“Tree-Based Symmetric Key Broadcast Encryption”

(Thesis Defense)

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ISI, Kolkata; ENS-Lyon

Supervisor: Prof. Palash Sarkar

ISI, Kolkata

Date: 8^{th} October, 2015

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Outline

Symmetric Key Encryption

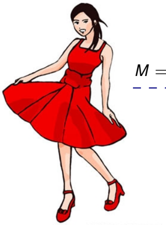
Alice

Bob

Symmetric Key Encryption

Alice

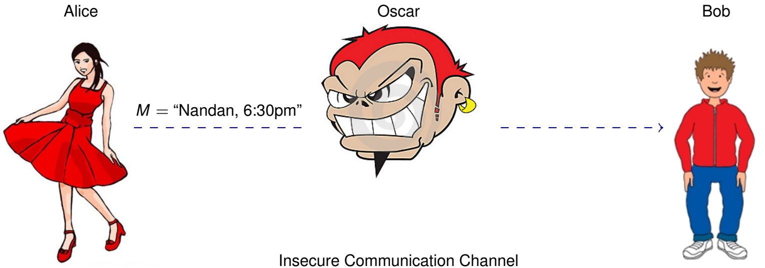
Bob



M = “Nandan, 6:30pm”

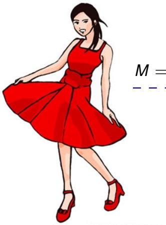
Insecure Communication Channel

Symmetric Key Encryption



Symmetric Key Encryption

Alice



Oscar

Bob

M = "Nandan, 6:30pm"

Insecure Communication Channel

K $\quad$ Enc $\qquad\qquad$ Dec $\quad$ K

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K

Enc

Dec

K

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$K \dashrightarrow$ $C \leftarrow E(M, K)$

K

Enc

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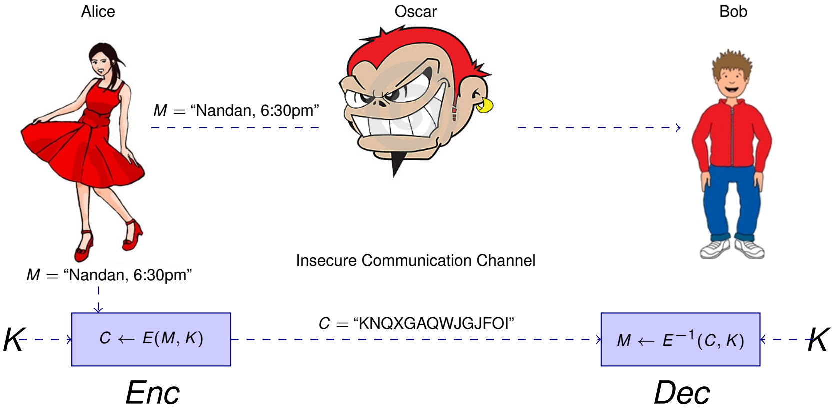
Insecure Communication Channel

M = “Nandan, 6:30pm”

K $\quad$ $C \leftarrow E(M, K)$ $\quad$ C = “KNQXGAQWJGJFOI” $\quad$ K

Enc $\quad$ *Dec*

Symmetric Key Encryption



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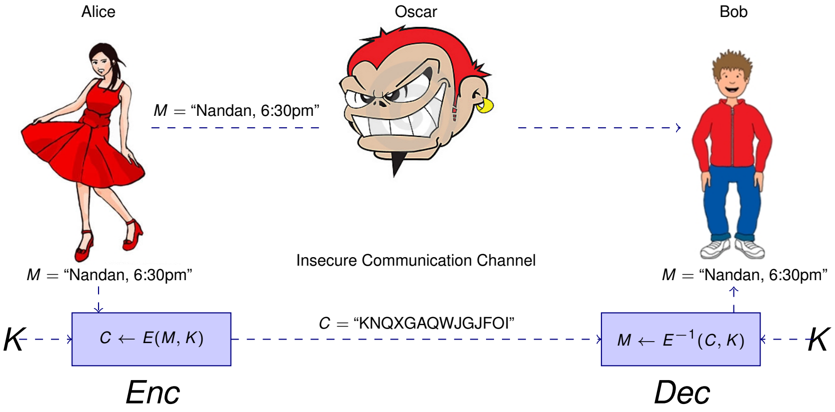

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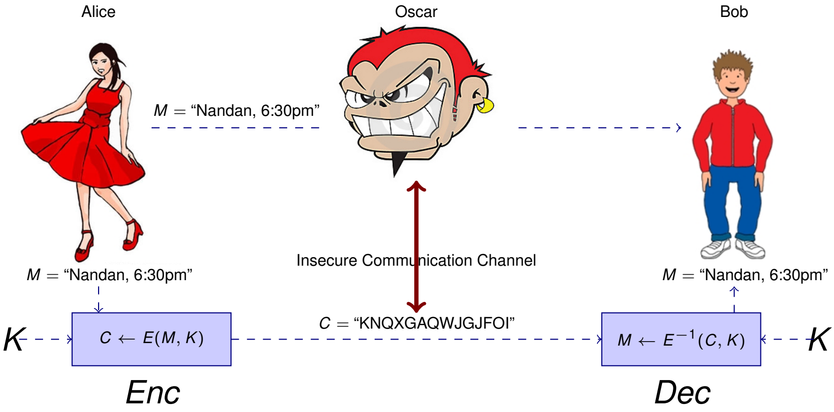
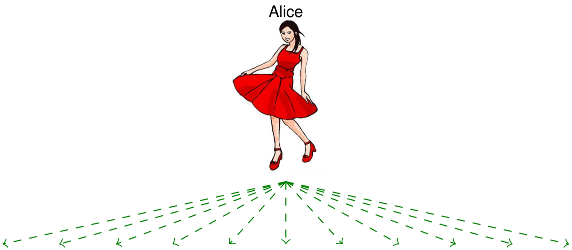


One Alice → Many Bobs?

Alice

One Alice $\rightarrow$ Many Bobs?



n (Bobs and Oscars)

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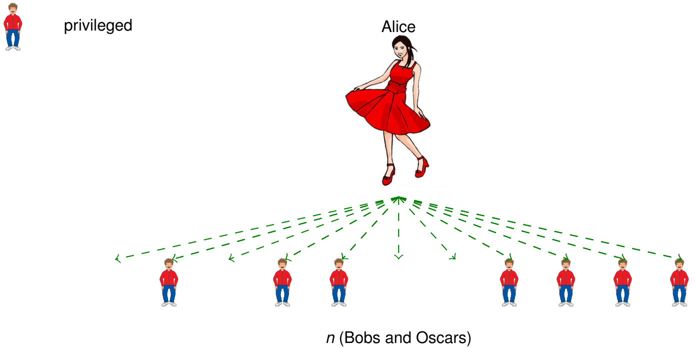

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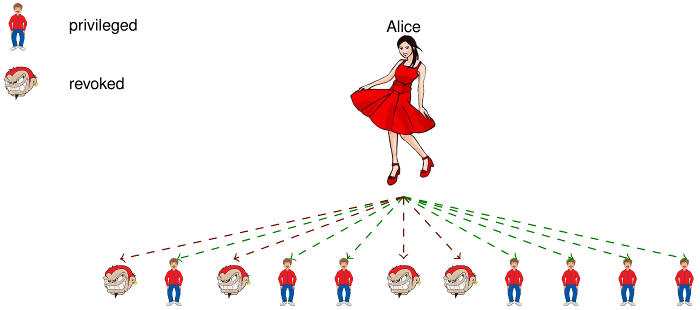

n (Bobs and Oscars)

Pay-TV center $\rightarrow$ Subscribers

Broadcasting Center
(Tata Sky, Dish TV, etc.)

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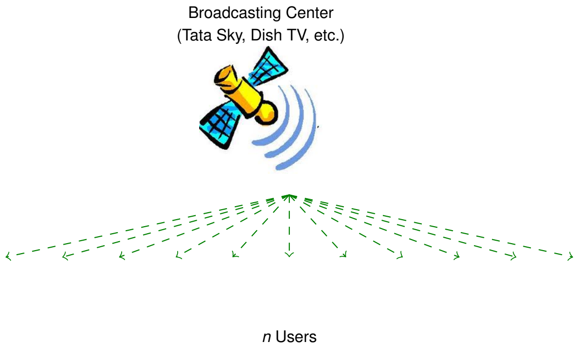
Pay-TV center $\rightarrow$ Subscribers



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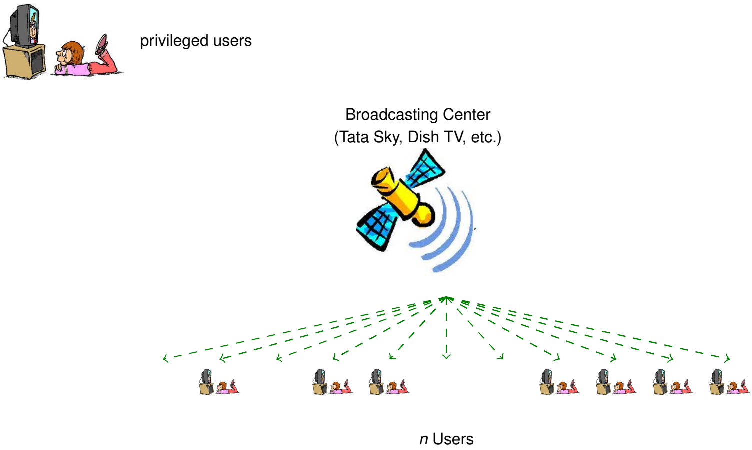

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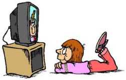

privileged users

revoked users

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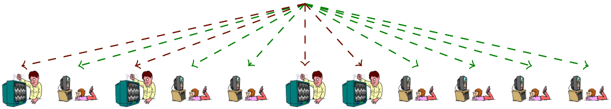

n Users

DRM in Blu-ray/DVD discs

(Copyrighted) Content Production House

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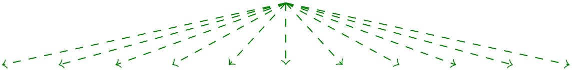

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legitimate player

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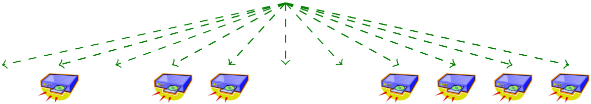

