Boosting

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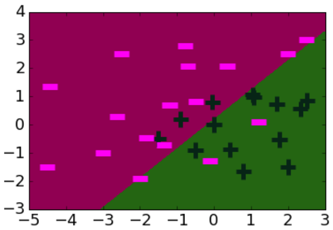
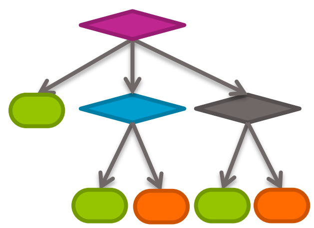
Simple (weak) classifiers are good!



Logistic regression w. simple features



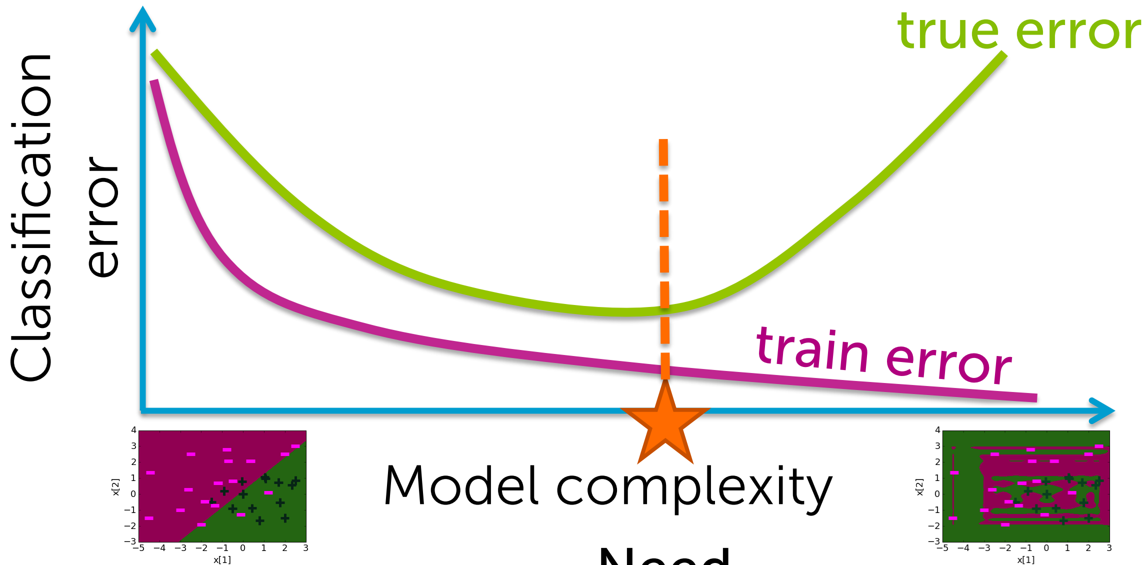
Shallow decision trees

Decision stumps

Low variance. Learning is fast!

But high bias…

Finding a classifier that's just right



Weak learner → Need stronger learner

Option 1: add more features or depth
Option 2: ?????

Boosting question

"Can a set of weak learners be combined to create a stronger learner?" *Kearns and Valiant (1988)*

↓

Yes! *Schapire (1990)*

↓

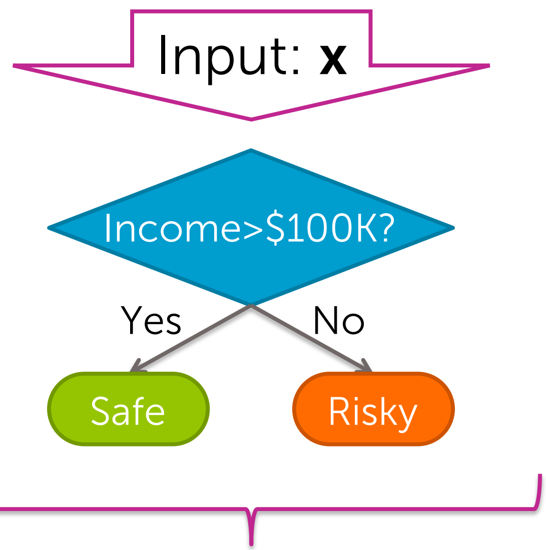
Boosting

↓

Amazing impact: • simple approach • widely used in industry • wins most Kaggle competitions

Ensemble classifier

A single classifier



Output: $\hat{y} = f(\mathbf{x})$

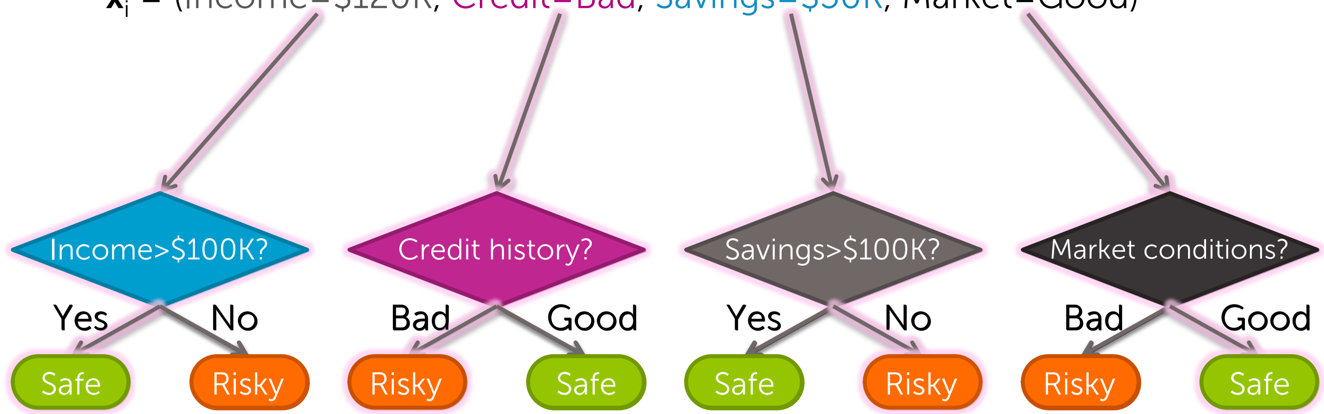
– Either +1 or -1

Classifier


Income>$100K?
Yes
No
Safe
Risky

Ensemble methods: Each classifier "votes" on prediction

$\mathbf{x}_i$ = (Income=\$120K, Credit=Bad, Savings=\$50K, Market=Good)



$f_1(\mathbf{x}_i) = +1$ $f_2(\mathbf{x}_i) = -1$ $f_3(\mathbf{x}_i) = -1$ $f_4(\mathbf{x}_i) = +1$

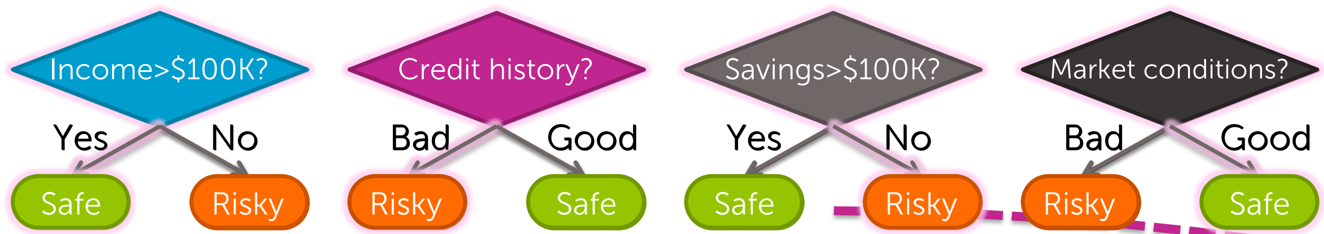
Combine?

