NOISE

Date: 14.03.1

Source → Tx → channel → Rx → Destination.

$\uparrow$
noise

→ SNR - Signal to Noise Ratio

① SNR_I ② SNR_O ③ SNR_C (Reference)

Reciever → Bandpass filter → (SNR_I) → Demodulator → (SNR_O) → LPF 0→W

Demodulator ↓ squarelaw detector, envelop detector } DSB FC

SNR (Input)

Ratio of (avg) Power of Modulated Signal to the avg power of Noise in to be signal bandwidth.

SNR (out)

Ratio of avg power of message signal at the demodulator to the avg power of Message Bandwidth.

SNR (channel)

Ratio of avg power of modulated signal to the average power of noise in message bandwidth.

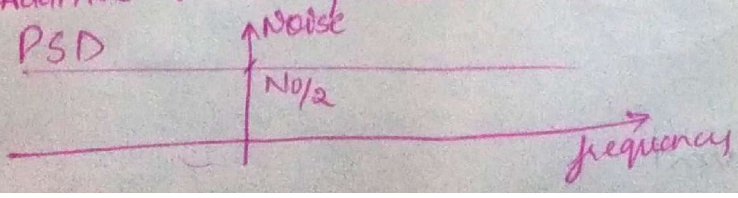
→ FOM : figure of Merit

FOM should not Be less than 1

$$FOM = \frac{SNR_O}{SNR_C}$$

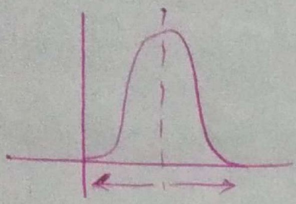
AWGN :- Additive White Gaussian Noise

Power Spectral Density



Graph for Calculation of power.

- Generally we use Gaussian Ratio for is Random variables.



- avg value : Zero mean value

Dad's Bday :)
16/03/17

Random Experiment

Any Experiment can be referred to as random if the outcome of the experiment cannot be predicted precisely.

The conditions for an Experiment to be Random are :-

1) For any trial, the outcome is unpredictable.
2) For a large no. of trials of an expt, the outcomes Exhibit statistical Regularity or an avg pattern.

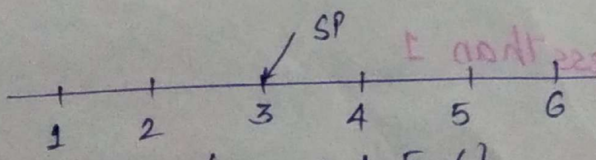
Random Expt

(1) Unpredictable

(2) Any pattern

① Sample Space : $S = \{H, T\}$

Total no. of possible outcomes or collection of identical Outcome. Each outcome is reffered as sample point.



$S = \{1, 2, 3, 4, 5, 6\}$

* Three types of Events :-

① Sure Event
② Null Event
③ Elementary Event

Sure Event :- Entire sample Space

Null Event : No sample point

Elementary Event :- 1 sample point

Probability

- Relative frequency approach n_A n = no. of times we are performing the Expt.

Relative freq $= \frac{n_A}{n}$

Probability $= \lim_{n \to \infty} \frac{n_A}{n}$

Q. Repeat a random Expt n no. of times with Event A as desired Event and Record the no. of times Event A has occured, the ratio $\frac{n_A}{n}$ is known as relative freq. occurance of event A.

Classical Def : $\frac{n_A}{n}$

The properties of probability of sample space $P(S) = 1$

Probability of A : $0 \leq P(A) \leq 1$

* If there are two events which are mutually exclusive events then

Prob of $P(A \cup B) = P(A) + P(B)$

Random Variable

While the meaning of the Outcome of random expt is clear but such outcomes are not the most convinient representation for the mathematical Analysis. Hence the random variable is defined.

R.V: assigns a numerical value to each possible Out come

Random Variable < Discreate value / continuous value

→ Probability Mass function:

It describes the probability of each possible Out come of the random variable. $P(X = x_i)$

Ex: Tossing a coin $S = \{H, T\}$

→ probability Mass function

S	X	$P(X=x_1)$
H	1	$P(1)=1/2$
T	0	$P(0)=1/2$

Q. Construct the probability mass function for a random variable which is specifying the no. of heads in the expt of a tossing a coin twice.

S		X	
H	H	2	$P(X=2)=1/4$
H	T	1	$P(X=1)=2/4=1/2$
T	H	1	
T	T	0	$P(X=0)=1/4$

* Probability mass function is for Discrete Random variable.

Cumulative Distribution function (CDF)

CDF is the probability that the Random Variable 'X' takes any value less than or equal to x $(X \le x)$

$$P(X \le x)$$

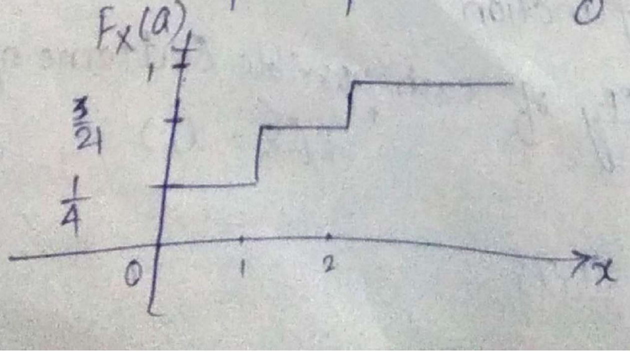
$$F_X(x) = P(X \le x)$$

Tossing two coins at same time. Desired Event hh

		X
H	H	2
H	T	1
T	H	1
T	T	0

P(no. of heads)

		X	
H	H	2	$1/4 = P(X=0)$
H	T	1	$1/2 = P(X=1)$
T	H	1	$1/4 = P(X=2)$
T	T	0	



A three digit message is transmitted over a noisy channel having probability of error = 2/5 per digit. Find out the corresponding CDF.

