Modeling the Spread of Rumors in Networks

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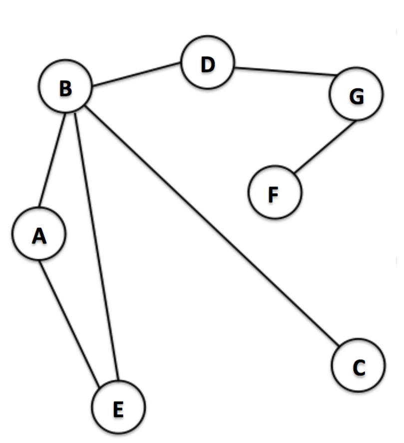
1. Introduction

A rumor is a piece of information or a story that is passed from person to person but has not been proven to be true. In this poster, we examine the propagation of rumors over network structures. We create a rumor spreading model by bridging concepts from graph theory, network theory, and epidemic modeling. These models are incorporated in three complex networks: Complete Graph, Small World, and Preferential Attachment. Then, we conduct numerical simulations to study the speed, intensity, duration, and extent of the spreading rumor through each structure. Finally, we compare the results and pinpoint the underlying characteristics of each network.

2. Network Theory

- Used to study complex interacting systems that can be represented by graphs.
- A *graph* is defined as $G = (V(G), E(G))$, where $V(G)$ are set of objects within graph called *nodes* and $E(G)$ are the links that connect the objects to each other called *edges*.
- *Triadic Closure* is the increased likelihood of two people who share a common friend to more likely be friends themselves.
- *Clustering Coefficient* is the probability that two randomly selected friends of a node are friends with each other. Let n be in individual in a graph G, then:

$$cc(n) = \frac{\text{\# of adjacent neighbors}}{\text{\# of neighbors}} = \frac{\text{\# of triadic closures containing n}}{\text{\# of possible triadic closures containing n}}.$$



- The average clustering coefficient of a graph G is defined as $CC(G) = \frac{1}{|V|}\sum_{n\in V} cc(n)$.
- *Path Length* between two nodes $m, n \in G$ is the length of the shortest path from m to n, denoted by $d(m, n)$.
- The average path length of a graph G is defined as $L(G) = \frac{1}{n(n-1)}\sum_{m,n\in V} d(m, n)$.
- The figure to the left is a graph with 7 nodes and 7 edges. Triadic closure exists between A, B, and E. The $CC(G) = 0.4$, $L(G) = 2.14$, and $\langle k\rangle = 2$.

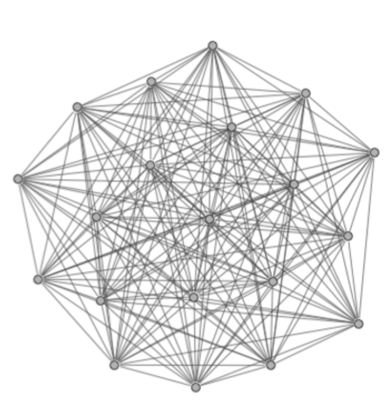
3. Networks

Complete Graph K_n

- Each individual, n, has an equal probability of interaction with every other individual, represented by a network containing all possible edges between nodes.
- The $CC(G) = 1$, $L(G) = 1$, $\langle k\rangle = n - 1$.

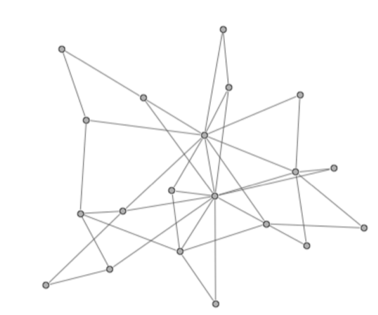
Small World $G(n, p, k)$

- Characterized by many small groups of tightly connected individuals with a few inter-community connections.
- Rewiring probability p, where $p = \frac{\langle k\rangle}{n}$.
- High $CC(G)$, Short $L(G)$, Degree distribution follows normal distribution.
- To the right is a Small World Graph $G(20, 0.1, 9)$ with $CC(G) = 0.94$, $L(G) = 1.05$, and edge count of 180.



Preferential Attachment $G(n, k)$

- Depicted by a few highly connected centralized hubs which form the bulk of the network.
- Defined by *growth*, where a node with k edges is added at each step .
- Power law degree distribution → Deemed the 'rich get richer' model .
- To the right is a Preferential Attachment Graph $G(20, 2)$ with $L(G) = 2.12$, $CC(G) = 0.38$, and edge count of 37.



