

The Financial Origins of Non-Fundamental Risk

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formalize idea that the financial sector can be a *source* of risk, rather than a means to manage fundamental risk (Rajan (2005), Danielsson and Shin (2003))

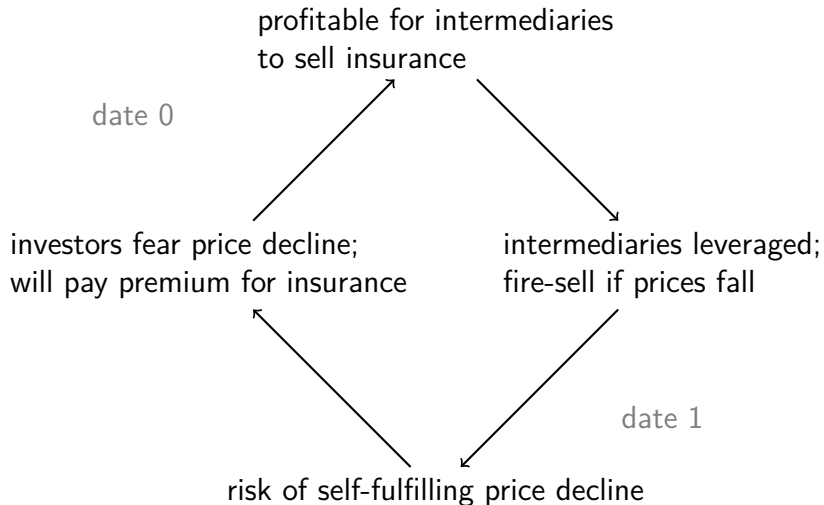
stylized 3-period model, key ingredients:

- risk-averse investors can buy insurance/safe assets from risk-neutral intermediaries
- no fundamental shocks (can relax this)

result: there exists a nonfundamental equilibrium in which

- asset prices sometimes fall below fundamental value as intermediaries fire-sell assets
- investors buy insurance against this risk
- but prices can only fall *because* intermediaries issue insurance

Key mechanism



Related literature

Sunspot eqba can arise from trade in assets w price-contingent payoffs (Bowman & Faust 1997) or sunspot-contingent payoffs (Hens 2000)

- we show trade in assets w *non-contingent payoffs* can also cause sunspot eqba

Pecuniary externalities with financial frictions (Lorenzoni 2008, Stein 2012, Dávila & Korinek 2018): mostly study fundamental shocks, rule out multiplicity

- multiple equilibria w sunspots

Multiple equilibria with financial frictions in small open economies (Bocola & Lorenzoni 2020, Schmitt-Grohé & Uribe 2020)

- closed economy, no binding financial constraint, different source of multiplicity

Demand and supply of safe assets (Caballero & Farhi 2018, Acharya & Dogra 2020,...)

- demand for safe assets \leftarrow nonfundamental risk has different (policy) implications

1. Baseline model w/o insurance: only fundamental equilibria, no price volatility
2. Add trading of insurance contracts \rightarrow non-fundamental equilibria w price volatility
3. Extend to non-state-contingent contracts
4. Policy
5. Conclusion

- 3 dates: 0, 1, and 2
- 3 agents:
 1. risk-averse households (HHs)
 2. risk-neutral financial intermediaries (FIs)
 3. outside investors (OIs) *do not* trade at date 0
- fixed endowment of cookies (c) at both dates
- unit endowment of trees (e) at date 0
- 1 tree \rightarrow 1 cookie (c) at date 2
- trees can be traded at dates 0 and 1
- **no exogenous source of risk**

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model roadmap:

1. baseline model w/o insurance: trees are only asset traded
2. add trading of insurance contracts
3. discuss non-state-contingent contracts

Households

Born with large endowment χ_0^h of cookies, all trees; consume only at dates 0 and 1

Risk-averse: Epstein-Zin utility with $\text{IES} = \infty$ (can generalize)

$$\max c_0^h + \left[\mathbb{E}(c_1^h)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \quad \gamma > 1$$

s.t.

