

# The Financial Origins of Non-Fundamental Risk

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# Question

Can the financial sector be a source of non-fundamental risk for the economy?

- Rajan (2005): “Has Financial Development Made the World Riskier?”
- Danielsson and Shin (2003): “Endogenous Risk”
- Often cited examples: Portfolio insurance (Oct 1987), LTCM 1998

A stylized model where non-fundamental volatility emerges with financial intermediation:

- mutual feedback between the risk of a fall in asset prices and HH's purchase of insurance
- insurance demand by pessimistic HHs fulfilled by use of leveraged contracts
- trading of leveraged contracts generates possibility of fire-sales
- no fundamental sources of risk present
- full-information rational expectations framework

# Outline for presentation

- 1 Baseline model: unique equilibrium, no price volatility.
- 2 Add trading of insurance contracts  $\rightarrow$  obtain non-fundamental volatility
- 3 Conclusion

# Environment

- two dates: 0 and 1
- three agents: households, financial intermediaries and outside investors
- fixed endowment of cookies ( $c$ ) at both dates
- fixed endowment of trees at date 0
- trees are claims to apples ( $a$ ) at date 1
- trees can be traded at date 0
- benchmark: trees are the only asset traded at date 0

# Households

unit mass of HHs

only consume cookies (c)

$$U^h(c_0^h, c_1^h) = c_0^h + \left[ \mathbb{E}(c_1^h)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \gamma > 1$$

- risk-averse over date 1 consumption
- born with  $\chi_0^h$  cookies, and all the trees,  $e_0 = 1$ .

Date-0 budget constraint:

$$c_0^h + p_0 e^h = \chi_0^h + p_0$$

Date-1 budget constraint:

$$c_1^h = p_1 e^h$$

where  $p_j$  is price of tree at date  $j \in [0, 1]$ .

Note:  $p_1$  can be stochastic.



optimize subject to budget constraints and non-negativity constraints on  $c_j^h, e^h$ .

optimality condition

$$p_0 = \frac{\mathbb{E} p_1 c_1^{-\gamma}}{\left[ \mathbb{E} c_1^{1-\gamma} \right]^{\frac{-\gamma}{1-\gamma}}} = \left[ \mathbb{E} p_1^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

Assume  $\chi_0^h$  large enough s.t. non-negativity constraint on  $c_j, e^h$  does not bind

# Financial Intermediaries

unit mass of FIs

consume apples ( $a_1$ ) or cookies ( $c_j$ )

$$U^f(c_0, c_1, a_1) = c_0 + \mathbb{E}(c_1 + a_1)$$

- risk-neutral over date 1 consumption
- born with  $\chi_0^f < 1$  cookies, no trees

Date-0 budget constraint:

$$c_0^f + p_0 e^f = \chi_0^f$$

Date-1 budget constraint:

$$c_1^f + p_1 a_1^f = p_1 e^f$$



optimize subject to budget constraints and non-negativity constraints on  $c_j^f$ ,  $e^f$ ,  $a_1^f$ .

Date 1:

Sell all trees if  $p_1 > 1$ .

Keep all trees if  $p_1 < 1$

indifferent at the equality.

Date 0:

Only buy trees if  $p_0 < \mathbb{E} \max\{1, p_1\}$ .

Dont buy trees if  $p_0 > \mathbb{E} \max\{1, p_1\}$ .

indifferent at the equality.

# Outside Investors

unit mass of OIs

only trade and consume at date 1

$$U^o(c_1, a_1) = v(a_1) + c_1$$

where  $v'(\cdot) > 0$  and  $v''(\cdot) < 0$ .

- only agents with cookies at date 1
- large amt of cookies  $\chi_1$
- Assume  $v'(0) > 1 > v'(1)$ : interior soln

Date-1 budget constraint:

$$c_1^o + p_1 a_1^o = \chi_1$$



optimal demand for trees by OIs

$$v'(a_1^o) \leq p_1, \quad a_1^o \geq 0 \\ a_1^o [v'(a_1^o) - p_1] = 0$$

Assume  $v'(0) > 1 > v'(1)$ : st  $a_1^o \in (0, 1)$  when  $p_1 = 1$ .

Let  $\bar{e}$  be s.t.  $v'(\bar{e}) = 1$ .