Ensemble model

Learn coefficients

$$F(\mathbf{x}_i) = \text{sign}(w_1 f_1(\mathbf{x}_i) + w_2 f_2(\mathbf{x}_i) + w_3 f_3(\mathbf{x}_i) + w_4 f_4(\mathbf{x}_i))$$

Prediction with ensemble



$f_1(\mathbf{x}_i) = +1$ $f_2(\mathbf{x}_i) = -1$ $f_3(\mathbf{x}_i) = -1$ $f_4(\mathbf{x}_i) = +1$

Combine?

Ensemble model

Learn coefficients

$= \text{sign}(2 \times +1 + 1.5 \times -1 + 1.5 \times -1 + 0.5 \times +1)$

$$F(\mathbf{x}_i) = \text{sign}(w_1 f_1(\mathbf{x}_i) + w_2 f_2(\mathbf{x}_i) + w_3 f_3(\mathbf{x}_i) + w_4 f_4(\mathbf{x}_i))$$

$= \text{sign}(-0.5) \Rightarrow \hat{y}_i = -1$

w_1	2
w_2	1.5
w_3	1.5
w_4	0.5

Ensemble classifier in general

- Goal:
  - Predict output y
    - Either +1 or -1
  - From input **x**
- Learn ensemble model:
  - Classifiers: $f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_T(\mathbf{x})$
  - Coefficients: $\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_T$
- Prediction:

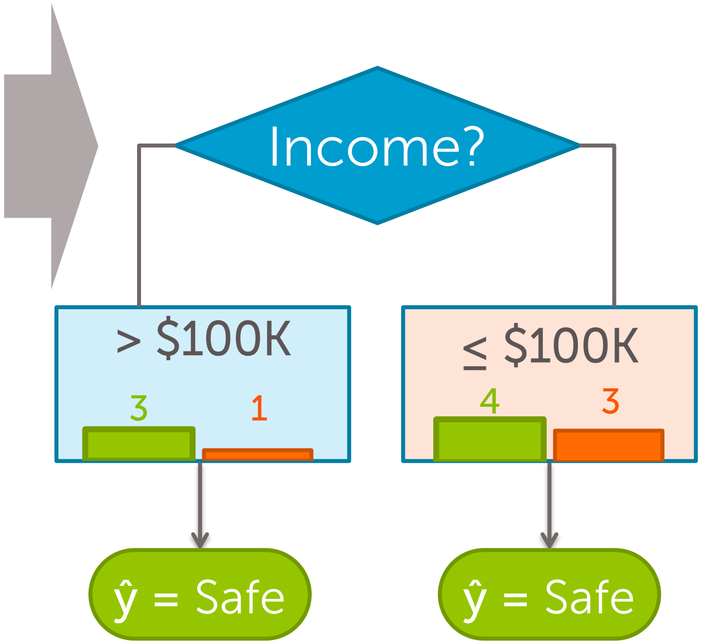
$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

Boosting

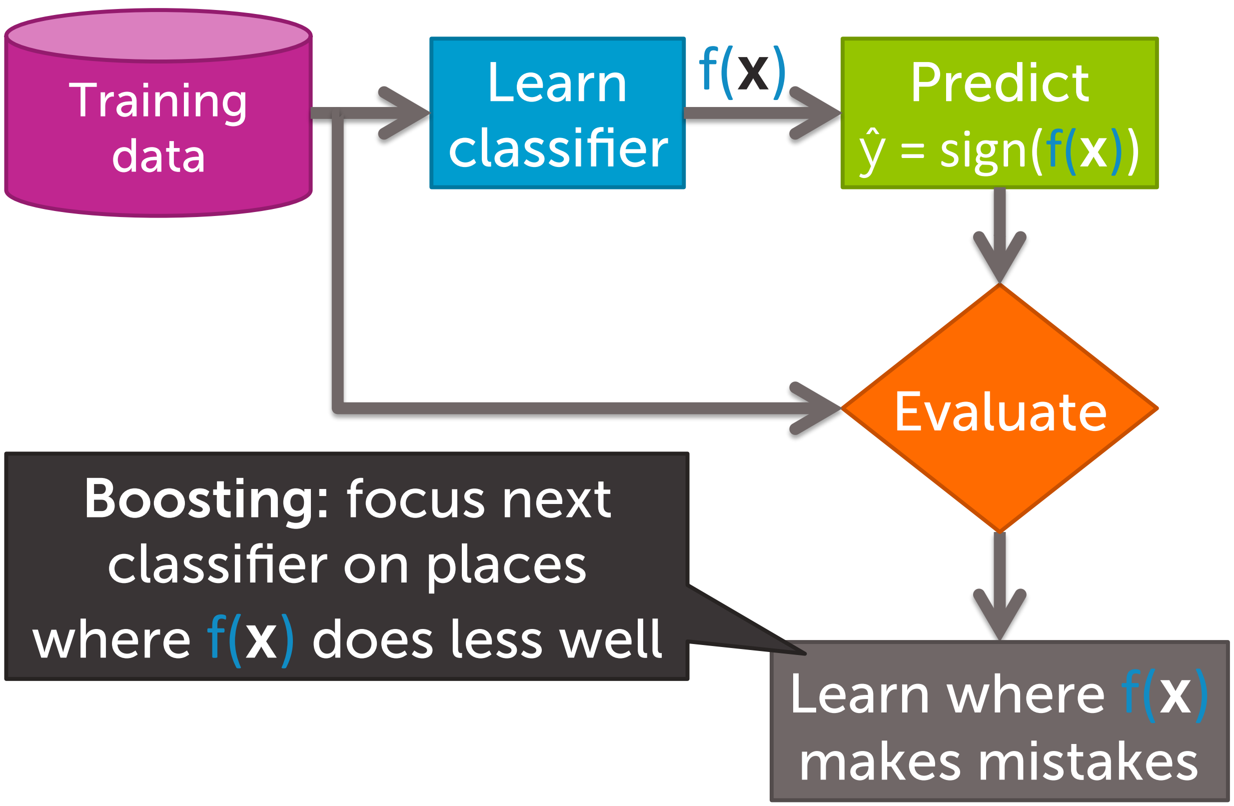
Training a classifier

Learning decision stump

Credit	Income	y
A	$130K	Safe
B	$80K	Risky
C	$110K	Risky
A	$110K	Safe
A	$90K	Safe
B	$120K	Safe
C	$30K	Risky
C	$60K	Risky
B	$95K	Safe
A	$60K	Safe
A	$98K	Safe



Boosting = Focus learning on "hard" points



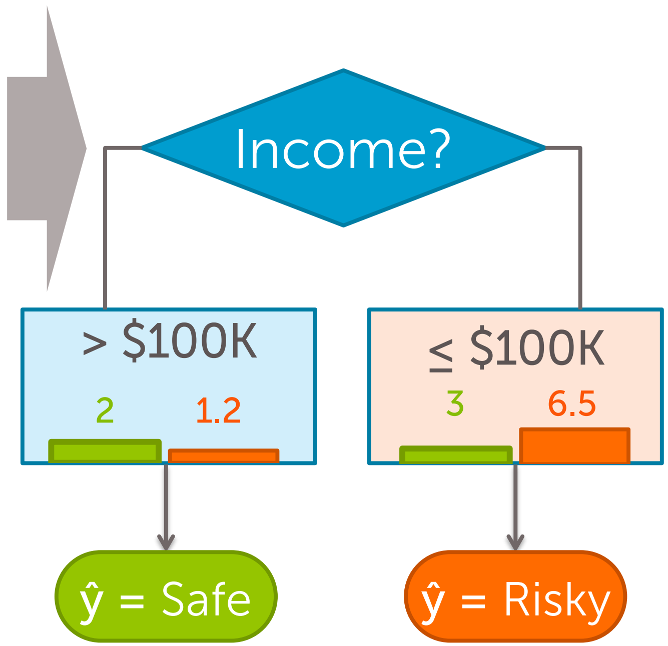
Learning on weighted data:
More weight on "hard" or more important points

