1. Problem Definition

Accurate state of charge (SOC) estimation is always a challenge since many external parameters can affect the accurate estimation of SOC of the battery. The key goal in this report is to represent the open circuit voltage (OCV) as a function of state of charge (SOC) accurately with very few parameters as possible. The OCV-SOC relationship is a nonlinear curve that is often represented by polynomial, logarithmic, and inverse functions that are not defined around the SOC ≜ [0,1].

Linear scaling approach of OCV-SOC characterization is a formal method to avoid the numeric instability by avoiding the substitution of s=0 and s=1. *Figure 1* shows the linear scaling approach. The OCV-SOC modelling can be improved significantly by properly selecting the scaling value "epsilon". This way s' is prevented by reaching 0 and 1 to avoid the instability.

$$s' = (1 - 2\varepsilon)s + \varepsilon \tag{1}$$

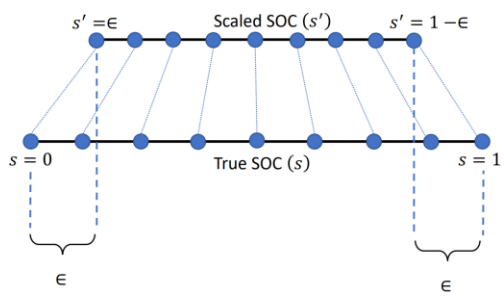

Figure 1 : Linear Scaling

Optimal epsilon is selected based on the model. In this report ε=0.175 is selected as it gives the optimal results in various models.

First step in OCV-SOC modeling is the selection of function that is suitable to capture the OCV-SOC relation of a particular battery chemistry. Models like linear model and polynomial model are well discussed models. One of the well adopted is the combined model. In this report four models such as linear model, polynomial model, combined model, and combined+3 model is discussed in the following sections.

The next method is to estimate the OCV parameters of all the above-mentioned models. For instance in combined+3 model OCV parameters k_0, k_1, k_2,…,k_7 are estimated. With the OCV, SOC data point collected spanning the entire range of OCV and SOC, these OCV parameters can be linearly estimated using least square estimation approach.

The terminal voltage, SOC, and charging and discharging current of a real battery during offline estimation is used in this report. With the available data, using the above mentioned four models, the OCV parameters are estimated. With the estimated OCV parameters, the open circuit voltage, resistance of a battery is estimated, and OCV-SOC curve is plotted.

In order to compute the real SOC at time k, coulomb counting approach is used. SOC calculated is the ration between the remaining coulombs and the battery capacity.

$$soc[k+1] = soc[k] + \frac{\Delta_k i[k]}{3600\ C_{batt}} \tag{2}$$

Where soc[k] is the soc at time k, and Δ_k is the sampling time, i[k] is the current through the battery, and C_{batt} is the capacity of the battery in Ah. Coulomb counting approach does not require offline estimation, but it is susceptible to following errors:

i. It requires initial SOC
ii. Current measurement error can affect the computed SOC
iii. Simplified approximation of current integration used in this approach will result in errors
iv. Uncertainty in battery capacity due to ageing, temperature, etc.
v. Timing oscillating error.

High precision measuring instrument such as Battery Tester Arbin machine is used to reduce the current measurement error and timing oscillating error. Further, the charge-discharge current is kept constant. This nullifies the current integration error. OCV characterization profile is chosen in a way such that initial SOC and battery capacity can be measured accurately before the parameters are obtained.

Novel contribution and results of this report can be summarized as follows:

i. Novel approach for OCV model parameter estimation. Linear model, polynomial model, combined model, and combined+3 model is derived and OCV parameters are estimated offline by simulation from the given real battery data loaded in “data.xls” file.
ii. An approach is developed formally to evaluate the accuracy of each model based on correlation coefficient R^2.
iii. Nominal OCV modelling where SOC is computed using coulomb counting method for provided battery capacity which is assumed to be same for all room temperature.
iv. Pros and cons are discussed for each model.

2. Solution Approach

Terminal voltages, current, and SOC values are given for a real battery. OCV-SOC characterization experiment was conducted for various temperature spanning from -25°C to 50°C and OCV-SOC characteristics for each temperature were analyzed. Further, the battery capacity is computed through slow discharge and slow charge. It is found that the OCV-SOC characterization curve remains same regardless of the temperature change and age.

