Summer Term 2015 Problem Set 3 30.04.2015

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## **Algorithms for Programming Contests**

This problem set is due by

Thursday, 07.05.2015, 6:00 a.m.

Try to solve all the problems and submit them at

http://judge.informatik.tu-muenchen.de/

This week's problems are:

SS15N03A	Hiking	2
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The following amount of points will be awarded for solving the problems.

Problem	SS15N03A	SS15N03B	SS15N03C	SS15N03D	SS15N03E
Difficulty	easy	easy	medium	medium	hard
Points	4	4	6	6	8

If the judge does not accept your solution but you are sure you solved it correctly, use the "request clarification" option. In your request, include:

- the name of the problem (by selecting it in the subject field)
- a verbose description of your approach to solve the problem
- the time you submitted the solution we should judge

We will check your submission and award you half the points if there is only a minor flaw in your code.

If you have any questions please ask by using the judge's clarification form.

## SS15N03A Hiking

Author: Philipp Hoffmann

Lea enjoys nature a lot, therefore she often goes hiking at the weekend. Last Sunday, she got up early, drove to the foot of a mountain and reached the top just at the right time for lunch. During her dessert, "Apfelstrudel", she suddenly remembered: She had a very important appointment this afternoon which she was about to miss. In a big hurry, she looked at the map to figure out the fastest way to her car. To her amazement, there were hundreds of walking tracks which crossed multiple times forming thousands of possible routes down! Lea was helpless. Luckily she had her satellite phone with her and called... you! After giving you the list of all the tracks she wants to know how far away from her car she currently is. Help her out!

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with two integers n and m where n is the number of intersections, those are numbered from 1 to n, and m is the number of walking trails, m lines follow. The i-th line contains three integers  $v_i$ ,  $w_i$  and  $c_i$ .  $v_i$  and  $w_i$  each denote an intersection of walking trails,  $c_i$  is the length of a walking track connecting those intersections. Walking trails are undirected. Lea is currently at intersection 1, her car at intersection n.

### Output

For each test case, output one line containing "Case #i: d" where i is its number, starting at 1, and d is the shortest distance of intersection 1 to intersection n. Each line of the output should end with a line break.

#### Constraints

- $1 \le t \le 20$
- $1 \le n \le 1000$
- 1 < m < 50000
- $1 \le c_i \le 1000$  for all  $1 \le i \le m$
- $1 \le v_i, w_i \le n$  for all  $1 \le i \le m$

# Sample Data

## Input

2	
3 2	
1 2	1
2 3	2
3 3	
1 2	1
1 3	3
2 3	1
	3 2 1 2 2 3 3 3 1 2 1 3

```
1 Case #1: 3
2 Case #2: 2
```

### SS15N03B Party Planning

Author: Chris Pinkau

Lea has a lot of things to do. Her birthday is coming up and she wants to host a big party. There are cakes to be baked, decoration to be set up, invitations to be sent, drinks to be cooled, and so on. As Lea likes to plan ahead and dislikes stress very much, she wants to find a perfect schedule to finish all her prep work as quickly and as calmly as possible. Luckily for her, she has lots of close friends (as much as she needs) that propose to help her set up the perfect birthday party. After all, they are the guests and want to enjoy the party as much as Lea does.

Unfortunately, not all needed tasks can be done simultaneously: For example, Lea cannot bake a cake unless all ingredients have been bought before, she cannot decorate the room unless it has been cleaned, and she cannot buy the right amount of drinks until she has checked all the answers on her RSVP cards. But from many years of experience she knows some things. She knows the exact amount of time needed to finish any certain task, and she knows the dependencies of all the needed tasks, i.e. which task has to be done before another can be started. Furthermore, she knows that the first task has to be to write a checklist, and the last task to be completed is to cross off all items on it. But as Lea's friends are not nearly as organized as she is, she needs to present them with an exact schedule so that everyone knows what he or she has to do. Can you help Lea to prepare her party as quickly as possible?

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a single line containing an integer n, which denotes the number of tasks (w.l.o.g. numbered from 1 to n). n lines follow. The i-th line consists of a sequence of integers  $p_i$ ,  $s_i$ , and  $j_{i,1}, \ldots, j_{i,s_i}$ , separated by spaces.  $p_i$  denotes the number of time units needed to finish task i,  $s_i$  is the number of tasks that are direct successors of task i, i.e. tasks that need task i to be finished before they can be started. The sequence  $j_{i,1}, \ldots, j_{i,s_i}$  lists all direct successors of task i, in no specific order.