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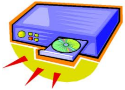
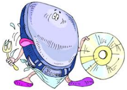
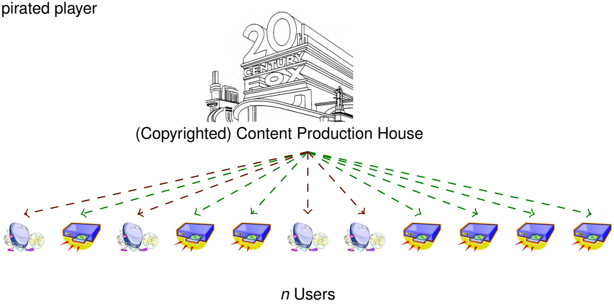
 legitimate player

 pirated player



Basic Schemes

$\mathcal{N}$: set of all users $u_1, \ldots, u_n$; $\quad n = |\mathcal{N}|$

$\mathcal{R}$: set of revoked users; $\quad r = |\mathcal{R}|$

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Singleton Set Scheme

$\mathcal{S} = \{\{u_1\}, \ldots, \{u_n\}\}$

- Each user is assigned a unique key. $\quad O(1)$
- M has to be encrypted for each user in $\mathcal{N} \setminus \mathcal{R}$. $\quad O(n - r)$

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Power Set Scheme

$\mathcal{S} = \{\{u_1\}, \ldots, \{u_1, u_2\}, \ldots, \{u_1, \ldots, u_{n-1}\}, \ldots, \mathcal{N}\}$

- Each subset of users is assigned a unique key. $O(2^n)$
- M is encrypted only once for the set $\mathcal{N} \setminus \mathcal{R} \in \mathcal{S}$. $O(1)$

Subset Cover Framework [NNL01]

1. Initiation

- Choose the collection

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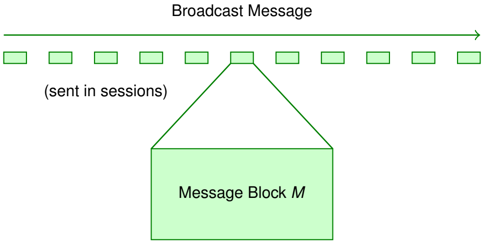

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$$\boxed{F_{K_s}(M)}$$

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$F_{K_s}(M)$	$E_{L_{i_1}}(K_s)$	$\dots$	$E_{L_{i_h}}(K_s)$

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body	header		

Subset Cover Framework [NNL01]

3. Decryption

$F_{K_s}(M)$	$E_{L_{i_1}}(K_s)$	$\cdots$	$E_{L_{i_h}}(K_s)$

For $u \in S_{i_j}$ where $S_{i,j} \in \mathcal{S}_c$

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- $M \leftarrow F^{-1}_{K_s}(F_{K_s}(M))$

Parameters of Interest

- $|\mathcal{S}_c| = h$: header length (costliest parameter)

  Example: Pay-TV bandwidth cost

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- Encryption time
- Decryption time

  Example: TV set-top box booting time

  

Applications of BE

- **Pay-TV**, CableLabs standard.
- **AACS:** Disney, Intel, Microsoft, Panasonic, Warner Bros., IBM, Toshiba and Sony.

Blu-ray Disc Manufacturer

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- File Sharing in Encrypted File Systems.
- Mailing list encryption: [BGW05] OpenPGP functions as a BE system
- Online content sharing and distribution [BBW06]
- eCommerce: trade secret broadcasts
- ...

Why NOT use Public-Key BE?

Efficiency!!!

(decryption time, hence cost)

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- determines the encryption and decryption time
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Two types of $\mathcal{S}$

- Subset Difference $\{1\}, \{3\}, \{6, 7, 8\}$
- Punctured Interval $\{1, 3, 6\}, \{7, 8\}$

Dalit Naor, Moni Naor, and Jeffery Lotspiech.
Revocation and tracing schemes for stateless receivers.
In Joe Kilian, editor, *CRYPTO*, volume 2139 of *Lecture Notes in Computer Science*, pages 41–62. Springer, 2001.

Nam-Su Jho and Jung Yeon Hwang and Jung Hee Cheon and Myung-Hwan Kim and Dong Hoon Lee and Eun Sun Yoo.
One-Way Chain Based Broadcast Encryption Schemes.
In Ronald Cramer, editor, *EUROCRYPT*, volume 3494 of *Lecture Notes in Computer Science*, pages 559–574. Springer, 2005.

Outline

Preliminaries

Background

NNL-SD: Initiation

Define $\mathcal{S}_{NNL-SD}$

Key Assignment

Key Distribution

NNL-SD: Encryption

Halevy-Shamir Layered SD

Other Related Works

Our Contributions

Paper 1: Arbitrary n; Detailed Analysis

Paper 2: Layering; Minimizing Storage

Paper 3: k-ary Generalization

Paper 4: Assured Savings on Communication

Conclusion

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Subset Difference (SD) Scheme [NNL01]

Naor-Naor-Lotspiech (2001)

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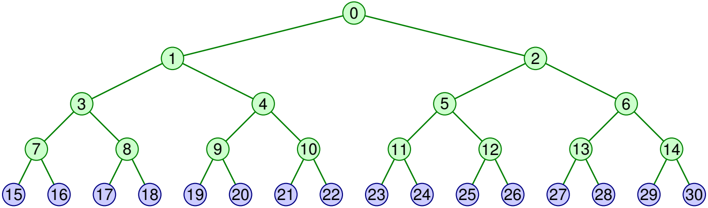

Assumed $n = 2^{\ell_0}$

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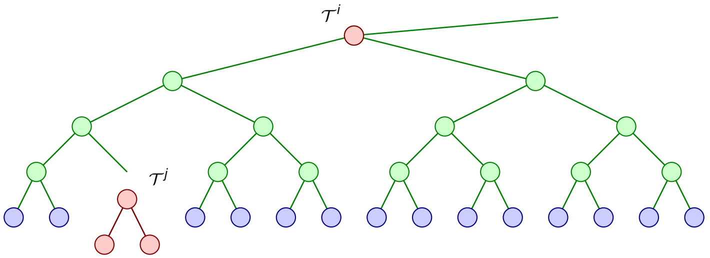

All users that are in $\mathcal{T}_i$ but not in $\mathcal{T}_j$

Key Assignment

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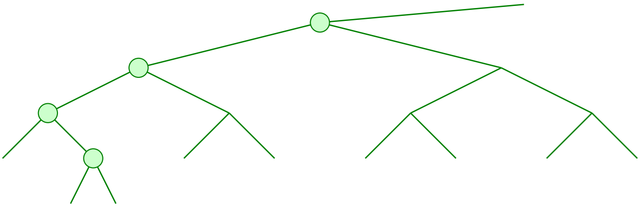

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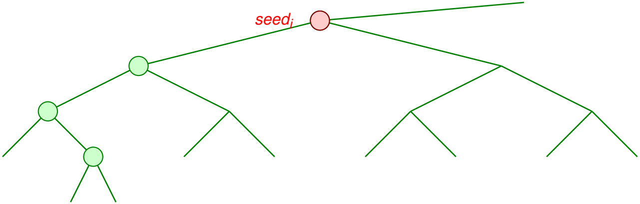

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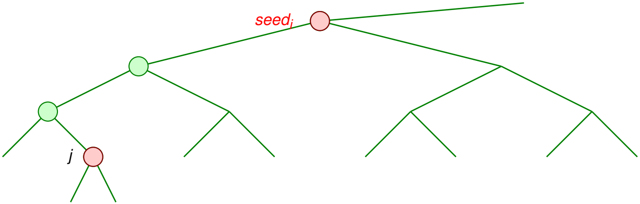

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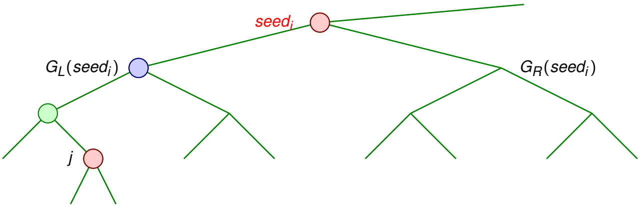

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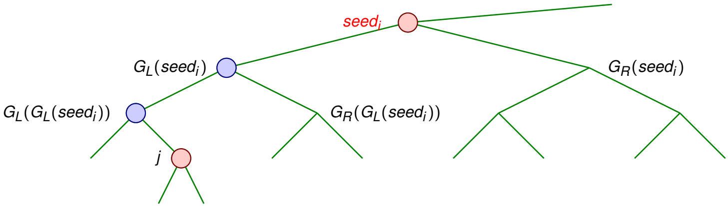
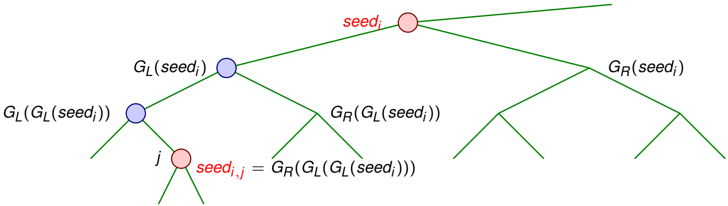


Key Assignment

Key for $S_{i,j}$:

- Assign random $seed_i$ to each internal node i
- Pseudo-random generator (PRG): $G : \{0,1\}^k \rightarrow \{0,1\}^{3k}$

$$G(seed) = G_L(seed) || G_M(seed) || G_R(seed)$$



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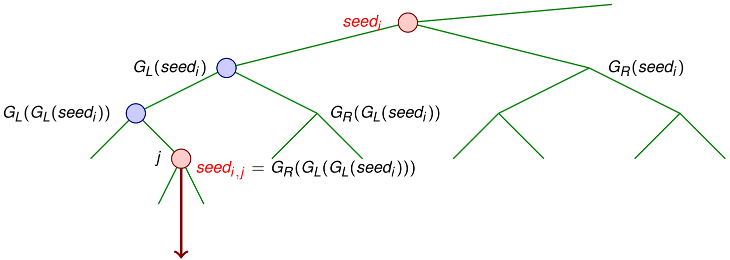

