Understanding Persistent ZLB:

Theory and Assessment[*]

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Abstract

Concerns of prolonged stagnation periods with near-zero interest rates and deflation have become widespread in many advanced economies. We build a theoretical framework that rationalizes two theories of low interest rates: *expectations-trap* and *secular stagnation* in a unified setting. We analytically derive contrasting policy implications under each hypothesis and identify robust policies that eliminate expectations-trap and reduce the severity of secular stagnation episodes. We provide a quantitative assessment of the Japanese experience from 1998:Q1-2020:Q4. We find evidence favoring the expectations-trap hypothesis and show that equilibrium indeterminacy is essential to distinguish between theories of low interest rates in the data.

"I believe that for the euro area there is some risk of *Japanification*, but it is by no means a foregone conclusion." — Mario Draghi (January, 2020).

1. INTRODUCTION

Since the global financial crisis of 2008-2009, concerns of prolonged near-zero interest rates and meager inflation became predominant across many advanced economies, most notably in Europe and the United States. Such concerns, dubbed as *Japanification*, relate to the decades-long stagnation of the Asian economy following the collapse of a real-estate bubble in the early 1990s. As a result, nominal interest declined to zero, and deflation emerged, leaving the central bank unable to fight recessions.[1]

Two predominant hypotheses rationalize interest rates near zero and inflation below the central bank's target. The first hypothesis is that of *expectation-driven* liquidity trap whereby pessimistic deflationary expectations become self-fulfilling in the presence of the zero lower bound (ZLB) constraint on short-term nominal interest rates (Benhabib, Schmitt-Grohé and Uribe, 2001, 2002). The second hypothesis is that of *secular stagnation* that entails a persistently negative natural interest rate constraining the central bank at the ZLB (Hansen, 1939; Summers, 2013). The theory and policy implications of these hypotheses have been developed in the literature using different frameworks.[2] Our unifying framework bridges this gap, facilitating analytical comparison and quantitative assessments.

In this paper, we build a theoretical framework that rationalizes *expectation-driven* liquidity traps and *secular stagnation* in a unified setting. We analytically derive contrasting policy implications under each hypothesis. For example, a payroll tax cut exacerbates the recession under secular stagnation but is expansionary in the expectations-trap model. Since conventional policy measures may have opposite

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[1]Financial Times, *"Japanification: investors fear malaise is spreading globally,"* August 26, 2019. ASSA Annual Meeting Panel Session: *"Japanification, Secular Stagnation, and Fiscal and Monetary Policy Challenges,"* January 2020.

[2]Expectations traps have been investigated using representative agent models (Benhabib et al., 2001; Schmitt-Grohé and Uribe, 2017), while secular stagnation is typically modeled using the overlapping generation framework (Eggertsson and Mehrotra, 2015; Eggertsson, Mehrotra, Singh and Summers, 2016).

implications, there is a need for robust policies to deal with liquidity traps (Bilbiie, 2018; Nakata and Schmidt, 2021). Our theoretical framework allows us to identify a set of robust policies that operate by placing a sufficiently high floor on the inflation rate. Imposing a lower bound on the inflation rate excludes the possibility of an expectations-driven liquidity trap. Policies that put a bound on deflation also reduce the severity of liquidity traps induced by exogenous declines in the natural rate of interest.

To obtain analytical results, we modify the textbook New Keynesian with endogenous discounting (Uzawa, 1968; Epstein, 1983). As a result, the representative agent's Euler equation in a steady-state equilibrium features a negative relationship between output and real interest rate, similar to a static IS curve. This modification breaks the tight connection between the natural interest rate and the discount factor, thus allowing for a permanently negative natural interest rate. On the supply side, we introduce tractable nominal frictions to obtain a linear relationship between inflation and output (Bhattarai, Eggertsson and Gafarov, 2019). Within this framework, we prove the existence of three steady-state equilibria. A *targeted-inflation* steady state at which the central bank can meet its inflation target, the economy is at full employment, and the nominal interest rate is positive. In addition, there are two liquidity trap steady states at which inflation is below the central bank's intended target. The level of output is below full employment, and the nominal interest rate is at the ZLB. Both liquidity trap steady states are the focus of our analysis.

The liquidity traps steady states arise in the presence of the ZLB constraint on short-term nominal interest rates. In one case, combined with long-run money non-neutrality, a shift in agent's inflation expectations makes the liquidity trap equilibrium self-fulfilling. For this reason we label it as *expectations-driven* liquidity trap. Alternatively, when prices are rigid, the economy can settle in a liquidity trap because of a permanent decline in the natural interest rate. In this case, the liquidity trap arises because of a change in the economy's fundamentals and not a shift in expectations. For this reason we label this situation as a *secular stagnation* steady-state or *fundamentals-driven* liquidity trap. In the absence of discounting, the natural rate of interest is constant and equal to

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the inverse of the household's discount factor, and the model cannot accommodate the *secular stagnation* hypothesis.

We use the Japanese experience from 1998-2020 as a laboratory to contrast the two hypotheses and offer the first quantitative assessment of expectation-driven liquidity traps versus secular stagnation. We embed our theory in a quantitative New Keynesian model and assess if a policymaker can use the data to discern the predominant hypothesis. Using bayesian prediction pools, we estimate the probabilistic assessment of the relevant model in real-time (Geweke and Amissano, 2011; Del Negro, Hasengawa and Schorfheide, 2016). Our quantitative analysis offers two main findings. First, we find evidence that Japan is more likely to be in an expectations-driven liquidity trap. Second, there is considerable real-time uncertainty in discerning between secular stagnation and the expectations-driven trap models, especially during Japan's first decade of near-zero interest rates.

We find that equilibrium indeterminacy is central to tilt our quantitative assessment in favor of the expectations-trap hypothesis. This result emerges because the dynamic properties of the ZLB equilibrium differ across the two narratives. Under secular stagnation, the ZLB equilibrium exhibits locally determinate dynamics. In contrast, the expectation traps model features locally indeterminate dynamics around the ZLB steady state. Thus the equilibrium dynamics are consistent with a multiplicity of stable paths. Because our quantitative analysis focuses on a long-lasting ZLB episode, equilibrium selection implies restrictions for the response of output and inflation to structural disturbances. Using procedures that maximize the model likelihood, we let the data select the best-fitting equilibrium. At the same time, our Bayesian procedure intrinsically penalizes the likelihood function for the presence of additional parameters in the expectations-trap equilibria (Schwarz, 1978; Lubik and Schorfheide, 2004).

What accounts for the better fit of the expectations-trap hypothesis in Japan? We find that the negative correlation between output growth and inflation in Japanese data is a central empirical moment for equilibrium selection and model fit. We find that the equilibrium dynamics around the secular stagnation steady state cannot deliver the observed negative correlation. With interest rates pegged at the ZLB, any shock that

generates a persistent increase in inflation rate lowers the real interest rate, increases consumption, and therefore output and inflation positively co-move. In contrast, local indeterminacy of the expectations-trap steady state implies that inflation can adjust in any direction. Our estimation procedure allows the data to pin down this response. The result is that expectation traps can generate an unconditional correlation of inflation and output close to that observed in the data.

We further investigate our empirical results along three dimensions. First, we restrict equilibrium selection using the minimal state variable (MSV) criterion (McCallum, 2004). We analytically show that the MSV solution implies a positive co-movement between inflation and output under expectations-trap. In this case, the prediction pool analysis cannot distinguish between secular stagnation and expectations-driven liquidity traps from the data. Second, we investigate the importance of non-fundamental i.i.d. shocks—known as sunspots—that emerge due to indeterminate model dynamics in the expectations-trap model. Our benchmark results index the multiplicity of equilibrium through the correlation between fundamental and sunspot shocks using the method of Bianchi and Nicolo (2021). We find that restricting the correlation between price-markup and sunspot shocks to non-negative values worsens the fit of the expectations-trap model and favors secular stagnation. Thus, using data to discipline equilibrium selection is central to our results. Third, we verify that the expectations-trap hypothesis generates a negative correlation between inflation and output in a calibrated medium-scale new Keynesian model, while the secular stagnation hypothesis does not. This final exercise implies that our analytical insights carry over to a wide class of models commonly used for policy analysis.

Relation to the literature. Our work complements the recent analyses of Michaillat and Saez (2021), Michau (2018), and Ono and Yamada (2018) who use the bonds-in-utility assumption to analyze a unique secular stagnation scenario.[3] We distinctly focus on understanding the differences between the two stagnation concepts analytically and quantitatively. These alternate micro-foundations essentially introduce discounting

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[3]Following Feenstra (1986) and Fisher (2015), a functional equivalence can be shown between using bonds in the utility and endogenous discounting.

into the Euler equation. Our paper jointly considers the two narratives of persistent ZLB and offers quantitative and analytical insights.

This paper is also related to the work by Mertens and Ravn (2014), Aruoba, Cuba-Borda and Schorfheide (2018), Bilbiie (2018), and Nakata and Schmidt (2021), who contrast expectations-driven and fundamental-driven liquidity traps using the standard Euler equation without discounting. Their setup can only accommodate a short-lived fundamentals-driven liquidity trap, while our modified Euler equation allows the possibility of secular stagnation as a competing hypothesis. Our paper is also complementary to Schmitt-Grohé and Uribe (2017), which analyzes the case of permanent expectations-driven liquidity traps. Coyle and Nakata (2019) characterizes optimal inflation target in the presence of expectations-liquidity traps. We build on these papers to show that policies that impose a lower bound on inflation preclude the expectations-driven traps. Benigno and Fornaro (2018)'s stagnation-trap, which focuses on the role of pessimism about the economy's growth rate, is complementary to the inflation pessimism we evaluate in this paper.[4]

Our framework allows agents in the model to expect ZLB episodes of permanent duration under both hypotheses. This feature stands in contrast to models that use transitory declines in the natural interest rate to generate ZLB episodes where agents' expectations have to be consistent with recovery to the full-employment steady state in the medium run (Bianchi and Melosi, 2017; Nakata, 2017; Nakata and Schmidt, 2019). Moreover, when modeling temporary liquidity traps, there is an equilibrium selection rule imposed, implicitly, by the assumed behavior of inflation at the end of the liquidity trap (Cochrane, 2017). We sidestep this issue by considering permanent ZLB episodes. Under secular stagnation, equilibrium dynamics are locally determinate. For expectations-driven liquidity traps, we use data to discipline equilibrium selection explicitly through the model's likelihood.

Our static prediction pool analysis is related to Lansing (2019) in which a model with endogenous switching regimes generates data from a time-varying mixture of

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[4]As they highlight, the possibility of a self-fulfilling expectations trap is more likely when multiple sources of pessimism (growth, deflation) are allowed in the same model. We leave the analysis of such models to future research.

two models. In this paper, we construct a time-varying probability on the predictive densities coming from two alternative models. Our paper also relates to Mertens and Williams (2021) that use the implications of changes in the natural interest rate on the distribution of interest rates and inflation in the options data from the U.S. financial markets, to discern between fundamentals- and expectations-driven liquidity. Instead, we explore the implications of changes in government spending, technology growth, and markups. We use the principal-agent decision framework of Del Negro et al. (2016) to identify the relevant hypothesis in Japan, combining the predictive densities derived from consumption, output and inflation data. The Bayesian nature of our approach allows us to measure the uncertainty about the contrasting hypotheses with a structural model.

A common theme of the papers that study expectations-driven liquidity traps is that policy implications may be opposite of the ones derived from fundamentals-driven liquidity traps. It becomes central, as a result, to assess what hypothesis is dominant in the data and come up with policies that can always be stabilizing.[5] Our robust policies prescribe a flattening of the aggregate supply curve to preclude the existence of expectations-driven liquidity traps. Our analysis is related to fiscal policy rules that prevent the decline of real marginal costs (Schmidt, 2016) or fiscal stabilization policies that eliminate expectation-traps (Nakata and Schmidt, 2021).[6] Similarly, research and development (R&D) subsidies advocated by Benigno and Fornaro (2018) that affect aggregate supply in an endogenous growth environment can eliminate expectations-driven liquidity traps.

Layout. Section 2 presents the main theoretical results of the paper. Section 3 presents the quantitative model to assess the Japanese experience, and Section 4 presents our quantitative findings. In Section 5 we investigate the role of equilibrium selection. Section 6 extends our analysis to a calibrated medium-scale DSGE environment. Section 7 concludes. All proofs and additional results are in the online appendix.

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[5]One can develop expectations-traps equilibria with similar comparative statics as the fundamentals-driven liquidity traps. Our analysis does not focus on those, as they may not generate policy dilemmas.

[6]Our minimum wage policy is also related to the work of Glover (2019) which introduces a tradeoff for employment stability through an allocative inefficiency.

2. Key insights in a two equations setup

We begin with a simple setup that analytically demonstrates our model's ability to entertain the expectations-driven liquidity trap and the fundamentals-driven liquidity trap. There are two main takeaways: a) a high degree of nominal rigidity may prohibit the existence of expectation traps, and b) a fundamentals-driven liquidity trap can exist for an arbitrarily long duration. The model features preferences with endogenous discounting and a particular variant of price setting by monopolistically competitive firms that generates analytical results. We characterize and formally define the different steady states: *targeted inflation* state, *secular stagnation* steady state, and the *expectations-driven trap* (also referred to as the BSGU steady state).

2.1. Household

Time is discrete and there is no uncertainty. For now, there is no government spending. A representative agent maximizes the following:

$$\max_{\{C_t, b_t\}} \sum_{t=0}^{\infty} \Theta_t \left[\log C_t - \omega h_t\right]$$
$$\Theta_0 = 1; \quad \Theta_{t+1} = \hat{\beta}(\tilde{C}_t)\Theta_t \ \forall t \geq 0$$

where Θ_t is an endogenous discount factor (Uzawa, 1968; Epstein and Hynes, 1983), C_t is consumption, $\tilde{C}_t$ is average consumption that the household takes as given, and h_t is hours. Such preferences are prominent in the small open economy literature (Schmitt-Grohé and Uribe, 2003).[7]

For tractability, we assume a linear functional form for $\hat{\beta}(\cdot) = \delta_t \beta \tilde{C}_t$, where $0 < \beta < 1$ is a parameter, $\tilde{C}_t$ is average consumption that the household takes as given, and $\delta_t > 0$ are exogenous shocks to the discount factor. In contrast to the conventional assumption in the endogenous discounting, we require $\hat{\beta}' > 0$. This is often referred to

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[7]What will be essential for our purpose is that there is a negative steady state relationship between consumption and real interest rate to generate discounting in the Euler equation. We can show a functional equivalence between preferences with endogenous discounting and a recent approach that employs bonds in utility (Michaillat and Saez, 2021).

as *decreasing marginal impatience* (DMI) in the literature.[8]

The household earns wage income $W_t h_t$, interest income on past bond holdings of risk-free government bonds b_{t-1} at gross nominal interest rate R_{t-1}, dividends Φ_t from firms' ownership and makes transfers T_t to the government. Π_t denotes gross inflation rate. The period by period (real) budget constraint faced by the household is given by $C_t + b_t = \frac{W_t}{P_t} h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + \Phi_t + T_t$. An interior solution to household optimization yields the consumption Euler equation, and intra-temporal labor supply condition

$$1 = \beta(\tilde{C}_t) \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] ; \quad \frac{W_t}{P_t} = \omega C_t$$

In equilibrium, individual and average consumption are identical, i.e. $C_t = \tilde{C}_t$. The Euler equation simplifies to:

$$1 = \delta_t \beta C_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$

Discussion of DMI:

The DMI assumption implies that as agents get wealthier, they become more patient. Several papers in the literature agree it is more realistic and intuitive to assume decreasing impatience rather than increasing impatience (Epstein and Hynes, 1983; Uzawa, 1968). Despite the realism, it is conventional to use increasing marginal impatience since it is consistent with the boundedness of wealth and stability in environments with an exogenous real interest rate. As shown by Das (2003), if the returns to savings diminish at a high enough rate, it is possible to guarantee stability in environments with decreasing marginal impatience.[9]

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[8]Our assumed functional form can be considered a special case of the more general functional: $\hat{\beta}(\cdot) = \delta_t \beta C_t^{\gamma_c}$. When $\gamma_c = 0$, this nests the textbook case of exogenous discounting. We consider $\gamma_c = 1$ for tractability.

[9]Following the insights of Das (2003), the decreasing marginal impatience assumption is consistent with the existence of stable equilibrium dynamics with capital accumulation. Results are available upon request.

2.2. Production

A perfectly competitive final-good producing firm combines a continuum of intermediate goods indexed by $j \in [0,1]$ using the CES Dixit-Stiglitz technology: $Y_t = \left(\int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$, where $1/\nu > 1$ is the elasticity of substitution across varieties. The price of the final good $P_t = \left(\int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$. Profit maximization gives The demand for intermediate good j can be derived from profit maximization as $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t$.

Intermediate good j is produced by a monopolist with a linear production technology: $Y_t(j) = h_t(j)$. Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs, and maximize profits $\Phi_t(j) = (1 + \tau)P_t(j)Y_t(j) - W_t h_t(j)$, where τ is a production subsidy.

We introduce nominal rigidities in the price-setting decision of these intermediate producers. To derive a tractable Phillips curve, we follow Bhattarai et al. (2019). A fraction α_p of the firms set prices flexibly every period to maximize per period profits $(1 + \tau)\frac{p_t^*}{P_t} = \frac{1}{1-\nu}\frac{W_t}{P_t}$. We set the production subsidy to eliminate markups so that (imposing $Y_t = C_t$), optimal price becomes $\frac{p_t^*}{P_t} = \omega Y_t$. The remaining fraction $1 - \alpha_p$ index their price p^n to the aggregate price level from previous period $\frac{p_t^n}{P_t} = \Gamma_t \frac{P_{t-1}}{P_t}$ where Γ_t is indexing variable to be defined shortly. The price index then becomes $P_t^{\frac{\nu-1}{\nu}} = \alpha_p (p_t^*)^{\frac{\nu-1}{\nu}} + (1 - \alpha_p)(p_t^n)^{\frac{\nu-1}{\nu}}$. With choice of $\nu = 1/2$ and $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - P_{t-1}) + P_{t-1}}$, we can derive the following relationship between gross inflation and aggregate output:[10]

$$\Pi_t = \kappa Y_t + (1 - \kappa \bar{Y})$$

where $\kappa \equiv \frac{\alpha_p}{1-\alpha_p}$ is slope of the Phillips curve and $\bar{Y}$ is steady state output in the absence of nominal rigidities (or zero price dispersion). We set $\omega = 1$ so as to normalize $\bar{Y} = 1$.

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[10]With $\Gamma_t = 1$, we get the neoclassical Phillips curve (See Ch 3.1 Woodford (2003)). Allowing for indexation to depend on current output allows us to derive Phillips curve that helps us derive insights analytically and make comparison with downward nominal wage rigidity assumption. More generally, the firms that are indexing prices follow an indexation rule $\Gamma(Y_t)$ where $\Gamma'(\cdot) > 0$ i.e. price reduction is increasing in unemployment. Similar analytical results with $\Gamma_t = Y_t$ can be shown.

2.3. Government and resource constraint

We close the model by assuming a government that balances budget every period and a monetary authority that sets the nominal interest rate on nominal risk-free one-period bonds using the following Taylor rule $R_t = \max\{1, (1 + r^*)\Pi_t^{\phi_\pi}\}$, where $1 + r_t^* \equiv \frac{1}{\delta_t \beta}$ is the natural interest rate, and $\phi_\pi > 1$. In equilibrium, bonds are in zero net supply. [11] We implicitly assumed the (gross) inflation target of the central bank to be one. The zero lower bound (ZLB) constraint on the short-term nominal interest rate introduces an additional non-linearity in the policy rule. Finally, we assume that the resource constraints hold in the aggregate: $C_t = Y_t$, and $b_t = 0$.

2.4. Equilibrium

The competitive equilibrium is given by the sequence of three endogenous processes $\{Y_t, R_t, \Pi_t\}$ that satisfy the conditions 1–3 for a given exogenous sequence of process $\{\delta_t\}_{t=0}^{\infty}$ and initial price level P_{-1}:

$$1 = \delta_t \beta Y_t \left[\frac{Y_t}{Y_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] \tag{1}$$

$$\Pi_t = \kappa Y_t + (1 - \kappa) \tag{2}$$

$$R_t = \max\{1, (1 + r_t^*)\Pi_t^{\phi_\pi}\} \tag{3}$$

where the exogenous sequence of natural interest rate is given by $1 + r_t^* \equiv \frac{1}{\delta_t \beta}$.

