The Financial Origins of Non-Fundamental Risk

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What we do

formalize idea that the financial sector can be a *source* of risk, rather than a means to manage fundamental risk (Rajan (2005), Danielsson and Shin (2003))

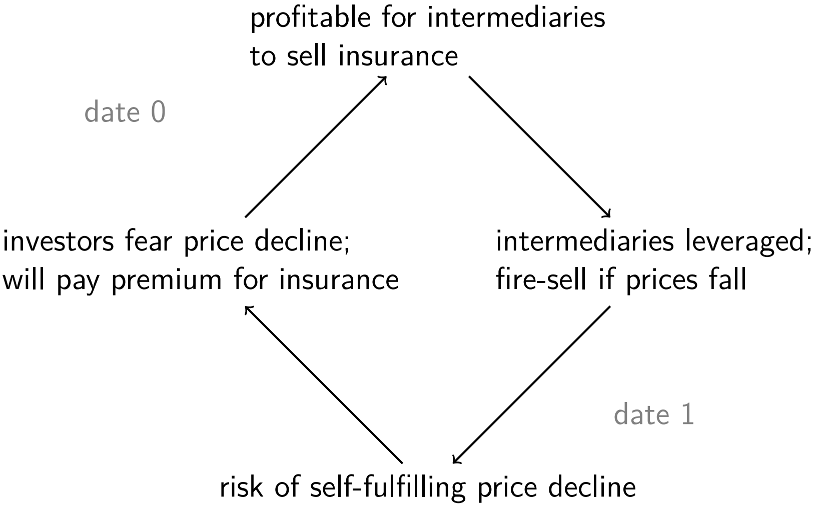
stylized 3-period model, key ingredients:

- risk-averse investors can buy insurance/safe assets from risk-neutral intermediaries
- no fundamental shocks (can relax this)

result: there exists a nonfundamental equilibrium in which

- asset prices sometimes fall below fundamental value as intermediaries fire-sell assets
- investors buy insurance against this risk
- but prices can only fall *because* intermediaries issue insurance

Key mechanism



Related literature

Sunspot eqba can arise from trade in assets w price-contingent payoffs (Bowman & Faust 1997) or sunspot-contingent payoffs (Hens 2000)

- we show trade in assets w *non-contingent payoffs* can also cause sunspot eqba

Pecuniary externalities with financial frictions (Lorenzoni 2008, Stein 2012, Dávila & Korinek 2018): mostly study fundamental shocks, rule out multiplicity

- multiple equilibria w sunspots

Multiple equilibria with financial frictions in small open economies (Bocola & Lorenzoni 2020, Schmitt-Grohé & Uribe 2020)

- closed economy, no binding financial constraint, different source of multiplicity

Demand and supply of safe assets (Caballero & Farhi 2018, Acharya & Dogra 2020,...)

- demand for safe assets $\leftarrow$ nonfundamental risk has different (policy) implications

Roadmap

1. Baseline model w/o insurance: only fundamental equilibria, no price volatility
2. Add trading of insurance contracts $\rightarrow$ non-fundamental equilibria w price volatility
3. Extend to non-state-contingent contracts
4. Policy
5. Conclusion

Environment

- 3 dates: 0, 1, and 2
- 3 agents:
  1. risk-averse households (HHs)
  2. risk-neutral financial intermediaries (FIs)
  3. outside investors (OIs) *do not* trade at date 0
- fixed endowment of cookies (c) at both dates
- unit endowment of trees (e) at date 0
- 1 tree $\rightarrow$ 1 cookie (c) at date 2
- trees can be traded at dates 0 and 1
- **no exogenous source of risk**

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model roadmap:

1. baseline model w/o insurance: trees are only asset traded
2. add trading of insurance contracts
3. discuss non-state-contingent contracts

Households

Born with large endowment χ_0^h of cookies, all trees; consume only at dates 0 and 1

Risk-averse: Epstein-Zin utility with IES $= \infty$ (can generalize)

$$\max c_0^h + \left[\mathbb{E}(c_1^h)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \gamma > 1$$

s.t.

