IIT Gandhinagar
CG501: Computation & Cognition
Assignment-II

Modeling Neuronal Dynamics: From Integrate-and-Fire to Hodgkin–Huxley

Sanjana Singh 25510110
24/11/2025

Leaky Integrate-and-Fire (LIF) Model

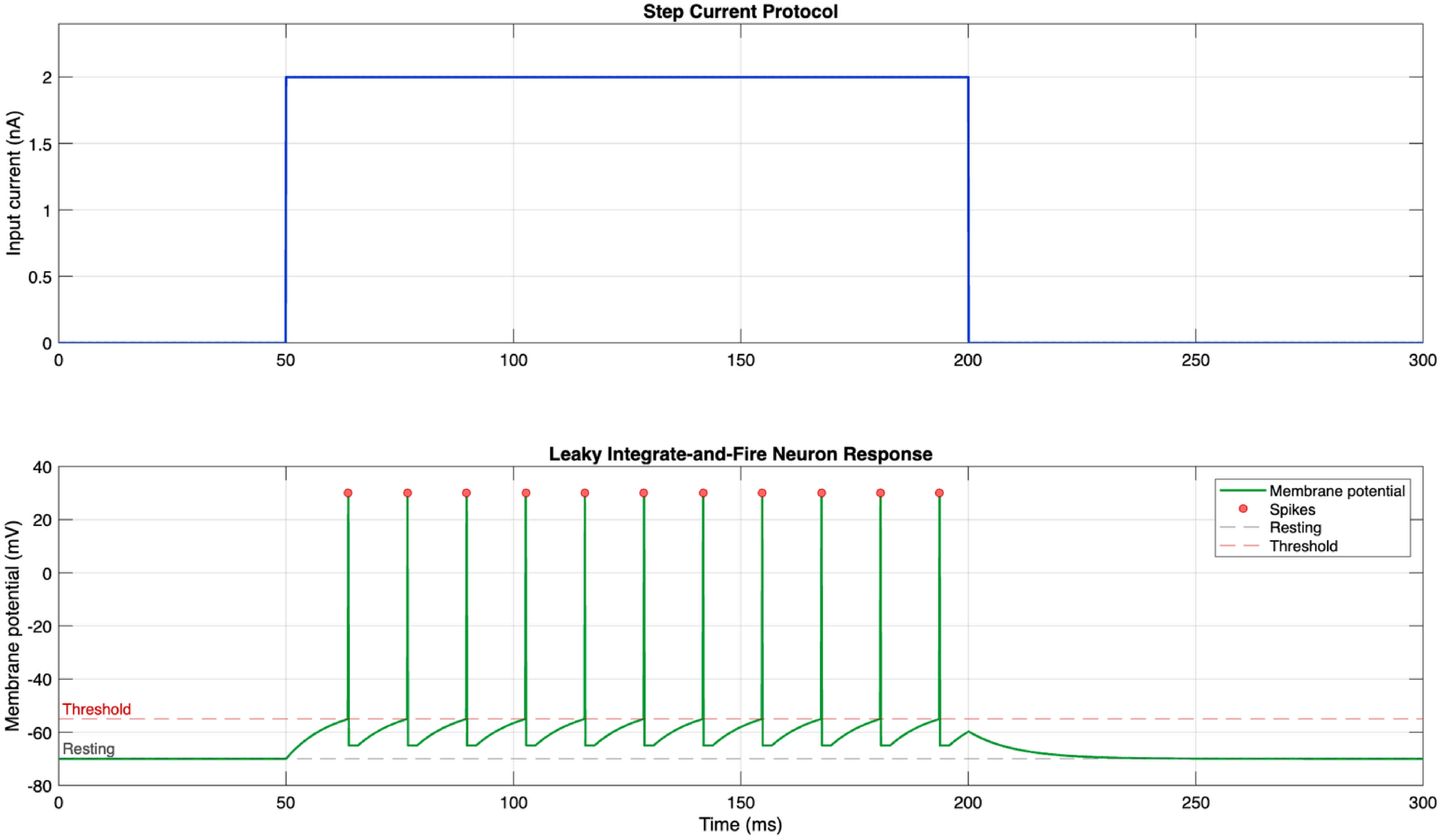


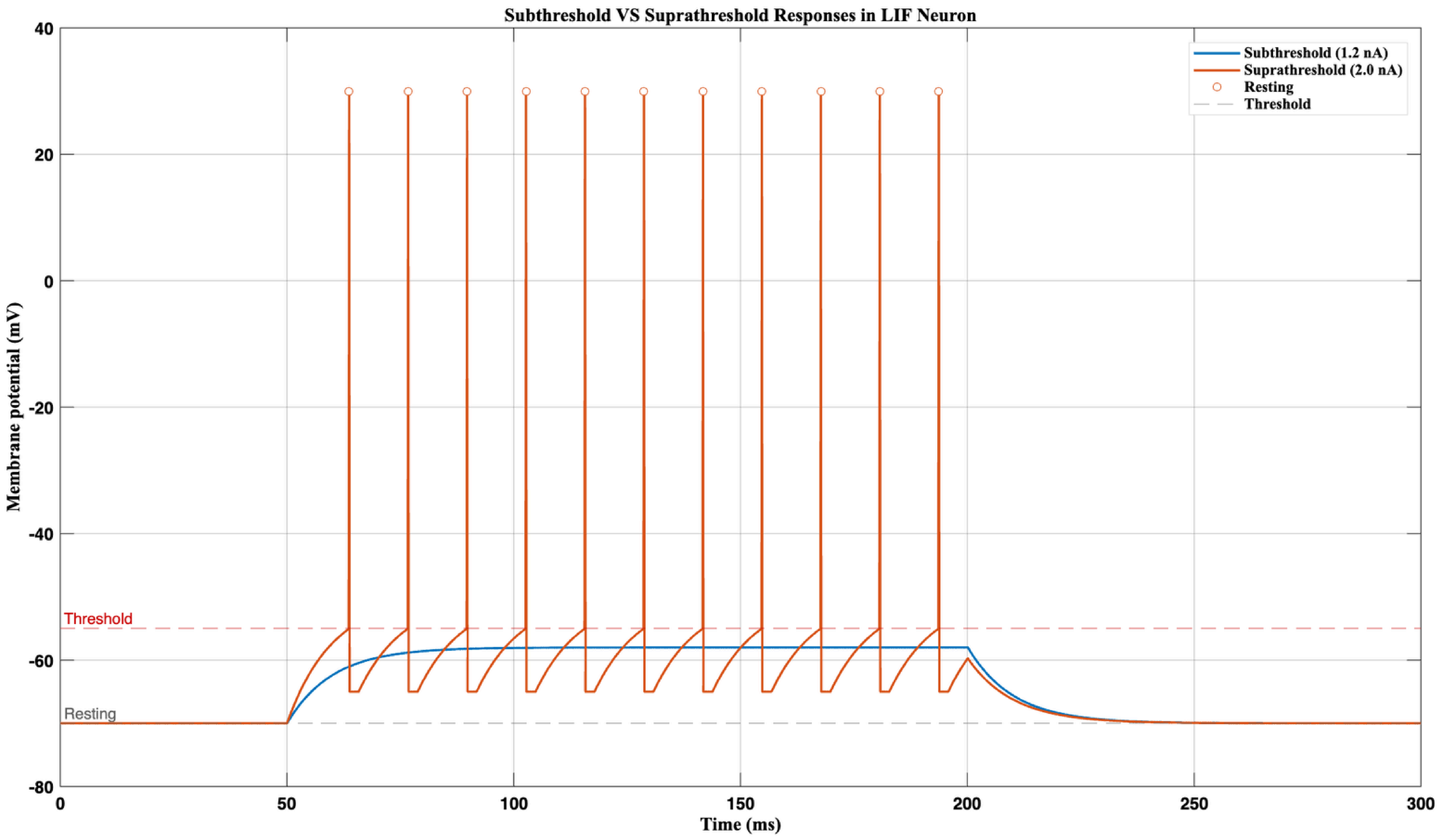
Figure-1a: A step current (top; 0→2 nA from 50–200 ms) elicits LIF spiking (bottom). Spikes occur when V crosses the dashed threshold line.

The **LIF neuron** behaves as a passive RC circuit that consolidtaes an incoming current. Its membrane voltage advances according to the differential equation:

$$\frac{\pi}{2} = \int_{-1}^{1} \sqrt{1 - x^2}\ dx$$

A **spike** is emitted when *V(t)* crosses a fixed threshold (e.g., –55 mV), after which the voltage is reset. Although the model omits the detailed spike shape, it correctly captures spike timing (Burkitt, 2006).