2.5. Non-stochastic steady state

We can represent the steady-state equilibrium with an aggregate demand block and an aggregate supply block.

Aggregate Demand (AD) is a relation between output and inflation and is derived by

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[11]The natural interest rate is the real interest rate on one-period government bonds that would prevail in the absence of nominal rigidities.

combining the Euler equation and the Taylor rule. The AD curve is given by

$$Y_{AD} = \frac{1}{\beta\delta} \begin{cases} \frac{1}{(1+r^*)\Pi^{\phi\pi-1}}, & \text{if R } > 1, \\ \Pi, & \text{if R } = 1 \end{cases} \qquad (4)$$

When the ZLB is not binding, the AD curve has a strictly negative slope; and it becomes linear and upward sloping when the ZLB constrains the nominal interest rate. Thus, the kink in the aggregate demand curve occurs at the inflation rate where the ZLB constrains monetary policy: $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi\pi}}$. When $1 + r^* > 1$, the kink in the AD curve occurs at an inflation rate below 1. For the natural interest rate to be positive, the patience parameter must be low enough i.e. $\delta < \frac{1}{\beta}$.

Aggregate Supply (AS) is given by $\Pi = \kappa Y + (1 - \kappa)$ in the steady state. When $Y = 1$, $\Pi = 1$. In this linear aggregate supply curve, the degree of nominal rigidity κ also determines the lower bound on inflation $= 1 - \kappa$. In the quantitative section, we will work with the standard forward-looking NK Phillips curve.

In this two-equation representation, we can characterize and prove the existence of different steady-state equilibria. Proposition 1 shows that a targeted steady state exists as long as the natural interest rate is positive.

Proposition 1. (Targeted Steady State): Let $0 < \delta < \frac{1}{\beta}$. There exists a unique positive interest rate steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$. It features output at efficient steady state, and inflation at the policy target. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

A steady state at which the central bank can meet its inflation target is *defined* as the *targeted-inflation steady state*. The presence of a targeted-inflation steady state is contingent on the natural interest rate and the monetary authority's inflation target. With a unitary inflation target, it must be the case that the natural interest rate is non-negative, which is implied by the assumption of $\delta < \frac{1}{\beta}$. In Proposition 2 we show that, a liquidity trap steady state (à la Schmitt-Grohé and Uribe, 2017) may jointly co-exist with the targeted steady state described above. However, with a flat enough

Figure 1: *Steady State Representation*

(a) *Expectations-Driven Trap*   **(b)** *Secular Stagnation*
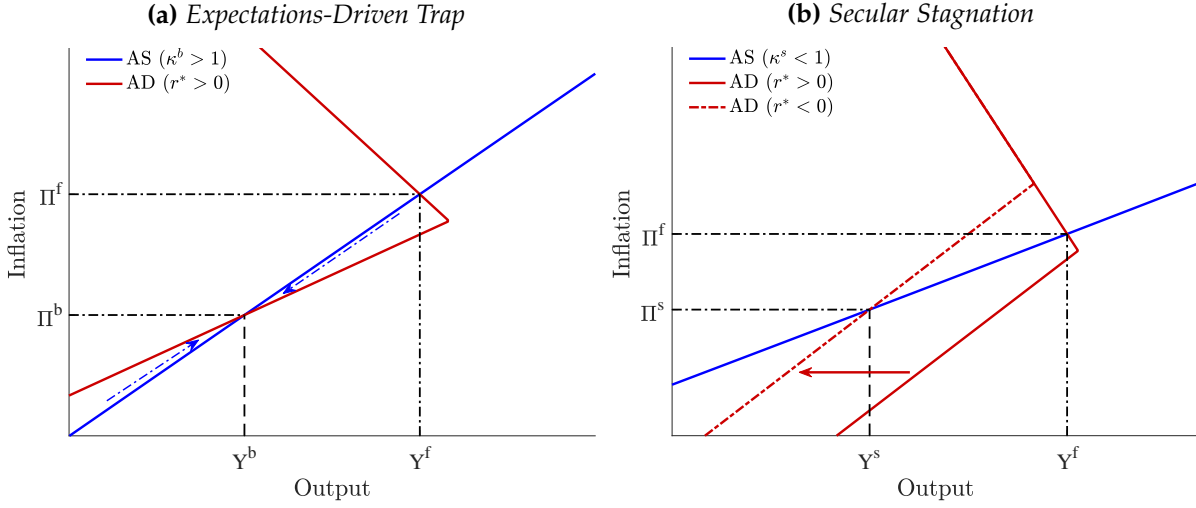
Phillips curve, a targeted steady state is the unique steady state in this economy. A high enough nominal rigidity prevents inflation from falling to levels such that self-fulfilling deflationary expectations do not manifest in the steady state.

Proposition 2. (Expectations trap steady state): Let $0 < \delta < \frac{1}{\beta}$. For $\kappa > 1$ (i.e. $\alpha_p > 0.5$) there exist two steady states:

1. The targeted steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$.

2. (*Expectations-driven* trap) A unique-ZLB steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. The local dynamics in a neighborhood around this steady state are locally indeterminate.

When prices are rigid enough, i.e., $\kappa < 1$, there exists a unique steady state, and it is the targeted inflation steady state. When prices are flexible $\alpha_p = 1$ ($\kappa \to \infty$), two steady states exist. A unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.

Panel a) in Figure 1 illustrates the unique targeted-steady state (Y^f, Π^f) and the unique ZLB steady state (Y^b, Π^b) with the modified Euler equation. We define the *expectations-driven* trap as the steady state with a positive natural interest rate, negative output gap, and deflation and in whose neighborhood the equilibrium dynamics are

locally indeterminate. Pessimistic inflationary expectations can push the economy to this steady state without any change in fundamentals.

We now consider the case where adverse fundamentals can push the economy to a permanent liquidity trap. If agents are sufficiently patient $\delta > \frac{1}{\beta}$, i.e., the natural rate of interest is negative, and the ZLB constrains monetary policy. In that case, the nominal interest rate is permanently zero while there is below-potential output and deflation in the economy. We characterize this possibility in Proposition 3.[12]

Proposition 3. (Secular Stagnation): Let $\delta > \frac{1}{\beta}$ and $\kappa < 1$. There exists a unique steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. It features output below the targeted steady state and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

See panel b) in Figure 1 for illustration of this unique steady state. The intersection of the solid red line(AD) with the solid blue line (AS) at (Y^f, Π^f) depicts the result of proposition 1, and the intersection of dashed red and blue lines at (Y^s, Π^s) depicts the liquidity trap steady state in proposition 3. We formally *define* the *secular stagnation steady state* as the steady state featuring negative output gap, zero nominal interest rate on short-term government bonds and exhibiting *locally determinate* equilibrium dynamics in its neighborhood. This local determinacy property is the main difference between the secular stagnation narrative and the expectations-driven narrative.

Note that the secular stagnation steady state exists in this model because of sufficient discounting in the modified Euler equation. Unlike the traditional new Keynesian model, an arbitrarily long ZLB episode driven by a negative natural rate can exist in equilibrium. In log-linearized new Keynesian models without discounting, *deflationary black holes* emerge as the duration of the temporary liquidity trap is increased, with inflation and output tending to negative infinity (Eggertsson, 2011). The solution remains bounded in our setup as the duration of ZLB is increased.

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[12]Note the efficient steady state is always an equilibrium of an economy without nominal rigidities.

2.6. Comparative Statics

The expectations-trap steady state and the secular stagnation steady state have different implications for policy. At the ZLB, a leftward shift in the aggregate demand graph lowers output and inflation under secular stagnation, but it is expansionary under expectations-trap. Similar policy reversals emerge due to a permanent increase in the nominal interest rate and positive supply shocks. We discuss comparative statics on the rise in the nominal interest rate and labor tax cuts. Because of the local determinacy properties of the secular stagnation steady state, the comparative static experiment is well-defined without the need for additional assumptions. In contrast, for an expectations trap, we assume that inflation expectations do not change drastically to push the economy to the full-employment steady state in response to the experiment.

2.6.1 Neo-Fisherian Exit

We now discuss the effects of a permanent increase in nominal interest rate. We model the policy as a permanent change in the intercept of the Taylor rule, a: $R^{new} = \max\{1 + a, a + R^* \left(\frac{\Pi}{\Pi^*}\right)^{\alpha_\pi}\} = a + R$, where a is increased to a positive number from zero, this policy simultaneously increases the lower bound on the nominal interest rate. It thus does not have any effect on the placement of the kink in the aggregate demand curve. it acts as a shifter for aggregate demand graph which is now given by: $Y_{AD} = \frac{1}{\beta\delta}\frac{\Pi}{a+R}$.

Given the inflation rate, an increase in a lowers the quantity of output demanded. This change induces deflationary pressures at the secular stagnation steady state. Lower inflation then increases the real interest rate gap and causes a further drop in production. In contrast, during an expectations trap, an increase in nominal interest rate anchors agents' expectations to higher levels of inflation, thus obtaining neo-Fisherian results (Schmitt-Grohé and Uribe, 2017; Uribe, 2021; Schmitt-Grohé and Uribe, Forthcoming).

2.6.2 Labor tax cuts

We now show the effects of a permanent reduction in payroll taxes financed by increase in lump-sum taxes. This policy reform is an aggregate supply shifter. Let workers' take home (real) wages be $(1 - \tau^w)\frac{W_t}{P_t}$ where τ^w are payroll taxes. In the steady state, aggregate supply curve is now given by: $\Pi = \kappa Y_{AS} + (1 - \frac{\kappa}{1-\tau^w})$.

Given the inflation rate, a reduction in τ^w increases workers' labor supply. The tax reduction induces deflationary pressures in the secular stagnation steady state, lower inflation increases the real interest rate gap and causes a further drop in output. This result corresponds to Eggertsson (2010)'s *paradox of toil*. In contrast, during an expectation trap, the increase in output dominates the deflationary pressures since the aggregate supply graph is steeper than the aggregate demand graph. Reductions in payroll taxes are expansionary in the case of an expectation trap. Similar results apply for structural reforms that reduce intermediate goods' markups (Eggertsson, Ferrero and Raffo, 2014).

2.7. A robust policy: appropriate price indexation

The disparate policy implications across the two steady states motivate the need for developing policies that may be robust to the source of recession. We introduce one such set of policy prescriptions to tackle these recessions. When a liquidity trap is transitory, inflationary pressures from a rise in price markups can be expansionary (Eggertsson, 2012). We use this insight to show that appropriate price indexation rule can increase output under secular stagnation and eliminate the expectation traps.

We prove that an appropriate indexation scheme can eliminate expectations-driven liquidity trap while also improving the output under secular stagnation. Recount that a fraction $1 - \alpha_p$ of firm index their price p^n to the aggregate price level from previous period $\frac{p_t^n}{P_t} = \Gamma_t \frac{P_{t-1}}{P_t}$. We now allow Γ_t to be somewhat general: $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - \lambda P_{t-1}) + P_{t-1}}$; $\lambda > 0$. The price Phillips curve is given by: $\Pi_t = \kappa Y_t + (\lambda - \kappa \bar{Y})$. We consider λ to be a policy tool that requires adjusting firms to index prices in a particular manner.

Proposition 4. Consider an indexation rule where the non-resetters index their prices to last period's price level with indexation coefficient: $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - \lambda P_{t-1}) + P_{t-1}}$. There does not exist expectations-driven liquidity trap $\forall\, \lambda > \kappa$. Output and inflation under secular stagnation are increasing in λ.

Under secular stagnation, higher values of λ act like higher price-markups, and increase output. A policy setting $\lambda > \kappa$ delivers a lower bound on deflation eliminating expectations-driven liquidity traps. Other policies that flatten the Phillips curve by strengthening labor unions during recessions can also preclude the possibility of expectation traps as well. A converse implication of this finding is that structural reforms that increase downward flexibility in prices make the economy vulnerable to swings in pessimistic expectations. We label this result as the *curse of flexibility*.[13]

2.8. Robust policy with downward nominal wage rigidity

We briefly discuss the robustness of our theoretical results with downward nominal wage rigidity. In particular, we discuss that minimum wage policies can also act like robust policies.[14] We make two changes to the model presented in Section 2. One, we assume an inelastic labor supply with a time endowment of one. Second, we assume nominal wages are downwardly rigid as in Schmitt-Grohé and Uribe (2017) i.e. $W_t \geq \gamma(h_t) W_{t-1}$, where $\gamma(0) = 1 - \kappa$ is a constant, $\gamma'(\cdot) > 0$, and $\gamma(1) > \delta\beta$. By choosing $\kappa < 1$, a policymaker can eliminate the expectations-driven liquidity trap. Since this policy lever is about the level of wages paid when employment approaches zero (an off-equilibrium limit), our model implies that policies similar to a *universal basic income*, can help fight expectations-driven recessions. We provide proof for these statements in Appendix F.

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[13]Our analysis does not imply that imposing a lower bound on deflation is enough to eliminate the expectations-trap steady state in more general settings. For example, in Benigno and Fornaro (2018) there is perfect downward nominal rigidity, but endogenous growth opens up the possibility of a stagnation trap. In Heathcote and Perri (2018), despite the presence of perfectly downward rigid wages, an expectation-trap steady state exists because of the precautionary savings motive.

[14]Furthermore, in Appendix G, we show that the expectations-driven trap can also emerge in the overlapping generations model of Eggertsson, Mehrotra and Robbins (2019) with appropriate wage flexibility.

3. A Quantitative Exploration

We now present a quantitative analysis based on a small-scale New Keynesian model that has been widely studied in the literature—see An and Schorfheide (2007). The critical difference is an Euler equation that features discounting. Relative to Section 2, we introduce a forward-looking Phillips curve and exogenous shocks to government spending, technology growth, and price-markups. Because the model is relatively standard, we focus on the log-linearized equilibrium conditions. We discuss the calibration that gives rise to the two ZLB hypotheses and discuss our strategy to take the model to the data.

3.1. Modified Euler Equation

The central piece that generates secular stagnation steady state is the modified Euler equation. As shown in equation 5, we use an specification that features an additive wedge, that arises from a bonds-in-utility specification, instead of the multiplicative wedge considered in Section 2.

$$1 = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1} \frac{R_t}{z_{t+1}\Pi_{t+1}} + \delta c_t \tag{5}$$

The term $\delta \geq 0$ corresponds to marginal utility of holding bonds (see Appendix C for derivation). We have three reasons for such choice: a) as $\delta \to 0$, this equation nests the textbook Euler equation as a special case; b) similar wedge can be derived from a wealth-in-utility argument (Michaillat and Saez, 2021), which is an alternate device to generate a persistent fundamentals-driven liquidity trap; and c) the additive wedge is related to models that emphasize flight-to-liquidity aspects of the Great Recession (Del Negro, Eggertsson, Ferrero and Kiyotaki, 2017).[15] We use the parameter δ to target empirical estimates of the natural interest rate in Japan. The calibration will depend on the particular hypothesis, and we describe our strategy shortly later in this section.

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[15]Some of the analytical results in Section 2 can be shown with this additive wedge. We discuss a comparison with the standard Euler equation in Appendix H.

3.2. Equilibrium Conditions

We approximate the equilibrium conditions around a permanent liquidity trap steady state. Let $\hat{y}_t$, $\hat{\pi}_t$ and $\hat{c}_t$ denote the log-deviations of output, inflation and consumption, respectively, relative to the steady state of interest. The following equations summarize the dynamics of consumption, inflation, output and the interest rate :[16]

$$\hat{c}_t = \bar{\mathcal{D}}\mathbb{E}_t(\hat{c}_{t+1} - \hat{R}_t + \hat{\pi}_{t+1} + \hat{z}_{t+1}) \tag{6}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \bar{\varphi}\mathbb{E}_t(\hat{g}_{t+1} - \hat{g}_t) + \bar{\kappa}\left[\left(\frac{1}{\eta} + 1\right)\hat{y}_t - \hat{g}_t\right] + \bar{\lambda}\hat{v}_t \tag{7}$$

$$\hat{y}_t = \hat{c}_t + \hat{g}_t \tag{8}$$

$$\hat{R}_t = 0 \tag{9}$$

The coefficients entering equations 6 and 7 are functions of the following structural parameters: $\bar{\mathcal{D}} = \frac{\beta}{\beta + \bar{\pi}z\delta\bar{c}}$; $\bar{\kappa} = \frac{(1-\nu)g\bar{c}\bar{y}^{1/\eta}}{\nu\phi(2\bar{\pi}-\pi_*)\bar{\pi}}$; $\bar{\varphi} = \frac{\bar{\pi}-\pi_*}{2\bar{\pi}-\pi_*}$; and $\bar{\lambda} = \frac{1-\phi(\bar{\pi}-\pi_*)\bar{\pi}(1-\beta)}{\phi(2\bar{\pi}-\pi_*)\bar{\pi}}$, where ϕ and δ correspond to the cost of price adjustment and the marginal utility of bonds, these are the only structural parameters specific to each model. The common parameters include β, the household's discount factor; η, the Frisch labor supply elasticity; $1/(1-\nu)$, the steady-state markup; z, the long-run growth of the economy; $1 - 1/g$, the steady-state share of government spending relative to output; and π_*, the central banks' inflation target. The rest of the terms correspond to the steady-state value of consumption ($\bar{c}$), output ($\bar{y}$), inflation ($\bar{\pi}$), technology growth (z), government spending (g). Variables with an over-line denote values in liquidity trap steady state.

We obtain equation 6 from log-linearizing the modified Euler equation 5. It resembles the dynamic IS relationship of the standard New Keynesian model but modified by the discount coefficient $\bar{\mathcal{D}}$. Since $\delta > 0$, the discounting coefficient $\bar{\mathcal{D}} < 1$. Discounting dampens the consumption response to changes in the ex-ante real interest rate. An increase in the preference for bonds, lower steady-state inflation, and lower long-run growth rate increase the discounting in the Euler equation conditional on $\delta > 0$. We

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[16]We denote all liquidity trap steady state parameters by $\bar{x}$. Appendix D provides the derivation of the log-linearized equations.

introduce shocks to growth rate of technology, z_t, to replicate the real interest rate observed in Japan.

Equation 7 is the forward-looking Phillips curve that depends on expected inflation and marginal costs $((1/\eta + 1)\,\hat{y}_t - \hat{g}_t)$, the growth in government expenditure $(\hat{g}_{t+1} - \hat{g}_t)$ and the price-markup shock $\hat{v}_t$. To generate this relationship we assume quadratic adjustment costs in price setting (Rotemberg, 1982). The growth in government expenditure appears in this equation because of we log-linearized the equation away from the targeted-inflation steady state.

Equation 8 is the resource constraint of the economy that specifies a time-varying wedge between consumption and output, corresponding to exogenous shocks in government spending. Equation 9 indicates that the economy operates under an interest rate peg. We can derive this equation from any policy rule in which the central bank faces an effective lower bound constraint.[17]

Exogenous shocks. There are three exogenous process in the model: (i) government expenditure g_t, (ii) the growth rate of productivity z_t, and (iii) changes in the inverse demand elasticity for intermediate goods, v_t, that translates into time-varying price markup. We assume that these exogenous components follow an AR(1) process around their deterministic mean (g, z, v), with persistence, ρ_g, ρ_z, ρ_v and innovations $\epsilon_g, \epsilon_z, \epsilon_v$, that are normally distributed with mean zero and standard deviations $\sigma_g, \sigma_z, \sigma_v$, respectively.

3.3. Calibration

Table 1 summarizes the parameters that are common across models. We fix the discount factor β to 0.942 consistent with structural estimates of Galı and Gertler (1999). While this estimate is lower than the standard calibrated value of 0.99 in the literature, a low β is needed for the model to generate a positive natural interest rate in the presence of a bond premium. In studies that have estimated the discount rate using field and laboratory experiments, the estimates for β are dispersed but point to high

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[17]As in Section 2, we assume that government runs a balanced budget every period. There is zero net supply of government bonds in the economy. We rebate the adjustment costs to the household to reduce the role of high adjustment costs in driving equilibrium dynamics.

discount rates. Surveys of these studies are conducted in Frederick, Loewenstein and O'Donoghue (2002, table 1), and Andersen, Harrison, Lau and Rutström (2014, table 3). Michaillat and Saez (2021) choose an annual discount rate of 43% from the median value of these estimates.