- Weighted dataset:
  - Each $\mathbf{x}_i, y_i$ weighted by α_i
    - More important point = higher weight α_i
- Learning:
  - Data point j counts as α_i data points
    - E.g., $\alpha_i = 2$ ➔ count point twice

Learning a decision stump on weighted data

Increase weight α of harder/ misclassified points

Credit	Income	y	Weight α
A	$130K	Safe	0.5
B	$80K	Risky	1.5
C	$110K	Risky	1.2
A	$110K	Safe	0.8
A	$90K	Safe	0.6
B	$120K	Safe	0.7
C	$30K	Risky	3
C	$60K	Risky	2
B	$95K	Safe	0.8
A	$60K	Safe	0.7
A	$98K	Safe	0.9



Learning from weighted data in general

- Usually, learning from weighted data
  - Data point i counts as α_i data points

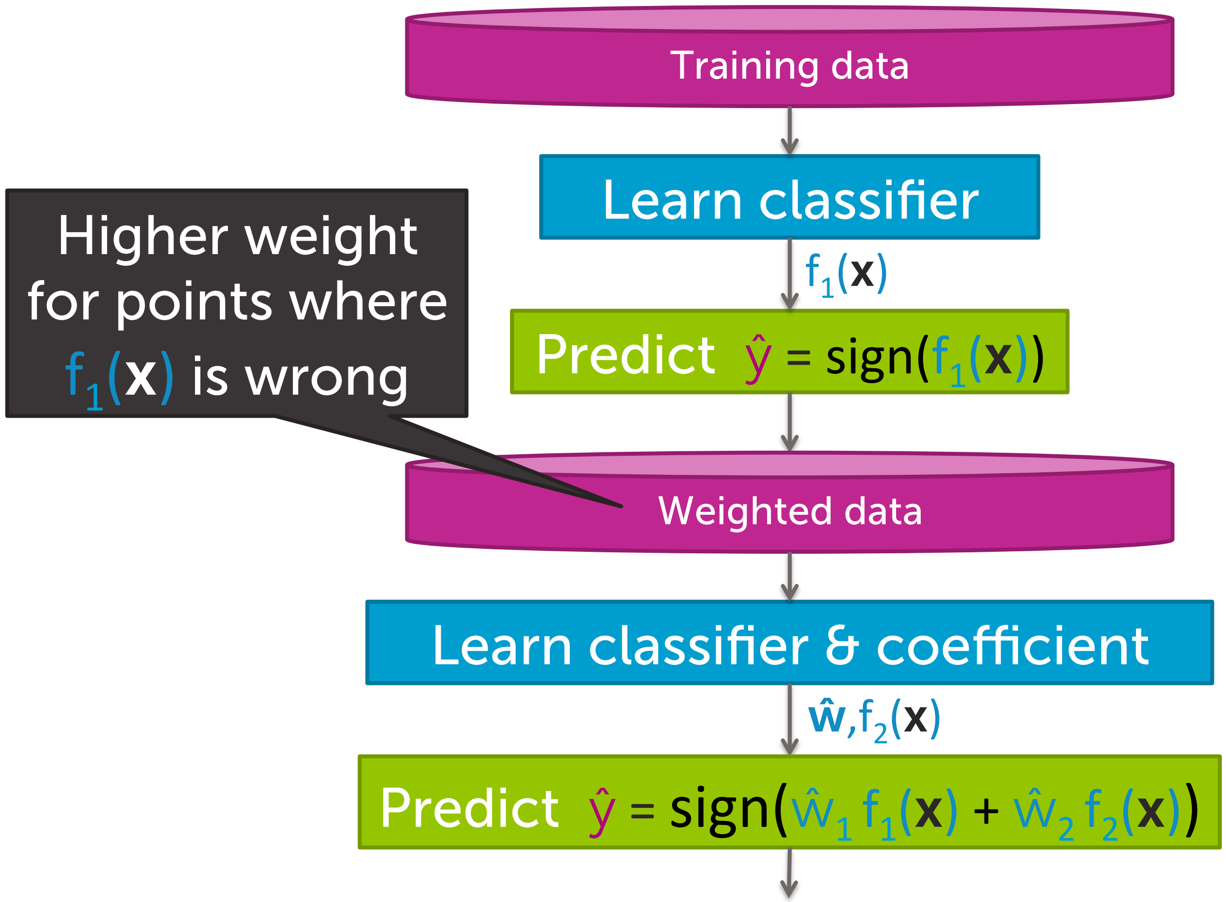
- E.g., gradient ascent for logistic regression:

Sum over data points

$$\mathbf{w}_j^{(t+1)} \leftarrow \mathbf{w}_j^{(t)} + \eta \sum_{i=1}^{N} \alpha_i h_j(\mathbf{x}_i)\Big(\mathbb{1}[y_i = +1] - P(y = +1 \mid \mathbf{x}_i, \mathbf{w}^{(t)})\Big)$$

Weigh each point by α_i

Boosting = Greedy learning ensembles from data



AdaBoost

AdaBoost: learning ensemble

[Freund & Schapire 1999]

- Start same weight for all points: $\alpha_i = 1/N$
- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$ with data weights α_i
  - Compute coefficient $\hat{w}_t$
  - Recompute weights α_i

Problem 1: How much do I trust f_t?

Problem 2: weigh mistakes more?

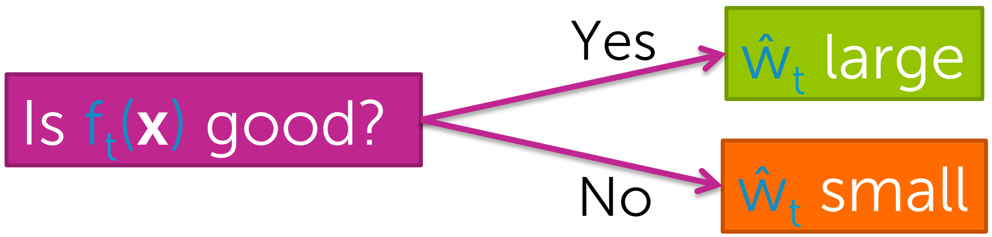
- Final model predicts by:

coefficients

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

Computing coefficient $\hat{w}_t$

AdaBoost: Computing coefficient $\hat{w}_t$ of classifier $f_t(\mathbf{x})$



- $f_t(\mathbf{x})$ is good ➔ f_t has low training error

- Measuring error in weighted data?
  - Just weighted # of misclassified points

Weighted classification error

Learned classifier

$\hat{y} = +$

Data point

(Sushi was great, +, α=1.2)

(Food was OK, –, α=0.5)

Correct!

Mistake!

Weight of correct	0 / 1.2
Weight of mistakes	0 / 0.5

Hide label

Weighted classification error

- Total weight of mistakes:

$$= \sum_{i=1}^{N} \alpha_i \underbrace{\mathbb{1}(\hat{y}_i \neq y_i)}_{\text{Mistake?}}$$

- Total weight of all points:

$$= \sum_{i=1}^{N} \alpha_i$$

- Weighted error measures fraction of weight of mistakes:

$$\boxed{\text{weighted\_error} = \frac{\text{Total weight of mistakes}}{\text{Total weight of all data points}}}$$

  – Best possible value is 0.0 → worst → 1.0 → Random classifier = 0.5

AdaBoost: Formula for computing coefficient $\hat{w}_t$ of classifier $f_t(\mathbf{x})$

$$\hat{w}_t = \underbrace{\frac{1}{2}\ln}\left(\underbrace{\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)}}\right)$$

Is $f_t(\mathbf{x})$ good?