# Equilibrium

prices  $\{p_0, p_1\}$  and quantities

$\{c_0^h, c_1^h, e^h, c_0^f, c_1^f, a_1^f, e^f, c_1^o, a_1^o\}$

- all agents optimize
- markets for cookies (🍪) and trees (🌲) at dates 0 and 1 clear,
- market for apples (🍏) at date 1 clears

$$c_0^h + c_0^f = \chi_0^h + \chi_0^f \quad (1)$$

$$c_1^h + c_1^f + c_1^o = \chi_1 \quad (2)$$

$$e^h + e^f = 1 \quad (3)$$

$$a_1^o + a_1^f = 1 \quad (4)$$

$e \doteq$  trees retained by HHs at date 0 ( $e^h$ )

## Lemma 1 (Date 1 price of trees)

*In equilibrium,  $p_1 = \min\{1, v'(e)\}$ .*

Proof:  $p_1 \leq 1$  since  $v'(1) < 1$ .

When  $p_1 < 1$ , OIs buy the trees from HHs  
 $\rightarrow p_1 = v'(e)$ .

HHs demand for trees: From FOC:

$$p_0 = p_1 = \min\{1, v'(e)\}. \quad (5)$$

FIs demand for trees Since  $p_1 \leq 1$ , FIs buy trees at date 0 if  $p_0 < 1$  - i.e.

$$p_0(1 - e) = \chi_0^f.$$

$$p_0 = \min\left\{\frac{\chi_0^f}{1 - e}, 1\right\} \quad (6)$$



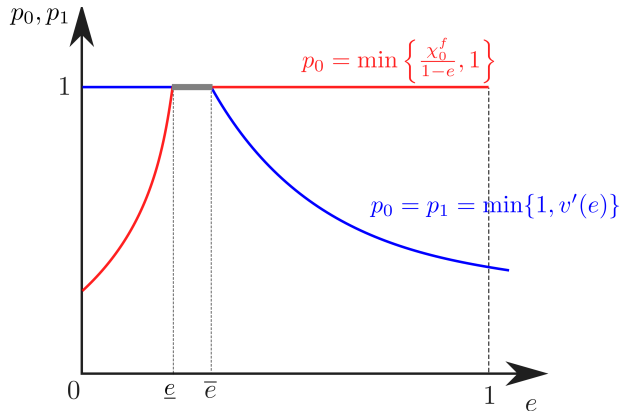
# Equilibrium

HHs demand for trees:

$$p_0 = p_1 = \min\{1, v'(e)\}$$

FIs demand for trees:

$$p_0 = \min\left\{\frac{\chi_0^f}{1-e}, 1\right\}$$



fundamental equilibria:  $p_0 = p_1 = 1$  and  $e^h \in [\underline{e}, \bar{e}]$ . where  $\underline{e} = 1 - \chi_0^f$ , and  $\bar{e}$  s.t.  $v'(\bar{e}) = 1$

welfare:  $U^h = \chi_0^h + 1, \quad U^f = \chi_0^f, \quad U^o = v(\bar{e}) - \bar{e}$

# Endogenous Fragility with Insurance Contracts

Only fundamental equilibria exist when trees are the only assets traded.

- 🌲🌲 are safe assets ( $p_1 = 1$ )

Allow FIs to sell insurance contracts  $z^f$  at date 0 at price  $q$

- pays out  $1 - p_1$  if  $p_1 < 1$
- equivalent to a put option on trees
- non-negative consumption constraint on FIs limit amt of insurance sold

$$\underbrace{(1 - p_1(s))z^f}_{\text{insurance payout}} \leq \underbrace{p_1(s)e^f}_{\text{value of trees}} \quad \text{in all states } s$$

- If HHs expect  $p_1 = 1$  in all states of the world, then no demand for insurance.
- 🌲🌲 continue to be safe assets
- Fundamental equilibria that we constructed exist, with  $q = z^f = 0$ .
- ... but not the only set of equilibria that exist

# Positive Insurance and Non-fundamental Volatility

$\lambda \in (0, 1)$  be probability that  $p_1 = \underline{p} < 1$  and  $1 - \lambda$  prob that  $p_1 = 1$ .  
FI's non-negative consumption constraint binds in the low state:

$$(1 - \underline{p})z^f = \underline{p}e^f \implies \frac{z^f}{e^f} = \frac{\underline{p}}{1 - \underline{p}} \equiv \phi$$

This is a date-1 equilibrium:

- A. When  $p_1 = \underline{p}$ , FIs sell all trees to payout on insurance contracts
  - OIs purchase all trees in the economy (only agents with cookies at date 1).
  - Since  $v'(1) < 1$ ,  $\underline{p} = v'(1) < 1$ .
- B. When  $p_1 = 1$ , FIs need not sell any trees
  - Can confirm  $p_1 = 1$  as in the benchmark economy (no insurance)

What happens at date 0?