Other standard parameters include the Frisch labor supply elasticity fixed at 0.85 (Kuroda and Yamamoto, 2008). We set the (inverse) elasticity of demand for intermediate goods, v, to 0.1 to generate a steady-state markup of 11%. Japan did not officially adopt an inflation target until 2013Q2, but the inflation rate averages 1.1% in the two decades before entering the ZLB. Thus we assume the central bank was pursuing an inflation target of 1% and use that target rate as the reference value for price adjustment($\Pi^* = 1.0025$).[18] We determine the values of z such that the model matches the average output growth over the estimation sample. The steady-state value of government spending matches a consumption-output ratio of 58% in the Japanese data.[19]

Table 1: *Fixed Parameters*

β	η	v	Π^*	g	$100\ln(z)$
Discount factor	Inverse Frisch elasticity	Price s.s. markup	Inflation target	Government spending parameter	TFP growth rate
0.942	0.85	0.1	1.0025	1.72	0.56

The remaining parameters δ and ϕ are chosen to jointly match targets for the natural interest rate and average inflation in Japan.[20] For the natural rate, we adopt two different targets depending on the regime. Under secular stagnation, we choose an annual rate of -1.1%. This choice is based on two studies by Fujiwara, Iwasaki, Muto, Nishizaki and Sudo (2016) and Iiboshi, Shintani and Ueda (2018) that separately estimate a series for the natural rate of interest in Japan based on Laubach and Williams

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[18]Our results are robust to choosing a zero inflation target as well.

[19]In our data, government expenditure is residually a combination of investment, net exports, and government spending. As an alternative, it is straightforward to make g_t in the model track actual government spending in the data by defining consumption appropriately. Results are available upon request.

[20]It may be worth noting that multiple steady states at zero lower bound may coexist if the Phillips curve is sufficiently non-linear. Alternately, it may be possible to model the possibility of secular stagnation steady state coexisting with the full-employment steady state as in Eggertsson and Mehrotra (2015) with a sufficiently high inflation target. We do not explore those exercises here.

Table 2: *Calibrated Parameters*

	δ Euler Equation Wedge	ϕ Adjustment cost parameter	
Secular Stagnation	0.1132	4825	
Expectations Trap	0.1088	2524	

	Natural Rate	Inflation	Output Gap
Secular Stagnation	-1.1	-1.06	-7.5
Expectations Trap	0	-1.06	-4.5

Notes: The table shows the parameter values of the model for the baseline calibration.

(2003). They find that the quarterly estimate was often -0.5% since the late 1990s and -2% at the lowest level. In contrast, we calibrate the expectations-trap steady state to imply an annualized long-run real interest rate of 0%. The calibration implies an inflation rate of -1.06% for both steady states, which is the average inflation rate in Japan over the period 1998Q4 – 2012Q4. Our calibration results in a somewhat larger value of the price adjustment parameter, ϕ, compared with econometric estimates of DSGE models for Japan (Iiboshi et al., 2018). Nonetheless, within the range of plausible estimates found in the literature—see Aruoba, Bocola and Schorfheide (2017) Finally, the implied output gap is close to the estimates of 5% in Hausman and Wieland (2014). Table 2 summarizes the parameters that are specific to each model.

3.4. Equilibrium dynamics

Similar to our analytical model in Section 2, the local dynamics of secular stagnation and expectations-trap are pretty different. The following proposition formalizes this result.

Proposition 5. (Local Determinacy) Assume $\beta < 1$. The system 6 - 9 is locally determinate if and only if $\frac{\bar{\pi}z\delta_m\tilde{c}}{\beta} > \frac{1+\eta}{\eta(1-\beta)}\kappa_m$.

The secular stagnation steady state exhibits local-determinacy. This requires a sufficiently flat Phillips curve (low $\bar{\kappa}$), or high enough discounting (high δ).[21] Our calibration satisfies this restriction. In contrast, an expectations-driven liquidity trap

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[21]Definition 1 in Michaillat and Saez (2021) impose a similar restriction for obtaining a permanent fundamentals-driven ZLB episode.

has locally indeterminate dynamics, with low enough discounting or sufficiently steep Phillips curve.

Equilibrium indeterminacy carries two distinguishing features (Lubik and Schorfheide, 2004; Canova and Gambetti, 2010). One is the presence of additional endogenous variables affecting the dynamics of agents' forecast errors. Second is the possibility of extraneous innovations, known as sunspot shocks (ζ_t), that is not part of the original description of agents' optimization problems.

To characterize the multiplicity of equilibrium, we apply the methods in Bianchi and Nicolo (2021). This allows lagged inflation expectations $\mathbb{E}_{t-1}\hat{\pi}_t$ to enter as an additional state variable in agent's decision rules, and sunspot shocks to affect the inflation forecast error, $\eta_\pi \equiv (\hat{\pi} - \mathbb{E}_{t-1}\hat{\pi}_t) = \zeta_t$. We estimate the correlation between sunspot and structural innovations $(\epsilon_g, \epsilon_z, \epsilon_\nu)$, thus we select the best-fitting equilibrium based on observed Japanese data.

3.5. Data and Estimation

Conditional on our calibration of steady state parameters, we are left to estimate the vector of parameters $\theta = [\rho_g, \rho_z, \rho_\nu, \sigma_g, \sigma_z, \sigma_\nu]'$ for the secular stagnation model. For the expectations-trap model, in addition to the parameters in θ, we estimate the standard deviation of the sunspot shock and the correlation between the structural and the sunspot shocks, denoted by $\rho_{x,\zeta}$, for $x = \{z, g, \nu\}$. Because our model is linear we can construct the true-likelihood and use a standard Bayesian approach to estimate the parameters of the model. We obtain draws from the posterior distribution by a single-block random walk Metropolis–Hastings (RWMH) algorithm (An and Schorfheide, 2007). Appendix D reports the prior distribution of parameters.

Data. For parameter estimation, we use quarterly data on output growth, consumption growth, and GDP deflator-based inflation rate in Japan during the period 1998:Q1 to 2012:Q4.[22] We focus on this sample period for two reasons. First, from 1995 to 1998 the Bank of Japan (BOJ) held the monetary policy rate at 0.5%, while struggling to

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[22]Our findings are robust to using data from 1998:Q1-2020:Q1 in estimation. We use the longer sample for our assessment of the mechanism in section 5.

boost the economy amidst turmoil in domestic and international financial markets (Ito and Mishkin, 2004). We start our analysis in 1998 to parallel the assumption in our model that the economy starts at the ZLB and agents expect near-zero interest rates for a prolonged period. The BOJ lowered its policy rate to zero in the first quarter of 1999, and it remained between 0% and 0.5%. Consequently, we consider the economy to be at the ZLB for the entire period. Second, in 2013, the BOJ introduced a new monetary policy program that included an explicit inflation target, asset and bond purchase programs as well as considering negative nominal interest rates (Gertler, 2017). None of these policies are explicitly modeled in our framework.

Measurement. To match the model to the data, we construct model implied output (Δy_t^o), consumption growth (Δc_t^o), as quarter-on-quarter percentages, and inflation measured in annualized percentages (π_t^A). We link the observed data series to the model counterparts through the following system of measurement equations:

$$\Delta y_t^o = 100 \log(z) + 100 \left(\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \right)$$
$$\Delta c_t^o = 100 \log(z) + 100 \left(\hat{c}_t - \hat{c}_{t-1} + \hat{z}_t \right)$$
$$\pi_t^o = 400 \log(\bar{\pi}) + 400 \hat{\pi}_t$$

4. RESULTS

4.1. Estimated parameters

Table 3 summarizes the estimated posterior distribution of parameters that fit the respective model to Japan's output, consumption, and inflation data in our sample. The marginal prior and posterior distributions for the estimated parameters are tabulated in the appendix. The posterior estimates for the common parameters are remarkably similar across model specifications. For the expectation traps model, the standard deviation of the sunspot shock is statistically different from zero and with a magnitude similar to that of the technology shock. In this specification, the estimated correlation between the fundamental and sunspot shocks varies substantially. The data favors a robust positive correlation between markup and sunspot shocks while picking up a

small correlation of the sunspot shock with the other two fundamental shocks.

Table 3: *Posterior DSGE estimates*

Parameters	Description	$\mathcal{M}_b$: Expectations Trap	$\mathcal{M}_s$: *SecularStagnation*
ρ_g	Persistence gov. spending shock	0.9562	0.8620
		[0.9197, 0.9947]	[0.7990, 0.9338]
ρ_v	Persistence markup shock	0.1571	0.1433
		[0.0334, 0.2718]	[0.0387, 0.2465]
ρ_z	Persistence technology. growth shock	0.7358	0.7824
		[0.5093, 0.9443]	[0.5873, 0.9383]
σ_g	Std dev. gov. spending shock	0.0045	0.0045
		[0.0039, 0.0051]	[0.0039, 0.0051]
σ_v	Std dev. markup shock	0.0172	0.0026
		[0.0021, 0.0026]	[0.0021, 0.0030]
σ_z	Std dev. markup shock	0.0040	0.0037
		[0.0015, 0.0066]	[0.0014, 0.0065]
		Sunspot shock parameters	
σ_ζ		0.0030	-
		[0.0026, 0.0035]	-
$\rho(\epsilon_z, \epsilon_\zeta)$		-0.0476	-
		[-0.1870, 0.0999]	-
$\rho(\epsilon_v, \epsilon_\zeta)$		0.9559	-
		[0.9263, 0.9853]	-
$\rho(\epsilon_g, \epsilon_\zeta)$		-0.2130	-
		[-0.2831, -0.1410]	-
$\log\left[p\left(Y^T \right) \right]$	Log-data density	-279.80	-284.80

Notes: The estimation sample is 1998:Q1 - 2012:Q4. We use $Y^T = [y_1, \ldots, y_T]$ to denote all the available data in our sample. For each model we report posterior means and 90% highest posterior density intervals in square brackets. All posterior statistics are based based on the last 25,000 draws from a RWMH algorithm, after discarding the first 25,000 draws.

4.2. Model Fit

We assess model fit using log-data density comparisons. The advantage of using log-data densities is that it intrinsically penalize the likelihood function for the presence of additional parameters. This approach is commonly used in the DSGE literature for model comparison (Lubik and Schorfheide, 2004; Justiniano and Primiceri, 2008; Cúrdia, Ferrero, Ng and Tambalotti, 2015; Ascari, Bonomolo and Lopes, 2019).

The last row in Table 3 shows that the log-data density favors the expectations-trap

hypothesis in terms of overall fit. To gauge the difference in fit, we construct Bayes factors of expectations-traps relative to secular stagnation, $\mathcal{F} = p(Y^T|\mathcal{M}_b)/p(Y^T|\mathcal{M}_s)$. As test statistic we compute $2 \times \log(\mathcal{F})$ because it resembles the familiar likelihood-ratio test. In our estimation, we find that this test statistic is equal to 10, implying "strong" evidence in favor of the expectations-trap hypothesis over secular stagnation according to standard criteria (Kass and Raftery, 1995).[23]

In our application, the sunspot shock and the correlation parameters necessary to select equilibria in the expectations-trap model come at a cost from the perspective the log-data density.[24] Nonetheless, one may still be concerned that the expectations-trap model always "edges-over" secular stagnation because of the multiplicity of equilibria. To allay this concern, we conduct an exercise where we simulate data from the secular stagnation model using the parameters in Table 3. Then, we re-estimate both models on simulated data and conduct a model comparison. We find that $2 \times \log(\mathcal{F})$ is equal to -28, which indicates that when data comes from secular stagnation, our estimation procedure finds "very strong" evidence in its favor.

4.3. Impulse Response Functions

We now illustrate the difference in dynamics of expectation traps and secular stagnation through impulse responses. Figure 2 shows the output and the inflation impulse response functions after a one-time unanticipated shock to government expenditure, aggregate productivity growth, and price-markups.

For the secular stagnation model, shown by red dashed lines, all structural shocks lead to a transitory increase in inflation. With nominal interest rates at the ZLB, higher inflation translates into lower real interest rates and higher aggregate demand through the inter-temporal substitution channel. Thus, all shocks induce a positive conditional correlation between inflation and output.

The dynamics under expectations-trap are depicted with solid blue lines. Temporary

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[23]According to Kass and Raftery (1995), values of $2 \times \log(\mathcal{F})$ above 10 can be considered very strong evidence in favor of model 1. Values between 6 and 10 represent strong evidence, between 2 and 6 positive evidence, while values below 2 are "not worth more than a bare mention."

[24]See the discussion in footnote 11 of Lubik and Schorfheide (2004).

increases in government spending or technology growth lower inflation. To understand how equilibrium indeterminacy affects the impact response of inflation, consider the expectation error $\eta_t = \hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t$. Before any shock realizes, at time $t = 0$, the economy is at steady state with $\pi_0 = \bar{\pi}$ and $\mathbb{E}_0\hat{\pi}_0 = 0$. Then, $\hat{\pi}_1 = \eta_1$, which in turn depends on the correlation of the sunspot and the structural shocks. If structural shocks have a negative correlation with η, inflation will fall. At the zero lower bound, the decline in inflation raises the real interest rate and weighs on aggregate demand.

Technology shocks also have a negative correlation with sunspot shocks. Hence, a positive technology shock leads to an initial decline in inflation. However, a higher real interest rate does not fully offset the effects of higher future productivity and output increase on impact. Nevertheless, output declines as deflationary expectations set in and the real interest rate rises due to lower inflation.

Price-markup shocks increase inflation in about the same magnitude in both models. However, in the expectations-trap model, inflation rises on impact due to the correlation with the sunspot shock. After the initial jump, expected inflation declines as the transitory increase in realized inflation cannot persist in the expectations-trap steady state. Lower inflation expectations depress aggregate demand and generate a negative (conditional) correlation between inflation and output.

## 4.4. Expectations trap or secular stagnation?

We now compare the relative importance of the two competing hypotheses in explaining the persistent liquidity trap episode in Japan. We use static prediction pools, as in Geweke and Amissano (2011) and Del Negro et al. (2016), that rely on predictive densities to construct recursive estimates of model weights. These time-varying model weights can be interpreted as a policymaker's views on the most relevant model using the information available in real-time.

We consider a policymaker that has access to the sequence of one-period-ahead predictive densities $p(y_t|y_{1:t-1}, \mathcal{M}_s)$ under secular stagnation and $p(y_t|y_{1:t-1}, \mathcal{M}_b)$ under the expectations-trap hypothesis.[25] We are interested in constructing an estimate

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[25]The predictive density is constructed sampling from the posterior distribution of the DSGE

Figure 2: *Impulse Responses: Expectations Trap vs Secular Stagnation*
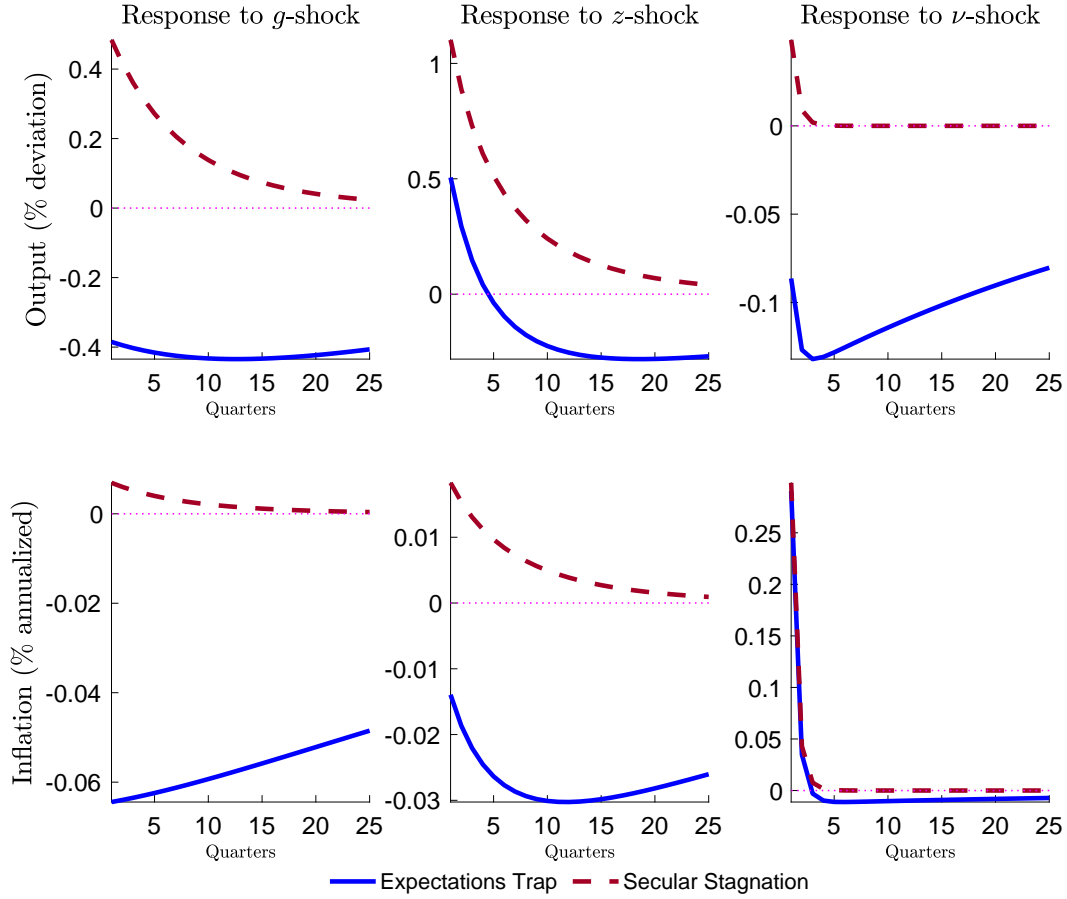
Notes: Impulse responses to one standard deviation shocks. All responses are computed at the posterior mean of the estimated parameters. The blue solid line corresponds to the expectations-driven traps model. The red dashed line corresponds to the secular stagnation model.

of the model weight, λ, that pools the information of each individual model:

$$p\left(y_t|\lambda,\mathcal{P}\right) = \lambda p\left(y_t|y_{1:t-1},\mathcal{M}_b\right) + (1-\lambda)p\left(y_t|y_{1:t-1},\mathcal{M}_s\right), \quad 0 \leq \lambda \leq 1 \qquad (10)$$

Where $p\left(y_t|\lambda,\mathcal{P}\right)$ is the predictive density obtained by pooling the two competing models for a given weight λ and pool $\mathcal{P} = \{\mathcal{M}_b,\mathcal{M}_s\}$. The policymaker is Bayesian and has a prior density $p(\lambda|\mathcal{P})$ of the weight assigned to each model in the pool. The posterior distribution of the model weights, $p(\lambda|\mathcal{I}_t^P,\mathcal{P})$, can be updated recursively

parameters of the baseline model of Section 4 and averaging the predictive densities across draws.

conditional on the information available to the pool in the previous period $\mathcal{I}^{\mathcal{P}}_{t-1}$:

$$p(\lambda|\mathcal{I}^{\mathcal{P}}_t, \mathcal{P}) \propto p\left(y_t|\lambda, \mathcal{P}\right) p(\lambda|\mathcal{I}^{\mathcal{P}}_{t-1}, \mathcal{P}) \qquad (11)$$

We estimate the posterior distribution in equation 11 recursively, starting in 1998:Q1. The estimated model weights are shown in Figure 3 together with posterior credible sets to capture model and parameter uncertainty. The Japanese data imply roughly similar weights on both models in the early part of the sample and through the early 2000s. Afterward, the data lean in favor of the specification $\mathcal{M}_b$, indicating a better fit of the expectations-trap hypothesis. Uncertainty about the model weight's posterior distribution is substantial but decreases later in the sample as more information favoring the expectations-trap model accumulates. From 2015, the data put at least 90% weight on the expectations-trap hypothesis as the best-fitting explanation.

Figure 3: *Model Weights: Expectations Traps vs Secular Stagnation*
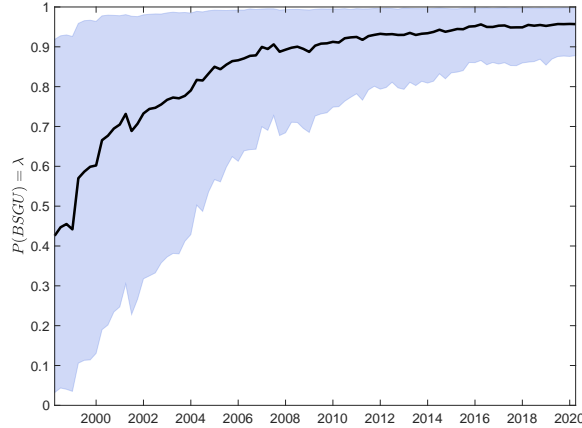
Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

5.   Inspecting the Mechanism

As a result of local-indeterminacy, inflation in the expectations-trap model is free to jump away from its steady state in response to shocks. In section 4, we introduced i.i.d. sunspot shocks as exogenous shifters of inflation's expectational errors to select among the multiple self-confirming equilibria. Moreover, we allowed sunspot shocks to be

correlated with structural shocks in the model. We now investigate why sunspot shocks matter? and why the correlation with structural shocks is essential for our results? We find that equilibrium indeterminacy relaxes the tight co-movement between inflation and output that afflicts the secular stagnation model.

