Practical – 4: Natural & Modified Control Limits

Q.1 An airplane manufacturer produces aluminum wings with a design length of 15m, following a normal distribution with mean = 15.02m and SD = 0.06m. Due to safety regulations, the wings must be within ±3 standard deviations of the process mean.

a) Compute natural tolerance limits (±3σ around the mean).

b) Check if the specification range (14.85m to 15.15m) covers all wings within safe limits.

```
> # Given process parameterws
> mu = 15.02
> sigma = 0.06
> spec_lower = 14.85
> spec_upper = 15.15
>
> #Compute Natural Tolerance Limits (NTL)
> NTL_lower = mu - 3*sigma
> NTL_upper = mu + 3*sigma
>
> #print the result
> NTL_lower
[1] 14.84
> NTL_upper
[1] 15.2
>
> #Check if the specification limits cover the natural tolerance range
> if (spec_lower <= NTL_lower & spec_upper >= NTL_upper){
+   cat("The specification limits are safe as they include all natural variations.\n")
+ } else {
+   cat("Warning : The specification limits may reject some acceptable wings.\n")
+ }
Warning : The specification limits may reject some acceptable wings.
```

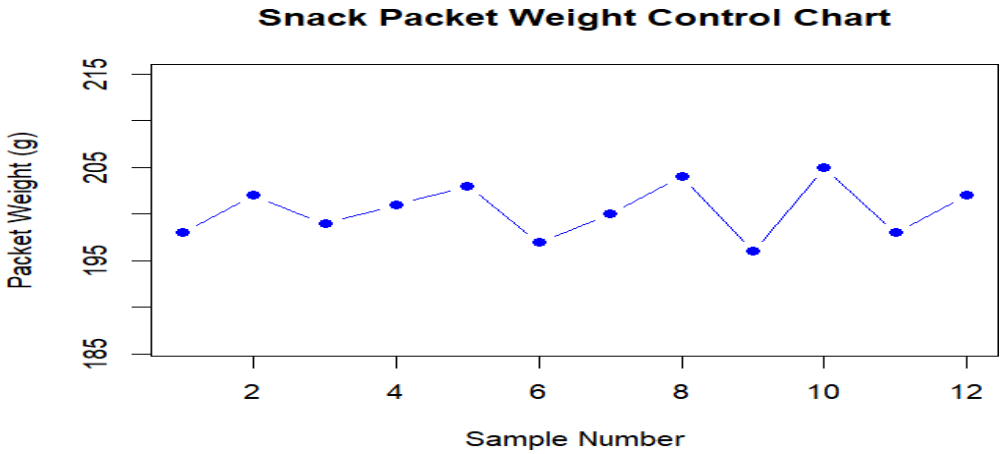
Q.2) A pharmaceutical company monitors the dissolution time (in minutes) of a new tablet formulation. They collect 25 samples of 4 tablets each, recording their average dissolution time and range. Historical data suggests a mean dissolution time of 30 minutes, a standard deviation of 1.5 minutes, and a sample range between 2 to 6 minutes. The A2 constant for n = 4 is 0.729. To reduce false alarms, the company modifies the control limits by widening them by 15%. Calculate and compare traditional vs. modified control limits, plot an $\bar{X}$ control chart, and identify out-of-control points based on the modified limits.

```
> #Given values
> sample_size = 4
> A2 = 0.729
>
> #Simulated sample data
> set.seed(123)
> Xbar = rnorm(25,mean=30 , sd=1.5 )
> R = runif(25, min=2 , max=6 )
> Rbar = mean(R)
> #Calculate Tradditional Control Limits
> Xbar_mean = mean(Xbar)
> UCL_trad = Xbar_mean + A2*Rbar
> LCL_trad = Xbar_mean - A2*Rbar
>
> #Apply modification factor (10% wider)
> mod_factor = 1.15
> UCL_mod =  Xbar_mean + mod_factor*A2*Rbar
> LCL_mod = Xbar_mean - mod_factor*A2*Rbar
>
> #Print result
> cat("Traditional Control Limits: [", round(UCL_trad, 2),",",round(LCL_trad,2),"],\n")
Traditional Control Limits: [ 32.86 , 27.04 ],
> cat("Modification Control Limits: [", round(UCL_mod, 2),",",round(LCL_mod,2),"],\n")
Modification Control Limits: [ 33.29 , 26.61 ],
```