C C C	0
C C E	1
C E C	1
C E E	2
E C C	1
E C E	2
E E C	2
E E E	3

$P(E) = \frac{2}{5}$

$P(C) = 3/5$

$P(X=0) = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$

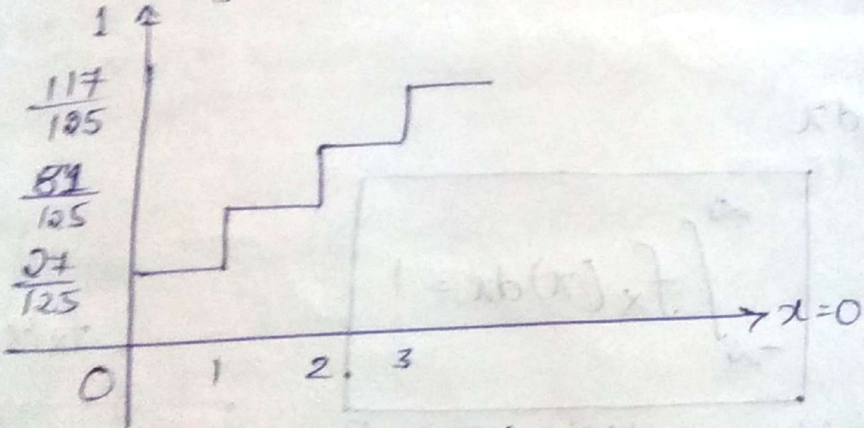
$P(X=1) = 54/125$

$P(X=2) = 36/125$

$P(X=3) = \frac{8}{125}$

$\left(\frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}\right)3 = \frac{36}{125}$ $\quad \left(\frac{3}{5} \times \frac{3}{5} \times \frac{2}{5}\right)3 = \frac{54}{125}$

$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$



Cummulative Dist function (CDF)

$x\{x_i\}$	-1	0	1	2
$P_x(x_i)$	0.3	0.2k	0.4	0.1

$0.3 + 0.2k + 0.4 + 0.1 = 1$

$0.2k = 0.2 \quad k = 1$

$k=1$

② $F_x(x)$ ① $F_x(-\infty) = 0$

② $F_x(\infty) = 1$

Properties of CDF

① $F_X(-\infty) = 0$

② $F_X(\infty) = 1$

③ $P(x_1 \le x \le x_2) = F_X(x_2) - F_X(x_1)$

④ $P(X > x) = 1 - F_X(x)$

21st March 2017

Probability ~~Distribution~~ Density function -

→ continuous Random variable

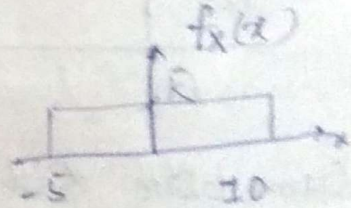
$$f_X(x) = \frac{d}{dx} F_X(x)$$

$\because P(X \le x) = F_X(x)$

* $P(X \le x) = F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

* $P(X > x) = \int_{x}^{\infty} f_X(x)\,dx$

Property of PDF : $\boxed{\int_{-\infty}^{\infty} f_X(x)\,dx = 1}$



Q. for a Random variable x pdf is given By

find ① Value of K

② what is the probability from -5 to +5.

③ plot CDF.

$P(-5 \le x \le 5)$

Area Under the Curve = 1

$$\int_{-5}^{10} K\,dx = 1 \qquad Kx\Big]_{-5}^{10} = 1$$

$$5K - 10K = 1$$

$$K = \frac{1}{15}$$

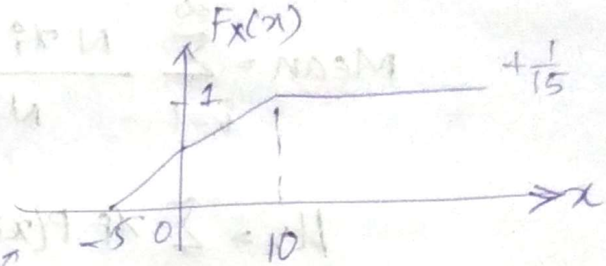
(ii) $F_X(x) = \int_{-5}^{5} f_X(x)\,dx$ $\quad \frac{1}{15}x\Big]_{-5}^{5} = \frac{2}{3}$

$\frac{x^2}{2}\Big]_{-5}^{5}$ $\quad \frac{5}{15} + \frac{5}{15} = \frac{10}{15} = \frac{2}{3}$

$\frac{25}{2}$

(iii) $F_X(x) = \int_{-5}^{x} f_X(x)\,dx$

$= \frac{x+5}{15}$ $\quad x=0$, $x=10$



Q. find a certain random variable has cummalative d.f is given by $F_X(x) = \begin{cases} 0 & ; x \le 0 \\ Kx^2 & ; 0 < x \le 10 \\ 100K & ; x > 10 \end{cases}$ ① calculate the value of K. ② probability of x. $P(5 \le x \le 7)$ ③ plot PDF.

① $100K = 1$

$K = \frac{1}{100} = 0.01$

② $P(x_1 \le x \le x_2) = F_X(x_2) - F_X(x_1)$

$F_X(7)^2 - F_X(5)$

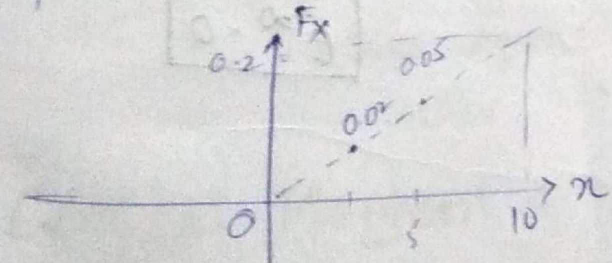
$\int_5^7 \frac{1}{100} = \frac{1}{100}7 - \frac{5}{100}$

$= 0.01(7)^2 - 0.01(5)^2 = 0.24$

③ plot PDF

$f_X(x) = \frac{d}{dx} F_X(x)$

$= 2Kx = 0 < x \le 10$



Conditional probability

$P[X/Y] = \frac{P[X;Y]}{P[Y]}$ or $P[X,Y] = P[X/Y]P[Y]$

$P[X,Y] = P[Y/X]P[X]$

Bayes Rule $\boxed{P[X/Y] = \frac{P[Y/X]P[X]}{P[Y]}}$

AVERAGE VALUE

$x_1 \to NP(x_1) \quad x_2 \to NP(x_2) \quad x_3 \to NP(x_3)$

N observation.

$$\text{Mean} = \sum_{i=1}^{\infty} \frac{N x_i P(x_i)}{N}$$

$$\mu_x = \sum x_i P(x_i)$$

Q. For a Continuous random variable, PDF is given by
$f_x(x) = ae^{-bx} \quad x \geq 0$ ① Relation between a and b
② plot CDF.

Ans→1 area under the curve is 1.

