

1)

- The initialization  $x = 1$  is  $O(1)$ .
- The nested loops iterate  $n \times n = n^2$  times.
- The statement  $x = x + 1$ ; executes inside the inner loop, contributing  $O(1)$  per iteration.

Summation Representation

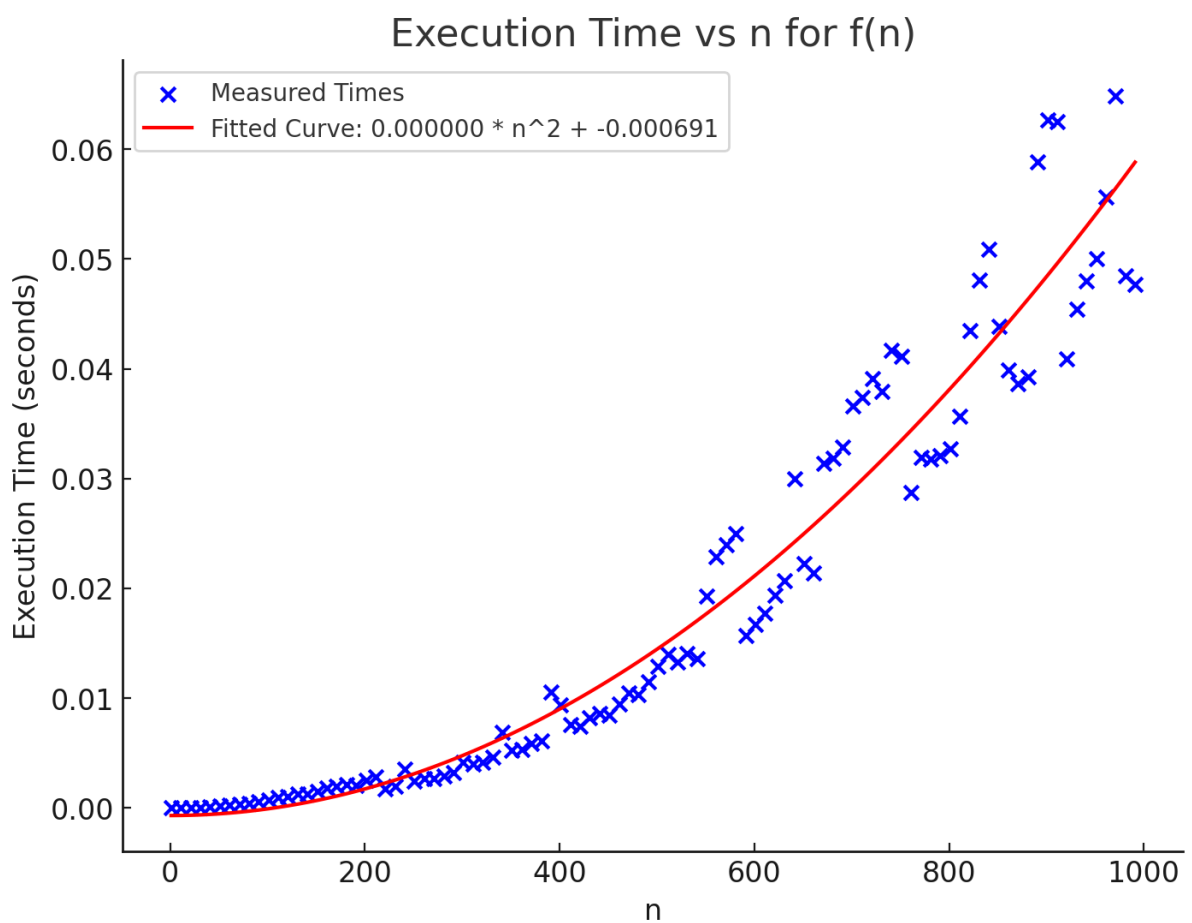
$$T(n) = 1 + \sum_{i=1}^n \sum_{j=1}^n 1$$

$$T(n) = 1 + \sum_{i=1}^n \sum_{j=1}^n O(1)$$

$$T(n) = 1 + O(n^2)$$

Thus,  $T(n) = O(n^2)$ .

2)



The plot shows the execution time of  $f(n)$  as  $n$  increases. The red curve is a quadratic fit, confirming the theoretical  $O(n^2)$  complexity.