Yes

No

weighted_error(f_t) on training data	$\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)}$	$\hat{w}_t$
0.01	$\frac{1-0.01}{0.01} = 99$	$\frac{1}{2}\ln 99 = 2.3$
0.5	$\frac{1-0.5}{0.5} = 1$	0
0.99	$\frac{1-0.99}{0.99} = 0.01$	-2.3

Terrible classifier, but $1-f_t$ is awesome !!

AdaBoost: learning ensemble

- Start same weight for all points: $\alpha_i = 1/N$

- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$ with data weights α_i
  - Compute coefficient $\hat{w}_t$
  - Recompute weights α_i

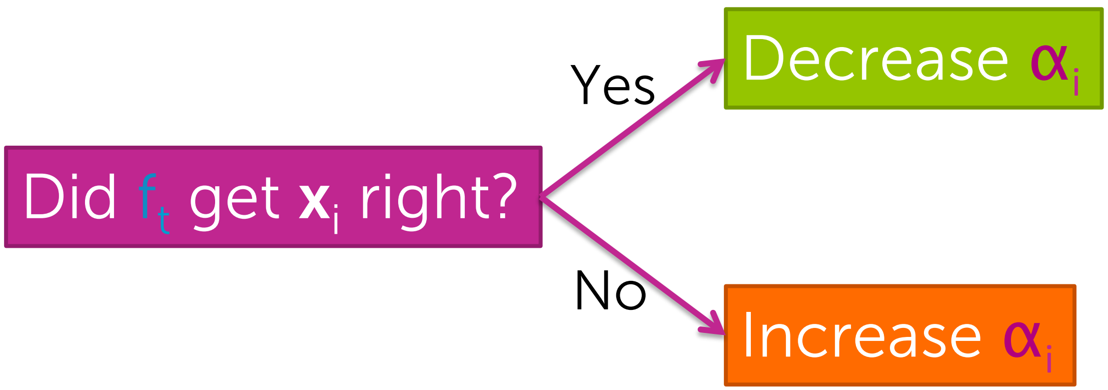
$$\hat{\mathbf{w}}_t = \frac{1}{2} \ln \left(\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)} \right)$$

- Final model predicts by:

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x}) \right)$$

Recompute weights α_i

AdaBoost: Updating weights α_i based on where classifier $f_t(\mathbf{x})$ makes mistakes



AdaBoost: Formula for updating weights α_i

$$\alpha_i \leftarrow \begin{cases} \alpha_i\, e^{-\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i)=y_i \leftarrow \text{correct} \\ \alpha_i\, e^{\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i)\neq y_i \leftarrow \text{mistake} \end{cases}$$

Did f_t get $\mathbf{x}_i$ right?

$f_t(\mathbf{x}_i)=y_i$?	$\hat{w}_t$	Multiply α_i by	Implication
Yes: Correct	2.3	$e^{-2.3} = 0.1$	Decrease importance of x_i, y_i
Yes: Correct	0	$e^{-0} = 1$	Keep importance the same
No: Mistake	2.3	$e^{2.3} = 9.98$	Increasing importance of x_i, y_i
No: Mistake	0	$e^{0} = 1$	Keep importance the same

AdaBoost: learning ensemble

- Start same weight for all points: $\alpha_i = 1/N$
- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$ with data weights α_i
  - Compute coefficient $\hat{w}_t$
  - Recompute weights α_i
- Final model predicts by:

$$\hat{\mathbf{w}}_t = \frac{1}{2}\ln\left(\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)}\right)$$

$$\alpha_i \leftarrow \begin{cases} \alpha_i e^{-\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i) = y_i \\ \alpha_i e^{\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i) \neq y_i \end{cases}$$

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

AdaBoost: Normalizing weights α_i

If $\mathbf{x}_i$ often mistake, weight α_i gets very **large**

If $\mathbf{x}_i$ often correct, weight α_i gets very **small**

Can cause numerical instability after many iterations

Normalize weights to add up to 1 after every iteration

$$\alpha_i \leftarrow \frac{\alpha_i}{\sum_{j=1}^{N} \alpha_j}$$

AdaBoost: learning ensemble

- Start same weight for all points: $\alpha_i = 1/N$
- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$ with data weights α_i
  - Compute coefficient $\hat{w}_t$
  - Recompute weights α_i
  - Normalize weights α_i
- Final model predicts by:

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

$$\hat{\mathbf{w}}_t = \frac{1}{2} \ln\left(\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)}\right)$$

$$\alpha_i \leftarrow \begin{cases} \alpha_i\, e^{-\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i)=y_i \\ \alpha_i\, e^{\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i)\neq y_i \end{cases}$$

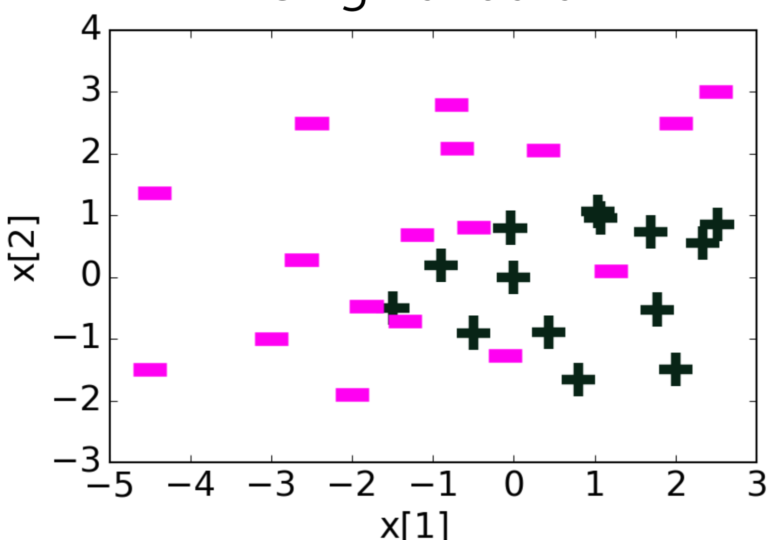
$$\alpha_i \leftarrow \frac{\alpha_i}{\sum_{j=1}^{N} \alpha_j}$$