For $f_x(x)$ = total area = 1

$$\int_0^{\infty} f_x(x)\,dx = 1$$

$$\int_0^{\infty} ae^{-bx} = a\left[-be^{-bx}\right]_0^{\infty} = 0\ 1$$

$$\frac{a}{-b} e^{-xb}\Big]_0^{\infty}$$

$\boxed{e^{-\infty} = 0}$

$$\frac{-a}{b}\left[e^{-\infty} - e^0\right]$$

$$\frac{a}{b} = 1 \qquad \underline{a = b}$$

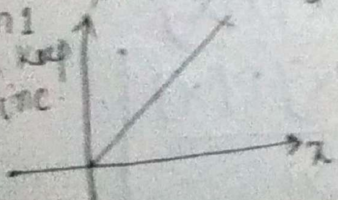
$e^{-ax} \to dx$ Ans → 2
sub it $F_x(x)$
from 1
will keep
on inc

$$\int_0^{\infty} ae^{-ax}$$

$$\frac{ae^{-ax}}{-a}\Big]_0^{x} = 1 - e^{-ax}$$

$$\int_{-\infty}^{x} \underbrace{F_x(x)}_{\text{General Expression}}$$

$\{ \because a = b \}$



Mean Value of Random Variable

$$\left.\begin{matrix} 0 & 1 & 2 \\ x_1 & x_2 & x_3 \\ P(x_1) & P(x_2) & P(x_3) \end{matrix}\right] \underline{X}$$

→ N observations

$$\mu_x = E(x) = \sum_{i=-\infty}^{\infty} x_i P(x_i)$$

↓

Expectation

Continuous Random Variable :-

$$E[x] = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

→ n^{th} moment $\quad E[X^n] = \int_{-\infty}^{\infty} x^n f_X(x)\, dx$

$n = 2$

$$\underbrace{E[X^2]}_{\text{Mean square value}} = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

Central Moment

It is Expected value of the difference b/w RV(x) & its mean Value (μ_x)

$$E[(X-\mu_x)^n] = \int_{-\infty}^{n} (x-\mu_c)^n f_X(x)\, dx$$

n^{th} Central Moment =

$n = 2$ is variance

$$\sigma_X^2 = E\left[(X-\mu_x)^2\right] = \int_{-\infty}^{\infty}(x-\mu_x)^2 f_X(x)\,dx$$

* The first R.X central moment of Random variable is always zero

$$E\left((X-\mu_x)\right) = E(X)-\mu_x = \mu_x - \mu_x = 0$$

$$\sigma_X^2 = E\left[(X-\mu_x)^2\right]$$

$$= E\left(X^2+\mu_x^2-2X\mu_x\right)$$

$$= E(X^2) + \mu_x^2 - 2\mu_x E[X]$$

$$= E(X^2) + \mu_x^2 - 2\mu_x(\mu_x)$$

$$= E(X^2) + \mu_x^2 - 2\mu_x^2$$

$$= E(X^2) - \mu_x^2$$

= Mean square value and square of the mean.

σ_X (standard deviation) = $\sqrt{\text{Variance}}$

UNIFORM DISTRIIBUTION

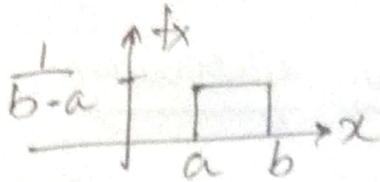
A Random Variable 'x' is Uniformly distributed in the interval (a,b) if its pdf is given by

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Calculate the mean and variance of Uniformly distributed random variable $E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$

$$= \int_a^b x \frac{1}{b-a}\,dx$$

$$\int_a^b x \frac{1}{b-a}\,dx$$



$$\frac{1}{b-a}\left.\frac{x^2}{2}\right]_a^b$$

$$\frac{1}{b-a}\,\frac{b^2-a^2}{2} = \frac{1}{\cancel{b-a}}\,\frac{(a+b)(a-b)}{2} = \frac{b+a}{2}$$

$$(b+a)(\cancel{b-a})$$

$$E\left[x^2\right] = \int_a^b x^2 \frac{1}{b-a}\,dx$$

$$\frac{1}{b-a}\left.\frac{x^3}{3}\right]_a^b$$

$$(b-a)(b^2+2ab+a^2)$$

$$\frac{1}{b-a}\,\frac{(b^3-a^3)}{3}$$

$$\frac{1}{\cancel{b-a}}\,\frac{(\cancel{b-a})(a^2+b^2+ab)}{3}$$

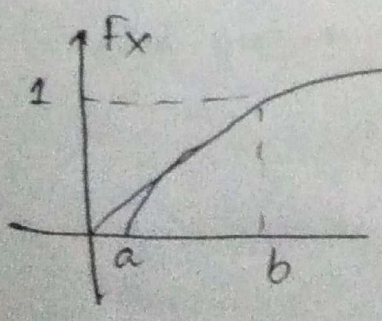
$$= \frac{a^2+b^2+ab}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{a^2+b^2+ab}{3} - \frac{a^2+b^2+2ab}{4}$$

$$\frac{4a^2+4b^2+4ab-(3a^2+3b^2+6ab)}{12}$$

$$= \frac{a^2+b^2-2ab}{12} = \frac{(a-b)^2}{12}$$

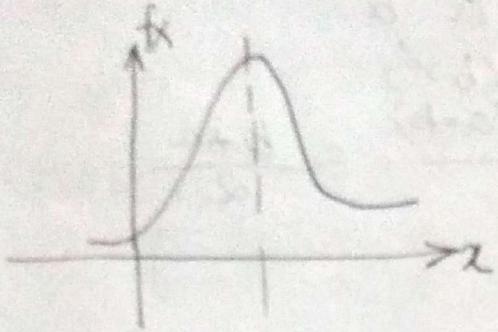
$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx \qquad = \int_a^b \frac{1}{b-a}\,dx = 1$$



$$= \int_a^x \frac{1}{b-a}\,dx = \frac{x-a}{b-a} \qquad (a \le x \le b)$$

24th March
[illegible]

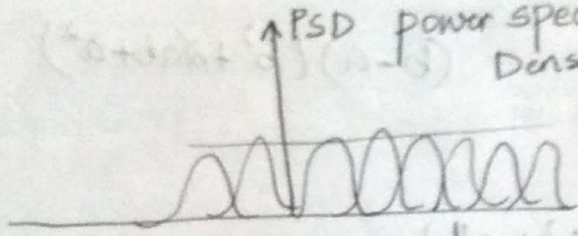
Gaussian - Distribution



AWGN
Additive white Gaussian Noise

$$f_X(x) = \frac{1}{\sqrt{2\pi \sigma_x^2}} \exp\left(-(x-\mu_x)^2 / 2\sigma_x^2\right)$$

PSD power spectral Density

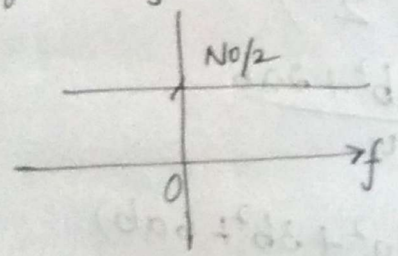


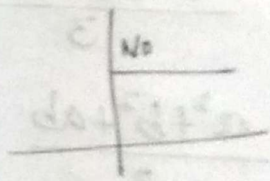
Overlapping each other

$\downarrow$
constant

→ present for all frequency.

