\%$$

- **Calculating Pot Odds**:

$$PO = \frac{B}{P + B} = \frac{150}{620} \approx 24\%$$

- **Calculating EV of decision**:

$$E[W_c] = Pq - B(1 - q) = 32 \cdot 470 - 68 \cdot 150 = 48.4$$

Since $q > PO$, in this scenario **calling will be +EV play** and the Hero should call.

Pot Odds: Drawing Hand Example

Let us alter the scenario, where the Villain **does not go all-in**, and bets the same \$150, making the pot a total of \$470. The procedure will be the same as the previous example except for the win percentage and the EV.

- **Calculating Win Percentage**:

$$q \approx 8 \cdot 2 = 16\%$$

- **Calculating Pot Odds**:

$$PO = \frac{B}{P + B} = \frac{150}{620} \approx 24\%$$

- **Calculating EV of decision**:

$$E[W_c] = Pq - B(1 - q) = 16 \cdot 470 - 84 \cdot 150 = -50.8$$

Since $q < PO$, in this scenario **calling will be -EV play** and the Hero should fold.

Implied Odds: Introduction

- Implied odds are the amount of money that you **expect to win** on later streets if you hit one of your outs, **based on the money put into the pot by the opponents**.
- During a poker game, a player can gauge the aggression of the opponents, which is defined as the amount of money bet, usually calculated as a fraction of money in the pot, and use this to their advantage to make $+EV$ plays.

Implied Odds: Introduction

- Typically the Hero is expected to fold whenever they have to make a $-EV$ play (according to pot odds), but using implied odds, the Hero can gauge taking a risk and making a $-EV$ play, which could eventually turn into a $+EV$ play by the end of the river.
- If you expect to win more money from your opponent after you hit your draw, then you have **good implied odds**. But if you anticipate not being able to get any more money from your opponent on future streets, then you have **little or no implied odds**.

Implied Odds: Calculating Implied Odds

- Implied Odds are calculated by figuring out what the pot would have to be after our call to make the x% chance of winning equal to the x% of the pot for the call.
- For example, if we have a **Flush Draw** (q = 18%), and we are facing a bet of \$180 into a pot of \$300, $PO = 180/660 = 27\%$ (i.e. too expensive to call)
- This would be a good call if we contributed 18% of the pot. So we need to find \$1000\$660 = \$340 in **dead money**.
- The additional \$340 after the draw makes our \$180 bet worth 18% of a \$1000 pot.

Implied Odds: Calculating Implied Odds

Given the scenario where the Hero is facing a bet of \$B with a pot of \$P. Let the win percentage of the Hero be q and the pot odds be PO. The Hero calling would be a $-EV$ play.

$$PO = \frac{B}{P + B}$$

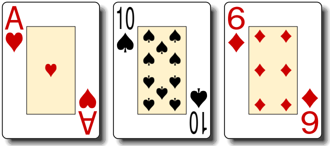
Since this is a $-EV$ play, $q < PO$. Therefore, for this play to become $+EV$, we need to calculate the amount of dead money that needs to be added to the pot in order for this play to become $+EV$.
Let the amount of extra dead money for the hero to break even ($0EV$) be \$X. At $0EV$, $q = PO$

$$q = \frac{B}{P + B + X}$$

$$\therefore X = \frac{B}{q} - P - B$$

Implied Odds: Example

For this example, the flop is as follows



The Hero has 

and the faces a bet of $100 with a pot of $375. The Hero knows that they are currently losing to the Villain and their hand needs to improve in order to win. Therefore they are drawing to a better hand in the hierarchy such as Two-pair or Three of a Kind.

- **Deduce Drawing Hand**: Two-Pair or Three of a Kind
- **Calculating Outs**: 3 Kings and 2 Tens, therefore **5 outs.**

Implied Odds: Example

- **Calculating Win Percentage for next street**:

$$q \approx 5 \cdot 2 = 10\%$$

- **Calculating Pot Odds**:

$$PO = \frac{100}{575} \approx 17\%$$

which is too expensive since $PO > q$.

- **Calculating EV of decision**:

$$E[W_c] = 575 \cdot 10\% - 100 \cdot 83\% = -25.5$$

Implied Odds: Example

- **Calculating Implied Odds**:

$$X = \frac{B}{q} - P - B = 1000 - 575 = \$425 \text{ more}$$

- Therefore, if we're making a call of \$100 on this street, our call is $+EV$ iff \$425 or more is extracted from other players on the next street.
- In this scenario, the Villain made a $\frac{100}{375} \cdot P = 0.26P$ bet on the flop.
- According to our implied odds the Villain would have to bet at least \$425 into a \$575 pot ($\frac{425}{575} \cdot P = 0.73P > 0.26P$) to make our call on the flop $+EV$ (assuming our draw hit on the turn).
- Since there is such a big discrepancy in the betting sizes between the street, it is quite unlikely that the Villain would cover our implied odds and therefore the $+EV$ play would be to fold on the flop itself.

