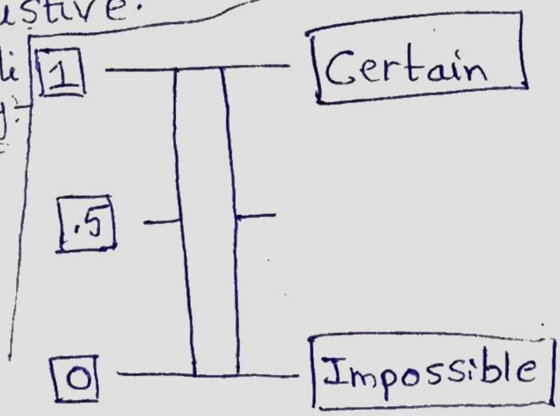
Introduction to probability

Probability:-

- Probability is the numerical measure of the likelihood that an event will occur.
- The probability of any event must be b/w 0 & 1 inclusively
  - $0 \le P(A) \le 1$ for any event A.
- The sum of probabilities of all mutually exclusive & collectively exhaustive events is 1.
  - $P(A) + P(B) + P(C) = 1$
  - A, B & C are mutually exclusive & Collectively exhaustive.

Range of probability:



Methods of Assigning Probabilities:-

- Classical method of assigning probability (rules and laws)
- Relative frequency of occurence (cumulated historical data).
- Subjective Probability (personal intuition (or) reasoning)

Classical Probability:-

- No. of outcomes leading to the event divided by the total no. of outcomes possible.
- Each outcome is equally likely.
- Determined a priori -- before performing the experiment.
- Applicable to games of chance.
- Objective -- everyone correctly using the method assigns an identical probability

$$\boxed{\begin{array}{l} P(E) = \dfrac{n_e}{N} \\ \text{where } N = \text{total no. of Outcomes} \\ \qquad n_e = \text{no. of Outcomes in } E \end{array}}$$

Relatively Frequency Probability:-

- Based on historical data
- Computed after performing the experiment.
- No. of times an event occured divided by the no. of trials.
- Objective -- everyone correctly using the method assigns an identical probability.

$$\boxed{\begin{array}{l} P(E) = \dfrac{n_e}{N} \\ \text{where } N = \text{total no. of } \cancel{\text{Outcomes}} \text{ Trials} \\ \qquad n_e = \text{no. of Outcomes Producing } E \end{array}}$$

Subjective Probability:-

- Comes from a Person's intuition (or) reasoning.
- Subjective -- diff. individuals may (correctly) assign diff. numeric probabilities to the same event.
- Degree of belief.
- Useful for unique (single-trial) experiments
  - New product introduction
  - Initial public offering of Common stock
  - Sporting events

Terminology:-

1) Experiment 2) Event 3) Elementary events
4) Sample Space 5) Unions & Intersections
6) Mutually Exclusive events 7) Independent events.
8) Collectively Exhaustive events 9) Complementary events.

Experiment, Trial, Elementary Event, Event:-

Experiment: a process that produces
- More than one possible outcome.
- only one Outcome per trial

Trial:- One repetition of the process.

Elementary event:- Cannot be decomposed (or) broken down into other events.

Event:- An outcome of an experiment
- may be an elementary event, or

- may be an aggregate of elementary events
- usually represented by an uppercase letter, e.g., A, E1

An Example Experiment:-

- Experiment:- randomly select, without replacement, two families from the residents of Tiny Town.
- Elementary Event:- The sample includes families A & C.
- Event:- Each family in the sample has children in the household.
- Event:- The sample families own a total of 4 automobiles

Tiny Town Population		
Family	Children in Household	No. of Automobiles
A	Yes	3
B	Yes	2
C	No	1
D	Yes	2

Sample Space:-

- The set of all elementary events for an experiment
- Methods for describing a sample space
  - roster (or) listing
  - set builder notation
  - tree diagram
  - Venn diagram.

