Incomplete Markets and Exchange Rates[*]

Emile A. Marin[†]   Sanjay R. Singh[§]

June 2025

Abstract

We show that allowing for imperfect risk-sharing *within* countries can reconcile the cyclicality of exchange rates with respect to aggregate consumption, i.e. the Backus-Smith puzzle, as long as exchange rates are risky with respect to idiosyncratic states. Within a tractable two-country, consumption-based asset pricing model, in equilibrium, we show that idiosyncratic risk must remain relatively high in the country experiencing higher consumption growth. Furthermore, we identify distinct roles for market incompleteness both *within* and *across* countries, to match key moments of exchange rates. Turning to household level data, we measure discount factor wedges which capture the effects of imperfect risk sharing and we find direct empirical support for the mechanism.

Keywords: Incomplete Markets, Heterogeneous agents, Exchange Rates.

---

1. INTRODUCTION

We revisit the Backus-Smith condition (Kollmann, 1991; Backus and Smith, 1993) to consider the implications of domestic and international market incompleteness for exchange rate dynamics. This condition is a centrepiece in the international literature as a benchmark for international risk-sharing in a flexible-price, representative agent economy and is also closely tied to other important moments (and puzzles) such as the volatility of exchange rates and the correlation of pricing kernels across countries. Under complete financial markets, assuming power utility:

$$\left(\frac{C_{t+1}}{C_{t+1}^*} \Big/ \frac{C_t}{C_t^*}\right)^\gamma = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \tag{1}$$

where C_t is Home aggregate consumption, C_t^* is Foreign aggregate consumption, $\mathcal{E}_t$ is the real exchange rate where an increase signifies a depreciation of Home currency, and γ is the inter-temporal elasticity of substitution. Condition (1) implies that exchange rate depreciations coincide with periods of low marginal utility growth for Home investors and exchange rates are counter-cyclical.[1] In the data, however, exchange rates usually appreciate when Home consumption rises and marginal utility growth is low, i.e. are pro-cyclical, constituting the Backus-Smith *puzzle*.

Our motivation is driven by two advancements in the literature. First, we extend a canonical two-country framework to allow imperfect domestic risk sharing— incomplete markets within countries.[2] A large class of incomplete markets models admits an "as-if" representative investor Euler with a discount factor wedge and we leverage recent innovations in the literature (Berger, Bocola and Dovis, 2023) to measures these discount factor wedges directly using household-level data on U.S. consumption, and

---

[1]This implication echoes closed-economy complete markets models featuring a correlation of -1 between the representative agent stochastic discount factor (SDF) and the market portfolio, see, e.g. (Duffie, 2001, Ch. 1), also Lettau (2002).

[2]Incomplete domestic markets and heterogeneity within countries has been shown to be important for asset prices (Constantinides and Duffie, 1996), monetary policy (Kaplan, Moll and Violante, 2018) and exchange rates (Kocherlakota and Pistaferri, 2007).

income data in a panel of advanced economies (Guvenen, Pistaferri and Violante, 2022).
Second, it has been shown that existing resolutions to the Backus-Smith puzzle based
on the representative agent paradigm rely on allowing cross-border trade *only* in a
single risk-free asset. Lustig and Verdelhan (2019) show that, no-arbitrage restrictions
from cross-border trade in at least one Home and one Foreign risk-free asset imply
incomplete international financial markets alone cannot resolve the puzzle of excessive
risk-sharing *regardless* of goods market frictions and other economy specifics. Our
takeaway result is that imperfect risk-sharing within countries is a strong confounding
factor in the determination of exchange rates.

We generalize the representative agent, two-country, consumption capital-based
asset pricing framework to allow for uninsurable idiosyncratic consumption risk within
each country. The extended framework ties exchange rates to the pricing kernels of an
"as-if" representative agent in each country, where a discount factor wedge captures a
precautionary motive due to idiosyncratic risk. When the as-if representative agent
(in each country) prices both Home and Foreign bonds, international no-arbitrage
conditions no longer imposes a strictly positive co-movement between exchange rates
and aggregate consumption growth.

We analytically derive conditions under which domestic market incompleteness
switches the cyclicality of exchange rates with respect to aggregate consumption, i.e.
the Backus-Smith covariance. Our key result is that pro-cyclical exchange rates can
be obtained if and only if the correlation of uninsurable idiosyncratic consumption risk
with exchange rate depreciations is sufficiently negative, relative to a threshold, even
with frictionless trade in at least two nominally risk-free bonds. The threshold is given
by the ratio of the volatility of exchange rates to that of the relative idiosyncratic
risk. This condition implies that foreign bonds must be a poor hedge for idiosyncratic
risk in each country, yielding high returns at a time when domestic investors face low
idiosyncratic risk (whether due to the volatility of permanent idiosyncratic risk or the
possibility of becoming constrained), thus making exchange rates *risky* with respect

3

to the *idiosyncratic* state.

Building on this, we derive the equilibrium exchange rate process in a framework building on Lucas (1982) and Cox, Ingersoll and Ross (1985) where we allow for idiosyncratic risk to be a linear function of the aggregate state.[3] We highlight two additional results arise in this setting. First, the model can deliver risky *equilibrium* exchange rates with respect to the idiosyncratic state, as required to resolve the Backus-Smith puzzle, while also allowing for counter-cyclical idiosyncratic risk (with respect to domestic aggregate consumption growth), see e.g. Storesletten, Telmer and Yaron (2004). Counter-cyclicality implies that idiosyncratic consumption uncertainty is high– the pricers' marginal utility growth is expected to rise by more than marginal utility growth from average consumption – at times when average consumption growth is expected to be low. To deliver risky equilibrium exchange rates, our model requires that despite higher aggregate consumption growth in the Home relative to the Foreign country, the Home country experiences a relatively weaker fall in their discount factor wedge–the Home pricer remains concerned with the future outlook and therefore patient (and vice versa for higher foreign aggregate consumption growth).

Second, we delineate the effects of domestic and international financial market incompleteness. In particular, we show that while imperfect risk sharing within countries is necessary for resolving exchange rate cyclicality, it is not generally sufficient when there are additional country-specific factors which are necessary to account for an imperfect correlation of international stochastic discount factors. In this case, incomplete financial markets across countries are also required. Moreover, international market incompleteness is necessary to account for the volatility of exchange rates which is much lower that that of stochastic discount factors when allowing for idiosyncratic risk or calibrating to equity prices. Our calibrated model matches moments from equity and bond market, as well the volatility and cyclicality of exchange rates with international spanning of 60% for the common factor, illustrating a role for both

---

[3]We use a discrete time version by Sun (1992), extended to two-countries environment in Backus, Foresi and Telmer (2001) and has been extensively used since, e.g. Lustig and Verdelhan (2019)

domestic and international market incompleteness.

We provide explicit examples for tractable models of within-country imperfect risk sharing, borrowing from (i) Constantinides and Duffie (1996), and (ii) Krusell, Mukoyama and Smith (2011) and Bilbiie (2024). In the former case, exchange rates are risky with respect to the idiosyncratic state if permanent risk faced by households is low in periods of depreciation (when the Foreign bond pays out). In the latter case, they are risky if the probability of earning low income, or becoming financially constrained, is low when exchange rates depreciate. We also motivate our focus on the Backus-Smith condition by showing that it remains the benchmark for welfare analysis, even when there is imperfect risk-sharing within countries.[4]

Turning to the data, our main objective is to test whether exchange rates are indeed "risky" with respect to idiosyncratic state. Following Berger et al. (2023), we construct discount factor wedges based on households' consumption shares from the Consumption Expenditure Survey, identifying high-income, low net worth individuals likely to assign the highest prices to bonds and act as the pricer. The resulting wedges capture the pricing implications of a lack of risk sharing within countries for a large class of incomplete market models. We begin by allowing for imperfect risk sharing only in the U.S., maintaining the representative agent assumption abroad. For a conservative IES of 0.2 (Best, Cloyne, Ilzetzki and Kleven, 2020), we robustly find that the $\beta-$wedge, on average, co-moves sufficiently negatively with real exchange rate growth in most countries and is itself sufficiently volatile, for our mechanism to go through. We confirm support for the sufficient condition both unconditionally and by constructing conditional moments, controlling for variables observed at time t. We analyze our finding further in two ways: i) the $\beta-$wedge dynamics are not driven by composition effects (i.e. the identity or characeristics of the pricer), but

---

[4]Moreover, we address a shortcoming common across consumption-based asset pricing models pertaining to goods market clearing. In general, these models solve for the exchange rate at which autarky interest rates are equated across countries, but allocations do not need to satisfy static goods-basket optimization. We detail trade costs (Fitzgerald, 2012), or equivalently shocks to home bias (Pavlova and Rigobon, 2007; Gabaix and Maggiori, 2015), required to clear goods markets.

rather by within group variation capturing a risk channel; ii) turning to data on permanent idiosyncratic risk from Bayer, Luetticke, Pham-Dao and Tjaden (2019), the constructed $\beta-$wedge is not volatile enough for our mechanism to drive exchange rate cyclicality pointing to the role of transitory risk or the possibility of becoming constrained instead.

To also allow for idiosyncratic risk in a panel of foreign countries, in the absence of detailed individual consumption growth data, we construct discount factor wedges using growth in the income shares of high-income individuals (top 2.5%, 5%, 10% of the population) across seven advanced economies from the Global Repository of Income Dynamics (GRID) (Guvenen et al., 2022), as a proxy for the growth in their consumption shares. While the volatility of these wedges is distorted (much higher) due to the lack of consumption smoothing, we present conditions under which the correlation of bilateral wedges with exchange rates is a valid proxy. We then compare these correlations to a 'worst case' threshold computed using only U.S. data. Our findings further confirm the relevance of imperfect sharing within countries for the cyclicality of exchange rates.

Finally, using our constructed β wedges (unilateral or bilateral), we find a positive covariance between depreciations and the relative valuation of the "as-if" representative investor, consistent with the international no-arbitrage conditions, even though the Backus Smith covariance is negative.

Related Literature   Our work closely relates to a literature which tries to confront exchange rate anomalies by allowing for wedges in Euler equations, e.g. Jiang, Krishnamurthy and Lustig (2023). Important precursors have shown that segmentation (Alvarez, Atkeson and Kehoe, 2002; Chien, Lustig and Naknoi, 2020) or intermediation frictions and UIP shocks (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021) can resolve various exchange rate puzzles. We build on the work of Lustig and Verdelhan (2019), see also Benigno and Küçük (2012), who show that introducing a second

internationally traded bond breaks the ability of canonical international macro models to reconcile the Backus-Smith puzzle. Relatedly, Chernov, Haddad and Itskhoki (2024) investigate how different financial market structures and the mix of locally, globally traded, and unspanned risks contribute to different exchange rate puzzles.

We extend these contributions to consider imperfect risk-sharing within countries, which can be represented in the form of discount factor wedges, see e.g. Nakajima (2005); Krueger and Lustig (2010); Werning (2015); Guerrieri and Lorenzoni (2017); Berger et al. (2023) for closed economy applications. In contemporaneous work, Kekre and Lenel (2024) show that *exogenous* permanent discount factor shocks can resolve the comovement and predictability puzzles.

More generally, our model with heterogeneous consumers ties to a large literature investigating whether idiosyncratic risk affects macro-finance aggregates (see, e.g. Mankiw, 1986; Weil, 1992; Guvenen, 2009; Kaplan et al., 2018; Auclert, Rognlie and Straub, 2024; Challe, 2020; Di Tella, Hébert and Kurlat, 2024). In the open economy macro literature, for papers with agent heterogeneity, see Ghironi (2006); Kocherlakota and Pistaferri (2007); Hassan (2013); De Ferra, Mitman and Romei (2020); Kollmann (2012); Auclert, Rognlie, Souchier and Straub (2021); Acharya and Challe (2025) amongst others. Our contribution relative to these models is to emphasize the riskiness of exchange rates with respect to idiosyncratic states and derive a condition which we directly take to household-level data.

A closely related contribution is Kocherlakota and Pistaferri (2007) who show that models with domestically incomplete (but internationally complete) markets only impose restrictions on the comovement of bilateral higher order moments of the consumption distribution and exchange rates.[5] Huang, Kogan and Papanikolaou (2025) connect cyclicality of exchange rates to agent heterogeneity mechanisms through technological innovation based displacement risk faced by shareholders. Acharya,

---

[5]Relatedly, Leduc (2002) using a calibrated open economy model featuring idiosyncratic and aggregate risk investigated the role of these risks in generating currency premia. See also Ramchand (1999) and Tessari (2021).

Challe and Coulibaly (2025) study the ability of international real business cycle model with heterogenous agents to resolve exchange rate puzzles.[6]

The rest of the paper is organized as follows. Section 2 derives our main proposition according to which imperfect domestic risk-sharing can result in pro-cyclical exchange rates. Section 3 solves for equilibrium exchange rates and provides analytical conditions under which our proposition is satisfied. Section 3.1 illustrates the quantitative properties of our model. Section 4 provides tractable examples of models with idiosyncratic risk satisfying our conditions, characterizes goods markets and discusses welfare. Section 5 evaluates the plausibility of our mechanism using household-level data from the U.S. and abroad. Section 6 concludes.

2. Two-country, Consumption Based Asset Pricing Model

Time is discrete and infinite. There are two types of states z_t, aggregate and idiosyncratic ν_t. Their respective histories are denoted by $z^t = (z_0, z_1, ..., z_t)$ and $\nu^t = (\nu_0, \nu_1, ..., \nu_t)$.[7] We use $s^t = (z^t, \nu^t)$ to summarize the joint history. We focus on time-separable CRRA utility. Each agent with joint history s^t derives per-period utility from consumption:

$$u(C(s^t)) = \beta \, \frac{C(s^t)^{\,1-\gamma}}{1-\gamma} \tag{2}$$

where β is the constant discount factor, γ is the inverse elasticity of intertemporal substitution, and we abstract from labour supply considerations. We define $M(z^{t+1})$ as the SDF based on Home aggregate consumption (which would correspond to the representative agent case) and M^*_{t+1} denotes representative SDF abroad such that $M^{(*)}(z^{t+1}) = \beta \left(\frac{C^{(*)}(z^{t+1})}{C^{(*)}(z^t)} \right)^{-\gamma}$, as in condition (1), where the discount factor is a constant.

---

[6]Relatedly, there is a large literature in international macro-finance that studies role of non-separabilities in reconciling various exchange rate puzzles. See eg: Verdelhan (2010); Karabarbounis (2014); Colacito and Croce (2013); Farhi and Gabaix (2016).

[7]We use the convention that states subsume Home and Foreign shocks, i.e. $z_t = \{u^z_t, u^{z*}_t\}, \nu_t = \{u^\nu_t, u^{\nu*}_t\}$.

Individual Home households budget constraint is given by:

$$C(s^t) - I(s^t) \leq R(z^t)B_H(s^{t-1}) - B_H(s^t) + \mathcal{E}_t(z^t)R^*(z^t)B_F(s^t) - \mathcal{E}(z^t)B_F(s^{t-1}) \quad (3)$$
$$+ \int_{j \in J} \tilde{R}^j(z^t)X^{i,j}(s^t) - \int_{j \in J} X^{i,j}(s^{t-1}),$$

where $I(s^t) = I(z^t, \nu^t)$ is the income drawn by an agent with individual history ν^t, $B_{H(F)}(s^t, \nu^t)$ is the position in Home (Foreign) risk-free bonds and $R^{(*)}(z^t)$ is the corresponding returns and $X^j(z^t, \nu^t)$ denote risky assets indexed by j held by agent with history ν^t. As we describe in Section 4, we allow for the possibility that C is a bundle of differentiated goods H and F with prices P_H and P_F, in which case $I(s^t) = \frac{P_H(z^t)}{P(z^t)} I_H(s^t, \nu^t) + \frac{P_F(z^t)}{P(z^t)} I_F(s^t, \nu^t)$. Finally, goods market clearing requires:

$$\int_{\nu^t} C_H(z^t, \nu^t) \, d\nu^t + \int_{\nu^t} C_H^*(z^t, \nu^t) \, d\nu^t = \int_{\nu^t} I_H(z^t, \nu^t) \, d\nu^t + \int_{\nu^t} I_H^*(z^t, \nu^t) \, d\nu^t, \quad (4)$$
$$\int_{\nu^t} C_F(z^t, \nu^t) \, d\nu^t + \int_{\nu^t} C_F^*(z^t, \nu^t) \, d\nu^t = \int_{\nu^t} I_F(z^t, \nu^t) \, d\nu^t + \int_{\nu^t} I_F^*(z^t, \nu^t) \, d\nu^t$$

We specify borrowing constraints in a general manner:

$$\mathcal{H}(B_H(s^t), B_F(s^t), \{X^j(s^t)\}_{j \in J}) \geq 0, \quad (5)$$

for some vector-valued function. We proceed using two assumptions. First, we assume $\frac{d\mathcal{H}}{dB} > 0$, i.e. that purchasing risk-free domestic bonds weakly relaxes the borrowing constraint. Second, we focus on the zero Foreign liquidity, so $B_F \to 0$. The borrowing constraint is analogously defined for Foreign households.

Individuals' SDF $M^{(*)}(s^{t+1})$ is instead defined on individual consumption growth $\frac{C^{(*)}(s^{t+1})}{C^{(*)}(s^t)}$, which is related to aggregate consumption growth as follows:

$$\frac{C(s^{t+1})}{C(s^t)} = \frac{\delta(s^{t+1})}{\delta(s^t)} \frac{C_{t+1}(z^{t+1})}{C_t(z^t)} \quad (6)$$

where $\delta(s^t)$ satisfies the law of large numbers $\int_{\nu^t} \delta(z^t, \nu^t) d\nu^t = 1 \; \forall z^t$.

Each Home and Foreign household trades domestic and foreign risk-free real bond with returns $R(z^t)$ and $R^*(z^t)$ respectively. We begin with the case where trade is

frictionless (i.e. no borrowing constraints) and relax this later. By no-arbitrage, the household Euler implies:

$$\mathbb{E}_t[M(s^{t+1})] = \frac{1}{R(z^t)},$$

where $\mathbb{E}_t[X(s^{t+1})] = \mathbb{E}[X(s^{t+1})|z^t] = \sum_{s^{t+1}} Pr(s^{t+1}|s^t)X(s^t, s_{t+1})]$ with $Pr(\cdot)$ denoting transition probabilities. Using (6), the aggregate Euler for Home investors investing in the Home bond can be expressed as:

$$\mathbb{E}\left[\underbrace{\beta\mathbb{E}\left[\left(\frac{\delta(z^{t+1}, \nu^{t+1})}{\delta(z^t, \nu^t)} \right)^{-\gamma} \bigg| \nu^t, z^{t+1} \right]}_{\beta(z^{t+1}, \nu^t)} \times \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\gamma} \bigg| z^t \right] = \frac{1}{R(z^t)} \tag{7}$$

where $\mathbb{E}[X(s^{t+1})|\nu^t, z^{t+1}] = \sum_{\nu^{t+1}} Pr(\nu^{t+1}|\nu^t, z^t)X(v^{t+1}, z^{t+1})$ and $E[X(z^{t+1}, \nu^t)|z^t] = \sum_{z^{t+1}} Pr(z^{t+1}|z^t)X(z^{t+1}, \nu^t)$. All agents trading the Home bond must agree on its price and we define $\beta(z^{t+1}, \nu^t) \times \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\gamma}$ as the "pricer's SDF".[8]

Borrowing Constraints. In practice, markets are far from frictionless and borrowing constraints are key for generating sufficient volatility of idiosyncratic risk within countries. Suppose now that agents are constrained according to (5). We assume there exists at least one agent who is unconstrained and this will be the most "patient" consumer who values the risk-free bond the most. Then, (7) is replaced by:[9]

$$\mathbb{E}_t\left[\underbrace{\{\max_{\nu^t} \beta(z^{t+1}, \nu^t)\} \times \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\gamma}}_{\widehat{M}(z^{t+1}, s^t)} \right] = \frac{1}{R(z^t)} \tag{8}$$

Note that, in the limit where only a few agents are active, this framework relates to models of intermediation such as Hassan (2013); Gabaix and Maggiori (2015); Itskhoki and Mukhin (2021).

---

[8]This is also sometimes referred to as the "as-if" representative agent, see Werning (2015), Berger et al. (2023).

[9]In a setting without assumptions on (5), Krusell et al. (2011) derive conditions under which it is the most patient –specifically the individual with the most to lose– as opposed to a different agent who prices the assets.

The "micro" module pertaining to idiosyncratic risk (ν^t) and financial market structure can be summarized by the discount wedge $\beta(z^{t+1}, \nu^t)$ referred to as a β-wedge. Crucially, we can measure the $\beta-$ wedge from the micro-data on consumption following Berger et al. (2023).[10]

Analogously, the aggregate Euler for the Foreign bond, suppressing dependence of $\mathbb{E}_t[\cdot]$ on z^t, can be expressed as:

$$\mathbb{E}_t[\underbrace{\{\max_{\nu^t} \beta^*(z^{t+1}, \nu^t)\} M^*(z^{t+1})}_{\widehat{M}^*_{t+1}(z^{t+1}, s^t)}] = \frac{1}{R^*(z^t)} \tag{9}$$

If the *same* Home (Foreign) households also trade the Foreign (Home) bond, and the real exchange rate at time t is denoted with $\mathcal{E}(z^t)$, such that an increase in $\mathcal{E}(z^t)$ corresponds to a depreciation of Home currency, then we additionally obtain the following two Euler conditions:

$$\mathbb{E}_t\left[\widehat{M}(z^{t+1}, s^t) \frac{\mathcal{E}(z^{t+1})}{\mathcal{E}(z^t)}\right] = \frac{1}{R^*(z^t)}, \tag{10}$$

$$\mathbb{E}_t\left[\widehat{M}^*(z^{t+1}, s^t) \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] = \frac{1}{R(z^t)} \tag{11}$$

Next, we assume beta wedges, SDFs and prices are jointly log-normal. Moreover, since equations (7)-(11) depend on the current history s^t but are only uncertain with respect to the future aggregate state z^{t+1}, we drop notation on histories and denote $X(s^t) = X_t, X(z_{t+1}, s^t) = X_{t+1}$. To close the model, we pin down an exchange rate process consistent with equations (7)–(11) above, which reduces to finding an exchange rate process that satisfies:

$$var_t(\Delta e_{t+1}) = cov_t(\widehat{m}^*_{t+1} - \widehat{m}_{t+1}, \Delta e_{t+1}) \tag{12}$$

where $x = \log(X)$. International no-arbitrage requires that, regardless of the specific structure of international financial markets (i.e. complete or incomplete), exchange

---

[10]In Section 5 we show that while the $\beta-$ wedge we estimate is sufficiently volatile to rationalize exchange rate dynamics, competing models with permanent risk and integrated markets cannot deliver sufficient volatility, see e.g. Lettau (2002).

rate depreciations coincide with a period where the "pricer's" valuation of returns is low, i.e. exchange rates are risky consistent with international no-arbitrage. While this may be considered a limitation of the model, we find evidence in the data of positive comovement between exchange rates and pricers' SDFs, see Section 5.