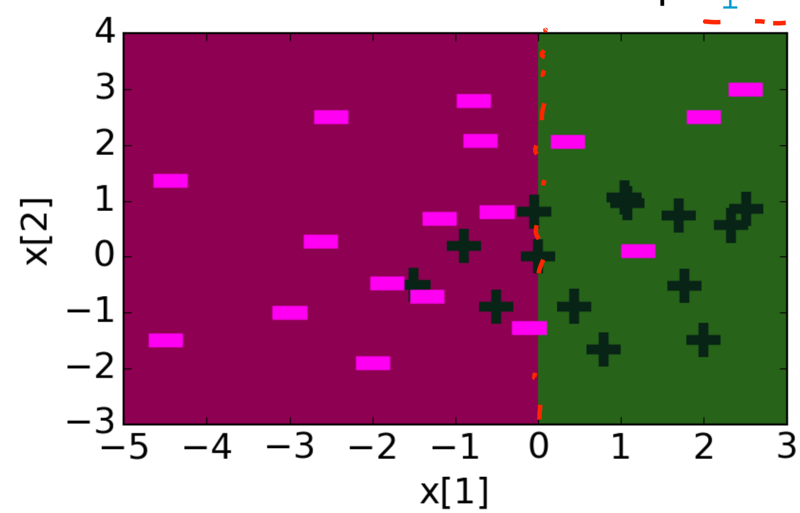
AdaBoost example

t=1: Just learn a classifier on original data

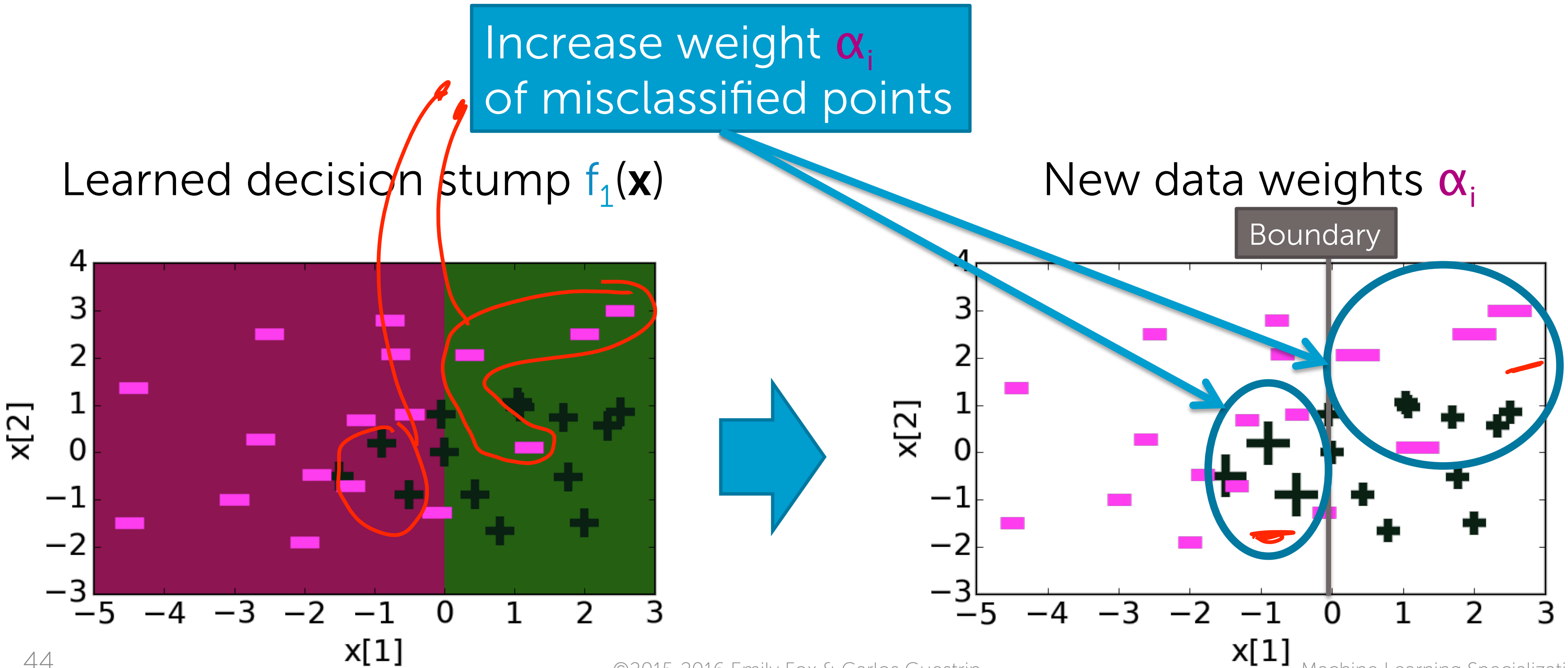
all points have same weight

standard learning



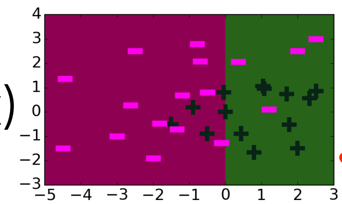


Updating weights α_i

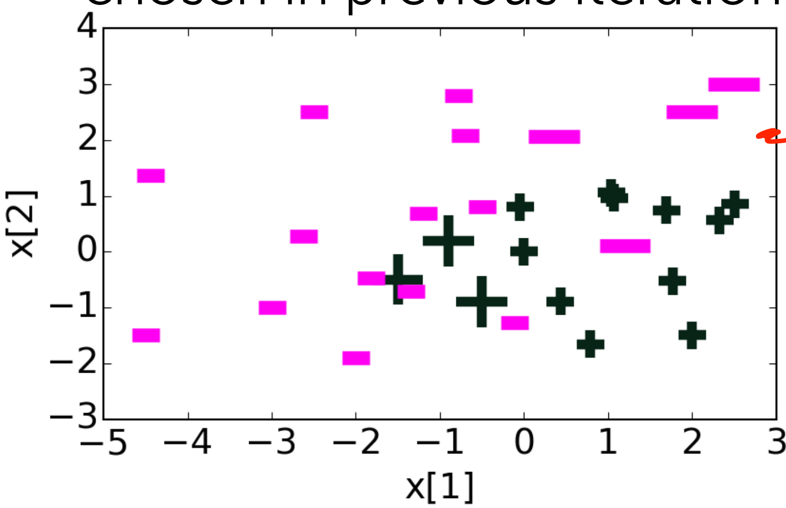


t=2: Learn classifier on weighted data

$f_1(\mathbf{x})$

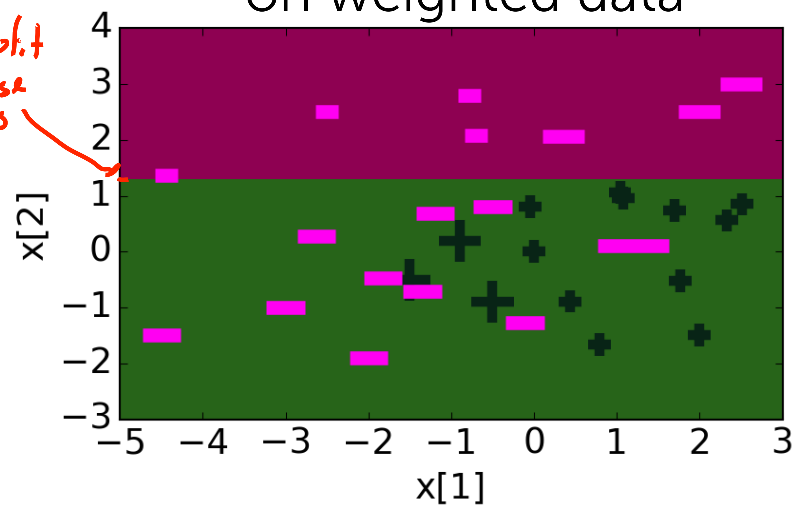


Weighted data: using α_i chosen in previous iteration

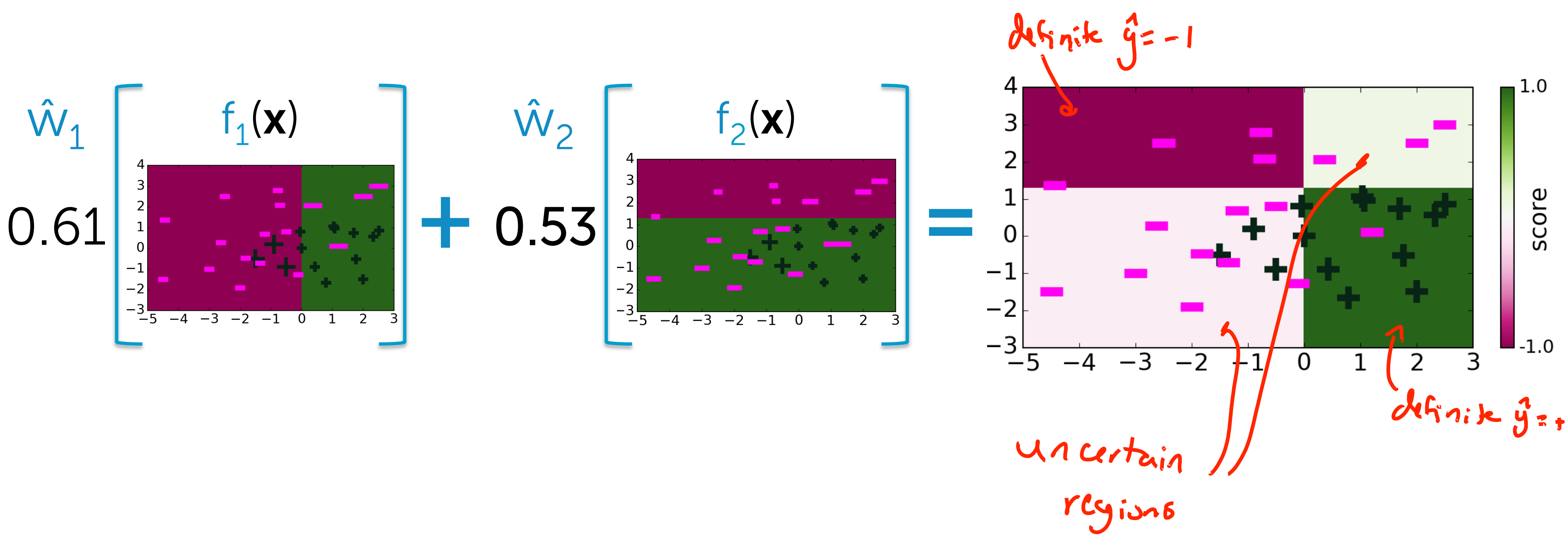


better split for these weights

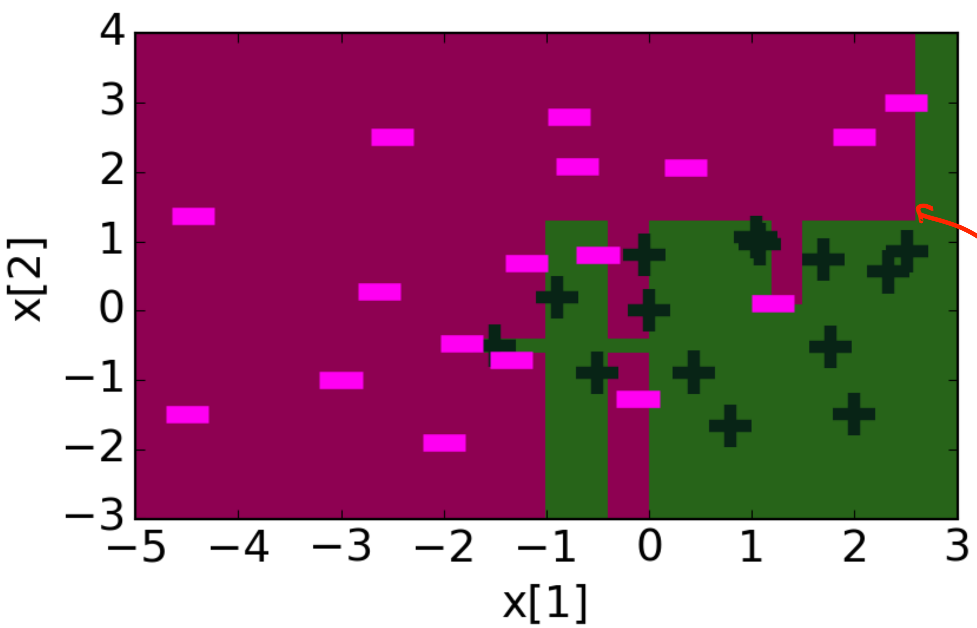
Learned decision stump $f_2(\mathbf{x})$ on weighted data



Ensemble becomes weighted sum of learned classifiers



Decision boundary of ensemble classifier after 30 iterations



Decision boundary is crazy!!

training_error = 0

Probably overfitting

AdaBoost summary

AdaBoost: learning ensemble

- Start same weight for all points: $\alpha_i = 1/N$

- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$ with data weights α_i
  - $\boxed{\text{Compute coefficient } \hat{w}_t}$
  - $\boxed{\text{Recompute weights } \alpha_i}$
  - $\boxed{\text{Normalize weights } \alpha_i}$

- Final model predicts by:

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

$$\boxed{\hat{\mathbf{w}}_t = \frac{1}{2} \ln\left(\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)}\right)}$$

$$\alpha_i \leftarrow \begin{cases} \alpha_i e^{-\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i) = y_i \\ \alpha_i e^{\hat{w}_t}, & \text{if } f_t(\mathbf{x}_i) \neq y_i \end{cases}$$

$$\boxed{\alpha_i \leftarrow \frac{\alpha_i}{\sum_{j=1}^{N} \alpha_j}}$$

Boosted decision stumps

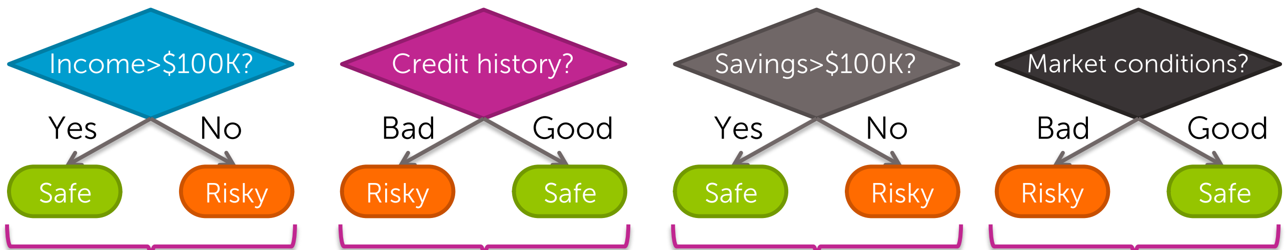