Subthreshold vs Suprathreshold Input



For a **subthreshold input** (for e.g., 1.2 nA), the membrane potential V(t) depolarizes but never attains the firing threshold, and therefore no spikes are initiated (blue trace). Conversely, a **suprathreshold input** (for e.g., 2.0 nA) continually drives V(t) above threshold, generating a regular train of action potentials (orange trace) (Burkitt, 2006).

This exhibits the core LIF principle i.e., the firing rate increases with input amplitude wherein the small currents generate only passive voltage deflections, whereas larger currents produce more of a sustained spiking. Empirical electrophysiology reinforces this behavior i.e., under steady input conditions, the cortical pyramidal neurons behave comparably to LIF units. Rauch et al. (2003) exhibited that the integrate-and-fire model dispenses "an adequate model reduction of cortical cells" throughout the stationary current injection.

Figure-1b: LIF responses to a weak (blue) vs. strong (red) step current; dashed line marks the firing threshold.

Learning Outcomes: LIF Model Usage and Extensions

- **The LIF model is a canonical spike model:** It is analytically tractable yet captures significant neuronal features (Burkitt, 2006). It permeates efficient simulation of large networks of spiking neurons. For example, Izhikevich designed his simple spiking model to retain the realism of Hodgkin–Huxley whilst complementing the computational efficiency of IF neurons (Izhikevich, 2003).

- **Extensions refine realism whilst preserving efficiency:** Adaptive LIF models i.e., the spike-frequency adaptation, and conductance-based LIF models include synaptic reversal potentials which all abide to the core integrate and threshold or reset principle.

Hodgkin–Huxley Model Overview

The **Hodgkin–Huxley (HH) model** (Hodgkin & Huxley, 1952) is a biophysically detailed conductance-based model of a neuron. The membrane is depicted as a capacitance analogous with voltage-gated sodium (Na^+), potassium (K^+), and leak conductances (Gerstner et al., 2014).

- **Gating dynamics:** Ion channels are inhibited by voltage-dependent gating variables *m, h, n,* which abide by first-order kinetics principles and further determine channel opening as well as closing probabilities.
- **Membrane equation:** The transmembrane voltage evolves according to:

$$\frac{\pi}{2} = \int_{-1}^{1} \sqrt{1 - x^2}\ dx$$

This system replicates the full action-potential waveform, including the rapid Na^+-driven upstroke, K^+-mediated repolarization, after-hyperpolarization, and refractory periods.

- **Historical significance:** It is derived from voltage-clamp experiments on squid giant axon, therefore the HH model remains an exemplar biophysical model of spike generation as well as a rudimentary framework for modern computational neuroscience (Hodgkin & Huxley, 1952).