# Positive Insurance and Non-fundamental Volatility

HH's problem:

$$\max_{c_0^h, e^h, z^h, c_1^h} \left[ c_0^h + \left( \mathbb{E}(c_1^h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right]$$

s.t.

$$c_0^h + p_0 e^h + q z^h = \chi_0^h + p_0$$

$$c_1^h = p_1 e^h + (1 - p_1) z^h$$

$$c_0^h, c_1^h, e^h \geq 0$$

FI's problem:

$$\max_{c_0^f, e^f, z^f, c_1^f} [c_0^f + \mathbb{E}(c_1^f + a_1)]$$

s.t.

$$c_0^f + p_0 e^f = \chi_0^f + q z^f$$

$$c_1^f + p_1 a_1^f + (1 - p_1) z^f = p_1 e^f$$

$$(1 - \underline{p}) z^f \leq \underline{p} e^f$$

$$c_0^f, c_1^f, e^f \geq 0$$

OIs problem unchanged + Additional market clearing ( $z^h = z^f$ ) condition

# FI's solution at date 0

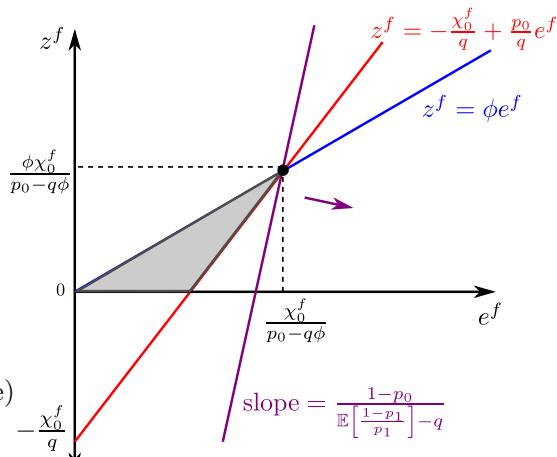
FIs' simplified problem:

$$\max_{e^f, z} \chi_0^f - p_0 e^f + q z^f + \mathbb{E} \left[ e^f - \frac{1-p_1}{p_1} z^f \right]$$

s.t.

$$\chi_0^f - p_0 e^f + q z^f \geq 0 \quad (c_0 \geq 0)$$

$$z \leq \phi e^f \quad (\text{insurance issuance})$$



If  $\frac{q}{p_0} > \mathbb{E} \left[ \frac{1-p_1}{p_1} \right]$ , lever up to the max and purchase  $e^f = \frac{\chi_0^f}{p_0 - \phi q}$ . Multiplier due to levered contracts

## HH's solution at date 0

Solution to HHs problem yields:

$$p_0 - \frac{\underline{p}}{1 - \underline{p}} q = \frac{(1 - \lambda)(e^h)^{-\gamma}}{[\lambda \underline{p}^{1-\gamma} + (1 - \lambda)(e^h)^{1-\gamma}]^{-\frac{\gamma}{1-\gamma}}}$$

Buying  $\phi$  insurance claims  $\equiv$  HHs selling a security that pays out when  $p_1 = 1$ .

- HHs sell trees to FIs at price  $p_0$
- use the proceeds to buy insurance claims  $\phi$

LHS is the price of this synthetic security.

RHS is the cost weighted by HHs' marginal utility.

# Equilibrium with Insurance

Using  $e^h = 1 - e^f$ ,

$$\frac{(1 - \lambda)(e^h)^{-\gamma}(1 - e^h)}{[\lambda \underline{p}^{1-\gamma} + (1 - \lambda)(e^h)^{1-\gamma}]^{-\frac{\gamma}{1-\gamma}}} = \chi_0^f \quad (7)$$

If  $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$ , for every  $\lambda \in (0, \bar{\lambda})$  where  $\bar{\lambda}$  is implicitly defined by:

$$\chi_0^f = \frac{(1 - \bar{\lambda}) \left[ 1 - \underline{p}^{\frac{\gamma-1}{\gamma}} \right]}{\left[ \bar{\lambda} \underline{p}^{\frac{1-\gamma}{\gamma}} + 1 - \bar{\lambda} \right]^{\frac{\gamma}{\gamma-1}}}$$

$\exists$  an equilibrium in which  $p_1 = 1$  with probability  $1 - \lambda$  and  $p_1 = \underline{p} = v'(1) < 1$  with probability  $\lambda$ .  $e^h$  is implicitly defined by equation (7).  $p_0$  and  $q$  are defined by (8) and (9) and  $z_h = \frac{\underline{p}}{1-\underline{p}} e^h$ .

