Low Risk Sharing with Many Assets

Emile Marin[1] Sanjay R. Singh[§]

[1]UC Davis

[§]UC Davis & FRBSF

May 2024

The views expressed herein are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

Motivation

When financial markets are complete: relative consumption and real exchange rates co-move positively [Kollman (1991), Backus-Smith (1993)]

Motivation

When financial markets are complete: relative consumption and real exchange rates co-move positively [Kollman (1991), Backus-Smith (1993)]

- Strongly rejected in data $\implies$ low risk-sharing

Motivation

When financial markets are complete: relative consumption and real exchange rates co-move positively [Kollman (1991), Backus-Smith (1993)]

- Strongly rejected in data $\implies$ low risk-sharing

When financial markets are incomplete: non-traded risk can generate low risk-sharing in models consistent with data [Corsetti, Dedola and Leduc (2009) & others]

Motivation

When financial markets are complete: relative consumption and real exchange rates co-move positively [Kollman (1991), Backus-Smith (1993)]

- Strongly rejected in data $\Longrightarrow$ low risk-sharing

When financial markets are incomplete: non-traded risk can generate low risk-sharing in models consistent with data [Corsetti, Dedola and Leduc (2009) & others]

But not with cross-border trade in two risk-free bonds i.e. many assets:

- Arbitrage restrictions on process for non-traded risk [Lustig and Verdelhan (2019)]

Motivation

When financial markets are complete: relative consumption and real exchange rates co-move positively [Kollman (1991), Backus-Smith (1993)]

- Strongly rejected in data $\implies$ low risk-sharing

When financial markets are incomplete: non-traded risk can generate low risk-sharing in models consistent with data [Corsetti, Dedola and Leduc (2009) & others]

But not with cross-border trade in two risk-free bonds i.e. many assets:

- Arbitrage restrictions on process for non-traded risk [Lustig and Verdelhan (2019)]

Our paper: within-country SDF heterogeneity

Revisiting the Backus Smith Condition

When markets are complete:

$$\underbrace{\left(\frac{C_{t+1}}{C^*_{t+1}} / \frac{C_t}{C^*_t}\right)^s}_{M^*_{t+1}/M_{t+1}} = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \qquad \text{(Kollman; Backus-Smith)}$$

Trade in only risk-free assets : FX must move to restore risk-sharing

Revisiting the Backus Smith Condition

When markets are complete:

$$\underbrace{\left(\frac{C_{t+1}}{C^*_{t+1}} \Big/ \frac{C_t}{C^*_t}\right)^s}_{M^*_{t+1}/M_{t+1}} = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \qquad \text{(Kollman; Backus-Smith)}$$

Trade in only risk-free assets : FX must move to restore risk-sharing

- risk sharing $\implies$ risky FX movement $cov_t(m_{t+1}, \Delta e_{t+1}) < 0$
- e.g. productivity $\uparrow$ $(C \uparrow)$, depr. $(\mathcal{E}_t \uparrow)$, FX reallocate wealth from H to F $(C^* \uparrow)$

Revisiting the Backus Smith Condition

When markets are complete:

$$\underbrace{\left(\frac{C_{t+1}}{C^*_{t+1}} / \frac{C_t}{C^*_t}\right)^s}_{M^*_{t+1}/M_{t+1}} = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$$

(Kollman; Backus-Smith)

Trade in only risk-free assets : FX must move to restore risk-sharing

- risk sharing $\implies$ <u>risky</u> FX movement $cov_t(m_{t+1}, \Delta e_{t+1}) < 0$
- e.g. productivity $\uparrow$ $(C \uparrow)$, depr. $(\mathcal{E}_t \uparrow)$, FX reallocate wealth from H to F $(C^* \uparrow)$

When markets are incomplete, condition only holds in $\mathbb{E}$

This paper

Low risk-sharing with “many" assets is a puzzle for theory

This paper

Low risk-sharing with “many" assets is a puzzle for theory

1. how does one get low risk sharing in an IM macro model?

This paper

Low risk-sharing with “many" assets is a puzzle for theory

1. how does one get low risk sharing in an IM macro model?
   - only one int’l traded risk-free bond traded
   - "goods-market" mechanisms generate non-traded component of FX which is “safe” from a domestic investor perspective: $\implies cov_t(m_{t+1}, \Delta e_{t+1}) > 0$

This paper

Low risk-sharing with “many" assets is a puzzle for theory

1. how does one get low risk sharing in an IM macro model?
    - only one int’l traded risk-free bond traded
    - "goods-market" mechanisms generate non-traded component of FX which is “safe” from a domestic investor perspective: $\implies cov_t(m_{t+1}, \Delta e_{t+1}) > 0$

Adding a second (F) risk-free bond $\implies cov_t(m_{t+1}, \Delta e_{t+1}) < 0$

This paper

2. Generalize two-country model to allow heterogeneous SDFs in the Home country

This paper

2. Generalize two-country model to allow heterogeneous SDFs in the Home country

   (i) limited participation in foreign markets

   (ii) heterogeneous consumers in integrated financial markets

   $\rightarrow$ <u>low risk sharing possible with two risk-free assets</u>

This paper

2. Generalize two-country model to allow heterogeneous SDFs in the Home country
   (i) limited participation in foreign markets
   (ii) heterogeneous consumers in integrated financial markets

