The Financial Origins of Non-Fundamental Risk

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Question

Can the financial sector be a source of non-fundamental risk for the economy?

- Rajan (2005): “Has Financial Development Made the World Riskier?”
- Danielsson and Shin (2003): “Endogenous Risk”
- Often cited examples: Portfolio insurance (Oct 1987), LTCM 1998

A stylized model where non-fundamental volatility emerges with financial intermediation:

- mutual feedback between the risk of a fall in asset prices and HH’s purchase of insurance
- insurance demand by pessimistic HHs fulfilled by use of leveraged contracts
- trading of leveraged contracts generates possibility of fire-sales
- no fundamental sources of risk present
- full-information rational expectations framework

Outline for presentation

1. Baseline model: unique equilibrium, no price volatility.
2. Add trading of insurance contracts $\rightarrow$ obtain non-fundamental volatility
3. Conclusion

Environment

- two dates: 0 and 1
- three agents: households, financial intermediaries and outside investors
- fixed endowment of cookies (c) at both dates
- fixed endowment of trees at date 0
- trees are claims to apples (a) at date 1
- trees can be traded at date 0
- benchmark: trees are the only asset traded at date 0

Households

unit mass of HHs
only consume cookies (c)

$$U^h(c_0^h, c_1^h) = c_0^h + \left[\mathbb{E}(c_1^h)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \gamma > 1$$

- risk-averse over date 1 consumption
- born with χ_0^h cookies, and all the trees, $e_0 = 1$.

Date-0 budget constraint:

$$c_0^h + p_0 e^h = \chi_0^h + p_0$$

Date-1 budget constraint:

$$c_1^h = p_1 e^h$$

where p_j is price of tree at date $j \in [0, 1]$.
Note: p_1 can be stochastic.

optimize subject to budget constraints and non-negativity constraints on c_j^h, e^h.

optimality condition

$$p_0 = \frac{\mathbb{E} p_1 c_1^{-\gamma}}{\left[\mathbb{E} c_1^{1-\gamma}\right]^{\frac{-\gamma}{1-\gamma}}} = \left[\mathbb{E} p_1^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

Assume χ_0^h large enough s.t. non-negativity constraint on c_j, e^h does not bind

Financial Intermediaries

unit mass of FIs
consume apples (a_1) or cookies (c_j)

$$U^f(c_0, c_1, a_1) = c_0 + \mathbb{E}\,(c_1 + a_1)$$

- risk-neutral over date 1 consumption
- born with $\chi_0^f < 1$ cookies, no trees

Date-0 budget constraint:

$$c_0^f + p_0\, e^f = \chi_0^f$$

Date-1 budget constraint:

$$c_1^f + p_1\, a_1^f = p_1\, e^f$$

optimize subject to budget constraints and non-negativity constraints on c_j^f, e^f, a_1^f.

Date 1:
Sell all trees if $p_1 > 1$.
Keep all trees if $p_1 < 1$
indifferent at the equality.

Date 0:
Only buy trees if $p_0 < \mathbb{E}\,\mathsf{max}\{1, p_1\}$.
Dont buy trees if $p_0 > \mathbb{E}\,\mathsf{max}\{1, p_1\}$.
indifferent at the equality.

Outside Investors

unit mass of OIs
only trade and consume at date 1

$$U^o(c_1, a_1) = v(a_1) + c_1$$

where $v'(\cdot) > 0$ and $v''(\cdot) < 0$.

- only agents with cookies at date 1
- large amt of cookies χ_1
- Assume $v'(0) > 1 > v'(1)$: interior soln

Date-1 budget constraint:

$$c_1^o + p_1 a_1^o = \chi_1$$

optimal demand for trees by OIs

$$v'(a_1^o) \leq p_1 \quad , \quad a_1^o \geq 0$$
$$a_1^o \left[v'(a_1^o) - p_1 \right] = 0$$

Assume $v'(0) > 1 > v'(1)$: st $a_1^o \in (0, 1)$ when $p_1 = 1$.

Let $\bar{e}$ be s.t. $v'(\bar{e}) = 1$.