$$p_0 = \frac{\lambda \underline{p}^{1-\gamma} + (1 - \lambda)(e^h)^{-\gamma}}{[\lambda \underline{p}^{1-\gamma} + (1 - \lambda)(e^h)^{1-\gamma}]^{-\frac{\gamma}{1-\gamma}}} \quad (8)$$

$$q = \frac{\lambda(1 - \underline{p})\underline{p}^{-\gamma}}{[\lambda \underline{p}^{1-\gamma} + (1 - \lambda)(e^h)^{1-\gamma}]^{-\frac{\gamma}{1-\gamma}}} \quad (9)$$

# Equilibrium with Insurance: in English

There exists an equilibrium in which,

- with non-zero probability, price decline at date 1 can be self-fulfilling
- when  $p_1$  is low, FIs sell trees to pay out on their insurance contracts, pushing down the price
- if households anticipate that prices might fall, they demand insurance from FIs
- issuance of insurance actually makes price declines possible.
- supply of private safe assets may create its own demand: *Say's law for risk*

Key market incompleteness: OIs are not allowed to participate at date 0



# Equilibrium with Insurance: Welfare

## 1. HHs

- worse off than in fundamental eqm
- welfare with insurance

$$\underbrace{\chi_0^f + \chi_0^h}_{c_0^h} + \left[ \lambda \underline{p}^{1-\gamma} + (1-\lambda) \left( e^h(\lambda) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

- $\lambda \rightarrow 0$ , welfare converges to no-insurance case



## 2. FIs



- weakly better off than in fundamental eqm
- have the option to consume their endowment  $\chi_0^f$  in the first period.

## 3. OIs



- benefit from fire-sales
- sell cookies for apples at steep discounts
- better off than in fundamental eqm
- welfare with insurance

$$(1-\lambda) \underbrace{[v(\bar{e}) - \bar{e}]}_{\text{no insurance welfare}} + \lambda \underbrace{[v(1) - v'(1)]}_{>0}$$

## Other private safe assets

allow FIs to issue risk-free non-state contingent bonds  $b$  at price  $q^b$

- pay one cookie to the holder at date 1
- bonds are backed by FIs' holdings of trees: *repo* transactions

HHs budget constraints

$$c_0^h + p_0 e^h + q^b b^h = \chi_0^h + p_0 \quad (10)$$

$$c_1^h = p_1 e^h + b^h, \quad (11)$$

FIs budget constraints

$$c_0^f + p_0 e^f = \chi_0^f + q^b b^f \quad (12)$$

$$c_1^f + p_1 a_1^f + b^f = p_1 e^f \quad (13)$$

non-negative consumption on FIs:

$$b^f = p_1 (e^f - a_1^f) - c_1^f \leq p_1 e^f \quad (14)$$

in all states of the world

## Other private safe assets

for every equilibrium that exists in insurance economy, a corresponding equilibrium exists in the bond economy

- FIs have to pay out in all states of the world
- *but* FIs sell more when  $p_1 = \underline{p} < 1$  to meet obligations

fundamental equilibrium:

- zero spread between expected return on bonds and trees
- both bonds and trees are riskless assets

non-fundamental equilibria

- date 0 price of bonds is higher
- That is, risk-free rate is lower in these equilibria
- safe rate endogenously falls as a *result* of private safe asset creation (contrast to a typical safe assets scarcity narrative)

# Policy to eliminate financial fragility

FIs should be the “natural” buyers of trees at date 1

- because of excessive leverage, they are forced to *sell* trees in some states
- explicit ban on such financial transactions would return the economy to a unique equilibrium setup (strict enough tax or leverage restrictions)
- or reduce the excess returns to leveraged investments in risky assets

Consider two sets of crisis-fighting policies

- 1 increase supply of publicly backed safe assets (issue debt, bailouts)
- 2 reduce demand for private safe assets (social insurance, market maker of last resort)

# Policy: reduce excess return to private safe asset creation

## 1. Crowd out private safe assets

- public safe assets by the government
  - i crowd out private provisioning of safe assets
  - ii prevent buildup of intermediary leverage
- ex-ante commitment to bail out financial intermediary in the bad state
  - i portion of private safe assets become “publicly backed” (Benigno & Robatto, 2019)
  - ii crowds out unbacked private safe assets.

## 2. Reduce demand for insurance

- social insurance to households, or *market maker of last resort* (Buiter and Sibert, 2008)
- eliminate possibility of fire-sales
- intervention not required in equilibrium

# Conclusion

Private creation of safe assets by leveraged intermediaries can lead to fragility

- Safe assets are produced due to demand for safety by households
- Demand for safety arises from fragility induced by the privately-supplied safe assets
- Economy becomes vulnerable to self-fulfilling fire sales

Novel contribution

- leverage is not being used to amplify exogenous fundamental shocks
- instead, financial system *generates* risk in an otherwise fundamentally safe economy