The first task to be done (the source) is task 1, it is a predecessor of all other tasks; the last task (the sink) is task n, it is a successor of all other tasks.

### Output

For each test case, output one line containing "Case #i:" where i is its number, starting at 1, and the total number of time units needed for the prep work.

#### Constraints

•  $1 \le t \le 20$ 

- $1 \le n \le 1000$
- $1 \le p_i \le 1000$  for all  $1 \le i \le n$
- $0 \le s_i \le n-1$  for all  $1 \le i \le n$
- $1 \le j_{i,k} \le n$  for all  $1 \le i \le n$  and  $1 \le k \le s_i$

## Sample Data

### Input

#### 3 1 2 4 3 5 3 2 3 4 4 3 1 4 5 4 1 4 2 0 6 7 8 5 9 6 4 2 3 4 5 10 7 3 3 4 5 11 3 2 4 5 12 2 1 5 2 0 13 14 2 15 16 3 1 2 17 | 1 0

```
1 Case #1: 11
2 Case #2: 20
3 Case #3: 4
```

### SS15N03C Snakes and Ladders

Author: Stefan Toman

Lea's favourite board game is "Snakes and Ladders", a simple but funny game. She wins most games when she plays with her family, but when she plays it with her neighbours, Lea always loses. Lea is sure that they are cheating since they always use separate dice for each player. This time, she wants revenge: Lea plans to take a manipulated die with her and win for the first time against the neighbours. She is able to manipulate her die in a way that it always shows the same number. But which number should she choose?

To help her, you will need some more information about the game. "Snakes and Ladders" is played on a board with several fields forming a long queue. All of them are labelled with an integer from 1 to n in order. When the game begins, everybody puts his piece on field 1. In each round every player rolls a six-sided die and moves his piece the according number of fields. It is possible that several pieces are on the same field.

To make it more interesting, there are snakes (most times pointing downwards) and ladders (most times pointing upwards) drawn on the board. If some piece steps on a field with the head of a snake, it will move to the snake's tail. On the other hand, if it steps on a piece with the beginning of a ladder, it may go to the field where the ladder ends. All players are allowed to choose whether they want to use ladders, but everybody must use snakes. Note that snakes and ladders are directed and that a player may use several of them in one turn.

The first one to reach field n wins (even if the move might need to be continued due to snakes). If some player has a higher number than needed to reach field n he wins, too.

Tell Lea which number her die should always show to win the game as fast as possible. Lea will always choose whether to use ladders or not in an optimal way. If there are several numbers with the same speed print all of them.

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a single line containing three integers n, s and l. n is the number of fields on the board, s the number of snakes and l the number of ladders. s lines follow: the i-th line contains two integers  $a_i$  and  $b_i$  describing a snake having its head at field  $a_i$  and its tail at field  $b_i$ . Similarly, l lines follow them, line j containing two integers  $c_j$  and  $d_j$  describing a ladder starting at field  $c_j$  and ending at field  $d_j$ .

### Output

For each test case, print a line containing "Case #i: x" where i is its number, starting at 1, and x is the number which lets Lea finish the game as fast as possible. If there are several fastest numbers, print each of them in increasing order separated by spaces. If Lea can't finish the game either way, print "impossible".

### Constraints

- $1 \le t \le 20$
- $1 \le n \le 10000$
- $0 \le s, l \le n$
- $1 < a_i, b_i < n$  for all  $1 \le i \le s$
- $1 < c_j, d_j < n$  for all  $1 \le j \le l$
- On each field, at most one snake or one ladder starts, but not both.

## Sample Data

### Input

```
1 3
2 100 0 0
3 4 13 0 1
5 2 12
6 7 200 1 0
8 121 61
```

```
1 Case #1: 6
2 Case #2: 1 6
3 Case #3: impossible
```

## SS15N03D Supermarkets

Author: Stefan Toman

It was a long time ago that Lea last saw Peter. She got to know him at school, but now she has not seen him for years. One day she met Peter by chance and he invited Lea to visit him at his new home.

A few days later when Lea wants to leave by car, she suddenly remembers that she forgot to buy a gift. Therefore, she decides to buy a bottle of wine at some supermarket on her way to Peter. She wants to be on time, so the extra way and time needed to buy the wine should be as short as possible. Some of the supermarkets are huge malls where she would need a lot of time to get her wine, some are known for long waiting times and others are very small and perfect for getting just one item. Lea knows the lengths of all roads, the locations of all supermarkets and the time she would need to buy the wine in each store. Where should she buy the wine to reach Peter as fast as possible?