Naturally, the process corresponding to complete international financial markets $(\Delta e_{t+1} = \widehat{m}^*_{t+1} - \widehat{m}_{t+1})$ is one candidate to satisfy (12). More generally, as shown in Backus, Foresi and Telmer (2001), the following process also satisfies equation (12):

$$\Delta e_{t+1} = \widehat{m}^*_{t+1} - \widehat{m}_{t+1} + \eta_{t+1} \tag{13}$$

where η_{t+1} is an international incomplete markets wedge which must satisfy certain conditions imposed by asset trade.[11] The special case of a representative agent economy corresponds to the limit $\tilde{\beta}^{(*)}_{t+1} \equiv \log \beta^{(*)}_{t+1} \to \log \beta^{(*)}$ which implies $\widehat{m}^{(*)}_{t+1} = m^{(*)}_{t+1}$. When international financial markets are complete and the economy features a representative agent, condition (12) restricts the covariance between relative consumption growth and exchange rate depreciations to be positive as in (1). In the data, this covariance is negative, hence the Backus-Smith *puzzle*.

For illustration, replacing the aggregate SDFs with the aggregate consumption growth and allowing for the beta wedge, the incomplete markets wedge is given by:

$$\eta_{t+1} = \log \underbrace{\left(\frac{P_{t+1}}{P_t} \frac{P^*_t}{P^*_{t+1}} \right)}_{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}} - \log \underbrace{\left(\frac{C_t}{C_{t+1}} \frac{C^*_{t+1}}{C^*_t} \right)^\gamma}_{\frac{M_{t+1}}{M^*_{t+1}}} + \left(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1} \right)$$

where P_t is the Home price level, C_t is aggregate consumption, γ^{-1} is the inter-temporal elasticity of substitution, $\tilde{\beta}_{t+1} = \log(\beta^{(*)}_{t+1})$ is the beta-wedge defined above and terms with asterisks denote the corresponding Foreign objects. The wedge, η, is often interpreted as the non-traded component of exchange rate movements or the wealth

---

[11]Cross-border trade by the pricer in Home and Foreign bonds yield restrictions (37) and (38) on η_{t+1} characterizing the risk-return trade-off for investors, see Appendix A.1. Using these conditions Lustig and Verdelhan (2019) show that international incompleteness cannot change the sign of the Backus-Smith covariance.

gap, see e.g. Pavlova and Rigobon (2007); Corsetti, Dedola and Leduc (2008, 2023)

The proposition below illustrates under what conditions imperfect risk sharing *within* countries can help reconcile the cyclicality of exchange rates with respect to the aggregate SDFs.

Proposition 1 (Two Int'l Traded Assets, Many Agents).

The two-country model with two internationally traded bonds and a continuum of heterogeneous Home consumers characterized by Equations (8)*,* (9)*,* (10) *and* (11) *delivers* $cov_t(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta e_{t+1}) < 0$ *if and only if:*

$$1 \geq -\rho_{\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1})} \tag{14}$$

where $\rho_{\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1}} \equiv \dfrac{cov_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1})}{\sigma_t(\Delta e_{t+1})\sigma_t(\tilde{\beta}_{t+1})}$.

Proof. See Appendix A.1. □

Consider the limiting case where the Foreign country is populated by a representative agent $(\beta^*_{t+1} = \beta)$. Then, Proposition 1 requires that Foreign bonds are a poor hedge for domestic *idiosyncratic* consumption risk– specifically, the $\beta-$wedge is low during periods of depreciation such that the "pricer" values returns less at times when the returns on foreign bonds are high. The poor hedging property of Foreign bonds is a possible solution to *diversification puzzle*, documented in Heathcote and Perri (2013). Intuitively, given that exchange rates must be counter-cyclical with respect to the pricers' SDFs (12), if exchange rates are sufficiently counter-cyclical with respect to the relative $\beta-$wedge, they can be pro-cyclical with respect to the SDF evaluated for aggregate consumption growth.

To illustrate what it means for exchange rates to be risky from an idiosyncratic perspective, in Section 4.1, we present two tractable models for the "pricer's" kernel, (i) a model where markets are fully integrated and yet agents do not want to trade any assets (Constantinides and Duffie, 1996), and (ii) a two agent model where the investor

13

faces a probability of becoming maximally borrowing constrained, see Krusell et al. (2011); Bilbiie (2024). In (i), Foreign bonds are risky with respect to the beta wedge if they pay returns (depreciation) when the (cross sectional) volatility of idiosyncratic income risk is low. In (ii), the beta wedge is risky because the probability of becoming constrained (and receiving a lower income) is lower in periods of depreciation. If there is also imperfect risk sharing abroad $\tilde{\beta}^*_{t+1} \neq \log \beta^*$, either Home bonds must also a poor hedge for foreigners, or if they are safe, that they are less so than Foreign bonds are risky for Home households.[12]

Limits to International Arbitrage. A prominent literature argues that limits to international arbitrage, i.e. intermediation subject to portfolio constraints Gabaix and Maggiori (2015); Itskhoki and Mukhin (2021, 2023), can explain many moments of exchange rates. To compare this mechanism to ours, we consider a variant of our model where both Home and Foreign households are subject to uncertain intermediation costs affecting their returns ($e^{u^f_{t+1}}$) *only* when they take a foreign portfolio position.[13] Allowing for costly intermediation, (14) is replaced by:

$$1 \geq -\rho_{\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1}} \geq \frac{(1 - u^f)\sigma_t(\Delta e_{t+1})}{\sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1})} \tag{15}$$

where $u^f \in [0, 1]$ is the share of exchange rate volatility stemming from intermediation shocks $var_t(u^f_{t+1}) = u^f \times var_t(\Delta e_{t+1})$. Importantly, the u^f_{t+1} shock is assumed to be orthogonal to other shocks in the economy, hence it does not affect the co-movement of relative consumption and exchange rate on its own. However, since a fraction of exchange rate volatility now does not stem from $\widehat{m}, \widehat{m}^*$, intermediation shocks dampens the threshold required, reinforcing the ability of our mechanism in resolving the Backus-Smith puzzle.

When we turn to the data in Section 5, we find that idiosyncratic risk alone seems

---

[12]Moreover, we note that (14) is a sufficient condition as long as (at least) two risk-free bonds are traded internationally and we detail a generalization of the framework with trade in risky assets in Appendix B.1.

[13]Full details of the model and derivation in Appendix B.3.

to be volatile enough to reconcile the cyclicality of exchange rates even when $u^f = 0$, and since the two mechanisms are reinforcing, we focus on the limit where there are no limits to international arbitrage for the remainder of the paper.

In the remainder of the paper, we expand on (14) both theoretically and emprically. In Section 3, we solve for equilibrium exchange rates in a two-country Lucas (1982); Cox et al. (1985) framework with beta wedges and derive conditions under which this is satisfied. In this setting we also detail that the completeness of international financial markets *also* matters for exchange rate dynamics. In Section 5, we test condition (14) using household level data on consumption and income.

2.1.   Representative agent limit $(\tilde{\beta}^{(*)} \to \log \beta)$

Before proceeding, we contrast our results to the representative agent limit where the correlation between the relative beta wedge and exchange rates is zero. In this case, the Backus-Smith covariance can *never* be negative (Lustig and Verdelhan, 2019): (14) becomes an impossibility. To get around this stark result, we show that classical contributions restrict attention to the case where only a single bond is internationally traded, i.e. Home or Foreign currency denominated.

Corollary 1 (One Int'l Traded Asset, Representative Agent No-Arbitrage).

When only Foreign bonds are internationally traded such that equations (8), (9) *and* (11) *hold, and* $\tilde{\beta}^{(*)} \to \log \beta$, *then* $cov_t(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta e_{t+1}) < 0$ *if and only if*

$$cov_t(m_{t+1}, \eta_{t+1}) + \log \mathbb{E}_t[e^{\eta_{t+1}}] \geq var_t(m^*_{t+1} - m_{t+1}) \tag{16}$$

where,

$$cov_t(m_{t+1}, \eta_{t+1}) = cov_t(m_{t+1}, \Delta e_{t+1}) - cov_t(m_{t+1}, m^*_{t+1}) + var_t(m_{t+1}) \tag{17}$$

Proof. See Appendix A.2. □

Condition (16) provides a general characterization of mechanisms in the litera-

Figure 1

(a) *Corollary 1 Statistic* (16)   **(b)** *Backus-Smith Correlation*
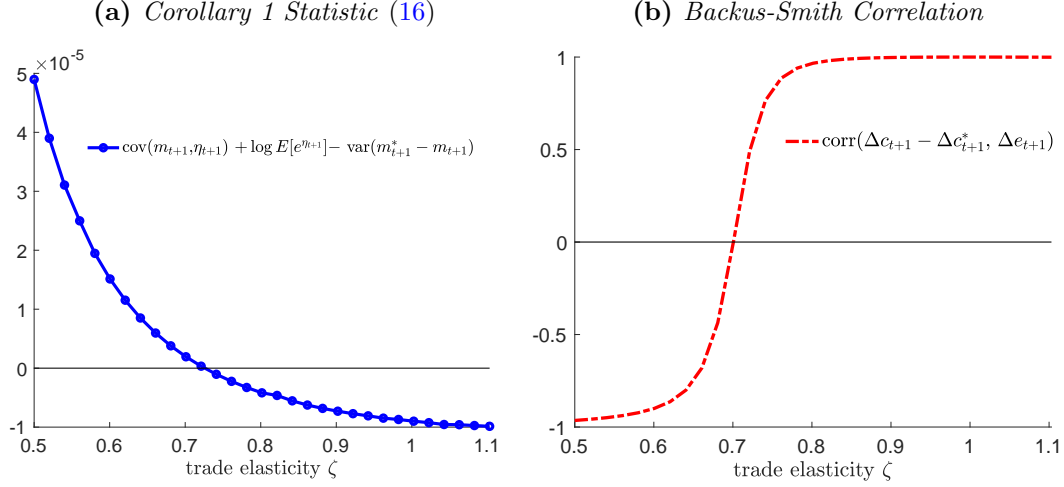
Calibration: Discount factor parameters $\omega = 0.96$, $u = 0.005$, EIS coefficient $\gamma^{-1} = 1$, trade elasticity $\zeta \in [0.5, 1.1]$, home-bias $\alpha = 0.6$, $\alpha^* = 1 - \alpha$, persistence of Home endowment shock $\rho = 0.964$, and steady state Home endowment $Y_H = 1$. Unconditional moments calculated from second-order simulation with one million draws. Within the representative agent model, we employ $\beta(C_{t-1}) = \omega C_{t-1}^{-u}$, with $\omega \in (0,1)$, as the discount factor used as a stationarity-inducing device following Bodenstein (2011), C_t is the aggregate consumption bundle at Home.

ture developed to resolve the Backus-Smith puzzle, e.g. consumption or production complementarities (Corsetti et al., 2008; Benigno and Thoenissen, 2008) reflected in (17). The RHS of (16) corresponds to the volatility of the exchange rate growth under complete markets and therefore is strictly positive. Condition (16) is satisfied if either the non-traded component η_{t+1} leads to relative price fluctuations which are sufficiently safe from the perspective of a Home investor ($cov_t(m_{t+1}, \eta_{t+1}) > 0$) or the volatility of the non-traded component is high.[14] Figure 1a plots the condition (16) from simulating a representative agent, two country, two good endowment version of our economy, allowing for internationally incomplete markets. Figure 1b displays the corresponding Backus-Smith correlation. The correlation is negative for low values of trade elasticity ζ, positive for higher values and intersects 0 at approximately the

---

[14]Equation (17) shows that non-traded risk results in relative price fluctuations which are particularly *safe* when the Home SDF is very volatile or when international comovement in SDFs is low relative to the comovement of exchange rates and the Home SDF, which is counterfactual (Brandt, Cochrane and Santa-Clara, 2006).

same value for ζ as Panel (a).[15]

In sum, in the representative agent limit, there is a stark contrast between the economy with one internationally traded asset (Corollary 1) and two assets (Proposition 1) because exchange rate risk becomes spanned when households trade in both Home and Foreign real bonds across borders, see also Chernov et al. (2024).[16] Allowing for idiosyncratic risk within countries which co-moves with the exchange rate recovers a non-traded component which can deliver negative cyclicality of exchange rates.

3. A Closed-Form Incomplete Markets Wedge

In this section, we evaluate the relationship between the relative beta wedge — which reflects imperfect risk sharing within countries— and *equilibrium* exchange rates (13). We also emphasize a separate role for international financial market incompleteness for both the cyclicality of exchange rates and their volatility. To do so, we specify laws of motion for aggregate consumption growth and the incomplete markets wedge as in Lustig and Verdelhan (2019), which builds on Cox et al. (1985, CIR henceforth). We introduce two further ingredients: first, we allow for countercyclical idiosyncratic risk as in Constantinides and Duffie (1996); Herskovic, Kelly, Lustig and Van Nieuwerburgh (2016); Berger et al. (2023) amongst others; second, using a combination of a common factor and country-specific factor, we allow for a correlation of SDFs across countries which is positive but below 1 (Brandt et al., 2006), consistent with the international comovement of asset prices and aggregates.

Suppose that a common factor z_t drives aggregate consumption growth globally

$$z_{t+1} = (1 - \rho)\theta + \rho z_t + \sqrt{z_t}u_{t+1}$$

---

[15]While SDFs and prices in the model (away from the Cole and Obstfeld (1991) are log-normally distributed only at the autarky limit, we find that the intersections in Figures (a) and (b) roughly coincide.

[16]A further limitation of these representative agents models is that they rely on a low volatility of exchange rates display very low risk premia (Lustig and Verdelhan, 2019, pp 2241).

Aggregate consumption growth in the Home country is given by $\Delta c_{t+1} = \sqrt{z_t}u_{t+1}$ while consumption growth abroad is given by $\Delta c_{t+1}^* = \xi^*\sqrt{z_t}u_{t+1} + \sigma_\nu u_{t+1}^\nu$, such that $\xi^* > 1$ ($\xi^* < 1$) corresponds to an environment where the shock u_t has a relatively larger effect on the Foreign (Home) aggregate consumption growth. Note that $\xi^* = 1$ implies $proj(\Delta c_{t+1} - \Delta c_{t+1}^*|u_{t+1}) = 0$. To prevent a perfect correlation of SDFs across countries, we allow Foreign consumption to also be driven by a country specific shock u_{t+1}^ν. Both u_{t+1} and u_{t+1}^ν are mean zero normal i.i.d innovations with unit standard deviations.

In the presence of uninsurable idiosyncratic risk, the relevant pricing kernels are instead characterized by $\widehat{m}_{t+1}^{(*)}$, defined in (8) and (9), and additionally depend on the beta wedges ($\tilde{\beta}_{t+1}^{(*)}$) for which we specify linear processes:

$$\tilde{\beta}_{t+1} = \phi\sqrt{z_t}u_{t+1}, \quad \tilde{\beta}_{t+1}^* = \phi^*(\xi^*\sqrt{z_t}u_{t+1} + \sigma_\nu u_{t+1}^\nu) \tag{18}$$

Using the $\beta-$ wedges, and definitions of SDFs in (8) and (9), we can write the Home and Foreign pricers' SDFs as follows:[17]

$$-\widehat{m}_{t+1} = \log\beta + \chi z_t + (\gamma - \phi)\sqrt{z_t}u_{t+1}, \tag{19}$$

$$-\widehat{m}_{t+1}^* = \log\beta^* + \chi^* z_t + (\gamma - \phi^*)(\xi^*\sqrt{z_t}u_{t+1} + \sigma_\nu u_{t+1}^\nu) \tag{20}$$

The volatility of the pricers' SDF is given by $var_t(\widehat{m}_{t+1}) = (\gamma - \phi)^2 z_t$, $var_t(\widehat{m}_{t+1}^*) = (\gamma - \phi^*)^2(\xi^{*2}z_t + \sigma_\nu^2)$. We define $\phi^\Delta = \phi - \phi^*\xi^*$ as the relative sensitivity of idiosyncratic risk to the common factor.

Throughout this analysis, we assume ϕ, $\phi^* < 0$, i.e. counter cyclical wedges in both countries. We also restrict attention to $\gamma - \phi^{(*)} > 0$ such that the pricer's SDF falls in response to an increase in domestic consumption growth (i.e. after an increase in u_{t+1}).

---

[17]If $\chi = \chi^* = 0$, this corresponds to power utility over consumption as in the canonical representative agent Lucas model. Allowing $\chi > 0$ is a reduced form way to capture a connection between the conditional mean and variance of SDFs which may arise due to habit formation or recursive preferences, see e.g. Hassan, Mertens and Wang (2024).

Lemma (Equilibrium Incomplete Markets Wedge)

In the model with heterogeneous consumers, satisfying (8)-(11) *and* (13), *the incomplete markets wedge is given by:*[18]

$$\eta_{t+1} = -\frac{1}{2}(\gamma(1-\xi^*) - \phi^\Delta)\sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}\ z_t$$

$$+\frac{1}{2}(\gamma - \phi^*)\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu}\ \sigma_\nu^2 - \sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}\sqrt{z_t}u_{t+1}$$

$$+\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu}\ \sigma_\nu u_{t+1}^\nu + \sqrt{\lambda - \kappa}\ \sqrt{z_t}\epsilon_{t+1} + \sqrt{\lambda^\nu - \kappa^\nu}\ \sigma_\nu \epsilon_{t+1}^\nu,$$

where $\lambda \geq \kappa, \lambda^\nu \geq \kappa^\nu$ *are parameters governing cross border spanning and*

$$\kappa = (\gamma(1-\xi^*) - \phi^\Delta)^2 - (\gamma(1-\xi^*) - \phi^\Delta)\sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda} \geq 0,$$

$$\kappa^\nu = (\gamma - \phi^*)^2 - (\gamma - \phi^*)\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu} \geq 0$$

and $var_t(\Delta e_{t+1}) = \kappa z_t + \kappa^\nu \sigma_\nu^2$.

Proof. See Appendix A.2. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

where $u_{t+1}^{(\nu)}$ are spanned innovations and $\epsilon_{t+1}^{(\nu)}$ are unspanned innovations. The exposure of the incomplete markets wedge to spanned shocks is given by $\sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}$ and $\sqrt{\lambda - \kappa}$ is the exposure to unspanned shocks. To illustrate the effects of incomplete spanning, we parametrize $\lambda = \alpha \times (\gamma(1-\xi^*) - \phi^\Delta)^2$ for $\alpha \in (0, 1)$. It can be verified that $\alpha \to 1$ coincides with the complete markets benchmark such that $\lambda_{CM} = \kappa_{CM} = (\gamma(1-\xi^*) - \phi^\Delta)^2$. Instead, $\alpha < 1$ implies increasing levels of unspanned risk. We similarly define $\lambda^\nu = \alpha^\nu(\gamma - \phi^*)^2$ and then $\lambda_{CM}^\nu = \kappa_{CM}^\nu = (\gamma - \phi^*)^2$. When both $\alpha, \alpha^\nu \to 1$, necessitated by trade in additional risky assets (see Appendix B.1), the wedge tends to zero $\eta_{t+1} = 0$.

Armed with a closed form expression for the international incomplete markets

---

[18]Square roots refer to the positive root only unles we include the $\pm$ ahead. For the z_t factor, while the negative root is a potential solution, conditional on $\phi < 0$, it violates $\lambda > \kappa$ which we require for a real solution.

wedge, we revisit moments of exchange rates conditional on $u_{t+1} = u_{t+1}^\nu = 1$.

Proposition 2 (Exchange rate cyclicality in Cox et al. (1985) model)

In the model with heterogeneous consumers, satisfying (8)-(11) and (13), using the processes for SDFs (19) and (20), $cov_t(\Delta c_{t+1} - \Delta c_{t+1}^*, \Delta e_{t+1}) < 0$ if and only if:

$$(1 - \xi^*) \left[(\gamma(1 - \xi^*) - \phi^\Delta \right] \leq \underbrace{-(\gamma - \phi^*)\tilde{\alpha}\frac{\sigma_\nu^2}{z_t}}_{\geq 0}, \tag{21}$$

where $\tilde{\alpha} = \frac{1-\sqrt{1-\alpha^\nu}}{1-\sqrt{1-\alpha}}$ and $\lim_{\alpha^\nu \to 0} \tilde{\alpha} = 0$.

Proof. See Appendix A.2. $\qquad\square$

Condition (21) ensures that, in equilibrium, the correlation between $\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*$ and Δe_{t+1} is both negative and larger than the volatility of exchange rates, i.e. foreign bonds are relatively riskier for the domestic consumer. The model is able to deliver a negative aggregate Backus-Smith coefficient and can accommodate counter-cyclical idiosyncratic risk in one or both countries.

It is instructive to begin with the limit where the country-specific shock is small ($\sigma_\nu^2 \to 0$). Suppose that $\xi^* > 1$ such that following $u_{t+1} \uparrow$, Foreign consumption growth outpaces Home consumption growth $(\Delta c - \Delta c^*) \downarrow$. Then, in order to generate an exchange rate depreciation $\Delta e_{t+1} \uparrow$, we require $\phi^\Delta \leq \gamma(1 - \xi^*) \leq 0$, implying Home idiosyncratic risk is more counter-cyclical than Foreign. This implies that the Home $\beta-$wedge falls relatively more than the Foreign $\beta-$wedge. Under Constantinides and Duffie (1996) interpretation, a higher Foreign beta wedge implies Foreign consumers remains relatively more risk averse than Home such that exchange rate compensates Foreign consumers for their risk-aversion and their increased wealth from positive u shock doesn't get redistributed to Home consumers.

Conversely, if $\xi^* < 1$, Home aggregate consumption rises in relative terms $(\Delta c - \Delta c^*) \uparrow$. For exchange rates to appreciate $\Delta e_{t+1} \downarrow$, the model requires that $\phi^\Delta > \gamma(1 - \xi^*) > 0$ (either due to procyclical idiosyncratic risk or because $\phi^*\xi^* < \phi < 0$),

such that the Home beta wedge remains high, i.e. valuations of returns remain high due to idiosyncratic risk at a time when losses occur on foreign portfolios. In both cases, the country experiencing faster consumption growth must have a relatively muted fall in the $\beta-$wedge.[19]

Consider the empirically relevant case where idiosyncratic risk is counter-cyclical $sign(\phi) = sign(\phi^*) < 0$ and consumption growth is positively correlated across countries ($\xi^* > 0$). Once we take into account the equilibrium exchange rate process, this rules out the possibility that both Foreign (Home) bonds are risky with respect to idiosyncratic states for Home (Foreign) households. Instead, the cases above describe under what conditions exchange rates are relatively *riskier* for the Home country.