   $\rightarrow$ <u>low risk sharing possible with two risk-free assets</u>

   * domestic incompleteness must be large relative to the volatility of exchange rates

This paper

2. Generalize two-country model to allow heterogeneous SDFs in the Home country
   (i) limited participation in foreign markets
   (ii) heterogeneous consumers in integrated financial markets
   $\rightarrow$ <u>low risk sharing possible with two risk-free assets</u>

   * domestic incompleteness must be large relative to the volatility of exchange rates

   Resolve Backus-Smith puzzle, without worsening volatility or predictability puzzles

Roadmap

1. Representative agent
2. A model with George Soros
3. A model with heterogeneous consumers
4. Conclusion

A quartet of Eulers

- Two-country model, SDFs $M_{t+1}^{(*)}$ denotes Home (Foreign) RA SDF

A quartet of Eulers

- Two-country model, SDFs $M_{t+1}^{(*)}$ denotes Home (Foreign) RA SDF
- Trade in assets $\implies$

$$\mathbb{E}_t[M_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M_{t+1}^*] = 1/R_{t+1}^* \qquad \text{(Trade in dom. bonds)}$$

A quartet of Eulers

- Two-country model, SDFs $M_{t+1}^{(*)}$ denotes Home (Foreign) RA SDF
- Trade in assets $\implies$

$$\mathbb{E}_t[M_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M_{t+1}^*] = 1/R_{t+1}^* \qquad \text{(Trade in dom. bonds)}$$

$$\mathbb{E}_t\left[M_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1/R_{t+1}^* \qquad \text{(Int'l Trade in F bond)}$$

A quartet of Eulers

- Two-country model, SDFs $M_{t+1}^{(*)}$ denotes Home (Foreign) RA SDF
- Trade in assets $\implies$

$$\mathbb{E}_t[M_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M_{t+1}^*] = 1/R_{t+1}^* \qquad \text{(Trade in dom. bonds)}$$

$$\mathbb{E}_t\left[M_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1/R_{t+1}^* \qquad \text{(Int'l Trade in F bond)}$$

$$\mathbb{E}_t\left[M_{t+1}^*\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] = 1/R_{t+1}, \qquad \text{(Int'l Trade in H bond)}$$

Exchanges Rate in Incomplete Markets

Assume SDFs, allocations and prices are jointly log-normal 4 Eulers $\Longrightarrow$

$$cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) = var_t(\Delta e_{t+1})$$

Exchanges Rate in Incomplete Markets

Assume SDFs, allocations and prices are jointly log-normal 4 Eulers $\implies$

$$cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) = var_t(\Delta e_{t+1})$$

- CM: $\Delta e_{t+1} = m^*_{t+1} - m_{t+1}$
- IM: $\Delta e_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$

where η_{t+1} is the IM/ non-traded risk wedge

▸ η restr.

Exchanges Rate in Incomplete Markets

Assume SDFs, allocations and prices are jointly log-normal 4 Eulers $\implies$

$$cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) = var_t(\Delta e_{t+1})$$

- CM: $\Delta e_{t+1} = m^*_{t+1} - m_{t+1}$
- IM: $\Delta e_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$

where η_{t+1} is the IM/ non-traded risk wedge ▸ η restr.

Assuming time-separable, CRRA preferences, classical int'l macro models assume:

$$\eta_{t+1} = log \underbrace{\left(\frac{P_{t+1}}{P_t}\frac{P^*_t}{P^*_{t+1}}\right)}_{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}} - log \underbrace{\left(\frac{C_t}{C_{t+1}}\frac{C^*_{t+1}}{C^*_t}\right)^s}_{\frac{M_{t+1}}{M^*_{t+1}}}$$

Risk-Sharing with a Single Risk-Free Bond

Suppose only F bonds are internationally traded (3 Euler model).

Risk-Sharing with a Single Risk-Free Bond

Suppose only F bonds are internationally traded (3 Euler model).

Any IM int'l macro model delivers low risk-sharing $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$cov_t(m_{t+1}, \eta_{t+1}) + \log\ \mathbb{E}_t[e^{\eta_{t+1}}] > \underbrace{var_t(m^*_{t+1} - m_{t+1})}_{FX \text{ vol. in CM}}$$

Risk-Sharing with a Single Risk-Free Bond

Suppose only F bonds are internationally traded (3 Euler model).

Any IM int'l macro model delivers low risk-sharing $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$cov_t(m_{t+1}, \eta_{t+1}) + \log\ \mathbb{E}_t[e^{\eta_{t+1}}] > \underbrace{var_t(m^*_{t+1} - m_{t+1})}_{FX \text{ vol. in CM}}$$

- non-traded risk causes FX fluctuations which are sufficiently safe

Risk-Sharing with a Single Risk-Free Bond

Suppose only F bonds are internationally traded (3 Euler model).