$$\begin{aligned} c_0^h + p_0 e^h &= \chi_0^h + p_0 \\ c_1^h &= p_1 e^h \\ c_0^h, c_1^h, e^h &\geq 0 \end{aligned}$$

don't consume at date 2 $\Rightarrow$ date 0 valuation of tree depends on expected date 1 price:

$$p_0 = \frac{\mathbb{E} p_1 c_1^{-\gamma}}{\left[\mathbb{E} c_1^{1-\gamma}\right]^{\frac{-\gamma}{1-\gamma}}} = \left[\mathbb{E} p_1^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

Born with small endowment χ_0^f of cookies, no trees; can consume at all dates

Risk-neutral

$$\max c_0^f + \mathbb{E}\left(c_1^f + c_2^f\right)$$

s.t.

$$\begin{aligned} c_0^f + p_0 e_0^f &= \chi_0^f \\ c_1^f + p_1 e_1^f &= p_1 e^f \\ c_2^f &= e_1^f \\ c_0^f, c_1^f, e_0^f, e_1^f &\geq 0 \end{aligned}$$

Optimality conditions:

- Date 1: sell all trees if $p_1 > 1$, consume all trees if $p_1 < 1$
- Date 0: buy only trees if $p_0 < \mathbb{E}\max\{1, p_1\}$, don't buy any if $p_0 > \mathbb{E}\max\{1, p_1\}$

Outside Investors

only agents w cookies χ_1 at date 1; trade and consume at dates 1 and 2 (Stein, 2012)

$$\max c_1^o + v(c_2^o)$$

s.t.

$$\begin{aligned} c_1^o + p_1 e_1^o &= \chi_1 \\ c_2^o &= e_1^0 \end{aligned}$$

where $v'(\cdot) > 0$, $v''(\cdot) < 0$, $v'(0) > 1 > v'(1) := \underline{p}$

- Optimal demand for trees implies $p_1 = v'(e_1^o)$
- Define $\overline{e}$ s.t. $v'(\overline{e}) = 1$

Equilibrium

prices $\{p_0, p_1\}$ and quantities $\{c_0^h, c_1^h, e_0^h, c_0^f, c_1^f, e_1^f, e_0^f, c_1^o, e_1^o\}$ s.t. all agents optimize and prices clear:

$$
\begin{aligned}
c_0^h + c_0^f &= \chi_0^h + \chi_0^f \\
c_1^h + c_1^f + c_1^o &= \chi_1 \\
e_0^h + e_0^f &= 1 \\
e_1^o + e_1^f &= 1
\end{aligned}
$$

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$$\begin{aligned} c_0^h + c_0^f &= \chi_0^h + \chi_0^f \\ c_1^h + c_1^f + c_1^o &= \chi_1 \\ e_0^h + e_0^f &= 1 \\ e_1^o + e_1^f &= 1 \end{aligned}$$

Lemma (Date 1 price of trees)
In equilibrium, $p_1 = \min\{1, v'(e_0^h)\}$.

Equilibrium in the market for trees

HHs' demand: From FOC:

$$p_0 = p_1 = \min\{1, v'(e_0^h)\}$$

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FIs' demand: Since $p_1 \leq 1$, FIs spend everything on trees at date 0 ($p_0(1 - e_0^h) = \chi_0^f$) if $p_0 < 1 \Rightarrow$

$$p_0 = \min\left\{\frac{\chi_0^f}{1 - e_0^h}, 1\right\}$$

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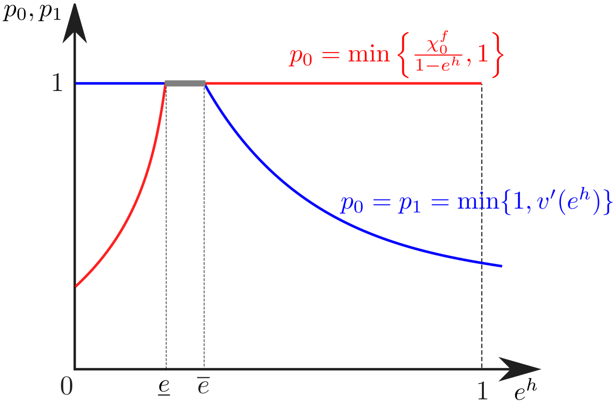