$$c_0^h + p_0 e^h = \chi_0^h + p_0$$

$$c_1^h = p_1 e^h$$

$$c_0^h, c_1^h, e^h \geq 0$$

don't consume at date 2 \Rightarrow date 0 valuation of tree depends on expected date 1 price:

$$p_0 = \frac{\mathbb{E} p_1 c_1^{-\gamma}}{\left[\mathbb{E} c_1^{1-\gamma} \right]^{\frac{-\gamma}{1-\gamma}}} = \left[\mathbb{E} p_1^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

Financial Intermediaries

Born with small endowment χ_0^f of cookies, no trees; can consume at all dates

Risk-neutral

$$\max c_0^f + \mathbb{E} (c_1^f + c_2^f)$$

s.t.

$$\begin{aligned}c_0^f + p_0 e_0^f &= \chi_0^f \\c_1^f + p_1 e_1^f &= p_1 e^f \\c_2^f &= e_1^f \\c_0^f, c_1^f, e_0^f, e_1^f &\geq 0\end{aligned}$$

Optimality conditions:

- Date 1: sell all trees if $p_1 > 1$, consume all trees if $p_1 < 1$
- Date 0: buy only trees if $p_0 < \mathbb{E} \max\{1, p_1\}$, don't buy any if $p_0 > \mathbb{E} \max\{1, p_1\}$

only agents w cookies χ_1 at date 1; trade and consume at dates 1 and 2 (Stein, 2012)

$$\max c_1^o + v(c_2^o)$$

s.t.

$$\begin{aligned} c_1^o + p_1 e_1^o &= \chi_1 \\ c_2^o &= e_1^o \end{aligned}$$

where $v'(\cdot) > 0$, $v''(\cdot) < 0$, $v'(0) > 1 > v'(1) := \underline{p}$

- Optimal demand for trees implies $p_1 = v'(e_1^o)$
- Define \bar{e} s.t. $v'(\bar{e}) = 1$

Equilibrium

prices $\{p_0, p_1\}$ and quantities $\{c_0^h, c_1^h, e_0^h, c_0^f, c_1^f, e_1^f, e_0^o, c_1^o, e_1^o\}$ s.t. all agents optimize and prices clear:

$$\begin{aligned}c_0^h + c_0^f &= \chi_0^h + \chi_0^f \\c_1^h + c_1^f + c_1^o &= \chi_1 \\e_0^h + e_0^f &= 1 \\e_1^o + e_1^f &= 1\end{aligned}$$

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Lemma (Date 1 price of trees)

In equilibrium, $p_1 = \min\{1, v'(e_0^h)\}$.

Equilibrium in the market for trees

HHs' demand: From FOC:

$$p_0 = p_1 = \min\{1, v'(e_0^h)\}$$

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$$p_0 = \min \left\{ \frac{\chi_0^f}{1 - e_0^h}, 1 \right\}$$

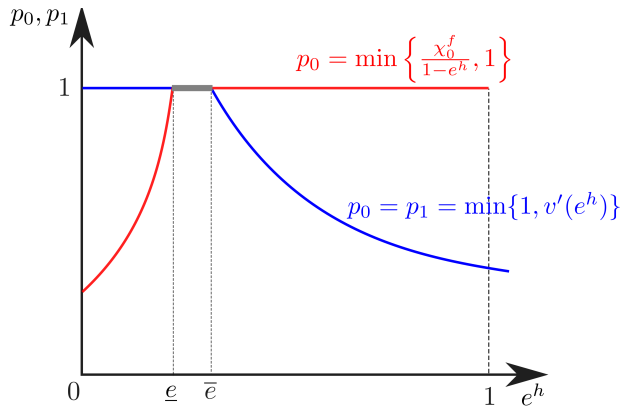
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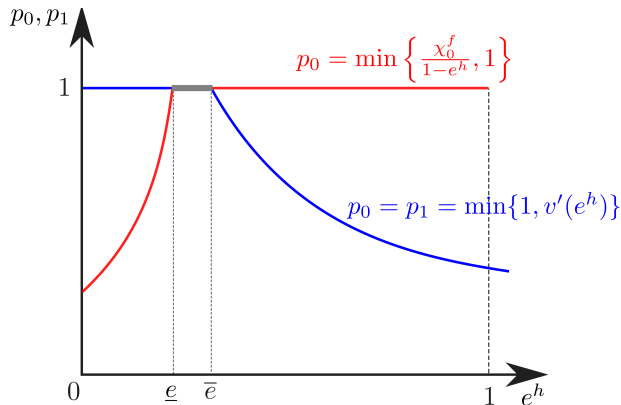
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fundamental equilibria: $p_0 = p_1 = 1$ and $e_0^h \in [\underline{e}, \bar{e}]$, where $\underline{e} = 1 - \chi_0^f$, $v'(\bar{e}) = 1$