The Role for Market Incompleteness. Away from the limit $\sigma_\nu \to 0$, the cyclicality of exchange rates critically depends on the degree of international financial market completeness. This is because country-specific shocks always induce a positive Backus Smith covariance as long as $\gamma - \phi^{(*)} > 0$.[20] In the limit where volatility of country specific shocks is large enough, the Proposition 2 can never be satisfied for values of $\alpha^\nu > 0$. As $\alpha^\nu \to 0$, the country-specific shock has zero effect on the volatility of exchange rate (and therefore the covariance). The shock simply serves to induce an imperfect correlation of consumption growth across countries. The Backus Smith covariance can be resolved in response to these combined shocks under same conditions as when $\sigma_\nu = 0$. Allowing for positive α_ν adds to volatility of exchange rates, while also directly reducing the ability of the model to resolve the Backus Smith covariance.

Additionally, the cyclicality and the volatility of exchange rates are intricately connected. While introducing idiosyncratic risk can recover negative Backus Smith co-

---

[19]Contrast this to the representative agent limit (Lustig and Verdelhan, 2019), which coincides with $\phi^\Delta = 0$. The LHS of condition (21) reduces to $\gamma(1 - \xi^*)^2$ which is strictly positive, therefore $cov_t(\Delta c_{t+1} - \Delta c_{t+1}^*, \Delta e_{t+1}) > 0$.

[20]It is important to note there is nothing special about the Foreign country specific shock, and these results will generally apply to any uncorrelated factor driving only the volatility of consumption in one country. In Appendix A.3, we show the same results follow if an uncorrelated shock is added to the Home SDF instead.

variance, in the next Proposition we show that this is at the cost of further exacerbating exchange rate volatility.

Proposition 3 (Idiosyncratic Risk and Exchange Rate Volatility)

Assume $\gamma(1 - \xi^*) - \phi^\Delta > 0$. Exchange rate volatility is higher under internationally complete markets ($\kappa^{CM} > \kappa^{IM}$ and $\kappa^{\nu CM} > \kappa^{\nu IM}$) and is also more sensitive to the cyclicality of idiosyncratic risk $\left(\frac{d\kappa^{CM}}{d(-\phi^\Delta)} > \frac{d\kappa^{IM}}{d(-\phi^\Delta)} > 0 \text{ and } \frac{d\kappa^{\nu CM}}{d(-\phi^*)} > \frac{d\kappa^{\nu IM}}{d(-\phi^*)} > 0 \right)$

Proof. See Appendix A.2. □

In the calibration we pursue below, if SDFs are sufficiently volatile to explain (e.g.) stock prices, models with complete markets struggle to reconcile the low exchange rate volatility observed in the data.[21]

3.1. Quantitative Exercise

Calibration We calibrate z_t using the process for U.S. interest rates as in (Backus et al., 2001). Starting from: $r_t = [\chi - \frac{1}{2}(\gamma - \phi)^2]z_t$, we choose $\chi = -1 + \frac{1}{2}(\gamma - \phi)^2$ to deliver counter-cyclical interest rates needed to match the uncovered interest parity puzzle (Verdelhan, 2010; Lustig and Verdelhan, 2019). We use average U.S. interest rates of 2.67% over 1991Q1–2017Q12, sample consistent with our later empirical exercises, to calibrate $\mathbb{E}_t[z_{t+1}] = \theta = 0.002225$ (monthly) and select $\log \beta^{(*)}$ to match average interest rates. We target a correlation of SDFs across countries of 0.8 (Brandt et al., 2006) which implies $\rho_{\widehat{m},\widehat{m}^*} = 0.8 = \theta\xi^*/\sqrt{\theta}\sqrt{\theta\xi^* + \sigma_\nu}$, from which, given ξ^* discussed below, we back out $\sigma_\nu^2 = 0.00168$. We take $\gamma = 5$ as a conservative estimate from Best et al. (2020); Di Tella, Hébert, Kurlat and Wang (2023) who find an elasticity of substitution between 0.1 and 0.2 best explains individual level data. We calibrate the asymmetry in $\gamma - \phi$ and $\gamma - \phi^*$, using Sharpe ratios on equity from the

---

[21]Consistent with Brandt et al. (2006), only in the limit where we shut down σ_ν such that SDFs are (counterfactually) perfectly correlated, can internationally complete markets be consistent with observed exchange rate volatility.

Figure 2

(a) *Backus-Smith Covariance*   **(b)** *Exchange Rate Volatility*
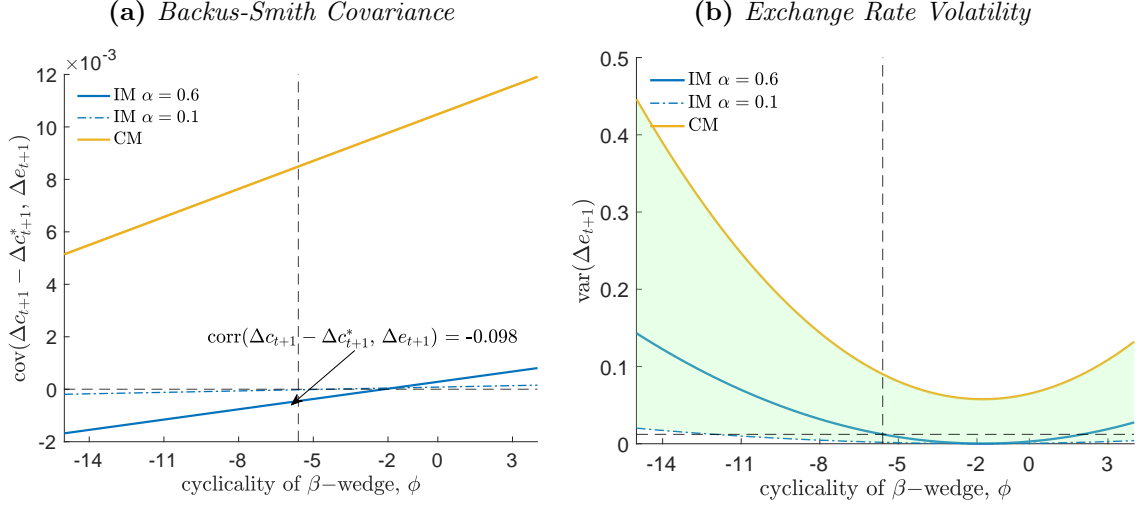
Solid yellow line reflects complete markets equilibrium, dotted blue line corresponds to incomplete markets with spanning summarised by $\alpha = 0.1, \alpha^\nu = 0.01$. The solid blue line is an example which seems to fit the data well corresponding to $\alpha = 0.6$. Horizontal dashed lines reflect the zero-line in the left panel, $var(\Delta e)$ from the data in the right panel. Vertical dashed lines mark the calibration for ϕ in both panels.

U.S. and abroad. We use a Sharpe ratio of 0.5 for the U.S. based on (S&P 500).[22] Using $(\gamma - \phi)\sqrt{\theta} = 0.5$, we back out $\phi = -5.6$ which is comparable to the calibration in Acharya, Challe and Dogra (2023) for the U.S. We repeat the exercise abroad with a Sharpe ratio of 0.4, which implies $\phi^* = -0.848$.

Calibrating the degree of international spanning is a challenge. We assume that the Foreign shock is unspanned $\alpha^\nu = 0.01$, and we choose $\alpha = 0.6$ to match exchange rate volatility. The remaining parameter is ξ^*, which must be greater than 1, given other parameters, to deliver a negative Backus-Smith covariance (see Proposition 2). We choose a $\xi^* = 1.16$ which results in a correlation of consumption differentials and exchange rates depreciation of approximately -0.1, consistent with the cross-country average of our sample, see Table 3.

Figure 2 illustrates the main results, plotting the relationship between the Home beta wedge cyclicality ϕ, the Backus-Smith covariance and exchange rate volatility for

---

[22]These Sharpe ratios are comparable to the literature and slightly higher than the historical averages in Jordà, Schularick, Taylor and Ward (2018).

three levels of spanning. Complete markets $\alpha = \alpha^\nu = 1$, our preferred calibration of intermediate spanning $(\alpha = 0.6, \alpha^\nu = 0.01)$ and, for comparison, very little spanning $(\alpha = 0.1, \alpha^\nu = 0.01)$. The left panel shows that for sufficiently counter-cyclical idiosyncratic risk the Backus-Smith covariance is negative, consistent with Proposition 2. While spanning of the common factor (higher values of α) leads to more negative values for the covariance, spanning of the Foreign shock implies even lower values of ϕ are required to generate a negative covariance.

The right panel illustrates that the exchange rate volatility under internationally complete markets is too high when we realistically calibrate to an imperfect correlation of SDFs across countries, consistent with the literature. This problem is exacerbated further as ϕ becomes increasingly negative– resulting in extremely high volatility when international markets are complete. However, any level of volatility in the shaded area can be attained by varying α and α^ν. Our calibration of $\alpha = 0.6$ targets the empirical exchange rate volatility of 0.11, which is the cross-country average standard deviation of bilateral exchange rates in our sample, see Section 5.

Figure 3 in Appendix B.4 illustrates comparative statics with respect to spanning of u^ν (i.e. varying α^ν parameter). Larger values of α^ν reduce the ability of the model to generate negative Backus Smith covariance and also increase exchange rate volatility. Consistent with the analysis above, the Figure shows international market incompleteness is crucial for both the cyclicality and the volatility of exchange rates when we allow for imperfect risk-sharing within countries. Figure **??** in Appendix B.4 plots the associated Backus Smith correlation and shows comparative statics with respect to market incompleteness of u^ν shock.

4. Equilibrium in a Two-Country Two-Good Economy

Before turning to the micro-data, we address three further issues for the equilibrium model underlying our analysis so far. In Section 4.1, we provide two tractable examples

for the equilibrium $\beta-$ wedge, or the pricer's SDF. The first is based on a model with no borrowing constraints while the latter allows for occasionally binding borrowing constraints and focuses on the limit where these are maximally binding. In Section 4.2, we address international goods market clearing. Section 4.3 highlights the relevance of the Backus-Smith condition for international risk sharing in our environment with heterogeneity within countries.

4.1. Two Models for the Stochastic Discount Factor

Constantinides and Duffie (1996)   The first tractable example we look at is one where financial markets are perfectly integrated, i.e. there are no borrowing constraints. Following Constantinides and Duffie (1996), we construct a no-trade equilibrium where all agents choose to consume their endowment. We assume individual endowments are given by $I(s^t) = \delta(s^t)C(z^t) + D(z^t)$, where $D(z^t)$ denotes the aggregate dividend in the economy. We adopt the following process $\frac{\delta(z^{t+1},\nu^{t+1})}{\delta(z^t,\nu^t)} = \exp(\xi(\nu^{t+1})\sqrt{y(z^{t+1})} - y(z^{t+1})/2)$ where $\xi(\nu^{t+1})$ are the uninsurable idiosyncratic shocks which are distributed as standard normal for all individual histories ν^t, independently distributed from the aggregate state z_t; and y_t is interpreted as cross-sectional volatility of idiosyncratic risk.[23] No-arbitrage pricing requires that the Euler equations for household i investing in an arbitrary security with return $\tilde{R}(z^{t+1})$:

$$\mathbb{E}\left[\beta e^{y(z^{t+1})\frac{\gamma(\gamma+1)}{2}}\left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\gamma}\tilde{R}(z^{t+1})\right] = 1 \tag{22}$$

where $\tilde{\beta}_{t+1} = \log\left(\beta e^{y(z^{t+1})\frac{\gamma(\gamma+1)}{2}}\right)$. Since all agents agree on the valuation of assets, standard arguments imply $C(s^t) = I(s^t)$ i.e. agents consume their own endowments and there is no trade in equilibrium.

In this environment, a risky foreign return implies that the volatility of permanent

---

[23]The law of large numbers follows from properties of the normal distribution for ξ. Treating y_{t+1} as a constant, and using the moment generating function $M_{xi}(h) = e^{h\xi}$ for $h \in \mathbb{R}$, $\mathbb{E}[e^{\xi\sqrt{y}-y/2}] = M_\xi(h)e^{-y/2} = e^0 = 1$.

risk is low in periods of depreciation, i.e. foreign bonds yield higher return at times when households face low idiosyncratic risk:[24]

$$cov(\tilde{\beta}_{t+1}, \Delta e_{t+1}) < 0 \implies cov(y_{t+1}, \Delta e_{t+1}) < 0 \tag{23}$$

Krusell et al. (2011), Bilbiie (2024). The second example, considers a model where households earn high ($v_t = h$) or low ($v_t = l$) levels of income, facing an exogenous probability $1 - s$ of becoming a low type (Krusell et al., 2011) which we allow to be a function of (aggregate) output $s(Y(z^t))$ (Bilbiie, 2024). In equilibrium, when these households have a low income draw they would like to borrow, but we restrict focus on the zero liquidity limit (binding borrowing constraints) which precludes this smoothing.

The first-order condition for the Home saver purchasing a domestic risk free bond is given by:

$$1 = R(z^t) \, E_t \left[\beta \, \frac{s(z^{t+1}) \left(\frac{C(z^{t+1},h)}{C(z^{t+1})} \right)^{-\gamma} + (1 - s(z^{t+1})) \left(\frac{C(z^{t+1},l)}{C(z^{t+1})} \right)^{-\gamma}}{\left(\frac{C(z^t,h)}{C(z^t)} \right)^{-\gamma}} \times \left(\frac{C(z^t)}{C(z^t)} \right)^{-\gamma} \right],$$

where $C(z^{t+1}, l)$ is the consumption of (low type) constrained households and $C(z^{t+1}, h)$ is that of the (high type) saver. The $\beta-$wedge, which premultiplies marginal utility growth from aggregate consumption, arises from time-varying probability of becoming a low type, and the difference in the marginal utility of consumption across two states. Because of incomplete domestic markets, marginal utility in the low state is higher than marginal utility in the high state, therefore the saver attaches a premium on the risk-free bond.[25] Similarly, the saver also attaches a premium on foreign bonds, adjusted for exchange rate risk as per equation (10).

For further illustration, we assume a transfer scheme such that saver's consumption

---

[24]In Section 5 we use estimates of permanent risk from Bayer et al. (2019) and find that it is not volatile enough to satisfy (14), highlighting the role of transitory risk or borrowing constraints described in the second example.

[25]One could add further frictions to the model by, e.g., making foreign bonds less liquid in the low state, generating "convenience yield" properties for the domestic bond, see for example Di Tella et al. (2024).

is $C(z^{t+1}, h) = \omega C(z^{t+1})$, and hand-to-mouth agents' consumption is $C(z^{t+1}, l) = (1 - \omega)C(z^{t+1})$ with $0.5 < \omega \leq 1$ so that saver's consumption is larger than the hand-to-mouth household's consumption.[26] The beta wedge simplifies to: $\tilde{\beta}_{t+1} = \log\left(\beta\left[s(z^{t+1})\left(1 - \left(\frac{1-\omega}{\omega}\right)^{-\gamma}\right) + \left(\frac{1-\omega}{\omega}\right)^{-\gamma}\right]\right)$. In this environment, a risky foreign return $cov(\tilde{\beta}_{t+1}, \Delta e_{t+1}) < 0$ implies that the probability of becoming hand to mouth $1 - s(z^{t+1})$ is low in periods of depreciation:

$$cov(\tilde{\beta}_{t+1}, \Delta e_{t+1}) < 0 \implies cov(s, \Delta e_{t+1}) > 0 \tag{24}$$

4.2. International Equilibrium and Goods Market Clearing

Taking any foreign aggregate endowment process as given, we select a process for aggregate domestic endowments such that agents optimally choose not to trade across borders. More generally, the endowment processes we consider can be interpreted as the wealth of consumers after all gains from trade have been exhausted.[27] The adjusted risk-sharing condition is then given by:

$$\mathbb{E}_t\left[\beta_{t+1}(C_{t+1})\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = \mathbb{E}_t\left[\beta_{t+1}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\right]. \tag{25}$$

Replacing C_t with $Y_t = \int_i Y_t^i$ and C_t^* with $Y_t^* = \int_{i^*} Y_t^{i\,*}$, (25) implies equalization of the autarky rates across countries resulting in a no trade equilibrium (Svensson, 1988).

Goods Market Equilibrium. Real exchange rate fluctuations necessitate differentiated consumption bundles across countries. The Home consumption bundle and associated price index are given by

$$C_t^i = \left[\alpha^{\frac{1}{\zeta}}C_{H,t}^{i\,\frac{\zeta-1}{\zeta}} + (1-\alpha)^{\frac{1}{\zeta}}C_{F,t}^{i\,\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \quad P_t = \left[\alpha P_{H,t}^{\frac{\zeta-1}{\zeta}} + (1-\alpha)P_{F,t}^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}} \tag{26}$$

---

[26]As well as improving tractability, this transfer scheme also eliminates composition-driven explanations for Backus Smith covariance (e.g. Kollmann, 2012).

[27]We follow the zero liquidity approach popular in the heterogeneous agents literature, e.g. Krusell et al. (2011); Werning (2015); Challe (2020); Bilbiie (2024) among others; for its analytical tractability in characterizing SDFs and prices.

where ζ is the trade elasticity and α is the measure of home-bias. Abroad, Foreign is defined symmetrically, with home bias $1 - \alpha$. Foreign analogues C^{i*} and P^* are defined symmetrically. The real exchange rate is given by $\mathcal{E} = P^*/P$.

The equilibrium characterized by (25) does not always lie on the static Pareto frontier, i.e. is not consistent with consumption bundle optimization at prices given by (26) (see Appendix B.5.1), but can always be supported with static goods market wedges, isomorphic to home bias shocks (Pavlova and Rigobon, 2007; Gabaix and Maggiori, 2015), described below.[28]

Proposition 4 (Goods Market Clearing)

Given processes for $\{Y_t, Y_{H,t}, Y_{F,t}, \tau_t\}$ and the Pareto Frontier $\{C_H(C_t), C_F(C_t)\}$, goods markets are efficient and clear if and only if:

$$\left[Y_t^{\frac{\zeta-1}{\zeta}} - \alpha^{\frac{1}{\zeta}} C_H(Y_t)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \left(\widehat{b} Y_{H,t} + (1 - \widehat{b}) C_H(Y_t) \right) = C_H(Y_t) Y_{F,t} (1 - \alpha)^{\frac{1}{\zeta-1}} \quad (27)$$

where $\widehat{b} = (\frac{\alpha}{1-\alpha})^2 (1 + \tau_t)^\zeta$ and $\tau_t > -1$. At the limit of full home bias $\alpha \to 1$ $\tau_t = 0$ satisfies the condition.

4.3. Backus-Smith, Risk Sharing and Welfare

We conclude our theoretical exposition by discussing the relevance of the Backus-Smith condition as a measure for risk sharing and welfare in our environment featuring i) heterogeneity within countries, ii) static trade wedges. To do so, we compare the decentralized allocation of our model to a planner problem, detailed in Appendix B.5. In general, the Backus-Smith condition shown in equation (1) characterizes the planner's optimal allocation of consumption across countries with $\beta_{t+1} = \beta^*_{t+1} = \beta$ is the first best outcome when the planner can distribute income uniformly across agents

---

[28]While this may sound undesirable, such wedges are non-zero in the data and play a significant role in impeding risk sharing (Fitzgerald, 2012; Bodenstein, Cuba-Borda, Gornemann and Presno, 2024). Moreover, as illustrated in Section 2, and consistent with Lustig and Verdelhan (2019), such goods markets frictions alone are unable to explain the Backus Smith puzzle with trade in many assets.

within each country. As we show in the Appendix, trade wedges implied by equation (27) are an additional impediment to risk-sharing, but if they are technological and also faced by the planner, deviations from equation (1) reflect deviations from a constrained efficient allocation. In the case where the planner is unable (or unwilling) to eliminate heterogeneity within countries, detailed in Appendix B.5, (1) is no longer the sufficient measure for risk sharing.

5. Empirical Evidence

In this section, we measure the discount factor wedge from the data for U.S. households following Berger et al. (2023), henceforth BBD. Then we assess the plausibility of our mechanism for reconciling international risk sharing patterns by assessing the plausibility of condition (14): how risky are exchange rates with respect to the relative $\beta-$wedge?

Wedge measurement. Beginning with the U.S., we use the Consumption Expenditure Survey. We want to measure the discount factor wedge defined in equations (7) and (8) which requires constructing the conditional expectation over idiosyncratic states of marginal utility growth for a currently unconstrained individual facing idiosyncratic risk. We take this measurement to the data following BBD who measure this wedge using cross-sectional average of marginal utility growth of currently unconstrained individuals. First, for each household i, we define the consumption share $\varphi_{it} = \frac{C_t^i}{C_t}$ and construct the growth rate of relative marginal utility of a household as $\left(\frac{\varphi_{it+1}}{\varphi_{it}}\right)^{-\gamma}$.[29] Then, at each date t, we collect households with similar levels of income and net worth into distinct groups. For each group of households, we compute a $\beta-$wedge as the average across N_g individuals in group g:

---

[29]As in BBD, we use residualized log consumption constructed by partialling out fixed effects for sex, race, education, age of the head of household, and the state of residence. All nominal variables are converted into 2000 dollars with CPI-U.

Table 1: *Summary Statistics for $\tilde{\beta}_g$*

γ (Inverse of EIS)	1	5	7.5	10
$\mathrm{Corr}(\tilde{\beta}_g, \Delta \log Y)$	-0.05	-0.61	-0.60	-0.61
$\mathrm{Corr}(\tilde{\beta}_g, \Delta \log C)$	0.03	-0.54	-0.55	-0.56
$\sigma(\tilde{\beta}_g)$	0.04	1.29	2.21	3.05

$$\beta_{g,t+1} \equiv \frac{1}{N_g} \sum_{i=1}^{N_g} \left(\frac{\varphi_{it+1}}{\varphi_{it}} \right)^{-\gamma} \tag{28}$$

Finally, we take logs and demean such that $\tilde{\beta}_{g,t+1} \equiv \log (\beta_{g,t+1}) - \overline{\log(\beta_g)}$.

In the presence of borrowing constraints, the theory suggests to select the maximal $\beta-$wedge, see (8). As in BBD, we choose high income, low net worth individuals who are likely to be unconstrained today, but face the possibility of becoming constrained in the future. As such, these households have a high incentive to save for precautionary reasons. We deviate from BBD who assume an EIS of 1 and we construct wedges for $\gamma \in \{5, 7.5, 10\}$. Best et al. (2020) find that $\gamma = 10$ best fits household level data and this is further supported by evidence in Di Tella et al. (2023). We use $\gamma = 5$ in our baseline and report robustness in Appendix C. For the U.S., we obtain an annual time-series of $\tilde{\beta}_{g,t+1}$ spanning 1992–2017. Table 1 provides summary statistics on the cyclicality and volatility of this wedge.

Measurement of the $\beta-$wedge requires panel data on consumption, income, and a measure of net worth. We are not aware of similar micro-data for other country from where we can construct similar wedge. We proceed in two ways. First, we present results assuming that time-varying idiosyncratic risk in the U.S. only, i.e. $\tilde{\beta}_{t+1}^* = \log \beta$ for the foreign country. Second, Section 5.1 generalizes to allow for idiosyncratic risk abroad using cross-country distributional income statistics from the Global Repository in Income Dynamics, provided by Guvenen et al. (2022).

We construct real exchange rates from annual nominal exchange rates (US dollar/ Foreign currency) and consumer price indices using the Macro-History database (Jordà,

Schularick and Taylor, 2017) for seventeen advanced economies.