→ Gaussian frequency





Negative freq : Mathematical Analysis
contour part → negative side
$\downarrow$
for power.

Cummulative Distribution frequency

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

$$e^{-(x-\mu_x)^2/2\sigma^2}$$

NOISE

- filter Required frequency

antenna - wire
- does not have any ability
- Recieve and pass.

Bandpass filter - filter and reciever the Required frequency

after demodulation → lowpass filter

Antenna

General Reciever: Bandpass filter → Demodulator → LPF → message

Rx

Antenna
↓
BPF —X (Pre Detection)→ Demodulator → LPF —Y (Post Detection)→ Message

SNR_0 → post Detection.
SNR_I →
SNR_C (Ref) → Pre Detection.

$$FOM = \frac{Post}{Pre} \qquad Y > X$$

Demodutor - Extract the frequency variations

FOM>1 ⇒ system is very good

for Frequency Modulation.

FOM<1 - DSBFC

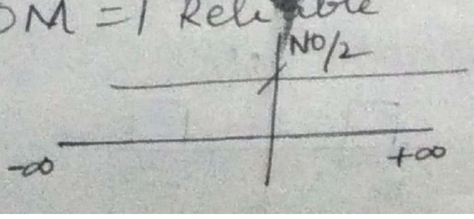
High frequency signal - High Harmonic Noise

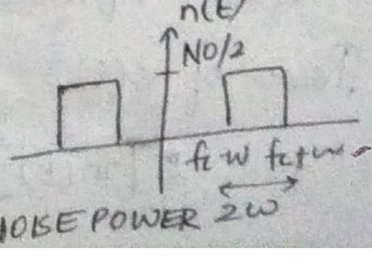
FOM = 1 Reliable

$\frac{No}{2} \times 2W +$

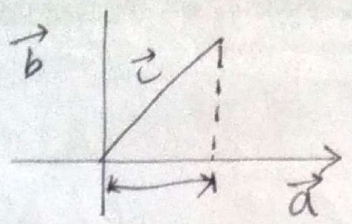
$\frac{No2W}{2}$

$2NoW$





NOISE POWER

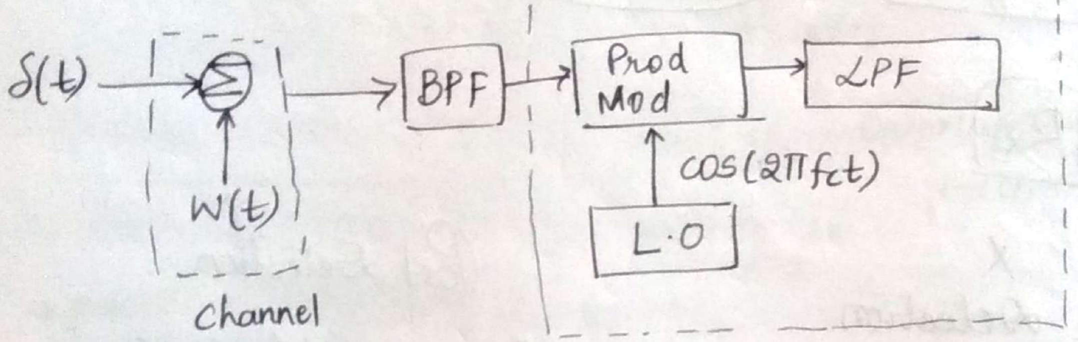


Pre detection of DSB-SC Noise Analysis

→ No numericals

→ Derivations / theory

Pre detection SNR



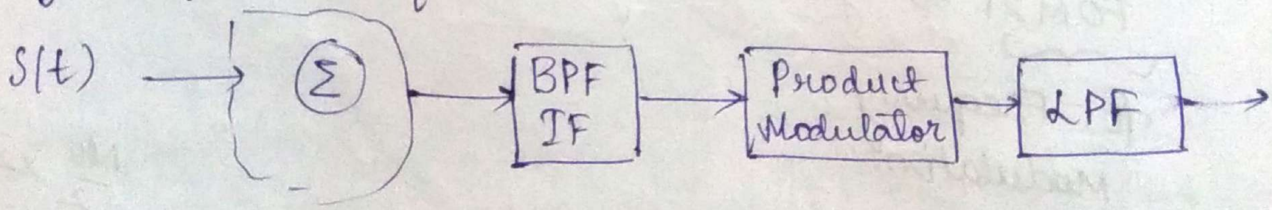
Signal power =

$s(t) = A\, m(t) \cos(2\pi f_c t)$

Signal power

Power of $m(t) = P$

Figure of Merit of DSB-SC :

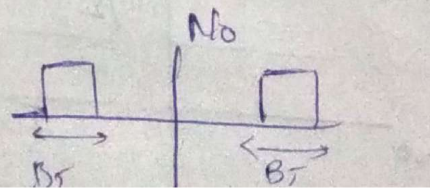


Pre detection SNR

$s(t) = A_c m(t) \cos(2\pi f_c t)$ (DSB-SC)

Power of $s(t) = \frac{A_c^2 P}{2}$

$P[m(t)] = P$



Noise power $= N_0 B_T$

$B_T = W$

1) Pre detection SNR $= \dfrac{A_c^2 P}{2N_0 B_T} = \dfrac{\text{Power of } s(t)}{\text{power of Noise}}$

Post Detection SNR

$x(t) = s(t) + n(t)$

$= A_c\, m(t)\cos(2\pi f_c t) + n_I t \cos(2\pi f_c t) - n_Q(t)\cos(2\pi f_c t)$

$v(t)$ (o/p of product modulator)

$= x(t)\cos(2\pi f_c t)$

$= [A_c m(t)\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q t \sin(2\pi f_c t)]\cos(2\pi f_c t)$

$\dfrac{A_c m(t)}{2}\left[\cos^2(2\pi f_c t)\right] + n_I t\left[\cos^2(2\pi f_c t)\right] - \dfrac{n_Q t}{2}\left(\sin(4\pi f_c t)\right)$

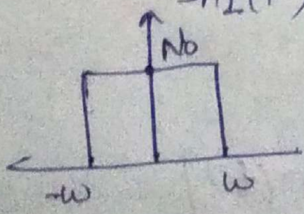
$= \dfrac{A_c m(t)}{2}\left[\cos(4\pi f_c t) + \dfrac{A_c m(t)}{2} + \dfrac{n_I(t)}{2}\cos 4\pi f_c t + \dfrac{n_I t}{2} - \dfrac{n_Q t}{2}\sin(4\pi f_c t)\right]$

