Supply or Demand? Policy Makers' Confusion in the Presence of Hysteresis

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Abstract: *Policy makers need to separate between temporary demand-driven shocks and permanent shocks in order to design optimal aggregate demand policies. In this paper we study the case of a central bank that ignores the presence of hysteresis when identifying shocks. By assuming that all low-frequency output fluctuations are driven by permanent technology shocks, monetary policy is not aggressive enough in response to demand shocks. In addition, we show that errors in assessing the state of the economy can be self-perpetuating if seen through the lens of the mistaken views of the policy maker. We show that a central bank that mistakes a demand shock for a supply shock, will produce permanent effects on output through their suboptimal policies. Ex-post, the central bank will see an economy that resembles what they had forecast when designing their policies. The shock is indeed persistent and this persistence validates their assumption that the shock was a supply-driven one. The interaction between forecasts, policies and hysteresis creates the dynamics of self-perpetuating errors that is the focus of this paper.*

1 Introduction

The concept of potential output is central to the study of business cycles. In most macroeconomic models, potential output is linked to the existence of an equilibrium or steady state level around which fluctuations take place (Lucas Jr, 1977; Kydland and Prescott, 1990; Galí and Gertler, 2007). This logic extends to other concepts such as, for example, unemployment through measures such as the natural rate of unemployment, the non-accelerating rate of unemployment or full employment (Friedman, 1968). When the economy is hit with a cyclical shock, it temporarily moves away from this long-term equilibrium level and these transitory deviations of output from potential are what constitutes the business cycle. Within this framework, potential output can also be seen as a long-term forecast of GDP.

Potential output is not observable so its measurement requires the use of macroeconomic models. Traditional macroeconomic models, like the Real Business Cycle or the New Keynesian

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models, are based on similar concepts of potential output, as both typically include permanent technology shocks as one source of fluctuations. In all these models, potential output is associated to the behavior of the supply side of the economy. This steady state level is stochastic, driven by technology as well as the steady-state level of inputs and it will change in the presence of, for example, permanent technology shocks. There is, however, a difference in the way fluctuations are seen from a welfare point of view. In models with imperfect competition and price rigidities, GDP deviates from its efficient level and these deviations need to be minimized by economic policy, either monetary or fiscal policy (Blanchard, 2018). In this environment, policies are designed to minimize deviations of output from potential, what we refer to as the output gap. This logic can be seen through the inclusion of the output gap in monetary policy reaction functions, such as in the Taylor rule (Woodford, 2001). In the case of fiscal policy, cyclically adjusted measures of government balances, an indicator of the policy stance, are also based on a measure of the output gap (Fatás and Mihov, 2012). In addition, debt sustainability analysis requires an understanding of the long-term level or trend to which the economy will return and, for this reason, we commonly measure debt levels as a ratio to potential output (Giorno et al., 1995; Gaspar, 2020).

Policy makers need to measure potential output in real time, which requires identifying the type of shocks causing fluctuations. Once GDP deviates from previous trends, policy makers need to assess whether the change in output is temporary or permanent. Temporary deviations require action while permanent ones simply represent a shift in potential. This requires knowledge not only about the specific model generating the stochastic movements of output but also about the shocks that are hitting the economy.

Most common methods to measure potential output make use of the notion that potential output can be thought of as the long-term forecast of output, under the assumption that output eventually returns to potential, which will always be the case in the standard RBC or NK models (Nelson and Plosser, 1982). They rely on statistical filters that separate high frequency from low frequency movements in output. These filters make use of the historical record of fluctuations to produce a methodology that is consistent with a certain set of assumptions about shocks (Barnett, Kozicki, and Petrinec, 2009). In practice, different filters will put different weights to changes in output that are driven by either supply-side or demand-side dynamics with implications for optimal policy. As time passes, the data will validate or reject the model and corrections will be made. Using the wrong measure of potential output can lead to suboptimal monetary and fiscal policies.

But there is a separate set of models where potential output is harder to define and where the consequences of policy errors can be much larger. Broadly speaking we can think of two types of models in this category . First, in a world with multiple equilibria, there is no simple definition of potential output (Farmer, 2012). From a policy point of view we might want to move the economy towards the equilibrium with the highest level of welfare, assuming it can be identified. The second class of models is models with hysteresis, where the steady state or natural level of output is dependent on history (Cerra et al., 2022).[1] In these models, relying on a statistical filter is likely to lead to larger and more costly errors. Not only we face the usual uncertainty of identifying the type of shock that hit the economy we also need to anticipate how the cyclical dynamics affect the long-term forecast of GDP. In other words, unlike in standard models without hysteresis where the knowledge of the shocks is enough to produce an accurate forecast of long-term GDP, here we also need to anticipate all the future responses to the shock, as trend output is history dependent. And while some of those dynamics might be under the control of policy makers some might not, for example the behavior of the private sector or the policy actions undertaken by other countries. In addition, while some of this uncertainty is also present in a world without hysteresis, here the potential errors in measuring potential output will have a much larger costs. Without hysteresis, measurement of potential leads to additional and unnecessary volatility. With hysteresis, it might lead to a permanently lower level of output.

There is a second complication that can make these policy errors persist over time. There is a potential vicious cycle in the way policy makers construct their forecasts, design their policies and learn from the consequences on economic activity. This vicious cycle might lead to lack of learning about the true model because policy makers ignore the consequences of their actions. This lack of learning comes from the fact that there are two possible economic models that are observationally equivalent. Imagine a policy maker that puts too much weight on supply shocks as the cause of a recession. This will lead to inaction on her side and via hysteresis effects it will

[1] See, for example, Acharya et al. (2022); Annicchiarico and Pelloni (2016); Anzoategui et al. (2019); Benigno and Fornaro (2018); Bianchi et al. (2019); Fornaro and Wolf (2020); Garga and Singh (2021); Guerron-Quintana and Jinnai (2019); Moran and Queraltó (2018); Queralto (2020); Schmöller and Spitzer (2021); Vinci and Licandro (2021).

cause permanent lower levels of GDP. Ex post, the persistence of GDP will validate the erroneous forecasts of policy makers, making it impossible for the policy maker to learn from her mistakes. This is not just a hypothetical scenario. There is evidence that the use of filters in the estimation of potential output tends to make policy makers significantly review their estimates of potential output any time there is a surprise in GDP growth (Fatás, 2019; Coibion et al., 2018).