Results. We highlight five empirical findings. First, using U.S. household data on consumption, the constructed β−wedge is sufficiently negatively correlated with depreciations and sufficiently volatile relative to exchange rates for condition (14) to be satisfied, giving support to our main results. Second, we show that depreciations correlate positively with the relative pricing kernels when adjusting for this β−wedge, even though they correlate negatively with pricing kernels constructed only on aggregate consumption, consistent with no-arbitrage from international trade in two risk-free assets (eg. (12)). Third, we highlight that the correlation with depreciations and variance of the wedge are not driven by a composition effect (i.e. time-varying relative consumption of pricers) but arise because of within-group dispersion capturing risk. Fourth, we show that an alternative construction of the β−wedge relying on permanent risk (22) cannot deliver our results, and we rely on transitory risk and borrowing constraints. Last but not least, we leverage international household-level income data and show the constructed bilateral β−wedges also satisfy condition (14).

Table 2 reports the key moments in the data relevant for Proposition 1 (condition 14). Namely, the (negative of the) correlation of the wedge with real exchange rate growth and the threshold $\frac{\sigma(\Delta e)}{\sigma(\beta)}$, for seventeen advanced economies. We report both conditional and unconditional moments. To construct conditional moments we control for date t values of relative log consumption in US and Foreign ($c_t - c_t^*$), relative short term nominal interest rate ($i_t - i_t^*$), relative CPI based inflation rates ($\pi_t - \pi_t^*$), and log bilateral real exchange rate (e_t).

For fifteen of the seventeen bilateral pairs, the exceptions being Japan and Switzerland, we find that bilateral exchange rates are risky with respect to the $\tilde{\beta}$ wedge – i.e. a negative correlation indicating depreciations tend to happen during periods of low valuation by the pricer ($\tilde{\beta}$). Our results are robust to lower degrees of risk aversion (specifically any $\gamma \geq 3$), to alternate conditioning sets, and are also similar

Table 2: *Empirical Moments: Real Exchange Rate Growth and Berger et al. (2023) Wedge*

ISO	Unconditional		Conditional	
	$-\mathrm{Corr}(\tilde{\beta}, \Delta e)$	Threshold	$-\mathrm{Corr}_t(\tilde{\beta}, \Delta e)$	$\mathrm{Threshold}_t$
AUS	0.05	0.10	0.07	0.08
BEL	0.24	0.08	0.25	0.09
CAN	0.28	0.08	0.14	0.07
CHE	-0.03	0.07	-0.01	0.06
DEU	0.23	0.08	0.31	0.07
DNK	0.25	0.08	0.36	0.09
ESP	0.32	0.09	0.19	0.08
FIN	0.25	0.09	0.25	0.08
FRA	0.26	0.08	0.27	0.08
GBR	0.36	0.08	0.58	0.08
IRL	0.23	0.07	0.06	0.06
ITA	0.36	0.09	0.23	0.09
JPN	-0.39	0.08	-0.44	0.08
NLD	0.25	0.08	0.17	0.09
NOR	0.23	0.09	0.35	0.09
PRT	0.30	0.09	0.06	0.08
SWE	0.28	0.10	0.24	0.08
AVERAGE	0.20	0.08	0.18	0.08

from a pooled regression with country fixed effects instead of constructing conditional moments using country level regressions. See Appendix Section C.1.[30]

Next, we show that while the Backus-Smith correlation is, on average, slightly negative in our sample consistent with the literature, we find that an adjusted condition defined on the pricer's SDF delivers a positive correlation providing support for our theoretical framework, i.e. (7)-(11). We construct the as-if representative agent log SDF in the U.S. as implied by the model: $\widehat{m} = -\gamma \Delta c + \tilde{\beta}$ and continue to assume the Foreign SDF is the scaled consumption growth $m^* = -\gamma \Delta c^*$. A positive correlation provides support for a framework where there is international no-arbitrage for the pricers' kernels (12).

Table 3 lists correlations for all seventeen bilateral pairs, and cross-country average is noted in the last row. Consistent with theoretical prediction, the correlation of

---

[30]We briefly discuss why our results are robust to existing concerns relating to incomplete market based explanations (e.g. Kocherlakota and Pistaferri (2007)), see Kollmann (2021): (i) the discount factor wedge can be more volatile than exchange rates due to incomplete international markets, see Section 3, and (ii) Table 2 shows that correlation between the wedge and exchange rates is sufficiently higher than the relevant threshold in condition (14).

Table 3: *Correlations between Pricing Kernels and Real Exchange Rate Growth*

	Unconditional		Conditional	
ISO	$\mathrm{Corr}(\Delta c - \Delta c^*, \Delta e)$	$\mathrm{Corr}(m^* - \widehat{m}, \Delta e)$	$\mathrm{Corr}_t(\Delta c - \Delta c^*, \Delta e)$	$\mathrm{Corr}_t(m^* - \widehat{m}, \Delta e)$
AUS	-0.39	0.03	-0.40	0.06
BEL	-0.30	0.23	-0.44	0.23
CAN	-0.12	0.27	-0.18	0.14
CHE	-0.09	-0.03	-0.23	-0.02
DEU	0.08	0.23	-0.03	0.30
DNK	-0.34	0.22	-0.42	0.33
ESP	-0.01	0.32	-0.00	0.19
FIN	-0.38	0.22	-0.32	0.23
FRA	-0.25	0.25	-0.34	0.25
GBR	0.07	0.37	-0.09	0.58
IRL	0.08	0.24	0.52	0.09
ITA	0.01	0.35	0.06	0.23
JPN	0.19	-0.37	0.05	-0.44
NLD	0.19	0.26	-0.12	0.17
NOR	-0.27	0.21	-0.63	0.33
PRT	0.21	0.31	0.09	0.07
SWE	-0.27	0.27	-0.28	0.23
AVERAGE	-0.09	0.20	-0.16	0.18

relative SDF with exchange rate growth is positive for all pairs with the exception of Japan and Switzerland. However, our results fail the stricter test for the theory would be that the covariance of the relative SDFs is exactly equal to variance of the real exchange rate growth (12), suggesting need for a richer structure of idiosyncratic risk.[31]

Composition vs. Risk. We decompose our main findings further to see if our mechanism is driven by a composition effect (i.e. the changing relative consumption of the pricer) or risk term captured by the within-group dispersion. As shown in BBD, the beta wedge can be decomposed into following terms:

$$\underbrace{\log \frac{1}{N_g} \sum_{i=1}^{N_g} \left(\frac{C_{t+1}^i/C_t^i}{C_{t+1}/C_t} \right)^{-\gamma}}_{\tilde{\beta}} = \underbrace{\log \left(\frac{C_{t+1}^g/C_t^g}{C_{t+1}/C_t} \right)^{-\gamma}}_{\text{Composition term } \tilde{\beta}^C} + \underbrace{\log \frac{1}{N_g} \sum_{i=1}^{N_g} \left(\frac{C_{t+1}^i/C_t^i}{C_{t+1}^g/C_t^g} \right)^{-\gamma}}_{\text{Jensen's term } \tilde{\beta}^J},$$

---

[31]In appendix Table 12, we report these covariances of pricing kernels with exchange rate growth for detail.

where C^i denotes individual consumption and C^g denotes the group average of consumption. To disentangle the two channels, we construct the beta wedge using the consumption of an average unconstrained agent (C^g) thus capturing only the composition channel, and we also consider the residual Jensen's term. Table 4 reports the correlations of the respective wedges with exchange rates and the thresholds required for Proposition 1 to be satisfied.

Table 4: *Decomposition of the Berger et al. (2023) Wedge*

ISO	Jensen's		Composition	
	$-\mathrm{Corr}(\tilde{\beta}^J, \Delta e)$	Threshold	$-\mathrm{Corr}(\tilde{\beta}^C, \Delta e)$	Threshold
AUS	0.03	0.10	0.29	1.37
BEL	0.22	0.08	0.29	1.08
CAN	0.25	0.08	0.37	1.00
CHE	-0.06	0.07	0.32	0.96
DEU	0.22	0.08	0.28	1.07
DNK	0.23	0.08	0.30	1.06
ESP	0.30	0.09	0.35	1.19
FIN	0.23	0.09	0.32	1.22
FRA	0.24	0.08	0.29	1.07
GBR	0.35	0.08	0.28	1.12
IRL	0.21	0.07	0.29	0.89
ITA	0.34	0.09	0.27	1.17
JPN	-0.39	0.09	-0.04	1.13
NLD	0.23	0.08	0.29	1.10
NOR	0.20	0.10	0.36	1.26
PRT	0.28	0.09	0.28	1.14
SWE	0.26	0.10	0.42	1.35
AVERAGE	0.18	0.09	0.29	1.13

While an explanation based solely on composition of spending by households on the Euler equation offers the right sign, the time-variation in the composition of spending based wedge is insufficiently volatile to be a driver of negative consumption exchange rate co-movement across countries, resulting in high thresholds which cannot be met (final column). Instead, within-group variation in marginal utility growth appears to

be the main driver behind satisfying condition (14) (see second and third columns).

Permanent Income Risk   The literature on the asset pricing implications idiosyncratic risk initially focused on settings with no borrowing constraints and permanent income risk, e.g. Constantinides and Duffie (1996) and others. We use a measure of permanent income risk constructed by Bayer et al. (2019) using the Survey of Income Participation Program in the U.S. While we once again find that while the correlation of the permanent income risk based wedge co-moves negatively with exchange rates, the volatility of permanent risk is orders of magnitude too small to satisfy (14) in order to reconcile the Backus Smith puzzle. This echoes earlier negative findings by Lettau (2002) on the resolution of equity premium puzzle in the U.S with such measures of idiosyncratic risk. We report the results in Appendix C.2.

These results, on the inability of the composition channel and the permanent risk based wedge to satisfy condition (14), lend support to mechanisms based on transitory risk and possibility of becoming constrained.

5.1.   Extending to Foreign Idiosyncratic Risk

Shortcomings with $\tilde{\beta}^* = 0$ Until now we have assumed the representative agent Euler equation holds abroad, i.e. foreign idiosyncratic risk plays no role. We highlight two shortcoming of this approach. First, our symmetric two country framework suggests that the relevant condition (14) is on the correlation of the bilateral wedge ($\tilde{\beta} - \tilde{\beta}^*$) with the real exchange rate. For example, it could be the case that $cov_t(\tilde{\beta}^*, \Delta e) \leq cov_t(\tilde{\beta}, \Delta e) < 0$ such that $cov_t(\tilde{\beta} - \tilde{\beta}^*, \Delta e) \geq 0$ despite the evidence we present above, and therefore the Backus Smith puzzle cannot be resolved. Second, the denominator in the threshold is given by the volatility of the relative $\beta-$ wedge, which could, in principle, be less volatile. While we do not have micro-data for other countries to construct an equivalent time-series of $\tilde{\beta}^*$, we discuss below how we use cross-country distributional income growth statistics to construct a comparable bilateral measure.

Using Global Repository in Income Dynamics   We construct a proxy of the bilateral wedge using growth in income shares for high income groups over a balanced sample of 1998–2015 in seven countries (Canada, Denmark, France, Germany, Italy, Norway, and Sweden) against the United States. We use average residual log earnings growth for Male in age groups 25–55 in our calculation of income shares. Residuals are calculation from regressions of log earnings on age dummies. To capture unconstrained agents likely to be the most patient, we present results based on the top 2.5 and top 5 percentile group in each country's income distribution.[32] A detailed construction of the wedges is given in Appendix C.3 and robustness with the top 10 percentile, and top 1 percentile groups in the Appendix C.3.2.

First key shortcoming from using income instead of consumption data arises due to consumption smoothing and this concern becomes even more pertinent when we focus on patient individuals with a higher propensity to save than the average.[33] However, we show that income data can still be used to construct a good proxy the correlation of the relative (consumption) $\beta-$wedge with depreciation within a class of models characterized by following relationships:

$$\frac{C_{t+1}^i}{C_t^i} = \theta^i \frac{Y_{t+1}^i}{Y_t^i}, \quad \frac{C_{t+1}^{i*}}{C_t^{i*}} = \theta^{i*} \frac{Y_{t+1}^{i*}}{Y_t^{i*}}, \tag{29}$$

$$\frac{C_{t+1}}{C_t} = \theta \frac{Y_{t+1}}{Y_t}, \quad \frac{C_{t+1}^*}{C_t^*} = \theta^* \frac{Y_{t+1}^*}{Y_t^*} \tag{30}$$

where X_t^i, and X_t denote the individual and the economy-average income or consumption. Within this class, it follows that:

$$\rho_{\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*, \Delta e_{t+1}} \approx \rho_{\tilde{\beta}_{t+1}^y - \tilde{\beta}_{t+1}^{y*}, \Delta e_{t+1}}, \quad \text{if } \frac{\theta^i}{\theta} \approx \frac{\theta^{i*}}{\theta^*}$$

where $\tilde{\beta}_{t+1}^y = -\gamma \log\left(\frac{Y_{t+1}^i/Y_t^i}{Y_{t+1}/Y_t}\right)$. See Appendix C.3.1 for a full derivation.

Second, we construct an upper bound for the relevant threshold in (14), which this

---

[32]Figure 5 plots the GRID based wedges alongside the wedge constructed by Berger et al. (2023), revealing the two co-move strongly. On average, the country specific wedges we construct are counter-cyclical with respect to own output growth and consumption growth.

[33]Specifically, the volatility of consumption growth is orders of magnitude smaller than the volatility of income growth.

correlation must exceed. In general, the denominator of this threshold depends on the volatility of the difference in $\beta-$ wedges across countries, but we want to avoid using the volatility of a $\beta-$wedge constructed using income. To this end, we consider a (conservative) lower bound for the volatility of the relative $\beta-$wedge using only the consumption based wedge from BBD. Denote the volatility of the US specific wedge with $\sigma(\tilde{\beta})$, and the correlation between the two wedges by $\rho_{\tilde{\beta},\tilde{\beta}^*} \in (-1,1)$. For a fixed correlation, it can be shown that volatility of the foreign wedge that minimizes volatility of bilateral wedge is $\rho_{\tilde{\beta},\tilde{\beta}^*} \times \sigma(\tilde{\beta})$.[34] Then, the volatility of the bilateral wedge is given by $\sigma\sqrt{(1-\rho^2_{\tilde{\beta},\tilde{\beta}^*})}$. We choose $\rho_{\tilde{\beta},\tilde{\beta}^*} = 0.9$, larger than the highest observation for bilateral correlation of GDP growth in the sample ($\rho = 0.85$, GBR/USA), we get an implied volatility of bilateral wedge is about $0.435\sigma(\tilde{\beta})$. We use this implied volatility of the bilateral wedge along with the volatility of respective bilateral exchange rate to construct threshold for testing (14).

One additional challenge of the GRID dataset is that we can only construct a wedge based on log-income shares which differs from $\log \beta_{t+1}$, defined in (28), due to scaling of marginal utility growth with elasticity of substitution at the individual level. In Appendix C.4, we show that a log-income based wedge in the US data exhibits a weaker correlation with exchange rate than consumption based actual wedge, and is substantially less volatile than correctly-measured consumption based wedge, suggesting that this is again a conservative proxy for the true bilateral beta correlation.[35]

Results. Table 5 presents the main results from using GRID dataset. Results are shown for bilateral wedges constructed for top 5% and top 2.5% income groups, using $\gamma = 5$ as above. The set of controls for constructing conditional moments is the same

---

[34]Specifically, we solve $\min_{\sigma^*} Var(\beta - \beta^*) = \sigma^2(\tilde{\beta}) + \sigma^2(\tilde{\beta}^*) + 2\rho_{\tilde{\beta},\tilde{\beta}^*}\sigma(\tilde{\beta})\sigma(\tilde{\beta}^*)$ and derive $\sigma(\tilde{\beta}^*) = \rho_{\tilde{\beta},\tilde{\beta}^*}\sigma$.

[35]Indeed, the two effects are offsetting: using income instead of consumption increases the wedge volatility, whereas using differences in log income vs. log of income differences decreases the volatility, such that the GRID based wedge can be more or less volatile relative to a true wedge.

as the one used for the consumption based wedge.

Table 5: *Correlation and Thresholds with Real Exchange Rate Growth using GRID data*

Panel A: Correlation and Thresholds for Top 5% and 2.5% Groups

iso	*Unconditional* $-Corr(\tilde{\beta} - \tilde{\beta}^*, \Delta e)$ Top 5%	Top 2.5%	*Thresh.*	*Conditional* $-Corr_t(\tilde{\beta} - \tilde{\beta}^*, \Delta e)$ Top 5%	Top 2.5%	*Thresh.*
CAN	0.46	0.67	0.17	0.52	0.72	0.20
DEU	0.60	0.64	0.17	0.66	0.75	0.13
DNK	0.44	0.50	0.17	0.55	0.61	0.21
FRA	0.17	0.25	0.17	0.35	0.37	0.17
ITA	0.09	0.41	0.17	0.30	0.63	0.20
NOR	0.42	0.32	0.21	0.27	0.21	0.21
SWE	0.49	0.45	0.21	0.47	0.45	0.16
AVERAGE	0.38	0.46	0.18	0.45	0.54	0.18

Panel B: Correlation of Pricing Kernels and Real Exchange Rate Growth

iso	$Corr(\Delta c - \Delta c^*, \Delta e)$	$Corr(\widehat{m}^* - \widehat{m}, \Delta e)$ Top 5%	Top 2.5%	$Corr_t(\Delta c - \Delta c^*, \Delta e)$	$Corr_t(\widehat{m}^* - \widehat{m}, \Delta e)$ Top 5%	Top 2.5%
CAN	-0.03	0.28	0.51	-0.21	0.45	0.66
DEU	0.19	0.63	0.68	0.12	0.86	0.90
DNK	-0.29	0.26	0.34	-0.61	0.14	0.33
FRA	-0.15	0.15	0.24	-0.21	0.35	0.36
ITA	0.09	0.11	0.40	0.14	0.33	0.61
NOR	-0.38	0.29	0.24	-0.58	0.08	0.10
SWE	-0.06	0.45	0.43	-0.07	0.42	0.42
AVERAGE	-0.09	0.31	0.41	-0.20	0.37	0.48

Panel A reports the correlation of bilateral wedge with exchange rate as well as the relevant threshold. The correlation of bilateral wedge with real exchange rate growth is robustly negative, implying exchange rates are *relatively* risky with respect to this wedge. Furthermore, the correlation is sufficiently negative that it meets the conservative threshold computed using the implied volatility of the bilateral wedge. Results also go through if thresholds were constructed using the volatility of the log income based bilateral wedge instead of the implied volatility of the bilateral wedge.

Panel B reports the correlation of relative consumption growth with exchange

rate, and the correlation of the bilateral pricing kernel with exchange rate. As before, we find that the correlation of the relative SDFs with exchange rates is positive for all bilateral pairs, while the correlation is negative when we use relative aggregate consumption growth to construct SDFs.[36]

6. Conclusion

We generalize a two-country framework beyond the representative agent no-arbitrage benchmark to allow for heterogenrity and imperfect risk sharing both *within* and *across* countries. Building on Lucas (1982) and Cox et al. (1985), we derive the incomplete markets wedge in closed form and show that the cyclicality of within-country idiosyncratic risk is critical to reconcile the Backus-Smith correlation. Solving for equilibrium exchange rates, we show this necessitates that even in countries which experience relatively high consumption growth, the pricers (or marginal investors), remain concerned about their future prospects. We show that while internationally incomplete markets cannot alone result in procyclical exchange rates (Lustig and Verdelhan, 2019), they are necessary when we also allow for country-specific factors and are also needed to reconcile relatively smooth exchange rates with volatile domestic asset prices.

The conditions we derive on idiosyncratic risk can be directly tested using household level consumption data, with income and net worth data used to identify the pricer. For countries other than the U.S. where the micro-data is less rich, we rely only on income data and the ranking of households within the distribution. We robustly find that exchange rates are both risky with respect to the idiosyncratic state, and the idiosyncratic state is sufficiently volatile such that our mechanism is empirically relevant.

---

[36]Still, as with the US consumption based wedge, the covariance between pricing kernels is not equal to the variance of exchange rate (12).

References

Acharya, Sushant, and Edouard Challe. 2025. "Inequality and optimal monetary policy in the open economy." *Journal of International Economics*, 155: 104076.

Acharya, Sushant, Edouard Challe, and Keshav Dogra. 2023. "Optimal monetary policy according to HANK." *American Economic Review*, 113(7): 1741–1782.

Acharya, Sushant, Edouard Challe, and Louphou Coulibaly. 2025. "The International RBC model Finally Works." Plenary at CEPR ESSIM May 2025.

Aguiar, Mark, Oleg Itskhoki, and Dmitry Mukhin. 2025. "How Good is International Risk Sharing? Stepping outside the Shadow of the Welfare Theorems." Working paper.

Alvarez, Fernando, Andrew Atkeson, and Patrick J Kehoe. 2002. "Money, interest rates, and exchange rates with endogenously segmented markets." *Journal of Political Economy*, 110(1): 73–112.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2024. "The Intertemporal Keynesian Cross." *Journal of Political Economy*, 132(12): 4068–4121.

Auclert, Adrien, Matthew Rognlie, Martin Souchier, and Ludwig Straub. 2021. "Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel." National Bureau of Economic Research.

Backus, David K, and Gregor W Smith. 1993. "Consumption and real exchange rates in dynamic economies with non-traded goods." *Journal of International Economics*, 35(3-4): 297–316.

Backus, David K, Silverio Foresi, and Chris I Telmer. 2001. "Affine term structure models and the forward premium anomaly." *The Journal of Finance*, 56(1): 279–304.

Bayer, Christian, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden. 2019. "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk." *Econometrica*, 87(1): 255–290.

Benigno, Gianluca, and Christoph Thoenissen. 2008. "Consumption and real exchange rates with incomplete markets and non-traded goods." *Journal of International Money and Finance*, 27(6): 926–948.

Benigno, Gianluca, and Hande Küçük. 2012. "Portfolio allocation and international risk sharing." *Canadian Journal of Economics/Revue canadienne d'économique*, 45(2): 535–565.

Berger, David, Luigi Bocola, and Alessandro Dovis. 2023. "Imperfect Risk Sharing and the Business Cycle." *The Quarterly Journal of Economics*, 138(3): 1765–1815.

Best, Michael Carlos, James S Cloyne, Ethan Ilzetzki, and Henrik J Kleven. 2020. "Estimating the elasticity of intertemporal substitution using mortgage notches." *The Review of Economic Studies*, 87(2): 656–690.

Bilbiie, Florin O. 2024. "Monetary policy and heterogeneity: An analytical framework." *Review of Economic Studies*, rdae066.

Bodenstein, Martin. 2011. "Closing large open economy models." *Journal of International Economics*, 84(2): 160–177.

Bodenstein, Martin, Pablo Cuba-Borda, Nils Gornemann, and Ignacio Presno. 2024. "Exchange rate disconnect and the trade balance." *International Finance Discussion Paper*, , (1391).