welfare: $U^h = \chi_0^h + 1, \quad U^f = \chi_0^f, \quad U^o = v(\bar{e}) - \bar{e}$

Introducing insurance contracts

- When trees are the only asset traded, they are safe ($p_1 = 1$) and only fundamental equilibria exist
- Now allow FIs to sell insurance contracts z^f at date 0 at price q
 - 1 insurance contract pays $1 - p_1$ cookies at date 1 if $p_1 < 1$
 - 1 insurance contract + 1 tree is worth 1 cookie at date 1 for sure.

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- If HHs expect $p_1 = 1$ for sure, this belief is self-confirming, insurance is not used and has a price $q = 0$, and we have the same set of fundamental equilibria
- But there are other equilibria...

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HHs :
$$\max_{c_0^h, e_0^h, z_0^h, c_1^h} \left[c_0^h + \left(\mathbb{E}(c_1^h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right]$$
$$\text{s.t. } c_0^h + p_0 e_0^h + q z_0^h = \chi_0^h + p_0$$
$$c_1^h = p_1 e_0^h + (1 - p_1) z_0^h$$
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$$\begin{aligned} \text{HHs : } \max_{c_0^h, e_0^h, z_0^h, c_1^h} & \left[c_0^h + \left(\mathbb{E}(c_1^h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right] \\ \text{s.t. } & c_0^h + p_0 e_0^h + q z_0^h = \chi_0^h + p_0 \\ & c_1^h = p_1 e_0^h + (1 - p_1) z_0^h \\ & c_0^h, c_1^h, e_0^h \geq 0 \end{aligned}$$

$$\begin{aligned} \text{FIs : } \max_{c_0^f, e_0^f, z_0^f, c_1^f} & \left[c_0^f + \mathbb{E}(c_1^f + e_1^f) \right] \\ \text{s.t. } & c_0^f + p_0 e_0^f = \chi_0^f + q z_0^f \\ & c_1^f + p_1 e_1^f + (1 - p_1) z_0^f = p_1 e_0^f \\ & c_0^f, c_1^f, e_1^f \geq 0 \\ & (\text{implies } \underbrace{(1 - p_1) z_0^f}_{\text{insurance payout}} \leq \underbrace{p_1 e_0^f}_{\text{value of trees}}) \end{aligned}$$

- Ols' problem unchanged
- Insurance mkt clears ($z_0^h = z_0^f$)

- If HHs expect $p_1 = 1$ for sure, this belief is self-confirming, insurance is not used and has a price $q = 0$, and we have the same set of fundamental eqba

- We'll construct another eqm in which $p_1 = \begin{cases} v'(1) := \underline{p} < 1 & \text{w prob } \lambda \in (0, 1) \\ p_1 = 1 & \text{w prob } 1 - \lambda \end{cases}$
- Fls' nonnegativity constraint binds in the low state:

$$(1 - \underline{p})z^f = \underline{p}e_0^f \implies \frac{z^f}{e_0^f} = \frac{\underline{p}}{1 - \underline{p}} \equiv \phi$$

- This satisfies all date 1 eqm conditions:
 - When $p_1 = \underline{p}$, Fls sell all trees to pay out $(1 - \underline{p})z_0^f$ on insurance contracts
 - Ols must purchase all trees in eqm (only agents with cookies).
 - To induce them to do so, $p_1 = v'(1) := \underline{p} < 1$.
 - When $p_1 = 1$, Fls have no insurance liabilities, need not sell any trees, $p_1 = 1$ as in the benchmark economy

Non-fundamental equilibrium

Provided that FIs have limited initial capital (χ_0^f), we can construct such non-fundamental equilibria