5.1. Why Sunspots?

We construct the *Minimal State Variable* (MSV) solution corresponding to the expectations-trap model. This concept is common to select among solutions in models with equilibrium indeterminacy (Aruoba et al., 2018; Lansing, 2019). The idea is to restrict inflation and output dynamics to functions of the vector of fundamental state variables. In our model, the MSV criterion implies that the expectational error of inflation is determined by the exogenous disturbances $\hat{z}, \hat{g}, \hat{v}$ and will not respond to other endogenous variables nor i.i.d. sunspot shocks. The following proposition formalizes the MSV solution concept and derives an analytical expression for our application:

Proposition 6. (MSV Solution). Let $\mathbb{X} = \{g, v, z\}'$ collect all fundamental state variables of the model, and let a and b be vectors of unknown coefficients. Under the MSV criterion, a solution of the following form exists:

$$\hat{y}(\mathbb{X}) = a_1^j \hat{z}_t + a_2^j \hat{g}_t + a_3^j \hat{v}_t; \quad \hat{\pi}(\mathbb{X}) = b_1^j \hat{z}_t + b_2^j \hat{g}_t + b_3^j \hat{v}_t$$

for $j \in \{$Secular Stagnation, Expectations Trap$\}$. The coefficients (a_i^j, b_i^j) are reported in Appendix D.5.

To illustrate the MSV solution, figure 4 shows the impulse response of output and inflation to a markup shock. Naturally, for the secular stagnation model, the IRFs are identical to those shown in Figure 2. For the expectations-trap model, the MSV criterion rules out sunspots. The figure clarifies that the MSV solution is different from assuming a zero correlation between sunspot and structural shocks. The latter restricts inflation's forecast errors to zero. Thus inflation does not jump in response to structural shocks. In contrast, the MSV solution induces a contemporaneous correlation between inflation and output in response to structural shocks.

Under the MSV solution, the conditional correlation between inflation and output in the expectations-trap model is positive and similar to that in the secular stagnation model. This result shows that equilibrium selection directly affects the response of inflation to fundamental shocks and transmits through the economy through the inter-temporal substitution channel.

Price-markup shock increases realized as well as expected inflation. At the ZLB, a lower real interest stimulates aggregate demand. The following proposition analytically proves that this positive correlation result holds for all fundamental shocks in our model.

Figure 4: *Impulse Response Functions: MSV solutions*
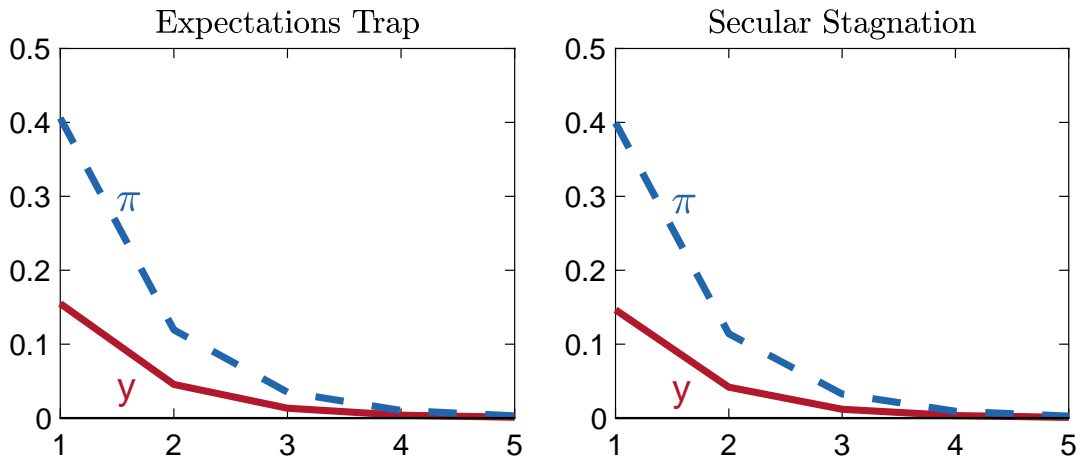
Notes: Impulse responses to one standard deviation shock to price markups. All responses are computed at the posterior mean of the estimated parameters. The red solid line corresponds to inflation. The blue dashed line corresponds to output.

Proposition 7. (Positive Correlation). Consider the MSV solution for BSGU and Secular Stagnation models. Output and inflation are positively correlated conditional on shocks to TFP growth rate $\hat{z}_t$ and price-markup $\hat{v}_t$. If $\bar{\kappa} < \frac{\bar{\pi} z \bar{c} \delta (1-\beta)}{\bar{R} \beta}$, the unconditional correlation of output growth and inflation is positive.

The intuition behind proposition 7 comes from the simple AS-AD graphs in section 2. Price-markup shocks and technology growth shocks shift only one schedule

simultaneously– either the Phillips curve or the Euler equation. These shifters un-equivocally induce a positive correlation between inflation and output. As long as the Phillips curve is sufficiently flat (low enough $\bar{\kappa}$ relative to other structural parameters), the government spending shock also induces a positive correlation between inflation and output. This restriction is satisfied by the parameters in our empirical exercise. Consequently, Proposition 7 implies that the expectations-trap model under the MSV criterion and the secular stagnation model yield qualitatively similar predictions.

The quantitative consequence of this proposition is that the likelihood of these two models is similar, thus making it challenging to identify the best-fitting model from the data. As we show in the Appendix D.5, the mean of the posterior distribution model weights $p(\lambda|.)$, is essentially constant at 50%. Our result shows that the equilibrium-multiplicity of solutions is not only theoretically relevant (Cochrane, 2011) but also quantitatively important.

5.2. Which Equilibrium?

The correlation between sunspot and structural shocks is crucial because it characterizes all admissible solutions under indeterminacy (Bianchi and Nicolo, 2021). Moreover, it helps discipline equilibrium selection using data.[26] Hence it is easy to study the equilibrium path that generates the quantitative success of the expectations-trap model by examining the correlation structure of the estimated model.

To understand which of the multiple equilibrium paths plays a role in discriminating between expectation traps and secular stagnation, we re-estimate the prediction pool under four restrictions on the correlation between the sunspot and fundamental shocks. Figure 5 displays the estimated time-varying model weights. Panel (a) sets to zero the correlation between the sunspot and productivity shocks. Panel (b) sets the correlation between the sunspot shock and the government expenditure shock to zero. Panel (c) sets the correlation of the sunspot shock and markup shock to zero. Lastly, panel (d) sets all the correlations to zero.

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[26]We focus on all linear rational expectations solutions. It is also possible to construct non-linear rational expectations solutions under indeterminacy—see Ascari et al. (2019).

When price-markups and sunspot shocks are uncorrelated as in panels (c) and (d), secular stagnation explains the Japanese experience better. Conversely, when the correlation between sunspot with productivity or government spending shocks is zero, we obtain results similar to our baseline specification, with expectation traps as the more likely explanation for the Japanese experience. We infer from these results that the correlation between price markups and sunspots shocks is crucial for the expectations trap hypothesis because it allows the model to generate a negative correlation between inflation and output. We elaborate on this empirical correlation in Section 5.3. This finding echoes the evidence presented in Wieland (2019) who shows that the oil supply shocks in Japan, which are equivalent to price markup shocks in our model, generate a negative correlation between inflation and output at the ZLB.

Figure 5: *Model Weights: Role of Sunspots*
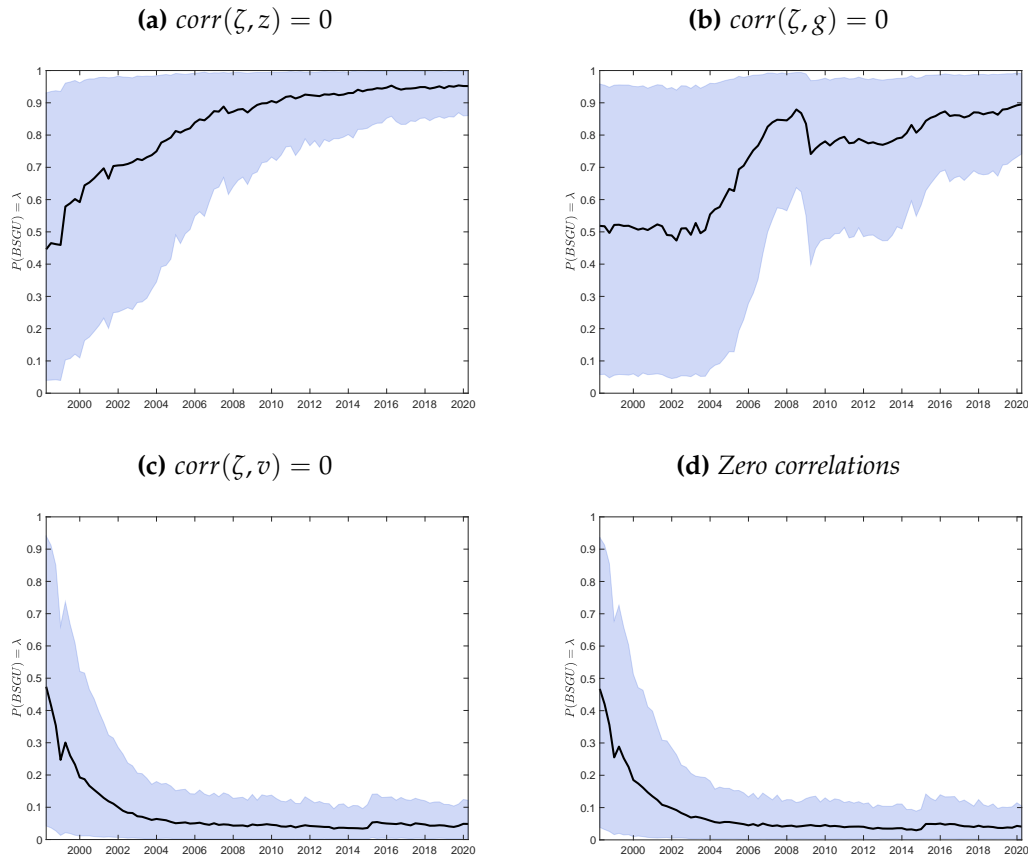
(a) $corr(\zeta, z) = 0$

(b) $corr(\zeta, g) = 0$

(c) $corr(\zeta, v) = 0$

(d) *Zero correlations*

Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

5.3. Inflation-Output Correlation

Now we turn to data moments that favor the expectations-trap hypothesis in our application. Our quantitative result is related to the ability of each model to generate an unconditional correlation between inflation and output that is consistent with the data.

Figure 6 shows the range of theoretical moments implied by the posterior parameter distribution of the expectation traps and secular stagnation model. The left panel shows the correlation between inflation and output growth. The right panel shows the volatility of inflation relative to the volatility of output growth. The blue shaded areas correspond to the theoretical range of moments generated by each model. The red dots in the figure represent the same moments in the Japanese data used in estimation.

The left panel shows the critical mechanism at play. The expectations-trap model can generate an unconditional negative correlation between inflation and output consistent with observed data. In contrast, the secular stagnation model cannot.[27] This result is the direct consequence of the conditional moments documented in the impulse response of Figure 2. The right panel shows that the expectations-trap model also generates relative volatility of inflation closer to the data. The tight co-movement of inflation and output moderates this relative volatility for the secular stagnation model.

Our results suggest that the fit of the secular stagnation hypothesis is impaired because the model has a restrictive set of exogenous shocks and cannot generate the empirical correlation of inflation and output. It is possible to relax model misspecification by allowing the correlation of fundamental shocks in the secular stagnation model, thus generating a negative inflation-output correlation. We do not see a clear economic interpretation to pursue such an approach. In contrast, in the expectations-trap model, the correlation between sunspot and fundamental shocks indexes an equilibrium as discussed in Bianchi and Nicolo (2021). Instead, in the next section, we show that relaxing misspecification through a more elaborate model structure does not overturn

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[27]Datta, Johannsen, Kwon and Vigfusson (2021) document a positive correlation between oil and equity prices in the U.S. post-2008. This measure is a proxy of the correlation between inflation and output in our model. We leave a formal quantitative assessment for the U.S. in our framework for future work.

(a) $corr(\Delta y_t^o, \pi_t^o)$ **(b)** $\sigma_{\pi_t^o} / \sigma_{\Delta y_t^o}$
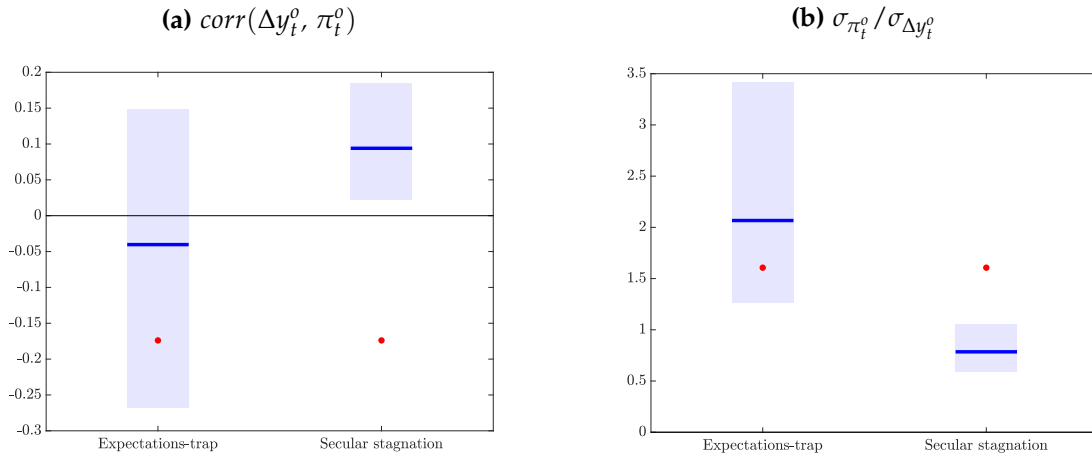
Notes: Dots correspond to sample moments in Japan's data. Solid horizontal lines indicate medians of theoretical moments of the posterior distributions for parameter estimates and the boxes indicate 90% credible associated with the posterior distributions.

our main result.

6. Persistent Stagnation in a Medium-Scale DSGE Model

We extend our set up along the lines of Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), or Justiniano, Primiceri and Tambalotti (2013). Our exercise confirms that our key finding of positive inflation-output correlation under secular stagnation also holds in a medium-scale DSGE model. This section briefly details the model structure and introduces new parameters used in our quantitative exercise. Relative to existing medium-scale DSGE models, the Euler equation central to our analysis is modified exactly in the same way as our three-equation setup in Section 3.

6.1. Model description

Compared to the model in Section 3, we introduce three changes. First, we allow for internal habits in consumption. Second, as in Gust, Herbst, López-Salido and Smith (2017), we incorporate a quadratic loss in household's utility due to wage adjustment and let monopolistically competitive households supply differentiated labor services, thus introducing nominal rigidities in wages. Third, we introduce

capital into production, with costly capital utilization and investment adjustment costs. Along each of these modifications we introduce three new structural shocks: time-varying wage-markups $(\frac{1}{1-\nu_{w,t}})$, a risk-premium shock (η_t^b), and an investment-specific shock (μ_t). A detailed description of the model is available in Appendix E.

We take most of the parameter values from Hirose (2020), who estimated a similar medium-scale model using Japanese data. The difference is that we approximate the model dynamics around the expectations-trap steady state. We also discuss how we re-calibrate some of the parameters to accommodate the secular stagnation hypothesis in our model.

Modified Euler-equation: Equation 12, is the consumption Euler equation, where λ_t represents the marginal utility of income and δ is the additive wedge induced by the preference for bonds. To be consistent with the secular stagnation calibration of the baseline model of Section 3, we set $\delta = 0.038$, such that the annualized natural rate of interest is -1.1%. With the presence of habits, marginal utility of income becomes a function of current and past consumption $\lambda_t = \frac{1}{c_t - \frac{hc_{t-1}}{z_t}} - \mathbb{E}_t \frac{h\beta}{z_{t+1}\left(c_{t+1} - \frac{hc_t}{z_{t+1}}\right)}$, where the parameter h controls the persistence of the consumption habit, and z_t denotes the growth rate of TFP. We set $h = 0.36$ and $\beta = 0.942$.

$$\lambda_t = \beta\eta_t^b \mathbb{E}_t \left[\frac{\lambda_{t+1}}{z_{t+1}}\frac{1}{\Pi_{t+1}}\right] + \delta \tag{12}$$

Wage-setting: Equation 13 defines household's optimal wage-setting decisions and determines wage inflation (Π_t^w). The parameter ψ_w defines the cost of wage adjustment, and $\nu_{w,t}$ represents time-varying wage markup parameter. The cost of wage adjustment depends on a reference inflation rate $\tilde{\Pi}_t^w = \bar{z}\bar{\Pi}^{1-\iota_w}\left(\exp(\epsilon_{z,t})\Pi_t\right)^{\iota_w}$, which is a geometric average between steady-state wage inflation $\bar{\Pi}^\omega$ and price inflation Π_{t-1}^ω, with indexation weight given by the parameter $\iota_\omega = 0.30$. We set the disutility of hour worked $\omega = 0.50$ to hit the steady state labor hours in Hirose (2020), and the inverse of the Frisch labor elasticity $\frac{1}{\eta} = 2.3$. We re-calibrate the parameter ψ_w such that inflation

in the secular stagnation steady state equals an annualized rate of -1.06%.

$$v_{w,t}\psi_w \left[\frac{\Pi_t^w}{\tilde{\Pi}_{t-1}^w} - 1 \right] \frac{\Pi_t^w}{\tilde{\Pi}_{t-1}^w} = v_{w,t}\psi_w \beta \mathbb{E}_t \left[\frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w} - 1 \right] \frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w} + L_t \lambda_t \left[\omega \frac{L_t^{\frac{1}{\eta}}}{\lambda_t} - (1 - v_{w,t})w_t \right]$$

(13)

Price of capital and investment: Equation 14 defines the value of an additional unit of capital relative to consumption, (q_t), as a function of the expected marginal return to capital. The quarterly depreciation rate of capital is $\delta_k = 0.015$. To capture the cost of capital utilization we use $a(u_t) = (r^k/\sigma_a)(e^{(\sigma_a(u_t-1))} - 1)$, and set $\sigma_a = 2.246$.

$$q_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t z_{t+1}} \left(r_{t+1}^K u_{t+1} - a(u_{t+1}) + q_{t+1}(1 - \delta_k) \right) \right],$$

(14)

Equation 15 defines the optimal investment decision. To capture investment adjustment costs, we use the convex function $S(x_t) = \frac{\varphi_I}{2}(x_t - 1)^2$, and set the parameter $\varphi_I = 5.2$. The investment decision is also influenced by an exogenous investment specific shock μ_t. This shock changes the resource cost of transforming investment into installed capital.

$$q_t \mu_t \left[1 - S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}} \frac{z_t}{\bar{z}} \right) - S'\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}} \frac{z_t}{\bar{z}} \right) \frac{\mathbb{I}_t}{\mathbb{I}_{t-1}} \frac{z_t}{\bar{z}} \right]$$
$$+ \beta \mathbb{E}_t \left[\mu_{t+1} \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \frac{z_{t+1}}{\bar{z}} \left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t} \right)^2 S'\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t} \frac{z_{t+1}}{\bar{z}} \right) \right] = 1$$

(15)

Structural shocks: The six structural shocks driving the model economy are assumed to follow first order auto-regressive processes of the form $\log(x_t) = (1 - \rho_x)\log(x) + \rho_x log(x_{t-1}) + \sigma_x \epsilon_{x,t}$, with $\epsilon_{x,t} \sim N(0,1)$, and x denoting steady-state values, for $x_t = z_t, g_t, \eta_t^b, \mu_t, v_{p,t}, v_{w,t}$ (technology growth, government spending, risk-premium, investment-specific shock, price-markup and wage-markup respectively).

Equilibrium conditions We provide the full set of equilibrium conditions and additional parameter values in Appendix E.

6.2. Secular stagnation

We use the medium-scale DSGE model to generate two sets of moments: (i) impulse response functions of inflation and output to various shocks, and (ii) the model implied unconditional correlation between inflation and output.

