De Broglie hypothesis.

The universe is made of Radiation (light) and matter (particle). The light exhibits the dual nature (ie) it can behave both as a wave (Interference, diffraction phenomenon) and as a particle (Compton effect, photo-electric effect etc). Since the nature loves symmetry in 1924 Louis de-Broglie suggested that an electron (or) any other material particle must exhibit wave like properties in addition to particle nature. The waves associated with a material particles are called Matter waves.

de-Broglie wave length.

From the theory of light, considering a photon as a particle the total energy of the photon is given by

$$E = mc^2 \longrightarrow (1)$$

where $m \rightarrow$ Mass of the particle

$c \rightarrow$ Velocity of light

Considering the photon as a wave, the total energy is given by

$$E = h\nu \longrightarrow (2)$$

where $h \rightarrow$ Planck's constant

$\nu \rightarrow$ Frequency of radiation.

From equations (1) and (2) we can write

$$E = mc^2 = h\nu \longrightarrow (3)$$

we know momentum = mass × velocity

$$p = mc$$

∴ Equations ③ becomes $h\nu = pc$

$$p = \frac{h\nu}{c}$$

Since $\frac{c}{\nu} = \lambda$ we can write $p = \frac{h}{\lambda}$

(or) The wavelength of a photon $\lambda = \frac{h}{p}$

de-Broglie suggested that equation ④ can be applied both for photons and materials particles. If m is the mass of the particle and v is the velocity of the particle then

Momentum $p = mv$

∴ de-Broglie wavelength $\lambda = \frac{h}{mv}$

Other forms of de-Broglie wavelength.

① de-Broglie wavelength interms of Energy

we know kinetic energy $E = \frac{1}{2}mv^2$

Multiplying by m on both sides we get

$$Em = \frac{1}{2}m^2v^2$$

$$m^2v^2 = 2Em$$

$$mv = \sqrt{2Em}$$

∴ de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

(ii) de-Broglie wavelength interms of voltage

If a charged particle of charge 'e' is accelarated through a potential difference 'V'

Then the kinetic energy of the particle $= \frac{1}{2}mv^2 \rightarrow (7)$

Also we know energy $= eV \rightarrow (8)$

Equating equations (7) and (8) we get

$$\frac{1}{2}mv^2 = eV$$

Multiplying by 'm' on both sides we get

$$m^2v^2 = 2meV$$

$$\text{(or) } mv = \sqrt{2meV} \rightarrow (9)$$

Substituting equations (9) in (6) we get

$$\text{de-Broglie wavelength } \lambda = \frac{h}{\sqrt{2meV}} \rightarrow (10)$$

(iii) de-Broglie wavelength interms of Temperature

When a particle like neutron is in thermal equilibrium at temperature T, then they possess Maxwell distribution of velocities.

$\therefore$ Their kinetic energy $E_K = \frac{1}{2}mv_{rms}^2 \rightarrow (11)$

where V_{rms} is the Root mean square velocity of the particle

Also we know Energy $= \frac{3}{2}K_BT \rightarrow (12)$

where K_B is the Boltzmann constant

$\therefore$ Equating equations (11) and (12) we get

$$\frac{1}{2}mv^2 = \frac{3}{2}K_BT$$

$$m^2v^2 = 3mK_BT$$

$$mv = \sqrt{2mK_BT}$$

$\therefore$ de-Broglie wavelength $\lambda = \frac{h}{mv}$

$$\lambda = \frac{h}{\sqrt{3mK_BT}} \longrightarrow (13)$$

Properties of Matter waves.

① Matter waves are not electromagnetic waves.

② Matter waves are new kind of waves in which due to the motion of the charged particles. electromagnetic waves are produced.

③ The wave and particle aspects cannot appear together.

④ Locating the exact position of the particle in the wave is uncertain.

⑤ Lighter particles will have high wavelength.

Heisenberg's Uncertainty Principle

In 1927, Heisenberg proposed a very interesting principle, which is a direct consequence of the dual nature of matter known as uncertainty principle. In classical mechanics, a moving particle at any instant has fixed position in space and a definite momentum which can be determined if the initial values are known. However, in wave mechanics the particle is described interms of a wavepacket. According to Born's probability interpretation the particle may be found anywhere within the wavepacket. when the wavepacket is small, the position of the particle may be fixed but the particle will spread rapidly and hence the velocity becomes indeterminate. On the other hand, when the wavepacket is large, the velocity can be fixed but there is large, indefiniteness in position. In this way the certainty in position involves uncertainty in momentum or velocity and certainty of momentum involves the uncertainty in position. This shows that it is impossible to know where within the wave packet the particle is and what is its exact momentum.

According to Heisenberg uncertainty principle " It is impossible to specify precisely and simultaneously the values of both members of particular pairs of physical variables that describe the behaviour of an atomic system " Qualitatively this principle states that " the order of magnitude of the product of the uncertainties in the knowledge of two variables must be at least Plank's constant h.

Considering the pair of physical variables as position and momentum we have

$$\Delta p \cdot \Delta x \simeq h \longrightarrow (1)$$

where Δp is the uncertainty in determining the momentum and Δx is the uncertainty in determining the position of the particle, Similarly we have

$$\Delta E \, \Delta t \simeq h \longrightarrow (2)$$

$$\Delta J \, \Delta \theta \simeq h \longrightarrow (3)$$

where ΔE and Δt are uncertainties in determining the energy and time while ΔJ and $\Delta \theta$ are uncertainties in determining the angular momentum and angle.

Physical Significance of a wave function (ψ)

wave function :

It is the variable quantity that is associated with a moving particle at any position (x, y, z) and at any time 't' and it relates the probability of finding the particle at that point and at that time.

* It relates the particle and the wave statistically

$$\Psi(x, y, z, t) = A e^{-i\omega(t - x/v)}$$

$$\Psi = \psi e^{-i\omega t}$$

* Wave function gives the information about the particle behaviour.
* Ψ is a complex quantity and individually it does not have any meaning.
* $|\psi|^2 = \psi^* \psi$ is real and positive, it has physical meaning. This concept is similar to light. In light, amplitude may be positive (or) negative but the intensity, which is the square of amplitude is real and is measurable.
* $|\Psi|^2$ represents the probability density (or) probability of finding the particle per unit volume

* For a given volume $d\tau$, the probability of finding the particle is given by

$$\text{Probability } (P) = \iiint |\psi|^2 \, d\tau$$

where $d\tau = dx \cdot dy \cdot dz$.

* The probability will have any value between zero to one.

① If $P=0$ then there is no chance for finding the particle (ie) there is no particle, within the given limits.

② If $P=1$ then there is 100% chance for finding the particle (ie) the particle is definitely present within the given limits.

③ If $P=0.7$, then there is 70% chance for finding the particle and 30% there is no chance for finding the particle, within the given limits.

Example: If a particle is definitely present within a one dimensional box (x-direction) of length 'l', then the probability of finding the particle can be written as

$$P = \int_0^l |\psi|^2 \, dx = 1$$