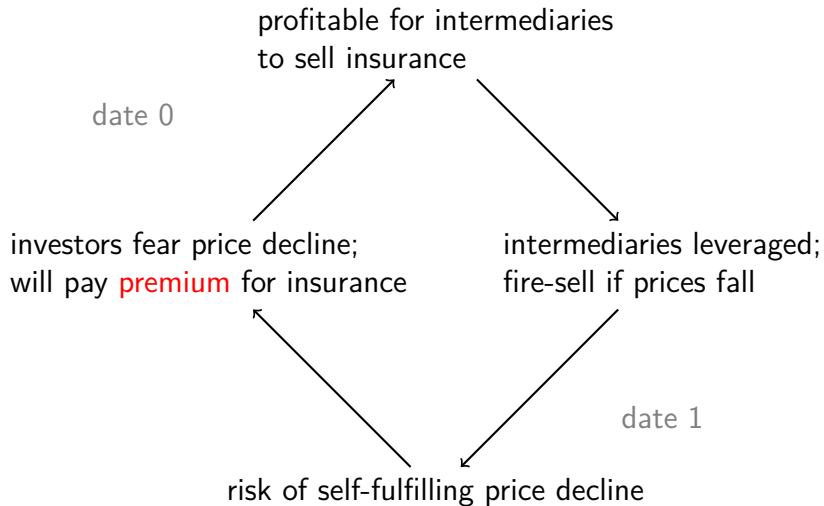
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Proposition: If $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$, a nonfundamental eqm with $Pr(p_1 = \underline{p}) = \lambda$ exists for every $\lambda \in (0, \bar{\lambda})$ where $\bar{\lambda}$ is defined by:

$$\chi_0^f = \frac{(1 - \bar{\lambda}) \left[1 - \underline{p}^{\frac{\gamma-1}{\gamma}} \right]}{\left[\bar{\lambda} \underline{p}^{\frac{1-\gamma}{\gamma}} + 1 - \bar{\lambda} \right]^{\frac{\gamma}{\gamma-1}}}$$

- If FIs lever up to the max, there can be a self-fulfilling price decline at date 1
- Risk-neutral FIs lever up - even though they're wiped out when prices fall - because it's profitable to sell insurance to risk-averse HHs who fear the price decline
 - provided that **risk premium** (difference between physical and risk-neutral probability of bad state) is large enough...
 - which is the case when FIs' capital (χ_0^f) is small and they cannot buy many trees, so HHs still hold most trees and are heavily exposed to fall in prices ($\underline{p} \ll e^h$)
- **issuance of insurance makes price declines possible, rationalizing households' decisions to buy insurance**
- *'supply of safe assets creates its own demand'*



- HHs worse off than in fundamental eqm: welfare with insurance

$$\underbrace{\chi_0^f + \chi_0^h}_{c_0^h} + \left[\lambda \underline{p}^{1-\gamma} + (1-\lambda) \left(e_0^h(\lambda) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

decreasing in λ , $\rightarrow \chi_0^f + 1$ as $\lambda \rightarrow 0$

- FIs better off: always have option to consume χ_0^f and get same welfare as fundamental eqm
- OIs better off: benefit from fire sales

Efficiency and market incompleteness

Pareto Efficiency

- Any fundamental equilibrium is Pareto efficient.

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Markets are complete for all agents who participate.

- If we allow outside investors to participate in asset markets at date 0, then the fundamental equilibrium is the unique equilibrium.

Market incompleteness: OLG

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If OI's allowed to participate in date-0 market,

$$\max_{e_0^o, z_0^o, c_1^o, c_2^o} \mathbb{E} [c_1^o + v(c_2^o)]$$

subject to:

$$\begin{aligned} p_0 e_0^o &= q z_0^o \\ c_1^o + p_1 e_1^o + (1 - p_1) z_0^o &= \chi_1 + p_1 e_0^o \\ c_2^o &= e_1^o \\ (1 - p_1) z_0^o &\leq \chi_1 + p_1 e_0^o \end{aligned}$$

where e_0^o denotes the OI's purchase of trees at date 0 and z_0^o denotes the OI's issuance of insurance.