Brandt, Michael W, John H Cochrane, and Pedro Santa-Clara. 2006. "International risk sharing is better than you think, or exchange rates are too smooth." *Journal of Monetary Economics*, 53(4): 671–698.

Challe, Edouard. 2020. "Uninsured unemployment risk and optimal monetary policy in a zero-liquidity economy." *American Economic Journal: Macroeconomics*, 12(2): 241–283.

Chernov, Mikhail, Valentin Haddad, and Oleg Itskhoki. 2024. "What do financial markets say about the exchange rate?" Working Paper, UCLA.

Chien, YiLi, Hanno Lustig, and Kanda Naknoi. 2020. "Why are exchange rates so smooth? A household finance explanation." *Journal of Monetary Economics*, 112: 129–144.

Colacito, Riccardo, and Mariano M Croce. 2013. "International asset pricing with recursive preferences." *The Journal of Finance*, 68(6): 2651–2686.

Cole, Harold L, and Maurice Obstfeld. 1991. "Commodity trade and international risk sharing: How much do financial markets matter?" *Journal of Monetary Economics*, 28(1): 3–24.

Constantinides, George M, and Darrell Duffie. 1996. "Asset pricing with heterogeneous consumers." *Journal of Political Economy*, 104(2): 219–240.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2008. "International risk sharing and the transmission of productivity shocks." *The Review of Economic Studies*, 75(2): 443–473.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2023. "Exchange rate misalignment and external imbalances: What is the optimal monetary policy response?" *Journal of International Economics*, 103771.

Costinot, Arnaud, Guido Lorenzoni, and Iván Werning. 2014. "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation." *Journal of Political Economy*, 122(1): 77–128.

Cox, J.C, J.E. Ingersoll, and S.A. Ross. 1985. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53(2): 385–407.

De Ferra, Sergio, Kurt Mitman, and Federica Romei. 2020. "Household heterogeneity and the transmission of foreign shocks." *Journal of International Economics*, 124: 103303.

Di Tella, Sebastian, Benjamin M Hébert, and Pablo Kurlat. 2024. "Aggregation, Liquidity, and Asset Prices with Incomplete Markets." National Bureau of Economic Research.

Di Tella, Sebastian, Benjamin M Hébert, Pablo Kurlat, and Qitong Wang. 2023. "The Zero-Beta Interest Rate." National Bureau of Economic Research Working Paper 31596.

Duffie, Darrell. 2001. *Dynamic Asset Pricing Theory.* . Third ed., Princeton, NJ:Princeton University Press.

Farhi, Emmanuel, and Xavier Gabaix. 2016. "Rare disasters and exchange rates." *The Quarterly Journal of Economics*, 131(1): 1–52.

Fitzgerald, Doireann. 2012. "Trade costs, asset market frictions, and risk sharing." *American Economic Review*, 102(6): 2700–2733.

Gabaix, Xavier, and Matteo Maggiori. 2015. "International liquidity and exchange rate dynamics." *The Quarterly Journal of Economics*, 130(3): 1369–1420.

Ghironi, Fabio. 2006. "Macroeconomic interdependence under incomplete markets." *Journal of International Economics*, 70(2): 428–450.

Guerrieri, Veronica, and Guido Lorenzoni. 2017. "Credit crises, precautionary savings, and the liquidity trap." *The Quarterly Journal of Economics*, 132(3): 1427–1467.

Guvenen, Fatih. 2009. "A parsimonious macroeconomic model for asset pricing." *Econometrica*, 77(6): 1711–1750.

Guvenen, Fatih, Luigi Pistaferri, and Giovanni L Violante. 2022. "Global trends in income inequality and income dynamics: New insights from GRID." *Quantitative Economics*, 13(4): 1321–1360.

Hassan, Tarek A. 2013. "Country size, currency unions, and international asset returns." *The Journal of Finance*, 68(6): 2269–2308.

Hassan, Tarek A, Thomas Mertens, and Jingye Wang. 2024. "A currency premium puzzle." Federal Reserve Bank of San Francisco.

Heathcote, Jonathan, and Fabrizio Perri. 2013. "The International Diversification Puzzle Is Not as Bad as You Think." *Journal of Political Economy*, 121(6): 1108–1159.

Herskovic, Bernard, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh. 2016. "The common factor in idiosyncratic volatility: Quantitative asset pricing implications." *Journal of Financial Economics*, 119(2): 249–283.

Huang, Qiushi, Leonid Kogan, and Dimitris Papanikolaou. 2025. "Tech Dollars and Exchange Rate Reconnect." Northwestern University.

Itskhoki, Oleg, and Dmitry Mukhin. 2021. "Exchange rate disconnect in general equilibrium." *Journal of Political Economy*, 129(8): 2183–2232.

Itskhoki, Oleg, and Dmitry Mukhin. 2023. "What Drives the Exchange Rate?" Working Paper, UCLA.

Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig. 2023. "Implications of Asset Market Data for Equilibrium Models of Exchange Rates." National Bureau of Economic Research.

Jordà, Òscar, Moritz Schularick, Alan M Taylor, and Felix Ward. 2018. "Global financial cycles and risk premiums." National Bureau of Economic Research.

Jordà, Òscar, Moritz Schularick, and Alan M Taylor. 2017. "Macrofinancial history and the new business cycle facts." *NBER macroeconomics annual*, 31(1): 213–263.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. 2018. "Monetary policy according to HANK." *American Economic Review*, 108(3): 697–743.

Karabarbounis, Loukas. 2014. "Home production, labor wedges, and international business cycles." *Journal of Monetary Economics*, 64: 68–84.

Kekre, Rohan, and Moritz Lenel. 2024. "Exchange Rates, Natural Rates, and the Price of Risk." Working Paper.

Kocherlakota, Narayana R, and Luigi Pistaferri. 2007. "Household heterogeneity and real exchange rates." *The Economic Journal*, 117(519): C1–C25.

Kollmann, Robert. 1991. "Essays on international business cycles." PhD diss. The University of Chicago.

Kollmann, Robert. 2012. "Limited asset market participation and the consumption-real exchange rate anomaly." *Canadian Journal of Economics/Revue canadienne d'économique*, 45(2): 566–584.

Kollmann, Robert. 2021. "The real exchange rate and household consumption heterogeneity: Testing Kocherlakota and Pistaferri's (2007) model." *Economics Letters*, 209: 110110.

Krueger, Dirk, and Hanno Lustig. 2010. "When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)?" *Journal of Economic Theory*, 145(1): 1–41.

Krusell, Per, Toshihiko Mukoyama, and Anthony A. Smith. 2011. "Asset prices in a Huggett economy." *Journal of Economic Theory*, 146(3): 812–844. Incompleteness and Uncertainty in Economics.

Leduc, Sylvain. 2002. "Incomplete markets, borrowing constraints, and the foreign exchange risk premium." *Journal of International Money and Finance*, 21(7): 957–980.

Lettau, Martin. 2002. "Idiosyncratic risk and volatility bounds, or Can models with idiosyncratic risk solve the equity premium puzzle?" *The Review of Economics and Statistics*, 84(2): 376–380.

Lloyd, Simon P., and Emile A. Marin. 2024. "Capital controls and trade policy." *Journal of International Economics*, 151: 103965.

Lucas, Robert E. 1978. "Asset prices in an exchange economy." *Econometrica: journal of the Econometric Society*, 1429–1445.

Lucas, Robert E. 1982. "Interest rates and currency prices in a two-country world." *Journal of Monetary Economics*, 10(3): 335–359.

Lustig, Hanno, and Adrien Verdelhan. 2019. "Does incomplete spanning in international financial markets help to explain exchange rates?" *American Economic Review*, 109(6): 2208–2244.

Mankiw, N Gregory. 1986. "The equity premium and the concentration of aggregate shocks." *Journal of Financial Economics*, 17(1): 211–219.

Nakajima, Tomoyuki. 2005. "A business cycle model with variable capacity utilization and demand disturbances." *European Economic Review*, 49(5): 1331–1360.

Pavlova, Anna, and Roberto Rigobon. 2007. "Asset prices and exchange rates." *The Review of Financial Studies*, 20(4): 1139–1180.

Ramchand, Latha. 1999. "Asset pricing in open economies with incomplete markets: implications for foreign currency returns." *Journal of International Money and Finance*, 18(6): 871–890.

Storesletten, Kjetil, Chris I Telmer, and Amir Yaron. 2004. "Cyclical dynamics in idiosyncratic labor market risk." *Journal of Political Economy*, 112(3): 695–717.

Sun, Tong-sheng. 1992. "Real and nominal interest rates: A discrete-time model and its continuous-time limit." *The Review of Financial Studies*, 5(4): 581–611.

Svensson, Lars E. O. 1988. "Trade in Risky Assets." *The American Economic Review*, 78(3): 375–394.

Tessari, Cristina. 2021. "Common idiosyncratic volatility and carry trade returns." Chapter in Dissertation Thesis "Essays in International Finance and Central Bank Policy", Columbia University.

Verdelhan, Adrien. 2010. "A habit-based explanation of the exchange rate risk premium." *The Journal of Finance*, 65(1): 123–146.

Weil, Philippe. 1992. "Equilibrium asset prices with undiversifiable labor income risk." *Journal of Economic Dynamics and Control*, 16(3-4): 769–790.

Werning, Iván. 2015. "Incomplete markets and aggregate demand." National Bureau of Economic Research.

A. Appendix

A.1. Additional Derivations for Section 2.

To find the admissible set of exchange rate processes, consider the log expansions of (7)-(11), assuming joint log normality of SDFs and prices:

$$\mathbb{E}_t[\widehat{m}_{t+1}] + \frac{1}{2}var_t(\widehat{m}_{t+1}) = -r_{t+1}, \quad (31)$$

$$\mathbb{E}_t[\widehat{m}^*_{t+1}] + \frac{1}{2}var_t(\widehat{m}^*_{t+1}) = -r^*_{t+1} \quad (32)$$

$$\mathbb{E}_t[\widehat{m}^*_{t+1}] + \frac{1}{2}var_t(\widehat{m}^*_{t+1}) - \mathbb{E}_t[\Delta e_{t+1}] + \frac{1}{2}var_t(\Delta e_{t+1}) + cov_t(\widehat{m}^*_{t+1}, -\Delta e_{t+1}) = -r_{t+1}, \quad (33)$$

$$\mathbb{E}_t[\widehat{m}_{t+1}] + \frac{1}{2}var_t(\widehat{m}_{t+1}) + \mathbb{E}_t[\Delta e_{t+1}] + \frac{1}{2}var_t(\Delta e_{t+1}) + cov_t(\widehat{m}_{t+1}, \Delta e_{t+1}) = -r^*_{t+1}, \quad (34)$$

where lower case levels denote logs, e.g. $\log(\widehat{M}_{t+1}) = \widehat{m}_{t+1}$ and $\Delta e_{t+1} = e_{t+1} - e_t$. Using (31) and (34), and (32) and (33) respectively, yields:

$$\mathbb{E}_t[\Delta e_{t+1}] + r^*_{t+1} - r_{t+1} = -cov_t(\widehat{m}_{t+1}, \Delta e_{t+1}) - \frac{1}{2}var_t(\Delta e_{t+1}), \quad (35)$$

$$\mathbb{E}_t[\Delta e_{t+1}] + r^*_{t+1} - r_{t+1} = cov_t(\widehat{m}^*_{t+1}, -\Delta e_{t+1}) + \frac{1}{2}var_t(\Delta e_{t+1}) \quad (36)$$

Combining the above yields (12).

Restriction on η wedge from cross-border trade in assets. Combining (7) and (10) for the Home pricer and Combining (9) and (11) for the foreign pricer, combing with (13) yields the following conditions:

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}var_t(\eta_{t+1}) - cov_t(\widehat{m}_{t+1}, \eta_{t+1}) \quad (37)$$

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}var_t(\eta_{t+1}) - cov_t(\widehat{m}^*_{t+1}, -\eta_{t+1}) \quad (38)$$

This is a straightforward generalization of the conditions in Lustig and Verdelhan (2019) to our environment with imperfect domestic risk sharing.

A.2. Proofs to Propositions

Proof to Proposition 1. Condition (12) can be expanded as follows:

$$var_t(\Delta e_{t+1}) = cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) + cov_t(\tilde{\beta}^*_{t+1} - \tilde{\beta}_{t+1}, \Delta e_{t+1})$$

Then $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) < 0$ if and only if:

$$var_t(\Delta e_{t+1}) + cov_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1}) \leq 0.$$

By the Cauchy-Schwarz inequality:

$$|cov_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1})| \leq \sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1})\sigma_t(\Delta e_{t+1})$$

Combining the inequalities:

$$\sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1})\sigma_t(\Delta e_{t+1}) \geq -cov_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1}) \geq var_t(\Delta e_{t+1})$$

Dividing through by $\sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1})\sigma_t(\Delta e_{t+1})$ yields the result. $\qquad\square$

Proof to Corollary 1 Consider again the expansion of (12):

$$var_t(\Delta e_{t+1}) = cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) + cov_t(\tilde{\beta}^*_{t+1} - \tilde{\beta}_{t+1}, \Delta e_{t+1})$$

and set $\tilde{\beta}_{t+1} = \tilde{\beta}^*_{t+1} = 0$ corresponding to the representative agent limit within each country. The Backus-Smith covariance $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1})$ which can be rewritten as:

$$cov_t(m^*_{t+1} - m_{t+1}, m^*_{t+1} - m_{t+1}) + cov_t(m^*_{t+1} - m_{t+1}, \eta_{t+1}) \tag{39}$$

$$= var_t(m^*_{t+1} - m_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) - cov_t(m_{t+1}, \eta_{t+1}) \tag{40}$$

Imposing (38) (international trade in the Foreign asset), but not (37) (international trade in the Home asset), remembering that $\widehat{m}^{(*)}_{t+1} = m^{(*)}_{t+1}$ in the representative agent limit yields:

$$cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) = var_t(m^*_{t+1} - m_{t+1}) - \log \mathbb{E}_t[\eta_{t+1}] - cov_t(m_{t+1}, \eta_{t+1}) \leq 0$$

and rearranging delivers the result. Note further that taking the limit $cov_t(m_{t+1}, \eta_{t+1}) \rightarrow$

(16) would imply $var_t(\Delta e_{t+1}) < 0$ which cannot be an equilibrium. $\qquad\square$

Proof to Lemma The individual Euler equation can be expressed as:

$$\mathbb{E}_t\left[\tilde{\beta}_{t+1}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\tilde{R}_{t+1}\right] = 1, \tag{41}$$

for any (risky) asset with return $\tilde{R}_{t+1}$, so that e.g. foreign nominally risk-free bonds are captured by $(\mathcal{E}_{t+1}/\mathcal{E}_t)R_{t+1}$. We define

$$\Delta e_{t+1} = \widehat{m}^*_{t+1} - \widehat{m}_{t+1} + \eta_{t+1}$$

From Equations (12), (13), (37), and (38), we have that

$$var_t(\eta_{t+1}) = var_t(\widehat{m}^*_{t+1} - \widehat{m}_{t+1}) - var_t(\Delta e_{t+1}) \tag{42}$$

We assume a process for the incomplete markets wedge:

$$\eta_{t+1} = \Gamma_0 z_t + \Gamma_1\sqrt{z_t}u_{t+1} + \Gamma_2\sqrt{z_t}\epsilon_{t+1} + \Gamma_0^\nu\sigma_\nu^2 + \Gamma_1^\nu\sigma^\nu u_{t+1}^\nu + \Gamma_2^\nu\sigma^\nu\epsilon_{t+1}^\nu \tag{43}$$

where $u^{(\nu)}$ are spanned innovations and $\epsilon^{(\nu)}$ are unspanned innovations. We proceed to determine the coefficients of the IM wedge consistent with no arbitrage. Substituting (41) and (18) into (42), and denoting $var_t(\Delta e_{t+1}) = \kappa z_t + \kappa^\nu\sigma_\nu^2$:

$$\underbrace{(\Gamma_1^2 + \Gamma_2^2)z_t + (\Gamma_1^{\nu 2} + \Gamma_2^{\nu 2})\sigma_\nu^2}_{var_t(\eta_{t+1})} = \underbrace{(\gamma(1-\xi^*) - \phi^\Delta)^2 z_t + (\gamma - \phi^*)^2\sigma_\nu^2}_{var_t(\widehat{m}^*_{t+1} - \widehat{m}_{t+1})} - \underbrace{(\kappa z_t + \kappa^\nu\sigma_\nu^2)}_{var_t(\Delta e_{t+1})}, \tag{44}$$

which yields:

$$\Gamma_1 = \pm\sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}, \quad \Gamma_1^\nu = \pm\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu} \tag{45}$$

$$\Gamma_2 = \pm\sqrt{\lambda - \kappa}, \quad \Gamma_2^\nu = \pm\sqrt{\lambda^\nu - \kappa^\nu} \tag{46}$$

where for real solutions, the following restrictions are required:

$$(\gamma(1-\xi^*) - \phi^\Delta)^2 \geq \lambda \geq \kappa, \tag{47}$$

$$(\gamma - \phi^*)^2 \geq \lambda^\nu \geq \kappa^\nu \tag{48}$$

Using (37)-(38) imply:

$$\Gamma_0 = \frac{1}{2}\left\{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \kappa\right\} - (\gamma - \phi)\sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}, \tag{49}$$

$$\Gamma_0^\nu = \frac{1}{2}\left\{(\gamma - \phi^*)^2 - \kappa^\nu\right\}, \tag{50}$$

and

$$\Gamma_0 = -\frac{1}{2}\left\{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \kappa\right\} - \xi^*(\gamma - \phi^*)\sqrt{(\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}, \tag{51}$$

$$\Gamma_0^\nu = +\frac{1}{2}\left\{(\gamma - \phi^*)^2 - \kappa^\nu\right\} + (\gamma - \phi^*)\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu} = 0, \tag{52}$$

respectively. Adding (49) and (51) for z_t and (50) and (52) for σ_ν^2 respectively, this can be rewritten as:

$$\Gamma_0 = -\frac{1}{2}(\gamma - \phi + \xi^*(\gamma - \phi^*))\sqrt{\gamma(1-\xi^*) - \phi^\Delta)^2 - \lambda}, \tag{53}$$

$$\Gamma_0^\nu = +\frac{1}{2}(\gamma - \phi^*)\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu} \tag{54}$$

Subbing Γ_0 and Γ_0^ν back into (49) and (51) respectively:

$$\kappa = (\gamma(1-\xi^*) - \phi^\Delta)^2 - (\gamma(1-\xi^*) - \phi^\Delta)\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu} \geq 0,, \tag{55}$$

$$\kappa^\nu = (\gamma - \phi^*)^2 - (\gamma - \phi^*)\sqrt{(\gamma - \phi^*)^2 - \lambda^\nu} \geq 0 \tag{56}$$

To ensure all square roots are real, we only choose the square root signs which satisfy (47) and (48). As a result, we select positive roots for all except Γ_1 where we select the negative root. This completes the characterization of the η process.

$\square$

Proof to Proposition 2. First consider:

$$cov_t(\widehat{m}_{t+1}^* - \widehat{m}_{t+1}, \Delta e_{t+1}) = var_t(\Delta e_{t+1}) =$$

$$cov_t(m_{t+1}^* - m_{t+1}, \Delta e_{t+1}) + cov_t(\tilde{\beta}_{t+1}^* - \tilde{\beta}_{t+1}, \Delta e_{t+1}) \tag{57}$$

Then, the Backus-Smith covariance is negative if and only if:

$$var_t(\Delta e_{t+1}) + cov_t(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*, \Delta e_{t+1}) \leq 0 \tag{58}$$

Using the Lemma above:

$$\kappa z_t + (\phi - \phi^* \xi^*)(\gamma(1 - \xi^*) - \phi^\Delta - \sqrt{(\gamma(1 - \xi^*) - \phi^\Delta)^2(1 - \alpha)} z_t \qquad (59)$$

$$+ \kappa^\nu \sigma_\nu^2 - \phi^* \left(-(\gamma - \phi^*) + \sqrt{(\gamma - \phi^*)^2(1 - \alpha^\nu)} \right) \sigma_\nu^2 \leq 0$$

We deal with the terms premultiplying z_t and σ_ν^2 separately. We substitute κ and κ^ν from the Lemma. Beginning with z_t:

$$(\gamma(1 - \xi^*) - \phi^\Delta)^2 - (\gamma(1 - \xi^*) - \phi^\Delta)\Gamma_1 + (\phi - \phi^* \xi^*)(\gamma(1 - \xi^*) - \phi^\Delta - \Gamma_1)$$

$$= (\gamma(1 - \xi^*) - \phi^\Delta)(\gamma(1 - \xi^*) - \phi^\Delta - \Gamma_1) + \phi^\Delta(\gamma(1 - \xi^*) - \phi^\Delta - \Gamma_1)$$

$$= (\gamma(1 - \xi^*))(\gamma(1 - \xi^*) - \phi^\Delta - \Gamma_1)$$

$$= (\gamma(1 - \xi^*))(\gamma(1 - \xi^*) - \phi^\Delta) \underbrace{(1 - \sqrt{1 - \alpha})}_{\geq 0}$$

Then, turning to σ_ν^2:

$$(\gamma - \phi^*)^2 - (\gamma - \phi^*)\sqrt{(\gamma - \phi^*)^2(1 - \alpha^\nu)} - \phi^*(-(\gamma - \phi))$$

$$= (\gamma - \phi^*)(\gamma - \phi^* + \phi^*)(1 - \sqrt{1 - \alpha^\nu}) =$$

$$(\gamma - \phi^*)\gamma \underbrace{(1 - \sqrt{1 - \alpha^\nu})}_{\geq 0}$$

which is strictly positive as long as $\phi^* < \gamma$.

Using both terms and multiplying by z_t and σ_ν^2 respectively delivers the Proposition.

$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \square$

Proof to Proposition 3. We focus on u_{t+1} and u_{t+1}^ν in turn. Consider κ for $\phi < 0$ and denote this as κ^{IM}:

$$\kappa^{IM} = (\gamma(1 - \xi^*) - \phi^\Delta)^2 - (\gamma(1 - \xi^*) - \phi^\Delta))^2 \sqrt{1 - \alpha} \qquad (60)$$

Imposing complete markets, $\alpha = 1$, $\lambda^{CM} = (\gamma(1 - \xi^*) - (\phi^\Delta))^2$ and define this as κ^{CM}:

$$\kappa^{CM} = (\gamma(1 - \xi^*) - \phi^\Delta)^2 \qquad (61)$$

Since $(\gamma(1 - \xi^*) - \phi^\Delta))^2\sqrt{1 - \alpha} > 0$, $\kappa^{IM} < \kappa^{CM}$. Second, denoting $\kappa^{CM} - \kappa^{IM} = (\gamma(1 - \xi^*) - \phi^\Delta))^2\sqrt{1 - \alpha}$ and taking the derivative with respect to $-\phi^\Delta$:

$$\frac{d(\kappa^{CM} - \kappa^{IM})}{d(-\phi^\Delta)} = 2((\gamma(1 - \xi^*) - \phi^\Delta))\sqrt{1 - \alpha} > 0$$

As long as $(\gamma(1 - \xi^*) - \phi^\Delta) > 0$, required to satisfy Proposition 3 for $\xi^* > 1$, $\phi^\Delta \downarrow$ (or $\phi \downarrow$), $\kappa^{CM} - \kappa^{IM} \uparrow$ delivering the result.

