Incomplete Markets and Exchange Rates

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Motivation

When financial markets are complete: relative consumption and real exchange rates co-move positively [Kollman (1991), Backus-Smith (1993)]

- Strongly rejected in data. Corr(Δe, $\Delta c - \Delta c^*$) ≤ 0

When financial markets are incomplete: non-traded risk can generate negative correlation in models [Corsetti, Dedola and Leduc (2009) & others]

But not with cross-border trade in two risk-free bonds:

- Arbitrage restrictions on process for non-traded risk [Lustig and Verdelhan (2019)]

Our paper: domestic $\times$ intn'l market incompleteness matters for exchange rates

This paper

Two-country asset pricing model with heterogeneous consumers

* analytical heterogeneity ala Constantinides & Duffie (1996)
* integrated financial markets (with “many" assets)
* presence of uninsurable idiosyncratic risk $\implies$ wedge in the aggregate Euler eqn

Resolve Backus-Smith puzzle if

<u>exchange rates are risky</u>

- domestic risk goes down when exchange rate depreciates

Closed form solutions

two-country factor model Cox, Ingersoll, & Ross (1985) with heterogenous consumers

* closed form solution for incomplete markets wedge.
* complete international markets + incomplete domestic markets fix Backus Smith
* but exchange rates too volatile under complete intn'l markets

Resolve Backus-Smith puzzle + Match FX volatility when

international + domestic markets are incomplete

Roadmap

1. Representative agent asset pricing model
2. Heterogenous consumers model
3. Testable empirical conditions
4. Conclusion

A quartet of Eulers

- Two-country model, SDFs $M^{(*)}_{t+1}$ denotes Home (Foreign) RA SDF
- Trade in assets $\implies$

$$\mathbb{E}_t[M_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M^*_{t+1}] = 1/R^*_{t+1}$$ (Trade in dom. bonds)

$$\mathbb{E}_t\left[M_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1/R^*_{t+1}$$ (Int’l Trade in F bond)

$$\mathbb{E}_t\left[M^*_{t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] = 1/R_{t+1},$$ (Int’l Trade in H bond)

Exchanges Rate in Incomplete Markets

Assume SDFs, allocations and prices are jointly log-normal 4 Eulers $\implies$

$$cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) = var_t(\Delta e_{t+1})$$

- CM: $\Delta e_{t+1} = m^*_{t+1} - m_{t+1}$
- IM: $\Delta e_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$

where η_{t+1} is the IM/ non-traded risk wedge

Assuming time-separable, CRRA preferences, classical int'l macro models assume:

$$\eta_{t+1} = log \underbrace{\left(\frac{P_{t+1}}{P_t} \frac{P^*_t}{P^*_{t+1}} \right)}_{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}} - log \underbrace{\left(\frac{C_t}{C_{t+1}} \frac{C^*_{t+1}}{C^*_t} \right)^s}_{\frac{M_{t+1}}{M^*_{t+1}}}$$

IM wedge

Int'l trade in bonds disciplines IM wedge [Lustig and Verdelhan (2019)]:

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) - cov_t(m_{t+1}, \eta_{t+1}), \qquad \text{(H bond traded)}$$

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) \qquad \text{(F bond traded)}$$

But what does this mean for macro? Trade in asset means:

- Higher volatility compensated by expected return or change in cyclicality of non-traded risk

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Heterogeneous Consumers, Integrated Markets 1/2

2-country CAPM with heterogeneity [see e.g. Constantinides and Duffie (1996)]:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-s}, \quad \Delta c_{t+1} = w_{t+1} \sim \mathcal{N}(\mu_{C_t}, \sigma^2_{C_t}),$$

$$\Delta c^i_{t+1} = \log \left(\frac{\delta^i_{t+1}}{\delta^i_t} \frac{C_{t+1}}{C_t} \right) \sim \mathcal{N}(\mu_{c^i_t}, \sigma^2_{i,t})$$

Individual consumption draw related to aggregate:

$$\int_i \delta^i_t \, di = 1, \quad \log(\delta^i_{t+1}/\delta^i_t) \sim \mathcal{N}(-\tfrac{y_{t+1}}{2}, y_{t+1})$$

where y is the variance of log of idiosyncratic consumption risk.