Boosted decision stumps

- Start same weight for all points: $\alpha_i = 1/N$
- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$: pick decision stump with lowest weighted training error according to α_i
  - Compute coefficient $\hat{w}_t$
  - Recompute weights α_i
  - Normalize weights α_i
- Final model predicts by:

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

Finding best next decision stump $f_t(\mathbf{x})$

Consider splitting on each feature:



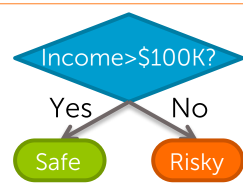
weighted_error = 0.2

weighted_error = 0.35

weighted_error = 0.3

weighted_error = 0.4

f_t =



$$\hat{w}_t = \frac{1}{2} \ln \left(\frac{1 - weighted\_error(f_t)}{weighted\_error(f_t)} \right) = 0.69$$

Boosted decision stumps

- Start same weight for all points: $\alpha_i = 1/N$
- For t = 1,…,T
  - Learn $f_t(\mathbf{x})$: pick decision stump with lowest weighted training error according to α_i
  - Compute coefficient $\hat{w}_t$
  - Recompute weights α_i
  - Normalize weights α_i
- Final model predicts by:

$$\hat{y} = sign\left(\sum_{t=1}^{T} \hat{\mathbf{w}}_t f_t(\mathbf{x})\right)$$

Updating weights α_i

$$\alpha_i \leftarrow \begin{cases} \alpha_i e^{-\hat{w}_t} = \alpha_i e^{-0.69} = \alpha_i/2, & \text{if } f_t(\mathbf{x}_i) = y_i \\ \alpha_i e^{\hat{w}_t} = \alpha_i e^{0.69} = 2\,\alpha_i, & \text{if } f_t(\mathbf{x}_i) \neq y_i \end{cases}$$

Credit	Income	y	ŷ	Previous weight α	New weight α
A	$130K	Safe	Safe	0.5	0.5/2 = 0.25
B	$80K	Risky	Risky	1.5	0.75
C	$110K	Risky	Safe	1.5	2 * 1.5 = 3
A	$110K	Safe	Safe	2	1
A	$90K	Safe	Risky	1	2
B	$120K	Safe	Safe	2.5	1.25
C	$30K	Risky	Risky	3	1.5
C	$60K	Risky	Risky	2	1
B	$95K	Safe	Risky	0.5	1
A	$60K	Safe	Risky	1	2
A	$98K	Safe	Risky	0.5	1