4. Rumor Spreading Model

- The *Ignorant-Spreader-Stifler* model simulates rumor propagation using a modified Susceptible-Infected-Recovered compartmental model.
- The rumor is propagated through the population by pair-wise contacts. A spreader who contacts an ignorant attempts to infect the ignorant with the rumor determined by a probability of infection λ. If successful the ignorant becomes a spreader. When a spreader interacts with another spreader or stifler, the initial spreader will become a stifler with a probability σ, due to the initial spreader realizing the rumor has lost its shock value. With a certain probability δ, the spreader will forget the rumor, thus becoming a stifler.

$$\boxed{\text{Ignorants } S} \xrightarrow{\lambda} \boxed{\text{Spreaders } I} \xrightarrow{\delta} \boxed{\text{Stiflers } R}$$

$$\boxed{\text{Spreaders } I} \xrightarrow{\sigma} \boxed{\text{Stiflers } R}$$

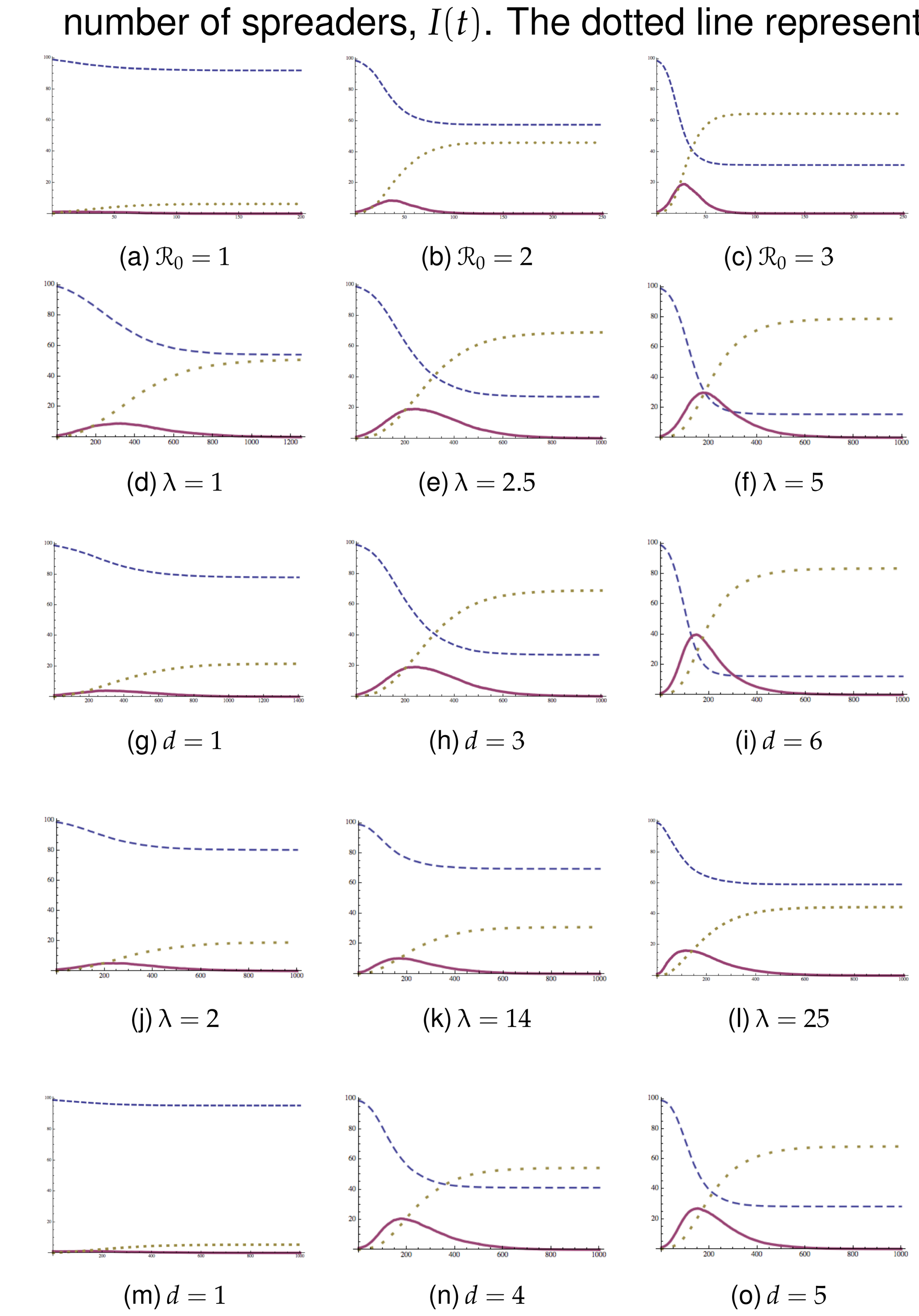
- Model assumptions: closed population, homogeneity of hosts.
- Basic Reproductive rate $\mathcal{R}_r$.