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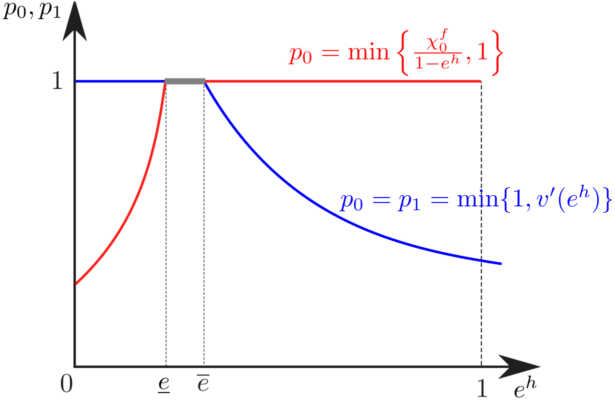

fundamental equilibria: $p_0 = p_1 = 1$ and $e_0^h \in [\underline{e}, \overline{e}]$, where $\underline{e} = 1 - \chi_0^f$, $v'(\overline{e}) = 1$

welfare: $U^h = \chi_0^h + 1, \quad U^f = \chi_0^f, \quad U^o = v(\overline{e}) - \overline{e}$

Introducing insurance contracts

- When trees are the only asset traded, they are safe ($p_1 = 1$) and only fundamental equilibria exist
- Now allow FIs to sell insurance contracts z^f at date 0 at price q
    - 1 insurance contract pays $1 - p_1$ cookies at date 1 if $p_1 < 1$
    - 1 insurance contract $+$ 1 tree is worth 1 cookie at date 1 for sure.

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- If HHs expect $p_1 = 1$ for sure, this belief is self-confirming, insurance is not used and has a price $q = 0$, and we have the same set of fundamental equilibria
- But there are other equilibria...

Introducing insurance contracts

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$$\textbf{HHs}: \max_{c_0^h, e_0^h, z_0^h, c_1^h} \left[c_0^h + \left(\mathbb{E}(c_1^h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right]$$

$$\begin{aligned} \text{s.t. } & c_0^h + p_0 e_0^h + q z_0^h = \chi_0^h + p_0 \\ & c_1^h = p_1 e_0^h + (1 - p_1) z_0^h \\ & c_0^h, c_1^h, e_0^h \geq 0 \end{aligned}$$

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FIs : $$\max_{c_0^f, e_0^f, z_0^f, c_1^f} \left[c_0^f + \mathbb{E} \left(c_1^f + e_1^f \right) \right]$$

$$\text{s.t.} \quad c_0^f + p_0 e_0^f = \chi_0^f + q z_0^f$$
$$c_1^f + p_1 e_1^f + (1 - p_1) z_0^f = p_1 e_0^f$$
$$c_0^f, c_1^f, e_1^f \geq 0$$
$$\text{(implies } \underbrace{(1 - p_1) z^f}_{\text{insurance payout}} \leq \underbrace{p_1 e_0^f}_{\text{value of trees}} \text{)}$$

- OIs' problem unchanged
- Insurance mkt clears ($z_0^h = z_0^f$)

Constructing a non-fundamental eqm: Date 1

- If HHs expect $p_1 = 1$ for sure, this belief is self-confirming, insurance is not used and has a price $q = 0$, and we have the same set of fundamental eqba

Constructing a non-fundamental eqm: Date 1

- We'll construct another eqm in which $p_1 = \begin{cases} v'(1) := \underline{p} < 1 & \text{w prob } \lambda \in (0,1) \\ p_1 = 1 & \text{w prob } 1-\lambda \end{cases}$
- FIs' nonnegativity constraint binds in the low state:

$$(1-\underline{p})z^f = \underline{p}e_0^f \implies \frac{z^f}{e_0^f} = \frac{\underline{p}}{1-\underline{p}} \equiv \phi$$