Output of LPF $= \dfrac{A_c m(t)}{2} + \dfrac{n_I(t)}{2}$

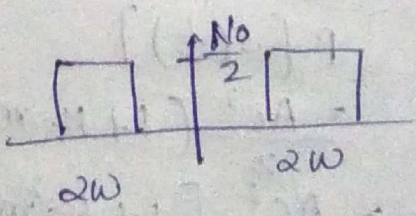
Power of signal $= \dfrac{A_c^2 P}{4}$

Noise power $(n_I(t)) \Rightarrow$

$S_{nI}(F) = S_{nQ}(t)$



$S_N(F)$



$n_I(t)$ and $n_Q(t)$ having same power spectral density

$$P(n_I(t)) = \left(\tfrac{1}{2}\right)^2 N_0 W (2) = \frac{N_0 W}{2}$$

2) post - Detection SNR $= \dfrac{A_c^2 P}{4(N_0 W/2)} = \dfrac{A_c^2 P}{2 N_0 W}$

3) FOM $= \dfrac{\text{Post DSNR}}{\text{Reference SNR}}$

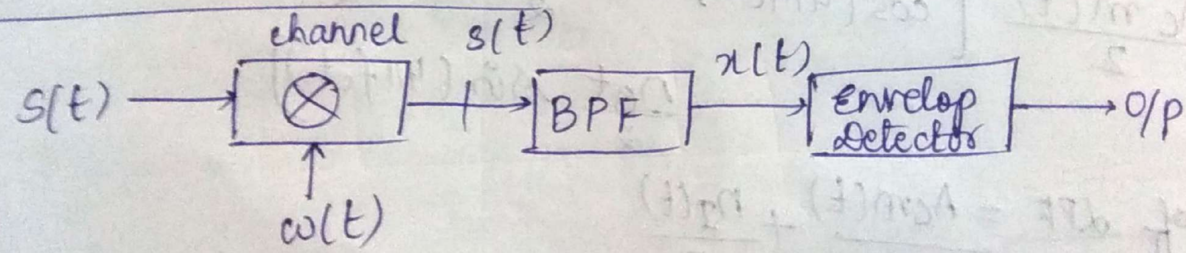
$$= \frac{A_c^2 P/2}{N_0 W} \times \frac{N_0 W}{A_c^2 P/2} = 1$$

Reference SNR: Ratio of power of modulated signal to Power of noise to message signal

$$\text{Ref SNR} = \frac{A_c^2 P/2}{N_0 W}$$

* FOM (DSB-SC) = 1

Noise in DSB-FC:



1) pre Detection

$s(t) = A_c(1 + K_a(m(t)) \cos 2\pi f_c t$

Power of $s(t) = A_c\left[1 + K_a m(t) \cos 2\pi f_c t\right)$

$P[s(t)]$

$$= \frac{A_c^2}{2}(1 + K_a m(t))^2$$

$$= \frac{A_c^2}{2}\left[1 + K_a^2 m^2(t) + 2K_a m(t)\right]$$

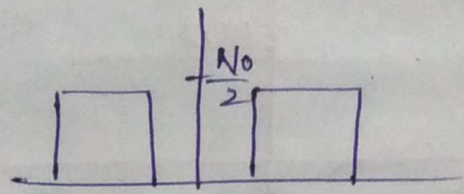
$\because$ power of $m(t) = P$ $\quad \frac{A_c^2}{2}[1 + K_a^2 P]$ modulated signal

$2K_a m(t) = 0$ (?)

$$\text{Predetection} = \frac{\frac{A_c^2}{2}(1 + K_a^2 P)}{N_0 B_T}$$

$$= \frac{A_c^2(1 + K_a^2 P)}{2(N_0 B_T)} \qquad \boxed{B_T = 2W}$$

PF & PF



$\frac{N_0}{2}$

$-f_c-W \quad -f_c+W \quad f_c-W \quad f_c+W$

$\xleftrightarrow{B_T}$

power $= N_0 B_T$

2) post detection

$x(t) = s(t) + n(t)$

$A_c(1 + K_a m(t))\cos(2\pi f_c t) + n_I t \cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$

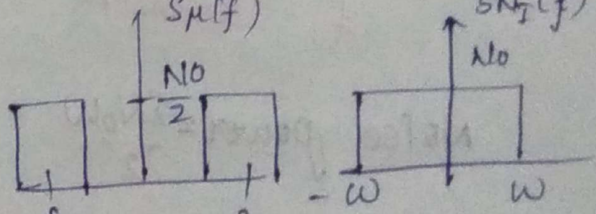
$Y(t) = \sqrt{A_c(1 + K_a m(t) + n_I t)^2 + n_Q^2 t}$

Envelop of signal $= \sqrt{A_c^2(1 + K_a m(t))^2 + n_I^2(t) + 2A_c(1 + K_a m(t)) n_I(t) + n_Q^2(t)}$

$= \sqrt{A_c^2(1 + K_a^2 m^2(t)) + n_I^2 t + 2A_c(1 + K_a m(t)) n_I t + n_Q^2 t}$

$= A_c(1 + K_a m(t) + n_I t$

$= \underbrace{A_c + A_c K_a m(t)}_{\text{signal}} + \underbrace{n_I(t)}_{\text{Noise}}$



$S_N(f)$ $\quad \frac{N_0}{2}$ $\quad -f_c \quad f_c$

$S_{N_I}(f)$ $\quad N_0$ $\quad -W \quad W$

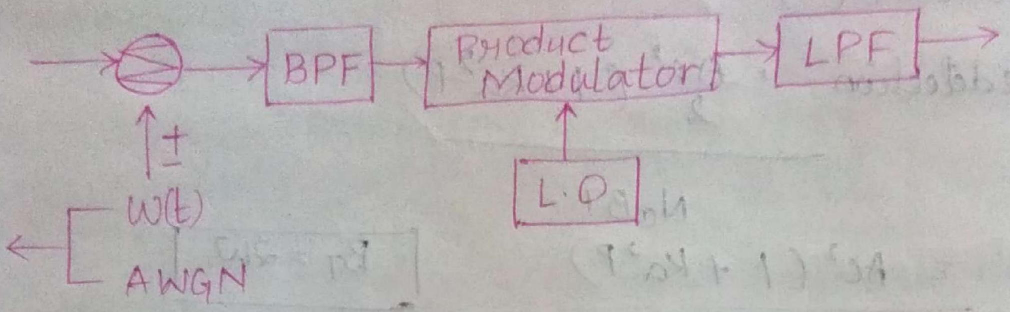
$$\text{Post detection SNR} = \frac{A_c^2 K_a^2 P}{N_0 B_T}$$

$$\text{FOM} = \frac{A_c^2 K_a^2 P / N_0 B_T}{\frac{A_c^2}{2}(1 + K_a^2 P)/N_0 B_T} = \frac{K_a^2 P}{1 + K_a^2 P} < 1$$