Turning to the first set of moments, the impulse response functions in figure 7 show that, in our calibration, all the conditional correlations between inflation and output are positive. This result emerges from the positive co-movement of production and inflation in the absence of an active policy rule that responds to inflation deviations. In contrast, when the natural interest rate is non-negative, and nominal rates react to changes in inflation, higher markups or lower investment efficiency tend to reduce output and increase inflation. This mechanism is absent in the model linearized around the permanent liquidity trap.

Our second result is immediate from impulse response functions. The medium-scale model implies a positive correlation between inflation and output. The theoretical correlation for annualized inflation and output growth is 0.13, well within the range of implied theoretical correlations from the baseline model displayed in figure 6. The result confirms that secular stagnation is an unlikely candidate to explain the Japanese experience at the ZLB. The underlying reason is the limitation of the secular stagnation hypothesis to generate an inflation-output tradeoff in the absence of an active policy rule.

6.3. Expectation-traps

Finally, we briefly turn to the implications of the expectation-traps hypothesis in the context of the calibrated medium-scale DSGE model. Re-calibrating the model to an annualized natural interest rate of 0% and an annualized inflation rate of -1.06%, as in Section 3, we obtain a steady-state output gap of -4.3%. This calibration implies that the dynamics near the ZLB steady state are locally indeterminate. To select an equilibrium, we introduce a sunspot shock that is correlated with our six structural shocks, and calibrate the correlations between sunspot shocks and structural shocks

Figure 7: *Secular Stagnation Dynamics in Medium-Scale NK Model*
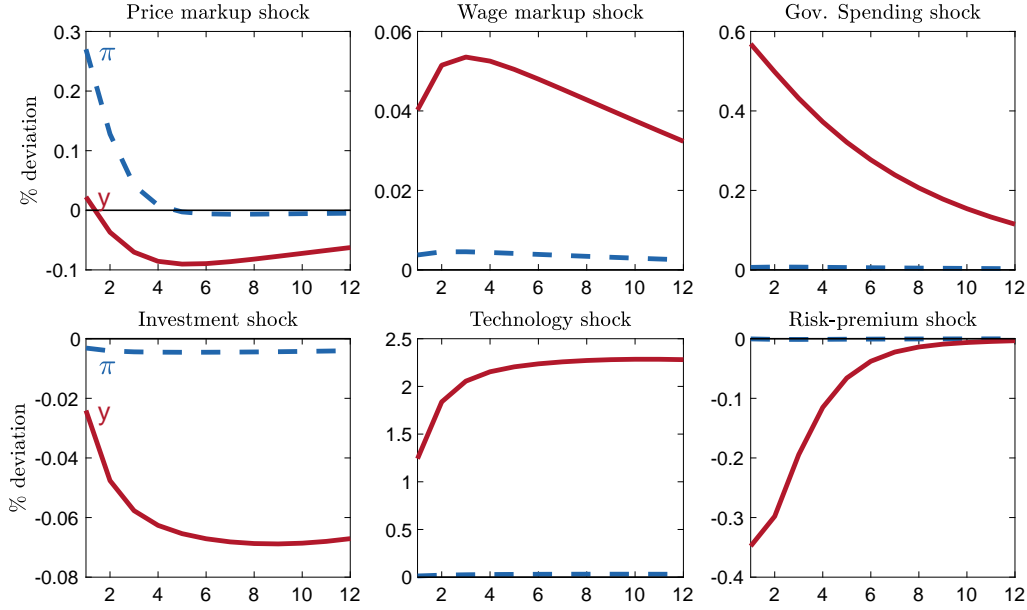
Notes: Impulse responses to one standard deviation shocks. The red solid lines correspond to inflation response. The blue dashed lines correspond to output responses.

using estimates from Hirose (2020).

In this case, the unconditional correlation between inflation and output growth is -0.12, consistent with our findings in Figure 6. Thus, as in Section 3, the expectations-trap model can fit a critical data moment of the Japanese data, and equilibrium indeterminacy remains essential to generate the negative correlation between inflation and output in a model of permanent liquidity traps.

7. CONCLUSION

In this paper, we developed a framework to formally model two hypotheses of stagnation: expectations-driven traps and secular stagnation. Our framework hinges on a modified Euler equation with discounting. We provide a tractable version of such modification that allows us to obtain analytical results. In this setting, we show the conditions under which expectations traps and secular stagnation emerge as steady-state equilibria of the model. We also show that the two hypotheses differ in the local determinacy properties. Because of contrasting predictions from traditional policies

implemented at ZLB, we argue that there is a need for robust policies that can stimulate the economy regardless of the stagnation hypothesis. Our robust policies prescribe a flattening of the aggregate supply curve through price-indexation or minimum wage schemes.

We conduct an assessment of Japan's experience near the ZLB using a quantitative new-Keynesian model that embeds both hypotheses. We construct a time-varying probability of the relevant theory of stagnation. Our results show that the Japanese experience is consistent with the expectations-trap model. We find that the real-time assessment of both models has considerable uncertainty. We document that equilibrium indeterminacy of the expectations-trap model is central to account for empirical moments in Japanese data. In particular, the negative correlation between inflation and output growth. Our findings extend to a medium-scale model DSGE model of the Japanese economy.

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A. Proofs for Propositions in Section 2

Proposition (Proposition 1: Targeted Steady State). Let $0 < \delta < \frac{1}{\beta}$. There exists a unique positive interest rate steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$. It features output at efficient steady state, and inflation at the policy target. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

Proof. The downward sloping portion of aggregate demand always goes through $Y = 1$, $\Pi = 1$. When $\delta < \frac{1}{\beta}$, $1 + r^* > 1$. The kink in the AD curve occurs at $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi_\pi}} < 1$ and $Y_{AD,kink} = (1 + r^*)^{1-\frac{1}{\phi_\pi}} > 1$. There always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there does not exist another steady state at positive interest rates, note the AS curve is linear and upward sloping. For $\Pi > 1$, $Y_{AD} < 1 < Y_{AS}$. And for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. There does not exist another steady state at positive nominal interest rate.

To prove local-determinacy, log-linearize the equilibrium conditions (1) - (3) around the unique non-stochastic steady state $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$. The system of equations can be simplified to:

$$\hat{Y}_t = \frac{1 + \kappa}{2 + \phi_\pi \kappa} \mathbb{E}_t \hat{Y}_{t+1} + \hat{r}_t^n$$

where hat variables represent log-deviations from steady state. Given, $\kappa > 0$ and $\phi_\pi > 1$, this system satisfies Blanchard-Kahn conditions for determinacy of linear rational-expectations equilibria. $\square$

Proposition (Proposition 2: Expectations trap steady state). Let $0 < \delta < \frac{1}{\beta}$. For $\kappa > 1$ (i.e. $\alpha_p > 0.5$) there exist two steady states:

1. The targeted steady state with $Y = 1$, $\Pi = 1$ and $R = \frac{1}{\beta\delta} > 1$.

2. (*Expectations-driven* trap) A unique-ZLB steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. The local dynamics in a neighborhood around the unemployment steady state are locally in-determinate.

When prices are rigid enough, i.e. $\kappa < 1$, there exists a unique steady state and it is the targeted inflation steady state. When prices are flexible $\alpha_p = 1$ ($\kappa \to \infty$), there always exist two steady states: a unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.

Proof. <u>For $\kappa > 1$</u>:

When $\delta < \frac{1}{\beta}$, $1 + r^* > 1$. The kink in the AD curve occurs at $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi_\pi}} < 1$ and $Y_{AD,kink} = (1 + r^*)^{1-\frac{1}{\phi_\pi}} > 1$. There always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there doesn't exist another steady state at positive interest rates, note that the AS curve is linear and upward sloping. For $\Pi > 1$, $Y_{AD} < 1 < Y_{AS}$. And for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. There does not exist another steady state at positive nominal interest rate. The proof for local-determinacy of this targeted steady state follows similar steps as in Proposition 1.

To prove that there exists a unique intersection at zero nominal interest rates, we note that AS and AD are linear for $\Pi < \Pi_{kink}$. When $\Pi = \Pi_{kink}$, $Y_{AD} > 1 > Y_{AS}$. And when $\Pi = 0$,

$Y_{AD} = 0 < Y_{AS} = \frac{\kappa - 1}{\kappa} > 0$. This is because of the assumption that $\alpha_p > 0.5$. Hence, there exists a unique intersection at zero nominal interest rates. To prove local-indeterminacy, log-linearize the equilibrium conditions (1) - (3) around the unique non-stochastic steady state $Y_S = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi_S = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R_S = 1$. The system of equations can be simplified to:

$$\hat{Y}_t = \frac{1 + \kappa_b}{2} \mathbb{E}_t \hat{Y}_{t+1} + \hat{r}_t^n$$

where hat variables represent log-deviations from steady state and $\kappa_b \equiv \frac{\kappa Y_S}{\kappa Y_S + 1 - \kappa} > 1$ (because $1 - \kappa < 0$ and $\beta\delta < 1$). Hence, this system does not have a unique bounded rational expectations equilibrium.

For $\kappa < 1$:

That there exists a unique non-ZLB steady state follows from Proposition 1. Remains to show that there does not exist a ZLB steady state. Note that, AD is linear and upward sloping when ZLB is binding and AS is always linear. Furthermore for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. And for $\Pi = 1 - \kappa$, $Y_{AS} = 0 < Y_{AD}$. Thus there does not exist a steady state with zero nominal interest rate.

$\square$

Proposition (Proposition 3: Secular Stagnation). Let $\delta > \frac{1}{\beta}$ and $\kappa < 1$. There exists a unique steady state with $Y = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R = 1$. It features output below the targeted steady state and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

Proof. When $\delta > \frac{1}{\beta}$, $1 + r^* < 1$, thus, the kink in the AD occurs at $\Pi_{kink} > 1$, $Y_{AD,kink} < 1$. For $\Pi > \Pi_{kink}$, $Y_{AD} < 1 < Y_{AS}$. Thus, no steady state exists at positive nominal interest rates. When $\Pi = \Pi_{kink}$, $Y_{AD} < 1 < Y_{AS}$. For $\Pi < \Pi_{kink}$, AS and AD are both and downward sloping. At $\Pi = 1 - \kappa < 1$, $Y_{AS} = 0 < Y_{AD}$. Hence there exists a unique steady state of the economy with nominal rigidities.

To prove local-determinacy, log-linearize the equilibrium conditions (1) - (3) around the unique non-stochastic steady state $Y_S = \frac{1-\kappa}{\beta\delta-\kappa} < 1$, $\Pi_S = \frac{\beta\delta(1-\kappa)}{\beta\delta-\kappa} < 1$ and $R_S = 1$. The system of equations can be simplified to:

$$\hat{Y}_t = \frac{1 + \kappa_a}{2} \mathbb{E}_t \hat{Y}_{t+1} + \hat{r}_t^n$$

where hat variables represent log-deviations from steady state and $\kappa_a \equiv \frac{\kappa Y_S}{\kappa Y_S + 1 - \kappa} < 1$. Given, $0 < \kappa_a < 1$, this system satisfies Blanchard-Kahn conditions for determinacy of linear rational-expectations equilibria.

$\square$

B. DERIVATION OF THE PRICE INDEXATION SCHEME

As shown in Section 2.2, the price index is given by

$$P_t^{\frac{\nu-1}{\nu}} = \alpha \left(p_t^*\right)^{\frac{\nu-1}{\nu}} + (1 - \alpha) \left(p_t^n\right)^{\frac{\nu-1}{\nu}}$$

It can be rewritten as

$$1 = \alpha \left(\frac{p_t^*}{P_t} \right)^{\frac{v-1}{v}} + (1 - \alpha) \left(\frac{p_t^n}{P_t} \right)^{\frac{v-1}{v}}$$

$$= \alpha \left(\frac{Y_t}{\bar{Y}} \right)^{\frac{v-1}{v}} + (1 - \alpha) \left(\Gamma_t \frac{P_{t-1}}{P_t} \right)^{\frac{v-1}{v}}$$

If $v = 1/2$, we get:

$$1 = \alpha \left(\frac{Y_t}{\bar{Y}} \right)^{-1} + (1 - \alpha) \left(\Gamma_t \frac{P_{t-1}}{P_t} \right)^{-1}$$

$$(1 - \alpha) \left(\Gamma_t^{-1} \Pi_t - 1 \right) = \alpha \left(\frac{Y_t - \bar{Y}}{Y_t} \right)$$

Define $\Gamma_t = \frac{P_t}{Y_t^{-1}(P_t - \lambda P_{t-1}) + P_{t-1}}$, to get

$$(\Pi_t - \lambda) = \frac{\alpha}{1 - \alpha} (Y_t - \bar{Y})$$

When $\lambda = 1$, the price Phillips curve simplifies to

$$(\Pi_t - 1) = \frac{\alpha}{1 - \alpha} (Y_t - \bar{Y})$$

$\forall \lambda > \kappa > 1$, there does not exist expectations trap. An indexation to Γ_t of yesterday's price with $\lambda > \kappa$ can eliminate expectations trap.

Another indexation in price setting that we can assume is $\Gamma_t = Y_t$. This gives rise to the following relationship between gross inflation and output:

$$\Pi_t = (1 + \kappa)Y_t - \kappa$$

where $\kappa = \frac{\alpha_p}{1 - \alpha_p} > 0$ as before. With this Phillips curve, there always exist two steady states as long as $0 < \delta \beta < 1$ and $\kappa > 0$. We can analytically derive the steady states as in Proposition 2. A takeaway of our analysis is that appropriate price/wage indexation schemes can eliminate expectations trap. These can also be shown to improve the outcome in secular stagnation (as a corollary of paradox of flexibility).

A set of policies that are robust to the kind of stagnation can be framed from our analysis:

Corollary 1. (robust policies) A downwardly rigid price/wage indexation scheme can eliminate expectations trap while also improving welfare under secular stagnation.

C. QUANTITATIVE MODEL

This section describes the micro-foundations behind the log-linearized setup presented in Section 3.

C.1. Households

$$\max_{C_t(k),H_t,B_t(k)} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\log\left(C_t(k)/A_t\right) - \omega \frac{H_t^{1+1/\eta}}{1+1/\eta} - \delta \frac{B_t}{A_t P_t} \right) \right],$$

subject to:

$$P_t C_t(k) + B_t(k) + T_t = W_t H_t + R_{t-1} B_{t-1}(k) + P_t D_t(k) + P_t SC_t,$$

The household derives utility from consumption C_t and from holding (real) stock of risk-free nominal bonds B_t, and disutility from hours worked H_t. The parameter ω scales the steady-state level of hours worked. The parameter δ regulates the marginal utility from holding bonds. The risk-free nominal bond pays a gross nominal interest rate R_t Each household supplies homogeneous labor services H_t in a competitive labor market taking the aggregate wage W_t as given. It collects interest payments on bond holdings, real profits D_t from intermediate good producers, pays lump sum taxes T_t, and receives payouts SC_t from trading a full set of state(k)-contingent securities.

Consumption decision. Let $\beta^t \lambda_t$ be the Lagrange multiplier on the household budget constraint, the first-order condition with respect to consumption and bond holdings are given by:

$$\left(\frac{C_t}{A_t}\right)^{-1} \frac{1}{A_t} = \beta R_t \left(\left(\frac{C_{t+1}}{A_{t+1}}\right)^{-1} \frac{1}{A_t}\right) \frac{P_t}{P_{t+1}} + \frac{\delta}{A_t} \tag{C.1}$$

Define:

$$Q_{t+1|t} = \frac{\lambda_{t+1} P_{t+1}}{\lambda_t P_t} \tag{C.2}$$

Using this definition, the first-order condition for bond holdings becomes:

$$1 = \beta \mathbb{E}_t \left[Q_{t+1|t} R_t \frac{P_t}{P_{t+1}} \right] + \delta \left(\frac{C_t}{A_t}\right) \tag{C.3}$$

The stochastic discount factor can be written as:

$$Q_{t+1|t} = \left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t}\right)^{-1} \frac{A_t}{A_{t+1}} = \left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t}\right)^{-1} \frac{1}{\gamma z_{t+1}} \tag{C.4}$$

Combining with the FOC for bond holdings we obtain the expression for the Euler equation:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t}\right)^{-1} \frac{R_t}{\gamma z_{t+1}} \frac{P_t}{P_{t+1}} \right] + \delta \left(\frac{C_t}{A_t}\right) \tag{C.5}$$

Labor supply. Taking first-order conditions with respect to H_t yields

$$-\omega H_t^{1/\eta} + W_t \lambda_t = 0 \tag{C.6}$$

Combine with the FOC for consumption and defining the real wage as $w_t = \frac{W_t}{P_t}$ to obtain:

$$\frac{w_t}{A_t} = \omega H_t^{1/\eta} \left(\frac{C_t}{A_t} \right)^{-1} \tag{C.7}$$

C.2. Final Good Firms

The perfectly competitive, representative, final-good producing firm combines a continuum of intermediate goods indexed by $j \in [0,1]$ using the technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu_t} dj \right)^{\frac{1}{1-\nu_t}}.$$

Here $1/\nu_t > 1$ represents the elasticity of demand for each intermediate good. Profit maximization implies that the demand for intermediate goods is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu_t} Y_t.$$

where the price of the final good P_t is given by :

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\nu_t - 1}{\nu_t}} dj \right)^{\frac{\nu_t}{\nu_t - 1}}.$$

C.3. Intermediate Good Producers

Intermediate good j is produced by a monopolist who has access to the following production technology:

$$Y_t(j) = A_t H_t(j), \quad \text{with} \quad A_t = A_{t-1} z_t,$$

where A_t denotes the aggregate level of technology that is common to all firms, and z_t represents the stochastic (stationary) movements in TFP.

Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs. These adjustment costs, expressed as a fraction of total output, are defined by the function $\Phi_p(.)$:

$$\Phi_p \left(\frac{P_t(j)}{P_{t-1}(j)} \right) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi^* \right)^2 Y_t$$

where Π^* is the inflation rate in the targeted steady state. Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its the price $P_t(j)$ to maximize the present value of future profits:

$$\max_{\{P_{t+s}(j)\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \Phi_p \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} \right) Y_{t+s} - \frac{W_{t+s}Y_{t+s}(j)}{z_{t+s}P_{t+s}} \right) ,$$

subject to

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu_t} Y_t.$$

Price setting decision . Denoting $\mu_{t+s}\beta^s Q_{t+s|t}$ as the Lagrange multiplier associated with this constraint. The first-order condition with respect to $P_t(j)$ is given by

$$0 = \frac{A_t H_t(j)}{P_t} - \Phi' \left(\frac{P_t(j)}{P_{t-1}(j)} \right) \frac{A_t H_t(j)}{P_{t-1}(j)} - \frac{\mu_t}{\nu_t} \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu_t-1} \frac{Y_t}{P_t}$$
$$+ \beta \mathbb{E}_t \left[Q_{t+1|t} \Phi' \left(\frac{P_{t+1}(j)}{P_t(j)} \right) A_{t+1} H_{t+1}(j) \frac{P_{t+1}(j)}{P_t^2(j)} \right].$$

Labor demand. Taking first-order conditions with respect to $H_t(j)$ yields

$$\frac{W_t}{A_t P_t} = \frac{P_t(j)}{P_t} - \Phi \left(\frac{P_t(j)}{P_{t-1}(j)} \right) - \mu_t$$

Symmetric equilibrium. Let $P_t(j) = P_t \ \forall j$, the firm's equilibrium condition become:

$$\frac{\mu_t}{\nu_t} + \Phi'_p \left(\frac{P_t}{P_{t-1}} \right) \frac{P_t}{P_{t-1}} - 1 = \beta \mathbb{E}_t \left[Q_{t+1|t} \frac{P_{t+1}}{P_t} \Phi' \left(\frac{P_{t+1}}{P_t} \right) \frac{Y_{t+1}}{Y_t} \right]$$
$$\mu_t = 1 - \Phi \left(\frac{P_t}{P_{t-1}} \right) - \frac{w_t}{A_t}$$

Combine both equilibrium conditions with the optimal labor supply condition from the household problem to obtain the FOC determining optimal price-setting:

$$(1 - \nu_t) - \omega H_t^{1/\eta} \left(\frac{C_t}{A_t} \right)^{-1} + \nu_t \Phi' \left(\frac{P_t}{P_{t-1}} \right) \frac{P_t}{P_{t-1}} = \nu_t \beta \mathbb{E}_t \left[Q_{t+1|t} \frac{P_{t+1}}{P_t} \Phi' \left(\frac{P_{t+1}}{P_t} \right) \frac{Y_{t+1}}{Y_t} \right] \quad \text{(C.8)}$$

Government Policies

The desired policy rate is set according to the following rule:

$$R_t^* = \left[r\Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\psi_1} \right],$$

Here r is the steady-state real interest rate, Π_t is the gross inflation rate defined as Π_t, and Π^* is the target inflation rate, which in equilibrium coincides with the steady state inflation rate. The actual policy rate relevant for agents decisions is subject to the zero lower bound constraint:

$$R_t \;=\; \max\{1, R_t^*\}$$

The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by:

$$P_t G_t + R_{t-1} B_{t-1} \;=\; T_t + B_t,$$

where $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ is the government expenditure.