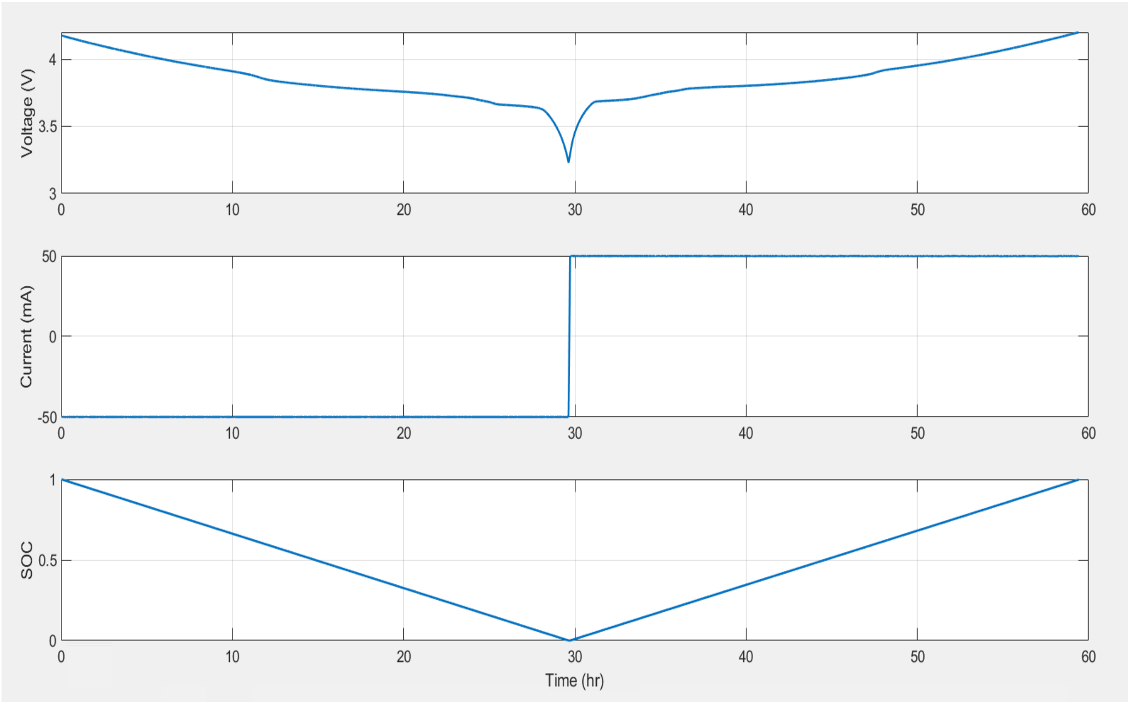

Figure 2 : Measured Voltage, Current, and SOC

Figure 2 shows the slow discharging and slow charging when low current (C/30) is used to drain and charge the battery respectively. Since OCV-SOC curve is constant for all temperatures as per the experiment, capacity of the given battery is calculated to be **1.4844Ah** from the given current and time data. C_{batt} is kept constant at 1.4844Ah and coulomb counting method can be used to compute the SOC with the given current measurements. With the computed SOC and the provided measured voltage and current details, OCV-SOC model parameters can be computed using least square estimation approach. With the computed OCV parameters, OCV-SOC curve can be plotted.

3. OCV Parameter Estimation Models

OCV parameter estimation is done with the provided real battery data. Four models are discussed in this report. The four models are: Linear Model, Polynomial model, Combined model, and Combined+3 model. Each model is derived mathematically and simulated using MATLAB to obtain the OCV parameters for each model.

3.1. Linear Model

$$V_o(s) = a_0 + a_1 s \tag{3}$$

Where a_0 and a_1 are the parameters of the linear model.

The measured voltage across the battery terminals is,

$$z_v[k] = v[k] + n_v[k] \tag{4}$$

The terminal voltage when the battery is slowly charged or discharged can be written as,

$$z_v[k] = V_o(s[k]) + h[k] + i[k]R_0 + n_v[k] \tag{5}$$

Where h[k] is the hysteresis which is a function of current and SOC of the battery. We assume hysteresis is proportional to current only, because the OCV test is performed at a very low current.

$$\text{So, } h[k] \propto i[k] \tag{6}$$

Equation (5) can be written as,

$$z_v[k] = V_o(s[k]) + i[k]R_{0,h} + n_v[k] \tag{7}$$

Where the effective resistance

$$R_{0,h} = R_0 + R_h \tag{8}$$

Let's rewrite (7) in vector notation,

$$z_v[k] = a_0 + a_1 s[k] + i[k]R_{0,h} + n_v[k]$$

$$z_v[k] = [1 \quad s[k] \quad i[k]] \begin{bmatrix} a_0 \\ a_1 \\ R_{0,h} \end{bmatrix} + n_v[k]$$

$$z_v[k] = \underbrace{[p_o(s[k])^T \quad i[k]]}_{p[k]^T} \underbrace{\begin{bmatrix} k_o \\ R_{0,h} \end{bmatrix}}_{k} + n_v[k] \tag{9}$$

Where

$$k_o = [a_0 \quad a_1]^T \tag{10}$$

$$p_o(s[k])^T = [1 \quad s[k]] \tag{11}$$

For a batch of N voltages, (9) can be written as,

$$v = Pk + n \tag{12}$$

Where

$$v = [z_v[1]\, z_v[2]\, z_v[3] \, \cdots \, z_v[t_n]]^T \tag{13}$$

$$P = [p[1]\, p[2]\, p[3] \, \cdots \, p[t_n]]^T \tag{14}$$

$$n = [n[1]\, n[2]\, n[3] \, \cdots \, n[t_n]]^T \tag{15}$$

$$k = [a_0 \quad a_1 \quad R_{0,h}]^T \tag{16}$$

The least square estimate of the parameter vector is given by

$$\hat{k} = (P^T P)^{-1} P^T v \tag{17}$$

$$\widehat{V_o}(s) = p_o(s)^T \widehat{k_o} \tag{18}$$

Where $\widehat{k_o}$ is formed by the first two elements of $\hat{k}$

3.2. Polynomial Model

$$V_o(s) = p_0 + p_1 s + \cdots + p_n s^n + p_{n+1} s^{-1} + \cdots + p_{n+m} s^{-m}$$

Order = 5 $$V_o(s) = p_0 + p_1 s + p_2 s^2 + p_3 s^3 + p_4 s^4 + p_5 s^5 + p_6 s^{-1} + p_7 s^{-2} + p_8 s^{-3} + p_9 s^{-4} + p_{10} s^{-5} \tag{19}$$

Where p_0, p_1,…,p_{10} are the parameter of the polynomial model.

Accuracy of polynomial model can be increased by recursively increasing the order of the polynomial. In this report, n=5 and m=5 (Order 5) is chosen and OCV is modelled.

