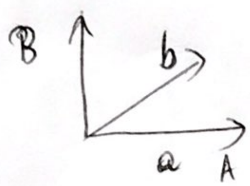
Part C:

$$a = \begin{bmatrix} 2/7 \\ 3/7 \\ 6/7 \end{bmatrix}$$

Let's assume $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

We need to make b as an orthogonal to a.



We can write $a = A$

We need to find B from A and b using Gram-Schmidt.

$$B = b - \frac{A^T b}{A^T A} A$$

$$A = \begin{bmatrix} 2/7 \\ 3/7 \\ 6/7 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2/7 , & 3/7 & 6/7 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 2/7 & 3/7 & 6/7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\cancel{= [2/7]} \quad \cancel{= [2/7 + 6/7 + 18/7]}$$

$$\cancel{= [26/7]}$$

$$= 2/7 + 6/7 + 18/7$$

$$= 26/7$$

$$A^T A = \begin{bmatrix} 2/7 & 3/7 & 6/7 \end{bmatrix} \begin{bmatrix} 2/7 \\ 3/7 \\ 6/7 \end{bmatrix}$$

$$= \frac{4+9+36}{49}$$

$$= 1$$

$$B = b - \frac{A^T b}{A^T A} A$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{26/7}{1} \begin{bmatrix} 2/7 \\ 3/7 \\ 6/7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 52/49 \\ 78/49 \\ 156/49 \end{bmatrix}$$

$$= \begin{bmatrix} -3/49 \\ 20/49 \\ -9/49 \end{bmatrix}$$

$$||B|| = \frac{\sqrt{490}}{49}$$

$$Q2 = B/||B|| = \begin{bmatrix} -3/\sqrt{490} \\ 20/\sqrt{490} \\ 9/\sqrt{490} \end{bmatrix}$$

$$||A_1|| = 1$$

$$Q_1 = \frac{A}{||A_1||}$$

$$= \begin{bmatrix} 2/7 \\ 3/7 \\ 6/7 \end{bmatrix}$$

So orthonormal basis is:

$$Q = [Q_1, Q_2]$$

$$= \begin{bmatrix} 2/7 & -3/\sqrt{490} \\ 3/7 & 20/\sqrt{490} \\ 6/7 & 9/\sqrt{490} \end{bmatrix}$$

Part E :—

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\text{Assume } b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

As per Gram - Schmidt →

$$B = b - \frac{A^T b}{A^T A} A$$

$$a = A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$A^T = [1, 2, 3]$$

$$A^T b = 4 + 10 + 18 = 32$$

$$A^T A = 1 + 4 + 9 = 14$$

$$B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \frac{32}{14} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} \frac{32}{14} \\ 64/14 \\ 96/14 \end{bmatrix}$$

$$= \begin{bmatrix} 24/14 \\ 6/14 \\ -12/14 \end{bmatrix}$$

So orthogonal of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is

$$\begin{bmatrix} 24/14 \\ 6/14 \\ -12/14 \end{bmatrix}$$

$$A.B = \frac{24}{14} + \frac{12}{14} - \frac{36}{14} = 0$$

Part F)

For a diagonal Matrix

$$A = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The pseudo inverse is nothing but reciprocals of the diagonal values |.

$$\text{Moore-Penrose } A^{-1} = \begin{bmatrix} 1/\sigma_1 & 0 & 0 & 0 \\ 0 & 1/\sigma_2 & 0 & 0 \\ 0 & 0 & 1/\sigma_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

M is a diagonal Matrix.

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$