Bluffing

- **Bluffing** in Poker is the act of making the Villain fold a better hand what the Hero holds by making them think the Hero has a good hand.
- Bluffing is the exact opposite of a value bet.
- When betting for value, Hero hopes to be called by a worse hand.
- When betting as a bluff, Hero hopes opponent folds a better hand.

- **Continuation Bet Bluff**: This is a bet where the Hero has the initiative and is relying on the Villain not being able to make a good hand from board.
- **Semi-Bluff**: This is a bet when the (drawing) hand is currently weak, but not totally devoid of showdown value. It has a decent number of outs to make it a strong hand. It is a bet with $+EV$ even though it has $E_f < 0$ because of sufficiently high q
- **0EV/Stone-Cold Bluff**: This is a bluff where you have almost no chance of improving to the best hand and it relies entirely on **fold equity**. It is a $+EV$ bet only because $E_f > 0$

Fold Equity: Introduction

- **Fold Equity**: refers to the probability that the Hero can win the pot by making the Villain fold their hand, assuming that a call will result in a loss.
- The most important use of this concept is for determining **profitable bluffs**.
- Fold equity helps shape the optimal strategy on all streets, considering you wont always have a good hand, it should be taken into account to make the highest EV decisions.
- **Fold Percentage**: refers to the percentage of times a player folds over the course of multiple hands. It is a property of the player and is calculated over multiple hands.

Fold Equity: Formula

Let us represent **Showdown Win Percentage** as q. This represents the probability of a hand winning in a showdown.

For $q = 0$,

$$E_f = P_i f - B(1 - f)$$

For $q > 0$,

$$E_f = P_i f + E[W_c](1 - f) = P_i f + EV_{SD}$$

Here,

- E_f refers to Fold Equity
- P_i refers to **Current Pot** before the Hero's bet. $P_i + B = P$
- B refers to the size of the **Bet**
- $E[W_c]$ refers to EV if called
- f represents the **Fold Percentage** of the Villain

Fold Equity: Formula

- **Bluffing**: A bet which has $+EV$ because $E_f > 0$

$$f > \frac{B}{P_i + B} = \frac{B}{P}$$

- **Semi-Bluffing**: A bet where $E_f < 0$ but has $+EV$ because of the high enough q

$$Pf < (1 - f)(B(1 - q) - (P_i + B)q)$$

$$f < \frac{E[W_c]}{E[W_c] - P_i}$$

$$f < \frac{(P + B)q - B}{(P + B)q - P}$$

Fold Equity: Maximum Possible Bet

Given that we know the Villain's fold percentage, we can calculate the maximum possible amount that the Hero can bet in bluff/semi-bluff.

- **Stone-cold Bluff**:

$$B_{\max} = fP$$

- **Semi-Bluff**:

$$fq(P + B_{\max}) - Pf = (P + B)q - B_{\max}$$

$$B_{\max} = \frac{P(q + f(1 - q))}{1 - q(1 - f)}$$

Fold Equity: Semi-Bluffing Formula Analysis

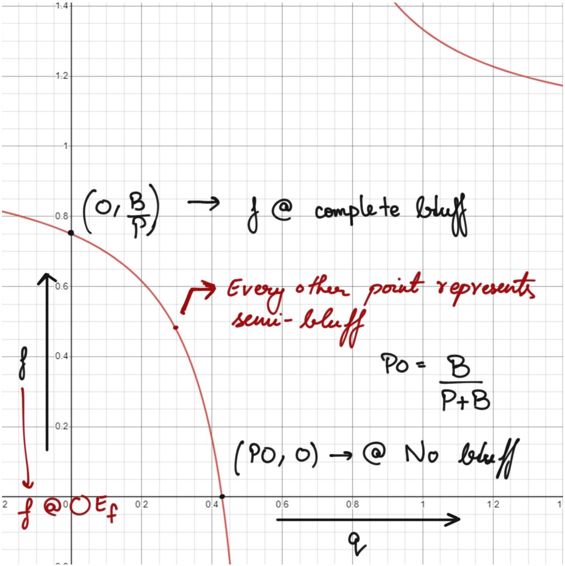
Let us look at the formula,

$$f = \frac{(P+B)q - B}{(P+B)q - P}$$

Here f would represent fold percentage at $E_f = 0$

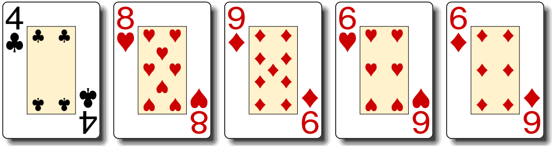
- The curve between f and q will be **hyperbolic.** The part of the curve that we're interested in is $0 \leq q \leq 1$ and $0 \leq f \leq 1$
- At $q = 0$, $f = \frac{B}{P} \implies 0EV$ bluff
- At $f = 0$, $q = \frac{B}{B+P} = PO \implies$ No bluff