Equilibrium

prices $\{p_0, p_1\}$ and quantities $\{c_0^h, c_1^h, e^h, c_0^f, c_1^f, a_1^f, e^f, c_1^o, a_1^o\}$

- all agents optimize
- markets for cookies () and trees () at dates 0 and 1 clear,
- market for apples () at date 1 clears

$$
\begin{aligned}
c_0^h + c_0^f &= \chi_0^h + \chi_0^f && (1)\\
c_1^h + c_1^f + c_1^o &= \chi_1 && (2)\\
e^h + e^f &= 1 && (3)\\
a_1^o + a_1^f &= 1 && (4)
\end{aligned}
$$

$e \doteq$ trees retained by HHs at date 0 (e^h)

Lemma 1 (Date 1 price of trees)

In equilibrium, $p_1 = \min\{1, v'(e)\}$.

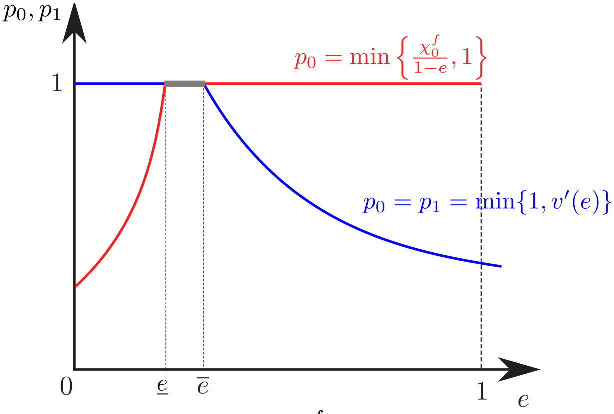
Proof: $p_1 \leq 1$ since $v'(1) < 1$.
When $p_1 < 1$, OIs buy the trees from HHs $\rightarrow p_1 = v'(e)$.
HHs demand for trees: From FOC:

$$p_0 = p_1 = \min\{1, v'(e)\}. \tag{5}$$

FIs demand for trees Since $p_1 \leq 1$, FIs buy trees at date 0 if $p_0 < 1$ - i.e. $p_0(1-e) = \chi_0^f$.

$$p_0 = \min\left\{\frac{\chi_0^f}{1-e}, 1\right\} \tag{6}$$

Equilibrium



HHs demand for trees:

$$p_0 = p_1 = \min\{1, v'(e)\}$$

FIs demand for trees:

$$p_0 = \min\left\{\frac{\chi_0^f}{1-e}, 1\right\}$$

fundamental equilibria: $p_0 = p_1 = 1$ and $e^h \in [\underline{e}, \overline{e}]$. where $\underline{e} = 1 - \chi_0^f$, and $\overline{e}$ s.t. $v'(\overline{e}) = 1$

welfare: $\quad U^h = \chi_0^h + 1, \quad U^f = \chi_0^f, \quad U^o = v(\overline{e}) - \overline{e}$

Endogenous Fragility with Insurance Contracts

Only fundamental equilibria exist when trees are the only assets traded.

- 🌲🌲 are safe assets ($p_1 = 1$)

Allow FIs to sell insurance contracts z^f at date 0 at price q

- pays out $1 - p_1$ if $p_1 < 1$
- equivalent to a put option on trees
- non-negative consumption constraint on FIs limit amt of insurance sold

$$\underbrace{(1 - p_1(s))z^f}_{\text{insurance payout}} \leq \underbrace{p_1(s)e^f}_{\text{value of trees}} \qquad \text{in all states } s$$

- If HHs expect $p_1 = 1$ in all states of the world, then no demand for insurance.
- 🌲🌲 continue to be safe assets
- Fundamental equlibria that we constructed exist, with $q = z^f = 0$.
- ... but <u>not the only set of equilibria</u> that exist

Positive Insurance and Non-fundamental Volatility

$\lambda \in (0,1)$ be probability that $p_1 = \underline{p} < 1$ and $1-\lambda$ prob that $p_1 = 1$.
FI's non-negative consumption constraint binds in the low state:

$$(1-\underline{p})z^f = \underline{p}e^f \implies \frac{z^f}{e^f} = \frac{\underline{p}}{1-\underline{p}} \equiv \phi$$

This is a date-1 equilibrium:

A. When $p_1 = \underline{p}$, FIs sell all trees to payout on insurance contracts
   - OIs purchase all trees in the economy (only agents with cookies at date 1).
   - Since $v'(1) < 1$, $\underline{p} = v'(1) < 1$.

B. When $p_1 = 1$, FIs need not sell any trees
   - Can confirm $p_1 = 1$ as in the benchmark economy (no insurance)

What happens at date 0?