C.4. Resource constraint

We assume that the price adjustment costs are rebated back to the household in lump-sum fashion as part of the government transfers.[28] Hence, the market-clearing resource constraint is given by:

$$C_t + G_t = Y_t$$

Finally, we assume nominal bonds are in net zero supply

$$B_t = 0$$

D. DSGE Solution and Estimation

This section describes how we obtain the equations of the log-linearized model of Section 3, as well as proofs Propositions 5, 6 and 7.

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[28] An analogous interpretation would be to consider these costs as mental accounting costs for the firms or model these in the utility function of the representative agent. This assumption allows us to avoid unnatural results commonly associated with resource costs modeled in terms of output.

D.1. Log-linearized model

With a slight abuse of notation, we can write the system of equilibrium conditions as follow:

$$1 = \frac{\beta R}{\pi z} e^{-(c_{t+1}-c_t)+R_t-\pi_{t+1}-z_{t+1}} + \delta c e^{c_t}$$

$$(1 - v e^{v_t}) + v\phi e^{v_t} (\pi e^{\pi_t} - \pi_*) \pi e^{\pi_t} = \omega c y^{1/\eta} e^{c_t+(1/\eta)y_t}$$

$$+ v\phi\beta\pi e^{v_t} \left[e^{-(c_{t+1}-c_t)+y_{t+1}-y_t+\pi_{t+1}} (\pi e^{\pi_{t+1}} - \pi_*) \right]$$

$$c e^{c_t} = \frac{y}{g} e^{y_t-g_t}$$

Linearization around an arbitrary (R, π) point yields:

$$c_t = \frac{R\beta}{(R\beta + \pi z\delta c)} c_{t+1} - \frac{R\beta}{(R\beta + \pi z\delta c)} (R_t - \pi_{t+1} - z_{t+1})$$

$$\pi_t = \beta \frac{(\pi - \pi_*)}{2\pi - \pi_*} [(c_{t+1} - c_t) + y_{t+1} - y_t] + \beta \pi_{t+1} + \left(\frac{v - (1-\beta)v\phi\pi(\pi - \pi_*)}{v\phi\pi(2\pi - \pi_*)} \right) v_t$$

$$+ \left(\frac{(1-v)y^{1+1/\eta}}{v\phi\pi(2\pi - \pi_*)} \right) \left(c_t + \frac{1}{\eta} y_t \right)$$

$$c_t = y_t - g_t$$

D.2. Full employment

Around the full employment steady state we have $\pi = \pi_*$ and $R = \pi_* r_0$. In our calibration for the full employment steady state we have $r_0 = \frac{z(1-\delta g^{-1})}{\beta} = \exp(1/400)$ and $\pi_* = \exp(1/400)$. Moreover, we choose ω to normalize the full employment level of output to $y = 1$.

$$c_t = \mathcal{D} c_{t+1} - \mathcal{D} (R_t - \pi_{t+1} - z_{t+1})$$

$$\pi_t = \beta \pi_{t+1} + \lambda v_t + \kappa c_t + \left(\frac{\kappa}{\eta} \right) y_t \qquad \text{(D.1)}$$

$$c_t = y_t - g_t$$

Where, $\mathcal{D} = \frac{R\beta}{(R\beta+\pi z\delta c)}, \lambda = \left(\frac{v}{v\phi\pi^2} \right)$, and $\kappa = \left(\frac{1-v}{v\phi\pi^2} \right)$.

D.3. Permanent Liquidity Trap

When the economy is at a permanent liquidity trap, we have R = 1. We denote by $\bar{x}$ the steady state values corresponding to the liquidity trap steady state.

$$c_t = \frac{\beta}{(\beta + \bar{\pi}z\delta\bar{c})}c_{t+1} - \frac{\beta}{(R\beta + \bar{\pi}z\delta\bar{c})}(-\pi_{t+1} - z_{t+1})$$

$$\pi_t = \beta\frac{(\bar{\pi} - \pi_*)}{2\bar{\pi} - \pi_*}\left[(c_{t+1} - c_t) + y_{t+1} - y_t\right] + \beta\pi_{t+1} + \left(\frac{v - (1-\beta)v\phi\bar{\pi}(\bar{\pi} - \pi_*)}{v\phi\bar{\pi}(2\bar{\pi} - \pi_*)}\right)v_t$$

$$+ \left(\frac{(1-v)\bar{y}^{1+1/\eta}}{v\phi\bar{\pi}(2\bar{\pi} - \pi_*)}\right)\left(c_t + \frac{1}{\eta}y_t\right)$$

$$c_t = y_t - g_t$$

Collecting terms and replacing the log-linearized resource constraint we have:

$$\hat{y}_t = \bar{\mathcal{D}}(\hat{y}_{t+1} - \hat{g}_{t+1}) + \bar{\mathcal{D}}(\hat{\pi}_{t+1} + \hat{z}_{t+1}) + \hat{g}_t \tag{D.2}$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \bar{\lambda}v_t + \bar{\kappa}(y_t - g_t) + \frac{\bar{\kappa}}{\eta}y_t + 2\bar{\Gamma}(y_{t+1} - y_t) - \bar{\Gamma}(g_{t+1} - g_t)$$

Where $\bar{\mathcal{D}} = \frac{\beta}{(\beta + \bar{\pi}z\delta\bar{c})}$, $\bar{\lambda} = \left(\frac{v-(1-\beta)v\phi\bar{\pi}(\bar{\pi}-\pi_*)}{v\phi\bar{\pi}(2\bar{\pi}-\pi_*)}\right)$, $\bar{\kappa} = \left(\frac{(1-v)\bar{y}^{1+1/\eta}}{v\phi\bar{\pi}(2\bar{\pi}-\pi_*)}\right)$, and $\bar{\Gamma} = \beta\frac{(\bar{\pi}-\pi_*)}{2\bar{\pi}-\pi_*}$, we obtain the log-linearized equations presented in the main text.

D.4. Proof of Proposition 5

Without shocks, the system of equations around a permanent liquidity trap can be rewritten as:

$$\hat{y}_t = \bar{\mathcal{D}}\mathbb{E}_t(\hat{y}_{t+1} + \hat{\pi}_{t+1})$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \tilde{\kappa}\hat{y}_t$$

Where $\tilde{\kappa} = \left(\frac{1}{\eta} + 1\right)\left(\frac{(1-v)\bar{y}^{1+1/\eta}}{v\phi\bar{\pi}(2\bar{\pi}-\pi_*)}\right)$, $\bar{\mathcal{D}} = \frac{\beta}{\beta + \bar{\pi}z\delta\bar{c}}$, then $1/\bar{\mathcal{D}} = 1 + \frac{\bar{\pi}z\delta\bar{c}}{\beta}$.

To iterate the system forward, we write:

$$\begin{bmatrix} \bar{\mathcal{D}} & \bar{\mathcal{D}} \\ 0 & \beta \end{bmatrix}\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\tilde{\kappa} & 1 \end{bmatrix}\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix}$$

Then we can write:

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{\mathcal{D}}} & -\frac{1}{\beta} \\ -0 & \frac{1}{\beta} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\tilde{\kappa} & 1 \end{bmatrix}\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix}$$

Simplifying:

$$\begin{bmatrix} \hat{y}_{t+1} \\ s\hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{\mathcal{D}}} + \frac{\tilde{\kappa}}{\beta} & -\frac{1}{\beta} \\ -\frac{\tilde{\kappa}}{\beta} & \frac{1}{\beta} \end{bmatrix}\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix}$$

Define $\rho = 1/\beta, \phi = 1/\bar{\mathcal{D}}$,

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} (\phi + \tilde{\kappa}\rho) & -\rho \\ -\tilde{\kappa}\rho & \rho \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} \frac{1}{\bar{\mathcal{D}}} & -\frac{1}{\beta} \\ -0 & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} 0 \\ -\lambda \end{bmatrix} \hat{v}_t$$

From this system, define $M \equiv \begin{bmatrix} (\phi + \tilde{\kappa}\rho) & -\rho \\ -\tilde{\kappa}\rho & \rho \end{bmatrix}$. Then, we can derive the following properties of the matrix M:

$$det(M) = \phi\rho, \quad tr(M) = \phi + (1 + \tilde{\kappa})\rho$$

Proposition C1 in (Woodford, 2003, pp 670) provides the necessary and sufficient conditions for determinacy for a system of 2 equations. A 2×2 matrix M with positive determinant has both eigenvalues outside the unit circle if and only if

$$\det M > 1, \quad \det M - \operatorname{tr} M > -1, \quad \det M + \operatorname{tr} M > -1$$

Under our sign restrictions and the assumption that $\beta < 1$, first and third inequalities necessarily hold, do that both eigenvalues are outside the unit circle if and only if $\phi > \frac{1-\rho(1+\tilde{\kappa})}{1-\rho} = 1 - \frac{\rho\tilde{\kappa}}{1-\rho}$ for determinacy. This implies $1/\bar{\mathcal{D}} > \frac{\frac{\beta}{\beta} - \frac{1}{\beta}(1+\tilde{\kappa})}{\frac{\beta-1}{\beta}} = \frac{(\beta-1)-\tilde{\kappa}}{\beta-1} = 1 + \frac{\tilde{\kappa}}{1-\beta}$. We can rewrite this inequality to obtain $\frac{1-\beta}{1-\beta+\tilde{\kappa}} > \bar{\mathcal{D}}$, which yields the restriction in the proposition.

D.5. MSV solution

This section derives the MSV solution for the analysis in Section 5.

D.5.1 proof of Proposition 6

We guess that $\hat{y}_t = a_1\hat{z}_t + a_2\hat{g}_t + a_3\hat{v}_t$ and $\hat{\pi}_t = b_1\hat{z}_t + b_2\hat{g}_t + b_3\hat{v}_t$ and solve for the unknown $a's$ and $b's$. Replacing the guess into (D.2), collecting terms and using the method of undetermined coefficients, we obtain the following system of equations:

$$\hat{y}_t = \bar{\mathcal{D}}(\mathbb{E}\hat{y}_{t+1} - g_{t+1} + E\hat{\pi}_{t+1} + \hat{E}\hat{z}_{t+1}) + g_t$$
$$\hat{\pi}_t = \beta\mathbb{E}\hat{\pi}_{t+1} + \bar{\kappa}(\frac{1}{\eta} + 1)\hat{y}_t - \bar{\kappa}\hat{g}_t + \bar{\lambda}\hat{v}_t + \bar{\varphi}(E\hat{g}_{t+1} - \hat{g}_t),$$

where $\bar{\mathcal{D}} = \frac{\beta}{(\beta + \bar{\pi}z\delta\bar{c})}$, $\bar{\varphi} = \frac{(\bar{\pi} - \pi_*)}{2\bar{\pi} - \pi_*}$, $\bar{\lambda} = \left(\frac{\nu - (1-\beta)\nu\phi\bar{\pi}(\bar{\pi} - \pi_*)}{\nu\phi\bar{\pi}(2\bar{\pi} - \pi_*)}\right)$, and $\bar{\kappa} = \left(\frac{(1-\nu)\bar{y}^{1+1/\eta}}{\nu\phi\bar{\pi}(2\bar{\pi} - \pi_*)}\right)$. Guessing $y_t = a_1\hat{z}_t + a_2\hat{g}_t + a_3\hat{v}_t$ and $\pi_t = b_1\hat{z}_t + b_2\hat{g}_t + b_3\hat{v}_t$ we can replace back into the previous equations. Dropping time subscripts:

$$\begin{aligned} a_1\hat{z}_t + a_2\hat{g}_t + a_3\hat{v}_t &= \bar{\mathcal{D}}\left(a_1\rho_z\hat{z}_t + a_2\rho_g\hat{g}_t + a_3\rho_v\hat{v}_t - \rho_g g + b_1\rho_z\hat{z}_t + b_2\rho_g\hat{g}_t + b_3\rho_v\hat{v}_t + \rho_z\hat{z}_t\right) + \hat{g}_t \\ &= \bar{\mathcal{D}}(a_1\rho_z + b_1\rho_z + \rho_z)\hat{z}_t + (\bar{\mathcal{D}}a_2\rho_g - \bar{\mathcal{D}}\rho_g + \bar{\mathcal{D}}b_2\rho_g + 1)\hat{g}_t + \bar{\mathcal{D}}\left(b_3\rho_v + a_3\rho_v\right)\hat{v}_t \end{aligned}$$

$$\begin{aligned}
b_1\hat{z}_t + b_2\hat{g}_t + b_3\hat{v}_t &= \beta\left(b_1\rho_z\hat{z}_t + b_2\rho_g\hat{g}_t + b_3\rho_v\hat{v}_t\right) + \bar{\varphi}\left(\rho_g - 1\right)\hat{g}_t + \bar{\lambda}\hat{v}_t \\
&+ \bar{\kappa}(\frac{1}{\eta} + 1)\left(a_1\hat{z}_t + a_2\hat{g}_t + a_3\hat{v}_t\right) - \bar{\kappa}\hat{g}_t \\
&= \left(\beta b_1\rho_z + \bar{\kappa}(\frac{1}{\eta} + 1)a_1\right)\hat{z}_t + \left(\beta b_2\rho_g + \bar{\varphi}\left(\rho_g - 1\right) + \bar{\kappa}(\frac{1}{\eta} + 1)a_2 - \bar{\kappa}\right)\hat{g}_t \\
&+ \left(\beta b_3\rho_v + \bar{\kappa}(\frac{1}{\eta} + 1)a_3 + \bar{\lambda}\right)\hat{v}_t
\end{aligned}$$

Comparing terms we can write the following system of equations:

$$
\begin{bmatrix}
0 \\
-\bar{\varphi}\left(\rho_g - 1\right) + \bar{\kappa} \\
-\bar{\lambda} \\
-\bar{\mathcal{D}}\rho_z \\
\bar{\mathcal{D}}\rho_g - 1 \\
0
\end{bmatrix}
=
\begin{bmatrix}
(\beta\rho_z - 1) & 0 & 0 & \bar{\kappa}(\frac{1}{\eta} + 1) & 0 & 0 \\
0 & (\beta\rho_g - 1) & 0 & 0 & \bar{\kappa}(\frac{1}{\eta} + 1) & 0 \\
0 & 0 & (\beta\rho_v - 1) & 0 & 0 & \bar{\kappa}(\frac{1}{\eta} + 1) \\
\bar{\mathcal{D}}\rho_z & 0 & 0 & (\bar{\mathcal{D}}\rho_z - 1) & 0 & 0 \\
0 & \bar{\mathcal{D}}\rho_g & 0 & 0 & (\bar{\mathcal{D}}\rho_g - 1) & 0 \\
0 & 0 & \bar{\mathcal{D}}\rho_v & 0 & 0 & (\bar{\mathcal{D}}\rho_v - 1)
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
a_1 \\
a_2 \\
a_3
\end{bmatrix}
$$

The solution is:

$$
b_1 = \frac{-\bar{\mathcal{D}}\rho_Z\bar{\kappa}\left(\frac{1}{\eta} + 1\right)}{\bar{\mathcal{D}}\rho_z\left[\bar{\kappa}\left(\frac{1}{\eta} + 1\right) + (1 - \beta\rho_z)\right] - (1 - \beta\rho_z)}; \qquad a_1 = \frac{(1 - \beta\rho_z)b_1}{\bar{\kappa}\left(\frac{1}{\eta} + 1\right)} \tag{D.3}
$$

$$
b_2 = \frac{(1 - \bar{\mathcal{D}}\rho_g)\left[\bar{\varphi}(1 - \rho_g) - \bar{\kappa}\frac{1}{\eta}\right]}{\bar{\mathcal{D}}\rho_g\left[\bar{\kappa}\left(\frac{1}{\eta} + 1\right)\right] - (1 - \beta\rho_g)(1 - \bar{\mathcal{D}}\rho_g)}; \qquad a_2 = \frac{\bar{\kappa} + (1 - \beta\rho_g)b_2 + \bar{\varphi}(1 - \rho_g)}{\bar{\kappa}\left(\frac{1}{\eta} + 1\right)} \tag{D.4}
$$

$$
b_3 = \frac{-\bar{\lambda}(1 - \bar{\mathcal{D}})\rho_v}{\bar{\mathcal{D}}\rho_v\left[\bar{\kappa}\left(\frac{1}{\eta} + 1\right) + (1 - \beta\rho_v)\right] - (1 - \beta\rho_v)}; \qquad a_3 = \frac{\bar{\mathcal{D}}\rho_v b_3}{1 - \bar{\mathcal{D}}\rho_v} \tag{D.5}
$$

D.5.2 Unconditional Correlation: proof of Proposition 7

From the solution $\{a_i, b_i\}\ \forall i \in [1, 2, 3]$ derived in Proposition 6, we can see that inflation and output are positively correlated conditional on technology growth shocks and price-markup shocks. Positive correlation between inflation and output also obtains under government spending shocks if $\bar{\kappa} < \frac{\bar{\pi}z\bar{c}\delta(1 - \beta)}{\bar{R}\beta}$.

Proof. We can use the matrix equations to alternately rewrite output and inflation IRF to govt spending shock as follows.

$$a_2 = \frac{1 + \bar{\mathcal{D}}\rho_g(b_2 - 1)}{1 - \bar{\mathcal{D}}\rho_g}$$

Consequently, $a_2 > 0$ whenever $b_2 > 0$.

When $b_2 < 0$, a condition that guarantees that $a_2 < 0$ is $b_2 < -\frac{\bar{\kappa}}{(1-\beta)}$ (From D.4 and the fact that $\bar{\varphi} < 0$). Rewrite this condition, and substitute in the values of parameters to obtain the requirement that $\bar{\kappa} < \frac{\bar{\pi}z\bar{c}\delta(1-\beta)}{\bar{R}\beta}$ is sufficient for positive correlation between inflation and output.

□

D.5.3 Posterior distribution of model weights

Figure 8: *Model Weights Under MSV Criterio: Expectations Traps vs Secular Stagnation*
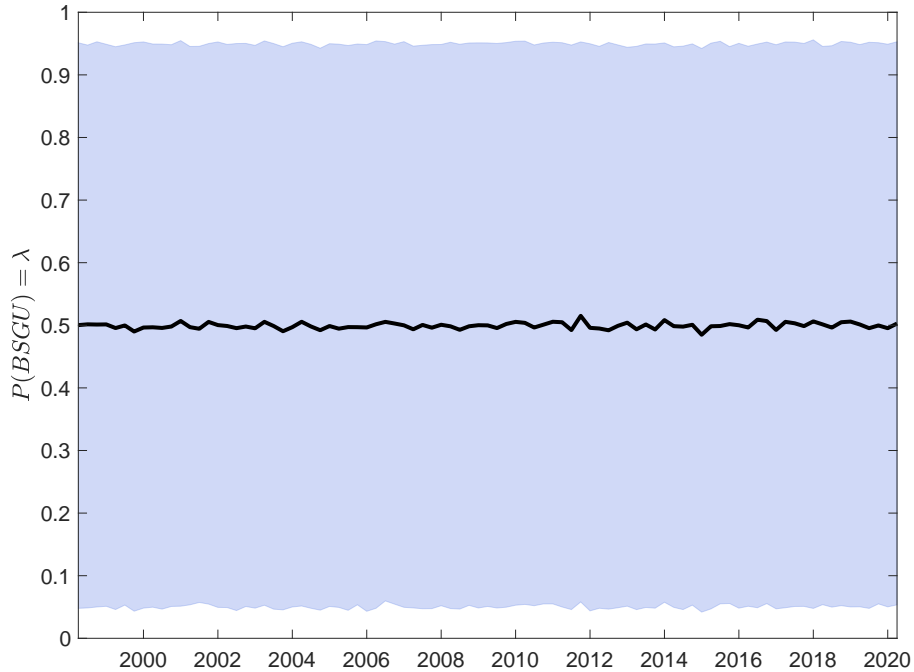
Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

D.6.   Prior distributions

Table 4 lists the priors used for estimation of the DSGE model of Section 4, including information on the marginal prior distributions for the estimated parameters. Under the prior, we

assume that all estimated parameters are distributed independently which implies that the joint prior distribution can be computed from the product of the marginal distributions.