In turn:

$$\kappa^{\nu\ IM} = (\gamma - \phi^*)^2 - (\gamma - \phi^*)^2\sqrt{1 - \alpha^\nu} \tag{62}$$

and

$$\kappa^{\nu\ CM} = (\gamma - \phi^*)^2 \tag{63}$$

Since $\sqrt{1 - \alpha^\nu} > 0$, $\kappa^{\nu\ IM} < \kappa^{\nu\ CM}$. Finally:

$$\frac{d(\kappa^{\nu\ CM} - \kappa^{\nu\ IM})}{d(-\phi^*)} = 2(\gamma - \phi^*)\sqrt{1 - \alpha} > 0 \tag{64}$$

which confirms the result. $\qquad\square$

Proof to Proposition 4. Consider the static problem (88). Aggregating (90) over all agents i, combining with market clearing (4) and the (first order homogeneous) aggregator (26) delivers the result. $\qquad\square$

A.3. Additional Results for Section 3

Country-specific shock applied to Home SDF. Consider the following pair of SDFs:

$$-\widehat{m}_{t+1} = \beta + \chi z_t + (\gamma - \phi)(\sqrt{z_t}u_{t+1} + \sigma_\nu u_{t+1}^\nu), \tag{65}$$

$$-\widehat{m}_{t+1}^* = \beta^* + \chi^* z_t + (\gamma - \phi^*)(\xi^*\sqrt{z_t}u_{t+1}) \tag{66}$$

We repeat our analysis focusing on the projection of moments on u_{t+1}^ν. The process for the incomplete markets wedge continues to be given by (43). Using (44):

$$\underbrace{\Gamma_1^{\nu 2} + \Gamma_2^{\nu 2}}_{var_t(\eta_{t+1}|u_{t+1}^\nu)} = (\gamma - \phi)^2 - \kappa^\nu)\sigma_\nu^2 \tag{67}$$

Then:

$$\Gamma_1^\nu = \pm\sqrt{(\gamma - \phi)^2 - \lambda^\nu}, \Gamma_2^\nu = \pm\sqrt{\lambda^\nu - \kappa^\nu}$$

Using (37) and (38) we derive:

$$\Gamma_0^\nu = \frac{1}{2}((\gamma - \phi)^2 - \kappa^\nu) - (\gamma - \phi)\sqrt{(\gamma - \phi)^2 - \lambda^\nu}, \tag{68}$$

$$\Gamma_0^\nu = 0\frac{1}{2}((\gamma - \phi)^2 - \kappa^\nu) \tag{69}$$

Combining:

$$\kappa^\nu = (\gamma - \phi)^2 - (\gamma - \phi)^2\sqrt{(\gamma - \phi)^2 - \lambda^\nu} \tag{70}$$

Finally, we derive

$$cov_t(m_{t+1}^* - m_{t+1}, \Delta e_{t+1}|u_{t+1}^\nu) = (\gamma - \phi)(1 - \sqrt{1 - \alpha^\nu})$$

which is strictly positive as long as $\phi < \gamma$.

B.   ONLINE APPENDIX

B.1.   Trade in Risky Assets

If instead of allowing for trade in both risk-free assets, we allow for trade in Home and Foreign risky assets, then equations (7)–(11) are replaced by:

$$\mathbb{E}_t[\widehat{M}_{t+1}\tilde{R}_{t+1}] = 1, \tag{71}$$

$$\mathbb{E}_t[\widehat{M}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\tilde{R}_{t+1}^*] = 1, \tag{72}$$

$$\mathbb{E}_t[\widehat{M}_{t+1}^*\tilde{R}_{t+1}^*] = 1, \tag{73}$$

$$\mathbb{E}_t[\widehat{M}_{t+1}^*\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)^{-1}\tilde{R}_{t+1}] = 1, \tag{74}$$

where $\tilde{R}$ and $\tilde{R}^*$ are returns on risky Home and Foreign assets respectively.

Suppose Home and Foreign households trade in Home and Foreign currency denominated risky assets $\tilde{R}_{t+1}$ such that (71)- (74) hold. Assuming joint log normality, the above Euler equations imply:

$$\mathbb{E}_t[\widehat{m}_{t+1}] + \frac{1}{2}var_t(\widehat{m}_{t+1}) + \mathbb{E}_t[\tilde{r}_{t+1}] + \frac{1}{2}var_t(\tilde{r}_{t+1}) + cov_t(\widehat{m}_{t+1}, \tilde{r}_{t+1}) = 0, (75)$$

$$\mathbb{E}_t[\widehat{m}_{t+1}] + \frac{1}{2}var_t(\widehat{m}_{t+1}) + \mathbb{E}_t[\tilde{r}_{t+1}^*] + \frac{1}{2}var_t(\tilde{r}_{t+1}^*) + \mathbb{E}_t[\Delta e_{t+1}] + \frac{1}{2}var_t(\Delta e_{t+1}) + \cdots$$

$$cov_t(\widehat{m}_{t+1}, \tilde{r}_{t+1}^*) + cov_t(\widehat{m}_{t+1}, \Delta e_{t+1}) + cov_t(\Delta e_{t+1}, \tilde{r}_{t+1}^*) = 0, (76)$$

$$\mathbb{E}_t[\widehat{m}_{t+1}^*] + \frac{1}{2}var_t(\widehat{m}_{t+1}^*) + \mathbb{E}_t[\tilde{r}_{t+1}^*] + \frac{1}{2}var_t(\tilde{r}_{t+1}^*) + cov_t(\widehat{m}_{t+1}^*, \tilde{r}_{t+1}^*) = 0, (77)$$

$$\mathbb{E}_t[\widehat{m}_{t+1}^*] + \frac{1}{2}var_t(\widehat{m}_{t+1}^*) - \mathbb{E}_t[\Delta e_{t+1}] + \frac{1}{2}var_t(\Delta e_{t+1}) + \mathbb{E}_t[\tilde{r}_{t+1}] + \frac{1}{2}var_t(\tilde{r}_{t+1}) + \cdots$$

$$cov_t(\widehat{m}_{t+1}^*, \tilde{r}_{t+1}) + cov_t(\widehat{m}_{t+1}^*, -\Delta e_{t+1}) + cov_t(-\Delta e_{t+1}, \tilde{r}_{t+1}) = 0 (78)$$

Combining (75) - (78)

$$var_t(\Delta e_{t+1}) = cov_t(\widehat{m}_{t+1} - \widehat{m}_{t+1}^*, \Delta e_{t+1}) + cov_t(\eta_{t+1}, \tilde{r}_{t+1}^* - \tilde{r}_{t+1}) \qquad (79)$$

Away from the representative agent limit, how many assets would it take to impose full risk-sharing? In practice, many (more than two) assets are traded across borders but few of these are risk-free in real terms, e.g. long-maturity bonds and equity. We extend our main findings to a framework with trading in multiple risky assets but when (nominally) risk-free bonds are not available.

Proposition A1 (Many Assets, Heterogeneous Consumers No-Arbitrage)

When Home and Foreign currency risky assets with returns $\tilde{r}_{t+1}^{(*)}$ are internationally traded (71)-(74), then $cov_t(m_{t+1}^* - m_{t+1}, \Delta e_{t+1}) < 0$ if and only if:

$$-\rho_{\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*, \Delta e_{t+1}} + \frac{cov_t(\eta_{t+1}, \tilde{r}_{t+1}^* - \tilde{r}_{t+1})}{\sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*)\sigma_t(\Delta e_{t+1})} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*)} \qquad (80)$$

Proof. Rearrange (79) and apply the Cauchy-Schwarz Inequality. $\qquad\square$

When only risky assets are traded, exchange rates need not be spanned, allowing further scope for pro-cyclical exchange rates. In particular, pro-cyclicality is recovered if the incomplete markets wedge co-varies positively with the differential return from

a risky foreign asset. If, however, we additionally allow for trade in two nominally risk-free assets the new term in Proposition A1 ($cov_t(\eta_{t+1}, \tilde{r}^*_{t+1} - \tilde{r}_{t+1})$) converges to 0. Note also that this is true for any $r^{\tilde{k}(*)}$ traded, see Section B.2. This implies that Proposition A1 in the main body is the necessary and sufficient condition even when there is trade in many assets. Nonetheless, $\rho_{\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1}, \Delta e_{t+1}}$ and $\sigma_t(\Delta e_{t+1})$ are themselves changing as the number of traded assets changes, see Section 3.

B.2. Degree of Market Completeness and Risk-Sharing

To illustrate that two nominally risk-free assets suffice to span exchange rates, regardless of market completeness, we turn to a framework in the tradition of Lucas (1978):

$$\Delta c_{t+1} = \sum_{k=1}^{N} g_{y_k, t+1}, \tag{81}$$

$$m_{t+1} = -\gamma \Delta c_{t+1}, \tag{82}$$

where $g_{y_k, t+1} = y_{k, t+1} - y_{k,t} \sim i.i.d\ \mathcal{N}(\mu_{y_k}, \sigma_{y_k})$ denotes the growth rate of k-th productive unit that comprises the consumption good. Corresponding variables for the Foreign economy are denoted with an asterisk. Start with the case of $N = 1$ productive units, discussed in (Lustig and Verdelhan, 2019, Sec III.A) and in Appendix B.1. Frictionless international trade in Home and Foreign risk-free bonds (37)- (38) and additional trade in a Home and a Foreign risky asset (a claim on $g_{y_k, t+1}, g^*_{y_k, t+1}$ respectively) will imply that the incomplete markets wedge η_{t+1} is orthogonal to $g_{y_k, t+1}, g^*_{y_k, t+1}$, and it then follows that the only equilibrium is $\eta_{t+1} = 0$– i.e. markets are complete. When $N > 1$, additional risky claims need to traded to complete the market. However, for any N, frictionless international trade in just the Home and the Foreign real bonds ensures that the sign of the relationship between the pricers' SDFs and exchange rate depreciations is positive.

B.3. Limits to International Arbitrage

Consider the following aggregate Euler equations capturing within-country idiosyncratic risk, now allowing for shocks to international returns (due to limits to international arbitrage) u_{t+1}:

$$\mathbb{E}_t \left[\widehat{M}_{t+1} \right] R_{t+1} = 1, \tag{83}$$

$$\mathbb{E}_t \left[\widehat{M}^*_{t+1} \right] R^*_{t+1} = 1, \tag{84}$$

$$\mathbb{E}_t \left[\widehat{M}^*_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] R_{t+1} = \mathbb{E}_t[e^{u^f_{t+1}}], \tag{85}$$

$$\mathbb{E}_t \left[\widehat{M}_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] R^*_{t+1} = \mathbb{E}_t[e^{u^f_{t+1}}] \tag{86}$$

where we assume intermediation shocks are mean 0 ($\mathbb{E}_t[u^f_{t+1}] = 0$). Taking logs, we derive:

$$var_t(\Delta e_{t+1}) - var_t(u^f_{t+1}) + cov_t(\widehat{m}_{t+1} - \widehat{m}^*_{t+1}, \Delta e_{t+1}) = 0$$

Substituting $var_t(u^f_{t+1}) = u^f var_t(\Delta e_{t+1})$ and assuming $u^f \in [0, 1]$, rearrranging:

$$var_t(\Delta e_{t+1})(1 - u^f) cov_t(\widehat{m}_{t+1} - \widehat{m}^*_{t+1}, \Delta e_{t+1}) \tag{87}$$

Then, $cov_t(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta e_{t+1}) < 0$ requires:

$$-cov_t \left(\tilde{\beta}_{t+1} - \tilde{\beta}^*_{t+1} \right) > var_t(\Delta e_{t+1})(1 - u^f)$$

Applying the Cauchy-Schwarz identity and dividing by standard deviations yields condition (15).

B.4. Additional results for Section 3

This section provides comparative statics with respect to the spanning of the country-specific factor α^ν. The solid blue line shows our baseline calibration of $\alpha^\nu = 0.01$. Spanning of the common shock α is set at baseline value of 0.6 across blue lines. Both the volatility of the exchange rate and the Backus Smith covariance are increasing in

the spanning of the country specific shock, for a given cyclicality of the $\beta-$wedge.

Figure 3

(a) *Backus-Smith Covariance*   **(b)** *Exchange Rate Volatility*
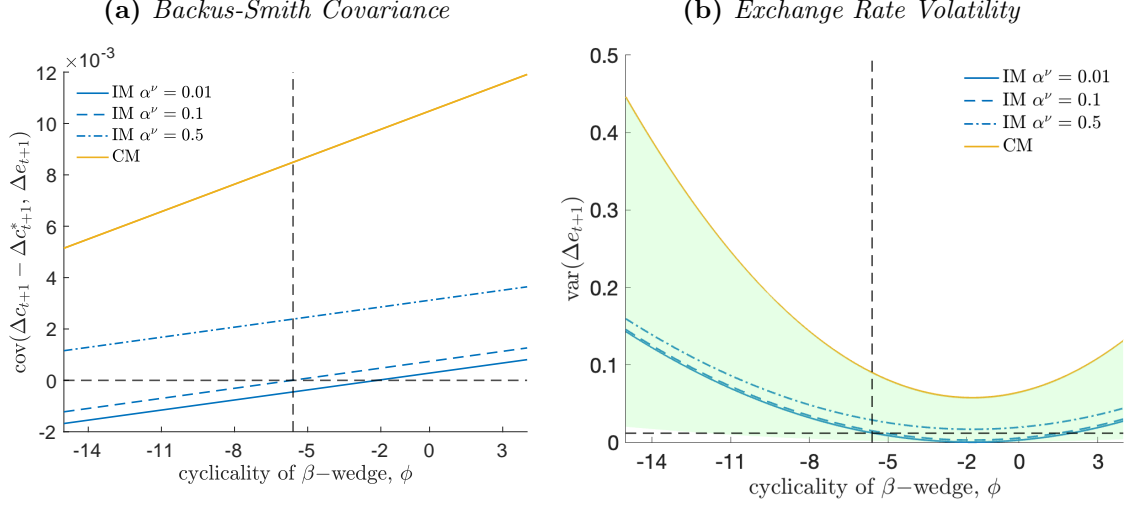
B.5. Planning Problem

B.5.1 Optimal Consumption Bundle with Heterogeneous Agents

Given a level of consumption $C(s^t)$, households optimally choose their consumption bundle $\{c_H(s^t), c_F(s^t)\}$. For ease of exposition, since the problem is static we supress dependence on histories and we denote with a superscript ν individual consumptions $c^\nu = c(z^t, \nu^t)$. The optimal consumption bundle satisfies:

$$\max_{\{c_H^\nu, c_F^\nu\}} \left\{ g^*(c_H^{\nu\,*}, c_F^{\nu\,*}) \quad \text{s.t.} \quad \int_\nu c_H^\nu d\nu + \int_\nu c_H^{\nu\,*} d\nu^* = I_H, \tag{88}\right.$$
$$\left. \int_\nu c_F^\nu (1+\tau_t) d\nu + \int_\nu c_F^\nu d\nu = I_F \text{ and } g(c_H^\nu, c_F^\nu) \geq C^\nu \right\},$$

where we define $g(c_H, c_F)$ by (26). See Costinot, Lorenzoni and Werning (2014) for a friction-less representative agent framework, and Lloyd and Marin (2024) for a treatment with static wedges. Notice that from first order homogeneity of $g(\cdot)$ it

follows that:

$$\frac{g_H(c_H^\nu, c_F^\nu)}{g_F(c_H^\nu, c_F^\nu)} = \frac{g_H(c_H, c_F)}{g_F(c_H, c_F)} = \frac{p_F}{p_H} \tag{89}$$

B.5.2 Risk Sharing and the Cyclicality of Exchange Rates

Consider the following planning problem where we assume i) equal weights for each agent within each country and ii) the planner potentially faces costs to allocate good F to Home households. As a result of (ii), if costs are positive, the allocation is constrained efficient.

The planning problem is given as follows:

$$\max_{\{\delta^\nu, \delta^{\nu*}, c_H, c_F, c_H^*, c_F^*\}} \mu \int_\nu \frac{1}{1-\gamma} (\underbrace{\delta_t^\nu g(c_H, c_F)}_{C^\nu})^{1-\gamma} d\nu + (1-\mu) \int_{\nu^*} \frac{1}{1-\gamma} (\delta_t^{\nu*} g^*(c_H^*, c_F^*))^{1-\gamma} d\nu,$$

$$\text{s.t.} \quad \int_i c_H^\nu d\nu + \int_\nu c_H^{*\,\nu} d\nu = Y_H, \quad \int_\nu c_F^\nu d\nu (1 + \tau_t^{plan}) + \int_\nu c_F^{*\,\nu} d\nu = Y_F,$$

$$\int \delta \nu d\nu = 1, \quad \int \delta^{*\,\nu} d\nu = 1$$

where $g(\cdot)$ is characterized by (26) and the constraints have associated multipliers λ_H, λ_F and $\lambda^\delta, \lambda^{\delta*}$.

There are three associated efficiency conditions. The first reveals that the planner wishes to equate marginal utility for every agent within countries:

$$\mu \left(C^\nu\right)^{-\gamma} C = \lambda^\delta, \quad (1-\mu) \left(C^{*\nu}\right)^{-\gamma} C = \lambda^{*\delta}$$

Since λ^δ and $\lambda^{*\delta}$ do not depend on inidividual histories ν, the planner chooses $\delta^\nu = \delta$ which must imply from the law of large numbers $\delta = 1$, and analogously $\delta^* = 1$.

The second condition relates relative goods consumption across the two countries:

$$\frac{c_H}{c_F} = \left(\frac{\alpha}{1-\alpha}\right)^2 (1 + \tau_t^{plan})^\zeta \frac{c_H^*}{c_F^*}, \tag{90}$$

which coincides with the decentralized allocation.

The final condition concerns the role of real exchange rates. Equating the first

order conditions with respect to c_H and c_H^* implies:

$$\frac{\int_\nu (\delta^\nu)^{1-\gamma} \, d\nu}{\int_\nu (\delta^{\nu*})^{1-\gamma} \, d\nu} \left(\frac{C}{C^*}\right)^{-\gamma} = \frac{1-\mu}{\mu} \frac{1}{\mathcal{E}} \tag{91}$$

where we have derived the real exchange rate as the ratio of multipliers as follows $\frac{dC^*}{dC} = \frac{1}{\mathcal{E}}$.[37],[38] Condition (91) illustrates how domestic market incompleteness affects the international transmission of risk. Assuming $\mu = 1 - \mu$, (91) differs from (1) because, absent borrowing constraints:

$$\int_\nu (\delta^\nu)^{1-\gamma} \, d\nu \neq \int_\nu (\delta^\nu)^{-\gamma} \, d\nu \tag{93}$$

reflecting a Jensen's term which arises because the planner cares about inequality. To see this, we can rewrite the above as

$$\int_\nu (\delta^\nu)^{1-\gamma} \, d\nu = \int_\nu (\delta^\nu)^{-\gamma} \, (\delta^\nu d\nu) = \mathbb{E}_\delta[(\delta^\nu)^{-\gamma}]$$

reflecting that the planner weights the discount factor adjustment $(\delta^\nu)^{-\gamma}$ by the dispersion of consumptions across agents.

Moreover, in the presence of borrowing constraints (5), the decentralized equilibrium exchange rate process reflects the maximal beta wedge (i.e. that of the most patient investors), not the average

$$\frac{\int_\nu \left(\delta_{t+1}^\nu\right)^{-\gamma} d\nu}{\int_\nu \left(\delta_t^\nu\right)^{-\gamma} d\nu} \neq \max_\nu \beta_{t+1}^\nu$$

yielding an additional inefficiency of the decentralized equilibrium.

To recover the first best, substitute $\delta_t^i = 1 \; \forall i = 1$, and evaluate (91). In general, comparing the planner conditions with the decentralized allocation reveals two im-

---

[37]This equation follows because:

$$\left(\frac{C}{C^*}\right)^{-\gamma} = \frac{1-\mu}{\mu} \frac{g_H^*}{g_H} = \frac{1-\mu}{\mu} \frac{g_F^*}{g_F}(1+\tau) \tag{92}$$

and $\dfrac{g_F^*}{g_F} = \dfrac{\frac{dC^*}{dc_F^*}}{\frac{dC}{dc_F}} = \dfrac{\frac{dC^*}{dC}}{\frac{dc_F^*}{dc_F}} = \dfrac{\frac{dC^*}{dC}}{\frac{dY_F - c_F(1+\tau)}{dc_F}} = -\dfrac{dC^*}{dC}(1+\tau)^{-1}$.

[38]Away from the flex price limit, Aguiar, Itskhoki and Mukhin (2025) argue that, due to pricing to market frictions, the real exchange rate does not correspond to the ratio of multipliers, and therefore the Backus-Smith condition is not a sufficient condition for risk-sharing.

pediments to risk-sharing, τ and δ^ν. If trade wedges are technological $\tau = \tau^{plan}$, then idiosyncratic risk would be the only reason why risk sharing is low.

C. EMPIRICAL RESULTS AND ROBUSTNESS

C.1. Berger et al. (2023) wedge

Figure 5 plots the time-series of the consumption based wedge from Berger et al. (2023) dataset, reconstructed for different values of $\gamma \in \{1, 3, 5, 7.5\}$.

Figure 4: *Time-series of Consumption based Wedge*

(a) *Inverse of EIS $\gamma = 1$*   **(b)** *Inverse of EIS $\gamma = 3$*

(c) *Inverse of EIS $\gamma = 5$*   **(d)** *Inverse of EIS $\gamma = 7.5$*

Tables 6 -8 list the volatility of bilateral real exchange rate, the (negative of the) correlation of the wedge with real exchange rate growth and the threshold in equation (14) for different values of $\gamma \in \{3, 7.5, 10\}$ for seventeen advanced economies. Both conditional and unconditional moments are presented in the Tables.