Any IM int'l macro model delivers low risk-sharing $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$cov_t(m_{t+1}, \eta_{t+1}) + \log \ \mathbb{E}_t[e^{\eta_{t+1}}] > \underbrace{var_t(m^*_{t+1} - m_{t+1})}_{FX \text{ vol. in CM}}$$

- non-traded risk causes FX fluctuations which are sufficiently safe
- trade in H as well : $cov_t(m_{t+1}, \eta_{t+1}) \to \frac{1}{2} var_t(\eta_{t+1})$ so $var_t(\Delta e_{t+1}) < 0$

Risky Assets

Risk-Sharing with a Single Risk-Free Bond

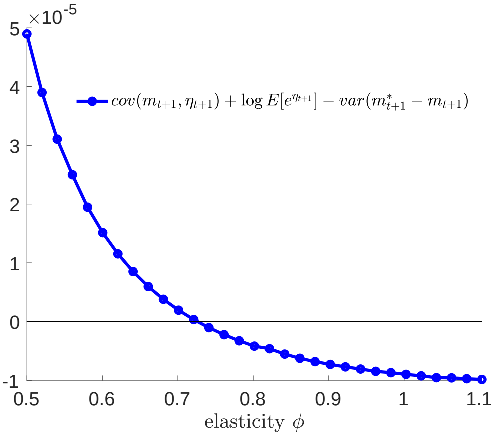
Suppose only F bonds are internationally traded (3 Euler model).

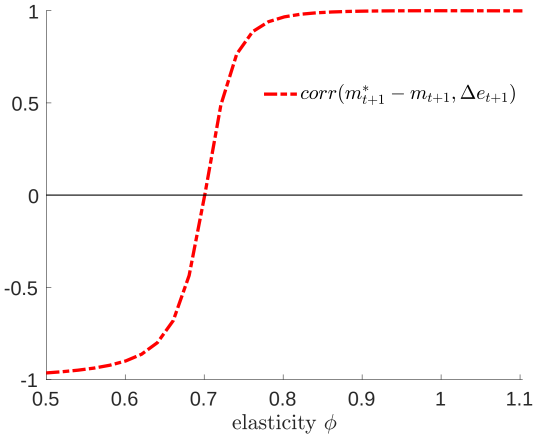
Any IM int'l macro model delivers low risk-sharing $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$cov_t(m_{t+1}, \eta_{t+1}) + \log\ \mathbb{E}_t[e^{\eta_{t+1}}] > \underbrace{var_t(m^*_{t+1} - m_{t+1})}_{FX \text{ vol. in CM}}$$

- non-traded risk causes FX fluctuations which are sufficiently safe
- trade in H as well : $cov_t(m_{t+1}, \eta_{t+1}) \to \frac{1}{2} var_t(\eta_{t+1})$ so $var_t(\Delta e_{t+1}) < 0$ Risky Assets
- flesh out covariance restr. with a canonical macro model [Corsetti, Dedola & Leduc (2008)]

Calibrated Example





Unconditional moments calculated from second-order simulation with one million draws.

Risk-Sharing with Heterogeneous Marginal Investors

- We generalize the model to allow for heterogeneous marginal investors in the Home country for H and F assets $\implies$ low risk sharing with many assets

Risk-Sharing with Heterogeneous Marginal Investors

- We generalize the model to allow for heterogeneous marginal investors in the Home country for H and F assets $\implies$ low risk sharing with many assets
- Define $\hat{M}_{t+1} = M_{t+1} D_{t+1}$

Risk-Sharing with Heterogeneous Marginal Investors

- We generalize the model to allow for heterogeneous marginal investors in the Home country for H and F assets $\implies$ low risk sharing with many assets
- Define $\hat{M}_{t+1} = M_{t+1} D_{t+1}$

$$\mathbb{E}_t[M_{t+1}] = \mathbb{E}_t[\hat{M}_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M^*_{t+1}] = 1/R^*_{t+1} \qquad \text{(Trade in dom. bonds)}$$

Risk-Sharing with Heterogeneous Marginal Investors

- We generalize the model to allow for heterogeneous marginal investors in the Home country for H and F assets $\implies$ low risk sharing with many assets
- Define $\hat{M}_{t+1} = M_{t+1} D_{t+1}$

$$\mathbb{E}_t[M_{t+1}] = \mathbb{E}_t[\hat{M}_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M^*_{t+1}] = 1/R^*_{t+1} \qquad \text{(Trade in dom. bonds)}$$

$$\mathbb{E}_t\left[\hat{M}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1/R^*_{t+1} \qquad \text{(Int'l Trade in F bond)}$$

Risk-Sharing with Heterogeneous Marginal Investors

- We generalize the model to allow for heterogeneous marginal investors in the Home country for H and F assets $\implies$ low risk sharing with many assets
- Define $\hat{M}_{t+1} = M_{t+1} D_{t+1}$

$$\mathbb{E}_t[M_{t+1}] = \mathbb{E}_t[\hat{M}_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M^*_{t+1}] = 1/R^*_{t+1} \qquad \text{(Trade in dom. bonds)}$$

$$\mathbb{E}_t\left[\hat{M}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1/R^*_{t+1} \qquad \text{(Int'l Trade in F bond)}$$