- This satisfies all date 1 eqm conditions:
  - When $p_1 = \underline{p}$, FIs sell all trees to pay out $(1-\underline{p})z_0^f$ on insurance contracts
    - OIs must purchase all trees in eqm (only agents with cookies).
    - To induce them to do so, $p_1 = v'(1) := \underline{p} < 1$.
  - When $p_1 = 1$, FIs have no insurance liabilities, need not sell any trees, $p_1 = 1$ as in the benchmark economy

Non-fundamental equilibrium

Provided that FIs have limited initial capital (χ_0^f), we can construct such non-fundamental equilibria

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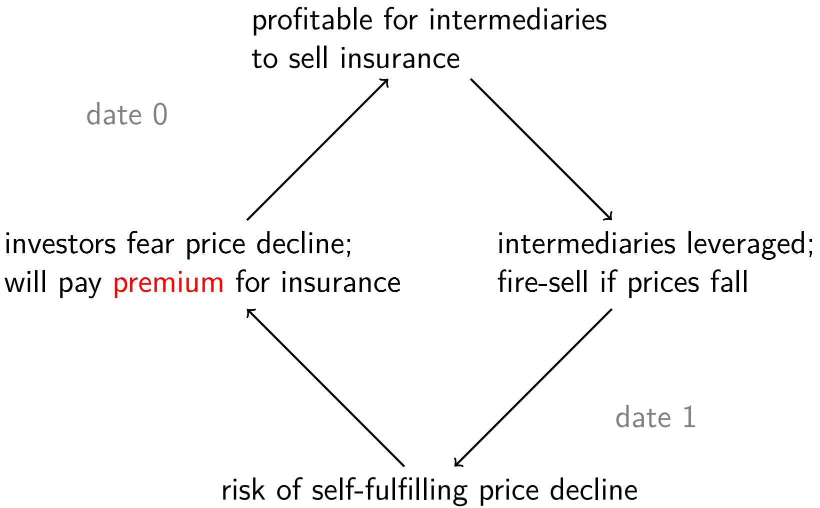
Proposition: If $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$, a nonfundamental eqm with $Pr(p_1 = \underline{p}) = \lambda$ exists for every $\lambda \in \left(0, \overline{\lambda}\right)$ where $\overline{\lambda}$ is defined by:

$$\chi_0^f = \frac{\left(1 - \overline{\lambda}\right)\left[1 - \underline{p}^{\frac{\gamma-1}{\gamma}}\right]}{\left[\overline{\lambda}\underline{p}^{\frac{1-\gamma}{\gamma}} + 1 - \overline{\lambda}\right]^{\frac{\gamma}{\gamma-1}}}$$

Intuition

- If FIs lever up to the max, there can be a self-fulfilling price decline at date 1
- Risk-neutral FIs lever up - even though they're wiped out when prices fall - because it's profitable to sell insurance to risk-averse HHs who fear the price decline
  - provided that risk premium (difference between physical and risk-neutral probability of bad state) is large enough...
  - which is the case when FIs' capital (χ_0^f) is small and they cannot buy many trees, so HHs still hold most trees and are heavily exposed to fall in prices ($\underline{p} \ll e^h$)
- **issuance of insurance makes price declines possible, rationalizing households' decisions to buy insurance**
- *'supply of safe assets creates its own demand'*

hopefully this picture makes more sense now



Welfare in non-fundamental eqm

- HHs worse off than in fundamental eqm: welfare with insurance

$$\underbrace{\chi_0^f + \chi_0^h}_{c_0^h} + \left[\lambda \underline{p}^{1-\gamma} + (1-\lambda)\left(e_0^h(\lambda)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

  decreasing in λ, $\to \chi_0^f + 1$ as $\lambda \to 0$

- FIs better off: always have option to consume χ_0^f and get same welfare as fundamental eqm

- OIs better off: benefit from fire sales

Pareto Efficiency

- Any fundamental equilibrium is Pareto efficient.

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Markets are complete for all agents who participate.

- If we allow outside investors to participate in asset markets at date 0, then the fundamental equilibrium is the unique equilibrium.