HH Model – Response to Step Currents

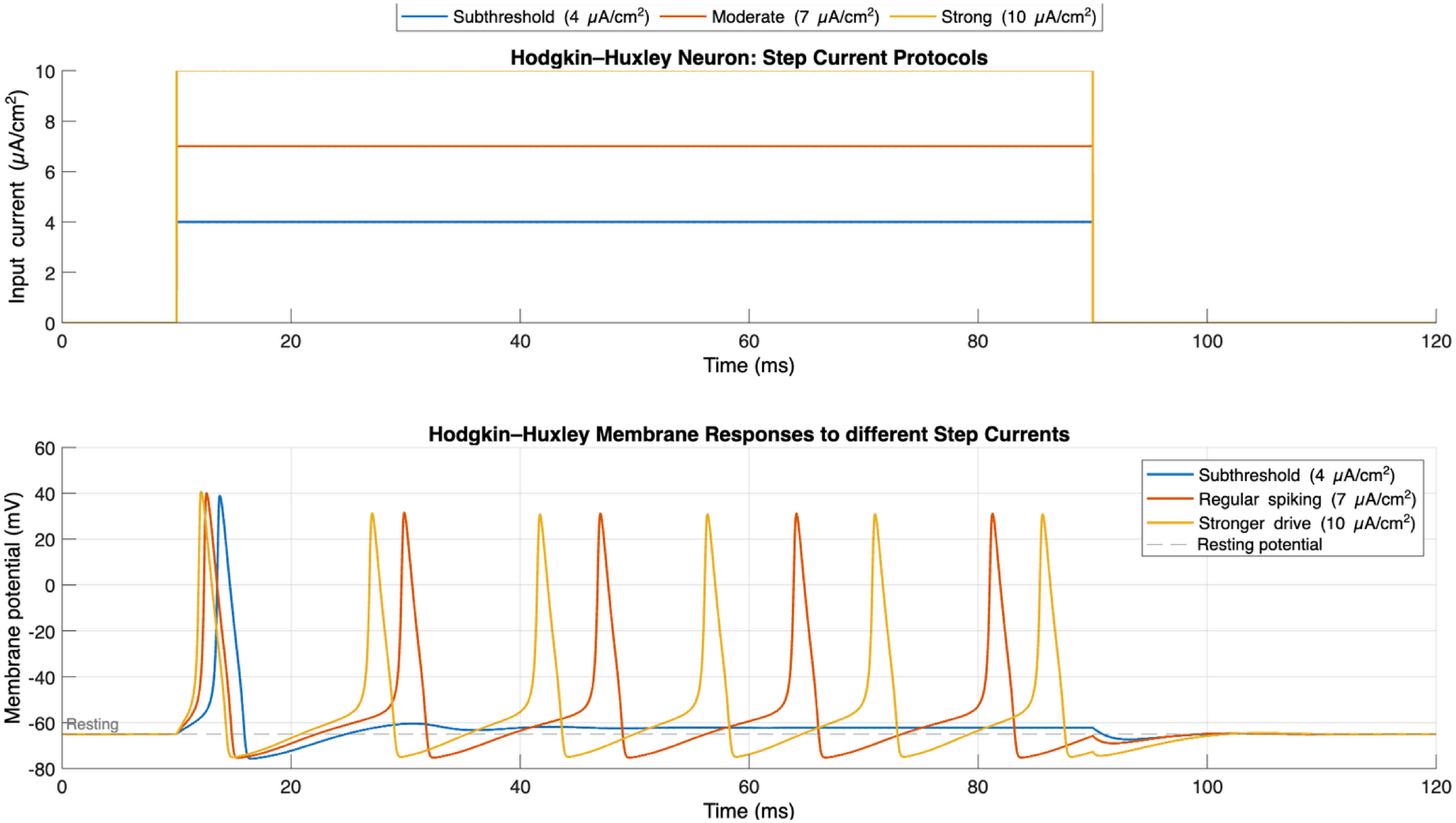


Figure-2: HH model responses to increasing step-current amplitudes (top) and resulting membrane potentials (bottom).

The Figure-2 exhibits the **HH membrane potential** as a reaction to three step currents (4, 7, 10 μA/cm²). The smallest current (4 μA/cm²) is below threshold and elicits no spikes while stronger currents (7 and 10 μA/cm²) generate regular trains of action potentials. Notably, larger input yields higher firing frequency further exemplifying the HH model's threshold and frequency–current relation (Hodgkin & Huxley, 1952).

Additionally, it should also be noted that **HH spikes reach a peak near +30 mV and reset to ~–65 mV**, exhibiting a rapid Na^+ influx and K^+-mediated repolarization. This compeers with the classic HH dynamics as well as the experimentally observed AP shape (Gerstner et al., 2014).

Learning Outcomes and References

Learning Outcomes: The distinctions between simple IF models and detailed HH models is pivotal. The chieft learning outcomes include how membrane integration as well as a threshold lead to spiking in LIF neurons, and how voltage-gated Na^+/K^+ currents produce the action potential in the HH model. This comparison exhibits the interchange amidst computational simplicity (IF) and biophysical realism (HH), and emphasizes how threshold nonlinearity shapes neuronal firing patterns.

References

Burkitt, A. N. (2006). A review of the integrate-and-fire neuron model: I. Homogeneous synaptic input. Biological Cybernetics, 95(1), 1–19.

Rauch, A., La Camera, G., Lüscher, H. R., Senn, W., & Fusi, S. (2003). Neocortical pyramidal cells respond as integrate-and-fire neurons to in vivo-like input currents. Journal of Neurophysiology, 90(4), 1598–1612.

Izhikevich, E. M. (2003). Simple model of spiking neurons. IEEE Transactions on Neural Networks, 14(6), 1569–1572.

Hodgkin, A. L., & Huxley, A. F. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. Journal of Physiology, 117(4), 500–544.

Gerstner, W., Kistler, W. M., Naud, R., & Paninski, L. (2014). Neuronal dynamics: From single neurons to networks and models of cognition (1st ed.). Cambridge University Press.