Boosting convergence & overfitting

Boosting question revisited

"Can a set of weak learners be combined to create a stronger learner?" *Kearns and Valiant (1988)*

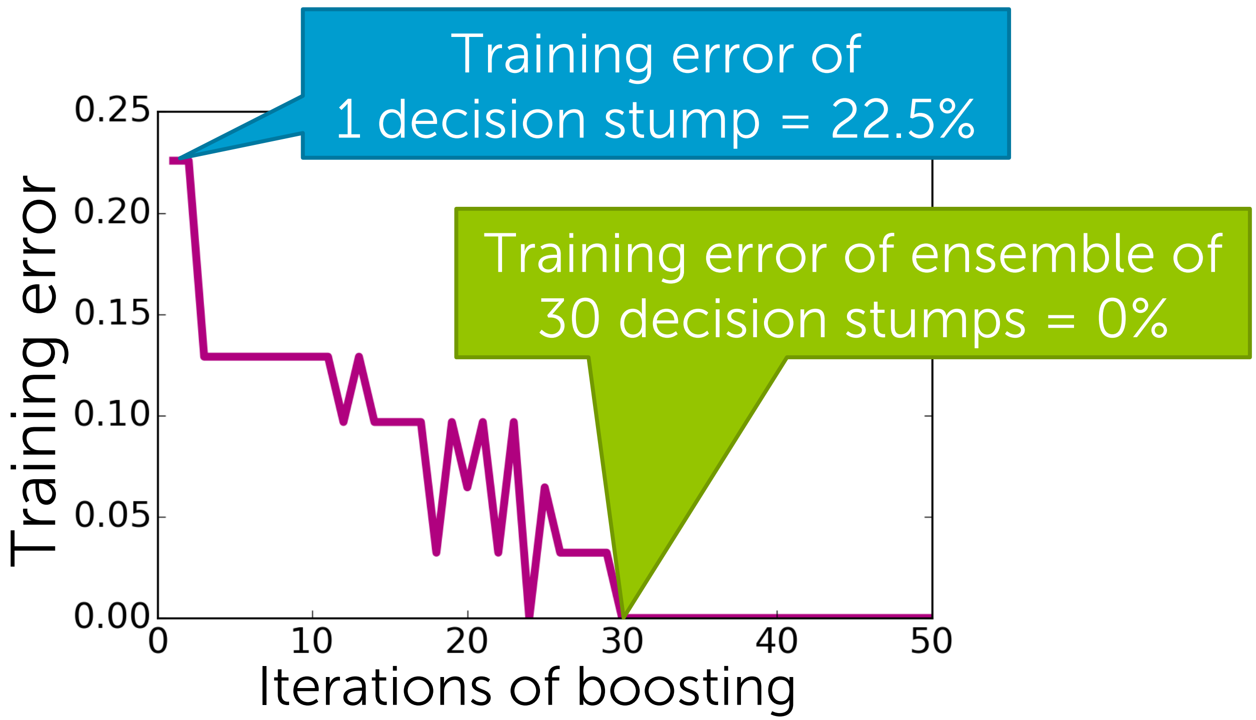
↓

Yes! *Schapire (1990)*

↓

Boosting

After some iterations, training error of boosting goes to zero!!!

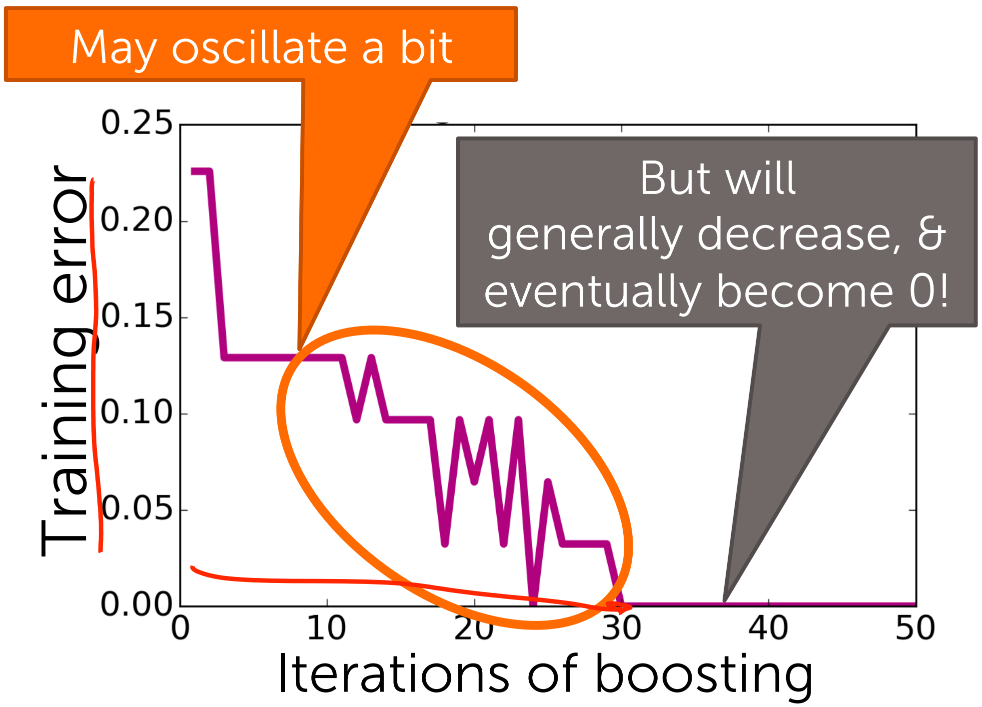


Boosted decision stumps on toy dataset

AdaBoost Theorem

Under some technical conditions…

Training error of boosted classifier → 0 as T→∞



Condition of AdaBoost Theorem

Under some technical conditions…

Training error of boosted classifier → 0 as $T \to \infty$

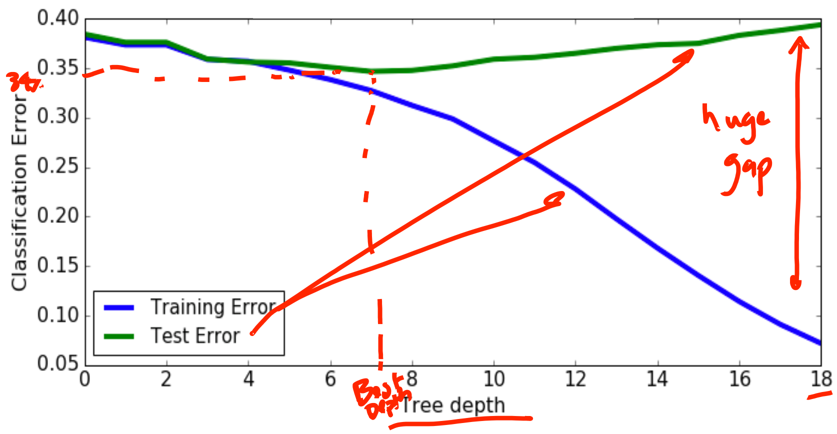
Condition = At every t, can find a weak learner with $\text{weighted\_error}(f_t) < 0.5$