Trade in non-state-contingent assets can also produce nonfundamental eqba

allow Fls to issue riskless bonds b at price q^b (instead of insurance)

- pay one cookie to the holder at date 1
- can interpret as *repo* (backed by holdings of trees)

HHs budget constraints

$$c_0^h + p_0 e_0^h + q^b b^h = \chi_0^h + p_0 \quad (1)$$

$$c_1^h = p_1 e_0^h + b^h, \quad (2)$$

Fls budget constraints

$$c_0^f + p_0 e_0^f = \chi_0^f + q^b b^f \quad (3)$$

$$c_1^f + p_1 e_1^f + b^f = p_1 e_0^f \quad (4)$$

as before, Fls' consumption must ≥ 0 whatever the realization of p_1 :

$$b^f = p_1 (e_0^f - e_1^f) - c_1^f \leq p_1 e_0^f \quad (5)$$

Trade in non-state-contingent assets can also produce nonfundamental eqba

for every equilibrium that exists in insurance economy, a corresponding equilibrium exists in the bond economy

- Fls have to pay out in all states of the world
- *but* Fls sell more when $p_1 = \underline{p} < 1$ to meet obligations

fundamental equilibrium:

- zero spread between expected return on bonds and trees
- both bonds and trees are riskless assets

non-fundamental equilibria

- date 0 price of bonds is higher (risk-free rate lower) in these equilibria
- safe rate endogenously falls as a *result* of private safe asset creation (contrast to a typical safe assets scarcity narrative)

Policy to eliminate financial fragility

- Simple multiple equilibrium model. Not surprisingly, various policies can eliminate nonfundamental eqba (benefiting HHs at expense of FIs & OIs)
 - ban trade in insurance contracts!
 - or tax them, impose leverage constraints...
 - richer models might have additional tradeoffs
- What's (hopefully) interesting is *how* some of the policies do so
- Distinguish between policies that
 - 1 increase supply of publicly backed safe assets (issue debt, bailouts)
 - 2 reduce demand for private safe assets (social insurance, market maker of last resort)

Conclusion

Private creation of safe assets by leveraged intermediaries can lead to fragility

- Safe assets are produced due to demand for safety by households
- Demand for safety arises from fragility induced by the privately-supplied safe assets
- Economy becomes vulnerable to self-fulfilling fire sales

Novel contribution: leverage does not just amplify fundamental shocks, but *generates* risk in a fundamentally safe economy

- adding fundamental shocks does not change results: financial sector can both amplify fundamental risk and create non-fundamental risk

END

Insurance Salesman of the Opera: Gary Larson



Scene from *Insurance Salesman of the Opera*

Constructing a non-fundamental eqm: Fls' date 0 optimality conditions

Since $p_1 \leq 1$, Fls spend all date 1 resources on trees \Rightarrow

$$\max_{e^f, z} \chi_0^f - p_0 e_0^f + q z_0^f + \underbrace{\mathbb{E} \left[e_0^f - \frac{1 - p_1}{p_1} z_0^f \right]}_{\text{spend everything on trees}}$$

$$\text{s.t. } \chi_0^f - p_0 e_0^f + q z_0^f \geq 0 \quad (c_0^f \geq 0)$$

$$z_0^f \leq \phi e_0^f \quad (c_1^f, e_1^f \geq 0)$$

$$\Rightarrow \text{ if } \underbrace{\frac{1}{p_0}}_{\text{return on tree}} > \underbrace{\frac{1}{q} \mathbb{E} \left[\frac{1 - p_1}{p_1} \right]}_{\text{return on insurance}}, \text{ lever up}$$