To illustrate these dynamics, we present in this paper an extreme form of errors in measuring potential output. A central bank designs optimal monetary policy under the assumption that persistence in GDP is the result of supply shocks. But this assumption is wrong, In reality, persistence is the outcome of hysteresis. Demand driven fluctuations lead, through hysteresis, to permanent changes in output. We show that in this environment, the mistaken actions of the central banker generate dynamics of GDP that are consistent with her mistaken beliefs. Shocks are as persistent as the central banker had assumed but the persistence is entirely due to the mistaken policy that she had implemented. Not only the mistakes are very costly but they might never get corrected because of the observational equivalent nature of the mistaken model and the actual behavior of GDP.

This paper is organized as follows. Section 2 discuss different definitions of potential output in the context of models with exogenous growth. Section 3 shows how these concepts change in an environment with hysteresis. Section 4 presents different methodologies to estimate potential output. Section 5 introduces introduces a standard new-Keynesian model as a framework to talk about optimal policies. Section 6 shows how the mistaken views of central bankers about the true model can generate suboptimal policies. Section 7 provides a numerical illustration of these dynamics and Section 8 concludes.

2 Potential Output, inefficiencies and trend growth in models with exogenous growth

The concept of potential output can be traced back to early descriptions of business cycles in the academic literature. The early macroeconomic literature saw business cycles as deviations of output from its natural state. Hicks (1933) saw business cycles as states of *"disequilibrium"* from the *"idealized state of dynamic equilibrium."* Burns and Mitchell (1946) referred to business cycles as short-term fluctuations around a trend driven by fundamentals. In their words: *"Defining business cycles as recurrent departures from and returns toward 'a normal state of trade', or 'a position of economic equilibrium'."* This notion of potential output remains in most macroeconomic models today. Those models tend to see fluctuations as deviations from a steady state.

There are two interpretations of potential output. One of them is related to the efficient level of output. This level can be defined at any point in time. The second one is more about the level of output to which the economy will return after cyclical dynamics die out. Under the assumption that the economy always return to this natural state, potential output can also be seen as a long-term forecast of GDP, i.e. the value of output to which the economy converges to in the long run. The literature refers to this second version of potential output as trend output (Vetlov et al., 2011).

The connection between the efficient level of output and trend output depends on the specifics of the model we have in mind. In models without inefficiencies, output is always at its efficient level, and we can think about that level as potential output at a particular point in time. Shocks might create transitional dynamics that move GDP away from its long-run steady-state level and, in that sense, output deviates temporarily from its pre-shock trend as defined by the steady-state values of all relevant variables. But in the long-run, it converges to the trend. Trend output will always be seen as the long-term value of potential (i.e. the long-term forecast of GDP, which is always at its efficient level).[2]

In these models, the transition dynamics can be caused by shocks to the trend. In that case, trend output itself shifts at the time of the shock. Beveridge and Nelson (1981) produce a trend decomposition using this approach in a standard RBC model. At time t the trend changes by the implied long-term shift in steady state. In this environment, trend output remains well defined and can be measured as a the long-term GDP forecast.

More precisely, and assuming long-term growth is exogenous, trend output is driven by technology and the steady state levels of relevant variables. We start with a Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

[2] Nelson and Plosser (1982).

and assume labor-augmenting technology growing at an exogenous rate g and preferences are such that in steady state time devoted to labor is a constant fraction of population. In these models, the capital to output ratio is constant and both variables growth at a rate g. Then trend GDP is simply the steady state level of GDP augmented by the stochastic trend

$$Y_t^T = Y_0 e^{(g+n)t}$$

In the presence of a technology shock the trend shifts by the size of the shock (as the value of steady state capital increases proportionally to the permanent shift in A_t. This value is equivalent to the change in the long-term forecast of GDP.

In the transition path, output will converge towards a new steady state. If there is no inefficiencies then output equals potential output

$$Y_t^P = Y_t$$

And as we look at an infinite horizon, potential output converges to trend output (Beveridge and Nelson, 1981).

But in many macroeconomic models, the steady state does not need to be the welfare maximizing level of output and during the transition dynamics output will not be at its efficient level either. How do we then define potential output? In standard New Keynesian models, we typically have two types of inefficiencies. First, those that make output always be below its efficient level, such as imperfect competition. But then there are other inefficiencies, such as price rigidities, that are responsible for the cyclical dynamics and, as such, they vanish in the long run when all prices adjust and the economy has gone back to its steady state.

In this environment we can think of potential output at any point in time as the level that would prevail if all prices were equal to the flexible price equilibrium level. The literature refers to this concept as *natural* output, which is different from the efficient level of output.[3] We can then measure the distance between actual GDP and natural GDP (the output gap) and use it as an indicator of the size and persistence of cycles. Under the assumption that those other

[3] We could also use as a reference the level of output that would prevail if all inefficiencies or price rigidities had never been in place, what Neiss and Nelson (2003) define as the unconditional equilibrium level of output.

inefficiencies (such as imperfect competition) are constant over the business cycle, the distance between the natural and efficient level of output will be constant (Blanchard and Galí, 2007). In this case, whether we measure the output gap relative to natural or potential the evolution will be identical even if there is a difference in levels.

When measuring the output gap in real time, a shock will create a gap between the natural level of output and actual output. In addition, as investment reacts to the shock, the natural rate of output will also change (Blanchard, 2018). As time passes, prices will adjust, unemployment will approach its natural level and the capital to output ratio will return to its steady state. In these models, output converges to natural output. This long-term level of natural output can also be associated to trend output. If there are inefficiencies that remain in place after the cyclical dynamics have gone away, this level of output will not be efficient but it remains the long-term forecast of output.