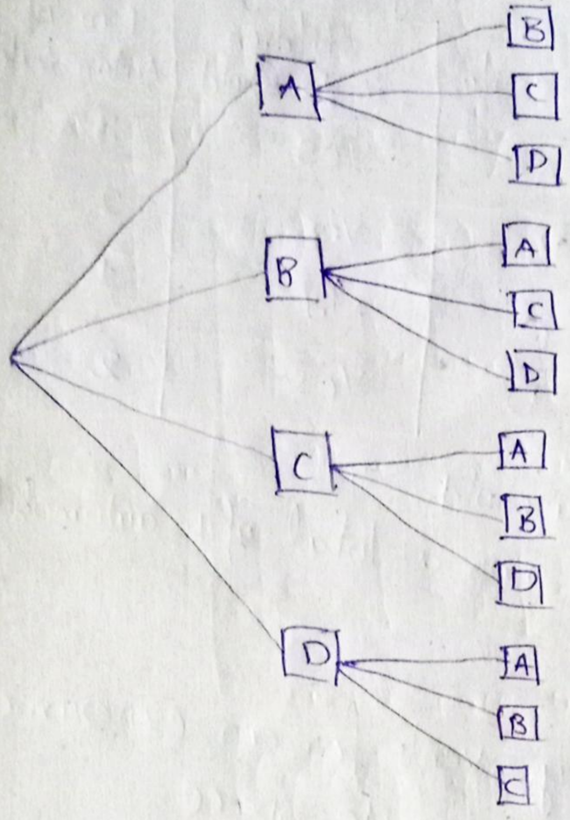
Roster Example of Sample Space:-

- Experiment: randomly select, without replacement, two families from the residents of Tiny Town.
- Each ordered pair in the sample space ~~elem~~ event is an elementary event, for example -- (D, C)

Listing of Sample Space:- (A, B), (A, C), (A, D), (B, A), (B, C), (B, D), (C, A), (C, B), (C, D), (D, A), (D, B), (D, C)

- The table considered above is used as data to list the sample space.

Sample Space: Tree Diagram for Random Sample of Two Families:-



Sample Space: Set Notation for Random Sample of Two Families:-

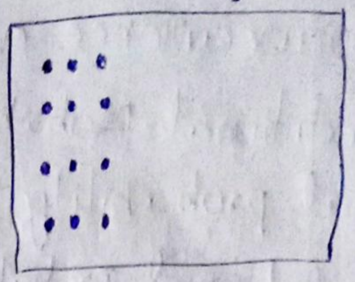
- $S = \{(x, y) \mid x$ is the family selected on the first draw and y is the family selected on the second draw
- Concise description of large sample spaces.

Sample Space:-

- Useful for discussion of general principles & concepts.

Listing of Sample Space
(A,B), (A,C), (A,D),
(B,A), (B,C), (B,D),
(C,A), (C,B), (C,D),
(D,A), (D,B), (D,C)

Venn diagram



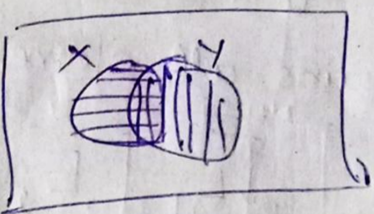
Union of Sets:-

- The union of two sets contains an instance of each element of two sets.

$X = \{1, 4, 7, 9\}$

$Y = \{2, 3, 4, 5, 6\}$

$X \cup Y = \{1, 2, 3, 4, 5, 6, 7, 9\}$



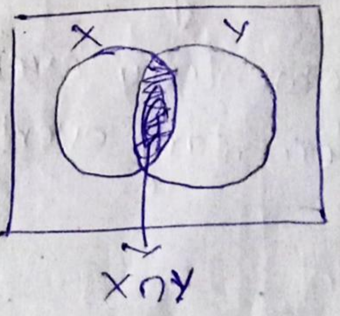
Intersection of Sets:-

- The intersection of two sets contain only those element common to both sets.

$X = \{1, 4, 7, 9\}$

$Y = \{2, 3, 4, 5, 6\}$

$X \cap Y = \{4\}$

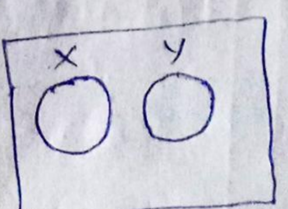


Mutually Exclusive Events:-

- Events with no common outcomes.
- Occurence of one event precludes the occurence of the other event