Eg: single tone : $m(t) = A_m \cos(2\pi f_c t)$

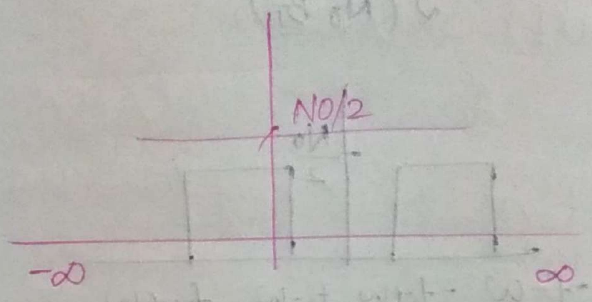
$$\text{FOM} = \frac{A_m^2 K_a^2 P}{N_0 B_T} \Big/ \frac{1 + A_m K_a^2 P}{N_0 B_T} = \frac{\mu^2}{2 + \mu^2}$$

SSB-SC



0 - zero mean

Variance $= \frac{N_0}{2}$

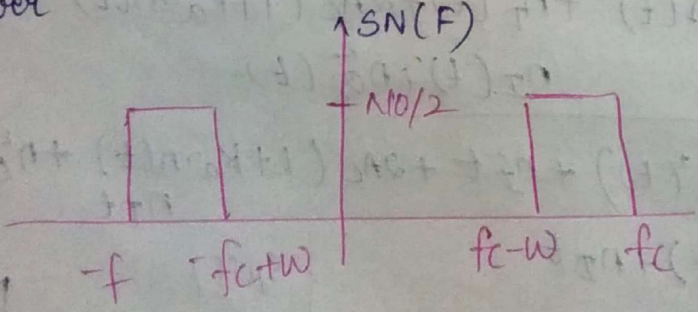


Predetection

$$S(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

$m(t)$ and $\hat{m}(t)$ have equal power

Signal Power $= \frac{A_c^2 P}{8} + \frac{A_c^2 P}{8} = \frac{A_c^2 P}{4}$



Noise power $= \frac{2 N_0 W}{2} = N_0 W$

Post Detection

$$x(t) = s(t) + n(t)$$

$$= \frac{A_c}{2} m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

$$+ n_I(t)\cos(2\pi f_c t) - n_Q t \sin(2\pi f_c t)) \cos(2\pi f_c t)$$

Scanned by CamScanner

$\frac{A_c}{2} m(t) \cos^2(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t) \cos(2\pi f_c t) +$

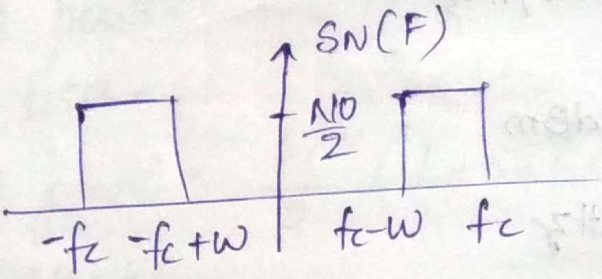
$n_I(t) \cos^2(2\pi f_c t) - n_Q t \sin(2\pi f_c t) \cos(2\pi f_c t)$

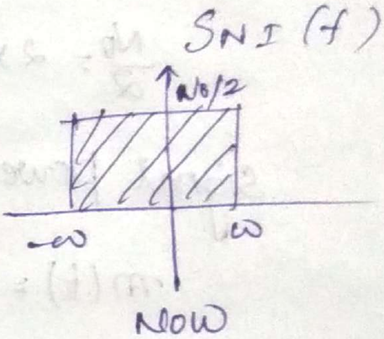
$\Rightarrow \frac{A_c m(t)}{2} \left[\frac{1 + \cos(4\pi f_c t)}{2} \right] + \frac{n_I t}{2} \left[1 + \frac{\cos 4\pi f_c t}{2} \right]$

O/P of LPF $= \frac{A_c m(t)}{4} + \frac{n_I t}{2}$

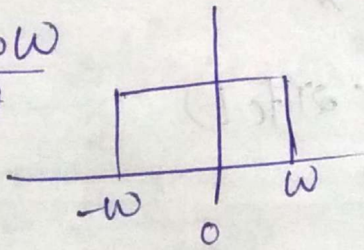
cos & sin Terms are removed because it pass through LPF (HPF)

Signal power $= \frac{A_c^2 P}{16}$





Noise power $= \frac{N_0 W}{4}$



FOM $= \frac{\text{post det SNR}}{\text{Ref SNR}}$

Ref SNR $= \frac{A_c^2 P}{4 N_0 W} = \frac{\text{Power of Modulated}}{\text{Power of Noise in } m(t) \text{ BW.}}$

April

A DSB-SC signal is transmitted over a noisy channel having noise spectral density $\frac{N_0}{2} = 2\times10^{-17}$ watts/Hz

The message bandwidth is 4KHz and carrier frequency is 200KHz. Assume the avg power recieved of signal −80dBm. Determine post detection SNR.

DSB-SC = 2W

$\frac{N_0}{2} = 2\times10^{-17}$ watts/Hz

signal power = −80dBm

$m(t) = W = 4\,KHz$

$f_c = 200KHz$]−x

$s(t) = A_c m(t) \cos(2\pi f_c t)$

$Power = \frac{A_c^2 P}{2}$

$\frac{A_c^2 P}{2} = -80dBm$

$$SNR_{Post} = \frac{A_c^2 P}{2N_0W} = \frac{10^{-11}}{4\times10^{-17}\times4\times10^{3}} = 62.5$$

$-80\,dBm = 10\log_{10}(X)$

$\log_{10}(X) = -8$ m

$X = 10^{-8}\,mW = 10^{-11}\,W$

In dB = $10\log_{10}(62.5) = 17.9dB$

For Signal power recived of -80 dBm compares the Post detection SNR of DSB-SC with DSB-FC assume the average message power is 1.

$$SNR_{post\ DSB-SC} = \frac{A_c^2 P}{2 N_0 W}$$

$$SNR_{post\ DSB-FC} = \frac{A_c^2 K_a^2 P}{2 N_0 W}$$

$$\text{Signal power}_{DSB-SC} = \frac{A_c^2 P}{2}$$

$$\text{Signal power}_{DSB-FC} = \frac{A_c'^2}{2}(1 + K_a^2 P)$$

$$\frac{A_c^2 P}{2} = \frac{A_c'^2}{2}(1 + K_a^2 P)$$

$$A_c'^2 = \frac{A_c^2 P}{1 + K_a^2 P}$$

$$\frac{A_c^2 K_a^2}{2 N_0 W (1 + K_a^2 P)} \quad \left(\because \frac{A_c^2 K_a^2 P}{2 N_0 W} \right)$$