From a policy point of view, standard New Keynesian models have a well-known prescription for stabilization policies: eliminate the output gap defined as the difference between actual GDP and natural GDP. In other words, bring GDP to the level associated to flexible prices (Blanchard and Galí, 2007). This takes care of the inefficiencies that are responsible for the cyclical component of GDP. Other inefficiencies might persist, such as those derived from imperfect competition, but those should be addressed by other policies. In some models, price rigidities might not be the only friction that is relevant to the business cycle. If more than one of these frictions is in place, policy makers need to eliminate the effects of all of them and the definition of natural output becomes more challenging (Blanchard and Galí, 2010; Queralto, 2022).

This prescription fits well with the frameworks used by policy makers. Policy makers attempt to eliminate the cyclical component, the one that can be controlled via aggregate demand policies and reduce deviations from a target that is implicitly associated to the natural level in New Keynesian models. In an environment with exogenous growth, policy mistakes result in deviations from these natural levels. Failure to act soon will open a larger output gap. These errors do not change the long-term forecast for GDP as deviations from trend will always disappear as prices adjust.

In this environment the challenge for policy makers is two fold. First they need to understand how much of the fluctuations in GDP are caused by a shift in the trend versus how much are due to cyclical conditions. In other words, estimate the output gap. Second, they need to decide how much to change their instruments (interest rates, taxes, spending) in order to correct for the cyclical dynamics. Errors in the estimation of potential output or the optimal change in their instruments will increase volatility but will not affect the long-term value of GDP.

There is also an element of learning, even if mistakes are made. The economic outcomes will help policy makers identify their errors and improve their knowledge about the model that drives cycles. As an example, if there is a negative supply shock that is interpreted as demand deficiency, this will be seen by policy makers as an opening of the output gap and signal to them that they need to engage in expansionary policies. But, as they do, those expansionary policies will generate price pressures and inflation that will suggest that the original assessment of the output gap was mistaken. In the long run, as GDP will always return to trend, policy makers will be able to correctly identify the magnitude of the permanent component of the shock.

3 Potential output and hysteresis

In models with endogenous growth and hysteresis the balanced-growth path is a function of the history of the economy. This represents a fundamental departure from traditional macroeconomic models. Because the balanced-growth path is history dependent, so are any of the definitions of potential output. And this has a fundamental effect in the way we think about long-term definitions of potential output. Trend output is now dependent on history and, in that sense, we cannot simply estimate it in real time by identifying the size and type of the shock. Trend output is now dependent on the future path of the economy including future actions of policy makers.

The earliest references to hysteresis in the business cycle literature were in the context of labor markets (Clark, 1989; Blanchard and Summers, 1986). The observation that unemployment did not quickly return to a low level after the crisis of the 1970s questioned the view of a stable steady state level of unemployment. A relevant question is whether these stubbornly high levels of unemployment were reflecting a permanent shift or just a very persistent deviation from equilibrium. In the case of labor markets, while at the individual level we can imagine permanent effects (short-term unemployment becoming long-term unemployment and possibly turning into an exit from the labor force), at the aggregate level it is harder to think about permanent effects as those worker will eventually be replaced by newer workers. If it is just higher persistence, Blanchard (2018) argues that in some sense the previous framework is still valid. Persistence in unemployment or GDP, either cyclical or structural, does not fundamentally change optimal stabilization, it just changes the horizon and magnitude over which it is relevant. But hysteresis can also be the result of temporary disruptions to the forces that drive long-term growth (investment, R&D, learning by doing). In this case, the effects are likely to be permanent as there is no recovery from the months or quarters lost during a recession (Cerra et al., 2022).

In the presence of hysteresis, any cyclical shock or any policy to be implemented as a result of the crisis needs to be considered to separate trend and cycle. Standard econometric methods that are based on the ideas of smoothing GDP series are not equipped to deal with this environment.

And there is one additional complication that is the focus of our paper: errors in assessing the state of the economy can be self-perpetuating if seen through the lens of the mistaken views of a policy maker. Consider a central bank that faces a recessionary shock. The central bank needs to separate how much of the crisis is structural versus cyclical, which will determine their optimal interest rate via their policy rule. Imagine the central bank puts too much weight on the possibility that the crisis is structural. Then the interest rate will not decrease as much, and policy will be too contractionary. This will worsen the recession and via hysteresis effects it will make some of the GDP changes permanent. Ex-post, the central bank will see an economy that resembles what they had forecast when designing their policies. The shock was indeed persistent and they will associate this persistence to the notion of a structural shock. The interaction between forecasts, policies and hysteresis creates the dynamics of self-perpetuating errors that is the focus of this paper.

4 From theory to empirical estimates of potential output

Estimating potential output in real time requires a full understanding of the economic model driving fluctuations and the particular shocks that are hitting the economy at any point in time. In that sense, one approach could be to estimate natural output by relying on a fully developed DSGE model to identify all shocks and their effects (Andrés et al., 2005; Edge et al., 2008). But such an exercise would require enormous amount of knowledge of the structure of the model and the parameter values as well as our ability to estimate in real time the current set of shocks. The alternatives are to use either simpler reduced-form models to identify particular empirical relationships that allow us to identify shocks or to rely on statistical filters to uncover their dynamics. In fact, in some contexts, a certain statistical filter might provide a good approximation to a model-based measure of potential output. For example, under certain assumptions about the type of shocks that are driving fluctuations, natural output can be a smooth series that can be well approximated by a standard econometric filter.[4] However, in the presence of markup shocks, Justiniano and Primiceri (2008) show that while the unconditional efficient output is similar to the HP filtered output of the CBO estimate, the natural level of output tends to be highly volatile.

Most estimates of potential output are produced by using a minimum set of assumptions about the true economic model (Basu and Fernald, 2009). The main framework is one where growth is exogenous and fluctuations are caused by both supply and demand shocks. Estimating potential output requires separating the two types of shocks or, alternatively, to produce a series of GDP that is removed from its cyclical dynamics. While this methodology produces an observation for potential output for every period and in that sense resemble the concept of natural output, the methodology used fits better with the notion of trend output. Trend output, while linked to theoretical concepts, can simply be thought of as a pure statistical exercise to smooth fluctuations and calculate low-frequency movements of output.