Table 4: *Prior Distribution of DSGE parameters*

Parameters	Description	Distribution	P(1)	P(2)
ρ_g	Persistence gov. spending shock	$\mathcal{B}$	0.6	0.2
ρ_v	Persistence markup shock	$\mathcal{B}$	0.6	0.2
ρ_z	Persistence technology. growth shock	$\mathcal{B}$	0.4	0.1
σ_g	Std dev. gov. spending shock	$\mathcal{IG}$	0.004	Inf
σ_v	Std dev. markup shock	$\mathcal{IG}$	0.004	Inf
σ_z	Std dev. markup shock	$\mathcal{IG}$	0.004	Inf
σ_ζ	Std dev. sunspot shock	$\mathcal{IG}$	0.004	Inf
$corr(\epsilon_z, \epsilon_\zeta)$	Correl. sunspot	$\mathcal{U}$	0	0.5774
$corr(\epsilon_v, \epsilon_\zeta)$	Correl. sunspot	$\mathcal{U}$	0	0.5774
$corr(\epsilon_g, \epsilon_\zeta)$	Correl. sunspot	$\mathcal{U}$	0	0.5774

Notes: $\mathcal{G}$ is Gamma distribution; $\mathcal{B}$ is Beta distribution; $\mathcal{IG}$ is Inverse Gamma distribution; and $\mathcal{U}$ is Uniform distribution. P(1) and P(2) are mean and standard deviations for Beta, Gamma, and Uniform distributions. .

D.7. Posterior sampler

We can solve the log-linearized system of equations of Section 4 using standard perturbation techniques. As a result, the likelihood function can be evaluated with the Kalman filter. We generate draws from the posterior distribution using the random walk Metropolis algorithm (RWM) described in An and Schorfheide (2007). We scale the covariance matrix of the proposal distribution in the RWM algorithm to obtain an acceptance rate of approximately 60%. For posterior inference we generated 50,000 draws from the posterior distribution and discard the first 25,000 draws.

D.8. Comparative Statics: Expectations Trap vs Secular Stagnation

The BSGU and the secular stagnation hypotheses have contrasting implications for shocks and policy. These differences stem from the local determinacy property of these steady states, which translate into differences in slopes of aggregate supply and aggregate demand in our model. We now demonstrate these properties with comparative static experiments. Because of local determinacy of the secular stagnation steady state, the comparative static experiment is well-defined without the need for additional assumptions. With the BSGU steady state, we assume that inflation expectations do not change drastically to push the economy to the full-employment steady state in response to the experiment.

In Figure 9, solid lines plot the steady-state AD-AS representation of the quantitative model under two parametrizations. Annualized inflation deviation relative to the central

(a) *Expectations-Driven Trap*

(b) *Secular Stagnation*
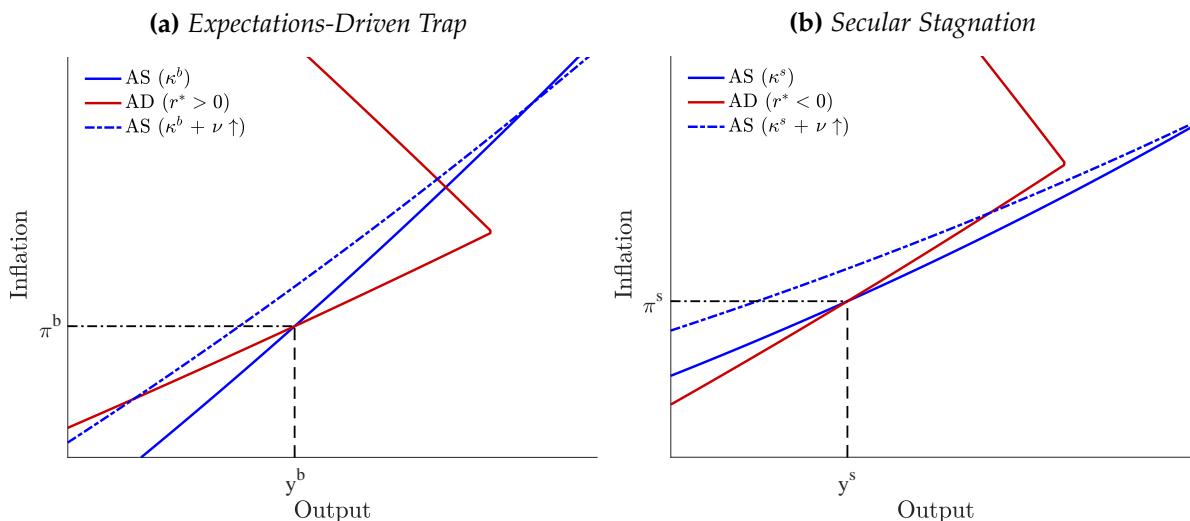
bank target inflation is on the vertical axes and output gap relative to target-steady state output (in percents) is on the horizontal axes. The coordinates (y^b, π^b) and (y^s, π^s) denote the expectations-driven and fundamentals-driven liquidity trap steady states respectively. The left panel plots AD-AS curves when prices are relatively flexible (κ^b) and the natural rate of interest is positive. The AD-AS intersection depicted at (y^b, π^b) is locally indeterminate, features zero nominal interest rates and output is permanently below potential. At this intersection, the AS curve is steeper than the AD curve. In the right panel, we plot the AS curve with relatively rigid prices (κ^s), and the AD curve with negative natural interest rate, $r^* < 0$. AD intersects AS at the secular stagnation steady state at the coordinate (y^s, π^s).

An upward shift in aggregate supply curve in Figure 9, denoted with dashed blue line, induced by permanent increase in steady state markups, translates into higher output under secular stagnation and lower output under BSGU. Under secular stagnation, the natural interest rate is too low for the central bank to stabilize the economy. An increase in markups through inflationary pressures helps lower real interest rate, thus reducing the real interest rate gap and expand output. Under BSGU, the problem is of pessimism about inflation expectations. If agents remain pessimistic about inflation undershooting its target, an increase in markups is further contractionary since the resource inefficiencies associated with increased markups dominate the increase in output demand due to higher prices (see also Mertens and Ravn, 2014).

An outward shift in aggregate demand in Figure 10, denoted with dashed red line, induced by permanent increase in steady state TFP growth, translates to higher output under secular stagnation but lower output under BSGU. Higher TFP growth signals higher income for households and leads to increased consumption demand. This increased impatience translates into higher output under secular stagnation. Under BSGU, in contrast, the increased TFP

Figure 10: *Permanent increase in TFP growth rate z*

(a) *Expectations-Driven Trap* **(b)** *Secular Stagnation*
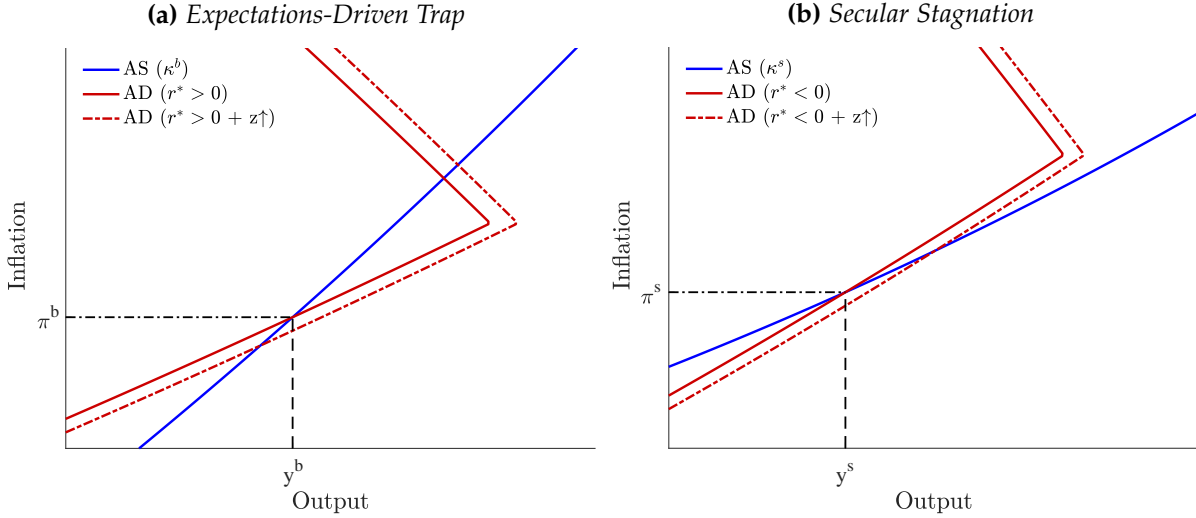
growth translates into higher reduction in prices by firms, which dominates the increased demand by households. As a result, there is lower output and inflation under BSGU.

Similarly, a neo-Fisherian exit policy of raising interest rates at the ZLB is contractionary under secular stagnation as it increases the real interest rate gap from natural rate, but it is expansionary at the BSGU steady state equilibrium (Schmitt-Grohé and Uribe, 2017) . Furthermore, an increase in government expenditure (financed by lumpsum taxes) or a permanent reduction in short term interest rates below the ZLB has inflationary effects under secular stagnation but deflationary effects under BSGU.[29]

These disparate policy implications raise the question whether it is possible to distinguish these two different kinds of liquidity traps in the data. We turn to this question next.

---

[29]We model the neo-Fisherian policy as a permanent change in the intercept of the Taylor rule, a: $R^{new} = \max\{1 + a, a + R^* \left(\frac{\Pi}{\Pi^*}\right)^{\phi_\pi}\} = a + R.$ where a is increased to a positive number from zero. This policy simultaneously increases the lower bound on nominal interest rate and thus does not have any effect on the placement of the kink in the aggregate demand curve. Given the inflation rate, an increase in a lowers output demanded. At the secular stagnation steady state, this induces deflationary pressures that increases the real interest rate gap and causes a further drop in output. In contrast, during a BSGU trap, an increase in nominal interest rate anchors agents' expectations to higher levels of inflation, thus obtaining the neo-Fisherian results (Schmitt-Grohé and Uribe, 2017). The effects of increased government spending on output are somewhat ambiguous because of elastic labor supply that also causes changes in the aggregate supply curve.

E. Medium-Scale DSGE Model

We present the equilibrium conditions of the model written in the form of stationary variables. Let A_t be the non-stationary level of TFP at time t. We normalize the following variables :

$$y_t = Y_t/A_t,$$

$$c_t = C_t/A_t,$$

$$k_t = K_t/A_t,$$

$$k_t^u = K_t^u/A_{t-1},$$

$$\mathbb{I}_t = I_t/A_t,$$

$$w_t = W_t/(A_tP_t),$$

$$r_t^k = R_t^k/P_t,$$

$$\lambda_t = \Lambda_t A_t,$$

Definition 1 (Normalized equilibrium). 17 endogenous variables $\{\lambda_t, i_t, c_t, y_t, \Pi_t, mc_t, \tilde{\Pi}_{t-1}, \Pi_t^w, \tilde{\Pi}_{t-1}^w, w_t, L_t, k_{t+1}^u, r_t^K, \mathbb{I}_t, q_t, u_t, k_t\}$, 6 endogenous shock processes $\{z_t, g_t, \eta_t^b, \mu_t, \nu_{p,t}, \nu_{w,t}\}$, 6 exogenous shocks $\{\epsilon_{z,t}, \epsilon_{g,t}, \epsilon_{\eta^b,t}, \epsilon_{\mu,t}, \epsilon_{\nu_p,t}, \epsilon_{\nu_w,t}\}$ given initial values of k_{t-1}^u.

Consumption Euler equation

$$\lambda_t = \beta(1+i_t)\mathbb{E}_t\left[\frac{\lambda_{t+1}}{z_{t+1}}\frac{1}{\Pi_{t+1}}\right] + \delta_t, \tag{E.1}$$

$$\lambda_t = \frac{1}{c_t - \frac{hc_{t-1}}{z_t}} - h\beta\mathbb{E}_t\frac{1}{z_{t+1}\left[c_{t+1} - \frac{hc_t}{z_{t+1}}\right]}, \tag{E.2}$$

Price-setting

$$(1-\nu_{p,t}) - mc_t + \nu_{p,t}\psi_p\left(\frac{\Pi_t}{\tilde{\Pi}_{t-1}} - 1\right)\frac{\Pi_t}{\tilde{\Pi}_{t-1}} - \nu_{p,t}\psi_p\beta\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\Pi_{t+1}}{\tilde{\Pi}_t} - 1\right)\frac{\Pi_{t+1}}{\tilde{\Pi}_t}\frac{y_{t+1}}{y_t} = 0 \tag{E.3}$$

$$\tilde{\Pi}_{t-1} = \bar{\Pi}^{1-\iota_p}\Pi_{t-1}^{\iota_p} \tag{E.4}$$

Wage-setting

$$\nu_{w,t}\psi_w\left[\frac{\Pi_t^w}{\tilde{\Pi}_{t-1}^w} - 1\right]\frac{\Pi_t^w}{\tilde{\Pi}_{t-1}^w} = \nu_{w,t}\psi_w\beta\mathbb{E}_t\left[\frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w} - 1\right]\frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w} + L_t\lambda_t\left[\omega\frac{L_t^{\frac{1}{\eta}}}{\lambda_t} - (1-\nu_{w,t})w_t\right] \tag{E.5}$$

$$\tilde{\Pi}_{t-1}^w = z\bar{\Pi}^{1-\iota_w} \left(\exp(\epsilon_{z,t})\Pi_{t-1}\right)^{\iota_w} \tag{E.6}$$

$$\Pi_{W,t} = \frac{w_t}{w_{t-1}}\Pi_t z_t, \tag{E.7}$$

Capital investment

$$k_{t+1}^u = \mu_t \left[1 - S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{z_t}{z}\right)\right]\mathbb{I}_t + (1-\delta_k)\frac{k_t^u}{z_t}, \tag{E.8}$$

$$q_t = \beta\mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t z_{t+1}}\left(r_{t+1}^K u_{t+1} - a(u_{t+1}) + q_{t+1}(1-\delta_k)\right)\right], \tag{E.9}$$

$$q_t\mu_t \left[1 - S\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{z_t}{z}\right) - S'\left(\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{z_t}{z}\right)\frac{\mathbb{I}_t}{\mathbb{I}_{t-1}}\frac{z_t}{z}\right]$$
$$+\beta\mathbb{E}_t \left[\mu_{t+1}\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}\frac{z_{t+1}}{z}\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t}\right)^2 S'\left(\frac{\mathbb{I}_{t+1}}{\mathbb{I}_t}\frac{z_{t+1}}{z}\right)\right] = 1 \tag{E.10}$$

Capital utilization rate

$$k_t = u_t\frac{k_t^u}{z_t}, \tag{E.11}$$

$$r_t^K = a'(u_t), \tag{E.12}$$

Production technologies

$$y_t = A_t k_t^\alpha L_t^{1-\alpha}, \tag{E.13}$$

$$r_t^k = \alpha mc_t\frac{y_t}{k_t}, \tag{E.14}$$

$$w_t = (1-\alpha)mc_t\frac{y_t}{L_t}, \tag{E.15}$$

Government

$$\frac{1+i_t}{1+i_{ss}} = \max\left(\frac{1}{1+i_{ss}}, \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\rho_R}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{y_t z_t}{z y_{t-1}}\right)^{\phi_{dy}}\right]^{1-\rho_R}\exp(\epsilon_{mp,t})\right), \tag{E.16}$$

Market clearing

$$y_t = c_t + \mathbb{I}_t + a(u_t)\frac{k_t^u}{z_t} + \left(1 - \frac{1}{g_t}\right)y_t, \tag{E.17}$$

Law of motion of Shocks   The six structural shocks driving the model economy are assumed to follow first order auto-regressive processes of the form $\log(x_t) = (1 - \rho_x) \log(x) + \rho_x log(x_{t-1}) + \sigma_x \epsilon_{x,t}$, with $\epsilon_{x,t} \sim N(0,1)$, and x denoting steady-state values, for $x = z, g, \eta^b, \mu, \nu_p, \nu_w$.

Parameters   Table 5 details the values of the parameters for the medium-scale DSGE model of section 6.

<div align="center">

Table 5: *Parameters Medium-Scale DSGE Model*

</div>

Parameter	Description	Value
β	Discount factor	0.942
δ	Marginal utility of bonds	0.109
ν_p	Price markup parameter	0.22
ν_w	Wage markup parameter	0.2
α	Capital elasticity of output	0.37
δ_k	Capital Depreciation rate	0.015
h	Consumption habit	0.358
ν	Inverse of Frisch labor elasticity	2.27
ω	Labor disutility	0.492
σ_a	Capital utilization elasticity	2.246
ψ_I	Investment adjustment cost	5.16
ψ_p	Price adjustment cost	1600
ψ_w	Wage adjustment cost	833.73
ι_p	Indexation on price inflation	-0.225
ι_w	Indexation on wage inflation	-0.295
ρ_w	Persistence wage-markup shock	0.252
ρ_p	Persistence price-markup shock	0.268
ρ_g	Persistence government spending shock	0.843
ρ_μ	Persistence investment shock	0.353
ρ_z	Persistence technology shock	0
ρ_b	Persistence risk premium shock	0.488
$100\sigma_w$	Standard deviation wage-markup shock	0.338
$100\sigma_p$	Standard deviation price-markup shock	0.394
$100\sigma_g$	Standard deviation government spending shock	0.45
$100\sigma_\mu$	Standard deviation investment shock	4.269
$100\sigma_z$	Standard deviation technology shock	1.641
$100\sigma_b$	Standard deviation risk premium shock	0.351

F.   SIMPLE MODEL WITH DOWNWARD NOMINAL WAGE RIGIIDTY

We now derive the results presented in Section 2 with wage-setting frictions and inelastic labor instead of price-rigidities and elastic labor. Our model is a variant of the downward-nominal

wage rigidity apparatus introduced by Schmitt-Grohé and Uribe (2017).

Time is discrete and there is no uncertainty. Suppose the representative agent supplies labor $\bar{h} = 1$ inelastically and maximizes the following utility function choosing consumption good C_t and one-period (real) risk-free government bonds b_t:

$$\max_{\{C_t, b_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t \left[\log C_t \right]$$
$$\theta_0 = 1;$$
$$\theta_{t+1} = \hat{\beta}(\tilde{C}_t)\theta_t \forall t \geq 0$$

where θ_t is an endogenous discount factor (Uzawa, 1968; Epstein and Hynes, 1983), C_t is consumption, $\tilde{C}_t$ is average consumption that the household takes as given, and h_t is hours. For tractability, we assume a linear functional form for $\hat{\beta}(\cdot) = \delta_t \beta C_t$, where $0 < \beta < 1$ is a parameter, and $\delta_t > 0$ are exogenous shocks to the discount factor. The household earns wage income $W_t h_t$, interest income on past bond holdings of risk-free government bonds b_{t-1} at gross nominal interest rate R_{t-1}, dividends Φ_t from firms' ownership and makes transfers T_t to the government. Π_t denotes gross inflation rate. The period by period (real) budget constraint faced by the household is given by

$$C_t + b_t = \frac{W_t}{P_t} h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + \Phi_t + T_t$$

An interior solution to household optimization yields the Euler equation:

$$1 = \beta(\tilde{C}_t)\mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$

In equilibrium, individual and average per consumption are identical, i.e. $C_t = \tilde{C}_t$. The Euler equation simplifies to:

$$1 = \delta_t \beta C_t \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right]$$

Consumption goods are produced by competitive firms with labor as the only input using the technology

$$Y_t = F(h_t) = h_t^{\alpha}, \quad \text{where } 0 < \alpha < 1$$

These firms set price of the final good P_t to equate marginal product of labor to the marginal cost.

$$F'(h_t) = \frac{W_t}{P_t}$$

We introduce a very stylized form of downward nominal wage rigidity (following Schmitt-Grohé and Uribe 2017):

$$W_t \geq (1 - \kappa + \kappa(1 - u_t)^{\alpha})W_{t-1} \equiv \tilde{\gamma}(u_t)W_{t-1}$$

where $\kappa > 0$, and $u_t \equiv 1 - \frac{h_t}{\bar{h}}$ is involuntary unemployment. This downward rigidity implies that employment cannot exceed the total labor supply in the economy i.e. $h_t \leq 1$. We further assume that the following slackness condition holds:

$$(\bar{h} - h_t)(W_t - \tilde{\gamma}(u_t)W_{t-1}) = 0$$

We close the model by assuming a government that balances budget,

$$b_t + T_t = \frac{R_{t-1}}{\Pi_t}b_{t-1}$$

and a monetary authority that sets nominal interest rate on the net zero supply of nominal risk-free one-period bonds using the following Taylor rule

$$R_t = \max\{1, (1+r^*)\Pi_t^{\phi_\pi}\}$$

where $(1+r^*) \equiv \frac{1}{\delta_t\beta}$ is the natural interest rate, and $\phi_\pi > 1$.[30] The zero lower bound (ZLB) constraint on the short-term nominal interest rate introduces an additional nonlinearity in the policy rule. Finally, we assume that the resource constraints hold in the aggregate:

$$C_t = Y_t, \quad \text{and} \quad b_t = 0.$$

F.1. Equilibrium

Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. The competitive equilibrium is given by the sequence of seven endogenous processes $\{C_t, Y_t, R_t, \Pi_t, h_t, w_t, u_t\}$ that satisfy the conditions (F.1) - (F.7) for a given exogenous sequence of process $\{\delta_t\}_{t=0}^\infty$ and the initial condition w_{-1}:

$$1 = \delta_t\beta C_t \mathbb{E}_t\left[\frac{C_t}{C_{t+1}}\frac{R_t}{\Pi_{t+1}}\right] \tag{F.1}$$

$$Y_t = h_t^\alpha. \tag{F.2}$$

$$\alpha h_t^{\alpha-1} = w_t \tag{F.3}$$

$$h_t \leq \bar{h}, \quad w_t \geq (1 - \kappa + \kappa h_t^\alpha)\frac{w_{t-1}}{\Pi_t}, \quad (\bar{h} - h_t)\left(w_t - (1 - \kappa + \kappa h_t^\alpha)\frac{w_{t-1}}{\Pi_t}\right) = 0 \tag{F.4}$$

$$u_t = 1 - \frac{h_t}{\bar{h}} \tag{F.5}$$

$$R_t = \max\{1, (1+r^*)\Pi_t^{\phi_\pi}\}\} \tag{F.6}$$

$$Y_t = C_t \tag{F.7}$$

where the exogenous sequence of natural interest rate is given by $1 + r_t^* \equiv \frac{1}{\delta_t\beta}$.