- CD96 choose endowment processes to support a no trade (autarky) equilibrium

Heterogeneous Consumers, Integrated Markets 2/2

- Individual Euler equations hold

$$\mathbb{E}_t\left[\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-s}\right]=\frac{1}{R_t},$$

but aggregate Euler holds with inequality

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}\left(\frac{\delta^i_{t+1}}{\delta^i_t}\right)^{-s}\right]=\frac{1}{R_t}$$

analogously for Foreign bond

- Segmentation no longer required — but can be a complementary mechanism

[see e.g. Chernov, Haddad, and Itskhoki (2024)]

Aggregate Euler with a Wedge

Individual Euler holds

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}\left(\frac{\delta^i_{t+1}}{\delta^i_t}\right)^{-s}\right]=\frac{1}{R_t}$$

With the CD'96 income share process $\left[\frac{\delta^i_{t+1}}{\delta^i_t}=\exp(\xi^i_{t+1}\sqrt{y}_{t+1}-y_{t+1}/2)\right]$

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}e^{\frac{s(s+1)}{2}y_{t+1}}\right]=\frac{1}{R_t}$$

Aggregate Euler with a Wedge

Individual Euler holds

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}\left(\frac{\delta^i_{t+1}}{\delta^i_t}\right)^{-s}\right]=\frac{1}{R_t}$$

With the CD'96 income share process $\left[\frac{\delta^i_{t+1}}{\delta^i_t}=\exp(\xi^i_{t+1}\sqrt{y}_{t+1}-y_{t+1}/2))\right]$

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-s}\underbrace{e^{\frac{s(s+1)}{2}y_{t+1}}}_{e^{\tilde{y}_{t+1}}}\right]=\frac{1}{R_t}$$

$\tilde{y}_{t+1}\equiv\frac{s(s+1)}{2}y_{t+1}$ denotes the log of the variance of the idiosyncratic permanent shocks

Comovement with Heterogeneous Consumers

The model with H & F traded bonds and heterogeneous Home consumers delivers $cov_t(m^*_{t+1} - \log(\int_i e^{\Delta c^i_t} di)^{-s}, \Delta e_{t+1}) < 0$ ***if and only if:***

$$1 \geq -\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(\tilde{y}_{t+1})}$$

where $\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} \equiv \dfrac{cov_t(\tilde{y}_{t+1}, \Delta e_{t+1})}{\sigma_t(\Delta e_{t+1})\sigma_t(\tilde{y}_{t+1})}$

- Idiosyncratic risk which co-moves with FX recovers a non-traded component
  - F bonds are a poor hedge if $\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} < 0$
- relaxed when only one bond intn'lly traded
- preserved when adding trade in more assets

Cox-Ingersoll Ross for Closed form

Let a common factor z_t drive aggregate consumption globally

$$z_{t+1} = \rho z_t + u_{t+1}$$

Home and Foreign aggregate consumption

$$\Delta c_{t+1} = \sqrt{z_t} u_{t+1}; \quad \Delta c^*_{t+1} = \xi^* \sqrt{z_t} u_{t+1}, \quad \xi^* > 1$$

SDF:

$$m_{t+1} = \log \beta - \gamma \Delta c_{t+1}; \quad m^*_{t+1} = \log \beta - \gamma^* \Delta c^*_{t+1};$$

uninsurable idiosyncratic risk process

$$\tilde{y}_{t+1} = \tilde{\alpha}_y + \tilde{\phi} \sqrt{z_t} u_{t+1},$$

$\tilde{\phi} \equiv \phi \frac{\gamma(\gamma+1)}{2}$: the cyclicality of idiosyncratic risk

Equilibrium Incomplete Markets Wedge

In the model with heterogeneous consumers, with H & F bonds traded, the incomplete markets wedge is given by:

$$\eta_{t+1} = -\frac{1}{2}\left[(\gamma - \gamma^* - \phi)^2 - \kappa\right] + \left[-\phi - \sqrt{(\phi - \gamma - \gamma^*)^2 - \lambda}\right]\sqrt{z_t}u_{t+1} - \left[\sqrt{\lambda - \kappa}\right]\sqrt{z_t}\epsilon_{t+1},$$

where $\lambda \geq \kappa$ is a parameter governing cross border spanning and

$$\kappa = (\gamma - \gamma^* - \phi)^2 - (\gamma - \gamma^* - \phi)\sqrt{(\gamma - \gamma^* - \phi)^2 - \lambda} \geq 0$$

where κz_t is the exchange rate volatility.

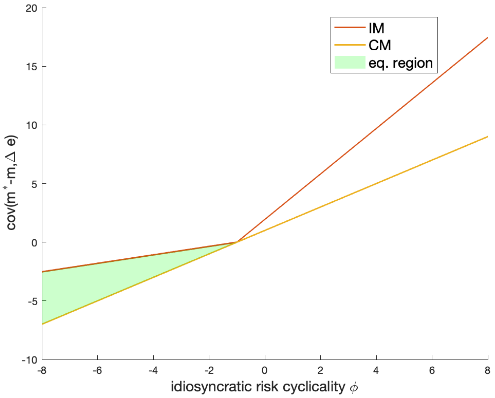
Exchange rate cyclicality

The model with heterogeneous consumers, with H & F bonds traded, delivers $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) < 0$ *if and only if* $var(r_{t+1}) < var(r^*_{t+1})$.