Risk-Sharing with Heterogeneous Marginal Investors

- We generalize the model to allow for heterogeneous marginal investors in the Home country for H and F assets $\implies$ low risk sharing with many assets
- Define $\hat{M}_{t+1} = M_{t+1} D_{t+1}$

$$\mathbb{E}_t[M_{t+1}] = \mathbb{E}_t[\hat{M}_{t+1}] = 1/R_{t+1}, \qquad \mathbb{E}_t[M^*_{t+1}] = 1/R^*_{t+1} \qquad \text{(Trade in dom. bonds)}$$

$$\mathbb{E}_t\left[\hat{M}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1/R^*_{t+1} \qquad \text{(Int'l Trade in F bond)}$$

$$\mathbb{E}_t\left[M^*_{t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] = 1/R_{t+1}, \qquad \text{(Int'l Trade in H bond)}$$

- Domestic risk-sharing imposes tight constraints on D_{t+1}, η_{t+1}.

Details

Roadmap

1. Representative agent
2. A model with George Soros
3. A model with heterogeneous consumers
4. Conclusion

Risk-Sharing with Heterogeneous Investors and Multiple Assets

Assume $\mu(\hat{M}) \to 0$. The model with H & F traded bonds and heterogeneous Home investors delivers $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ if and only if:

$$1 \geq \rho_{d_{t+1}, -\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(d_{t+1})}$$

Risk-Sharing with Heterogeneous Investors and Multiple Assets

Assume $\mu(\hat{M}) \to 0$. The model with H & F traded bonds and heterogeneous Home investors delivers $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ if and only if:

$$1 \geq \rho_{d_{t+1}, -\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(d_{t+1})}$$

- RA: FX risk becomes traded (spanned) with trade in both H and F bonds
- Heterogeneity (d) which co-moves (+) with FX recovers non-traded component

Risk-Sharing with Heterogeneous Investors and Multiple Assets

Assume $\mu(\hat{M}) \to 0$. The model with H & F traded bonds and heterogeneous Home investors delivers $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ if and only if:

$$1 \geq \rho_{d_{t+1}, -\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(d_{t+1})}$$

- RA: FX risk becomes traded (spanned) with trade in both H and F bonds
- Heterogeneity (d) which co-moves (+) with FX recovers non-traded component
- Measure $\sigma_t(\Delta d_{t+1})$ using asset prices (Sharpe Ratio on international vs. U.S. portfolio $\times 2$)

Risk-Sharing with Heterogeneous Investors and Multiple Assets

Assume $\mu(\hat{M}) \to 0$. The model with H & F traded bonds and heterogeneous Home investors delivers $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ if and only if:

$$1 \geq \rho_{d_{t+1},-\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{\sigma_t(d_{t+1})}$$

- RA: FX risk becomes traded (spanned) with trade in both H and F bonds
- Heterogeneity (d) which co-moves (+) with FX recovers non-traded component
- Measure $\sigma_t(\Delta d_{t+1})$ using asset prices (Sharpe Ratio on international vs. U.S. portfolio $\times 2$) $\implies \rho_{d_{t+1},-\Delta e_{t+1}} \geq \frac{0.33}{1.2} \approx 0.28$

Calibration

Other FX puzzles

Model with heterogeneous SDFs explain cyclicality puzzle without worsening:

1. Volatility puzzle, FX not volatile enough [Brandt, Cochrane & Santa Clara (2006)]
2. Predictability puzzle, FX movements should not be predictable [e.g. Chernov & Creal (2019)]

Other FX puzzles

Model with heterogeneous SDFs explain cyclicality puzzle without worsening:

1. Volatility puzzle, FX not volatile enough [Brandt, Cochrane & Santa Clara (2006)]
2. Predictability puzzle, FX movements should not be predictable [e.g. Chernov & Creal (2019)]

1. Need $\frac{\sigma_{\Delta e_{t+1}}}{\sigma_{d_{t+1}}}$ suff. small, but $\sigma_{\Delta e_{t+1}}$ order of magnitude smaller than $\sigma_{d_{t+1}}$

Other FX puzzles

Model with heterogeneous SDFs explain cyclicality puzzle without worsening:

1. Volatility puzzle, FX not volatile enough [Brandt, Cochrane & Santa Clara (2006)]
2. Predictability puzzle, FX movements should not be predictable [e.g. Chernov & Creal (2019)]

1. Need $\frac{\sigma_{\Delta e_{t+1}}}{\sigma_{d_{t+1}}}$ suff. small, but $\sigma_{\Delta e_{t+1}}$ order of magnitude smaller than $\sigma_{d_{t+1}}$
2. d_{t+1} drives Δe_{t+1} but not spanned by r_{t+1}, i.e. $proj(r_{t+1}|\Delta d_{t+1}) = 0$

$$\mathbb{E}_t[M_{t+1}] = \mathbb{E}_t[\underbrace{M_{t+1} D_{t+1}}_{\hat{M}_{t+1}}] = 1/R_{t+1}$$

