SMOOTH DIAGNOSTIC EXPECTATIONS BIANCHI, ILUT, AND SAIJO

Discussant: Sanjay R. Singh FRBSF & UC Davis

> January 3, 2025 ASSA Meetings

The views expressed herein are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

Introduction

- Growing interest in psychological foundations for belief formation in Economics.
- ▶ Diagnostic Expectations (DE): Overreaction driven by the representativeness heuristic.
- ► Smooth DE: A new paradigm linking representativeness and uncertainty in dynamic models.

NOVEL CONTRIBUTIONS

- High-uncertainty environments amplify overreaction.
- Smooth DE bridges Diagnostic Expectations and Uncertainty literatures.
- ► Comprehensive explanation of survey data patterns.

KEY CONCEPTS

- ► Representativeness heuristic: Selective memory recalls events that are representative of current news.
- ► Smooth DE:
 - Overreaction depends on uncertainty.
 - ▶ Joint foundation for survey evidence and business cycle properties.

EMPIRICAL EVIDENCE

- Survey forecasts show:
 - Overreaction to news.
 - Stronger overreaction at longer horizons.
 - Overconfidence in subjective uncertainty.
- ▶ Novel finding: Overreaction amplifies under high uncertainty.

THEORETICAL FRAMEWORK

- Smooth DE connects belief distortions with time-series dynamics.
- ► Business cycle implications:
 - Asymmetric cycles: Recessions sharper than expansions.
 - ► Countercyclical volatility at micro and macro levels.
- Policy insights: Reducing idiosyncratic uncertainty stabilizes macroeconomy.

COMPARISON: SMOOTH DE VS. STANDARD DE

Standard DE

(Bordalo, Gennaioli, and Shleifer, 2018; Bianchi, Ilut, and Saijo, 2024; L'Huillier, Singh, and Yoo, 2024):

- Captures overreaction but lacks flexibility.
- ► Ignores interaction with uncertainty.
- ► Smooth DE:
 - Explains overconfidence.
 - Adapts dynamically to uncertainty levels.

KEY EQUATIONS AND COROLLARY

Key Equations:

$$R_{t+h|t,t-J} = \frac{\sigma_{t+h|t}^2}{\sigma_{t+h|t-J}^2}$$
 (11)

$$E_t^{\theta}(x_{t+h}) = \mu_{t+h|t} + \underbrace{\theta \frac{R_{t+h|t,t-J}}{1 + \theta(1 - R_{t+h|t,t-J})}}_{\equiv \tilde{\theta}_t} (\mu_{t+h|t} - \mu_{t+h|t-J})$$
(12)

$$V_t^{\theta}(x_{t+h}) = \frac{\sigma_{t+h|t}^2}{1 + \theta(1 - R_{t+h|t,t-I})}$$
(13)

Corollary 1 (Condensed):

- Overreaction increases with relative uncertainty: $\frac{\partial \bar{\theta}_t}{\partial R_{t+hlt,t-l}} > 0$.
- ▶ Overconfidence when $R_{t+h|t,t-I}$ < 1.
- ▶ Underconfidence when $R_{t+h|t,t-I} > 1$.

8

AR(1) PROCESS: KEY INSIGHTS

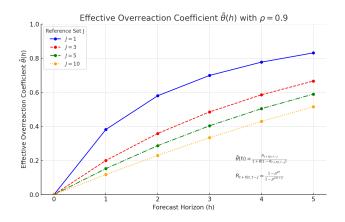
- ▶ Smooth DE applied to an AR(1) process: $x_{t+1} = \rho x_t + \epsilon_{t+1}$.
- ▶ Effective overreaction $(\tilde{\theta}_t)$ depends on the ratio of conditional variances:

$$R_{t+h|t,t-J} = \frac{1 - \rho^{2h}}{1 - \rho^{2(h+J)}}.$$

- Predictions:
 - Overreaction increases with forecast horizon h.
 - ► Subjective overconfidence (V_t^{θ} < V_t) typical in stationary settings.
 - Explains survey findings: stronger overreaction for long-term forecasts.

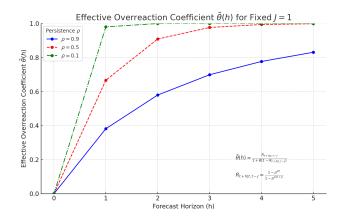
$R_{t+h|t,t-J}$ as a Function of J and h

- Vary J (e.g., J = 1, 3, 5, 10).
- Fix $\rho = 0.9$ and compute $\tilde{\theta}_t^{h,J}$.



$R_{t+h|t,t-J}$ as a function of ho and h

- Fixed J = 1.
- Vary ρ across [0.1, 0.9] and compute $\tilde{\theta}_t^{h,J}$.



Sign of
$$\frac{\partial R}{\partial \rho}$$
 for $h=1$

- ► For J = 1:
 - $ightharpoonup \frac{\partial R}{\partial \rho} < 0$: R decreases with ρ .
 - Sensitivity decreases at higher ρ values.
- ▶ For J = 2:
 - ▶ $\frac{\partial R}{\partial \rho}$ < 0: *R* continues to decrease with ρ .
 - Negative derivative becomes more pronounced for small ρ .
- ► For J = 3 and J = 4:
 - $ightharpoonup \frac{\partial R}{\partial \rho} < 0$: Similar trends observed.
 - Higher J values amplify the rate of decrease.
- Implication: For small horizons h, R is consistently decreasing in ρ across all J.

effective overreaction is decreasing in ρ

CONCLUSION

- Smooth DE provides a unified framework for overreaction and overconfidence.
- Empirical evidence supports uncertainty-dependent overreaction.
- Implications for macroeconomic modeling and policy design.

Overall:

- Very rich paper with lot of applications
- Quibble: I want more on each of the applications.

DERIVATION OF CONDITIONAL VARIANCE

- Starting from the AR(1) process: $x_{t+1} = \rho x_t + \epsilon_{t+1}$.
- ▶ Conditional variance of x_{t+h} at t:

$$\sigma_{t+h|t}^2 = \sigma_{\epsilon}^2 \sum_{i=0}^{h-1} \rho^{2i} = \sigma_{\epsilon}^2 \frac{1-\rho^{2h}}{1-\rho^2}.$$

Conditional variance at reference set horizon *J*:

$$\sigma_{t+h|t-J}^2 = \sigma_{\epsilon}^2 \frac{1 - \rho^{2(h+J)}}{1 - \rho^2}.$$

► Ratio of conditional variances:

$$R_{t+h|t,t-J} = \frac{\sigma_{t+h|t}^2}{\sigma_{t+h|t-J}^2} = \frac{1 - \rho^{2h}}{1 - \rho^{2(h+J)}}.$$

CONDITIONAL VARIANCE DYNAMICS

- ▶ Why conditional variance changes with horizon:
 - Persistence of shocks: Influence decays with $|\rho| < 1$.
 - ► Cumulative uncertainty: Accumulates over longer horizons.
 - ▶ Stationarity: Variance stabilizes as $h \to \infty$.
- $ightharpoonup R_{t+h|t,t-J}$ encapsulates how uncertainty varies with horizon and reference set.