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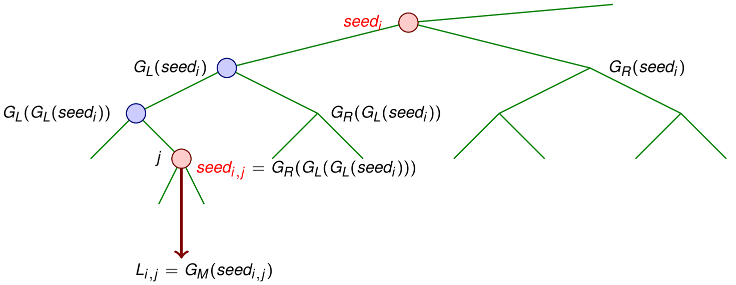
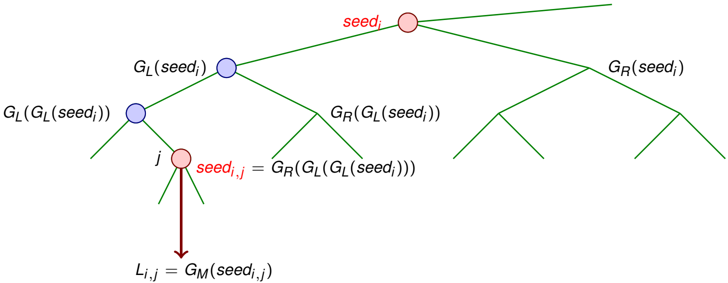


Key Assignment

Key for $S_{i,j}$:

- Assign random $\color{red}{seed_i}$ to each internal node i
- Pseudo-random generator (PRG): $\color{red}{G : \{0,1\}^k \rightarrow \{0,1\}^{3k}}$

$$\color{blue}{G(seed) = G_L(seed) || G_M(seed) || G_R(seed)}$$



Key of $S_{i,j}$: $\color{red}{L_{i,j}} = G_M(seed_{i,j})$

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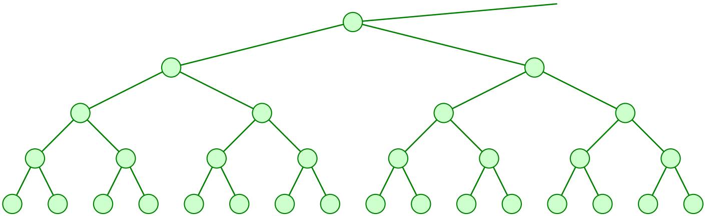

Figure: Secrets stored by u

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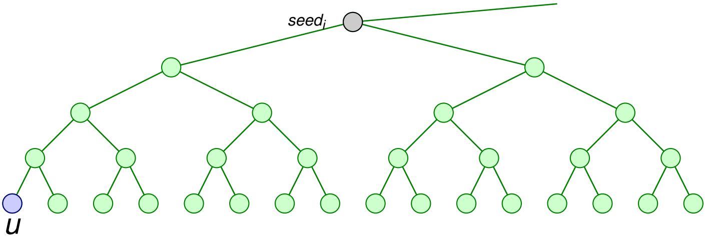

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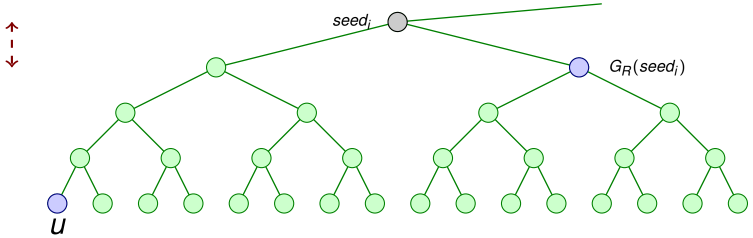

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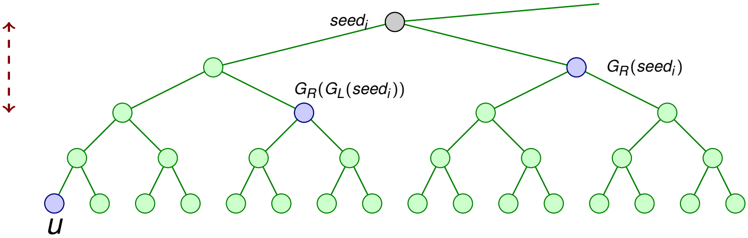

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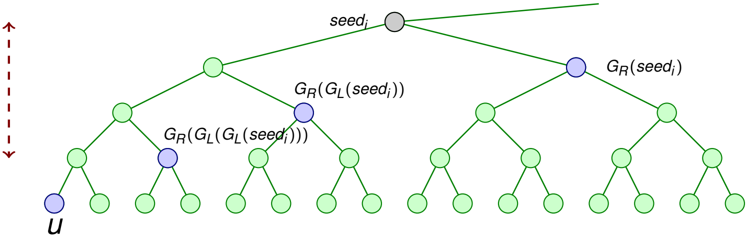

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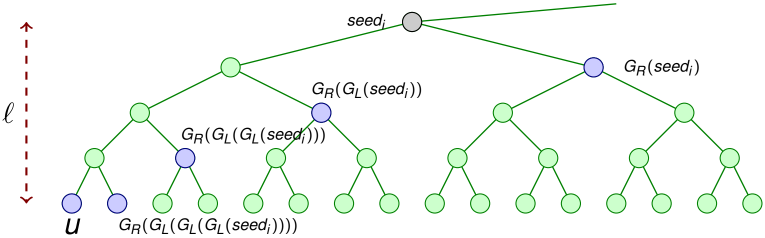

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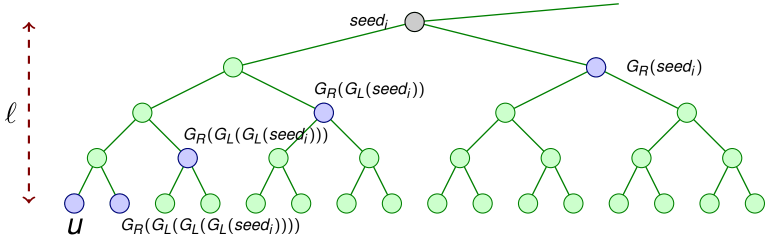

$1 +$

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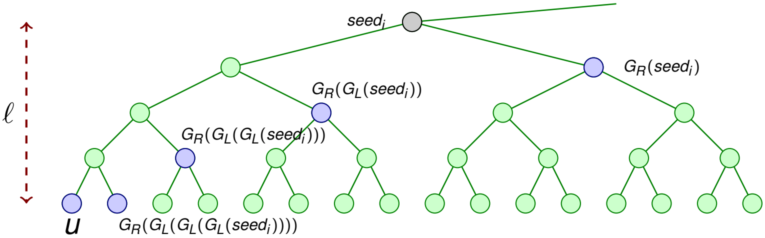

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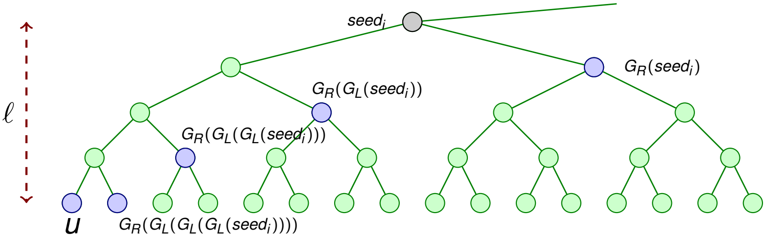

$$1 + 2 + \cdots + \ell_0 =$$

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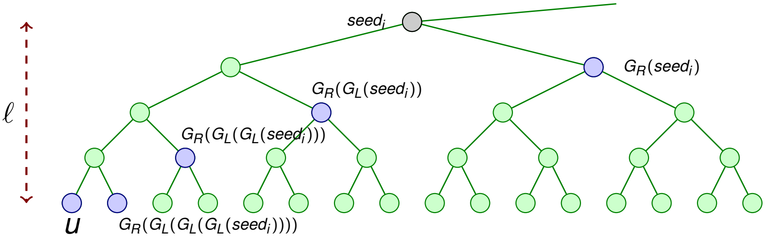

$$1 + 2 + \cdots + \ell_0 = \frac{\ell_0(\ell_0+1)}{2}$$

Figure: Secrets stored by u

Outline

Subset Cover Finding Algorithm

Given $\mathcal{R}$, find

$$\mathcal{S}_c = \{S_{i_1,j_1}, \ldots, S_{i_h,j_h}\}$$

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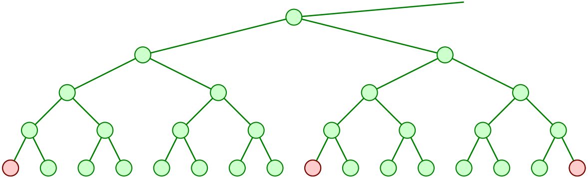

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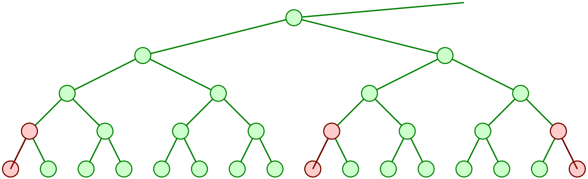

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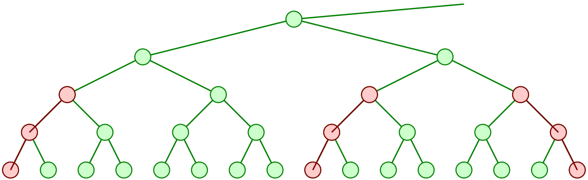

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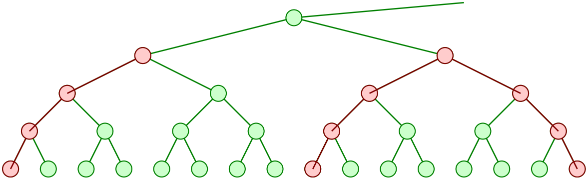

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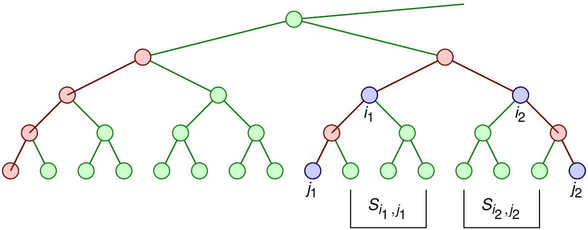

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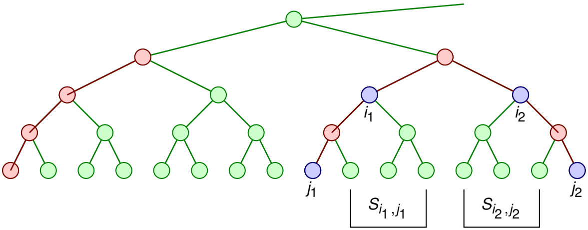

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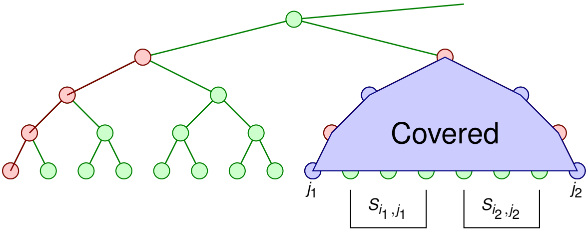