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[30]The natural interest rate is defined as the real interest rate on one-period government bonds that would prevail in the absence of nominal rigidities.

F.2. Non-stochastic steady state

In the steady state, we can simplify the system of equations to an aggregate demand block and an aggregate supply block.

Aggregate Demand (AD) is a relation between output and inflation and is derived by combining the Euler equation and the Taylor rule. Mathematically, the AD curve is given by

$$Y_{AD} = \frac{1}{\beta\delta} \begin{cases} \frac{1}{(1+r^*)\Pi^{\phi\pi-1}}, & \text{if } R > 1, \\ \Pi, & \text{if } R = 1 \end{cases} \tag{F.8}$$

When ZLB is not binding, the AD curve has a strictly negative slope, and it becomes linear and upward sloping when the nominal interest rate is constrained by the ZLB. The kink in the aggregate demand curve occurs at the inflation rate at which monetary policy is constrained by the ZLB: $\Pi_{kink} = \left(\frac{1}{(1+r^*)}\right)^{\frac{1}{\phi\pi}}$. When $1 + r^* > 1$, the kink in the AD curve occurs at an inflation rate below 1. For the natural interest rate to be positive, the patience parameter must be low enough i.e. $\delta < \frac{1}{\beta}$. The dashed red line in panel a) of Figure 11 plots the aggregate demand curve with a positive natural rate.

Aggregate Supply (AS): Because of the assumptions of downward nominal wage rigidity and capacity constraints on production, the AS curve features a kink at full employment level of output and gross inflation rate equal to one. When inflation rate is less than one, the downward wage rigidity constraint becomes binding. As a result, inflation cannot fall to completely adjust any demand deficiency and firms layoff workers. The aggregate supply curve can be summarized by:

$$Y_{AS} \leq 1, \quad \Pi \geq (1-\kappa) + \kappa Y_{AS}, \quad (Y_{AS}-1)\left(1 - (1-\kappa+\kappa Y_{AS})\frac{1}{\Pi}\right) = 0 \tag{F.9}$$

When $h = \bar{h} \equiv 1$, $\Pi \geq 1$. The AS curve is a vertical line at full employment. For $h < 1$, $\Pi = (1 - \kappa + \kappa h^\alpha)$. The AS curve is linear and upward sloping with slope=κ for $y < 1$. The kink in the AS curve occurs at the coordinate $Y = 1, \Pi = 1$. Because of this assumed linear aggregate supply curve under deflation, the degree of nominal rigidity κ also determines the lower bound on inflation $(1 - \kappa)$. The solid blue line in both panels of Figure 11 plots the aggregate supply curve.

Note that the equilibrium conditions are similar to those presented in Section 2, with the major exception being that the steady state AS graph is vertical at $h = 1$ due to the upper bound on labor endowment. One can derive similar results as in Section 2. We only note the following proposition to complete proofs for statements regarding robust minimum wage policy in Section 2.8.

Proposition 8. (Minimum wage policy): Let $0 < \delta < \frac{1}{\beta}$. A minimum income policy that installs a lower bound $1 - \kappa$ on nominal wage growth can preclude the expectations trap.

Figure 11: *Steady-State Representation*

(a) *Secular Stagnation*

(b) *Expectations-Driven Trap*
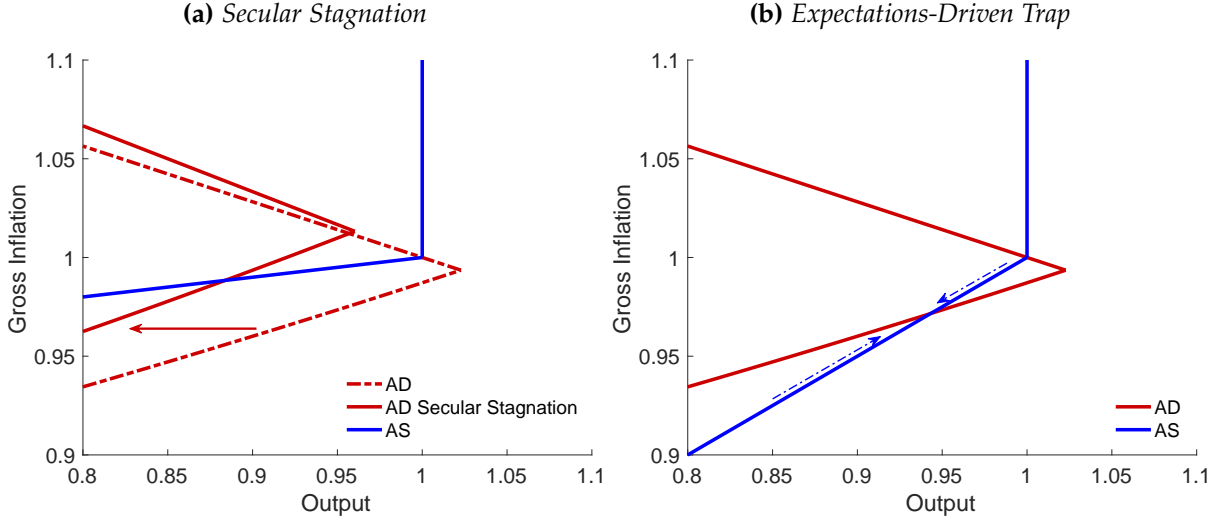
Proof. The downward sloping portion of AD curve goes through $Y = 1$, $\Pi = 1$, and so does the vertical portion of the AS curve. When $0 < \delta < \frac{1}{\beta}$, $1 + r^* > 1$. The kink in the AD curve occurs at inflation rate below 1. Thus, there always exists an intersection between the AS and the AD at $Y = 1$ and $\Pi = 1$. To show that there does not exist another equilibrium, note that, AD is linear and upward sloping when ZLB is binding and AS is also linear and upward sloping for gross inflation below 1. Furthermore for $\Pi_{kink} \leq \Pi < 1$, $Y_{AD} > 1 > Y_{AS}$. And for $\Pi = 1 - \kappa$, $Y_{AS} = 0 < Y_{AD}$. Thus there does not exist a steady state with zero nominal interest rate.

□

G.   Expectations trap in an OLG model

The degree of nominal rigidities also plays a key role in eliminating the locally indeterminate stagnation steady state in the overlapping generations model of Eggertsson et al. (2019) (EMR). We outline the key message here while referring the reader to EMR for a detailed model. Agents live for three periods: young, middle, and old. Young are borrowing constrained and derive no income. Middle supply labor inelastically to perfectly competitive firms for wages and save for retirement. Old consume the savings made when middle. Supply and demand for savings results in the following bond market-clearing condition : $1 + r_t = \frac{1+\beta}{\beta} \frac{D_t}{Y_t - D_{t-1}}$, where D is the exogenous debt limit faced by the young borrowers. It is further assumed that households do not accept nominal wages below a particular wage norm i.e. $W_t = \max\{\bar{W}_t, W_t^{flex}\}$ where $\bar{W}_t = \gamma W_{t-1} + (1 - \gamma)W_t^{flex}$ and $W_t^{flex} = P_t\alpha$. Perfectly competitive firms hire workers to produce final output using production function $Y_t = h_t^\alpha$, taking wages as given. The policy rule is the same as in our baseline model in Section 2.

Given inflation target $\Pi^* = 1$, the aggregate demand and the aggregate supply blocks in the steady state are given by:

$$Y_{AD} = D + \begin{cases} \frac{1+\beta}{\beta} \frac{D}{\Gamma^*} \Pi^{1-\alpha_\pi}, & \text{if R} > 1, \\ \frac{1+\beta}{\beta} D\Pi, & \text{if R} = 1 \end{cases} \tag{G.1}$$

where $\Gamma^* = (1 + r^*)^{-1}$.

$$Y_{AS} = \begin{cases} 1, & \text{if } \Pi \geq 1, \\ \left(\frac{1 - \frac{\gamma}{\Pi}}{1-\gamma} \right)^{\frac{\alpha}{1-\alpha}}, & \text{if } \Pi < 1 \end{cases} \tag{G.2}$$

If $\Pi^* = 1$, $r^* > 0$, and $\gamma < 0$, then there exists a unique liquidity trap steady state with positive unemployment, deflation, and zero nominal interest rate. The dynamics around this steady state are locally indeterminate. A negative value of γ implies that nominal wages increasingly fall with unemployment as in Schmitt-Grohé and Uribe (2017).[31]

H. Comparison with the textbook Euler equation

We provide a brief comparison of results for the reader with the textbook Euler equation (Woodford, 2003). We illustrate the role of two central elements in our framework - a) the modified Euler equation and b) bounds on deflation.

In the textbook model, the natural interest rate is always fixed at $\frac{1}{\beta} > 1$. As a result, the aggregate demand relationship is a horizontal line at $\Pi = \beta < 1$ when the ZLB constrains the nominal interest rate. However, the existence of an *unintended* deflationary steady state is contingent on the assumptions regarding the supply side of the economy. Suppose the y-intercept of the aggregate supply curve is large enough. In that case, there does not exist a deflationary steady state.[32] Setting this y-intercept is analogous to a minimum wage policy discussed in Section 2.8.

While modifying the Euler equation does not eliminate the expectations-trap steady state, it is necessary for the secular stagnation steady state to exist. The modified Euler equation with an endogenous long-run natural interest rate opens up the possibility of a secular stagnation steady state. This steady state cannot arise in the standard model because of a violation of the transversality condition of the representative household.

Bonds-in-utility is an alternative way to introduce discounting in the Euler equation (Michaillat and Saez, 2021; Michau, 2018; Ono and Yamada, 2018). Time variation in the preference for bonds, captured by δ in our model, has a functional equivalence with risk-premium in medium-scale DSGE models (Fisher, 2015).[33] Another interpretation of the shocks to δ is

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[31] Ascari and Bonchi (2020) study the use of income or wage growth policies to reflate an economy experiencing persistent ZLB in the EMR model.

[32] In the notation of Schmitt-Grohé and Uribe 2017, if $\gamma(1) > \tilde{\beta}$ there does not exist an unemployment steady state in their baseline model.

[33] It is straightforward to show similar functional equivalence between preferences with endogenous

that these capture the *flight to liquidity* episode of the recent financial crisis (Krishnamurthy and Vissing-Jorgensen, 2012). A similar wedge in the Euler equation can be associated with the deterioration in liquidity properties of AAA-rated corporate bonds in contrast to Treasury securities during the 2008-09 financial crisis (Del Negro et al., 2017).

While these different interpretations follow naturally from extensive work in the secular stagnation literature. We view this wedge in the Euler equation as a reduced-form representation of microfoundations such as population aging, savings glut, reserve accumulation, inequality, or debt deleveraging (Eggertsson et al. 2016, 2019; Auclert and Rognlie 2018). Remaining agnostic about the origin of this wedge allows researchers to easily introduce the secular stagnation hypothesis in DSGE models.[34]

I. Decreasing marginal impatience with capital accumulation

For completeness, we sketch the setup to show existence of a stable equilibrium with Uzawa-Epstein preferences under decreasing marginal impatience. We switch to continuous time formulation to analytically show stability

I.1. Simple model of Secular Stagnation

First, we abstract from capital and shut down any asset accumulation. Household maximizes the following lifetime utility

$$\int_0^\infty e^{-\int_0^t \rho(c_v)dv} u(c_t)dt$$

where $\int_0^t \rho(c_v)dv$ is the time-preference term that depends on the past and present consumption function through the function ρ. The function $\rho(c_t)$ denotes the instantaneous rate of time preference and it depends on c_t alone.

Assumption 1. (Das 2003) The functions $u(c)$ and $\rho(c)$ are real valued bounded above and twice continuously differentiable on $(0, \infty)$, and

1. for all $c > 0$, $u(c) > 0$; $u'(c) > 0$; $u''(c) < 0$

2. for all $c > 0$, $\rho(c) > 0$, $\rho'(c) < 0$; and $\rho''(c) \leq 0$. Also $\rho(0) = \bar{\rho}$; the upper bound on ρ.

These assumptions are standard, except for decreasing marginal impatience $\rho'(c) < 0$. Note that the upper bound on ρ will be crucial to guarantee the existence of a unique equilibrium.

The budget constraint of the household is:

$$\dot{a}_t = r_t a_t + w_t - c_t$$

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discounting, and preferences with wealth in the utility function.

[34]In the recent literature that augments DSGE models with endogenous growth, (mean zero) shocks to preference for bonds are added to get co-movement of investment and consumption as well to derive the *divine coincidence* benchmark (Garga and Singh, 2021).

where $a(0) = 0$. Let $\theta(t) = \int_0^t \rho(c_v)dv$. Then $\dot{\theta} = \rho(c_t)$ and $\theta(0) = 0$. Thus the optimization exercise of the household is

$$\max \int_0^\infty u(c_t)e^{-\theta(t)}dt$$

subject to the budget constraint $\dot{a}_t = r_t a_t + w_t - c_t$, $\dot{\theta} = \rho(c_t)$, $a(0) = 0$ and $\theta(0) = 0$.

The Hamiltonian function is

$$H = u(c_t)e^{-\theta(t)} + \gamma (r_t a_t + w_t - c_t) - \lambda(\rho(c))$$

where γ and $-\lambda$ are the co-state variables associated with the two variables a and θ, respectively. Rescaling the co-state variables so that the terms involving θ get eliminated, we can write the first-order conditions as

$$u'(c) - \mu - \phi\rho'(c) = 0 \tag{I.1}$$

$$\dot{\mu} = \mu [\rho(c) - r_t] \tag{I.2}$$

$$\dot{a}_t = r_t a_t + w_t - c_t \tag{I.3}$$

$$\dot{\phi} = \phi\rho(c) - u(c) \tag{I.4}$$

where $\mu = \gamma e^\theta$ and $\phi = \lambda e^\theta$ are the new co-state variables. Differentiating the first condition wrt time and rewriting:

$$\dot{c} = \frac{u'(c) - \phi\rho'(c)}{-u''(c) + \phi\rho''(c)} \left[r - \rho(c) - \rho'(c)\frac{\phi\rho(c) - u(c)}{u'(c) - \phi\rho'(c)} \right] \tag{I.5}$$

Assuming $\rho(C) = \bar{\rho} - \frac{\kappa}{u'(C)}$ and $u(c) = \log(c)$. Thus $\rho''(c) = 0$, $\rho(0) = \bar{\rho}$, and $\rho'(c) = \kappa$. thus, Equations I.3, I.4 and I.5, represent a system of differential equations involving three variables a, c, and ϕ. In the steady state. $\dot{c} = 0$, $\dot{a} = 0$, and $\dot{\phi} = 0$. The steady state is thus characterized by the following equations:

$$\phi = \frac{u(c)}{\rho(c)} \tag{I.6}$$

$$c = ra + w, \tag{I.7}$$

$$r - \rho(c) - \rho'(c)\frac{\phi\rho(c) - u(c)}{u'(c) - \phi\rho'(c)} = 0 \tag{I.8}$$

These simplify to give us:

$$r = \rho(c) = \bar{\rho} - \kappa C$$

This is exactly same as equation 4 in Michau (2018). A strictly positive increase in κ implies a finite elasticity of steady state consumption c with respect to steady state interest rate r. Note that we shut down wealth accumulation $a = 0$ in our model to generate this steady state. As in Michau, we also assume zero pure profits. Presence of pure profits will slightly modify the household budget constraint without any loss of results shown here. Following

the arguments in Section 2 of Michau (2018), one can show the existence of secular stagnation steady state.

I.2. A model of Secular Stagnation with capital

Household wealth is composed of physical capital K_t, and government bonds B_t .

$$\dot{a}_t = r_t a_t + w_t l_t + \psi_t - c_t$$

where ψ_t are real lump-sum transfers. household wealth will also include capital K_t, with real price $p_t^K = \frac{P_t^K}{P_t}$. Households are indifferent between holding risk-free capital and risk-free government bonds. Hence no-arbitrage implies that the two assets must yield the same return

$$r_t = \frac{R_t/P_t}{p_t^K} - \delta + \frac{\dot{p}_t^K}{p_t^K}$$

where δ is the capital depreciation rate. The intertemporal budget constraint prevents households from running Ponzi schemes:

$$\lim_{t \to \infty} e^{-\int_0^t r_s ds} a_t \geq 0$$

We will have the same utility function for the representative household as in the above sub-section:

$$\max \int_0^\infty u(c_t) e^{-\theta(t)} dt$$

The first-order conditions by the the maximum principle are:

$$\dot{c} = \frac{u'(c) - \phi \rho'(c)}{-u''(c) + \phi \rho''(c)} \left[r - \rho(c) - \rho'(c) \frac{\phi \rho(c) - u(c)}{u'(c) - \phi \rho'(c)} \right] \tag{I.9}$$

$$\dot{\phi} = \phi \rho(c) - u(c) \tag{I.10}$$

$$\dot{a}_t = r_t a_t + w_t l_t + \psi_t - c_t \tag{I.11}$$

along with the transversality condition

$$\max \int_0^\infty u(c_t) e^{-\theta(t)} dt$$

Steady state equilibria

$$\phi = \frac{u(c)}{\rho(c)} \tag{I.12}$$

$$c = F(L, K) - \delta K, \tag{I.13}$$

$$r - \rho(c) - \rho'(c)\frac{\phi\rho(c) - u(c)}{u'(c) - \phi\rho'(c)} = 0 \tag{I.14}$$

$$r = i - \pi \tag{I.15}$$

$$r_t = F_K - \delta \tag{I.16}$$

$$w_t = \frac{W_t}{P_t} = \begin{cases} F_L, & \text{if } \Pi \geq 1, \\ \frac{\kappa}{\Pi + \kappa - 1}, & \text{if } \Pi \in [1 - \kappa, 1] \end{cases} \tag{I.17}$$

Let $f(k) = F(K, L)/L$, where production function F is assumed to be homogenous in labor. As in Das (2003), we assume that

$$-f''(k) > -\rho'(f(k) - \delta k)(f'(k) - \delta).$$

This condition states that the marginal return to capital decreases faster than marginal impatience. As in Schmitt-Grohé and Uribe (2003), if we further assume that the discounting is exogenous to individual agents, the Euler equation simplifies to

$$\dot{c} = \frac{u'(c)}{-u''(c)}[r - \rho(c)]$$

The resulting system is observationally equivalent to that of Michau (2018) which shows existence of secular stagnation steady state.