In order to strip away the cyclical component of GDP, different organizations use a variety of methodologies. International organizations such as the OECD, the European Commission or the US CBO rely on a production function approach that identifies the relevant inputs and makes use of statistical techniques or additional structural models to determine the trend and the cycle of these variables (Chalaux and Guillemette, 2019; De Resende, 2014). Other estimates of potential output rely on much simpler models or empirical relationships. For example, Okun's Law can be used to estimate cyclical changes in GDP observing the unemployment rate (Shackleton, 2018). Or the Phillips Curve framework can make use of inflation to estimate potential output.

In all these cases, these estimates need to make use of information from previous cycles to produce statistical filters that will separate the cyclical from the trend dynamics of GDP or its

[4] Basistha and Nelson (2007)

individual inputs (capital or labor). There is no attempt to identify specific shocks in real time, but instead a decomposition into a permanent and a cyclical component is established following historical patterns. This assumes that history is a good indicator of the frequency and impact of these two types of shocks. And the key assumption in this separation is that permanent shocks can be associated to structural, supply-side events while cyclical ones are associated to temporary demand-side shocks.

In recent years, these methodologies have come under criticism because their estimates of potential output were too sensitive to changes in GDP. Cyclical shocks were interpreted as permanent ones leading to large revisions of potential output estimates in response to unexpected movements in output. For example, IMF or European Commission estimates of potential output are revised by 0.6-0.8% for every 1% surprise in GDP (Fatás, 2019). These estimates tend to be too optimistic during good years and too pessimistic during bad years (Mc Morrow et al., 2017; Coibion et al., 2018; Kuusi, 2017).

Coibion et al. (2018) provide a detailed critique of these methodologies. Their starting point is that identified demand shocks have a clear effect on estimates of potential output. In order to deal with the shortcomings of some these methodologies they propose a real-time estimate of potential output that makes use of the Blanchard and Quah (1989) framework. This framework makes use of the identifying assumption that only supply shocks can have permanent effects on GDP. Coibion et al. (2018) show that their proposed real-time estimate of potential output reacts much less to identified demand shocks, while it shows the expected response to supply shocks. When their methodology is applied to the global financial crisis episode, their real-time estimate of potential output is slow to react to the shock but a few years into the crisis it starts showing a decline in potential output. This is not surprising given the long-run restriction required to estimate potential output. Alternative estimates of potential output during this crisis reacted much stronger in the early days because they simply allocate some of the change in output to a permanent component. This is also true for the HP filter estimate. Coibion et al. (2018) also show how model-basesd estimates of potential output, such as those using the Phillips Curve (Galí, 1999) or the permanent income hypothesis relationship between consumption and GDP can generate slower dynamics in the early days of the cycles.[5]

[5] Anzoategui and Kim (2021) provide conditions under which a structural VAR estimation can be used to recover

Despite the methodological success of Coibion et al. (2018) technique in constructing a real time indicator that may be less reactive to GDP movements, it still fails to allow for the presence of hysteresis. This is inconsistent with the evidence presented in the hysteresis literature (Cerra et al., 2022; Jordà et al., 2021).[6] In the presence of hysteresis not only we need to work with a different model but, as GDP is path dependent, there is no well-defined measure of potential or trend output. Trend output does not just depend on the shock, it also depends on all current and future policies as well as the reaction of the private sector.

In addition, just adjusting policies based on outcomes might be difficult because learning requires more than just separating supply from demand shocks, we also need to learn about the correct model under which these outcomes are produced. Statistical methods are not enough because they rely on a specific framework that allows to separate two types of shocks based on the long-term response of some specific variables.[7] The dynamics of output are consistent wit a variety of models that are observationally equivalent.

In the next sections we present an illustrative example of the confusion of policy makers when estimating potential output. We start with an environment where central bankers believe that a combination of supply and demand shocks are causing cycles. Central banks set optimal monetary policy based on these assumptions. The ex-post observed persistence of output validates the existence of permanent supply shocks. We then present a second environment characterized by only demand shocks but with hysteresis. We assume central bankers still believe that the true model of the economy is the first one. Their policy is suboptimal as they do not react strong enough to the demand shocks. Ex-post, we manage to generate the same persistence of output in this environment as in the first one, through the existence of hysteresis. But all the persistence is due to the timid response of a confused central banker. Given that the two environments are observationally equivalent, there is not ex-post learning. Policy makers who believe that persistence is caused by supply shocks will continue believing in their assumptions and will make the same mistake again in the future. Suboptimal policy regimes self perpetuate.

an estimate of potential output that is consistent with a class of DSGE models.

[6] Cerra and Saxena (2008) show evidence for hysteresis in emerging economies. Jordà et al. (2013) for evidence on deep and protracted recovery/non-recovery from financial crises relative to normal recessions for advanced economies. Fatás and Mihov (2013) investigate slow recovery post the Great-Recession for the US.

[7] Furlanetto et al. (2020) recently make progress with statistical methods to estimate hysteresis effects using a combination of long-run and sign restrictions.

5 Monetary policy in a standard new-Keynesian model

Our starting point is an environment where growth is exogenous and business cycles are driven by a combination of permanent technology shocks and temporary demand shocks. The central bank conducts monetary policy using a rule that is optimal in this context.

We follow Giannoni (2014) and consider a stylized new Keynesian framework, variables expressed as log-linear deviations from respective steady state values. x_t is output gap measured in terms of deviations from stationary level of natural output, π_t is log-deviation of inflation from the target rate, i_t is the log-deviation of level of nominal interest rate from the steady state, and r_t^e is the log-deviation of natural real interest rate from the steady state. Equation (1) is derived from inter-temporal consumption Euler equation, combined with economy's resource constraint.[8] Equation (2) describes the new Keynesian Phillips curve.