Roadmap

1. Representative agent
2. A model with George Soros
3. A model with heterogeneous consumers
4. Conclusion

Heterogeneous Consumers, Integrated Markets

2-country CAPM with heterogeneity [see e.g. Constantinides and Duffie (1996)]:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-s}, \quad \Delta c_{t+1} = w_{t+1} \sim \mathcal{N}(\mu_{C_t}, \sigma^2_{C_t}),$$

Heterogeneous Consumers, Integrated Markets

2-country CAPM with heterogeneity [see e.g. Constantinides and Duffie (1996)]:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-s}, \quad \Delta c_{t+1} = w_{t+1} \sim \mathcal{N}(\mu_{C_t}, \sigma^2_{C_t}),$$

$$\Delta c^i_{t+1} = \log \left(\frac{\delta^i_{t+1}}{\delta^i_t} \frac{C_{t+1}}{C_t} \right) \sim \mathcal{N}(\mu_{c^i_t}, \sigma^2_{i,t})$$

Heterogeneous Consumers, Integrated Markets

2-country CAPM with heterogeneity [see e.g. Constantinides and Duffie (1996)]:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-s}, \quad \Delta c_{t+1} = w_{t+1} \sim \mathcal{N}(\mu_{C_t}, \sigma^2_{C_t}),$$

$$\Delta c^i_{t+1} = \log \left(\frac{\delta^i_{t+1}}{\delta^i_t} \frac{C_{t+1}}{C_t} \right) \sim \mathcal{N}(\mu_{c^i_t}, \sigma^2_{i,t})$$

Individual consumption draw related to aggregate:

$$\int_i \delta^i_t \, di = 1, \quad \log \left(\frac{\delta^i_{t+1}}{\delta^i_t} \right) \sim \mathcal{N}(\tfrac{x^2_{t+1}}{2}, x^2_{t+1}),$$

- Segmentation no longer required — but can be a complementary mechanism [see e.g. Chernov, Haddad, and Itskhoki (2024)]

Risk Sharing with Heterogeneous Consumers

The model with H & F traded bonds and heterogeneous Home consumers delivers $cov_t(m^*_{t+1} - \log(\int_i e^{\Delta c^i_t} di)^{-s}, \Delta e_{t+1}) < 0$ *if and only if:*

$$1 \geq \rho_{-\delta^i_{t+1}, -\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{s\sigma_t(\log(\delta^i_{t+1}))}$$

- Idiosyncratic risk which co-moves with FX recovers a non-traded component
  - F bonds are a poor hedge if $\rho_{-\delta^i_{t+1}, -\Delta e_{t+1}} > 0$

Risk Sharing with Heterogeneous Consumers

The model with H & F traded bonds and heterogeneous Home consumers delivers $cov_t(m^*_{t+1} - \log(\int_i e^{\Delta c^i_t} di)^{-s}, \Delta e_{t+1}) < 0$ *if and only if:*

$$1 \geq \rho_{-\delta^i_{t+1}, -\Delta e_{t+1}} \geq \frac{\sigma_t(\Delta e_{t+1})}{s\sigma_t(\log(\delta^i_{t+1}))}$$

- Idiosyncratic risk which co-moves with FX recovers a non-traded component
  - F bonds are a poor hedge if $\rho_{-\delta^i_{t+1}, -\Delta e_{t+1}} > 0$

* Exact same condition as before! d_{t+1} replaced by $\log(\delta^{-s}_{t+1})$
* Does not exacerbate volatility or predictability puzzles

Calibrating HA model

* $x_{t+1}^2 = \sigma_\delta^2 + \phi \Delta c_{t+1}$, $c_t = \alpha \Delta e_t + \nu_t$

- $\sigma_\delta = 0.4$ [Constantinides (2021)]
- $s = 10$ [Best, Cloyne, Ilzetzki and Kleven (2020)]
- $\rho_{-\delta_{t+1}^i, -\Delta e_{t+1}} \geq \frac{0.33}{4} = 0.0825$
- $\phi = -5.76$ [e.g. Acharya et al., 2023]
- Back of the envelope: [Verner and Gyongyosi (2020)] 30% depreciation of Hungarian forint $\rightarrow$ increase in debt of 10% of disposable income (MPC=0.22): $\rho_{-\delta_{t+1}^i, -\Delta e_{t+1}} \approx 0.175$

Conclusion

Dual role of FX : risk-sharing and portfolio returns

Conclusion

Dual role of FX : risk-sharing and portfolio returns

- IM int'l macro models formulate "goods-market" mechanism : $cov(m^* - m, \Delta e) < 0$

Conclusion

Dual role of FX : risk-sharing and portfolio returns

- IM int'l macro models formulate "goods-market" mechanism : $cov(m^* - m, \Delta e) < 0$
  - rely on a non-traded component to FX which is "safe" for domestic inv. $cov(m, \eta) > 0$