Market incompleteness: OLG

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If OI's allowed to participate in date-0 market,

$$\max_{e_0^o, z_0^o, c_1^o, c_2^o} \mathbb{E}\left[c_1^o + v(c_2^0)\right]$$

subject to:

$$\begin{aligned} p_0 e_0^o &= q z_0^o \\ c_1^o + p_1 e_1^o + (1-p_1) z_0^o &= \chi_1 + p_1 e_0^o \\ c_2^o &= e_1^o \\ (1-p_1) z_0^o &\leq \chi_1 + p_1 e_0^o \end{aligned}$$

where e_0^o denotes the OI's purchase of trees at date 0 and z_0^o denotes the OI's issuance of insurance.

Trade in non-state-contingent assets can also produce nonfundamental eqba

allow FIs to issue riskless bonds b at price q^b (instead of insurance)

- pay one cookie to the holder at date 1
- can interpret as *repo* (backed by holdings of trees)

HHs budget constraints

$$
\begin{aligned}
c_0^h + p_0 e_0^h + q^b b^h &= \chi_0^h + p_0 && (1)\\
c_1^h &= p_1 e_0^h + b^h, && (2)
\end{aligned}
$$

FIs budget constraints

$$
\begin{aligned}
c_0^f + p_0 e_0^f &= \chi_0^f + q^b b^f && (3)\\
c_1^f + p_1 e_1^f + b^f &= p_1 e_0^f && (4)
\end{aligned}
$$

as before, FIs' consumption must ≥ 0 whatever the realization of p_1:

$$
b^f = p_1 \left(e_0^f - e_1^f \right) - c_1^f \leq p_1 e_0^f \qquad (5)
$$

Trade in non-state-contingent assets can also produce nonfundamental eqba

for every equilibrium that exists in insurance economy, a corresponding equilibrium exists in the bond economy

- FIs have to pay out in all states of the world
- *but* FIs sell more when $p_1 = \underline{p} < 1$ to meet obligations

fundamental equilibrium:

- zero spread between expected return on bonds and trees
- both bonds and trees are riskless assets

non-fundamental equilibria

- date 0 price of bonds is higher (risk-free rate lower) in these equilibria
- safe rate endogenously falls as a *result* of private safe asset creation
  (contrast to a typical safe assets scarcity narrative)

Policy to eliminate financial fragility

- Simple multiple equilibrium model. Not surprisingly, various policies can eliminate nonfundamental eqba (benefiting HHs at expense of FIs & OIs)
  - ban trade in insurance contracts!
  - or tax them, impose leverage constraints...
  - richer models might have additional tradeoffs

- What's (hopefully) interesting is *how* some of the policies do so

- Distinguish between policies that
  1. increase supply of publicly backed safe assets (issue debt, bailouts)
  2. reduce demand for private safe assets (social insurance, market maker of last resort)

Conclusion

Private creation of safe assets by leveraged intermediaries can lead to fragility

- Safe assets are produced due to demand for safety by households
- Demand for safety arises from fragility induced by the privately-supplied safe assets
- Economy becomes vulnerable to self-fulfilling fire sales

Novel contribution: leverage does not just amplify fundamental shocks, but *generates* risk in a fundamentally safe economy

- adding fundamental shocks does not change results: financial sector can both amplify fundamental risk and create non-fundamental risk

END

Insurance Salesman of the Opera: Gary Larson

Scene from *Insurance Salesman of the Opera*

Constructing a non-fundamental eqm: FIs' date 0 optimality conditions

Since $p_1 \leq 1$, FIs spend all date 1 resources on trees $\Rightarrow$

$$\max_{e^f, z} \chi_0^f - p_0 e_0^f + q z_0^f + \underbrace{\mathbb{E}\left[e_0^f - \frac{1-p_1}{p_1} z_0^f\right]}_{\text{spend everything on trees}}$$

$$\text{s.t. } \chi_0^f - p_0 e_0^f + q z_0^f \geq 0 \quad (c_0^f \geq 0)$$

$$z_0^f \leq \phi e_0^f \quad (c_1^f, e_1^f \geq 0)$$

$$\Rightarrow \text{ if } \underbrace{\frac{1}{p_0}}_{\text{return on tree}} > \underbrace{\frac{1}{q}\mathbb{E}\left[\frac{1-p_1}{p_1}\right]}_{\text{return on insurance}}, \text{ lever up}$$

to the max and purchase $e_0^f = \frac{\chi_0^f}{p_0 - \phi q}$

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s.t. $\chi_0^f - p_0 e_0^f + q z_0^f \ge 0 \quad (c_0^f \ge 0)$