Fold Equity: Semi-Bluffing Formula Analysis



Fold Equity: Bluffing Example

For this example, the **river** is as follows



The Hero has 10♦5♠. The pot is currently \$375 and the Villain has checked on their turn. We also know that the Villain calls 25% of the time.

- Deduce Drawing Hand: High Card
- Number of Outs: 0 $\implies$ $q = 0\%$
- Type of Bluff: Stone-cold Bluff

Fold Equity: Bluffing Example

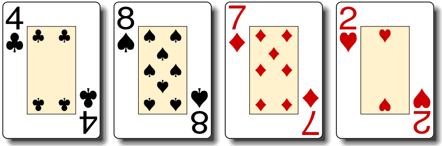
Calculate Breakeven Bet:

$$B_{\text{max}} = fP = \$468.75$$

Therefore, the maximum amount that the Hero can safely bet in this bluff would $468.75

Fold Equity: Semi-Bluffing Example

For this example, the **turn** is as follows



The Hero has 9♦10♠. The pot is currently $775 and the Villain has checked on their turn. We also know that the Villain calls 80% of the time.

- Deduce Drawing Hand: Open-ended Straight Draw
- Number of Outs: 8 $\implies$ $q = 16\%$
- Type of Bluff: Semi-Bluff

Fold Equity: Bluffing Example

Calculate Breakeven Bet:

$$B_{\max} = \frac{P(q + f(1-q))}{1 - q(1-f)} = \frac{775(0.16 + 0.2(1 - 0.16))}{1 - 0.16(1 - 0.2)} = \$305.5$$

Therefore, the maximum amount that the Hero can safely bet in this bluff would $305.5

Preflop Betting

- **Preflop Betting** is an important aspect of NLH poker which should be played by the Hero as close to optimal as possible in order to be in good position during the game.
- Making the decision of calling/raising-reraising/folding after the Hero recieves their starting hand depends on the following factors:
  - The Hero’s position at the table
  - The Playing Style of the Villain (Call Range)
  - The Hero’s Stack (M value)
  - Hero’s Starting Hand Ranking

Preflop Hand Rankings

- **Preflop Hand Rankings** is a method of assigning a relative value or strength to different starting hands.
- It's purpose is to help player make informed decisions about which hands to play and which hands to fold in different scenarios.
- The **Sklansky-Karlson Hand Rankings** is used to group together certain hands based on their strength and takes into account factors such as:
  - Likelihood of making a strong hand
  - Ability to bluff effectively
  - Which starting hands are profitable in the long run

Preflop Hand Rankings: Sklansky-Karlson Hand Ranking

- A **range strand** is represented as $XXba$ where $a = \{+, -\}$, which represents the set of hands better or worse than the given hand.
- For example, JJ+, this is a range strand that says "select pocket Jacks and all pocket pairs above it" so JJ, QQ, KK and AA. The plus sign after a starting hands tell you to include all similar hands that are higher than it.
  - **22+** means you should include all pocket pairs (22, 33, 44, ..., QQ, KK, AA)
  - **98s+** means you should include suited connectors 98s and higher (so 98s, T9s, etc.)
  - **AQ+** doesnt have an "s" (suited) or "o" (offsuit) qualifier, so you would include **all versions** of AQ and also AK.

Preflop Hand Rankings: Sklansky-Karlson Hand Ranking

The Sklansky-Karlson Rankings are as follows:

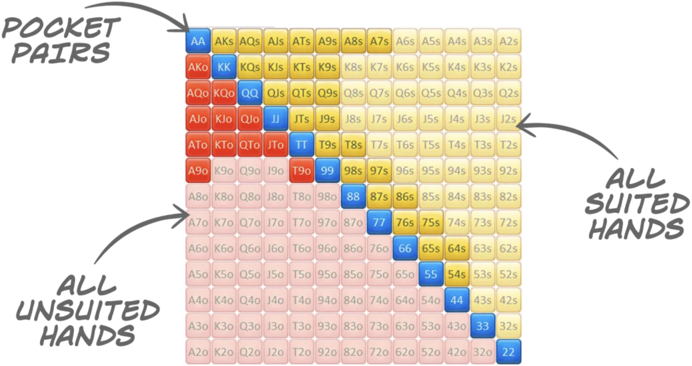
- **Top 10 hands**: AA, KK, AKs, QQ, AKo, JJ, AQs, TT, AQo, 99
- **Top 2.5%**: QQ+, AK
- **Top 5%**: TT+, AQs+, AQo+
- **Top 10%**: 44+, AJ+, KQ, KJs
- **Top 20%**: 22+, ATB, 54s+
- **Top 33%**: 22+, ATB, A2s+, A7o+, T9+, 43s+, 53s+, J8s+, K8s