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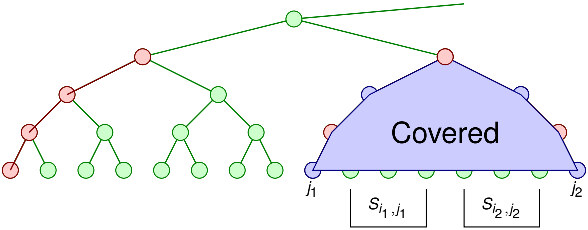

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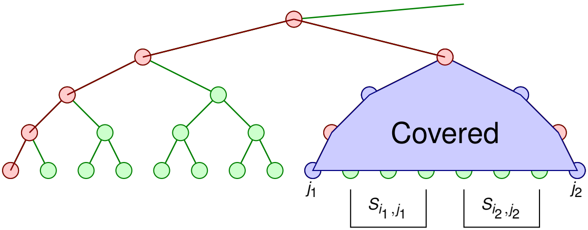

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Subset Cover Finding Algorithm

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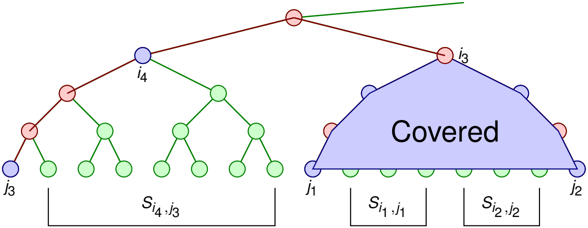

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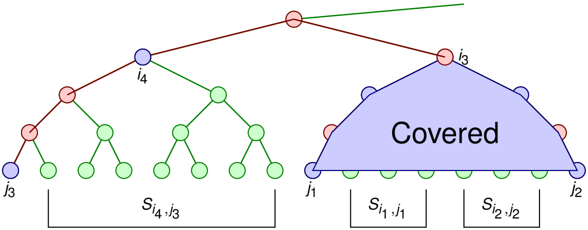

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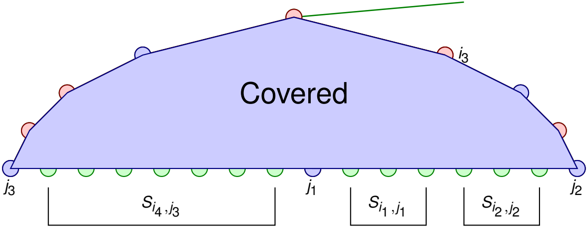

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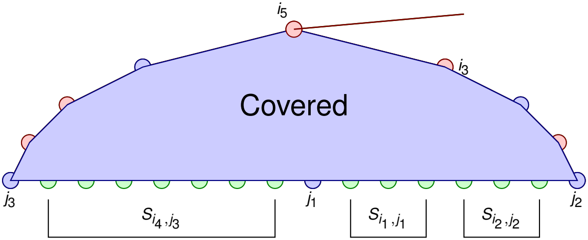

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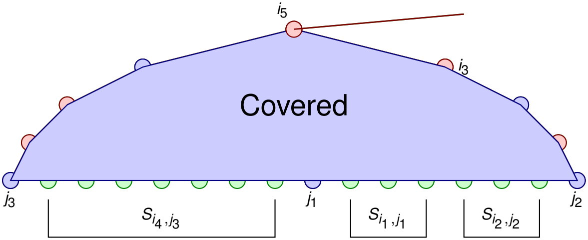

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NNL-SD Parameters

For n users out of which r are revoked:

- User storage: $O(\log^2(n))$.
- Maximum header length: $2r - 1$.
- Maximum decryption time: $O(\log n)$.

Outline

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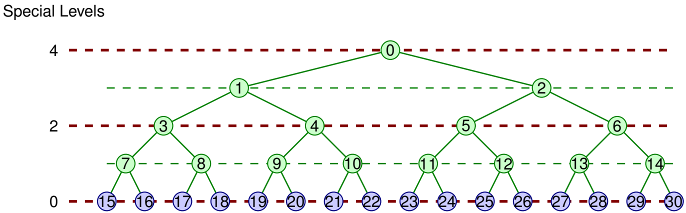
Layered Subset Difference Scheme [HS02]

Halevy-Shamir (CRYPTO, 2002): *"special levels"*



Using layering (with special levels), $\mathcal{S}_{HS-LSD} \subset \mathcal{S}_{NNL-SD}$.

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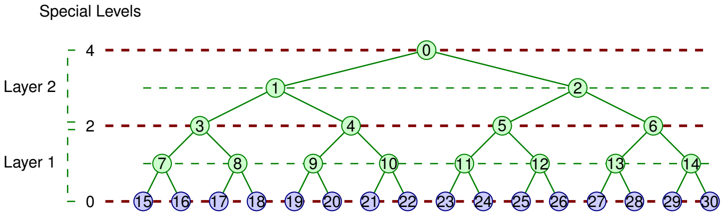

Using layering (with special levels), $\mathcal{S}_{HS-LSD} \subset \mathcal{S}_{NNL-SD}$.

Layered SD subsets

Which $S_{i,j} \in \mathcal{S}_{HS-LSD}$?

- If i is at a special level:
  for all j in $\mathcal{T}^i$, $S_{i,j} \in \mathcal{S}_{HS-LSD}$
- If i is not at a special level:
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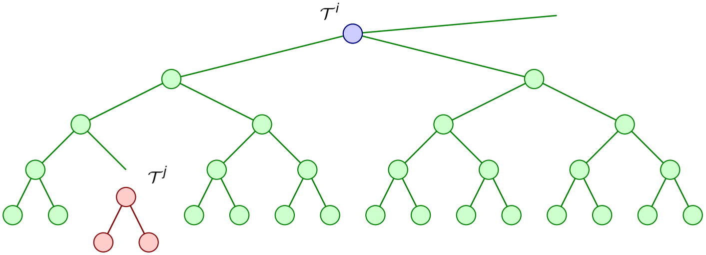

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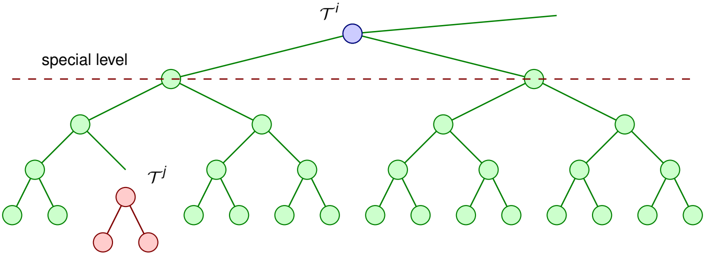

Layered SD subsets

$S_{i,j} \in \mathcal{S}_{NNL-SD} \setminus \mathcal{S}_{HS-LSD}$ if

- i is not at a special level
- and i and j are not in the same layer

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How to cover these subsets?

Layered SD subsets

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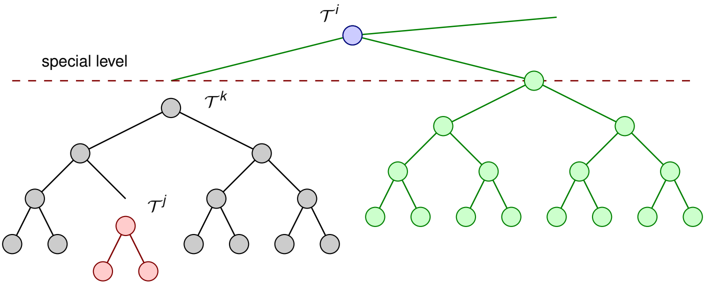
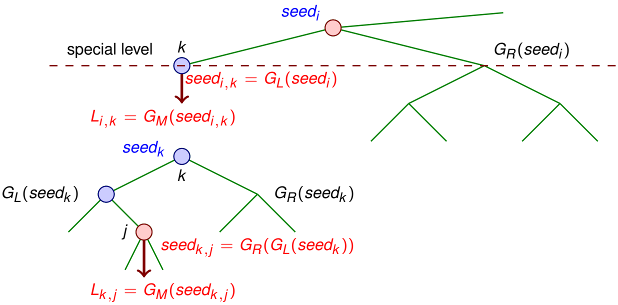
How to cover these subsets? SPLIT!!!



Subsets in $\mathcal{S}_{SD} \setminus \mathcal{S}_{LSD}$ are split into: $S_{i,j} = S_{i,k} \cup S_{k,j}$.

Layered SD Scheme

- Key for $S_{i,k}$ is $L_{i,k} = G_M(G_L(seed_i))$
- Key for $S_{k,j}$ is $L_{k,j} = G_M(G_R(G_L(seed_k)))$



LSD Parameters

NNL-SD scheme:

- User storage needed: $O(\log^2(n))$.
- Maximum Header Length: $2r - 1$.
- Decryption Time: $O(\log n)$.

HS-LSD scheme:

- User Storage needed: $O(\log^{3/2} n)$.
- Maximum header length: $4r - 2$.
- Decryption Time: $O(\log n)$.

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Outline

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Other SD-based Schemes

[GoodrichST04] Stratified SD

- Key assignment: Left and right preorder tree traversals
- $O(\log n)$ storage; $O(n)$ decryption time
- Double header length

[FukushimaKTS08] 3-ary tree SD

"However, in a general a-ary tree with $a \geq 4$,... our hash chain approach fails... Thus, the construction of a coalition resistant a-ary SD method with reasonable communication, computation, and storage overhead is an open issue."

[WangYL14] Balanced Double SD

- Published after I submitted my thesis
- We have better results now

Analysis of SD scheme

[ParkB06]

- Generating function for $N(n, r, h)$
- Mean header length: “complex to compute and difficult to gain insight from”

[EagleOPR08]

- Small standard deviations

[MartinMW09]

- Maximum header length

Outline

Preliminaries

Background

- NNL-SD: Initiation
  - Define $\mathcal{S}_{NNL-SD}$
  - Key Assignment
  - Key Distribution
- NNL-SD: Encryption
- Halevy-Shamir Layered SD
- Other Related Works

Our Contributions

- Paper 1: Arbitrary n; Detailed Analysis
- Paper 2: Layering; Minimizing Storage
- Paper 3: k-ary Generalization
- Paper 4: Assured Savings on Communication

Conclusion

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Complete Tree SD (CTSD) Scheme

Question: What happens when $n \neq 2^{\ell_0}$?
Answer: Add dummy users to get to the next power of two.

- If the dummy users are considered revoked, then the effect on the header length is disastrous.
- If the dummy users are privileged, the situation is better but, there is still a measurable effect on the header length.

Solution: Use a complete binary tree.