$$\text{to the max and purchase } e_0^f = \frac{\chi_0^f}{p_0 - \phi q}$$

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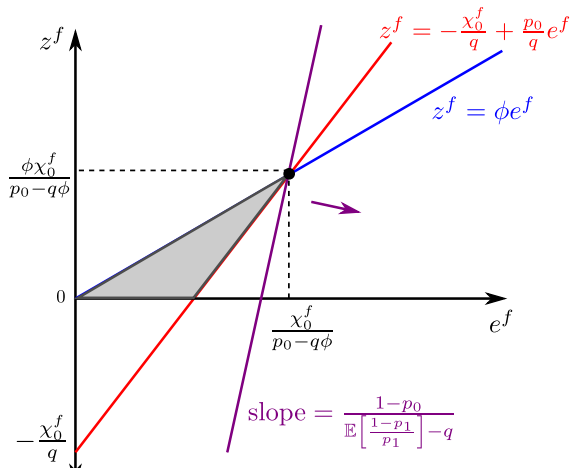
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	Nature of market incompleteness:	
Asset payoffs:	complete absent sunspots, incomplete w sunspots, no OLG	dynamically complete but OLG
sunspot-contingent	Hens (2000)	Cass & Shell (1983)
price-contingent	Bowman & Faust (1997)	Our insurance economy
'real'	Gottardi & Kajii(1999)	Our bond economy

Constructing a non-fundamental eqm: HHs' date 0 optimality conditions

- Buy 1 tree, sell ϕ insurance $\rightarrow p_1 - \phi(1 - p_1) = \begin{cases} 0 & \text{w prob } \lambda \\ 1 & \text{w prob } 1 - \lambda \end{cases}$ at date 1:
'synthetic Arrow security' with price $p_0 - \phi q$
- Fls spend their whole endowment to buy $e_0^f = \frac{\chi_0^f}{p_0 - \phi q}$ Arrow securities from HHs, betting that $p_1 = 1$
- HHs 'sell' Arrow securities (bet that $p_1 = \underline{p}$); price this security using Euler eqs:

$$p_0 - \frac{\underline{p}}{1 - \underline{p}} q = \frac{(1 - \lambda)}{\left[\lambda \left(\frac{\underline{p}}{e_0^h} \right)^{-(\gamma-1)} + (1 - \lambda) \right]^{\frac{\gamma}{\gamma-1}}} = \frac{\chi_0^f}{1 - e_0^h} \text{ in eqm}$$

- to the extent that consumption falls in bad state ($\frac{\underline{p}}{e_0^h} < 1$), payoffs in good state are less valuable, securities cheaper, Fls can buy more of them

Non-fundamental equilibrium

This is a valid eqm provided that the solution e_0^h satisfies

$$\frac{q}{p_0} = \frac{\lambda(1 - \underline{p}) \left(\frac{p}{e_0^h} \right)^{-\gamma}}{\lambda \left(\frac{p}{e_0^h} \right)^{-\gamma} \underline{p} + (1 - \lambda)} > \lambda \frac{1 - \underline{p}}{\underline{p}} \left(\text{true iff } e_0^h > \underline{p}^{\frac{\gamma-1}{\gamma}} \right)$$

- a higher risk premium (lower $\left(\frac{p}{e_0^h} \right)^{\gamma} < 1$) increases price of insurance (which only pays off in bad state) relative to trees (which pay less in bad state)...
- ... and increases Fls' incentive to sell insurance, buy trees

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Proposition: If $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$, a nonfundamental eqm with $Pr(p_1 = \underline{p}) = \lambda$ exists for every $\lambda \in (0, \bar{\lambda})$ where $\bar{\lambda}$ is defined by:

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Constrained Pareto Efficiency

Take any non-fundamental equilibrium, and let \mathbf{U}^i denotes agent i 's utility in this equilibrium. An allocation $\{c_0^h, c_0^f, e_0^h, e_0^f, z\}$ is a constrained Pareto improvement relative to this non-fundamental equilibrium if there exists a date 1 price p_1 such that

$$c_0^h + p_1 e_0^h + (1 - p_1)z^f \geq \mathbf{U}^h \quad (\text{HH PC})$$

$$c_0^f + [p_1 e_0^f - (1 - p_1)z^f] \max\left(\frac{1}{p_1}, 1\right) \geq \mathbf{U}^f \quad (\text{FI PC})$$

$$V_1(p_1) \geq \mathbf{U}^o \quad (\text{OI PC})$$

$$\frac{p_1 e_0^f - (1 - p_1)z^f}{p_1} + v'^{-1}(p_1) \geq 1, p_1 \leq 1, \text{ at least one strict equality} \quad (\text{IC})$$

$$c_0^h + c_0^f = \chi_0^h + \chi_0^f \quad (\text{RC1})$$

$$e_0^h + e_0^f = 1 \quad (\text{RC2})$$

where at least one of the first three inequalities (participation conditions) is strict.