* specifically, when $\phi < \gamma - \gamma^* < 0$

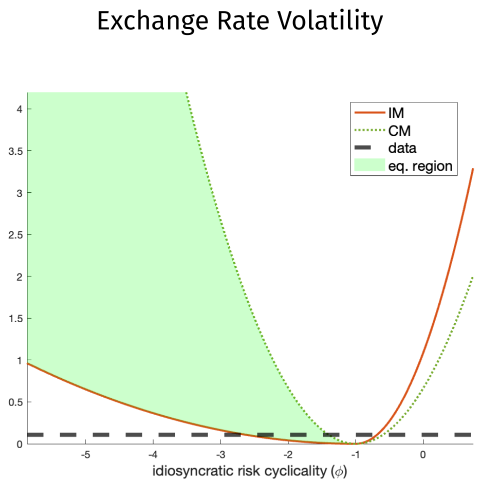
Exchange rate cyclicality: Illustration

Backus-Smith Covariance



Shaded regions capture the combinations of ϕ and moments of exchange rates admissible under incomplete markets for $\gamma = 1, \xi^* = 2$.

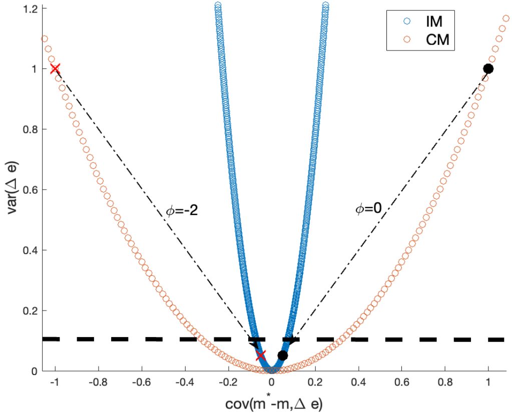
Exchange rate volatility: Illustration



Shaded regions capture the combinations of ϕ and moments of exchange rates admissible under incomplete markets for $\gamma = 1, \xi^* = 2$.

Exchange rate volatility: International Markets

If income risk is counter-cyclical $(\phi < 0)$*, exchange rate volatility is always higher under international complete markets* $\kappa^{CM} > \kappa^{IM}$.



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Testable Empirical Condition

$$1 \geq -\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(\tilde{y}_{t+1})}$$

Data over 1979Q1–2013Q1

* bilateral real exchange rate of U.S. dollar against seven advanced economies.
* estimate of income risk volatility (Bayer, Luetticke,Pham-Dao,& Tjaden 2019)

$$s = 10; \quad \sigma(\Delta e_{t+1}) = 0.11; \quad \sigma(\tilde{y}_{t+1}) = 4.4 \implies RHS = 0.025$$

The measured correlation: $-\rho_{\tilde{y}_{t+1},\Delta e_{t+1}} = 0.05$ substantially higher than needed

Conclusion

Incomplete Domestic and Int’l Markets

- Domestic incompleteness generates empirical exchange rate cyclicality
  - make exchange rates “risky"
- Incomplete International Markets hit FX volatility

Heterogeneous consumers with uninsurable risk offers a plausible mechanism for exchange rates

Equilibrium Incomplete Markets Wedge with Internationally complete markets

$$\eta_{t+1} = -\frac{1}{2}\left[(\gamma - \gamma^* - \phi)^2 - \kappa\right] + \left[-\phi - \sqrt{(\phi - \gamma - \gamma^*)^2 - \lambda}\right]\sqrt{z_t}u_{t+1} - \left[\sqrt{\lambda - \kappa}\right]\sqrt{z_t}\epsilon_{t+1},$$

where $\lambda \geq \kappa$ is a parameter governing cross border spanning and

$$\kappa = (\gamma - \gamma^* - \phi)^2 - (\gamma - \gamma^* - \phi)\sqrt{(\gamma - \gamma^* - \phi)^2 - \lambda} \geq 0$$

Internationally Complete Markets limit:

$$\eta^{CM}_{t+1} = -\tilde{y}_{t+1} = -\phi\sqrt{z_t}u_{t+1} \iff \lambda^{CM} = \kappa^{CM} = (\gamma - \gamma^* - \phi)^2$$

when exchange rates move to reflect differences in the as-if representative agent SDFs

Scatter Plot

Figure: Backus-Smith Correlation against the US Income Shock Volatility Correlation with Exchange Rate Growth