Table 6: *Empirical Moments: Real Exchange Rate Growth and Berger et al. (2023) Wedge*

$\gamma = 3$	Unconditional			Conditional		
ISO	$\sigma(\Delta e)$	$-\text{Corr}(\tilde{\beta}, \Delta e)$	Threshold	$\sigma_t(\Delta e)$	$-\text{Corr}_t(\tilde{\beta}, \Delta e)$	Threshold$_t$
AUS	0.13	0.11	0.30	0.10	0.20	0.24
BEL	0.10	0.25	0.24	0.09	0.32	0.25
CAN	0.10	0.35	0.22	0.08	0.29	0.20
CHE	0.09	0.01	0.21	0.08	0.05	0.19
DEU	0.10	0.24	0.24	0.08	0.33	0.21
DNK	0.10	0.26	0.23	0.09	0.43	0.26
ESP	0.11	0.33	0.26	0.09	0.21	0.22
FIN	0.12	0.27	0.27	0.09	0.29	0.23
FRA	0.10	0.27	0.24	0.09	0.33	0.23
GBR	0.11	0.37	0.25	0.09	0.58	0.22
IRL	0.09	0.26	0.20	0.06	0.09	0.17
ITA	0.11	0.37	0.26	0.10	0.31	0.27
JPN	0.11	-0.32	0.25	0.10	-0.31	0.24
NLD	0.11	0.25	0.24	0.09	0.25	0.25
NOR	0.12	0.27	0.28	0.10	0.50	0.29
PRT	0.11	0.31	0.25	0.09	0.12	0.23
SWE	0.13	0.33	0.30	0.09	0.34	0.24
AVERAGE	0.11	0.23	0.25	0.09	0.25	0.23

Notes: Table lists the negative of the correlation between the US discount factor wedge ($\tilde{\beta}$) the bilateral real exchange rate growth (Δe), the threshold for exchange rate cyclicality described in equation (14) assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$), and the standard deviation of the real exchange rate growth for pair of seventeen advanced economies. Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 3$ from Consumer Expenditure Survey in the US. Standard deviation of the discount factor wedge is provided in Table 1. Sample: 1992–2017 (annual).

There are two main takeaways. First, for fifteen of the seventeen bilateral pairs, the exceptions being Japan and Switzerland, and for different values of γ, we find that the exchange rates are risky with respect to the $\tilde{\beta}$ wedge – the correlation with the bilateral real exchange rate is negative. Second, starting from values of $\gamma = 3$, the threshold is low enough such that the $\tilde{\beta}$ wedge can satisfy the inequality (14) without a need for segmentation shocks i.e. we can set $u = 0$ and still reconcile the sign of the Backus Smith puzzle. When $\gamma = 1$, exchange rates are still risky with respect to the $\tilde{\beta}$ wedge, but the wedge is not volatile enough and as a result the inequality with the threshold is not met. As the EIS increases, the within group dispersion in marginal utility of consumption share growth is amplified due to a Jensen's term.

Table 7: *Empirical Moments: Real Exchange Rate Growth and Berger et al. (2023) Wedge*

$\gamma = 7.5$	Unconditional			Conditional		
ISO	$\sigma(\Delta e)$	$-\text{Corr}(\tilde{\beta}, \Delta e)$	Threshold	$\sigma_t(\Delta e)$	$-\text{Corr}_t(\tilde{\beta}, \Delta e)$	Threshold$_t$
AUS	0.13	0.01	0.06	0.10	0.01	0.05
BEL	0.10	0.24	0.05	0.09	0.22	0.05
CAN	0.10	0.24	0.04	0.08	0.09	0.04
CHE	0.09	-0.03	0.04	0.08	-0.02	0.04
DEU	0.10	0.24	0.05	0.08	0.30	0.04
DNK	0.10	0.24	0.05	0.09	0.34	0.05
ESP	0.11	0.31	0.05	0.09	0.19	0.04
FIN	0.12	0.25	0.05	0.09	0.23	0.05
FRA	0.10	0.26	0.05	0.09	0.25	0.05
GBR	0.11	0.35	0.05	0.09	0.56	0.04
IRL	0.09	0.21	0.04	0.06	0.05	0.03
ITA	0.11	0.34	0.05	0.10	0.20	0.05
JPN	0.11	-0.40	0.05	0.10	-0.48	0.04
NLD	0.11	0.25	0.05	0.09	0.15	0.05
NOR	0.12	0.20	0.06	0.10	0.29	0.05
PRT	0.11	0.29	0.05	0.09	0.04	0.05
SWE	0.13	0.26	0.06	0.09	0.21	0.05
AVERAGE	0.11	0.19	0.05	0.09	0.15	0.05

Notes: Table lists the negative of the correlation between the US discount factor wedge ($\tilde{\beta}$) the bilateral real exchange rate growth (Δe), the threshold for exchange rate cyclicality described in equation (14) assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$), and the standard deviation of the real exchange rate growth for pair of seventeen advanced economies. Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 7.5$ from Consumer Expenditure Survey in the US. Standard deviation of the discount factor wedge is provided in Table 1. Sample: 1992–2017 (annual).

The within-group dispersion in marginal utility growth proxies for the dispersion in marginal utility growth across idiosyncratic states. As we increase γ, the correlation of the wedge with exchange rate growth does not change as much as volatility of the wedge, which directly affects the threshold, making it easier to satisfy the inequality (14).

We can conduct a stronger empirical test of theory and verify if the correlation of difference in as-if representative agent log SDFs with real exchange rate growth is positive. We construct the as-if representative agent log SDF in the U.S. as implied by the model: $\widehat{m} = -\gamma \Delta c + \tilde{\beta}$. Since we have assumed the wedge is constant in Foreign, the Foreign SDF is the scaled consumption growth $m^* = -\gamma \Delta c^*$.

Table 8: *Empirical Moments: Real Exchange Rate Growth and Berger et al. (2023) Wedge*

$\gamma = 10$	Unconditional			Conditional		
ISO	$\sigma(\Delta e)$	$-\text{Corr}(\tilde{\beta}, \Delta e)$	Threshold	$\sigma_t(\Delta e)$	$-\text{Corr}_t(\tilde{\beta}, \Delta e)$	Threshold$_t$
AUS	0.13	-0.02	0.04	0.10	-0.02	0.03
BEL	0.10	0.23	0.03	0.09	0.20	0.04
CAN	0.10	0.21	0.03	0.08	0.06	0.03
CHE	0.09	-0.03	0.03	0.08	-0.03	0.03
DEU	0.10	0.23	0.03	0.08	0.29	0.03
DNK	0.10	0.24	0.03	0.09	0.32	0.04
ESP	0.11	0.30	0.04	0.09	0.18	0.03
FIN	0.12	0.24	0.04	0.09	0.21	0.03
FRA	0.10	0.25	0.03	0.09	0.23	0.03
GBR	0.11	0.34	0.04	0.09	0.55	0.03
IRL	0.09	0.19	0.03	0.06	0.05	0.02
ITA	0.11	0.32	0.04	0.10	0.17	0.04
JPN	0.11	-0.41	0.04	0.10	-0.50	0.03
NLD	0.11	0.25	0.03	0.09	0.13	0.04
NOR	0.12	0.19	0.04	0.10	0.26	0.04
PRT	0.11	0.28	0.04	0.09	0.03	0.03
SWE	0.13	0.24	0.04	0.09	0.19	0.03
AVERAGE	0.11	0.18	0.04	0.09	0.14	0.03

Notes: Table lists the negative of the correlation between the US discount factor wedge ($\tilde{\beta}$) the bilateral real exchange rate growth (Δe), the threshold for exchange rate cyclicality described in equation (14) assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$), and the standard deviation of the real exchange rate growth for pair of seventeen advanced economies. Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 10$ from Consumer Expenditure Survey in the US. Standard deviation of the discount factor wedge is provided in Table 1. Sample: 1992–2017 (annual).

Tables 9 - 11 lists these correlation (both conditional and unconditional) for all seventeen bilateral pairs for values of $\gamma \in \{3, 7.5, 10\}$, and cross-country average is noted in the last row. Consistent with theoretical prediction, the correlation of relative SDF with exchange rate growth is positive for all pairs with the exception of Japan and Switzerland.

Overall, we robustly find that (i) exchange rates are risky with respect to the wedge, (ii) inequality (14) is satisfied for values of $\gamma >= 3$, and (iii) correlation of relative pricing kernels (constructed using the wedge) with exchange rate is positive while correlation of relative consumption growth with exchange rate is negative.

Table 9: *Correlation between Pricing Kernels and Real Exchange Rate Growth*

$\gamma = 3$	Unconditional		Conditional	
ISO	$\text{Corr}(\Delta c - \Delta c^*, \Delta e)$	$\text{Corr}(m^* - \widehat{m}, \Delta e)$	$\text{Corr}_t(\Delta c - \Delta c^*, \Delta e)$	$\text{Corr}_t(m^* - \widehat{m}, \Delta e)$
AUS	-0.39	0.08	-0.40	0.17
BEL	-0.30	0.23	-0.44	0.28
CAN	-0.12	0.34	-0.18	0.28
CHE	-0.09	-0.00	-0.23	0.03
DEU	0.08	0.23	-0.03	0.32
DNK	-0.34	0.22	-0.42	0.38
ESP	-0.01	0.33	-0.00	0.21
FIN	-0.38	0.21	-0.32	0.26
FRA	-0.25	0.25	-0.34	0.30
GBR	0.07	0.38	-0.09	0.59
IRL	0.08	0.29	0.52	0.15
ITA	0.01	0.36	0.06	0.31
JPN	0.19	-0.30	0.05	-0.30
NLD	0.19	0.27	-0.12	0.24
NOR	-0.27	0.25	-0.63	0.46
PRT	0.21	0.32	0.09	0.13
SWE	-0.27	0.31	-0.28	0.33
AVERAGE	-0.09	0.22	-0.16	0.24

Notes: Table lists the correlation of relative consumption growth and exchange rate, and correlation of the relative SDF and real exchange rate growth assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$). Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 3$ from Consumer Expenditure Survey in the US. Sample: 1992–2017 (annual). See text for details.

Table 10: *Correlation between Pricing Kernels and Real Exchange Rate Growth*

$\gamma = 7.5$	Unconditional		Conditional	
ISO	$\mathrm{Corr}(\Delta c - \Delta c^*, \Delta e)$	$\mathrm{Corr}(m^* - \widehat{m}, \Delta e)$	$\mathrm{Corr}_t(\Delta c - \Delta c^*, \Delta e)$	$\mathrm{Corr}_t(m^* - \widehat{m}, \Delta e)$
AUS	-0.39	-0.01	-0.40	-0.00
BEL	-0.30	0.23	-0.44	0.20
CAN	-0.12	0.23	-0.18	0.08
CHE	-0.09	-0.03	-0.23	-0.03
DEU	0.08	0.23	-0.03	0.29
DNK	-0.34	0.23	-0.42	0.31
ESP	-0.01	0.32	-0.00	0.19
FIN	-0.38	0.22	-0.32	0.21
FRA	-0.25	0.25	-0.34	0.23
GBR	0.07	0.36	-0.09	0.57
IRL	0.08	0.23	0.52	0.08
ITA	0.01	0.34	0.06	0.20
JPN	0.19	-0.39	0.05	-0.48
NLD	0.19	0.26	-0.12	0.14
NOR	-0.27	0.19	-0.63	0.27
PRT	0.21	0.30	0.09	0.05
SWE	-0.27	0.25	-0.28	0.20
AVERAGE	-0.09	0.19	-0.16	0.15

Notes: Table lists the correlation of relative consumption growth and exchange rate, and correlation of the relative SDF and real exchange rate growth assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$). Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 7.5$ from Consumer Expenditure Survey in the US. Sample: 1992–2017 (annual). See text for details.

Table 11: *Correlation between Pricing Kernels and Real Exchange Rate Growth*

$\gamma = 10$	Unconditional		Conditional	
ISO	$\text{Corr}(\Delta c - \Delta c^*, \Delta e)$	$\text{Corr}(m^* - \widehat{m}, \Delta e)$	$\text{Corr}_t(\Delta c - \Delta c^*, \Delta e)$	$\text{Corr}_t(m^* - \widehat{m}, \Delta e)$
AUS	-0.39	-0.03	-0.40	-0.04
BEL	-0.30	0.22	-0.44	0.18
CAN	-0.12	0.21	-0.18	0.06
CHE	-0.09	-0.03	-0.23	-0.04
DEU	0.08	0.23	-0.03	0.29
DNK	-0.34	0.22	-0.42	0.30
ESP	-0.01	0.31	-0.00	0.18
FIN	-0.38	0.22	-0.32	0.20
FRA	-0.25	0.24	-0.34	0.22
GBR	0.07	0.34	-0.09	0.56
IRL	0.08	0.21	0.52	0.08
ITA	0.01	0.32	0.06	0.17
JPN	0.19	-0.40	0.05	-0.50
NLD	0.19	0.25	-0.12	0.12
NOR	-0.27	0.18	-0.63	0.24
PRT	0.21	0.29	0.09	0.03
SWE	-0.27	0.23	-0.28	0.18
AVERAGE	-0.09	0.18	-0.16	0.13

Notes: Table lists the correlation of relative consumption growth and exchange rate, and correlation of the relative SDF and real exchange rate growth assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$). Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 10$ from Consumer Expenditure Survey in the US. Sample: 1992–2017 (annual). See text for details.

Table 12: *Covariances between Pricing Kernels and Real Exchange Rate Growth*

$\gamma = 5$	Unconditional (100 $\times$)		Conditional (100 $\times$)	
ISO	$\mathrm{Cov}(\Delta c - \Delta c^*, \Delta e)$	$\mathrm{Cov}(m^* - \widehat{m}, \Delta e)$	$\mathrm{Cov}_t(\Delta c - \Delta c^*, \Delta e)$	$\mathrm{Cov}_t(m^* - \widehat{m}, \Delta e)$
AUS	-0.06	0.57	-0.04	0.75
BEL	-0.03	3.07	-0.03	1.95
CAN	-0.01	3.38	-0.01	1.29
CHE	-0.01	-0.42	-0.02	-0.15
DEU	0.01	3.19	-0.00	3.11
DNK	-0.06	2.90	-0.06	2.96
ESP	-0.00	4.69	-0.00	2.13
FIN	-0.11	3.35	-0.03	2.47
FRA	-0.02	3.37	-0.03	2.63
GBR	0.01	5.07	-0.01	6.08
IRL	0.02	2.58	0.05	0.65
ITA	0.00	5.18	0.01	2.42
JPN	0.03	-5.25	0.01	-5.22
NLD	0.03	3.58	-0.01	1.46
NOR	-0.04	3.33	-0.07	3.77
PRT	0.05	4.48	0.02	0.72
SWE	-0.05	4.51	-0.02	2.57
AVERAGE	-0.02	2.80	-0.02	1.74

Notes: Table lists the 100 $\times$ covariance of relative consumption growth and exchange rate, and 100 $\times$ covariance of the relative SDF and real exchange rate growth assuming domestic incomplete markets only within the US ($\tilde{\beta}^* = 0$). Real exchange rate growth is constructed from Jordà et al. (2017) database. US discount factor wedge is constructed as in Berger et al. (2023) for $\gamma = 5$ from Consumer Expenditure Survey in the US. Sample: 1992–2017 (annual). See text for details.

C.1.1   Alternate Conditioning Set

Table 13: *Empirical Moments: Real Exchange Rate Growth and Berger et al. (2023) Wedge with a Large Conditioning Set*

ISO	$\sigma_t(\Delta e)$	$-\mathrm{Corr}_t(\tilde{\beta}, \Delta e)$	Threshold$_t$
AUS	0.08	0.18	0.10
BEL	0.07	0.18	0.09
CAN	0.07	0.33	0.08
CHE	0.06	0.29	0.07
DEU	0.08	0.68	0.10
DNK	0.08	0.49	0.09
ESP	0.08	0.49	0.11
FIN	0.08	0.36	0.09
FRA	0.09	0.34	0.10
GBR	0.07	0.67	0.09
IRL	0.05	-0.13	0.08
ITA	0.08	0.32	0.10
JPN	0.09	-0.48	0.09
NLD	0.08	0.24	0.11
NOR	0.09	0.16	0.10
PRT	0.08	0.49	0.12
SWE	0.09	0.28	0.10
AVERAGE	0.08	0.29	0.09

We conduct robustness with a larger conditioning set to construct moments of Δe_{t+1}, and $\tilde{\beta}_{t+1}$. We control for date t values of log consumption in US and Foreign (c_t, c_t^*), short term nominal interest rate (i_t, i_t^*), level of long term interest rate in each country, log CPI in each country, and log bilateral real exchange rate (e_t). Relative to the baseline, we allow each country variables to have a separate loading in the conditioning set. Table 13 shows that the correlation remains sufficiently negative to meet the threshold. Results are shown for $\gamma = 5$.

C.1.2 Panel Fixed Effects Regressions

Table 14: *Panel Fixed Effects Empirical Moments: Real Exchange Rate Growth and Berger et al. (2023) Wedge*

ISO	$\sigma_t(\Delta e)$	$-\mathrm{Corr}_t(\tilde{\beta}, \Delta e)$	$\mathrm{Threshold}_t$
AUS	0.12	0.05	0.09
BEL	0.09	0.21	0.07
CAN	0.09	0.17	0.08
CHE	0.09	0.07	0.07
DEU	0.09	0.23	0.07
DNK	0.09	0.21	0.08
ESP	0.10	0.15	0.08
FIN	0.11	0.20	0.09
FRA	0.09	0.24	0.08
GBR	0.10	0.43	0.07
IRL	0.09	0.00	0.07
ITA	0.10	0.20	0.08
JPN	0.10	-0.46	0.07
NLD	0.09	0.23	0.08
NOR	0.11	0.22	0.09
PRT	0.10	0.16	0.08
SWE	0.11	0.27	0.09
AVERAGE	0.10	0.15	0.08

We now construct the conditional moments by residualizing the beta wedge and real exchange rate using a panel fixed effects regression. In the baseline exercise reported in the main text, the residuals were constructed from each country's own regression, allowing both the intercept and slope to be different across bilateral pair of countries. We continue to set $\gamma = 5$.

Table 14 reports the results. The conditional correlation of the beta wedge with exchange rate is sufficiently negative that it meets the threshold.[39]

---

[39]We briefly note why the conditional correlation estimated here may differ from the conditional correlation in the main text. Fixed effect regression based residual correlations emphasize the co-movement of deviations relative to common global relationships. If the true underlying relationships differ across countries (heterogeneous effects), imposing common slopes reduces correlation or even distorts it, because residuals become mixed with systematic differences in responsiveness.

C.2. Permanent income risk based wedge

Bayer et al. (2019) construct a measure of very persistent income risk using Survey of Income Participation Program (SIPP). In the Constantinides and Duffie (1996) model, the discount factor wedge $\tilde{\beta}^{CD}$ is a function of the cross-sectional variance of the permanent income process y^2. Namely:

$$\tilde{\beta}^{CD} = \frac{\gamma\,(\gamma+1)}{2}y^2$$

Using $\gamma = 5$, and the variance of income risk series estimated in Bayer et al. (2019), we reconstruct the beta wedge assuming only permanent income risk. We keep the sample fixed at annual 1992–2013 (last year for which Bayer et al. (2019) series is available).

Table 15: *Empirical Moments: Real Exchange Rate Growth and Bayer et al. (2019) Wedge*

ISO	$\sigma(\Delta e)$	$-\mathrm{Corr}(\tilde{\beta}^{\mathrm{CD}}, \Delta e)$	$\sigma(\tilde{\beta}^{CD})$	Threshold
AUS	0.13	0.19	0.03	4.89
BEL	0.12	0.06	0.03	4.33
CAN	0.09	0.39	0.03	3.19
CHE	0.12	-0.09	0.03	4.54
DEU	0.12	0.09	0.03	4.38
DNK	0.12	0.06	0.03	4.26
ESP	0.12	0.11	0.03	4.38
FIN	0.12	0.09	0.03	4.59
FRA	0.12	0.07	0.03	4.28
GBR	0.13	0.31	0.03	4.65
IRL	0.09	-0.10	0.03	3.49
ITA	0.12	0.07	0.03	4.48
JPN	0.13	-0.39	0.03	4.69
NLD	0.12	0.08	0.03	4.41
NOR	0.12	0.25	0.03	4.52
PRT	0.12	0.09	0.03	4.25
SWE	0.13	0.17	0.03	4.80
AVERAGE	0.12	0.09	0.03	4.36

Table 15 reports the results for the condition (14) with beta wedge constructed as

shown in equation (22) in Section 4.1. While the discount factor wedge measured only with permanent income risk also comoves negatively with exchange rate, the volatility of this wedge is of two orders of magnitude lower than the discount factor wedge constructed from Berger et al. (2023) with $\gamma = 5$ (see Table 1 for summary statistics of the Berger et al. (2023) wedge). Consequently, relying solely on permanent income risk based wedge is insufficient to explain the Backus Smith cyclicality puzzle.

C.3. Bilateral Wedge from GRID

There is another major difference in the way the wedge is calculated using the GRID data relative to the Berger et al. (2023). The data in the GRID is on $\frac{1}{N_g} \sum_{i=1}^{N_g} \log \left(\frac{Y_{t+1}^i}{Y_t^i} \right)$, where Y^i is individual i's income. We can therefore only construct $\frac{1}{N_g} \sum_{i=1}^{N_g} \log \left(\frac{Y_{t+1}^i}{Y_t^i} \right)^{-\gamma}$. True wedge and the proxy wedge can be connected with a Jensen's gap term:

$$\underbrace{\log \frac{1}{N_g} \sum_{i=1}^{N_g} \left(\frac{Y_{t+1}^i/Y_{t+1}}{Y_t^i/Y_t} \right)^{-\gamma}}_{\text{True wedge}} = \underbrace{\frac{1}{N_g} \sum_{i=1}^{N_g} \log \left(\frac{Y_{t+1}^i/Y_{t+1}}{Y_t^i/Y_t} \right)^{-\gamma}}_{\text{Proxy Wedge}} + \text{Jensen gap}$$

where Y_t is the average income in the economy. We can decompose either of the wedges into within group term and a composition term:

$$\log \frac{1}{N_g} \sum_{i=1}^{N_g} \left(\frac{Y_{t+1}^i/Y_{t+1}}{Y_t^i/Y_t} \right)^{-\gamma} = \log \left(\frac{Y_{t+1}^g/Y_{t+1}}{Y_t^g/Y_t} \right)^{-\gamma} + \log \frac{1}{N_g} \sum_{i=1}^{N_g} \left(\frac{Y_{t+1}^i/Y_{t+1}^g}{Y_t^i/Y_t^g} \right)^{-\gamma}$$

$$\frac{1}{N_g} \sum_{i=1}^{N_g} \log \left(\frac{Y_{t+1}^i/Y_{t+1}}{Y_t^i/Y_t} \right)^{-\gamma} = \log \left(\frac{Y_{t+1}^g/Y_{t+1}}{Y_t^g/Y_t} \right)^{-\gamma} + \frac{1}{N_g} \sum_{i=1}^{N_g} \log \left(\frac{Y_{t+1}^i/Y_{t+1}^g}{Y_t^i/Y_t^g} \right)^{-\gamma}$$

where Y_t^g denotes average income of the group g. Both the true and the proxy wedges have the identical composition term capturing change in powered income share of the group in the economy. Both also capture a within group dispersion of income shares across households. Jensen gap in the true wedge captures the convexity component that comes from averaging before taking logs. Taking a second order approximation of the wedge, it can be shown that the Jensen gap term increases in γ

since the degree of convexity goes up with γ. There are no a priori reasons to believe that the Jensen gap term would not affect the correlation of the wedge with exchange rate. However, when we computed the correlation of beta wedges computed with different EIS parameters for the U.S., we found that the correlation was relatively stable as we increased γ. In Table 19, we show the correlation with exchange rate when measured using log-income based wedge and the consumption based wedge for $\gamma = 5$. We discuss this issue further in Section C.4.

