Hindi Vidya Prachar Samiti's Ramniranjan Jhunjhunwala College of Arts, Science and Commerce(Autonomous)

Programme: MSc. (Statistics) Part-1 Semester-2

Practical- 2.3.1 Types of Error and Power function

- Q.1 A sample of size 1 is taken from a population distribution $P(\lambda)$. To test H_0 : $\lambda=1$ against H_1 : $\lambda=2$, consider the nonrandomized test $\varphi(x)=1 \ \ \text{if} \ \ x>3, \ and =0 \ \text{if} \ \ x\leq 3.$ Find the probabilities of type I and type II errors and the power of the test against $\ \lambda=2$. If it is required to achieve a size equal to 0.05, how should one modify the test $\ \varphi$? Plot the power functions for $\ H_1$: $\lambda>2$.
- Q.2 A sample of size 1 is taken from an exponential PDF with parameter θ ,that is, $X \sim G(1,\theta)$. To test $H_o: \theta=1$ against $H_1: \theta>1$, the test to be used is the nonrandomized test

$$\varphi(x) = \begin{cases} 1 & \text{if } x > 2 \\ 0 & \text{if } x \le 2. \end{cases}$$

Find the size of the test. What is the power function? Plot the power functions.

Q.3 Let X_1 , X_2 be iid observations from

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}; \ 0 < x < \infty, \theta > 0$$

Consider the acceptance region as $w = \{(x_1, x_2)/x_1 + x_2 < 9.448\}$ for testing $H_o: \theta = 1 \ against \ H_1: \theta = 5$. Determine type I and II errors.

Q.4 Let X_{1}, X_{2} be a random sample drawn from

$$f(x,\theta) = \theta \quad x^{\theta-1} ; \quad 0 < x < 1$$

If we test $H_o: \theta = 2 \text{ against } H_1: \theta = 4$ with the critical region $w = \{(x_1, x_2)/x_1.x_2 \ge 2/3\}$

Find the size and power of the test.

Q.5 Let the random variable X is distributed as $U(0,\theta)$. We are testing hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$ based on a single observation. Calculate the type-I-Error and type-II Error based on the following critical regions. Also obtain the power of the test. A1={x/x<0.9}, A2={x/1<x<1.5}.