Ranges in Poker

- A **range** is a collection of all possible hands a player can have at the given moment.
- During preflop, it is important to consider the range of cards the Villain can have before considering betting.
- The Hero can start building the Villain's range preflop and refine it at subsequent streets.
- Ranges are used for developing rules to determine plays (call/raise/fold).

Range Matrix

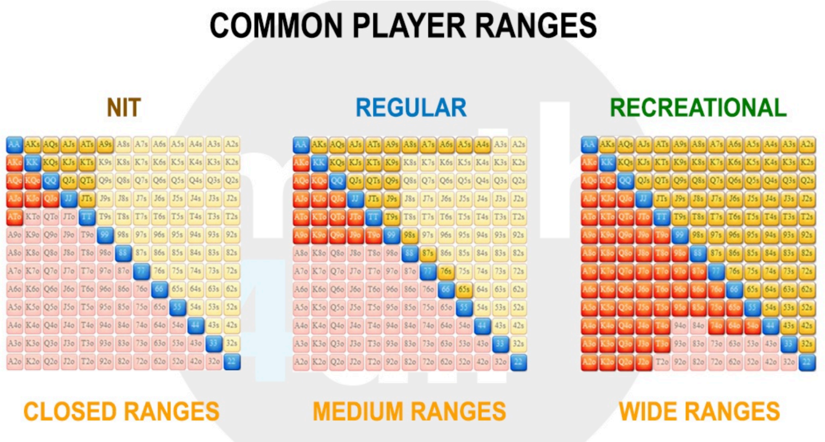
When analysing ranges, we look at the **range matrix** which is the possible hands in poker and determine the percentage of all possible starting hands which are within that range.



Range Calculation

- Using the parameters player position, stack size, calling and raising percentages the Hero can make a good approximation of the range that the Villain has.
- The steps for range calculation are as follows:
    1. Define the action the opponent is making (calling/raising)
    2. Estimate the frequency (percentage of hands they play in that position and the action that they are proceeding with)
    3. Remove the possibilities of hands that are not included in the range.

Common Player Ranges



Betting based on Fold Percentage

- Given the range of a player, if the player is loose then they would tend of have a lot of hands in their range that they are unlikely to have.
- If these extra hands are likely to fold if they face a raise or re-raise, the Hero is heavily incentivized to **bluff** against the opponents to apply pressure.
- If the Villain refuses to fold those extra hands, the Hero's focus should be on getting thinner value since they will continue onward with weak-marginal hand and will most likely have a higher win percentage when the flop arrives.

Ranges Application

- Ranges help the Hero deduce the possible hands the Villain can have based on their betting behaviour.
- This comprises of the frequency at which they are calling/raising a bet, i.e, their **call percentage** and **raise percentage**
- "The range you assign is a bi-product of *who* your opponent is, *what* action they are taking, *where* they are making that action, and *how* they might craft that specific range."
- Knowing whats included in the top-X% of hands can help the Hero remove hands from the Villain's range that could prove to be problematic later. Ex. A player that open-raises 15% of hands almost certainly is not going to have Q6s in their preflop range.

Hand Assessment: Introduction

The following section discusses about our implementations of various **poker hand assessment tools** that have been directly implemented from this paper.

The tools that we have implemented are as follows:

1. Preflop Analysis using Monte Carlo Simulation
2. Hand Strength Estimation
3. Hand Potential:
    - One-card lookahead
    - Two-card lookahead
4. Effective Hand Strength

Preflop Evaluation

- Preflop Evaluation is a fundamental aspect of poker that involves evaluating and making decision about the strength of a player's staring hand before any community cards are dealt.
- The evaluation returns a statistical measure of the approximate **income rate**, i.e, profit expectation for each starting hand.