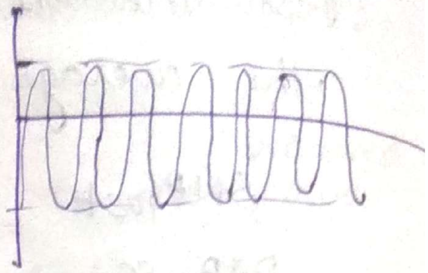
$$\boxed{SNR(DSB\text{-}FC) = \frac{K_a^2 P}{1 + K_a^2 P} \; SNR(DSB\text{-}SC)}$$

$$\frac{\left(\frac{2}{10}\right)^2}{1 + \left(\frac{2}{10}\right)^2} \qquad \frac{(0.2)^2}{1 + (0.2)^2} = 0.03$$

NOISE IN FM

$S(t) \rightarrow \Sigma \rightarrow$ BPF $\rightarrow$ Limiter $\rightarrow$ Discriminator $\rightarrow$ Envelop detector

Limiter: Removing amplitude variations



Discriminator: Demodulator for FM. It acts like a discriminator.

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int m(t)\, dt\right)$$

$$\left|\frac{d}{dt}(s(t))\right| = A_c \sin\left(2\pi f_c t + 2\pi K_f (m(t)) dt\right) \left(2\pi f_c\right) + 2\pi K_f\, m(t))$$

Hybrid modulated signal (AM+FM)

6th April

Noise in Frequency Modulation

$S(t) \rightarrow \Sigma \rightarrow$ BPF $\rightarrow$ Limiter $\rightarrow$ Discrimination $\rightarrow$ LPF

$w(t)$ (input to Σ)

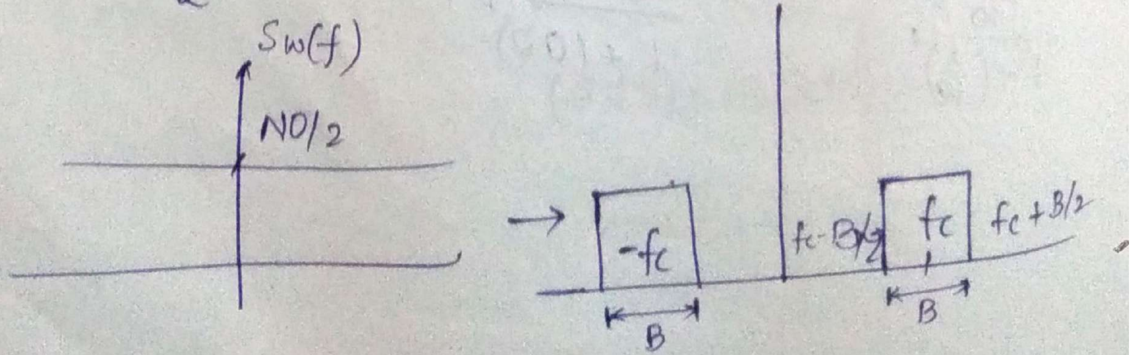
Limiter: Diff. & E.D

$$S(t) = A_c \cos(2\pi f_c t + \phi_t)$$

FM

$$\phi(t) = 2\pi K_f \int m(t)\, dt$$

Signal power $= \dfrac{A_c^2}{2}$



Noise power $= N_0 B$

Pre detection SNR $= \dfrac{A_c^2}{2 N_0 B}$

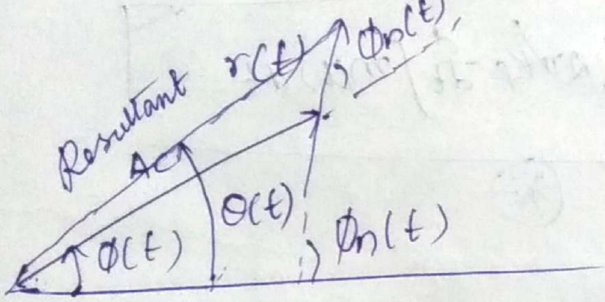
$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$n(t) = r(t)\cos(2\pi f_c t + \phi_n(t))$$

$$r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$$

$$\phi_n(t) = \tan^{-1}\left(\frac{-n_Q t}{n_I t}\right) - \phi t = \psi(t)$$

$\theta_n > \theta_u$



$$\theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t)\sin\psi(t)}{A_c + r(t)\cos\psi(t)}\right]$$

→ Assume SNR to be good.

$$A_c >> r(t)\cos\psi(t)$$

$$\theta(t) \simeq \phi(t) + \tan^{-1}\left(\frac{r(t)\sin\psi(t)}{A_c}\right)$$

$$\boxed{\theta << \tan^{-1}\theta \simeq \theta}$$

$$\theta(t) \simeq \phi(t) + \frac{r(t)}{A_c}\sin(\phi_n(t) - \phi(t))$$

$\phi_n t$ is phase of noise is uniformly distributed over 2π.

Therefore, we can assume that $\phi_n t$ is very much greater than $\phi(t)$.

$$\phi_n t >> \phi(t)$$

$$\therefore \theta t \simeq \phi(t) + \frac{r(t)}{A_c} \sin(\phi_n(t))$$

$$O(t) \simeq \phi(t) + \frac{n_Q(t)}{A_c}$$

O/p of discrimination $= \frac{1}{2\pi} \frac{d}{dt} O(t)$

Output $= k_f m(t) + n_d(t)$

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_Q(t)$$

$$\phi(t) = 2\pi k_f \int m(t)\,dt$$

$$\frac{1}{2\pi} \frac{d}{dt}(\phi(t)) = \frac{1}{2\pi} 2\pi k_f \frac{d}{dt} \int m(t)\,dt$$

$$\frac{d}{dt} \rightleftharpoons j2\pi f$$

NOISE IN FM

O/p $= k_f m(t) + n_d(t)$

(discriminator)

$$\frac{1}{2\pi} \frac{d}{dt} O(t) \quad \Big| \quad O(t) = \phi(t) + \frac{1}{2\pi f_c} n_Q(t)$$

$$\phi(t) \rightleftharpoons j2\pi f \int m(t)\,dt$$

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_Q(t)$$

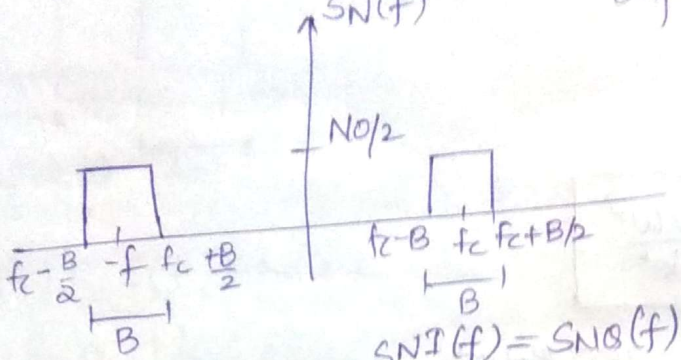
$$\frac{d}{dt} \rightleftharpoons j2\pi f$$