Conclusion

Dual role of FX : risk-sharing and portfolio returns

- IM int'l macro models formulate "goods-market" mechanism : $cov(m^* - m, \Delta e) < 0$
  - rely on a non-traded component to FX which is “safe” for domestic inv. $cov(m, \eta) > 0$
- IM RA model with trade in H and F curr. denominated risk-free bonds imposes $cov(m^* - m, \Delta e) > 0$

Conclusion

Dual role of FX : risk-sharing and portfolio returns

- IM int'l macro models formulate "goods-market" mechanism : $cov(m^* - m, \Delta e) < 0$
  - rely on a non-traded component to FX which is "safe" for domestic inv. $cov(m, \eta) > 0$
- IM RA model with trade in H and F curr. denominated risk-free bonds imposes $cov(m^* - m, \Delta e) > 0$
  - FX fully insured ex-ante $\implies$ redistribution & ex-post risky

Conclusion

Dual role of FX : risk-sharing and portfolio returns

- IM int'l macro models formulate "goods-market" mechanism : $cov(m^* - m, \Delta e) < 0$
  - rely on a non-traded component to FX which is "safe" for domestic inv. $cov(m, \eta) > 0$
- IM RA model with trade in H and F curr. denominated risk-free bonds imposes $cov(m^* - m, \Delta e) > 0$
  - FX fully insured ex-ante $\implies$ redistribution & ex-post risky
- Propose model w heterogeneous SDFs in IM $\implies$ <u>low risk-sharing with many assets</u>

Conclusion

Dual role of FX : risk-sharing and portfolio returns

- IM int'l macro models formulate "goods-market" mechanism : $cov(m^* - m, \Delta e) < 0$
    - rely on a non-traded component to FX which is "safe" for domestic inv. $cov(m, \eta) > 0$
- IM RA model with trade in H and F curr. denominated risk-free bonds imposes $cov(m^* - m, \Delta e) > 0$
    - FX fully insured ex-ante $\implies$ redistribution & ex-post risky
- Propose model w heterogeneous SDFs in IM $\implies$ low risk-sharing with many assets
    - Empirically plausible because heterogeneity high relative to FX volatility
    - ∗ recover FX cyclicality without compromising volatility or introducing predictability

IM wedge

Int’l trade in bonds disciplines IM wedge [Lustig and Verdelhan (2019)]:

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}var_t(\eta_{t+1}) - cov_t(m_{t+1}, \eta_{t+1}), \qquad \text{(H bond traded)}$$

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}var_t(\eta_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) \qquad \text{(F bond traded)}$$

IM wedge

Int'l trade in bonds disciplines IM wedge [Lustig and Verdelhan (2019)]:

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}var_t(\eta_{t+1}) - cov_t(m_{t+1}, \eta_{t+1}), \qquad \text{(H bond traded)}$$

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}var_t(\eta_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) \qquad \text{(F bond traded)}$$

But what does this mean for macro? Trade in asset means:

- Higher volatility compensated by expected return or change in cyclicality of non-traded risk

IM wedge

Int'l trade in bonds disciplines IM wedge [Lustig and Verdelhan (2019)]:

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) - cov_t(m_{t+1}, \eta_{t+1}), \qquad \text{(H bond traded)}$$

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) \qquad \text{(F bond traded)}$$

But what does this mean for macro? Trade in asset means:

- Higher volatility compensated by expected return or change in cyclicality of non-traded risk
- If H bond not traded, $cov_t(m_{t+1}, \eta_{t+1})$ determined by "goods-market" mechanisms (e.g. complementarities in consumption/production etc.)

Back

Model Setup

- Utility: $u(C_t) = \beta(C_t)\frac{1}{1-s}C_t^{1-s}$
- Goods aggregation: $C_t = [\alpha^{\frac{1}{\phi}} C_{H,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} C_{F,t}^{\frac{\phi-1}{\phi}}]^{\frac{\phi}{\phi-1}}$
- Endowments and assets: $P_t C_t - P_{H,t} Y_{H,t} \leq R_t B_{t-1} - B_t + \mathcal{E}_t(R_t^* B_{t-1}^F - B_t^F)$
- Goods market clearing: $C_{H,t} + C_{H,t}^* = Y_{H,t} C_{F,t} + C_{F,t}^* = Y_{F,t}^*$
- Asset market clearing: $B_t = 0, \quad B_t^* + B_t^F = 0$

A Macro Model at the Autarky Limit 1/2

- RA consumes goods H and F with home bias α & earns endowments y_H, y_F
- Home trades H and F bonds, Foreign trades F

A Macro Model at the Autarky Limit 1/2

- RA consumes goods H and F with home bias α & earns endowments y_H, y_F
- Home trades H and F bonds, Foreign trades F

In the autarky limit $\alpha \to 1$, $B, B^* \to 0$, the model is summarized by the following:

$$
\begin{aligned}
m_{t+1} &= -s g_{y_{H,t+1}}, \quad m^*_{t+1} = -s g_{y_{F,t+1}}, \\
\Delta e_{t+1} &= \frac{1}{1-2(1-\phi)}(g_{y_{H,t+1}} - g_{y_{F,t+1}}), \\
\eta_{t+1} &= (g_{y_{H,t+1}} - g_{y_{F,t+1}})\frac{1-s}{1-2(1-\phi)}
\end{aligned}
$$

where $g_{y_{i,t+1}} = y_{i,t+1} - y_{i,t}$. It follows that if $Y_{H,t}, Y_{F,t}$ are normally distributed, then $m_{t+1}, m^*_{t+1}, \eta_{t+1}$ and Δe_{t+1} are jointly log-normally distributed.