Preflop Evaluation Algorithm

Given that a poker player has n players. The exact implementation for preflop evaluation for a given hand is as follows:

- Counters for wins, losses and ties are initialized to 0
- For a given starting hand (Hero's hand), initialize a deck of cards and remove the starting hand cards from the deck.
- For all possible $n - 1$ starting hands that the opponents can have:
  - Distribute the inital hands to all $n - 1$ opponents
  - For every possible set of 5 cards that can appear on the board from the cards remaining in the deck
    - If Hero's hand wins, increment the number of wins
    - If the Hero ties, increment the number of ties
    - Else, increment the number of losses
- Income rate (IR) can be calculated as $\mathrm{IR} = \frac{\text{wins}+\text{ties}/2}{\text{wins}+\text{ties}+\text{losses}}$

Preflop Evaluation Algorithm

- For the given algorithm, let us count the number of hand + board combinations over which the calculation will take place.
- Given a deck of 52 cards, after removing the Hero's starting hand, there are a total of $\binom{50}{2}$ possible opponent hands.
- For each possible opponent hand, there are a total of $\binom{48}{2}$ possible boards that could appear in the round.
- Therefore the total number of cases that would have to be simulated to calculate IR would be $\binom{50}{2} \cdot \binom{48}{5} = 2097572400$
- It is not possible to simulate these many cases in a reasonable amount of time, which is why we have implemented preflop evaluation using a **Monte Carlo Simulation**.

Preflop Evaluation using Monte Carlo Simulation

ideas could be generalized to post-flop play.

For the initial two cards, there are {52 choose 2} = 1326 possible combinations, but only 169 distinct hand types. For each one of the 169 possible hand types, a simulation of 1,000,000 poker games was done against nine random hands. This produced a statistical measure of the approximate *income rate* (profit expectation) for each starting hand. A pair of aces had the highest income rate; a 2 and 7 of different suits had the lowest. There is a strong correlation between our simulation results and the pre-flop categorization given in Sklansky and Malmuth (1994).

The image above is taken from the paper which explains preflop analysis using Monte Carlo Simulation

Preflop Evaluation using Monte Carlo Simulation

- Instead of calculating all possible cases, we can use **random sampling** of cards for both the opponent cards and the board to obtain an **approximate value** of *IR* for the starting hand.
- Using the Monte Carlo approach, We calculated the value of *IR* for all possible 169 starting hands with the number of opponents ranging from 1 to 10.
- For verification, we compared our computed *IR* for 2 players with the results from this source
- Visualization are done by plotting a line plot the output of our preflop evaluation function for hands at an interval of 10 in the Sklansky-Karlson rankings.

Preflop Evaluation using Monte Carlo Simulation Pseudocode

```
PreflopMonteCarlo( ourcards, numPlayers, iterations=100000 ){
  wins = ties = losses = 0
  for i in range( iterations ){
    boardcards = deck.draw(5)
    ourrank = Rank( ourcards, boardcards )
    oppRanks = [ ]
    for i in range( numPlayers ){
      oppCards = deck.draw(2)
      oppRanks.append(Rank(oppCards, board))
    }
    if ( ourrank > max(oppRanks) { wins += 1 }
    else if ( ourrank == max(oppRanks) { wins += 2 }
    else { losses += 1 }
  }
  IR = ( wins + ties/2 ) / (wins + ties + losses )
  return(handstrength)
}
```

Preflop Evaluation using Monte Carlo Simulation Output

Cards	1	2	3	4	5	6	7
AAo	100.00%	85.30%	73.40%	63.90%	55.90%	49.20%	43.60%
AKs	100.00%	67.00%	50.70%	41.40%	35.40%	31.10%	27.70%
AKo	100.00%	65.40%	48.20%	38.60%	32.40%	27.90%	24.40%
AQs	100.00%	66.10%	49.40%	39.90%	33.70%	29.40%	26.00%
AQo	100.00%	64.50%	46.80%	36.90%	30.40%	25.90%	22.50%
AJs	100.00%	65.40%	48.20%	38.50%	32.20%	27.80%	24.50%
AJo	100.00%	63.60%	45.60%	35.40%	28.90%	24.40%	21.00%
ATs	100.00%	64.70%	47.10%	37.20%	31.00%	26.70%	23.50%
ATo	100.00%	62.90%	44.40%	34.10%	27.60%	23.10%	19.80%
A9s	100.00%	63.00%	44.80%	34.60%	28.40%	24.20%	21.10%
A9o	100.00%	60.90%	41.80%	31.20%	24.70%	20.30%	17.10%
A8s	100.00%	62.10%	43.70%	33.60%	27.40%	23.30%	20.30%
A8o	100.00%	60.10%	40.80%	30.10%	23.70%	19.40%	16.20%
A7s	100.00%	61.10%	42.60%	32.60%	26.50%	22.50%	19.60%
A7o	100.00%	59.10%	39.40%	28.90%	22.60%	18.40%	15.40%
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