Positive Insurance and Non-fundamental Volatility

HH's problem:

$$\max_{c_0^h, e^h, z^h, c_1^h} \left[c_0^h + \left(\mathbb{E}(c_1^h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right]$$

s.t.

$$c_0^h + p_0 e^h + qz^h = \chi_0^h + p_0$$
$$c_1^h = p_1 e^h + (1 - p_1) z^h$$

$$c_0^h, c_1^h, e^h \geq 0$$

FI's problem:

$$\max_{c_0^f, e^f, z^f, c_1^f} \left[c_0 + \mathbb{E}\left(c_1 + a_1 \right) \right]$$

s.t.

$$c_0^f + p_0 e^f = \chi_0^f + qz^f$$
$$c_1^f + p_1 a_1^f + (1 - p_1) z^f = p_1 e^f$$
$$(1 - \underline{p}) z^f \leq \underline{p} e^f$$
$$c_0^f, c_1^f, e^f \geq 0$$

OIs problem unchanged + Additional market clearing ($z^h = z^f$) condition

FI's solution at date 0

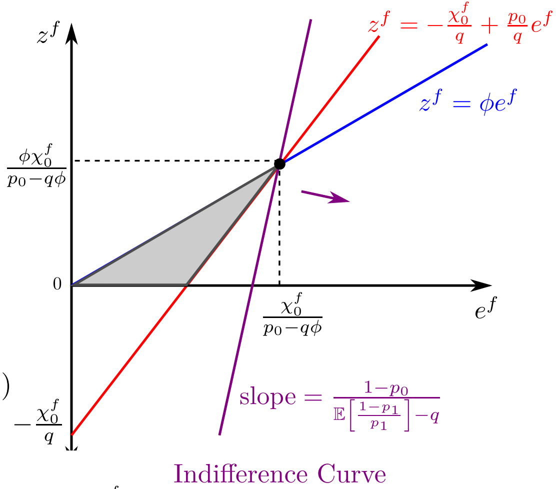
FIs' simplified problem:

$$\max_{e^f, z} \chi_0^f - p_0 e^f + q z^f + \mathbb{E}\left[e^f - \frac{1-p_1}{p_1} z^f\right]$$

s.t.

$$\chi_0^f - p_0 e^f + q z^f \quad \geq \quad 0 \quad (c_0 \geq 0)$$

$$z \quad \leq \quad \phi e^f \quad \text{(insurance issuance)}$$



If $\frac{q}{p_0} > \mathbb{E}\left[\frac{1-p_1}{p_1}\right]$, lever up to the max and purchase $e^f = \frac{\chi_0^f}{p_0 - \phi q}$. Multiplier due to levered contracts

HH's solution at date 0

Solution to HHs problem yields:

$$p_0 - \frac{\underline{p}}{1-\underline{p}} q = \frac{(1-\lambda)(e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}}$$

Buying ϕ insurance claims $\equiv$ HHs selling a security that pays out when $p_1 = 1$.

- HHs sell trees to FIs at price p_0
- use the proceeds to buy insurance claims ϕ

LHS is the price of this synthetic security.
RHS is the cost weighted by HHs' marginal utility.

Equilibrium with Insurance

Using $e^h = 1 - e^f$,

$$\frac{(1-\lambda)(e^h)^{-\gamma}(1-e^h)}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} = \chi_0^f \qquad (7)$$

If $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$, for every $\lambda \in \left(0, \overline{\lambda}\right)$ where $\overline{\lambda}$ is implicitly defined by:

$$\chi_0^f = \frac{(1-\overline{\lambda})\left[1 - \underline{p}^{\frac{\gamma-1}{\gamma}}\right]}{\left[\overline{\lambda}\underline{p}^{\frac{1-\gamma}{\gamma}} + 1 - \overline{\lambda}\right]^{\frac{\gamma}{\gamma-1}}}$$

$\exists$ an equilibrium in which $p_1 = 1$ with probability $1-\lambda$ and $p_1 = \underline{p} = v'(1) < 1$ with probability λ. e^h is implicitly defined by equation (7). p_0 and q are defined by (8) and (9) and $z_h = \frac{\underline{p}}{1-\underline{p}} e^h$.

$$p_0 = \frac{\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} \qquad (8)$$

$$q = \frac{\lambda(1-\underline{p})\underline{p}^{-\gamma}}{\left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)(e^h)^{1-\gamma}\right]^{-\frac{\gamma}{1-\gamma}}} \qquad (9)$$