Not always possible

Extreme example: No classifier can separate a +1 on top of -1

Nonetheless, boosting often yields great training error

Decision trees on loan data



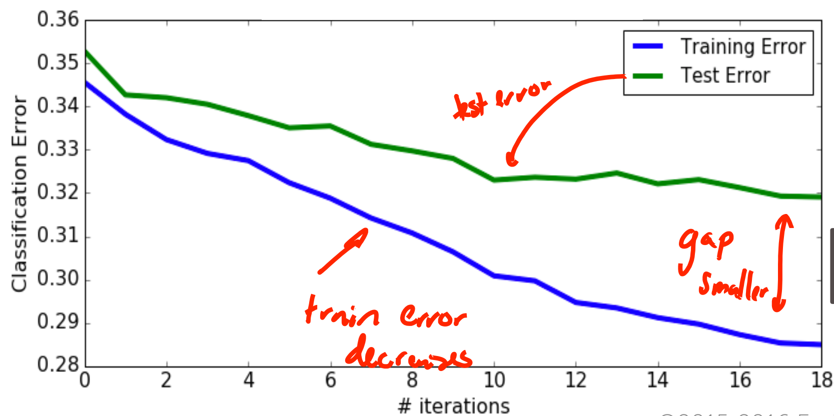
39% test error

Overfitting

8% training error

better

Boosted decision stumps on loan data

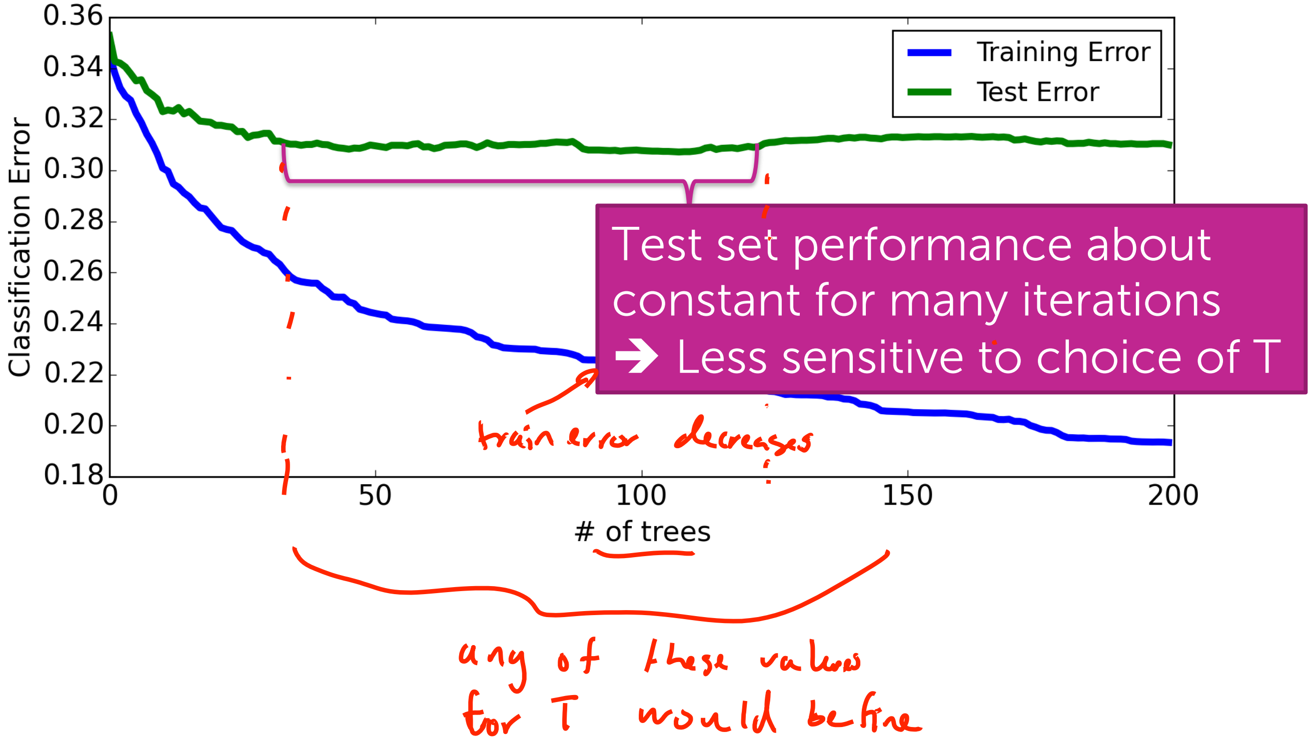


32% test error

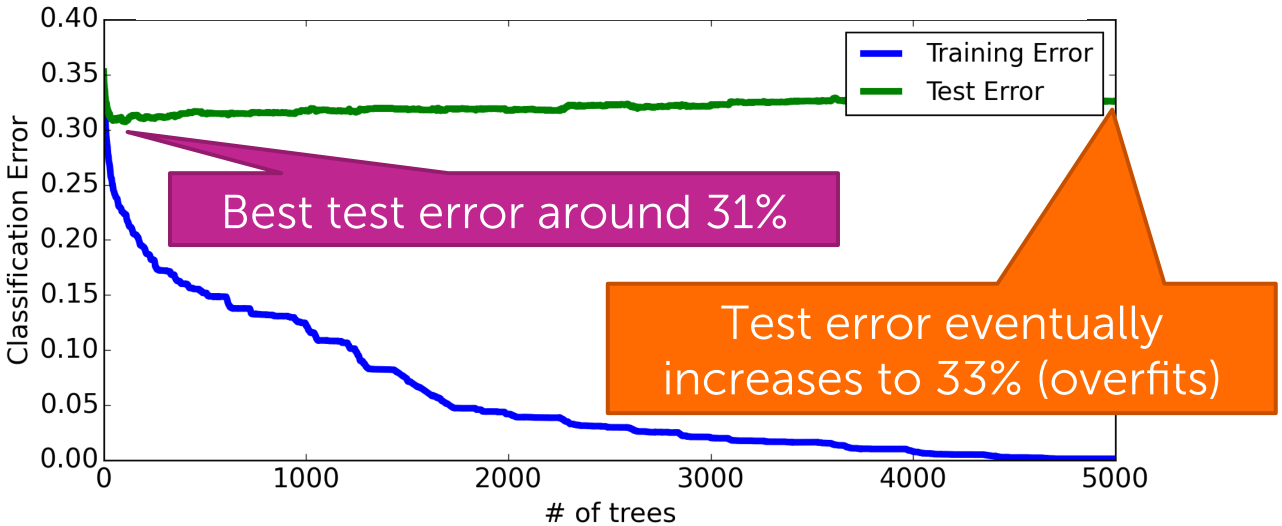
Better fit & lower test error

28.5% training error

Boosting tends to be robust to overfitting



But boosting will eventually overfit, so must choose max number of components T



How do we decide when to stop boosting?

Choosing T ?

Like selecting parameters in other ML approaches (e.g., λ in regularization)

- Not on training data
  Not useful: training error improves as T increases
- Never ever ever ever on test data
- Validation set
  If dataset is large
- Cross-validation
  For smaller data

Summary of boosting

Variants of boosting and related algorithms

There are hundreds of variants of boosting, most important:

Gradient boosting

- Like AdaBoost, but useful beyond basic classification

Many other approaches to learn ensembles, most important:

Random forests

- **Bagging**: Pick random subsets of the data
  - Learn a tree in each subset
  - Average predictions
- Simpler than boosting & easier to parallelize
- Typically higher error than boosting for same number of trees (# iterations T)

Impact of boosting
(spoiler alert... HUGE IMPACT)

Amongst most useful ML methods ever created

Extremely useful in computer vision

- Standard approach for face detection, for example

Used by most winners of ML competitions (Kaggle, KDD Cup,...)

- Malware classification, credit fraud detection, ads click through rate estimation, sales forecasting, ranking webpages for search, Higgs boson detection,...

Most deployed ML systems use model ensembles

- Coefficients chosen manually, with boosting, with bagging, or others

What you can do now…

- Identify notion ensemble classifiers
- Formalize ensembles as the weighted combination of simpler classifiers
- Outline the boosting framework – sequentially learn classifiers on weighted data
- Describe the AdaBoost algorithm
  - Learn each classifier on weighted data
  - Compute coefficient of classifier
  - Recompute data weights
  - Normalize weights
- Implement AdaBoost to create an ensemble of decision stumps
- Discuss convergence properties of AdaBoost & how to pick the maximum number of iterations T