Back

A Macro Model at the Autarky Limit 2/2

Assuming $var_t(g_{y_{H,t+1}} - g_{y_{F,t+1}}) = var_t(g_{y_{H,t+1}})$*, the model at the autarky limit delivers* $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *conditional on shocks to* $y_{H,t}$:

- *with int'l trade in F risk-free asset only*:

$$\frac{-s(1-s)}{1-2(1-\phi)} \geq s^2 - \frac{1}{2}\left[\frac{1-s}{1-2(1-\phi)}\right]^2$$

A Macro Model at the Autarky Limit 2/2

Assuming $var_t(g_{y_{H,t+1}} - g_{y_{F,t+1}}) = var_t(g_{y_{H,t+1}})$*, the model at the autarky limit delivers* $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *conditional on shocks to* $y_{H,t}$:

- *with int'l trade in F risk-free asset only*:

$$\frac{-s(1-s)}{1-2(1-\phi)} \geq s^2 - \frac{1}{2}\left[\frac{1-s}{1-2(1-\phi)}\right]^2$$

- *with trade in no assets* $\phi \leq 1/2$ [Corsetti, Dedola and Leduc (2008)]

A Macro Model at the Autarky Limit 2/2

Assuming $var_t(g_{y_{H,t+1}} - g_{y_{F,t+1}}) = var_t(g_{y_{H,t+1}})$, *the model at the autarky limit delivers* $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *conditional on shocks to* $y_{H,t}$:

- *with int'l trade in F risk-free asset only*:

$$\frac{-s(1-s)}{1-2(1-\phi)} \geq s^2 - \frac{1}{2}\left[\frac{1-s}{1-2(1-\phi)}\right]^2$$

- *with trade in no assets* $\phi \leq 1/2$ [Corsetti, Dedola and Leduc (2008)]
- *with int'l trade in H and F risk-free asset:* $\phi \to \infty : var(\Delta e_{t+1}) = 0$

Back

Risk-Sharing with risky assets

When F risk-free bonds are internationally traded, as well as a H risky asset, then $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$var_t(\Delta e_{t+1}) - cov_t(\eta_{t+1}, \tilde{r}_{t+1}) \leq 0$$

Risk-Sharing with risky assets

When F risk-free bonds are internationally traded, as well as a H risky asset, then $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$var_t(\Delta e_{t+1}) - cov_t(\eta_{t+1}, \tilde{r}_{t+1}) \leq 0$$

So trade in additional assets need not imply risk-sharing

- nominal bonds
- long maturity bonds (term premia)
- convenience yields

Risk-Sharing with risky assets

When F risk-free bonds are internationally traded, as well as a H risky asset, then $cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) \leq 0$ *if and only if:*

$$var_t(\Delta e_{t+1}) - cov_t(\eta_{t+1}, \tilde{r}_{t+1}) \leq 0$$

So trade in additional assets need not imply risk-sharing

- nominal bonds
- long maturity bonds (term premia)
- convenience yields

Naturally, in the limit of CM $cov_t(\eta_{t+1}, \tilde{r}_{t+1}) \rightarrow 0$

Prop1

Further Details on Heterogeneous SDF model

FX process must satisfy:

- $var_t(\Delta e_{t+1}) = cov_t(m^*_{t+1} - m_{t+1}, \Delta e_{t+1}) - cov_t(d_{t+1}, \Delta e_{t+1})$

Domestic risk sharing implies:

- $\mathbb{E}_t[d_{t+1}] + \frac{1}{2} var_t(d_{t+1}) + cov_t(m_{t+1}, d_{t+1}) = 0$

Moreover, if $E_t[d_{t+1}] = 0 \implies var_t(m_{t+1}) = var_t(\hat{m}_{t+1})$

No arbitrage conditions modified:

- $-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2} var_t(\eta_{t+1}) + cov_t(m^*_{t+1}, \eta_{t+1}) + cov_t(\Delta e_{t+1}, d_{t+1})$

▸ Back

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup\left(\frac{\mathbb{E}_t[R^e_{t+1}]-R_{t+1}}{\sqrt{var(R^e_{t+1})}}\right)^2$$

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup\left(\frac{\mathbb{E}_t[R^e_{t+1}] - R_{t+1}}{\sqrt{var(R^e_{t+1})}}\right)^2$$