$z_0^f \le \phi e_0^f \quad (c_1^f, e_1^f \ge 0)$

$\Rightarrow$ if $\underbrace{\frac{1}{p_0}}_{\text{return on tree}} > \underbrace{\frac{1}{q}\mathbb{E}\left[\frac{1-p_1}{p_1}\right]}_{\text{return on insurance}}$, lever up

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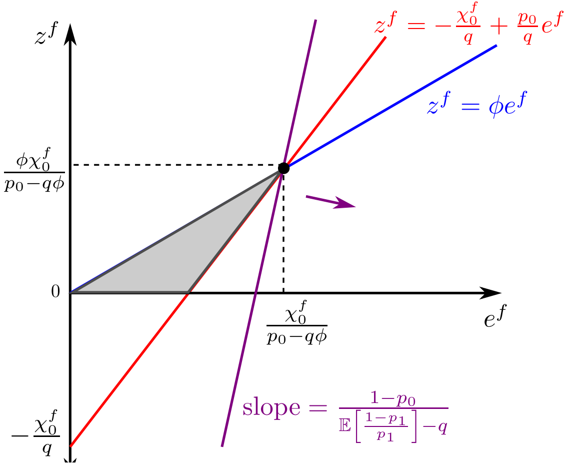

Relation to sunspots literature

	Nature of market incompleteness:	
Asset payoffs:	complete absent sunspots, incomplete w sunspots, no OLG	dynamically complete but OLG
sunspot-contingent	Hens (2000)	Cass & Shell (1983)
price-contingent	Bowman & Faust (1997)	Our insurance economy
‘real’	Gottardi & Kajii(1999)	Our bond economy

Constructing a non-fundamental eqm: HHs' date 0 optimality conditions

- Buy 1 tree, sell ϕ insurance $\rightarrow p_1 - \phi(1-p_1) = \begin{cases} 0 & \text{w prob } \lambda \\ 1 & \text{w prob } 1-\lambda \end{cases}$ at date 1: 'synthetic Arrow security' with price $p_0 - \phi q$

- FIs spend their whole endowment to buy $e_0^f = \dfrac{\chi_0^f}{p_0 - \phi q}$ Arrow securities from HHs, betting that $p_1 = 1$

- HHs 'sell' Arrow securities (bet that $p_1 = \underline{p}$); price this security using Euler eqs:

$$p_0 - \frac{\underline{p}}{1-\underline{p}} q = \frac{(1-\lambda)}{\left[\lambda\left(\frac{\underline{p}}{e_0^h}\right)^{-(\gamma-1)} + (1-\lambda)\right]^{\frac{\gamma}{\gamma-1}}} = \frac{\chi_0^f}{1-e_0^h} \text{ in eqm}$$

- to the extent that consumption falls in bad state ($\frac{\underline{p}}{e^h} < 1$), payoffs in good state are less valuable, securities cheaper, FIs can buy more of them

Non-fundamental equilibrium

This is a valid eqm provided that the solution e_0^h satisfies

$$\frac{q}{p_0} = \frac{\lambda(1-\underline{p})\left(\frac{\underline{p}}{e_0^h}\right)^{-\gamma}}{\lambda\left(\frac{\underline{p}}{e_0^h}\right)^{-\gamma}\underline{p} + (1-\lambda)} \qquad > \lambda\frac{1-\underline{p}}{\underline{p}} \left(\text{true iff } \; e_0^h > \underline{p}^{\frac{\gamma-1}{\gamma}} \right)$$