- "Completes" (and also subsumes) the NNL-SD scheme to work for any number of users.
- Conceptually simple; working out the details is a bit involved.

CTSD Scheme: Header Length Analysis

(n, r)-revocation

A choice of r revoked users out of total n users

For each (n, r)-revocation,

$$h \in \{1, \dots, h_{max}\}$$

$N(n, r, h)$

$\#(n, r)$-revocations for which the the header length is h.

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$\#(n, r)$-revocations for which the the header length is h.

How to compute $N(n, r, h)$?

The only known method would

- enumerate all possible $\binom{n}{r}$ (n, r)-revocations
- run the cover finding algorithm for each
- count the number of (n, r)-revocations leading to a header of size h.

Recurrence relation for $N(n, r, h)$

- $N(\lambda_i, r_1, h_1) = T(\lambda_i, r_1, h_1) + \sum_{j \in \mathrm{IN}(i)} T(\lambda_j, r_1, h_1 - 1)$ where $\mathrm{IN}(i)$ is the set of all internal nodes in the subtree $\mathcal{T}^i$ excluding the node i.
- $T(\lambda_i, r_1, h_1) = \sum_{r'=1}^{r_1-1} \sum_{h'=0}^{h_1} N(\lambda_{2i+1}, r', h') \times N(\lambda_{2i+2}, r_1 - r', h_1 - h')$ where λ_{2i+1} (respectively λ_{2i+2}) is the number of leaves in the left (respectively right) subtree of $\mathcal{T}^i$.

$T(\lambda_i, r_1, h_1)$	$r_1 < 0$	$r_1 = 0$	$r_1 = 1$	$2 \leq r_1 < n$	$r_1 = n$	$r_1 > n$
$h_1 = 0$	0	0	0	0	1	0
$h_1 \geq 1$	0	0	0	from rec.	0	0
$N(\lambda_i, r_1, h_1)$	$r_1 < 0$	$r_1 = 0$	$r_1 = 1$	$2 \leq r_1 < n$	$r_1 = n$	$r_1 > n$
$h_1 = 0$	0	0	0	0	1	0
$h_1 = 1$	0	1	n	from rec.	0	0
$h_1 > 1$	0	0	0	from rec.	0	0

Table: Boundary conditions on $T(n, r, h)$ and $N(n, r, h)$.

Computing $N(n, r, h)$

Dynamic Programming:

- $N(n, r, h)$ can be computed in $O(r^2h^2 \log n + rh \log^2 n)$ time and $O(rh \log n)$ space.
- $N(n, r, h)$ for all possible h can be computed in $O(r^4 \log n + r^2 \log n)$ time and $O(r^2 \log^2 n)$ space.
- $N(n, r, h)$ for all possible r and h can be computed in $O(n^4 \log n + n^2 \log^2 n)$ time and $O(n^2 \log n)$ space.
- $N(i, r, h)$ for $2 \leq i \leq n$ and all possible r and h can be computed in $O(n^5 + n^3 \log n)$ time and $O(n^3)$ space.

The combinatorics behind the cover generation algorithm was well captured!
(for n ~125)

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Using $N(n, r, h)$: Maximum Header Length

Theorem

The maximum header length in the CTSD method for n users is $h_{max} = \min(2r - 1, \lfloor \frac{n}{2} \rfloor, n - r)$.

- For the NNL-SD scheme, the bound of $2r - 1$ was known.
- Complete (refined) picture:
  - if $r \leq n/4$, $h_{max} = 2r - 1$;
  - if $n/4 < r \leq n/2$, $h_{max} = n/2$; and
  - for $r > n/2$, $h_{max} = n - r$.

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Using $N(n, r, h)$: More analysis

n_r

The value of n for which the header length of $2r - 1$ is achieved with r revoked users.

- Obtained a complete characterization of n_r.

Generating Function

- Similar to that of [PB06]

Probabilities and Expectation

- For n ~125
- Compute probabilities of $h \in \{1, \ldots, h_{max}\}$
- Compute expected value $H_{n,r}$

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Expected Header Length

Random experiment

Select a random subset of revoked users $\mathcal{R}$ from $\mathcal{N}$ (Select a random (n, r)-revocation).

Event: Node i generates a subset $S_{i,j}$

- $X^i_{n,r} = 1$ if $S_{i,j} \in \mathcal{S}_c$ for some j;
- $X^i_{n,r} = 0$ otherwise.

$$h = X^0_{n,r} + X^1_{n,r} + \cdots X^{n-1}_{n,r} = \sum_{i=0}^{n-1} X^i_{n,r}$$

$H_{n,r}$: expected header length for (n, r)-revocations.

$$H_{n,r} = \sum_{i=0}^{n-1} E[X^i_{n,r}] = \sum_{i=0}^{n-1} \Pr[X^i_{n,r} = 1]$$

$H_{n,r}$ for all SD based schemes

This technique has been useful for other SD-based schemes:

$$H_{n,r} = \sum_{i=0}^{n-1} \Pr[X^i_{n,r} = 1]$$

For the NNL-SD scheme:
Computing $H_{n,r}$ requires $O(r \log n)$ time and $O(1)$ space.

$H_{n,r}$ for the NNL-SD Scheme

Theorem: For all $n \geq 1$, $r \geq 1$, the expected header length $H_{n,r} \uparrow H_r$, as n increases through powers of two, where

$$H_r = 3r - 2 - 3 \times \sum_{i=1}^{r-1} \left(\left(-\frac{1}{2} \right)^i + \sum_{k=1}^{i} (-1)^k \binom{i}{k} \frac{(2^k - 3^k)}{(2^k - 1)} \right).$$

r	2	3	4	5	6
H_r/r	1.25	1.25	1.2455	1.2446	1.2448

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Outline

Preliminaries

Background

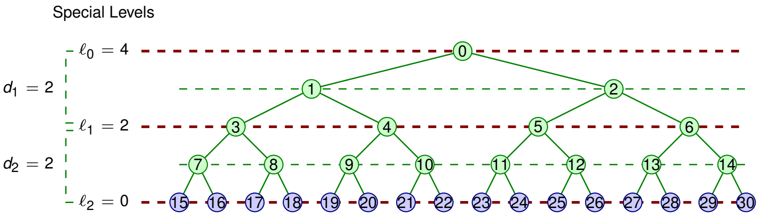
- NNL-SD: Initiation
  - Define $\mathcal{S}_{NNL-SD}$
  - Key Assignment
  - Key Distribution
- NNL-SD: Encryption
- Halevy-Shamir Layered SD
- Other Related Works

Our Contributions

- Paper 1: Arbitrary n; Detailed Analysis
- Paper 2: Layering; Minimizing Storage
- Paper 3: k-ary Generalization
- Paper 4: Assured Savings on Communication

Conclusion

Halevy-Shamir LSD Scheme



[HS02]: "The root is considered to be at a special level, and in addition we consider every level of depth $k \cdot \sqrt{\log(n)}$ *for* $k = 1 \ldots \log(n)$ *as special (wlog, we assume that these numbers are integers)."*

$n = 2^{\ell_0}$ with $\ell_0 = 4, 9, 16, 25$ only?

Layering Strategy

A choice of special levels is called a layering strategy.

General layering strategy $\boldsymbol{\ell}$

- Layering strategy $\boldsymbol{\ell} = (\ell_0, \ldots, \ell_e)$:
  - has $e + 1$ special levels
  - $\ell_0 > \ell_1 > \ldots > \ell_{e-1} > \ell_e = 0$.
- Layering strategy $\boldsymbol{d} = (d_1, \ldots, d_e)$
  - $d_i = \ell_i - \ell_{i-1}$ is a layer length
  - In general, the layer lengths need not be (almost) equal.

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Extending the HS Scheme

Residual bottom layer

Write $\ell_0 = d(e-1) + p$ where $1 \leq p \leq d$. Then the special levels are

$$\ell_0, \ell_0 - d, \ell_0 - 2d, \ldots, \ell - d(e-1), 0.$$

Balanced layering or extended-HS (eHS)

Write $\ell_0 = d(e-1) + p = (e-d+p)d + (d-p)(d-1)$. Define the layer lengths from the top to be

$$(\underbrace{d, \ldots, d}_{e-d+p}, \underbrace{d-1, \ldots, d-1}_{d-p}).$$

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Layering Strategy and User Storage

Layering strategy: $\boldsymbol{\ell} = (\ell_0, \ldots, \ell_e)$

$$\text{storage}_0(\boldsymbol{\ell}) \quad = \quad \sum_{i=0}^{e-1} \ell_i + \frac{1}{2} \sum_{i=0}^{e-1} (\ell_i - \ell_{i+1})(\ell_i - \ell_{i+1} - 1).$$

$$\text{storage}_0(\ell_0, \ell_1, \ldots, \ell_e)$$
$$= \quad \ell_0 + \frac{(\ell_0 - \ell_1)(\ell_0 - \ell_1 - 1)}{2} + \text{storage}_0(\ell_1, \ldots, \ell_e).$$

Storage Minimal Layering

$\mathrm{SML}_0(\ell_0)$

A layering strategy which minimizes the user storage among all layering strategies.

$\#\mathrm{SML}_0(\ell_0)$

User storage required by $\mathrm{SML}_0(\ell_0)$.

$$\begin{aligned}
\#\mathrm{SML}_0(\ell_0) &= \min_{1 \le e \le \ell_0} \#\mathrm{SML}_0(e, \ell_0); \\
\#\mathrm{SML}_0(e, \ell_0) &= \min_{(\ell_0, \ldots, \ell_e)} \mathrm{storage}_0(\ell_0, \ell_1, \ldots, \ell_e)
\end{aligned}$$

Dynamic programming algorithm to compute $\#\mathrm{SML}_0(\ell_0)$: $O(\ell_0^3)$ time and $O(\ell_0^2)$ space.

Root at a Non-Special Level

[HS02]: "The root is considered to be at a special level, and ..."

Making root level ℓ_0 non-special:

- $\text{storage}_1(\ell) = \text{storage}_0(\ell) - \ell_1$.
  Hence, user storage decreases.
- $\Pr[X^0_{n,r} = 1]$ is small.
  Hence, negligible increase in the expected header size.

$\text{SML}_1(\ell_0)$: SML with non-special root.
$\#\text{SML}_1(\ell_0)$: corresponding user storage.