Introduce government in the bond-economy.

- issues risk-free bonds w face value b^g ; buys e^g trees at date 0

$$q^b b^g = p_0 e^g$$

- sell trees, levy lumpsum taxes on outside investors at date 1

$$T + p_1 e^g = b^g$$

b^g can also be liability of central bank, e.g. interest bearing reserves or reverse repos (Greenwood Hanson Stein, 2016).

Fundamental equilibrium unchanged:

- both debt and trees are safe assets and trade at price of 1
- government never taxes Ols at date 1

Non-fundamental equilibrium

- trees are risky assets
- HH consumption when $p_1 = \underline{p}$ is now $\underline{p} + \underline{T}$.
 - in eqm, in bad state, HHs get cookies from Ols by both selling all trees at price \underline{p} , and taxing them
 - Higher b^g raises T , raises HH consumption when $p_1 = \underline{p}$, \downarrow risk premium ($\uparrow \frac{p+T}{e^h}$)
- If b^g high enough, risk premium is so low that FIs strictly prefer not to take leveraged position in trees

If $b^g \geq b^* \equiv \frac{p^{\frac{1}{\gamma}}}{1-\underline{p}} (1 - \chi_0^f) - \frac{p}{1-\underline{p}}$, no non-fundamental equilibrium exists

Transfers to Fls: “bailout policies”

Rather than issue debt ex-ante, transfer to Fls in a crisis.

- Farhi & Tirole (2012), Bianchi (2016), Jeanne & Korinek (2020): anticipated bailouts increase leverage and financial instability
- here too, bailouts increase Fls' borrowing in any non-fundamental equilibrium
- *but* generous bailouts rule out the existence of non-fundamental equilibrium!

Govt transfers $T^f \geq 0$ to Fls when $p_1 = \underline{p}$, taxes Ols. Fls budget constraint

$$c_1^f + \underline{p}a_1^f + b^f = \underline{p}e^f + T^f$$

large *unanticipated* transfer can prevent fire-sale because Fls can repay without selling trees. What if transfers are anticipated?

If FIs anticipate bailout, they borrow more so their borrowing constraint

$$b^f \leq \underline{p}e^f + T^f$$

holds with equality

HHs hold more 'publicly backed' safe assets – similar to effect of govt debt!

- can interpret as govt guarantees (deposit insurance, MMMF guarantee in Sep 08) (cf. Benigno & Robatto 2019)
- transfers 'pass through' FIs to households
- HH consumption when $p_1 = \underline{p}$ is $\underline{p} + T^f$

If $T^f \geq \underline{p}^{\frac{1}{\gamma}} (1 - \chi_0^f) - \underline{p}$, then no non-fundamental equilibrium exists

Difference from Farhi & Tirole (2012)

Farhi & Tirole (2012): anticipated bailouts make ex-post intermediaries' leverage decisions *strategic complements*

- if only a few banks lever up, a bailout is unlikely, so it is unprofitable to lever up
- if many banks lever up, policymakers will have to bailout, so profitable to lever up

Here: nonfundamental eqm exists *absent* bailout, large enough anticipated bailout can eliminate them:

- profitability of leveraging up depends on **risk premium** (HH demand for safe assets)
- large enough bailout/publicly backed safe asset supply satiates demand for safe assets, reduces **risk premium**
- making it privately unprofitable to lever up
- This channel's absent in Farhi & Tirole's risk neutral economy

Market maker of last resort

Stand ready to buy any quantity of trees at some price $p^\diamond > \underline{p}$

- the ECB's Outright Monetary Transactions
- the Municipal Liquidity Facility and the Secondary Market Corporate Credit Facility
- the Federal Reserve's standing repo facilities

Let $p^\diamond < 1$ be the price at which the government stands ready to buy.

$$p_1 e_1^g = T \tag{6}$$

$$p_1 \geq p^\diamond, \quad e_1^g \geq 0, \quad \text{with at least one equality} \tag{7}$$

$$a_1^f + a_1^o + e_1^g = 1 \tag{8}$$

Govt raises taxes T on Ols to fund purchases; apples from trees they buy are wasted