The measured voltage across the battery terminals is,

$$z_v[k] = v[k] + n_v[k] \tag{20}$$

The terminal voltage when the battery is slowly charged or discharged can be written as,

$$z_v[k] = V_o(s[k]) + h[k] + i[k]R_0 + n_v[k] \tag{21}$$

Where h[k] is the hysteresis which is a function of current and SOC of the battery. We assume hysteresis is proportional to current only, because the OCV test is performed at a very low current.

$$\text{So, } h[k] \propto i[k] \tag{22}$$

Equation *(21)* can be written as,

$$z_v[k] = V_o(s[k]) + i[k]R_{0,h} + n_v[k] \tag{23}$$

Where the effective resistance

$$R_{0,h} = R_0 + R_h \tag{24}$$

Let's rewrite *(23)* in vector notation,

$$z_v[k] = p_0 + p_1 s[k] + p_2 s[k]^2 + p_3 s[k]^3 + p_4 s[k]^4 + p_5 s[k]^5 + \frac{p_6}{s[k]} + \frac{p_7}{s[k]^2} + \frac{p_8}{s[k]^3} + \frac{p_9}{s[k]^4} + \frac{p_{10}}{s[k]^5} + i[k]R_{0,h} + n_v[k]$$

$$z_v[k] = \begin{bmatrix}1 & s[k] & s[k]^2 & s[k]^3 & s[k]^4 & s[k]^5 & \frac{1}{s[k]} & \frac{1}{s[k]^2} & \frac{1}{s[k]^3} & \frac{1}{s[k]^4} & \frac{1}{s[k]^5} & i[k]\end{bmatrix}\begin{bmatrix}p_0\\p_1\\p_2\\p_3\\p_4\\p_5\\p_6\\p_7\\p_8\\p_9\\p_{10}\\R_{0,h}\end{bmatrix} + n_v[k]$$

$$z_v[k] = \underbrace{[p_o(s[k])^T \quad i[k]]}_{p[k]^T}\underbrace{\begin{bmatrix}k_o\\R_{0,h}\end{bmatrix}}_{k} + n_v[k] \tag{25}$$

Where

$$k_o = [p_0\ p_1\ p_2\ p_3\ p_4\ p_5\ p_6\ p_7\ p_8\ p_9\ p_{10}]^T \tag{26}$$

$$p_o(s[k])^T = \begin{bmatrix}1 & s[k] & s[k]^2 & s[k]^3 & s[k]^4 & s[k]^5 & \frac{1}{s[k]} & \frac{1}{s[k]^2} & \frac{1}{s[k]^3} & \frac{1}{s[k]^4} & \frac{1}{s[k]^5}\end{bmatrix} \tag{27}$$

For a batch of N voltages, *(25)* can be written as,

$$v = Pk + n \tag{28}$$

Where

$$v = [z_v[1]\ z_v[2]\ z_v[3]\ \cdots\ z_v[t_n]]^T \tag{29}$$

$$P = [p[1]\ p[2]\ p[3]\ \cdots\ p[t_n]]^T \tag{30}$$

$$n = [n[1]\ n[2]\ n[3]\ \cdots\ n[t_n]]^T \tag{31}$$

$$k = [1 \quad s[k] \quad s[k]^2 \quad s[k]^3 \quad s[k]^4 \quad s[k]^5 \quad \frac{1}{s[k]} \quad \frac{1}{s[k]^2} \quad \frac{1}{s[k]^3} \quad \frac{1}{s[k]^4} \quad \frac{1}{s[k]^5} \quad R_{0,h}]^T \tag{32}$$

The least square estimate of the parameter vector is given by

$$\hat{k} = (P^T P)^{-1} P^T v \tag{33}$$

$$\widehat{V_o}(s) = p_o(s)^T \widehat{k_o} \tag{34}$$

Where $\widehat{k_o}$is formed by the first 11 elements of $\hat{k}$

3.3. Combined Model

$$V_o(s) = k_0 + k_1 s^{-1} + k_2 s + k_3 ln(s) + k_4 ln(1-s) \tag{35}$$

Where k_0,k_1,k_2,k_3 and k_4 are the parameters of the combined model.

The measured voltage across the battery terminals is,

$$z_v[k] = v[k] + n_v[k] \tag{36}$$

The terminal voltage when the battery is slowly charged or discharged can be written as,

$$z_v[k] = V_o(s[k]) + h[k] + i[k]R_0 + n_v[k] \tag{37}$$

Where h[k] is the hysteresis which is a function of current and SOC of the battery. We assume hysteresis is proportional to current only, because the OCV test is performed at a very low current.

$$\text{So, } h[k] \ \propto \ i[k] \tag{38}$$

Equation *(37)* can be written as,

$$z_v[k] = V_o(s[k]) + i[k]R_{0,h} + n_v[k] \tag{39}$$

Where the effective resistance

$$R_{0,h} = R_0 + R_h \tag{40}$$

Let's rewrite *(39)* in vector notation,

$$z_v[k] = k_0 + \frac{k_1}{s[k]} + k_2 s + k_3 ln(s) + k_4 ln(1-s) + i[k]R_{0,h} + n_v[k]$$

$$z_v[k] = \begin{bmatrix} 1 & \frac{1}{s[k]} & s[k] & ln(s) & ln(1-s) & i[k] \end{bmatrix} \begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ R_{0,h} \end{bmatrix} + n_v[k]$$

$$z_v[k] = \underbrace{[p_o(s[k])^T \quad i[k]]}_{p[k]^T} \underbrace{\begin{bmatrix} k_o \\ R_{0,h} \end{bmatrix}}_{k} + n_v[k] \tag{41}$$

Where

$$k_o = [k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4]^T \tag{42}$$

$$p_o(s[k])^T = \begin{bmatrix} 1 & \frac{1}{s[k]} & s[k] & ln(s) & ln(1-s) \end{bmatrix} \tag{43}$$

