Homework 8 answers

S320/520

Upload your answers as a PDF file or Word document through the Assignments tab on Canvas by 4pm, Thursday 29th October.

Please write "S320" or "S520" at the top of your homework. Trosset question numbers refer to the hardcover textbook. Show working and include all graphs you are asked to draw in R, and include R code as an appendix to your answers.

Trosset chapter 10.5 problem set A (10 points)

1. (a) That we have s but not σ points us toward the t-test, although the sample size is large enough that the z-test won't be much different.

$$t = \frac{3.194887 - 0}{\sqrt{104.0118/400}} = 6.26.$$

   (b) Either (iv) or (iii): (iv) is best, but (iii) gives the same answer to three sig figs.

   (c) It's less than α, so true.

2. Playing around with the binomial distribution, we get

```
> pbinom(5,20,0.5)
[1] 0.02069473
```

   So a confidence interval running from the 6th lowest to 6th highest observation will have level of confidence $1 - 2 \times 0.0207 \approx 96\%$, which is as close as we can get. The 96% confidence interval is 238 to 251.

Trosset chapter 10.5 problem set B (15 points)

1. (a)
```
sim1 = rnorm(19)
sim2 = rnorm(19)
sim3 = rnorm(19)
sim4 = rnorm(19)
```

   (b)
```
par(mfrow = c(2,2))
qqnorm(sim1)
qqnorm(sim2)
qqnorm(sim3)
```

```
qqnorm(sim4)
```

Graphs will depend on the random samples generated in (a). Figure 10.1 is not especially close to a straight line, but many normal QQ plots of 19 random normal data points aren't close to straight either.

(c)
```
log2CPA = c(1.1402, -1.8658, 0.8520, -1.8251, 0.8530, -0.0589, -1.6554,
 -1.7599, -1.4330, -1.3853, 2.9794, 2.4919, 2.1601, 2.2670,
 -0.5479, -0.7164, 0.6462, -0.8365, 1.1997)
(quantile(log2CPA, 0.75) - quantile(log2CPA, 0.25)) / sd(log2CPA)
```

For the observed data, the ratio of IQR to SD (using s) is about 1.59. Simulations show that for normal samples of size 19, the ratio of IQR to SD exceeds this about 10% of time.

(d) It's pretty borderline. It would be better to compare the data to more than four simulated samples, but even if we compared it to millions of simulations, it'd still be borderline.

2.
```
> t.stat = mean(log2CPA) / (sd(log2CPA)/sqrt(19))
> 1 - pt(t.stat, df=18)
[1] 0.3635348
> mean(log2CPA) - qt(0.95, df=18) * sd(log2CPA) / sqrt(19)
[1] -0.5131008
> mean(log2CPA) + qt(0.95, df=18) * sd(log2CPA) / sqrt(19)
[1] 0.7768166
```

With a P-value of 0.36, we do not reject the null hypothesis: The mean could be zero. A 90% confidence interval for the mean is -0.51 to 0.78. Section 10.4 uses a different test (Wilcoxon signed rank) but comes to similar conclusions.

3.
```
> y = sum(log2CPA > 0)
> 1 - pbinom(y-1, 19, 0.5)
[1] 0.6761971
```

With a P-value of 0.67, we do not reject the null hypothesis: The median could be zero. This is the same result as before, albeit with a bigger P-value.

```
> sort(log2CPA)
 [1] -1.8658 -1.8251 -1.7599 -1.6554 -1.4330 -1.3853
 [7] -0.8365 -0.7164 -0.5479 -0.0589  0.6462  0.8520
[13]  0.8530  1.1402  1.1997  2.1601  2.2670  2.4919
[19]  2.9794
> round(1 - 2*pbinom(0:9, 19, 0.5), 2)
 [1] 1.00 1.00 1.00 1.00 0.98 0.94 0.83 0.64 0.35
[10] 0.00
```

An interval from the 6th lowest to the 6th highest value gives a 94% confidence interval, which is as close as we can get. The interval is -1.39 to 1.14, much wider than before.

Trosset chapter 10.5 problem set D (15 points)

1.
```
ratios = c(.693, .662, .690, .606, .570, .749, .672, .628, .609, .844,
  .654, .615, .668, .601, .576, .670, .606, .611, .553, .933)
qqnorm(ratios)
qqnorm(log(ratios))
```

   Frankly, neither QQ plot looks like a straight line. Which scale to use is a matter of judgment — the logged data gives a plot slightly closer to a straight line, but transformations hinder interpretation. If you put a gun to my head I would go with the logged data, but reasonable statistician might differ.

2. Suppose we're working on the log scale. Let $\mu_0 = \log(2/(1+\sqrt{5}))$. We test $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_1$.

```
golden.ratio = 2 / (1 + sqrt(5))
t.stat = (mean(log(ratios)) - log(golden.ratio)) / (sd(log(ratios)) / sqrt(length(ratios))
2 * (1 - pt(abs(t.stat), df = length(ratios) - 1))
```

   The P-value is 0.058. If we use a strict significance level of 0.05, then we do not reject the null hypothesis. (If you did the test on the original scale, you get a P-value of 0.054, and the same conclusion.)

3. We need to find k such that `1 - 2 * pbinom(k, 20, 0.5)` ≈ 0.9. After some trial and error, I got:

```
> 1 - 2*pbinom(5, 20, 0.5)
[1] 0.9586105
> 1 - 2*pbinom(6, 20, 0.5)
[1] 0.8846817
```

   This means that an interval going from the *sixth* lowest to sixth highest value has a confidence level of about 96%, while an interval going from the *seventh* lowest to seventh highest value has a confidence level of about 88%. Sorting the data:

```
> sort(ratios)
 [1] 0.553 0.570 0.576 0.601 0.606 0.606 0.609 0.611 0.615
[10] 0.628 0.654 0.662 0.668 0.670 0.672 0.690 0.693 0.749
[19] 0.844 0.933
```

   An 96% confidence interval for the median goes from 0.606 to 0.672, while an 88% confidence interval for the median goes from 0.609 to 0.670. If you're asked for a 90% confidence interval, then which of these two you give is a matter of statistical philosophy — do you want to be closest to 90%, or do you want to err on the side of a little more confidence? Again, reasonable statisticians may differ.