We measure bilateral wedges using micro statistics from the Global Repository of Income Dynamics (Guvenen et al., 2022, GRID henceforth). We focus on sample of advanced economies for which we can obtain the longest panel to construct the bilateral wedge against the U.S. The annual data sample covers Canada, Denmark, France, Germany, Italy, Norway, Sweden, the UK, and the US spanning 1998–2015.

Variables used in our wedge construction are residual log earnings growth for top 1 percent, 2.5 percent, 5 percent, 10 percent of each country's log earnings distribution in year t, and the mean residual log earnings growth for that year. These statistics are computed for Male in age groups 25–55 population of each country. GRID constructs residual log earnings growth from first regressing log earnings on age dummies for each year t and then taking average of growth rate.[40]

Denote with $\overline{\log I_{gt}}$ the average (residual) log earnings for the group g, and $\overline{\log I_t}$ as the average (residual) log earnings in the country at time t. Then the income share at date t for a group g is defined as $\varphi_{gt}^I \equiv \exp\left(\overline{\log I_{gt}} - \overline{\log I_t}\right)$. The discount factor wedge for a group g is then constructed as:[41]

$$\beta_{gt}^I = \left(\frac{\varphi_{gt+1}^I}{\varphi_{gt}^I}\right)^{-\gamma} \tag{94}$$

where γ is the inverse of the intertemporal elasticity of substitution. We de-mean

---

[40]In unreported results, we verified the results are robust to using statistics for all genders, age 25–55 population.

[41]The data in GRID is averages at the percentile level. This proxy construction is thus different from Berger et al. (2023) in that we are taking average income shares to construct a wedge, whereas Berger et al. (2023) construct average of the individual wedges.

each country's wedge for (i) partially addressing measurement error as in Berger et al. (2023), and (ii) comparison across countries.[42] The bilateral wedge is then given by the difference in the de-meaned country specific wedges for group g:

$$\Delta\beta_{gt} \equiv \beta_{gt}^I - \beta_{gt}^{*I} \tag{95}$$

C.3.1 When can we use correlation of income wedge to proxy correlation of consumption wedge.

Ideally, we would use micro-data on consumption for each country and construct bilateral wedge distances. Instead, we need to rely on micro-data for income. The two key concerns are i) the correlation of consumption shares differs from the correlation of growth shares, ii) the volatility of consumption shares is significinatly lower than the volatility of income shares. Consider the following statistical model capturing the idea of consumption smoothing:

$$\frac{C_{t+1}^i}{C_t^i} = \theta^i \frac{Y_{t+1}^i}{Y_t^i}, \quad \frac{C_{t+1}^{i*}}{C_t^{i*}} = \theta^{i*} \frac{Y_{t+1}^{i*}}{Y_t^{i*}}, \tag{96}$$

$$\frac{C_{t+1}}{C_t} = \theta \frac{Y_{t+1}}{Y_t}, \quad \frac{C_{t+1}^*}{C_t^*} = \theta^* \frac{Y_{t+1}^*}{Y_t^*} \tag{97}$$

where $\theta^{(*)}, \theta^{i(*)} \leq 1$ such that consumption growth is less volatile than income growth, and for high income individuals on their Euler equations $\theta^{i(*)} < \theta^{(*)}$, i.e. they engage in more consumption smoothing than the aggregate. As long as $\theta^i/\theta \approx \theta^{i*}/\theta^*$, such that consumption smoothing of the marginal investor relative to the aggregate is similar across countries, then:

$$cov\left(\tilde{\beta}_{t+1} - \tilde{\beta}_{t+1}^*, \Delta e_{t+1}\right) = cov\left(-\gamma \log\left(\frac{\theta^i}{\theta} \frac{Y_{t+1}^i/Y_t^i}{Y_{t+1}/Y_t}\right) + \gamma \log\left(\frac{\theta^{i*}}{\theta^*} \frac{Y_{t+1}^{i*}/Y_t^{i*}}{Y_{t+1}^*/Y_t^*}\right), \Delta e_{t+1}\right)$$

$$\approx \frac{\theta^i}{\theta} cov\left(\tilde{\beta}_{t+1}^y - \tilde{\beta}_{t+1}^{y*}, \Delta e_{t+1}\right)$$

where $\tilde{\beta}_{t+1}^y = -\gamma \log\left(\frac{Y_{t+1}^i/Y_t^i}{Y_{t+1}/Y_t}\right)$.

We will need additionaly that $\dfrac{\theta^i}{\theta}$ is same within group for this to work for BBD

---

[42]Results are robust to using the non-deamened wedges.

wedge. Which means that differences in consumptoipn shares within group are entirely driven by income shares within group.

Moreover, in the correlation space, the ratio θ^i/θ does not affect scaling. Specifically, when $\theta^i/\theta \approx \theta^{i*}/\theta^*$, then:

$$\rho_{\tilde{\beta}_{t+1}-\tilde{\beta}^*_{t+1},\Delta e_{t+1}} = \frac{cov_t\left(\tilde{\beta}_{t+1}-\tilde{\beta}^*_{t+1},\Delta e_{t+1}\right)}{\sigma_t(\tilde{\beta}_{t+1}-\tilde{\beta}^*_{t+1})\sigma_t(\Delta e_{t+1})} \approx \frac{cov_t\left(\tilde{\beta}^y_{t+1}-\tilde{\beta}^{y*}_{t+1},\Delta e_{t+1}\right)}{\sigma_t(\tilde{\beta}^y_{t+1}-\tilde{\beta}^{y*}_{t+1})\sigma_t(\Delta e_{t+1})} = \rho_{\tilde{\beta}^y_{t+1}-\tilde{\beta}^{y*}_{t+1},\Delta e_{t+1}}$$

C.3.2 Summary Statistics and Additional Results with GRID dataset

Table 16: *Summary Statistics for GRID data set*

Panel A: Log-income based Country-specific Wedge

iso	Top 10% $\sigma(\beta_{gt}^I)$	Top 10% $\mathrm{Corr}(\beta_{gt}^I, \widehat{Y}_t)$	Top 5% $\sigma(\beta_{gt}^I)$	Top 5% $\mathrm{Corr}(\beta_{gt}^I, \widehat{Y}_t)$	Top 2.5% $\sigma(\beta_{gt}^I)$	Top 2.5% $\mathrm{Corr}(\beta_{gt}^I, \widehat{Y}_t)$	Top 1% $\sigma(\beta_{gt}^I)$	Top 1% $\mathrm{Corr}(\beta_{gt}^I, \widehat{Y}_t)$
CAN	0.06	0.03	0.13	0.09	0.15	0.06	0.15	-0.04
DEU	0.10	-0.36	0.19	-0.27	0.19	-0.21	0.15	-0.24
DNK	0.10	0.30	0.20	-0.07	0.23	-0.16	0.22	-0.26
FRA	0.11	-0.19	0.28	-0.29	0.35	-0.28	0.41	-0.27
ITA	0.20	0.33	0.50	0.10	0.55	-0.00	0.48	-0.04
NOR	0.10	0.52	0.18	0.43	0.28	0.32	0.43	0.30
SWE	0.09	-0.53	0.23	-0.58	0.33	-0.60	0.37	-0.60
USA	0.09	-0.17	0.14	-0.03	0.18	-0.01	0.19	-0.07
AVERAGE	0.11	-0.01	0.23	-0.08	0.28	-0.11	0.30	-0.15

Panel B: Log-Income based Bilateral Wedge

iso	$\sigma(\Delta e_t)$	$\sigma(\Delta \beta_{gt})$ Top 10%	Top 5%	Top 2.5%	Top 1%
CAN	0.09	0.05	0.06	0.08	0.09
DEU	0.10	0.14	0.24	0.26	0.26
DNK	0.10	0.13	0.20	0.24	0.24
FRA	0.10	0.12	0.21	0.24	0.28
ITA	0.11	0.19	0.39	0.38	0.40
NOR	0.11	0.13	0.22	0.34	0.52
SWE	0.13	0.09	0.17	0.24	0.28
AVERAGE	0.10	0.12	0.21	0.25	0.30

Notes: Panel A reports country-specific summary statistics for the standard deviation of the wedge, $\sigma(\beta_{gt}^I)$, and its correlation with output growth, $\mathrm{Corr}(\beta_{gt}^I, \widehat{Y}_t)$, across four percentile groups. Panel B lists the standard deviation of the bilateral real exchange rate (Δe) and the bilateral discount factor wedges constructed for different groups β_{gt}^I. Wedges are constructed using residual log earnings data for Male ages between 25–55 in GRID. Inverse of EIS, $\gamma = 5$. Real exchange rate growth is constructed from Jordà et al. (2017) database. Sample: 1998–2015 (annual). See text for details.

Table 17: *Correlation, Thresholds, and Covariances with Real Exchange Rate Growth*

Panel A: Correlation and Thresholds for Top 10% and 1% Groups

iso	Unconditional $-Corr(\tilde{\beta} - \tilde{\beta}^*, \Delta e)$ Top 10%	Top 1%	*Thresh.*	Conditional $-Corr_t(\tilde{\beta} - \tilde{\beta}^*, \Delta e)$ Top 10%	Top 1%	*Thresh.$_t$*
CAN	0.30	0.66	0.17	0.33	0.63	0.20
DEU	0.54	0.57	0.17	0.63	0.88	0.13
DNK	0.29	0.52	0.17	0.57	0.57	0.21
FRA	0.18	0.13	0.17	0.31	0.46	0.17
ITA	0.18	0.52	0.17	0.36	0.63	0.20
NOR	0.29	0.23	0.21	0.06	0.17	0.21
SWE	0.42	0.40	0.21	0.49	0.45	0.16
AVERAGE	0.31	0.43	0.18	0.39	0.54	0.18

Panel B: Correlation of Pricing Kernels and Real Exchange Rate Growth

iso	$Corr(\Delta c - \Delta c^*, \Delta e)$	$Corr(\widehat{m}^* - \widehat{m}, \Delta e)$ Top 10%	Top 1%	$Corr_t(\Delta c - \Delta c^*, \Delta e)$	$Corr_t(\widehat{m}^* - \widehat{m}, \Delta e)$ Top 10%	Top 1%
CAN	-0.03	0.13	0.55	-0.21	0.06	0.54
DEU	0.19	0.53	0.59	0.12	0.61	0.89
DNK	-0.29	0.06	0.38	-0.61	-0.38	0.36
FRA	-0.15	0.14	0.11	-0.21	0.23	0.42
ITA	0.09	0.19	0.51	0.14	0.42	0.62
NOR	-0.38	0.09	0.18	-0.58	-0.20	0.10
SWE	-0.06	0.32	0.39	-0.07	0.37	0.41
AVERAGE	-0.09	0.21	0.39	-0.20	0.16	0.48

C.3.3 Comparison with GRID based wedge with Consumption based wedge

Figure 5: *Consumption based Wedge and GRID's Income Based Wedge for the US (2.5 percentile)*
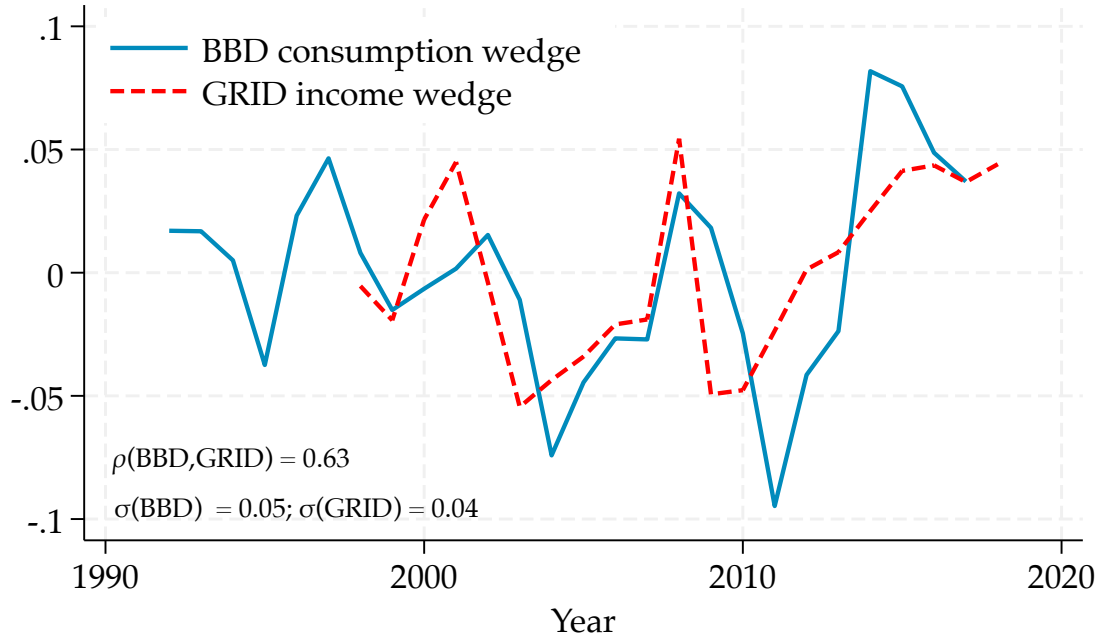
Notes: Figure plots the time-series of consumption based wedge in solid blue line, measured from Consumption expenditure survey for the US, and the log-income based wedge for the US measured from the GRID dataset in dashed red line. EIS is set at 1 in constructing both wedges.

C.3.4 Time-series plots of the bilateral wedges

Figure 6: *Bilateral Wedges*
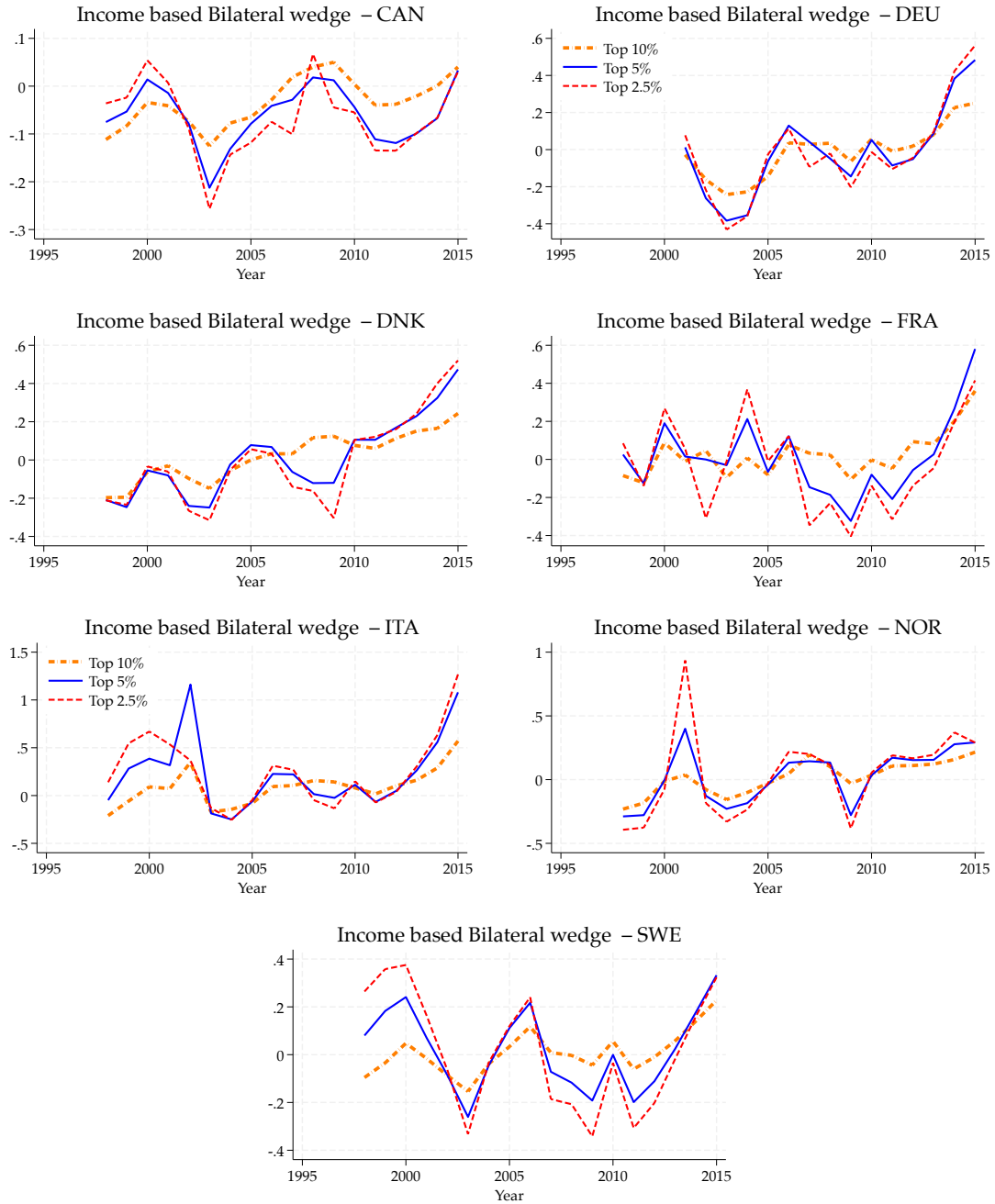
Notes: Figure plots the time-series of the bilateral income based wedges for US/Foreign country bilateral pairs for different percentile groups. Inverse of the EIS, γ is set to 5. Wedges are constructed from the GRID dataset.

C.3.5 Conditional Summary Statistics with GRID dataset

Table 18: *Summary Statistics for Country-specific Wedges (Conditional)*

Panel A: Log-income based Country-specific Wedge

iso	Top 10% $\sigma_t(\beta_{gt}^I)$	Top 10% $\mathrm{Corr}_t(\beta_{gt}^I, \widehat{Y}_t)$	Top 5% $\sigma_t(\beta_{gt}^I)$	Top 5% $\mathrm{Corr}_t(\beta_{gt}^I, \widehat{Y}_t)$	Top 2.5% $\sigma_t(\beta_{gt}^I)$	Top 2.5% $\mathrm{Corr}_t(\beta_{gt}^I, \widehat{Y}_t)$	Top 1% $\sigma_t(\beta_{gt}^I)$	Top 1% $\mathrm{Corr}_t(\beta_{gt}^I, \widehat{Y}_t)$
CAN	0.05	0.17	0.10	0.12	0.12	0.08	0.12	0.02
DEU	0.06	-0.40	0.14	-0.30	0.16	-0.26	0.14	-0.29
DNK	0.03	-0.53	0.13	-0.66	0.15	-0.70	0.17	-0.70
FRA	0.07	-0.80	0.17	-0.82	0.21	-0.77	0.17	-0.60
ITA	0.08	-0.08	0.23	-0.22	0.21	-0.41	0.24	-0.39
NOR	0.05	-0.25	0.13	-0.11	0.25	-0.07	0.42	0.02
SWE	0.07	-0.72	0.15	-0.68	0.20	-0.68	0.21	-0.65
USA	0.08	-0.32	0.13	-0.12	0.17	-0.06	0.20	-0.10
AVERAGE	0.06	-0.37	0.15	-0.35	0.19	-0.36	0.21	-0.34

Panel B: Log-Income based Bilateral Wedge

	$\sigma_t(\Delta e_t)$	$\sigma_t(\Delta\beta_{gt})$ Top 10%	$\sigma_t(\Delta\beta_{gt})$ Top 5%	$\sigma_t(\Delta\beta_{gt})$ Top 2.5%	$\sigma_t(\Delta\beta_{gt})$ Top 1%
CAN	0.10	0.02	0.05	0.07	0.08
DEU	0.08	0.05	0.14	0.16	0.14
DNK	0.09	0.03	0.10	0.13	0.17
FRA	0.09	0.06	0.16	0.21	0.21
ITA	0.09	0.08	0.22	0.20	0.28
NOR	0.11	0.08	0.16	0.28	0.44
SWE	0.09	0.07	0.15	0.20	0.21
AVERAGE	0.09	0.06	0.14	0.18	0.22

Notes: Table reports conditonal moments. Panel A reports country-specific summary statistics for the standard deviation of the wedge, $\sigma(\beta_{gt}^I)$, and its correlation with output growth, $\mathrm{Corr}(\beta_{gt}^I, \widehat{Y}_t)$, across four percentile groups. Panel B lists the standard deviation of the bilateral real exchange rate (Δe) and the bilateral discount factor wedges constructed for different groups β_{gt}^I. Wedges are constructed using residual log earnings data for Male ages between 25–55 in GRID. Inverse of EIS, $\gamma = 5$. Real exchange rate growth is constructed from Jordà et al. (2017) database. Sample: 1998–2015 (annual). See text for details.

C.4. Consumption-based wedge vs log income-based wedge in CEX

We construct log income based wedge in the CEX data:

$$\tilde{\beta}^{\ln y} = \frac{1}{N_g} \sum_{i=1}^{N_g} \log\left(\frac{Y_{t+1}^i/Y_t^i}{Y_{t+1}/Y_t}\right)^{-\gamma}$$

We find that the correlation of this wedge with exchange rate is similar to that of

Table 19: *Empirical Moments: Real Exchange Rate Growth and log income based Wedge*

ISO	$\sigma(\Delta e)$	Consumption based		Log-Income based	
		$-\mathrm{Corr}(\tilde{\beta}, \Delta e)$	Threshold	$-\mathrm{Corr}(\tilde{\beta}^{\ln y}, \Delta e)$	Threshold
AUS	0.13	0.05	0.10	0.17	0.83
BEL	0.10	0.24	0.08	0.03	0.66
CAN	0.10	0.28	0.08	0.28	0.61
CHE	0.09	-0.03	0.07	0.03	0.58
DEU	0.10	0.23	0.08	-0.02	0.66
DNK	0.10	0.25	0.08	0.05	0.64
ESP	0.11	0.32	0.09	0.18	0.72
FIN	0.12	0.25	0.09	0.20	0.75
FRA	0.10	0.26	0.08	0.04	0.66
GBR	0.11	0.36	0.08	0.22	0.68
IRL	0.09	0.23	0.07	0.15	0.54
ITA	0.11	0.36	0.09	0.28	0.71
JPN	0.11	-0.39	0.08	-0.17	0.69
NLD	0.11	0.25	0.08	0.02	0.67
NOR	0.12	0.23	0.09	0.27	0.77
PRT	0.11	0.30	0.09	0.04	0.70
SWE	0.13	0.28	0.10	0.28	0.82
AVERAGE	0.11	0.20	0.08	0.12	0.69

the true consumption based wedge. However, volatility of this wedge is of an order of magnitude smaller than the true wedge's volatility. Standard deviation of log income based wedge is 0.16, while that of the consumption based wedge is 1.29. While income based wedge should be more volatile than consumption based wedge, the volatility of log income based wedge is smaller than an actual income based wedge because of a missing Jensen's term as discussed in Section C.3. It turns out, on net, the log income based wedge is much less volatile than the true consumption based wedge. As a result, the threshold is not satisfied with the log-income wedge. The correlation of log income based wedge with exchange rate (0.12) is similar to the observed correlation of consumption based wedge with exchange rate (0.20).