To create a rumor spreading model over complex networks, we modify the simple rumor model by including the average degree of the network, $\langle k\rangle$, into our equations:

$$\frac{dS(t)}{dt} = -\lambda\langle k\rangle I(t)S(t)$$
$$\frac{dI(t)}{dt} = \lambda I(t)S(t) - \sigma\langle k\rangle I(t)(I(t) + R(t)) - \delta I(t)$$
$$\frac{dR(t)}{dt} = \sigma\langle k\rangle I(t)(I(t) + R(t)) + \delta I(t)$$

5. Simulations

- Simulations consisted of 1000 iterations with differing parameters, tracking the number of ignorant, spreader, and stifler nodes per time step for 1000+ time steps.
- Took the mean of each compartment per time step and plotted S, I, and R curves.
- The dashed line represents the number of ignorants, $S(t)$. The solid line represents the number of spreaders, $I(t)$. The dotted line represents the number of stiflers, $R(t)$.



(a) $\mathcal{R}_0 = 1$ (b) $\mathcal{R}_0 = 2$ (c) $\mathcal{R}_0 = 3$

(d) $\lambda = 1$ (e) $\lambda = 2.5$ (f) $\lambda = 5$

(g) $d = 1$ (h) $d = 3$ (i) $d = 6$

(j) $\lambda = 2$ (k) $\lambda = 14$ (l) $\lambda = 25$

(m) $d = 1$ (n) $d = 4$ (o) $d = 5$

Increasing the infection rate in CG results in an increase in $\mathcal{R}_0$. We see the same trend for each network.

In SW, λ is the rewiring probability. Increasing λ results in an increase of the mean degree and $\mathcal{R}_0$, and a decrease in the clustering coefficient and path length.

In SW, each node connects to d adjacent neighbors. Increasing d results in an increase of the mean degree, $\mathcal{R}_0$, and the clustering coefficient, while a decrease occurs in the path length.

PA grows from an initial complete graph of λ-nodes. Increasing λ results in an increase the mean degree, $\mathcal{R}_0$, and the clustering coefficient, while a decrease in path length.

In PA, each newly added node has d edges. Increasing d results in an increase of the mean degree, $\mathcal{R}_0$, and the clustering coefficient, while a decrease occurs in the path length.

6. Results

- For all three networks, increasing the infection rate results in faster, more intense, and larger final rumor sizes.
- In SW, increasing the rewiring probability leads to an increase in the number of cross graph connections between communities. Simultaneously this decreases the strong clustering held between small communities. In affect, this SW network becomes more like a PA network, leading to more severe rumors.
- By increasing the number of adjacent neighbor connections in SW, stronger clustering occurs within the network resulting in slower spreading rumors across the network.
- For PA, increasing the size of the initial complete graph leads to a greater number of hubs that emerge within the network. These hubs act like tightly knit communities, exhibiting stronger clustering and slower rumor spreading. In affect, this PA network mimics the properties of a SW network.
- We encounter a spectrum of network variation between SW and PA by tweaking the initial conditions of the simulation.

7. Conclusion

We find that the CG model is the most effective network for spreading rumors since it has the maximum number of connections, highest clustering, and lowest path length. In general the mean degree, path length, and clustering are lower in PA than SW, but the hubs in PA are more effective at rumor spreading than the communities in SW. The clustering coefficient and the path length are common features shared by all networks. These features can be manipulated to create network structures that mimic real life phenomenon. They ultimately determine the speed, duration, intensity, and final size of a rumor spreading over a network. Simply put, the more connections that exist within a network, the more avenues there are for a rumor to spread. Similarly, the fewer number of steps between individuals to get exposure to a rumor, the faster the rumor will be able to reach more individuals.