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Programme: MSc. (Statistics) **Part-1** **Semester-2**

Practical- 2.4.1 **Markov Chain -1**

Practical -1

Q.1. Let Xn be a Markov chain with state space {0,1,2} and initial probability vector p(0)=(1/4,1/2,1/4) and one step transition probability matrix

0	1	2
1/4	3/4	0
1/3	1/3	1/3
0	1/4	3/4

(a) Compute $P(X_0 = 0, X_1 = 1, X_2 = 1)$

(b) Compute $P(X_2 = 1)$

(c) Compute $P(X_1 = 1, X_2 = 1 \,/\, X_0 = 0)$

(d) Compute $P(X_4 = 1/X_2 = 2)$

(e) Compute $P(X_7 = 0/X_5 = 0)$

Q.2. Consider a communication system which transmits the two digits 0 and 1 through several stages. Let X0 be the digit transmitted initially (leaving)n 0th stage and Xn, $n \geq 1$be the digit leaving the nth stage. At each stage there is constant probability p that the digit which enters is transmitted unchanged and q otherwise with p+q=1. Find one-step transition probability matrix P and compute

(a) P^m

(b) $\lim_{n \to \infty} P^m$

Q.3. A factory has two machines and one crew. Assume that the probability of any one machine breaking down on a given day is α. Assume that if the repair crew is working on machine, the probability that they will complete the repair in one day is β. For simplicity, ignore the probability of a repair completion or a breakdown taking place except at the end of the day. Let Xn denote the number of machines in operation at the end of the nth day. Assume that the behavior of Xn can be a Markov chain and find a one step transition matrix of the chain.

Q.4. Find (under certain condition) whether the stochastic process with probability distribution given by

$P(X(t) = n) = \frac{(at)^{n-1}}{(1+at)^{n+1}}$, n=1,2,3,.....

$= \frac{at}{1+at}$, n=0

Is stationary.

Q.5. Classify the following random processes according to state space and parameter space.

(i) Water level in a tank at time t ≥ 0.

(ii) Number of customers in a shop at time t ≥ 0.

(iii) Number of breakdowns of a machinery in each week.

(iv) Water level in the tank at the end of each hour.