### Input

The first line of the input contains an integer t. t test cases follow, each of them separated by a blank line.

Each test case starts with a single line containing five integers n, m, s, a and b. n is the number of cities (labelled city 1 to city n), m is the number of roads and s is the number of supermarkets. Lea lives in city a whereas Peter lives in city b.

Next, there are m lines describing the roads. The i-th line contains three integers  $x_i$ ,  $y_i$  and  $z_i$  and implies that there is a road between city  $x_i$  and city  $y_i$  (which may be used in both directions) for which Lea will need  $z_i$  minutes. s lines follow describing the supermarkets. The j-th line contains two integers  $c_j$  and  $w_j$  describing a supermarket in city  $c_j$  where Lea will need  $w_j$  minutes to buy the wine. Note that there may be multiple roads between cities as well as multiple supermarkets per city.

### Output

For each test case, print a line containing "Case #i: x" where i is its number, starting at 1, and x is the time she needs to go to Peter formatted as "hours:minutes", for instance "5:23" (add leading zeros to the number of minutes if needed) or "impossible" if there is no way to Peter's house.

#### Constraints

- 1 < t < 20
- 1 < n < 10000
- $0 < m < n^2$

- $1 \le a \le n$
- $1 \le b \le n$
- $0 \le s \le n$
- $1 \le x_i, y_i \le n$  for all  $1 \le i \le m$
- $1 \le z_i \le 100$  for all  $1 \le i \le m$
- $1 \le c_j \le n$  for all  $1 \le j \le s$
- $1 \le w_i \le 1000$  for all  $1 \le j \le s$

## Sample Data

### Input

```
1 2
2 2 1 2 1 2
3 1 2 30
4 1 15
5 2 20
6
7 2 1 0 1 2
8 1 2 30
```

```
1 Case #1: 0:45
2 Case #2: impossible
```

#### SS15N03E Escher Stairs

Author: Christian Müller

Recently, Lea went to an art exhibition with many interesting pictures. She especially liked one part of the exhibition that dealt with non-euclidean geometry, for example buildings that can not be built in the real world. A famous example of this is "Relativity" by M. C. Escher.



Figure 1: M. C. Escher's "Relativity"

There, Lea found many pictures of buildings with stairs. For fun, she tried to imagine if this particular building could be built in the real world. She checked this by counting the amount of steps of every flight of stairs, positive for going up, negative for going down. Then she chose a random starting point and tried to get back to the same point. If she could find a sequence of stairs that led her back where the amount of steps did not equal 0, she could be sure that the building could not be built (assuming all steps are of equal height). Otherwise, Lea is certain that some genious architect will find a way to construct such a building.

Lea easily got lost in the picture while counting. Can you tell her if the building could be real?

### Input

The first line of the input contains an integer t. t test cases follow.

Each test case begins with a line containing three integers n m s, where n is the amount of places (indexed from 1 to n), m is the amount of connecting flights of stairs and s is the point Lea chose to start in. m lines follow. The i-th line consists of three integers  $a_i$ ,  $b_i$ ,  $c_i$  separated by spaces, meaning that there is a flight of stairs from place  $a_i$  to  $b_i$  with  $c_i$  steps. All flights of stairs can be used in both directions, but are only given going

upward, i.e. to go from  $a_i$  to  $b_i$  you would go  $c_i$  steps up, and to go from  $b_i$  from  $a_i$  you would go  $c_i$  steps down.

### Output

For each test case, print a line containing "Case #i: possible" if there is no path from s to s such that the sum of steps is different from 0. Otherwise, print "Case #i: k", where k is the minimal amount of flights of stairs leading back to s with a step-sum different from 0.

#### Constraints

- $1 \le t \le 20$
- $1 \le n \le 2000$
- $0 \le m \le 3 \cdot 10^5$
- $1 \le a_i, b_i \le n$  for all  $1 \le i \le m$
- $0 \le c_i \le 5000$  for all  $1 \le i \le m$
- The graph is connected.

### Sample Data

#### Input

#### 2 1 2 3 3 1 3 1 2 1 4 2 3 1 5 1 3 2 6 7 4 5 2 8 1 2 1 9 2 3 1 10 3 4 1 11 1 3 2 12 1 4 2

#### Output

Case #1: possible
Case #2: 4