- fundamental equilibrium unchanged (no intervention)
- non-fundamental equilibrium:
 - price cannot fall below p^\diamond
 - when prices fall, govt is the marginal buyer of trees, purchasing $e^g = 1 - v'^{-1}(p^\diamond)$ trees and levying taxes $T = p^\diamond e^g$ on Ols
 - Higher p^\diamond reduces **risk premium** and HHs' demand for insurance
 - effectively govt provides a certain amount of insurance at zero price

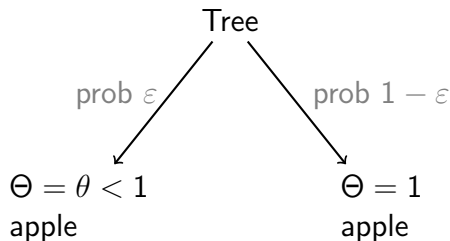
if $p^\diamond \geq (1 - \chi_0^f)^{\frac{\gamma}{\gamma-1}}$, no non-fundamental equilibrium exists.

Environment with fundamental risk

- 2 dates: 0 and 1
- 3 agents:
 1. risk-averse households (HHs)
 2. risk-neutral financial intermediaries (FIs)
 3. outside investors (OIs) who only trade at date 1
- fixed endowment of cookies (c) at both dates
- unit endowment of trees (e) at date 0
- trees can be traded at date 0

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Date 1 price of trees now also depends on fundamental state: $p_1(\Theta)$

Fundamental Equilibria with fundamental risk, only trees traded at date 0

Define \bar{e} s.t. $v'(\bar{e}) = 1$

Assume that $\chi_0^f \geq (1 - \bar{e}) \mathbb{E}[\Theta]$.

Then, the equilibrium date 0 and date 1 price of trees is given by

$$p_0 = \left[\mathbb{E} \Theta^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \quad p_1(\Theta) = \Theta$$

households retain

$$e^* = 1 - \frac{\chi_0^f}{p_0} \text{ trees.}$$

Households' face consumption risk at date 1:

$$c_1^h(\theta) = \theta e^* < e^* = c_1^h(1)$$

With the introduction of insurance, households' budget constraints now become

$$c_0^h + p_0 e^h + \sum_{p \in \mathbb{P}} q(p) z^h(p) = \chi_0^h + p_0 \quad (9)$$

$$c_1^h(\Theta) = p_1(\Theta) e^h + \left(1 - p_1(\Theta)\right) z^h, \quad \Theta \in \{\theta, 1\} \quad (10)$$

where

$q(p)$ is the date 0 price of the derivative which pays off $1 - p$ cookies at date 1 if the price realized is p ,

$z^h(p)$ denotes the quantity of that derivative purchased by households

the date 0 and date 1 budget constraints of Fls can be written as

$$c_0^f + p_0 e^f = \chi_0^f + \sum_{p \in \mathbb{P}} q(p) z^f(p) \quad (11)$$

$$c_1^f(\Theta) + \frac{p_1(\Theta)}{\Theta} a_1^f + \left(1 - p_1(\Theta)\right) z^f = p_1(\Theta) e^f \quad \Theta \in \{\theta, 1\} \quad (12)$$

where $z^f(p)$ denotes the quantity of the derivative sold by Fls.

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$$c_0^f + p_0 e^f = \chi_0^f + \sum_{p \in \mathbb{P}} q(p) z^f(p) \quad (11)$$

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where $z^f(p)$ denotes the quantity of the derivative sold by Fls.

Fls issuance of derivative z^f is limited by:

$$\left(1 - p_1(\Theta)\right) z^f \leq p_1(\Theta) e^f, \quad (13)$$

Fundamental Equilibria with fundamental risk and trade in insurance

For large enough χ_0^f , in the economy with insurance,

\exists a unique fundamental equilibrium with perfect hh consumption insurance

$$c^h(1) = c^h(\theta)$$

in which

$$p_0 = \varepsilon\theta + 1 - \varepsilon, \quad q = \varepsilon(1 - \theta)$$

$$p_1(\Theta) = \Theta$$

and $e^h = z^h$.

HHs better off with insurance and no sunspots than without insurance.

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but there also exist equilibria with sunspots.