Examples of SML

Suppose there are 2^{28} users, i.e., $\ell_0 = 28$
(a good estimate as per the CableLabs website)

Scheme Name	Layering ℓ	Storage $\|I_u\|$
NNL-SD:	(28,0)	406
eHS:	(28,22,16,10,5,0)	146
SML_0:	(28,21,15,10,6,3,1,0)	140
SML_1:	(22,16,11,7,4,2,0)	119

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Other Results

Complete Tree LSD scheme

Maximum Header Length

- $h_{max} = \min\left(4r - 2, \left\lceil \frac{n}{2} \right\rceil, n - r\right)$ if root is non-special.
- $h_{max} = \min\left(4r - 3, \left\lceil \frac{n}{2} \right\rceil, n - r\right)$ if root is special.

Expected Header Length:

- The splitting of subsets complicates the analysis.
- $O(r \log^2 n)$ time and $O(1)$ space.

Constrained Minimization

- For a given r, the contribution of level $\ell_{max} = \ell_0 - \log_2 r$ to the header is maximum.
- As $r \uparrow$, $\ell_{max} \downarrow$. Hence,
  - Depending on the application, fix a value of r_{min} and set $\ell_{max} = \ell_0 - \log_2 r_{min}$.
  - Let $\boldsymbol{\ell} = \{\ell_{max}, 0\}$.

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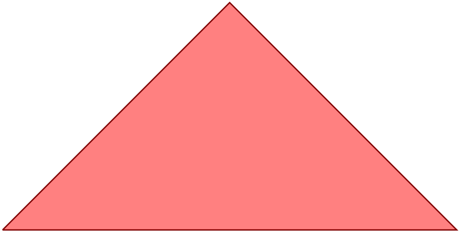

Result: $H_{n,r}$ close to that of NNL-SD, but, with lower user storage.

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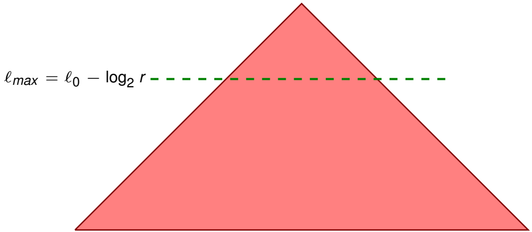

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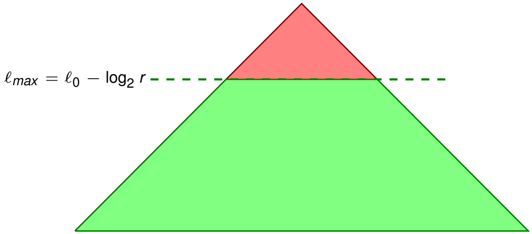

Result: $H_{n,r}$ close to that of NNL-SD, but, with lower user storage.

A CML Example

$n = 2^{28}$ and $r_{\min} = 2^{10}$.

Scheme	Layering ℓ	$\lvert I_u \rvert$	$H_{n,r}$ (normalized with NNL-SD)
NNL-SD:	(28,0)	406	(1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00)
eHS:	(28,22,16,10,5,0)	146	(1.69, 1.63, 1.64, 1.67, 1.69, 1.72, 1.73, 1.74, 1.75, 1.75)
CML:	(23, 18,0)	219	(1.14, 1.08, 1.04, 1.03, 1.01, 1.01, 1.00, 1.00, 1.00, 1.00)

Header lengths for 10 equispaced values of r from 2^{10} to 2^{14} normalized by the header length of the NNL-SD scheme.

Outline

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k-ary tree SD

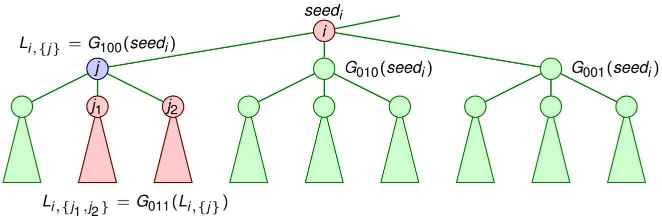

Figure: Key of $S_{i,\{j_1,j_2\}}$ is $G_{000}(L_{i,\{j_1,j_2\}}) = G_{000}(G_{011}(G_{100}(seed_i)))$.

User storage

$$1 + (2^{k-1} - 1) \sum_{\ell=1}^{\ell_0} \ell = 1 + \frac{\ell_0(\ell_0 + 1)}{2} (2^{k-1} - 1)$$

... reduced using additional tree structure
(constructed using cyclotomic cosets mod $2^k - 1$)

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k-ary tree SD: Results

- Why k-ary trees?
  - $|\mathcal{S}| \uparrow \implies (H_{n,r} \downarrow, \ |I_u| \uparrow)$

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- Header length analysis
  - $h_{max} = \min(2r - 1, \lceil n/k \rceil, n - r)$
  - Algorithm to compute $H_{n,r}$ (for $n = k^{\ell_0}$)
    $O(r \log n)$ space; $O(1)$ time
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  - An additional tree structure $T^{(k)}$
- Complete Tree for arbitrary number of users
- Layering
  - Storage Minimal Layering
- Header length simulation study (for $n \neq k^{\ell_0}$)

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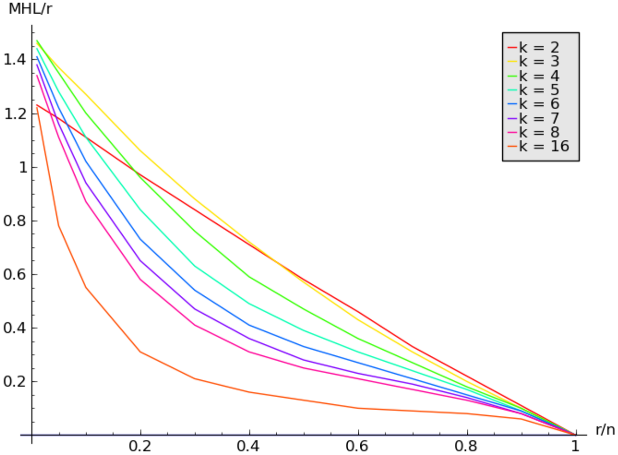
k-ary tree SD: Header length and user storage

n	k	us_k	MHL_k/r	n	k	us_k	MHL_k/r
10^3	2	55	(1.10, 0.98, 0.72)	10^4	2	105	(1.11, 0.97, 0.71)
	3	56	(1.27, 1.06, 0.72)		3	90	(1.26, 1.07, 0.72)
	4	60	(1.21, 0.96, 0.59)		4	112	(1.20, 0.96, 0.59)
	5	90	(1.11, 0.84, 0.50)		5	126	(1.11, 0.84, 0.49)
	6	120	(1.03, 0.73, 0.42)		6	252	(1.02, 0.73, 0.41)
	7	180	(0.95, 0.65, 0.36)		7	270	(0.94, 0.65, 0.36)
	8	340	(0.86, 0.58, 0.32)		8	510	(0.86, 0.58, 0.31)
10^5	2	153	(1.11, 0.97, 0.71)	10^6	2	210	(1.11, 0.97, 0.71)
	3	132	(1.27, 1.06, 0.72)		3	182	(1.27, 1.07, 0.72)
	4	180	(1.20, 0.96, 0.59)		4	220	(1.20, 0.96, 0.59)
	5	216	(1.11, 0.84, 0.49)		5	270	(1.11, 0.84, 0.49)
	6	336	(1.02, 0.73, 0.41)		6	432	(1.02, 0.73, 0.41)
	7	378	(0.94, 0.65, 0.36)		7	648	(0.94, 0.65, 0.36)
	8	714	(0.87, 0.58, 0.31)		8	952	(0.87, 0.58, 0.31)
10^7	2	300	(1.11, 0.97, 0.71)	10^8	2	378	(1.11, 0.97, 0.71)
	3	240	(1.27, 1.06, 0.72)		3	306	(1.27, 1.06, 0.72)
	4	312	(1.20, 0.96, 0.59)		4	420	(1.20, 0.96, 0.59)
	5	396	(1.11, 0.84, 0.49)		5	468	(1.11, 0.84, 0.49)
	6	540	(1.02, 0.73, 0.41)		6	792	(1.02, 0.73, 0.41)
	7	810	(0.94, 0.65, 0.36)		7	990	(0.94, 0.65, 0.36)
	8	1224	(0.87, 0.58, 0.31)		8	1530	(0.87, 0.58, 0.31)

Table: MHL_k/r for $r = (0.1n, 0.2n, 0.4n)$.

k-ary tree SD



k	3	4	5	6	7	8	16
δ_k	0.44	0.19	0.11	0.07	0.05	0.04	< 0.01

Table: Values of the threshold δ_k.

Outline

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a-ABTSD scheme

- $\mathcal{S}_{NNL-SD} \subset \mathcal{S}_{a-ABTSD}$
  - Augment trees of height a (with $k = 2^a$ leaf nodes)

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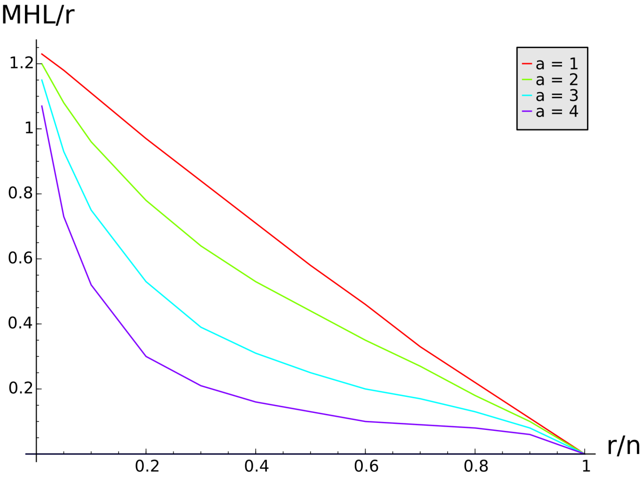
- Complete Tree for arbitrary number of users
- Header length simulation study

a-ABTSD: Header length and user storage

n	a	$us_a(n)$	MHL_a/r	n	a	$us_a(n)$	MHL_a/r
10^3	1	55	$(1.11, 0.97, 0.71)$	10^4	1	105	$(1.11, 0.97, 0.71)$
	2	145	$(0.96, 0.78, 0.53)$		2	287	$(0.96, 0.78, 0.53)$
	3	1279	$(0.75, 0.53, 0.31)$		3	2757	$(0.75, 0.53, 0.31)$
	4	115247	$(0.52, 0.31, 0.16)$		4	271629	$(0.52, 0.30, 0.16)$
10^5	1	153	$(1.11, 0.97, 0.71)$	10^6	1	210	$(1.11, 0.97, 0.71)$
	2	425	$(0.96, 0.78, 0.53)$		2	590	$(0.96, 0.78, 0.53)$
	3	4233	$(0.75, 0.53, 0.31)$		3	6024	$(0.75, 0.53, 0.31)$
	4	432123	$(0.52, 0.30, 0.16)$		4	629652	$(0.52, 0.30, 0.16)$
10^7	1	300	$(1.11, 0.97, 0.71)$	10^8	1	378	$(1.11, 0.97, 0.71)$
	2	852	$(0.96, 0.78, 0.53)$		2	1080	$(0.96, 0.78, 0.53)$
	3	8902	$(0.75, 0.53, 0.31)$		3	11428	$(0.75, 0.53, 0.31)$
	4	950634	$(0.52, 0.30, 0.16)$		4	1234578	$(0.52, 0.30, 0.16)$