2. Assume $\nexists$ good-deals $var_t(\hat{m}_{t+1}) \leq K var_t(m_{t+1})$ [Cochrane Saa-Requejo (2000)].

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup\left(\frac{\mathbb{E}_t[R^e_{t+1}]-R_{t+1}}{\sqrt{var(R^e_{t+1})}}\right)^2$$

2. Assume $\nexists$ good-deals $var_t(\hat{m}_{t+1}) \leq K var_t(m_{t+1})$ [Cochrane Saa-Requejo (2000)]. We show:

$$var(d_{t+1}) \leq var(m_{t+1})\left[1 + K(1 - \frac{2}{\sqrt{K}}\rho_t(\hat{m}_{t+1}, m_{t+1}))\right]$$

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup \left(\frac{\mathbb{E}_t[R^e_{t+1}] - R_{t+1}}{\sqrt{var(R^e_{t+1})}} \right)^2$$

2. Assume $\nexists$ good-deals $var_t(\hat{m}_{t+1}) \leq K var_t(m_{t+1})$ [Cochrane Saa-Requejo (2000)]. We show:

$$var(d_{t+1}) \leq var(m_{t+1}) \left[1 + K(1 - \frac{2}{\sqrt{K}} \rho_t(\hat{m}_{t+1}, m_{t+1})) \right]$$

There is more heterogeneity if:

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup\left(\frac{\mathbb{E}_t[R^e_{t+1}] - R_{t+1}}{\sqrt{var(R^e_{t+1})}}\right)^2$$

2. Assume $\nexists$ good-deals $var_t(\hat{m}_{t+1}) \leq K var_t(m_{t+1})$ [Cochrane Saa-Requejo (2000)]. We show:

$$var(d_{t+1}) \leq var(m_{t+1})\left[1 + K(1 - \frac{2}{\sqrt{K}}\rho_t(\hat{m}_{t+1}, m_{t+1}))\right]$$

There is more heterogeneity if:

- We allow for more risk $var(m_{t+1})$

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup \left(\frac{\mathbb{E}_t[R^e_{t+1}] - R_{t+1}}{\sqrt{var(R^e_{t+1})}} \right)^2$$

2. Assume $\nexists$ good-deals $var_t(\hat{m}_{t+1}) \leq K var_t(m_{t+1})$ [Cochrane Saa-Requejo (2000)]. We show:

$$var(d_{t+1}) \leq var(m_{t+1}) \left[1 + K(1 - \frac{2}{\sqrt{K}} \rho_t(\hat{m}_{t+1}, m_{t+1})) \right]$$

There is more heterogeneity if:

- We allow for more risk $var(m_{t+1})$
- We allow for good-deals $K > 1$ (beyond RA no-arbitrage [Cerny and Hodges (2002)])

Calibrating Model 1/2

1. Measure SDF volatility using excess returns [Hansen-Jaganathan (1991)]:

$$var(M_{t+1}) \geq sup\left(\frac{\mathbb{E}_t[R^e_{t+1}]-R_{t+1}}{\sqrt{var(R^e_{t+1})}}\right)^2$$

2. Assume $\nexists$ good-deals $var_t(\hat{m}_{t+1}) \leq K var_t(m_{t+1})$ [Cochrane Saa-Requejo (2000)]. We show:

$$var(d_{t+1}) \leq var(m_{t+1})\left[1 + K(1 - \frac{2}{\sqrt{K}}\rho_t(\hat{m}_{t+1}, m_{t+1}))\right]$$

There is more heterogeneity if:

- We allow for more risk $var(m_{t+1})$
- We allow for good-deals $K > 1$ (beyond RA no-arbitrage [Cerny and Hodges (2002)])
- Poor risk-sharing within countries $\rho(\hat{m}_{t+1}, m_{t+1})$, e.g. labour risk/ fin. frictions

Calibrating Model 2/2

- S&P 500 $\implies$ $var(m_{t+1}) = 0.5$ [Lustig and Verdelhan (2019), Babu et al. (2020)]
- Foreign returns $K \leq 2$ [Jorda and Taylor (2012), Barroso and Santa-Clara (2015)]
- Micro risk-sharing evidence $\rho(\hat{m}_{t+1}, m_{t+1}) \approx$ 0.21 [Zhang (2020)]
- $var(\Delta e_{t+1}) = 0.33$ [Lustig and Verdelhan (2019), Lloyd and Marin (2022)]

Calibrating Model 2/2

- S&P 500 $\implies var(m_{t+1}) = 0.5$ [Lustig and Verdelhan (2019), Babu et al. (2020)]
- Foreign returns $K \leq 2$ [Jorda and Taylor (2012), Barroso and Santa-Clara (2015)]
- Micro risk-sharing evidence $\rho(\hat{m}_{t+1}, m_{t+1}) \approx$ 0.21 [Zhang (2020)]
- $var(\Delta e_{t+1}) = 0.33$ [Lustig and Verdelhan (2019), Lloyd and Marin (2022)]

$$\rho^{K=2}_{d_{t+1},\Delta e_{t+1}} \leq -\frac{0.33}{1.2} \approx -0.28$$

Back