

**Hindi Vidya Prachar Samiti's**  
**Ramniranjan Jhunjunwala College of Arts, Science and Commerce**  
**(Empowered Autonomous)**

**Programme: MSc. (Statistics)**

**Part-1**

**Semester-2**

**Practical- 2.3.2**

**Most Powerful test**

- Q.1. Let  $X_1, X_2, \dots, X_{10}$  be a random sample from  $N(\mu, \sigma^2)$ . Find a best test of the hypothesis  $H_0: \mu = 0, \sigma^2 = 1$  against the alternative hypothesis  $H_1: \mu = 1, \sigma^2 = 4$

Data : 0.08, -1.69, -0.27, 2.67, -0.48, 0.52, 1.75, 0.75, 0.57, 0.60.

- Q.2. Let  $X_1, X_2, \dots, X_n$  be a random sample from a normal distribution with mean  $\mu$  and variance 100. It is designed to test  $H_0: \mu = 75$  against  $H_1: \mu = 78$ . Find test of level of significance 0.05 and with power equal to 0.90 approximately.

- Q.3. Let  $X_1, X_2, \dots, X_{10}$  be a random sample of size 10 from a  $N(0, \sigma^2)$ . Find a best critical region of size  $\alpha = 0.05$  for testing  $H_0: \sigma^2 = 1$  against  $H_1: \sigma^2 = 2$ .  
Is this a best critical region against alternative  $H_1: \sigma^2 > 1$ .

- Q.4. Find the Neyman-Pearson test of size  $\alpha$  to test  $H_0: \beta = 1$  against  $H_1: \beta = \beta_1 (> 1)$  based on a sample of size 1 from
- $$f(x; \beta) = \begin{cases} \beta x^{\beta-1} & ; \quad 0 < x < 1 \\ 0 & ; \quad otherwise \end{cases}$$

- Q.5. Let  $X_1, X_2, \dots, X_n$  be a random sample with common pdf

$$f_0(x) = \frac{1}{2\theta} \exp\left(-\frac{|x|}{\theta}\right); \quad x \in R, \theta > 0$$

Find a size  $\alpha$  MP test for testing  $H_0: \theta = \theta_0$  versus  $H_1: \theta = \theta_1 (> \theta_0)$

Q.6.

Let  $X$  be an observation in  $(0,1)$ . Find an MP size  $\alpha$  test to test

$$H_0 : X \sim f(x) = 4x \quad \text{if } 0 < x < \frac{1}{2}$$

$$= 4 - 4x \quad \text{if } \frac{1}{2} \leq x \leq 1$$

against

$$H_1 : X \sim f(x) = 1 \quad \text{if } 0 < x < 1.$$

Find the power of your test.

Q.7.

Let  $X \sim f_j, j = 0, 1$  where

	$x$				
	1	2	3	4	5
$f_0(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$f_1(x)$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{6}$

(a) Find the form of the MP test.

(b) Find the size and the power of your test for various values of the cutoff point.

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