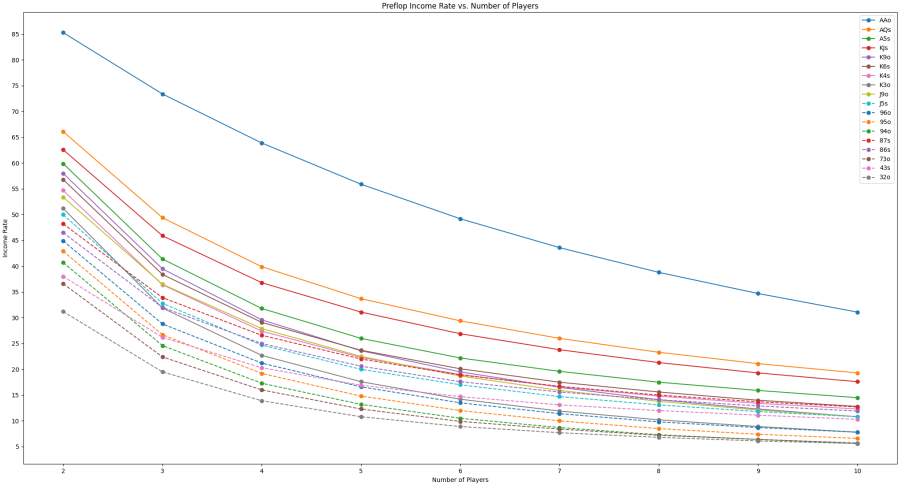
The image above is the output of our preflop evaluation monte carlo function for all possible hands with number of players ranging from 1 to 10

Preflop Evaluation using Monte Carlo Simulation Verification

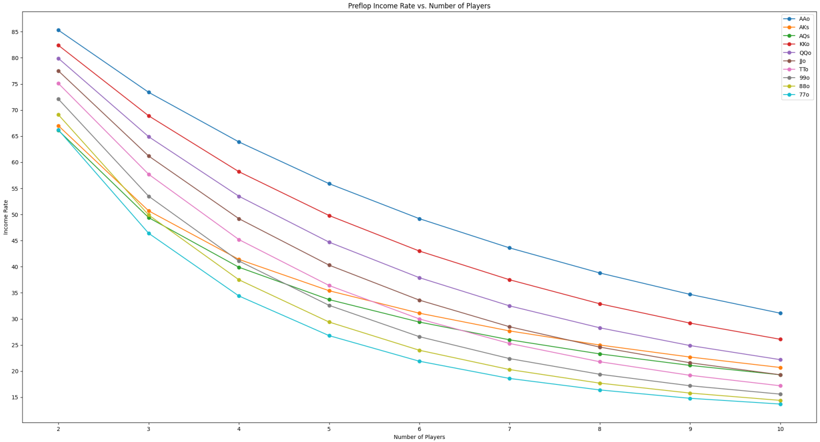
Cards	MC Preflop	Absolute(Source)	Difference
AAo	85.3	84.93	0.37
66o	63.3	62.7	0.6
QTs	59.5	58.17	1.33
K8o	56.3	54.43	1.87
Q8o	53.8	51.93	1.87
J6s	50.8	48.57	2.23
J6o	47.9	45.71	2.19
85s	44.8	41.99	2.81
54s	41.1	38.53	2.57
54o	37.9	35.07	2.83
32o	31.2	29.23	1.97

Table: Verification of Preflop Monte Carlo Output

Preflop Evaluation using Monte Carlo Simulation Graph



Preflop Evaluation using Monte Carlo Simulation Graph



Preflop Evaluation for top 10 hands according to Sklanskly-Karlson rankings

Hand Strength Estimation

- Hand strength estimation (HSE) is used to estimate the winning potential of a player’s hand by look at all possible games that could play out with that hand.
- At a minimum, it is a function of your cards and the current community cards.
- As compared to preflop evaluation, HSE is computed once the flop is revealed and calculated at every subsequent street.

Hand Strength Estimation

- It tells us what percentage chance our hand is better than any random hand in that moment.
- HSE estimates the probability of holding the best hand at any time can be accurately estimated using **enumeration techniques**, i.e, going through every possible hand.

Hand Strength Estimation

```
HandStrength(ourcards, boardcards){
    ahead = tied = behind = 0
    ourrank = Rank(ourcards, boardcards)
    for each case(oppcards){
        opprank = Rank(oppcards, boardcards)
        if(ourrank > opprank) ahead += 1
        else if(ourrank = opprank) tied += 1
        else behind += 1
    }
    handstrength = (ahead+tied/2) / (ahead+tied+behind)
    return(handstrength)
}
```

HSE Extrapolation

The hand strength calculation is with respect to one opponent but can be extrapolated to multiple opponents by raising it to the power of the number of active opponents. Against five opponents with random hands, the adjusted hand strength (HS_5) is $.585^5 = .069$. Hence, the presence of additional opponents has reduced the likelihood of our having the best hand to only 6.9%.