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^e \right) \tag{1}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{2}$$

The natural (and efficient) rate of interest is defined as[9]

$$r_t^e \equiv \sigma E_t \left[(y_{t+1}^e - y_t^e) + z_{t+1} + b_t \right] \tag{3}$$

where z_{t+1} is log-deviation of TFP growth rate in period $t+1$ relative to the steady state TFP growth rate, y_t^e is the stationary level of output (in log-deviations) that would prevail in the absence of nominal rigidities,[10] and b_t is a temporary demand disturbance in period t. The coefficient σ denotes the inter-temporal elasticity of substitution, κ is the slope of the Phillips curve, and β is the discount factor. Finally, $x_t \equiv y_t - y_t^e$ denotes the output gap.

We assume that the natural level of (stationary output) is constant, $y_t^e = 0$, and exogenous shocks to TFP growth rate z_{t+1}, and demand disturbances b_t are the only source of fluctuations

[8] There is no government spending or storage. Output is equal to consumption.

[9] We assume the presence of lumpsum taxes to finance production subsidy that offsets monopoly distortions in the intermediate goods sector.

[10] Actual output in the absence of nominal rigidities would grow at the rate of TFP growth in the steady state.

in this economy. We model the following process for the evolution of TFP growth, and demand disturbance respectively:

$$z_t = \rho z_{t-1} + \epsilon_t \tag{4}$$

$$b_t = \rho b_{t-1} + \epsilon_t^b \tag{5}$$

The central bank follows an interest rate rule. In Taylor-type interest rate rules typically interest rates depend on both the output gap and inflation. In our specification we write a rule that only includes the output gap and ignores inflation. We do this because we want to stress the influence of potential output estimates on monetary policy rather than issues related to the measurement of inflation. Although central banks consider inflation as a key indicator in their monetary policy decisions, output gap estimates are typically calculated independent of inflation. In addition, our logic is as relevant for monetary as for fiscal policy. In the case of fiscal policy, the output gap is the key variable that enters decisions on the cyclicality of budget balances or the sustainability of debt. And, as before, estimates of the output gap by central banks, the US CBO or international organization do not make use of inflation (Arnold, 2009).

It happens to be that, in this setting, a central bank can achieve optimality by only reacting to one of the two variables, so our rule is optimal in this case. The reason is that under the assumption that there are no shocks to the Phillips curve, optimal policy can be achieved just by central bank reacting to either the output gap or the inflation rate. The output gap because provides enough information about inflation so that central bankers do not need to put a weight on it (Giannoni, 2014).

The central bank interest rate rule can then be written as:

$$i_t = \psi_x x_t \tag{6}$$

where the weight ψ_x is chosen optimally to minimize a quadratic welfare loss criterion given by:

$$E[L] = (1-\beta)E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2\right] \tag{7}$$

λ_x, and λ_i are welfare weights in the objective function.

Proposition 1. $\psi_x = \frac{\lambda_x(1-\beta\rho)^2+\kappa^2}{\lambda_i(\sigma(1-\rho)(1-\beta\rho)-\rho\kappa)(1-\beta\rho)} > 0$ *solves the optimal policy problem of a central bank that minimizes the loss function in Equation (7).*

In Appendix A, we provide the derivation for the coefficient ψ_x under optimal policy following Giannoni (2014). The model can be solved using a method of undermined coefficients (Galí, 2015).[11]

The results so far are standard in the literature and we have established a simple Taylor-type rule that the central bank needs to follow in order to minimize the expected loss. The central bank reacts to the output gap in order to minimize volatility in inflation, interest rate and the output gap.

6 Endogenous growth model and policy makers' confusion

Having established a baseline for how central banks should behave in a standard New Keynesian environment with exogenous growth, we now put policy makers in an environment that is very different but we assume that their beliefs are that the first setting is still the relevant one. The second environment as growth being endogenous and where we only have demand shocks. The endogeneity of growth generates hysteresis effects in a way that demand shocks can have permanent effects on output. The reason to only have demand shocks is to keep the model as simple as possible.

If central banks were aware of the true model, they would need to react to demand shocks in order to close the output gap. But under our assumption that the central bank is mistaken about the true underlying model and believes that the economy is driven by the environment described in the previous section, we will see inaction to fluctuations caused by wrongly identified shocks. The central bank believes that growth is exogenous and the fluctuations in the long-run trend of the economy are outside of their influence, caused by supply shocks. Their inference about the existence of demand and supply shocks comes from their observation of the dynamics of output.

[11] While this optimal rule may not always guarantee equilibrium determinacy (Blanchard and Kahn, 1980), we solve the model using an equilibrium selection device commonly used in the literature (Eggertsson and Woodford, 2003; Werning, 2011; Cochrane, 2017). Under this device, the economy returns back to the same steady state after shocks have dissipated.

The more persistent output is, the more weight central bankers will put on supply shocks. In many ways this can be interpreted as if central banks makes use of a standard filter applied to GDP to separate permanent and transitory shocks.[12]

Our goal is to show that evolution of GDP under these two environments can be observationally equivalent, which will make the central bank be reassured that their beliefs about the model are correct. The persistence of GDP observed under the true model, when growth is endogenous, is the outcome of hysteresis and will mimic the one observed in the exogenous growth model with supply shocks. What is interesting is that the persistence of GDP in the endogenous growth environment is partially caused by the mistakes of the central bank. And this persistence, an outcome of mistakes of central banks, is used to reinforce the wrong assumptions that led to those mistakes in the first place.

6.1 The endogenous growth model with mis-measured output gap

The model is characterized by a similar set of equations as the previous with one main difference, growth is now endogenous. Productivity growth is assumed to react to the output gap via a hysteresis parameter (η). This reduced form relationship is intended to capture the effects of the cycle on productivity growth in a stylized manner. Micro-foundations for such a relationship, based on a learning-by-doing mechanism, can be found for example in Stadler (1990) or more recently in Queralto (2022), among other papers in the endogenous growth business cycle literature.

$$x_t = E_t x_{t+1} + E_t z_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - \tilde{r}^e_t \right) \tag{8}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{9}$$

$$z_{t+1} = \eta x_t \tag{10}$$

[12] Orphanides and Norden (2002) argue that including other variables such as inflation to estimate potential output does not deliver a more reliable estimate than univariate filters. However, Coibion et al. (2018) results suggest that in the context of their theoretical framework, with growth being exogenous, the use of other variables such as inflation could potentially improve the identification of shocks.

