Homework 7 solutions

S320/520

Upload your answers as a PDF file or Word document through the Assignments tab on Canvas by 4pm, Thursday 22nd October.

Please write "S320" or "S520" at the top of your homework. Trosset question numbers refer to the hardcover textbook. Include R code as an appendix to your answers.

1. Trosset exercise 9.6.6

   Hypotheses are $H_0 : \mu \geq 800, H_1 : \mu < 800$. The test statistic is $(745.1 - 800)/(238/\sqrt{100}) = -2.307$. Using a standard normal distribution, we get a P-value of 0.011. This is below the significance level of 0.05, so we reject the null hypothesis; there is strong evidence against the company. (You could also argue for a t-test here, in which case the P-value would be 0.012 — not substantively different.)

2. Trosset exercise 9.6.9

   We use the sample size formula $n = (2q\sigma/L)^2$ with $q = 2.576, \sigma = 6, L = 2$, giving $n = 239$.

3. Trosset exercise 9.6.11. Note: The book errata says to use $L = 0.001$. Either using this value of L or leaving L in your answer is acceptable.

   To use the sample size formula, we need to estimate σ. One way of doing this is to note that Y is approximately normal with mean 184 and standard error $\sqrt{585.6}$. This gives an approximate probability $p \approx 0.45$ of Y being in the interval $(170.5, 199.5)$ (see Exercise 8.4.5(b)). Let Z be 1 if Y is in $(170.5, 199.5)$ and 0 otherwise. The standard deviation of Z is $\sqrt{p(1-p)} \approx 0.498$; this is the σ we want. The required sample size is $(2 \times 1.96 \times 0.498/0.001)^2$, which is about 3.8 million. (Other acceptable options: (i) do a small simulation to estimate p; (ii) just use $p = 0.5$. All these methods give similar answers.)

4. *A person wishes to show beyond reasonable doubt that he has psychic powers. He takes the test described in Trosset exercise 4.5.14, except he tries to identify 100 symbols, with a 20% of being right for each symbol if he does not have psychic powers. The result is that he correctly identifies 25 out of 100 symbols.*

   (a) *Write down appropriate null and alternative hypotheses for this test.*

   Let p be the long-run proportion that he gets right. The null hypothesis is that $p = 0.2$ (or $p \leq 0.2$). The alternative hypothesis is that $p > 0.2$.

   (b) *For someone without psychic powers, what is the probability of correctly identifying 25 or more symbols out of 100? (Use the binomial.)*

   `1 - pbinom(24, 100, 0.2)` $\approx 13\%$.

(c) *Has the psychic demonstrated his powers beyond reasonable doubt?*

No. Getting 25 or more right happens 13% of the time even if you have no psychic powers — it's not rare enough an event to justify such a strong statement.

5. *In a recent Gallup poll, 55% of a sample of 1028 U.S. adults supported same-sex marriage.*

(a) *Treating the data as a simple random sample, find a 95% confidence interval for the percentage of all U.S. adults who support same-sex marriage.*

A 95% confidence interval is $.55 \pm 1.96 \times \sqrt{0.55 \times 0.45/1028} = (.52, .58)$ or 52% to 58%.

(b) *(Extra credit.) In fact, Gallup stated a 4 percentage point margin of error at the 95% confidence level, implying a confidence interval of 51% to 59%. Why does this differ from your result?*

Gallup's estimate is not the sample proportion of a true simple random sample, increasing the standard error. (Because some demographic groups are easier to reach than others, some responses are upweighted or downweighted when making the estimate. This makes the estimate more uncertain.)