Hand Potential

- In practice, just by being able to tell our current position in the game isn't enough to make the most optimal decision. HSE on its own is an incomplete criteria for decision making.
- We need to be able to tell which hands have how much potential to improve in the subsequent streets, i.e, **drawing hands**.

Hand Potential is a quantity that uses **enumeration techniques** compute the following values:

- **Positive Potential**(Ppot): The probability of a hand improving to a better hand in subsequent street given that the hand is behind right now.
- **Negative Potential**(Npot): The probability of a hand falling behind in the subsequent streets, i.e, the opponent's hand improves and we fall behind.

Hand Potential Pseudocode

```
HandPotential(ourcards,boardcards) {
  integer array HP[3][3]
  integer array HPTotal[3]
  ourrank = Rank(ourcards,boardcards)
  for each case(oppcards) {
    opprank = Rank(oppcards,boardcards)
    if(ourrank > opprank) index = ahead
    else if(ourrank = opprank) index = tied
    else index = behind
    for each case(turn,river) {
      HPTotal[index] += 1
      board = [boardcards,turn,river]
      ourbest = Rank(ourcards,board)
      oppbest = Rank(oppcards,board)
```

Hand Potential Pseudocode

```
        if(ourbest > oppbest) HP[index][ahead]+=1
        else if(ourbest = oppbest) HP[index][tied]+=1
        else HP[index][behind]+=1
      }
   }
```

$$\text{Ppot} = \frac{\text{HP[behind][ahead]+HP[behind][tied]/2+HP[tied][ahead]/2}}{\text{HPTotal[behind]+HPTotal[tied]}}$$

$$\text{Npot} = \frac{\text{HP[ahead][behind]+HP[tied][behind]/2+HP[ahead][tied]/2}}{\text{HPTotal[ahead]+HPTotal[tied]}}$$

```
   return(Ppot,Npot)
}
```

Effective Hand Strength

- HSE gives us information about the strength of our hand at a given moment and Hand Potential gives us information about the probability of the strength of our hand increasing.
- Combining these 2 criteria, we get **Effective Hand Strength**, which gives us information about both.

$$EHS = HS + (1 - HS) \cdot Ppot$$

Testcases

- For the testcases to test the hand assessment criteria that we have implemented, we are considering the most common possible hands that span the entire range of combination of hands.
- In our testcases, we follow a Hero VS Villain format, to assess the parameters of a specific hand, compared to another hand, given a fixed flop. This is first done preflop, and then with the flop, and then verified using intuition.

Testcases

1. Nut Flush vs Nut Low
2. Nut Straight vs Nut Low
3. Open-ended Straight Draw vs Top Pair
4. 2nd Pair vs 3rd Pair
5. High Card vs High Card
6. Full House vs Full House
7. Full House vs High Card
8. Three of a Kind vs Two Pair
9. Lower Straight Draw vs Higher Straight Draw
10. Flush Draw vs Flush Draw
11. Quad vs Full House
12. Middle Pair Cards with different kicker
13. Bottom 2 pair vs Top pair
14. Overpair vs Three of a Kind
15. Overpair vs Top pair Top kicker
16. Overpair vs Nut Flush Draw
17. Overpair vs Bottom Two pair

Testcases 1

Flop: 10♥ 8♥ 6♥

ID	Hand	vs Odds	MCP	HSE	*PPot2*	$Npot_2$	EHS	Prediction
1	9♥ 7♥	0.58	0.49	1	0	≈0	1	$P = 0, N \approx 0$
	3♣ 2♠	0.41	0.32	0.003	0.122	0.08	0.125	$P \downarrow, N \approx 0$
2	9♣ 7♠	0.63	0.46	0.95	0.001	0.1	0.954	$P \approx 0, N \downarrow$
	3♠ 2♦	0.36	0.32	0.003	0.122	0.08	0.125	$P \downarrow, N \approx 0$
3	J♣ 9♣	0.41	0.55	0.234	0.356	0.309	0.506	$P \sim, N \uparrow$
	A♠ 10♦	0.58	0.62	0.896	0.009	0.3	0.906	$P \approx 0, N \uparrow$
4	K♠ 8♣	0.437	0.559	0.775	0.139	0.314	0.806	$P \downarrow, N \uparrow$
	A♦ 6♣	0.562	0.575	0.668	0.154	0.324	0.720	$P \downarrow, N \uparrow$
5	A♦ 3♣	0.604	0.558	0.47	0.113	0.418	0.53	$P \downarrow, N \approx max$
	K♠ 4♣	0.396	0.525	0.373	0.129	0.421	0.453	$P \downarrow, N \approx max$
8	10♣ 8♠	0.625	0.497	0.938	0.163	0.237	0.948	$P \downarrow, N \sim$
	6♠ 6♦	0.375	0.634	0.939	0.315	0.179	0.958	$P \sim, N \sim$