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3)

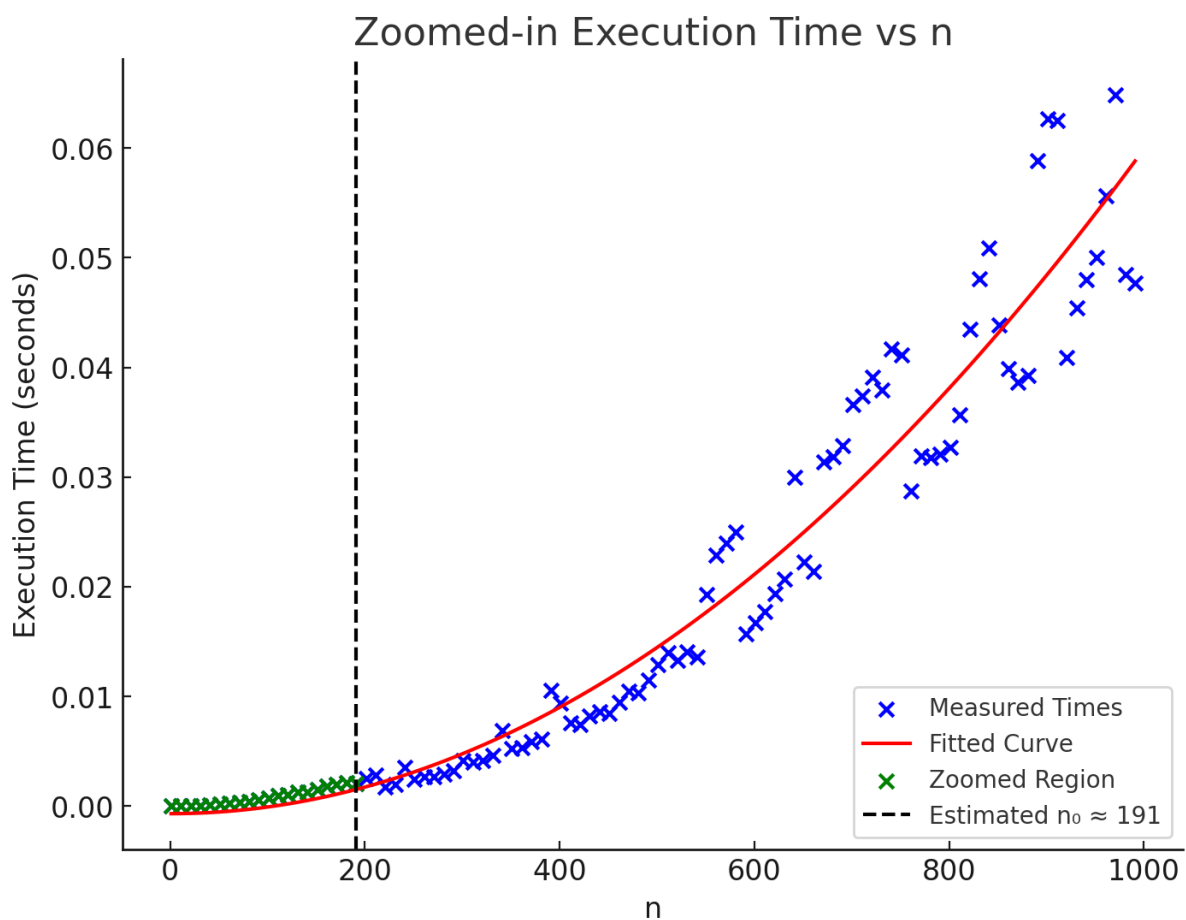
From our analysis:

- **Big-O (Upper Bound):**  $O(n^2)$ , since the worst-case growth is  $O(n^2)$ .
- **Big-Omega (Lower Bound):**  $\Omega(n^2)$ , as the function consistently executes  $n^2$  operations.
- **Big-Theta:** Since the upper and lower bounds match,  $\Theta(n^2)$ .

Thus,  $T(n) = \Theta(n^2)$ .

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4)



The zoomed-in plot highlights the region where the experimental data begins following the quadratic trend. The approximate location of  $n_0$  is around  $n \approx 200$ , where the execution times start aligning with the fitted curve.

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5)

Will this increase runtime?

- Yes, but only slightly. The additional operation  $y = i + j$ ; still executes in constant time  $O(1)$  per iteration.
- The number of iterations remains  $O(n^2)$ .
- Adding an extra constant-time operation does not change the asymptotic complexity.

Effect on Summation:

- The runtime now includes another  $O(1)$  operation per iteration.
- The summation expands to:

$$T(n) = 1 + \sum_{i=1}^n \sum_{j=1}^n (1 + 1)$$

$$T(n) = 1 + 2O(n^2)$$

$$T(n) = O(n^2)$$

- Big-O, Big-Omega, and Big-Theta remain the same ( $\Theta(n^2)$ ).