Roll NO : 904

2.3.1 Type of error and power function.

Q.3. let x_1 x_2 be iid obs from

$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}$; $0 < x < \infty$, $\theta > 0$

AR $\omega = \{(x_1, x_2) \mid (x_1 + x_2) < 9.448\}$

to test

$H_0 : \theta = 1$ vs $H_1 : \theta = 5$

i) P(Type I error) = P [reject H_0 | H_0 is true]

$= 1 - P$ [accept H_0 | H_0 is true]

$= 1 - P[x_1 + x_2 < 9.448 \mid H_0 = \theta = 1]$

$$= 1 - \int_0^{9.448} \int_0^{9.448 - x_2} e^{-x_1} e^{-x_2} \, dx_1 \, dx_2$$

$$= 1 - \int_0^{9.448} e^{-x_2} \left[1 - e^{-9.448 + x_2}\right] dx_2$$

$$= 1 - \left[\frac{e^{-x_2}}{-1} - e^{-9.448} x_2\right]_0^{9.448}$$

$$= 1 - \left[-e^{-9.448} - e^{-9.448} \, 9.448\right] + [-1 - 0]$$

$= 8.237945 \times 10^{-4}$

ii) P(Type II error) = 1 - Power of test

$= 1 - P$ [reject H_0 | H_1 is true]

$= P$ [accept H_0 | H_1 is true]

$= P[x_1 + x_2 < 9.448 \mid H_1 : \theta = 5]$

$$= \int_0^{9.448} \int_0^{9.448 - x_2} \frac{1}{5} e^{-x_1/5} \frac{1}{5} e^{-x_2/5} \, dx_1 \, dx_2$$

$$= \int_0^{9.448} \frac{1}{5} e^{-x_2/5} \left[\frac{1}{5} \frac{e^{-x_1/5}}{-1/5} \right]_0^{9.448 - x_2} dx_2$$

$$= \int_0^{9.448} \frac{1}{5} e^{-x_2/5} \left[1 - e^{-(9.448 - x_2)/5} \right] dx_2$$

$$= \left[\frac{1}{5} \frac{e^{-x_2/5}}{-1/5} - \frac{1}{5} e^{-9.448/5} x_2 \right]_0^{9.448}$$

$$= \left[-e^{-9.448/5} - \frac{1}{5} e^{-9.448/5} \, 9.448 \right] - (-1)$$

$$= 0.5632882514.$$

Q4. Let X_1 X_2 be r.s. from

$f(x, \theta) = \theta x^{\theta - 1}$; $0 < x < 1$

to test : $H_0 : \theta = 2$ vs $H_1 : \theta = 4$.

C.R. $= W = \{ (x_1 x_2) \mid x_1 x_2 \geq 2/3 \}$.

1) size of the test (α) :

$\alpha =$ P (reject H_0 | H_0 is true)

$= P(x_1 x_2 > 2/3 \mid \theta = 2)$

$$= \int_{2/3}^{1} \int_{2/3x_2}^{1} 4 x_1 x_2 \, dx_1 \, dx_2$$

$$= 4 \int_{2/3}^{1} x_2 \left[\frac{x_1^2}{2} \right]_{2/3(x_2)}^{1} dx_2$$

$$= 4 \int_{2/3}^{1} x_2 \left[-\frac{2}{9} 1/x_2^2 + \frac{1}{2} \right] dx_2$$

$$= 4\left[-\frac{2}{9}\ln(x_2) + \frac{1}{2}\frac{x_2^2}{2}\right]_{2/3}^{1}$$

$$= \left[1 - \frac{8}{9}\ln(1)\right] - \left[\frac{4}{9} - \frac{8}{9}\ln\left(\frac{2}{3}\right)\right]$$

$$= 0.19514212613$$

(ii) Power of test $(1-\beta) = P[\text{reject } H_0 \mid H_1 \text{ is true}]$

$$= P[x_1 x_2 > 2/3 \mid \theta = 4]$$

$$= \int_{2/3}^{1}\int_{2/3x_2}^{1} 4x_1^3\, 4x_2^3\, dx_1\, dx_2$$

$$= \int_{2/3}^{1} 16x_2^3\left[\frac{x_1^4}{4}\right]_{2/3x_2}^{1} dx_2$$

$$= \int_{2/3}^{1} 4x_2^3\left[1 - \frac{2^4}{3^4 x_2^4}\right]$$

$$= \left[4\frac{x_2^4}{4} - \frac{64}{81}\ln(x_2)\right]_{2/3}^{1}$$

$$= \left[1 - \frac{64}{81}\ln(1)\right] - \left[\frac{16}{81} - \frac{64}{81}\ln\left(\frac{2}{3}\right)\right]$$

$$= 0.48210164298$$

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Q.5. $X \sim U(0, \theta)$

to test $H_0 : \theta = 1$ vs $H_1 : \theta = 2$.

C.R. $A_1 = \{x \mid x < 0.9\}$. $A_2 = \{x \mid 1 < x < 1.5\}$.

(i) for C.R. $A_1 = \{x \mid x < 0.9\}$.

Type I - error = P [reject H_0 | H_0 is true]

$= P[x < 0.9 \mid \theta = 1]$

$= \int_0^{0.9} 1\, dx = [x]_0^{0.9} = 0.9$

Type 2 error = P [accep H_0 | H_1 is true]

$= \int_{0.9}^{2} \frac{1}{2}\, dx = \left[\frac{x}{2}\right]_{0.9}^{2} = 0.55$

Power of test = 1 − 0.55 = 0.45

(ii) for CR $A_2 = \{x \mid 1 < x < 1.5\}$.

Type - I error = P (reject H_0 | H_0 is true]

$= P(1 < x < 1.5 \mid \theta = 1)$

$= \int_1^{1.5} 1\, dx = 0 \quad \ldots \quad (0 < x < \theta)$

P Type II error = P (accept H_0 | H_1 is true)

$= 1 - P$(reject H_0 | H_1 is true)

$= 1 - P(1 < x < 1.5 \mid \theta = 2)$

$= 1 - \int_1^{1.5} \frac{1}{2}\, dx = 1 - \left[\frac{x}{2}\right]_1^{1.5}$

$= 1 - \frac{0.5}{2} = 0.75$.

Power of test = 1 − 0.75 = 0.25.