Only temporary demand shocks affect the natural interest rate in this economy. The natural rate process is given by:

$$\tilde{r}_t^e = \rho_e \tilde{r}_{t-1}^e + \epsilon_t^e$$

As for the policy rule, we use the same rule as before where the central bank is reacting to the output gap. The difference is that the central bank does not react to the true output gap. Instead, and because they are assuming the wrong economic model, they infer the output gap by observing the persistence of GDP, a signal of the importance of supply versus demand shocks. If we represent by x_t^{cb} the output gap as estimated by the central bank, we can write the policy rule as

$$i_t = \psi_x x_t^{cb}$$

Since this interest rate rule is effectively a peg, an ismorphism with an endogenous interest rate rule can be shown by modeling monetary policy shocks. The error in measuring the output gap can be considered a shock to the policy function.[13]

We will also consider an optimal rule under endogenous growth. In Appendix C, we provide the derivation for the optimal coefficient to the Taylor rule $i_t = \psi_x^{en} x_t$ in the endogenous growth environment.[14]

6.2 Output persistence equivalence and central bank confusion

We now show, in Proposition 2, that there exists a sequence of shocks in the exogenous growth and endogenous growth environment such that the output path is identical under the two settings. The central bank, under the belief that the true economic model is that of exogenous growth with supply and demand shocks will observe a path for output that is consistent with their beliefs. They will follow their policy rule and implement an interest rate that is optimal given their mistaken perception of the output gap. In reality, this interest rate is suboptimal, does not properly minimize the output gap, and this output gap via hysteresis generates that

[13] In fact, we will build the observational equivalence proof shortly using the following rule, under endogenous growth: $i_t = \psi_x x_t + \epsilon_t^i$, where x_t is now the actual gap and we can think of ϵ_t^i as the interest error that is introduce by the central bank because of their confusion about the economic model.

[14] Note there can be local indeterminacy for some parameterizations. We select an equilibrium using the minimum state variable criterion, as before, assuming the economy returns to the original stationary steady state when there are no shocks to the natural rate.

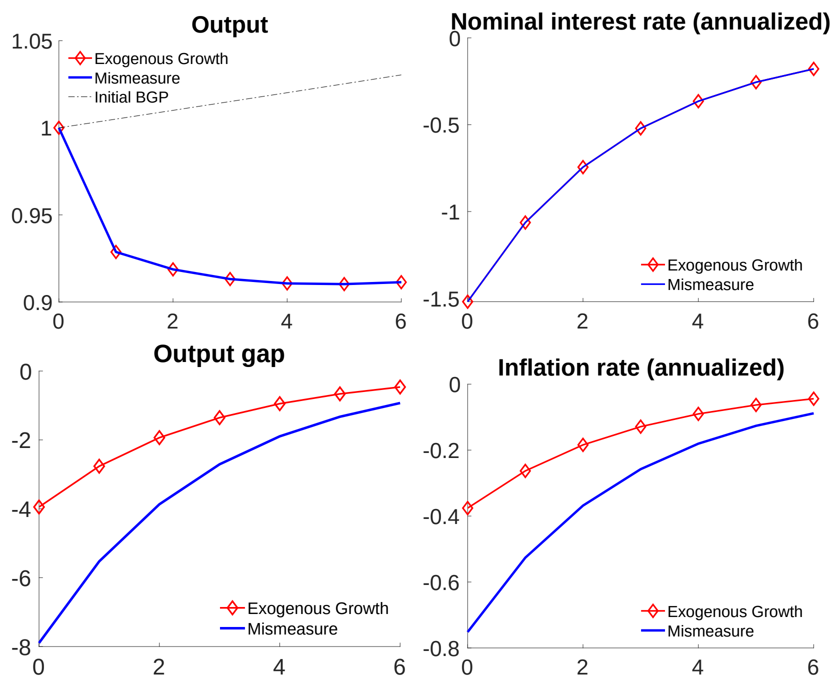
exact output path that the mistaken central bank was expecting.

Proposition 2. *Assume all shocks are iid. There exists a sequence of shocks* $\{\epsilon_t^b, \epsilon_t\}_{t=0}^T$ *in the exogenous growth environment, and shocks* $\{\epsilon_t^e\}$ *in the endogenous growth environment such that the output path is identical up to horizon* $T > 0$.

7 Numerical Illustration

We use the calibration of parameters as in Woodford (2003) and Giannoni (2014): $\beta = 0.99$, $\sigma = 0.1571$, $\kappa = 0.0238$, $\lambda_i = 0.236$, and $\lambda_x = 0.048$. Optimal Taylor rule coefficients imply: $\psi_x = 0.096$. We set $\eta = 0.2$ for illustration.

Figure 1: Model response of output, nominal interest rate and inflation rate to a decline in r-star in the endogenous growth model



Notes: “Exogenous Growth" refers to simulation of the economy under the baseline new Keynesian model, with exogenous growth, presented in Section 5 in response to TFP growth shock and stationary demand shocks. “Mismeasure" refers to simulation of the economy to a natural rate shock in the endogenous growth economy presented in Section 6.1. Under the “Mismeasure" scenario, policy maker incorrectly believes outpu gap is same as the one generated by the exogeneous growth model.