$\rightarrow$ $n_Q(t)$ is passed through a filter $H(f) = \frac{jf}{A_c}$

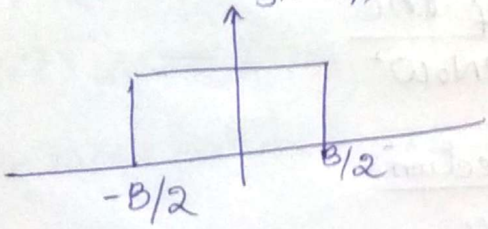
1. Output signal, $\therefore$ $S_{Nd}(f) = |H(f)|^2 S_{NQ}(f)$

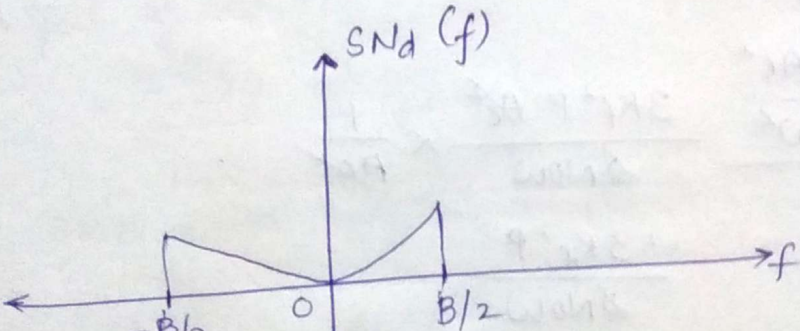
$= \frac{f^2}{A_c^2} S_{NQ}(f)$ — ?

$\propto f^2$



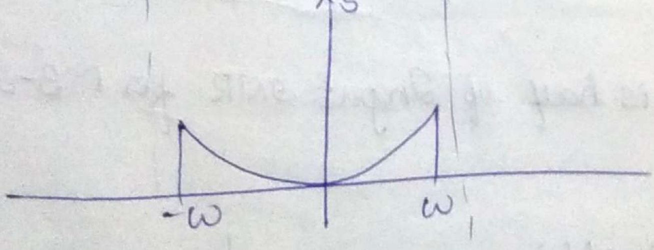
$S_{NI}(f) = S_{NQ}(f)$





$W < B/2$

$S_{Nd}(f)$ passed through $\propto PR$



Output of filter $= \frac{f^2}{A_c^2} S_{NQ}(f)$ $\quad -W \le f \le W$

Power of Noise $= \int_{-W}^{W} \frac{f^2}{A_c^2} S_{NQ}(f)\,df$

$$= \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 df$$

$$\frac{N_0}{A_c^2} \left. \frac{f^3}{3} \right]_{W}^{W}$$

$$\frac{N_0}{A_c^2} \quad \frac{W^3}{3} + \frac{W^3}{3}$$

$$\frac{N_0}{A_c^2} \left(\frac{\cancel{6} W^3}{\cancel{9} 3} \right) = \boxed{\frac{2}{3} \frac{N_0 W^3}{A_c^2}}$$

$$\text{Post Detection} = \frac{3K_f^2 P A_c^2}{2 N_0 W^2}$$

$$\text{FOM} = \frac{\text{Post Detection}}{\text{Reference}}$$

$$\frac{\dfrac{3K_f^2 P A_c^2}{\cancel{2} N_0 W^2}}{\dfrac{A_c^2}{\cancel{2} N_0 W}} \qquad \frac{3K_f^2 P \cancel{A_c^2}}{\cancel{2} N_0 W} \times \frac{1}{\cancel{A_c^2}}$$

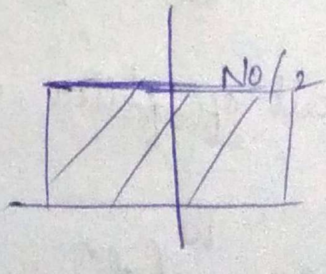
$$= \frac{3K_f^2 P}{\cancel{2} N_0 W}$$

Numericals: Unit-4

Derivations Unit-5

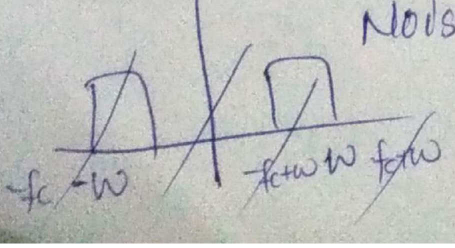
Q. prove that Output SNR is half of Input SNR for DSB-S reciever.

$$s(t) = A_c \cos(2\pi f_c t)\, m(t)$$



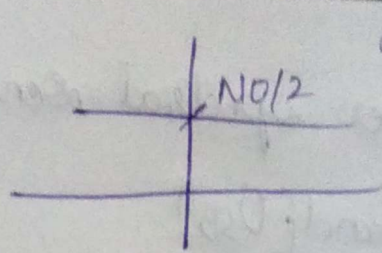
$$\text{Power} = \frac{A_c^2}{2\cancel{p}} \times P$$

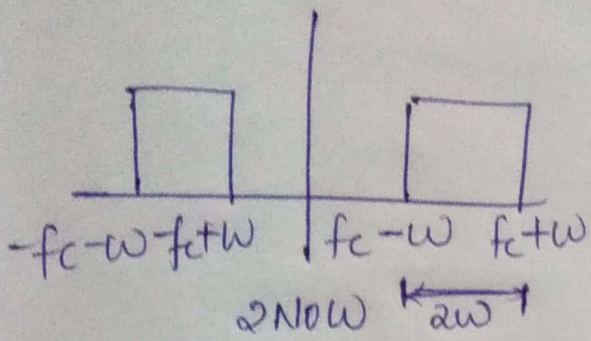
Noise power =



DSB - SC

Double Si





For det $x(t) = S(t) + N(t)$

$= A_c m(t) \cos 2\pi f_c t + N_I t \cos 2\pi f_c t - N_Q t \sin(2\pi f_c t)$

$Y(t) = X(t) \cos 2\pi f_c t$

$= A_c m(t) \cos^2 2\pi f_c t + N_I t \cos^2 2\pi f_c t - N_Q t \sin(2\pi f_c) \cos(2\pi f_c t)$

Output of the LPF,

$$\frac{n_I(t)}{2} + \frac{A_c m(t)}{2}$$

Noise power $= \frac{N_0 W}{2}$ $\qquad \frac{A_c^2}{4} P$

$$\frac{\frac{A_c^2 P}{4}}{\frac{N_0 W}{2}}$$