Equilibrium with Insurance: in English

There exists an equilibrium in which,

- with non-zero probability, price decline at date 1 can be self-fulfilling
- when p_1 is low, FIs sell trees to pay out on their insurance contracts, pushing down the price
- if households anticipate that prices might fall, they demand insurance from FIs
- issuance of insurance actually makes price declines possible.
- supply of private safe assets may create its own demand: *Say's law for risk*

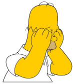
Key market incompleteness: OIs are not allowed to participate at date 0

Equilibrium with Insurance: Welfare

1. HHs
   - worse off than in fundamental eqm
   - welfare with insurance

$$\underbrace{\chi_0^f + \chi_0^h}_{c_0^h} + \left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left(e^h(\lambda)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

   - $\lambda \to 0$, welfare converges to no-insurance case



2. FIs 
   - weakly better off than in fundamental eqm
   - have the option to consume their endowment χ_0^f in the first period.

3. OIs 
   - benefit from fire-sales
   - sell cookies for apples at steep discounts
   - better off than in fundamental eqm
   - welfare with insurance

$$(1-\lambda) \quad \underbrace{[v(\overline{e}) - \overline{e}]}_{\text{no insurance welfare}} \quad +\lambda \underbrace{[v(1) - v'(1)]}_{>0}$$

Other private safe assets

allow FIs to issue risk-free non-state contingent bonds b at price q^b

- pay one cookie to the holder at date 1
- bonds are backed by FIs' holdings of trees: *repo* transactions

HHs budget constraints

$$\begin{aligned} c_0^h + p_0 e^h + q^b b^h &= \chi_0^h + p_0 && (10) \\ c_1^h &= p_1 e^h + b^h, && (11) \end{aligned}$$

FIs budget constraints

$$\begin{aligned} c_0^f + p_0 e^f &= \chi_0^f + q^b b^f && (12) \\ c_1^f + p_1 a_1^f + b^f &= p_1 e^f && (13) \end{aligned}$$

non-negative consumption on FIs:

$$b^f = p_1 \left(e^f - a_1^f \right) - c_1^f \leq p_1 e^f \qquad (14)$$

in all states of the world

Other private safe assets

for every equilibrium that exists in insurance economy, a corresponding equilibrium exists in the bond economy

- FIs have to pay out in all states of the world
- *but* FIs sell more when $p_1 = \underline{p} < 1$ to meet obligations

fundamental equilibrium:

- zero spread between expected return on bonds and trees
- both bonds and trees are riskless assets

non-fundamental equilibria

- date 0 price of bonds is higher
- That is, risk-free rate is lower in these equilibria
- safe rate endogenously falls as a *result* of private safe asset creation
  (contrast to a typical safe assets scarcity narrative)

Policy to eliminate financial fragility

FIs should be the "natural" buyers of trees at date 1

- because of excessive leverage, they are forced to *sell* trees in some states
- explicit ban on such financial transactions would return the economy to a unique equilibrium setup (strict enough tax or leverage restrictions)
- or reduce the excess returns to leveraged investments in risky assets

Consider two sets of crisis-fighting policies

1. increase supply of publicly backed safe assets (issue debt, bailouts)
2. reduce demand for private safe assets (social insurance, market maker of last resort)

Policy: reduce excess return to private safe asset creation

1. Crowd out private safe assets
   - public safe assets by the government
     - i crowd out private provisioning of safe assets
     - ii prevent buildup of intermediary leverage
   - ex-ante commitment to bail out financial intermediary in the bad state
     - i portion of private safe assets become "publicly backed" (Benigno & Robatto, 2019)
     - ii crowds out unbacked private safe assets.
2. Reduce demand for insurance
   - social insurance to households, or *market maker of last resort* (Buiter and Sibert, 2008)
   - eliminate possibility of fire-sales
   - intervention not required in equilibrium

Conclusion

Private creation of safe assets by leveraged intermediaries can lead to fragility

- Safe assets are produced due to demand for safety by households
- Demand for safety arises from fragility induced by the privately-supplied safe assets
- Economy becomes vulnerable to self-fulfilling fire sales

Novel contribution

- leverage is not being used to amplify exogenous fundamental shocks
- instead, financial system *generates* risk in an otherwise fundamentally safe economy