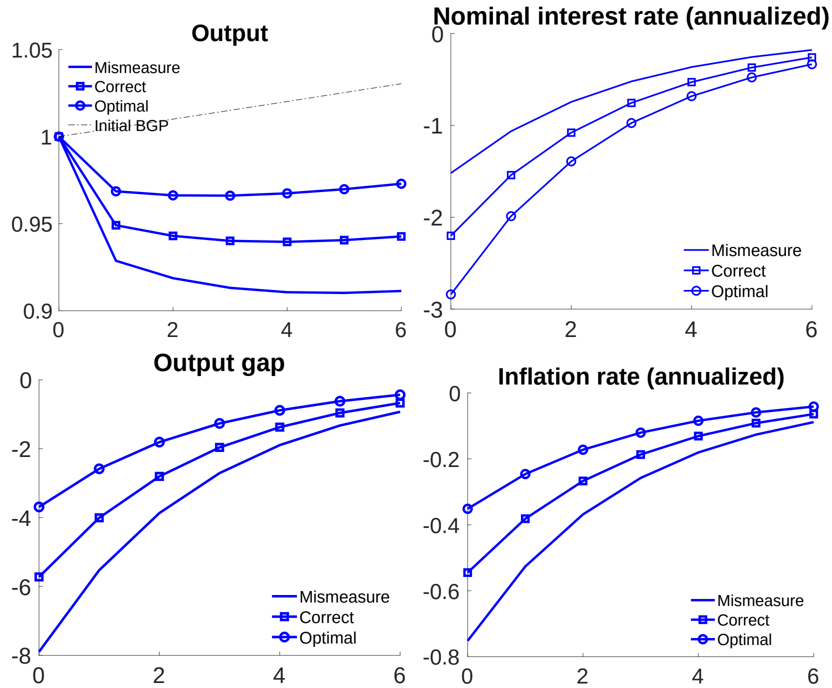
Figure 1 plots the baseline results. We shock the r-star in the endogenous model (the true model) and we compare the responses of output, the output gap, the interest rate and inflation in this scenario to our scenario where central banks believe in the exogenous growth model. By construction we have the same path for output. This identical path supports the confusion of the central bank that believes that they are responding to a combination of demand and supply shocks in an environment where growth is exogenous. The fact that the interest rates are identical means that the perceived output gap by the central bank is exactly the same as the output gap that would prevail if the path of output was coming from the first environment with exogenous growth.

But the central bank is wrong in its assessment of the source of shocks and as a result there is divergence in path of output gap across the two models. The observed output path in the endogenous growth environment is the result of an economy where shocks are only demand driven, growth is endogenous and persistence is the outcome of hysteresis. The true output gap, depicted in the bottom left corner in the solid blue line is higher than what the central bank estimates it to be. Given their policy rule, this implies that if they had properly estimated the output gap, they would have been more aggressive in their reaction with lower interest rates in response to the shock. And the path of output would be different.

We represent these alternative scenarios in Figure 2. We start with the solid line that comes from the previous exercise. This was the path followed in an environment where central banks were mistaken about the true model. We then introduce the actual output gap into their policy function, so we correct for their mistaken estimate. The solid line with square marks shows the new path. The central bank sets a more aggressive monetary policy with lower interest rates. As a result, the output gap ends up being smaller than in the previous scenario and output is less persistent.

Finally we produce an additional scenario (solid line with circles) where the central bank not only properly estimates the output gap but it also follows a rule that is optimal under the endogenous growth environment. In this third instance, the central bank reacts even more than before to the change in the output gap leading to an even smaller gap and a less persistent output response.

Figure 2: Model response of output, nominal interest rate and inflation rate to a decline in r-star in the endogenous growth model: Optimal Policy



Notes: Grpahs plot simulations of the economy to a natural rate shock in the endogenous growth economy presented in Section 6.1. Under the "Mismeasure" scenario, policy maker incorrectly believes output gap is same as the one generated by the exogeneous growth model of Section 5. In the "Correct" scenario, policy maker measures output gap correctly in the endogenous growth model, and sets interest rate to react to the correct measure of output gap but using the policy rule coefficient from Section 5. In the "Optimal" scenario (solid line with circles), policy maker measures output gap correctly in the endogenous growth model, and sets interest rate optimally for this environment.

8 Conclusions

In this paper we study the case of a policymakers whose errors in understanding the true model and estimating potential output leads to hysteresis through suboptimal policies. While suboptimal policies in the presence of information gaps by policy makers should not be a surprise (Orphanides and Norden, 2002), our setting is unique because of the large costs and the fact that policy makers cannot learn from their mistakes. We present an environment where policy makers estimate potential output by using historical data and a filter to separate demand and supply shocks under the assumption that growth is exogenous. Their policy rule would be optimal under their assumed environment, but the true environment is one where there are no supply shocks and all the observed persistence of output is due to hysteresis in the presence of endogenous growth. The fact that their assumed mistaken model generates dynamics that are equivalent to the true model means that, ex-post, they find in the data a validation of the mistaken assumptions.

Output is persistent partly because of the lack of inaction of the central bank and the fact that they underestimate the true output gap. The hysteresis caused by the excessive output gap leads to permanent output losses that produce the level of output persistence that policy makers expected.

Our example is clearly a special case where the behavior of output is identical between the two scenarios and we restrict policymakers to only focus on the path of output to estimate potential output. In more realistic settings there could be some ex-post adjustment to the models being used. However, as long as hysteresis is ignored, the learning will be limited and focused always on the mix of supply and demand shocks as opposed to the true driver of persistence.

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A Proof of Proposition 1

We only solve for optimal non-inertial plan, i.e. when policy maker optimizes every period and cannot commit to offsetting past variables. Let ϕ_{1t} and ϕ_{2t} be Lagrange multipliers on Euler equation and Phillips curve respectively. First order conditions wrt π_t, x_t and i_t are given by :

$$\pi_t + \phi_{2t} = 0$$

$$\lambda_x(x_t - x^*) + \phi_{1t} - \kappa\phi_{2t} = 0$$

$$\lambda_i(i_t - i^*) + \sigma^{-1}\phi_{1t} = 0$$

Since we only have shocks to the Euler equation, we obtain one linear restriction on coefficients ψ_x and ψ_π. This linear restriction is given by:

$$i_r = \psi_x x_r + \psi_\pi \pi_r$$

where

$$i_r = \lambda_x(1-\beta\rho)^2 + \kappa^2 > 0; \quad x_r = \lambda_i(\sigma(1-\rho)(1-\beta\rho) - \rho\kappa)(1-\beta\rho) > 0$$

$$\pi_r = \lambda_i(\sigma(1-\rho)(1-\beta\rho) - \rho\kappa)\kappa > 0$$

We make the assumption that $\sigma(1-\rho)(1-\beta\rho) > \rho\kappa$ to get conventional impulse responses.