For a batch of N voltages, *(41)* can be written as,

$$v = Pk + n \tag{44}$$

Where

$$v = [z_v[1] \, z_v[2] \, z_v[3] \, \cdots \, z_v[t_n]]^T \tag{45}$$

$$P = [p[1] \, p[2] \, p[3] \, \cdots \, p[t_n]]^T \tag{46}$$

$$n = [n[1]\; n[2]\; n[3]\; \cdots\; n[t_n]]^T \tag{47}$$

$$k = [k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4 \quad R_{0,h}]^T \tag{48}$$

The least square estimate of the parameter vector is given by

$$\hat{k} = (P^T P)^{-1} P^T v \tag{49}$$

$$\widehat{V_o}(s) = p_o(s)^T \widehat{k_o} \tag{50}$$

Where $\widehat{k_o}$ is formed by the first 5 elements of $\hat{k}$

3.4. Combined+3 Model

$$V_o(s) = k_0 + k_1 s^{-1} + k_2 s^{-2} + k_3 s^{-3} + k_4 s^{-4} + k_5 s + k_6 ln(s) + k_7 ln(1-s) \tag{51}$$

Where $k_0, k_1, \ldots, k_7$ are the parameters of the combined+3 model.

The measured voltage across the battery terminals is,

$$z_v[k] = v[k] + n_v[k] \tag{52}$$

The terminal voltage when the battery is slowly charged or discharged can be written as,

$$z_v[k] = V_o(s[k]) + h[k] + i[k]R_0 + n_v[k] \tag{53}$$

Where h[k] is the hysteresis which is a function of current and SOC of the battery. We assume hysteresis is proportional to current only, because the OCV test is performed at a very low current.

$$\text{So, } h[k] \propto i[k] \tag{54}$$

Equation (53) can be written as,

$$z_v[k] = V_o(s[k]) + i[k]R_{0,h} + n_v[k] \tag{55}$$

Where the effective resistance

$$R_{0,h} = R_0 + R_h \tag{56}$$

Let’s rewrite (55) in vector notation,

$$z_v[k] = \mathrm{k}_0 + \mathrm{k}_1\mathrm{s}^{-1} + \mathrm{k}_2\mathrm{s}^{-2} + \mathrm{k}_3\mathrm{s}^{-3} + \mathrm{k}_4\mathrm{s}^{-4} + \mathrm{k}_5\mathrm{s} + \mathrm{k}_6\ln(\mathrm{s}) + \mathrm{k}_7\ln(1-\mathrm{s}) + i[k]R_{0,h} + n_v[k]$$

$$z_v[k] = \begin{bmatrix} 1 & \frac{1}{s[k]} & \frac{1}{s[k]^2} & \frac{1}{s[k]^3} & \frac{1}{s[k]^4} & s[k] & ln(s) & ln(1-s) & i[k] \end{bmatrix} \begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \\ k_7 \\ R_{0,h} \end{bmatrix} + n_v[k]$$

$$z_v[k] = \underbrace{[p_o(s[k])^T \quad i[k]]}_{p[k]^T} \underbrace{\begin{bmatrix} k_o \\ R_{0,h} \end{bmatrix}}_{k} + n_v[k] \tag{57}$$

Where

$$k_o = [k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6 \quad k_7]^T \tag{58}$$

$$p_o(s[k])^T = \begin{bmatrix} 1 & \frac{1}{s[k]} & \frac{1}{s[k]^2} & \frac{1}{s[k]^3} & \frac{1}{s[k]^4} & s[k] & ln(s) & ln(1-s) \end{bmatrix} \tag{59}$$

For a batch of N voltages, (57) can be written as,

$$v = Pk + n \tag{60}$$

Where

$$v = [z_v[1]\, z_v[2]\, z_v[3]\, \cdots\, z_v[t_n]]^T \tag{61}$$

$$P = [p[1]\, p[2]\, p[3]\, \cdots\, p[t_n]]^T \tag{62}$$

$$n = [n[1]\, n[2]\, n[3]\, \cdots\, n[t_n]]^T \tag{63}$$

$$k = [k_0 \quad k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6 \quad k_7 \quad R_{0,h}]^T \tag{64}$$

The least square estimate of the parameter vector is given by

$$\hat{k} = (P^T P)^{-1} P^T v \tag{65}$$

$$\widehat{V_o}(s) = p_o(s)^T \widehat{k_o} \tag{66}$$

Where $\widehat{k_o}$ is formed by the first 8 elements of $\hat{k}$

4. Results and Discussion

4.1. OCV-SOC Parameter Estimation

Now the OCV parameter estimation models are derived, OCV parameters can be estimated using the real battery data provided using the least square estimation approach. The estimated OCV parameters are then used to plot the OCV-SOC curve.

Model	Least Square Approach	OCV Parameters	ECM Parameter $R_{0,h}$	Reference
Linear	$\hat{k} = (P^T P)^{-1} P^T v$	a_0= 3.58541 a_1= 0.544742	0.398474 ohms	*Figure*
Polynomial	$\hat{k} = (P^T P)^{-1} P^T v$	p_0 = -923.036 p_1 = 1330.92 p_2 = -1071.99 p_3 = 347.525 p_4 = 78.6651 p_5 = -62.7266 P6 = 393.742 P7 = -103.701 P8 = 16.4395 P9 = -1.4287 P10 = 0.0519272	0.399916 ohms	*Figure*
Combined	$\hat{k} = (P^T P)^{-1} P^T v$	k_0 = -1.04108 k_1 = -0.809928 k_2 = 7.12803 k_3 = -4.53476 k_4 = 0.31878	0.399405 ohms	*Figure*
Combined+3	$\hat{k} = (P^T P)^{-1} P^T v$	k_0 = -8.82392 k_1 = 101.377 k_2 = -17.8659 k_3 = 2.02379 k_4 = -0.0997196 k_5 = -75.3835 k_6 = 138.94 k_7 = -1.09904	0.399995 ohms	*Figure*