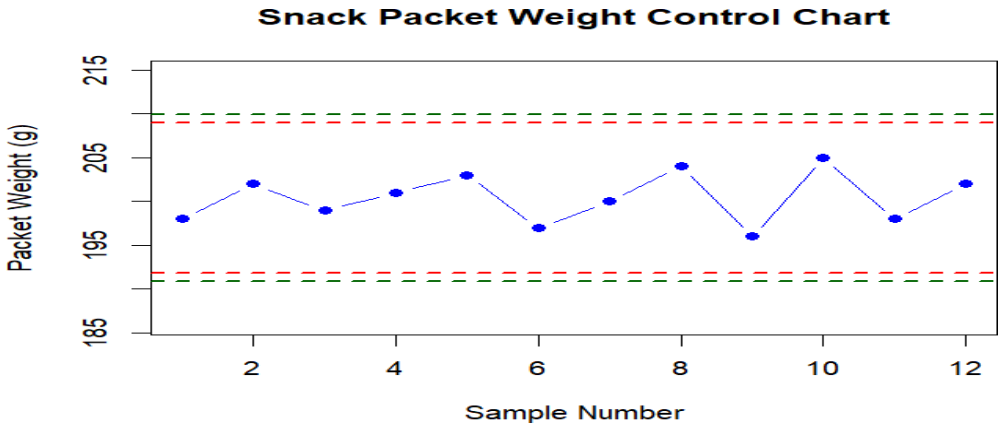
Q.3) A food packaging company produces snack packets with a target weight of 200g. A recent quality check measured the weights (in grams) of 12 packets:
198, 202, 199, 201, 203, 197, 200, 204, 196, 205, 198, 202
The company applies modified control limits, widening the traditional $\bar{X}$ chart limits by 10% to reduce false rejections.
Tasks: a) Calculate the traditional and modified control limits.
b) Plot an $\bar{X}$ control chart and identify out-of-control points.

```
> # Given data (packet weights in grams)
> weights <- c(198, 202, 199, 201, 203, 197, 200, 204, 196, 205, 198, 202)
```

```
> sample_size <- length(weights)
>
> # Calculate mean and standard deviation
> Xbar_mean <- mean(weights)
> sigma <- sd(weights)
>
> # Traditional control limits (±3σ)
> UCL_trad <- Xbar_mean + 3 * sigma
> LCL_trad <- Xbar_mean - 3 * sigma
>
> # Modified control limits (10% wider)
> mod_factor <- 1.1
> UCL_mod <- Xbar_mean + mod_factor * 3 * sigma
> LCL_mod <- Xbar_mean - mod_factor * 3 * sigma
>
> # Print control limits
> cat("Traditional Limits: [", round(LCL_trad, 2), ",", round(UCL_trad, 2), "]\n")
Traditional Limits: [ 191.79 , 209.04 ]
> cat("Modified Limits: [", round(LCL_mod, 2), ",", round(UCL_mod, 2), "]\n")
Modified Limits: [ 190.93 , 209.9 ]
>
> # Plot control chart
> plot(weights, type = "b", pch = 16, col = "blue", ylim = c(LCL_mod - 5, UCL_mod + 5),
+      main = "Snack Packet Weight Control Chart", xlab = "Sample Number", ylab = "Packet
Weight (g)")
```




```
> # Add control limits
> abline(h = c(UCL_trad, LCL_trad), col = "red", lty = 2, lwd = 2)  # Traditional Limits
> abline(h = c(UCL_mod, LCL_mod), col = "darkgreen", lty = 2, lwd = 2)  # Modified Limits
```




```
# Identify out-of-control points
> out_of_control <- which(weights > UCL_mod | weights < LCL_mod)
> points(out_of_control, weights[out_of_control], col = "red", pch = 19, cex = 1.5)

> # Add legend
> legend("topright", legend = c("Traditional Limits", "Modified Limits", "Out of Control"),
+        col = c("red", "darkgreen", "red"), lty = c(2, 2, NA), pch = c(NA, NA, 19), lwd = 2)
```