If we impose $\psi_\pi = 0$, then we get that the optimal Taylor rule is given by:

$$i_t = \psi_x x_t$$

where $\psi_x = \frac{\lambda_x(1-\beta\rho)^2+\kappa^2}{\lambda_i(\sigma(1-\rho)(1-\beta\rho)-\rho\kappa)(1-\beta\rho)} > 0$.

B Proof of Proposition 2

To generate the two observationally equivalent scenarios, we first consider an isomorphic rule in the exogenous growth environment. Let $i_t = \psi_x x_t + \epsilon_t^i$, where x_t is now the actual gap and we can think of ϵ_t^i as the interest error that is introduce by the central bank because of their confusion about the economic model

We then choose the values for all four shocks (demand shock and TFP growth rate shock in the exogenous growth model and demand and monetary policy shocks in the endogenous growth model).

Assume that all shocks are iid. Then, the solution for the output gap under *exogenous growth* is:

$$x_t^{ex} = \frac{\epsilon_t^b}{\sigma + \psi_x}$$

Output under exogenous growth will be

$$Y_1 = \epsilon_1 + x_1^{ex}; \quad Y_t = \epsilon_1 \ \forall t > 1$$

Interest rate:

$$i_1 = \psi_x x_1^{ex} = \frac{\psi_x \epsilon_1^b}{\sigma + \psi_x}$$

In the second environment, where we have *endogenous growth*, the output gap is equal to

$$x_t = \frac{\epsilon_t^e - \epsilon_t^i}{\sigma(1-\eta) + \psi_x}$$

Output is equal to:

$$Y_1 = x_1; \quad Y_t = \eta x_1 \ \forall t > 1$$

And the interest rate

$$i_1 = \psi_x x_1 + \epsilon_1^i = \frac{\psi_x(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x} + \epsilon_1^i = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x}$$

What we now need is:

- identical path of output (with identical long-term effects

$$Y_1 : \quad \epsilon_1 + \frac{\epsilon_1^b}{\sigma + \psi_x} = \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{11}$$

$$Y_t \ \forall t > 1 : \quad \epsilon_1 = \eta \frac{\epsilon_1^e - \epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{12}$$

- identical path of nominal interest rate, consistent with the mistaken views of the central bank on the output gap

$$\frac{\psi_x \epsilon_1^b}{\sigma + \psi_x} = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x} \tag{13}$$

Fix ϵ_1^e. From equations 11 and 12, we get:

$$\frac{\epsilon_1^b}{\sigma + \psi_x} = \frac{(1-\eta)(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x} \tag{14}$$

Equations 13 and 14 comprise a system of two equations in two unknowns ϵ_1^b, and ϵ_1^i, for a given fixed ϵ_1^e. We can solve:

$$\psi_x \frac{(1-\eta)(\epsilon_1^e - \epsilon_1^i)}{\sigma(1-\eta) + \psi_x} = \frac{\psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i}{\sigma(1-\eta) + \psi_x}$$

$$\iff \psi_x(1-\eta)(\epsilon_1^e - \epsilon_1^i) = \psi_x \epsilon_1^e + \sigma(1-\eta)\epsilon_1^i$$

$$\iff -\eta\psi_x \epsilon_1^e = (\sigma + \psi_x)(1-\eta)\epsilon_1^i$$

$$\iff \epsilon_1^i = -\frac{\eta\psi_x}{(\sigma + \psi_x)(1-\eta)}\epsilon_1^e$$

From equations 12 and 13, we can solve for ϵ_1 and ϵ_1^b respectively.

C Optimal plan under endogenous growth

$$x_t = E_t x_{t+1} + E_t z_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1} - \tilde{r}_t^e\right) \tag{15}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{16}$$

$$z_{t+1} = \eta x_t \tag{17}$$

where $\tilde{r}_t^e = \rho_e \tilde{r}_{t-1}^e + \epsilon_t^e$.

The optimal policy problem is then as follows:

$$\min\ (1-\beta)E\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_i(i_t - i^*)^2 + \lambda_z z_t^2\right]$$

$$(1-\eta)x_t = E_t x_{t+1} - \sigma^{-1}\left(i_t - E_t\pi_{t+1} - \tilde{r}_t^e\right) \tag{18}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \tag{19}$$

When $\lambda_i > 0$, optimal policy does not fully offset changes in r_t^e. Policy maker trades off welfare loss from inflation variability with smoother changes in nominal interest rate. Since we only have shocks to the Euler equation, we get only one linear restriction on coefficients ψ_x^{en} and ψ_π^{en}. This linear restriction is given by:

$$i_r^{en} = \psi_x^{en} x_r^{en} + \psi_\pi^{en} \pi_r^{en}$$

where

$$i_r^{en} = \lambda_x(1-\beta\rho)^2 + \kappa^2 > 0; \quad x_r^{en} = \lambda_i(\sigma(1-\eta-\rho)(1-\beta\rho) - \rho\kappa)(1-\beta\rho) > 0$$

$$\pi_r^{en} = \lambda_i(\sigma(1-\eta-\rho)(1-\beta\rho) - \rho\kappa)\kappa > 0$$

We make the assumption that $\sigma(1-\rho)(1-\beta\rho) > \rho\kappa$ to get conventional impulse responses.

If we impose $\psi_\pi^{en} = 0$, then we get that the optimal Taylor rule is given by:

$$i_t = \psi_x^{en} x_t$$

where $\psi_x^{en} = \frac{\lambda_x(1-\beta\rho)^2+\kappa^2}{\lambda_i(\sigma(1-\eta-\rho)(1-\beta\rho)-\rho\kappa)(1-\beta\rho)} > 0$.