Table 1: OCV Parameters, ECM Parameter $R_{0,h}$

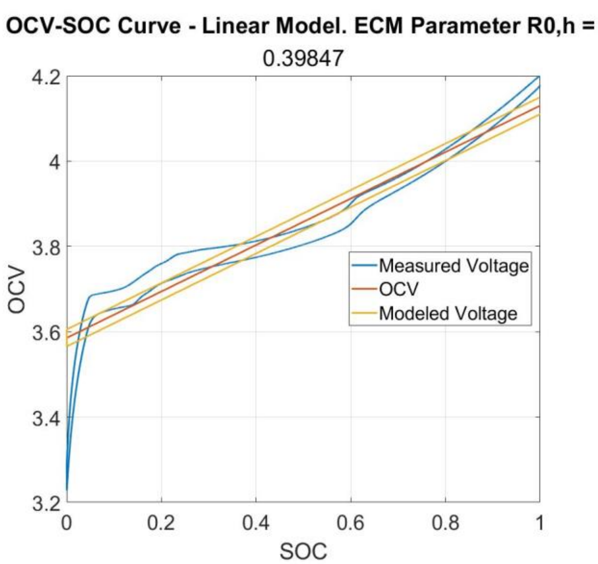

Figure 3: OCV-SOC Curve of Linear Model

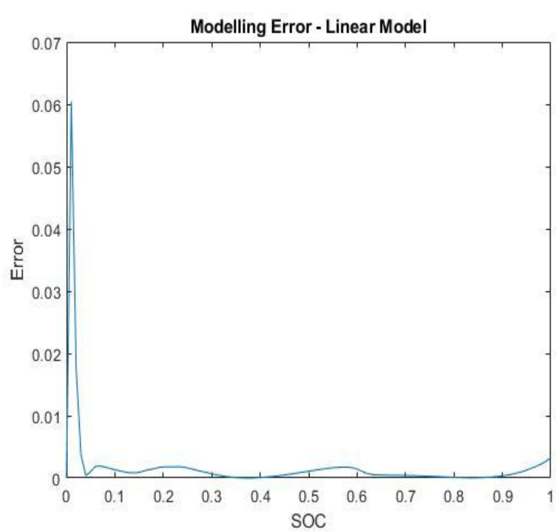

Figure 4: Modelling Error of Linear Model

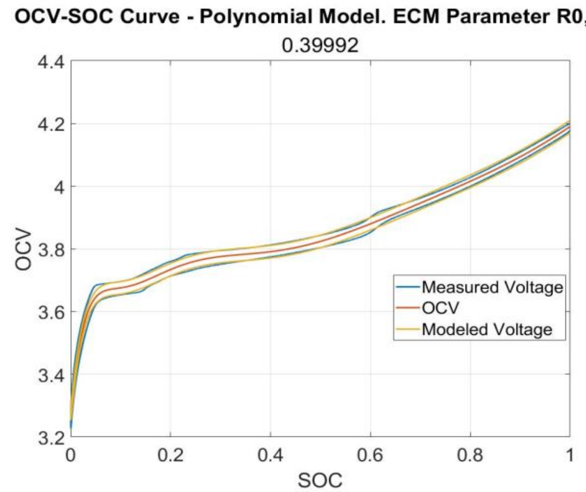

Figure 5: OCV-SOC Curve – Polynomial Mode

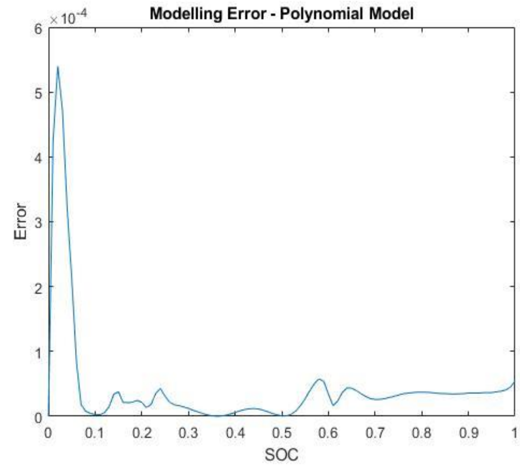

Figure 6: Modelling Error – Polynomial Model

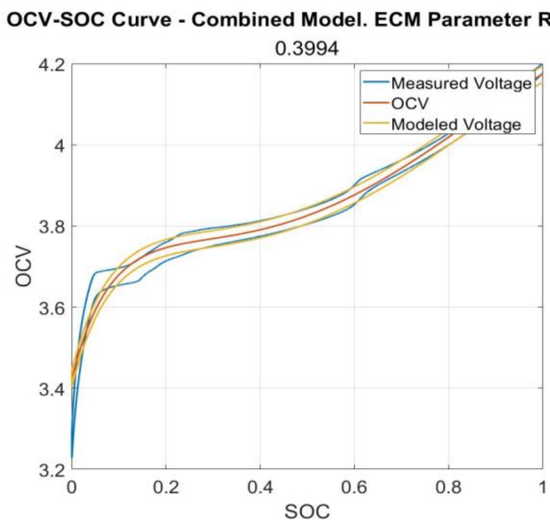

Figure 7: OCV-SOC Curve – Combined Model

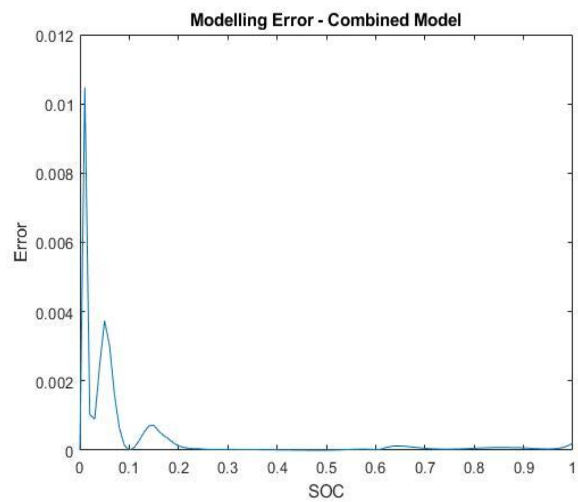

Figure 8: Modelling Error – Combined Model

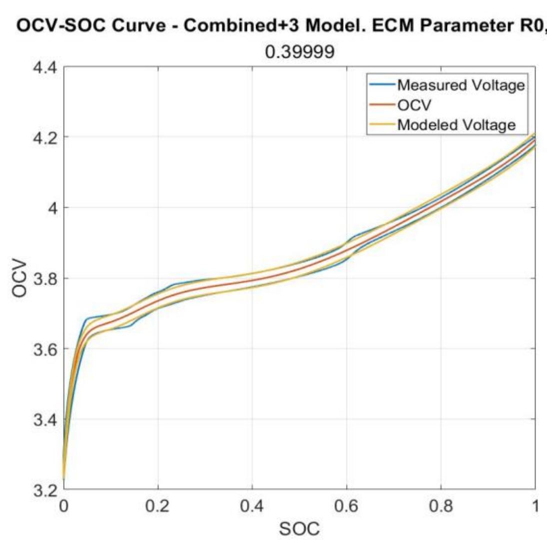

Figure 9: OCV-SOC Curve – Combined+3 Model

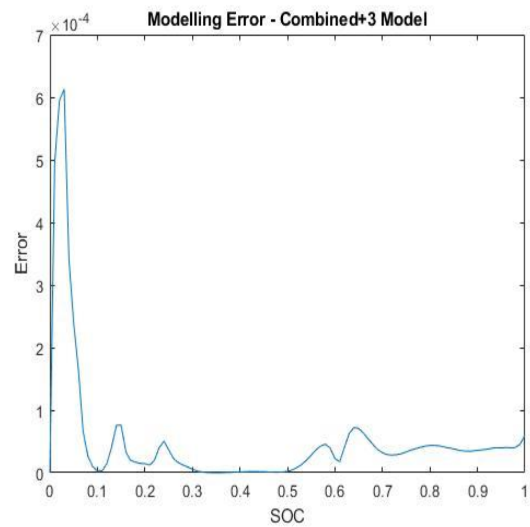

Figure 10: Modelling Error – Combined+3 Model

4.2. Accuracy of Each Model

Accuracy of each model based on correlation coefficient can be computed by the following method:

$$S_t = \sum (y_i - \bar{y})^2 \tag{67}$$

$$S_r = \sum (y_i - a_0 - a_i x_i)^2 \tag{68}$$

$$r = \sqrt{\frac{s_t - s_r}{s_t}} \tag{69}$$

The correlation coefficient R^2 denotes how well the function fits the OCV-SOC measurement. Best fit is when R^2 is closer to 1 and poor fit is when R^2 is closer to zero.

From *Table 2*, it is clear that polynomial model has the highest correlation coefficient which means polynomial model is very accurate and best fits the OCV-SOC measurement. Combined+3 model is second most accurate model and Linear model is the poorest of all and it doesn't fit the OCV-SOC measurement properly. Inference from the above data is the correlation coefficient increases with increasing number of OCV parameters.

S.No	Model	Accuracy
1	Linear Model	0.937482
2	Polynomial Model	0.998358
3	Combined Model	0.987789
4	Combined+3 Model	0.998091

Table 2: Correlation Coefficient of all Models

4.3. Coulomb Counting Method to compute SOC

SOC can be computed with the provided voltage and current measurement by coulomb counting method. The battery capacity is calculated to be 1.4844Ah. Coulomb counting equation can be given by (2),

$$soc[k+1] = soc[k] + \frac{\Delta_k i[k]}{3600\ C_{batt}} + n_s[k]$$

C_{batt} = 1.4844Ah

Noise factor is Gaussian in nature, and it can be neglected. The SOC calculated (*Figure 1*) using the above-mentioned method is then scaled to avoid numeric instability when using polynomial or combined models.

With the calculated scaled SOC values, OCV parameters for each model are obtained and then OCV of the battery is estimated. Computed OCV values are then plotted against SOC to obtain the OCV-SOC curve. *Table 3* shows the OCV parameters, ECM parameter $R_{0,h}$, and correlation coefficient estimated after calculating SOC using coulomb counting approach. *Figure - Figure 20* shows the OCV-SOC curve plotted with the computed OCV parameters, and its respective modelling error is plotted.