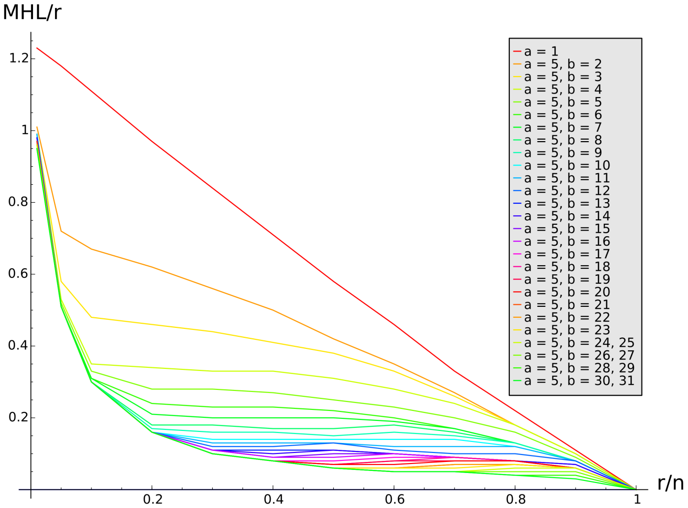
Table: MHL_a/r for three different choices of r namely, $r = (0.1n, 0.2n, 0.4n)$.

a-ABTSD performance



with $b = 2^a - 1$ and $c = \ell_0$.

(a, b, c)-ABTSD



with $a = 5$ and $c = \ell_0$.

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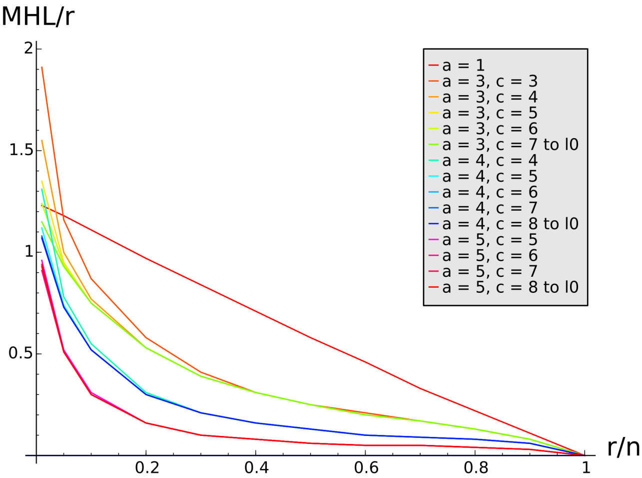
(a, b, c)-ABTSD



with $b = 2^a - 1$.

Preliminaries

Background

Our Contributions

Conclusion

Implementations

Schemes:

- NNL-SD, HS-LSD and all new schemes

Analysis:

- Header length algorithms
- User storage algorithms
- ...

What this thesis is NOT about

Asymptotic Improvements

What this thesis is ALL about

Combinatorial and Probabilistic Analysis

What this thesis is ALL about

Combinatorial and Probabilistic Analysis

Obtaining Hierarchies of Optimization

Summary of Contributions

- What if $n \neq 2^{\ell_0}$?

1, 2, 3, 4 Use dummy users or complete trees?

Summary of Contributions

- What if $n \neq 2^{\ell_0}$?

1, 2, 3, 4 Use dummy users or complete trees?

- Analysis of SD-based schemes?

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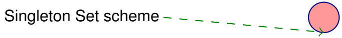

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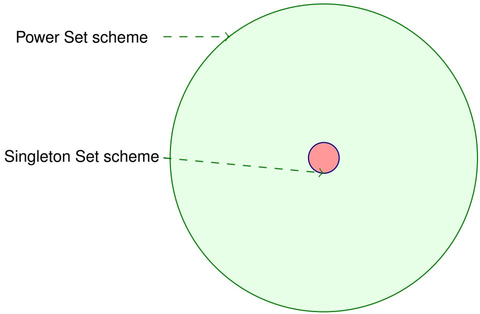

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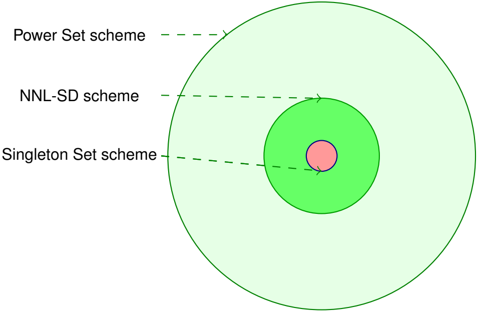

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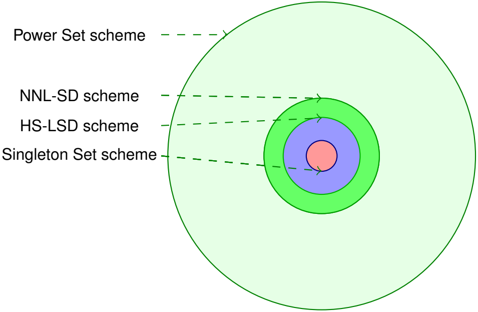

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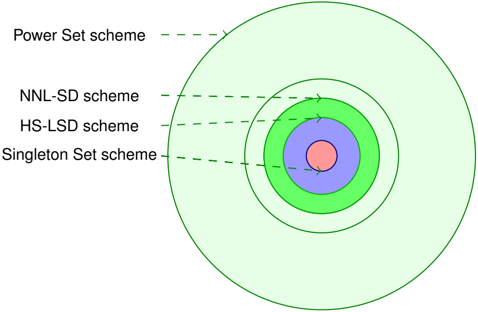

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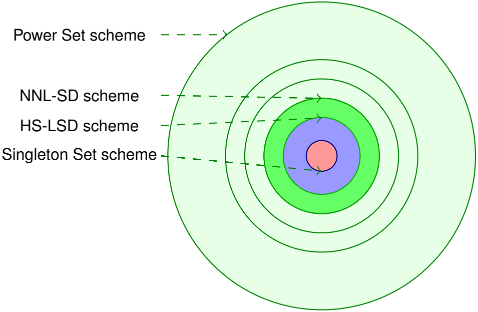

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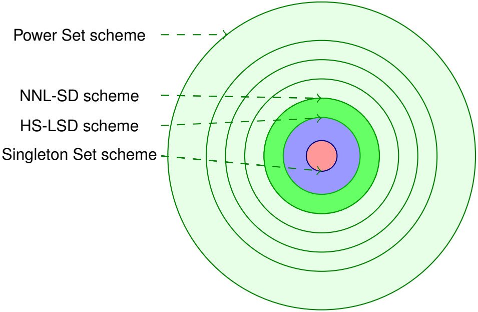

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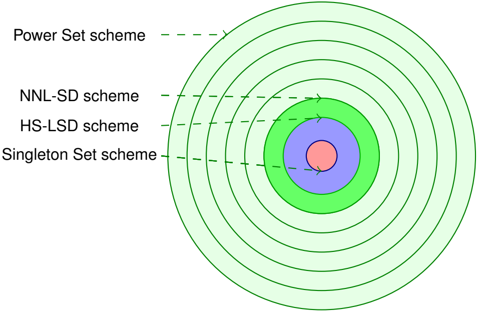

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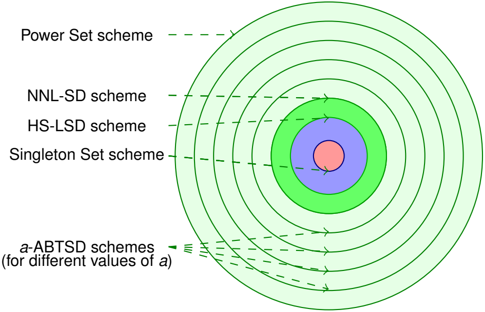

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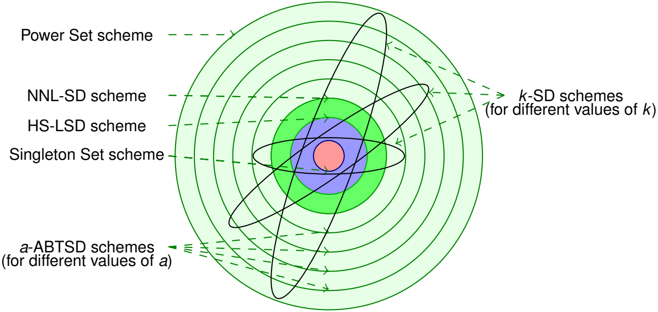

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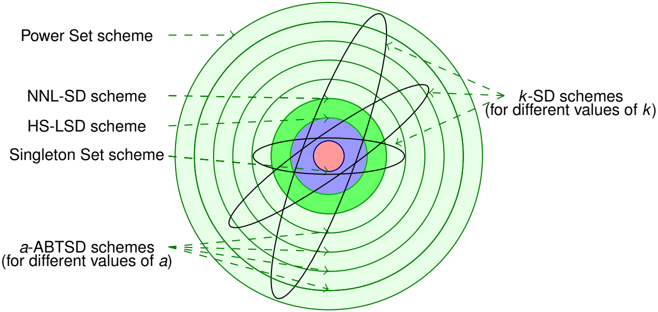

Publications

Sanjay Bhattacherjee and Palash Sarkar.
Complete tree subset difference broadcast encryption scheme and its analysis.
Des. Codes Cryptography, 66(1-3):335–362, 2013.

Sanjay Bhattacherjee and Palash Sarkar.
Concrete analysis and trade-offs for the (complete tree) layered subset difference broadcast encryption scheme.
IEEE Transactions on Computers, 63(7): 1709–1722, 2014.

Sanjay Bhattacherjee and Palash Sarkar.
Tree based symmetric key broadcast encryption.
J. Discrete Algorithms, 34: 78–107, 2015.

Sanjay Bhattacherjee and Palash Sarkar.
Reducing communication overhead of the subset difference scheme.
IEEE Transactions on Computers, to appear.