$X = \{1, 7, 9\}$, $Y = \{2, 3, 4, 5, 6\}$

$X \cap Y = \{\}$



$P(X \cap Y) = 0$

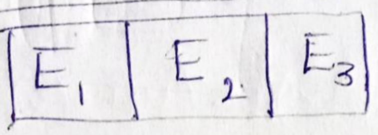
Independent Events :-

- Occurence of one event doesn't affect the Occurence (or) nonoccurence of the other event.
- The Conditional probability of X given Y is equal to the marginal probability of X.
- The conditional probability of Y given X is equal to the marginal probability of Y.

$$\boxed{P(X/Y) = P(X) \text{ and } P(Y/X) = P(Y)}$$

Collectively Exhaustive Events :-

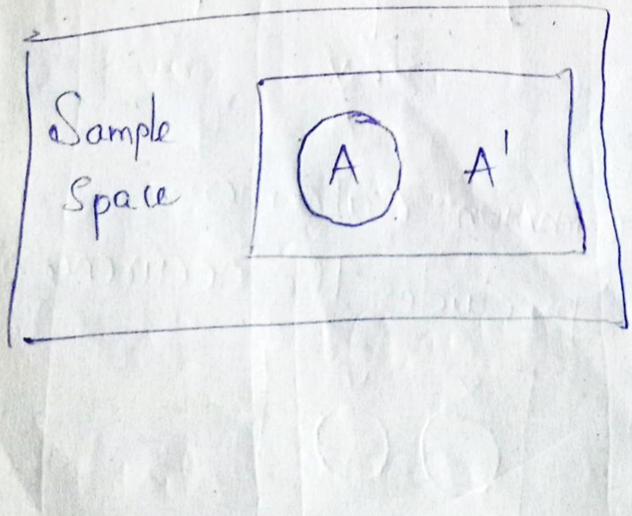
- Contains all elementary events for an experiment.



Sample Space with three Collectively exhaustive events

Complementary Events :-

- All elementary events not in the event 'A' are in its complementary event.



P(Sample Space) = 1

$$P(A') = 1 - P(A).$$

Counting the Possibilities:-

- mn Rule
- Sampling from a Population with Replacement.
- Combinations: Sampling from a Population without replacement.

mn Rule:-

- If an Operation can be done 'm' ways & a second operation can be done 'n' ways, then there are 'mn' ways for the two operations to occur in Order.
- This rule is easily extended to 'K' stages, with a no. of ways equal to $n_1 \cdot n_2 \cdot n_3 \cdots n_K$

Ex:- Toss two Coins. The total no. of Simple events is $2 \times 2 = 4$.

Sampling from a Population with Replacement:-

- A tray Contains 1,000 individual tax returns. If 3 returns are randomly selected with replacement from the tray, how many possible samples are there?

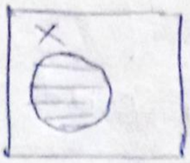
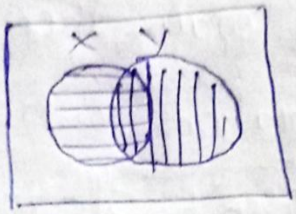
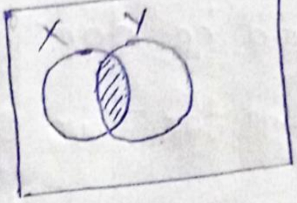
$$(N)^n = (1,000)^3 = 1,000,000,000$$

Combinations:-

- A tray Contains 1000 individual tax returns. If 3 returns are randomly selected without replacement from the tray, how many possible samples are there?

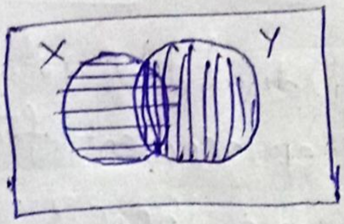
$$\left(\frac{N}{n}\right) = {}^N C_n = \frac{N!}{n!\,(N-n)!} = \frac{1000!}{3!\,(100-3)!} = 166,167,000$$

Four Types of Probability :-

Marginal	Union	Joint	Conditional
$P(X)$	$P(X \cup Y)$	$P(X \cap Y)$	$P(X/Y)$
	The probability of X or Y occuring 	The probability of X & Y occuring 	The probab of X occur given that has occur 

General law of Addition :-

$$\boxed{P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)}$$



Joint Probability Using a Contingency Table :-

Event	Event: B_1	Event: B_2	Total
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probabilities

Marginal (Simple) Probabili