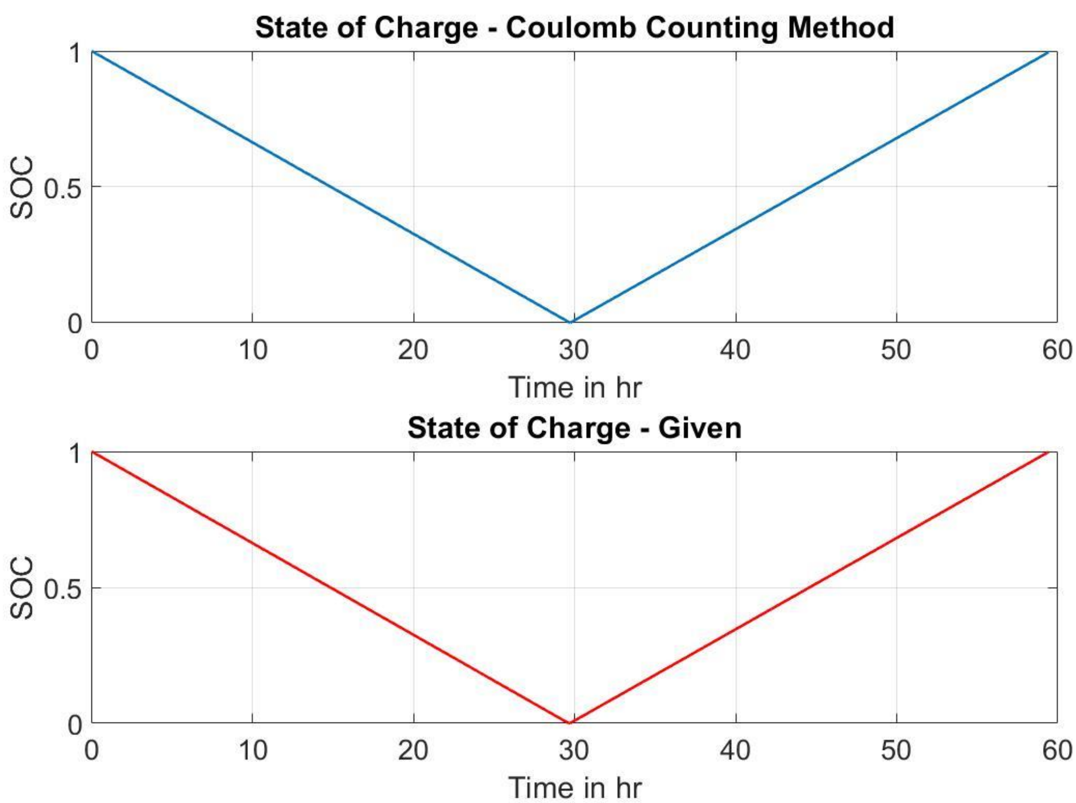

Figure 11: SOC calculated using Coulomb Counting Approach

Model	**OCV Parameter**	**ECM Parameter $R_{0,h}$**	**Correlation Coefficient R^2**	**Reference**
Linear	a_0= 3.58541 a_1= 0.544742	0.415305 ohms	0.937482	*Figure*
Polynomial	p_0 = -2239.36 p_1 = 4060.11 p_2 = -4837.07 p_3 = 3657.65 p_4 = -1597.77 p_5 = 309.152 P6 = 821.398 P7 = -196.117 P8 = 29.159 P9 = -2.43641 P10 = 0.0868672	0.427914 ohms	0.997321	*Figure*

Combined	k_0 = - 0.983941 k_1 = - 0.797388 k_2 = 7.0425 k_3 = - 4.47197 k_4 = 0.311966	0.422676 ohms	0.987469	*Figure*
Combined+3	k_0 = - 7.8494 k_1 = 92.8718 k_2 = - 16.35 k_3 = 1.84916 k_4 = - 0.0909629 k_5 = - 68.9488 k_6 = 127.295 k_7 = - 1.01428	0.428940 ohms	0.996907	*Figure*

Table 3: OCV Parameters, Internal Resistance, and Correlation Coefficient for SOC – Coulomb Counting Approach