Sanjay Bhattacherjee and Palash Sarkar.
Implementations related to the above papers, `https://drive.google.com/folderview?id=0B7azs7qqqdS0UnB5aHp3WmJwcDQ&usp=sharing_eil`.
Uploaded on 13th August, 2014.

Open Questions

Schemes:

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Acknowledgement

- Prof. Palash Sarkar
- Friends
- Family


A MONSTER
BIG
THANK YOU
x

Desirable Properties

Fully Collusion Resistant

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Dynamic revocation

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Stateless / Stateful

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Assigning seeds to users

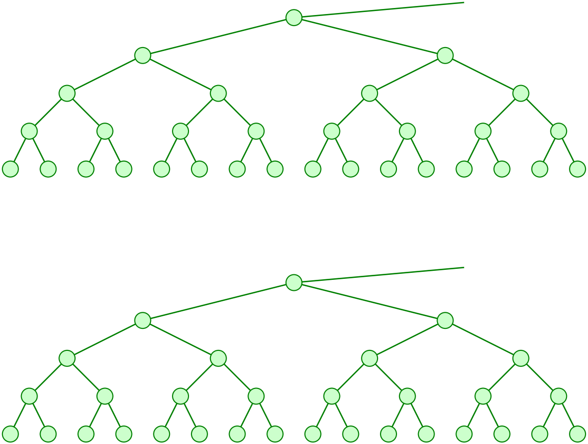

Figure: From one derived seed, keys of many subsets can be generated

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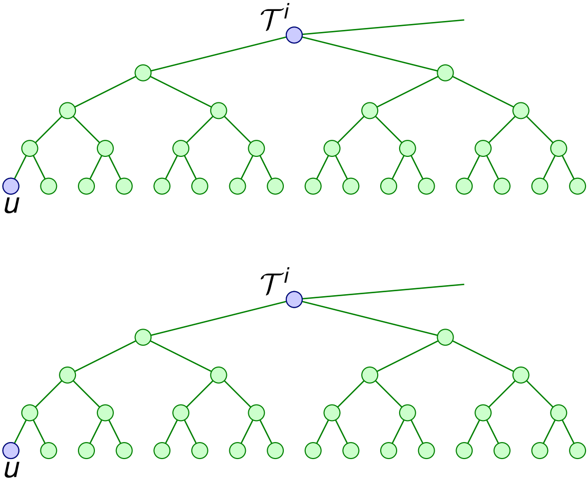

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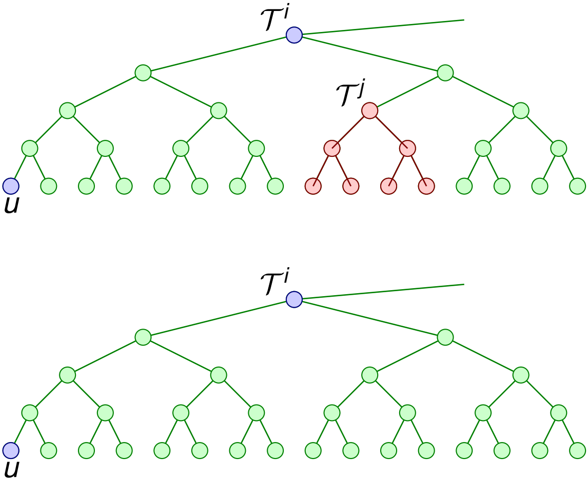

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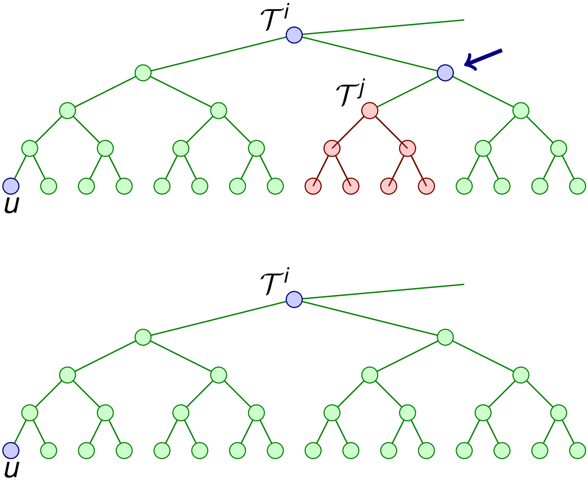

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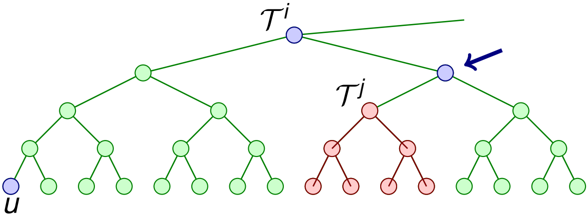

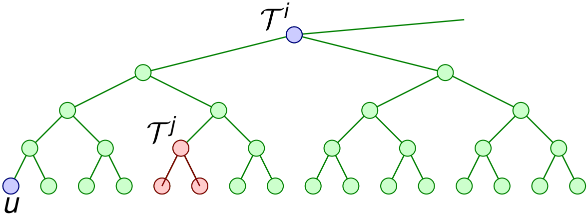

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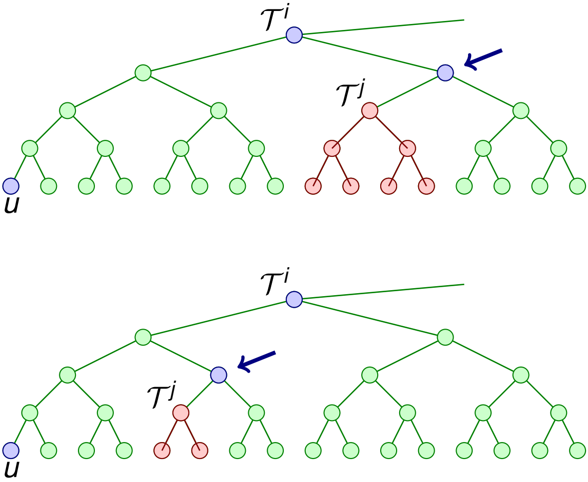

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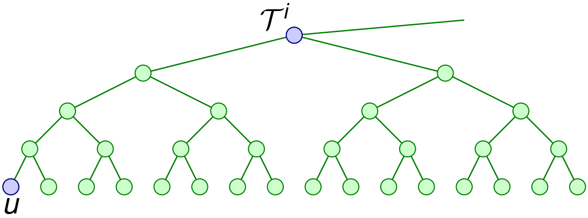

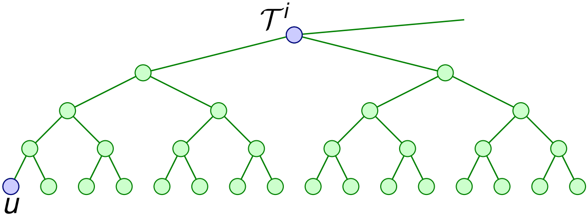

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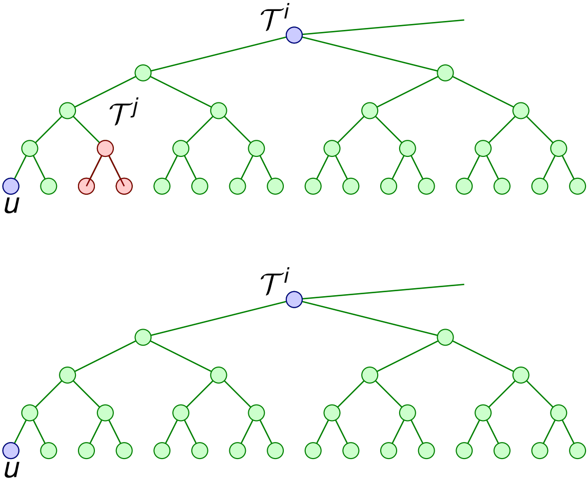

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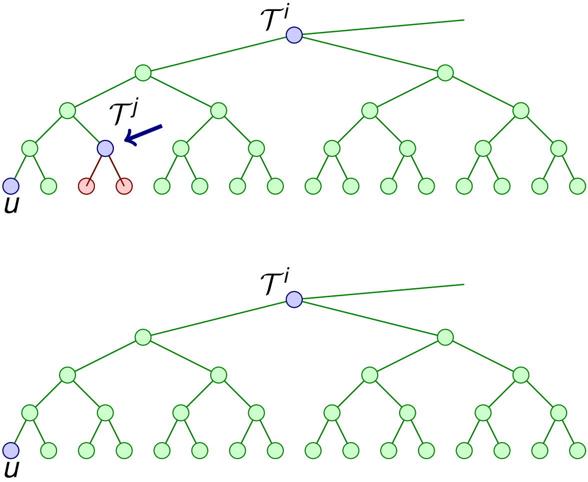

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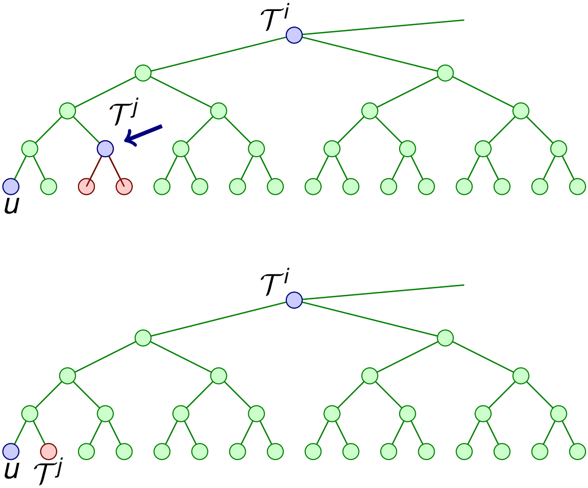

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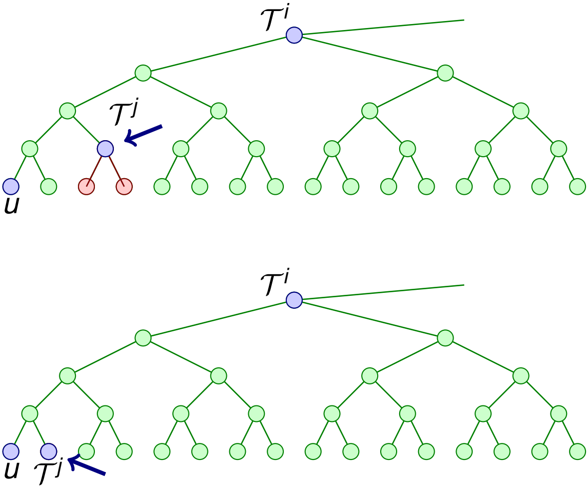

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