KALINGA INSTITUTE OF INDUSTRIAL TECHNOLOGY (KIIT)

(Deemed to be University)

DEPARTMENT OF MATHEMATICS,SAS

Mathematics-II[MA-1004], Quiz Test-1, Session-2020-21

SEC-B22

Sec- A is Compulsory and attain any one questions from Sec-B

Section-A

Time-1 hr **FM-10**

1. What is the radius of convergence of the series $\sum_{n=0}^{\infty}\frac{(-1)^n(x-1)^{3n}}{8^n}$?
   - A. 8
   - B. 3
   - C. 2
   - D. 4

2. If $P_m(x)$ is Legendre's polynomial then the value of $P_{100}(-1)$ is
   - A. 0
   - B. -1
   - C. 1
   - D. None of the above

3. The Bessel function $J_{\frac{1}{2}}(\frac{\pi}{2})$ is

   A. 0 B. 1 C. 2 D .5

4. What are the roots of the indicial equation of $(x^2-x)y'' - xy' + y = 0$?
   - A. 0 and ½
   - B. 1 and -1
   - C. 0 and 2
   - D. 0 and 1

5. What is one of the solutions of $(1-x^2)y'' - 2xy' + 6y = 0$?
   - A. $\frac{1}{2}(5x^3-3x)$
   - B. $\frac{1}{2}(3x^2-1)$
   - C. $\frac{1}{2}(3x^2-x)$
   - D. $\frac{1}{2}(5x^2-1)$

6. What is the interval of convergence of the series $\sum_{m=0}^{\infty}\frac{(-1)^m(x-1)^{2m}}{4^m}$?

   A) $-1 < x < 3$

   B) $1 < x < 4$

   C) $0 < x < 2$

   D) $-1 < x < 2$

7. If $P_m(x)$ is Legendre's polynomials then the value of $\int_{-1}^{1} P_7(x)P_9(x)\,dx$ is

   A) 0

   B) 1

C) $\frac{2}{16}$

D) $\frac{3}{16}$

8. Which one of the following is the perfect replacement of $P_2(x)$ in terms of $P_0(x)$& $P_1(x)$ if $P_m(x)$ is the Legendre's polynomials.

A) $\frac{5}{2}xP_1(x) - \frac{1}{2}P_0(x)$

B) $\frac{5}{2}xP_1(x) + \frac{1}{2}P_0(x)$

C) $\frac{5}{2}xP_1(x) - \frac{3}{2}P_0(x)$

D) $\frac{5}{2}xP_1(x) + \frac{1}{2}P_0(x)$

9. If $J_n(x)$ is the Bessel's function of the first kind of order n then what is $\frac{d}{dx}(-x^{-3}J_3(x))$?

A) $x^{-3}J_4(x)$

B) $x^3J_2(x)$

C) $x^2J_2(x)$

D) $-x^{-3}J_4(x)$

10. What is the value of the integral $\int_0^\infty e^{-t^2}t^2dt$?

A) $\frac{\sqrt{\pi}}{4}$

B) $\frac{\sqrt{\pi}}{2}$

C) π

D) $\frac{\pi}{2}$

11. Evaluate $\beta(3,4)$.

12. prove that for any real number n, $\gamma(n) = (n-1)\gamma(n-1)$.

13. Find first two terms of $J_0(x)$.

14. Express $5x^2 - 2x + 3$ as Legendre's polynomial.

Section-B

2. Find $P_4(0)$.

3. Find the solution of $(a^2 - x^2)y'' - 2xy' + 12y = 0, a \neq 0$.

4. Reduce the following equation into Bessel's equation using the substitution x=4z and find at least one of the solutions $x^2y'' + xy' + \frac{1}{16}(x^2-1)y = 0$.

5. Using Frobenius method find at least one of the solutions of the following equation $xy'' + (2x+1)y' + (x+1)y = 0$.

6. Show that $\frac{d}{dx}[x^nJ_n] = x^nJ_{n-1}(x)$

7. Find $J_{\frac{3}{2}}(x)$. **Or** Evaluate $\int J_3\,(x)dx$.

8. State and prove Rodrigues's formula.

9 Prove that the Bessel function $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.