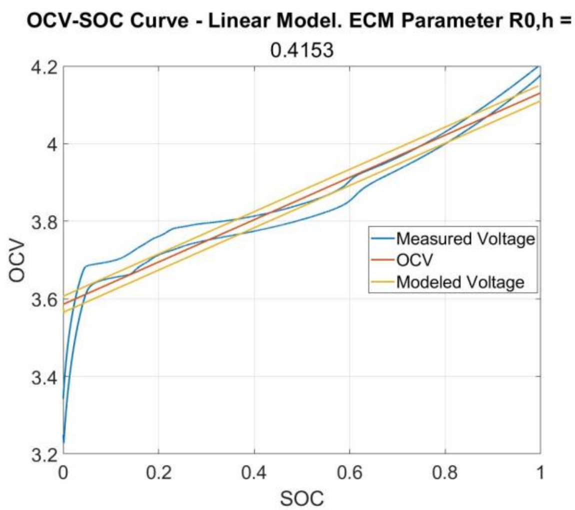

Figure 12: OCV-SOC Curve – Linear Model

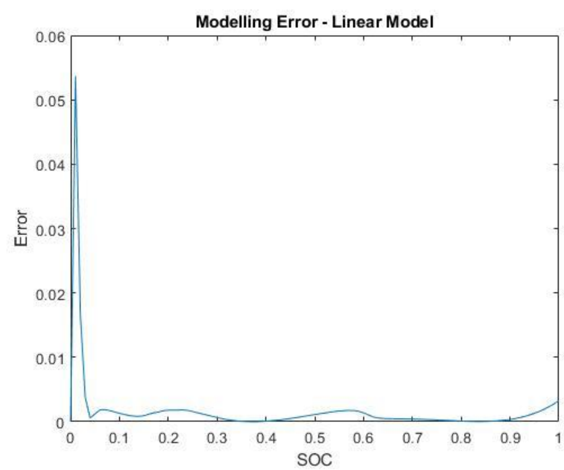

Figure 13: Modelling Error – Linear Model

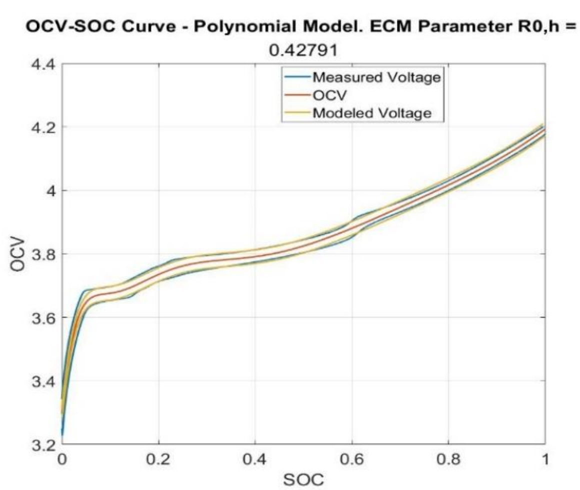

Figure 14 : OCV-SOC Curve – Polynomial Model

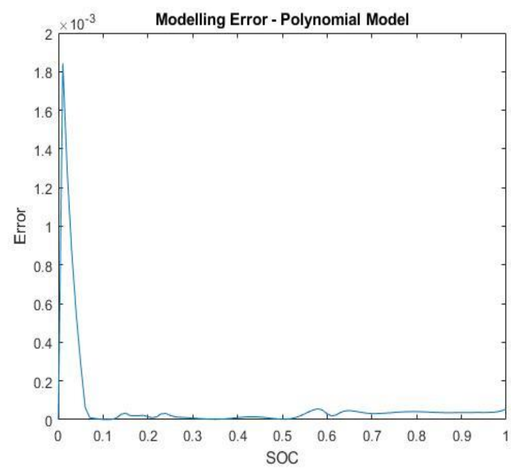

Figure 15: Modelling Error – Polynomial Model

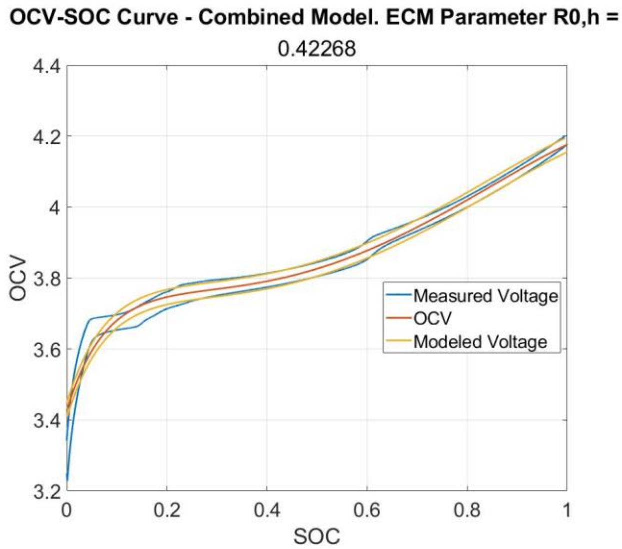

Figure 16: OCV-SOC Curve – Combined Model

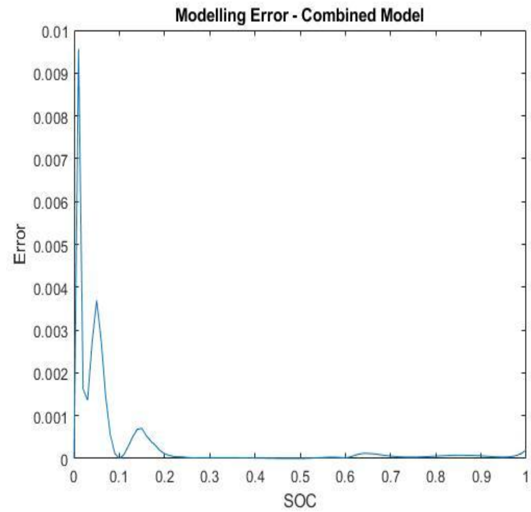

Figure 17: Modelling Error – Combined Model

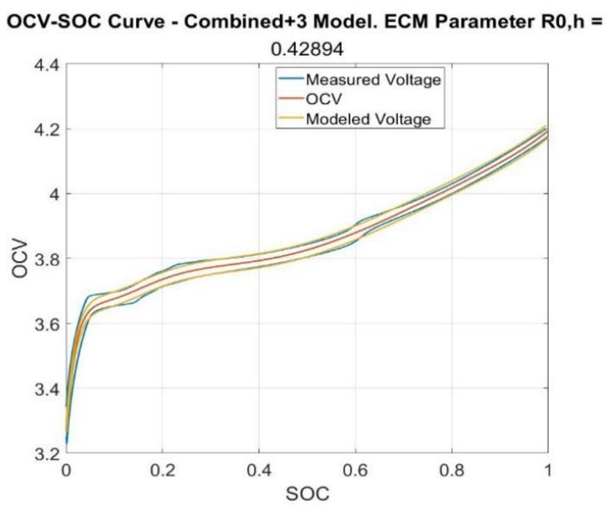

Figure 18: OCV-SOC Curve – Combined+3 Model

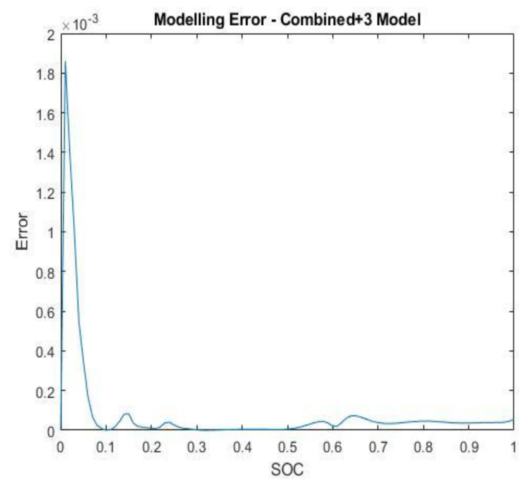

Figure 19: Modelling Error – Combined+3 Model