Testcases 1

Flop: 10♥ 8♥ 6♥

ID	Hand	vs Odds	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
9	J♣ 9♣	0.638	0.557	0.234	0.356	0.309	0.507	$P\sim, N\uparrow$
	7♦ 5♦	0.362	0.437	0.106	0.307	0.357	0.381	$P\sim, N\uparrow$
10	K♣ J♥	0.438	0.604	0.42	0.428	0.199	0.669	$P\uparrow, N\sim$
	A♥ 3♣	0.562	0.559	0.471	0.427	0.193	0.697	$P\uparrow, N\sim$
12	8♣ 3♠	0.152	0.376	0.721	0.146	0.334	0.761	$P\downarrow, N\uparrow$
	K♦ 8♦	0.848	0.585	0.775	0.14	0.314	0.807	$P\downarrow, N\uparrow$
13	Q♣ 10♦	0.532	0.572	0.882	0.106	0.299	0.894	$P\downarrow, N\uparrow$
	8♣ 6♠	0.468	0.432	0.927	0.15	0.262	0.938	$P\downarrow, N\uparrow$

Testcases 2

Flop: Q♠ Q♦ 10♣

ID	Hand	vs Odds	MCP	HSE	*PPot*$_2$	*Npot*$_2$	EHS	Prediction
6	Q♣ 10♦	0.273	0.572	0.999	0.0	0.005	0.999	$P = 0, N = 0$
	10♠ 10♥	0.727	0.753	0.997	0.0	0.028	0.997	$P = 0, N = 0$
7	Q♣ 10♦	0.458	0.569	0.999	0.0	0.005	0.999	$P = 0, N = 0$
	A♣ 8♠	0.542	0.598	0.704	0.153	0.244	0.749	$P \downarrow, N \sim$
11	Q♣ Q♥	0.896	0.797	1.0	0	0.0	1.0	$P = 0, N = 0$
	10♥ 10♠	0.104	0.748	0.997	0.0	0.028	0.997	$P = 0, N = 0$

Testcases 3

Flop: 9♣ 5♣ 2♠

ID	Hand	vs Odds	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
14	A♠ A♦	0.792	0.853	0.966	0.221	0.144	0.974	$P \sim, N \sim$
	2♣ 2♦	0.208	0.503	0.994	0.089	0.074	0.995	$P \approx 0, N \approx 0$
15	A♠ A♦	0.812	0.852	0.966	0.221	0.144	0.974	$P \sim, N \sim$
	K♥ 9♥	0.188	0.601	0.939	0.159	0.2	0.949	$P \downarrow, N \sim$
16	K♠ K♦	0.687	0.824	0.961	0.188	0.166	0.968	$P \sim, N \sim$
	A♣ Q♣	0.313	0.662	0.585	0.492	0.148	0.789	$P \uparrow, N \sim$
17	A♠ A♦	0.875	0.854	0.966	0.221	0.144	0.974	$P \sim, N \sim$
	5♠ 2♣	0.125	0.343	0.982	0.093	0.153	0.984	$P \approx 0, N \sim$

Testcase 4

Flop: 7♦ 10♦ A♣

ID	Hand	vs Odds	MCP	HSE	$PPot_2$	$Npot_2$	EHS	Prediction
14	8♦ 9♦	0.511	0.508	0.263	0.6	0.12	0.705	$P = max, N \downarrow$
	3♠ 3♣	0.489	0.537	0.605	0.081	0.314	0.637	$P \approx 0, N \uparrow$

Inferences

- As we can see, our predictions for the testcases match with the values outputted by the evaluation functions.
- Through our analysis, we found that the highest $Ppot_2$ came out to be ≈ 0.6 and the lowest being 0.
- On the other hand, the highest $Npot_2$ came out to be around 0.44 and the lowest being 0.
- When both quantities are 0, that EHS of that hand is extremely high, as the hand is extremely strong.

Inferences

- When *NPot* is high and *Ppot* is low, the hands are extremely strong. (high EHS)
- When both *Ppot* and *Npot* are close to 0, if *Ppot* is higher than *Npot*, then the hand is extremely weak, and if *Npot* is almost 0 and *Ppot* is greater, then the hand is extremely strong.
- Testcase 4 highlights the importance of *Ppot* which HSE doesnt take into account. Initially the HSE of the Hero was found to be significantly lower than the HSE of the Villain. However, after considering the *Ppots* for both, the EHS of Hero's hand is greater than that of the Villain. Therefore EHS serves as our main indicator of hand strength and aided us in our inferences.