- a higher risk premium (lower $\left(\frac{\underline{p}}{e^h}\right)^{\gamma} < 1$) increases price of insurance (which only pays off in bad state) relative to trees (which pay less in bad state)...
- ... and increases FIs' incentive to sell insurance, buy trees

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Proposition: If $\chi_0^f < 1 - \underline{p}^{\frac{\gamma-1}{\gamma}}$, a nonfundamental eqm with $Pr(p_1 = \underline{p}) = \lambda$ exists for every $\lambda \in \left(0, \overline{\lambda}\right)$ where $\overline{\lambda}$ is defined by:

$$\chi_0^f = \frac{\left(1-\overline{\lambda}\right)\left[1-\underline{p}^{\frac{\gamma-1}{\gamma}}\right]}{\left[\overline{\lambda}\underline{p}^{\frac{1-\gamma}{\gamma}} + 1 - \overline{\lambda}\right]^{\frac{\gamma}{\gamma-1}}}$$

Take any non-fundamental equilibrium, and let $\mathbf{U}^i$ denotes agent i's utility in this equilibrium. An allocation $\{c_0^h, c_0^f, e_0^h, e_0^f, z\}$ is a constrained Pareto improvement relative to this non-fundamental equilibrium if there exists a date 1 price p_1 such that

$$c_0^h + p_1 e_0^h + (1 - p_1) z^f \geq \mathbf{U}^h \qquad \text{(HH PC)}$$

$$c_0^f + \left[p_1 e_0^f - (1 - p_1) z^f\right] \max\left(\frac{1}{p_1}, 1\right) \geq \mathbf{U}^f \qquad \text{(FI PC)}$$

$$V_1(p_1) \geq \mathbf{U}^o \qquad \text{(OI PC)}$$

$$\frac{p_1 e_0^f - (1 - p_1) z^f}{p_1} + v'^{-1}(p_1) \geq 1, p_1 \leq 1, \text{at least one strict equality} \qquad \text{(IC)}$$

$$c_0^h + c_0^f = \chi_0^h + \chi_0^f \qquad \text{(RC1)}$$

$$e_0^h + e_0^f = 1 \qquad \text{(RC2)}$$

where at least one of the first three inequalities (participation conditions) is strict.

Public safe asset creation

Introduce government in the bond-economy.

- issues risk-free bonds w face value b^g; buys e^g trees at date 0

$$q^b b^g = p_0 e^g$$

- sell trees, levy lumpsum taxes on outside investors at date 1

$$T + p_1 e^g = b^g$$

b^g can also be liability of central bank, e.g. interest bearing reserves or reverse repos (Greenwood Hanson Stein, 2016).

Public safe asset creation

Fundamental equilibrium unchanged:

- both debt and trees are safe assets and trade at price of 1
- government never taxes OIs at date 1

Non-fundamental equilibrium

- trees are risky assets
- HH consumption when $p_1 = \underline{p}$ is now $\underline{p} + \underline{T}$.
    - in eqm, in bad state, HHs get cookies from OIs by both selling all trees at price $\underline{p}$, and taxing them
    - Higher b^g raises T, raises HH consumption when $p_1 = \underline{p}$, $\downarrow$ risk premium ($\uparrow \frac{\underline{p}+T}{e^h}$)
- If b^g high enough, risk premium is so low that FIs strictly prefer not to take leveraged position in trees

If $b^g \geq b^* \equiv \frac{\underline{p}^{\frac{1}{\gamma}}}{1-\underline{p}} \left(1 - \chi_0^f\right) - \frac{\underline{p}}{1-\underline{p}}$, no non-fundamental equilibrium exists

Transfers to FIs: "bailout policies"

Rather than issue debt ex-ante, transfer to FIs in a crisis.

- Farhi & Tirole (2012), Bianchi (2016), Jeanne & Korinek (2020): anticipated bailouts increase leverage and financial instability
- here too, bailouts increase FIs' borrowing in any non-fundamental equilibrium
- *but* generous bailouts rule out the existence of non-fundamental equilibrium!

Govt transfers $T^f \geq 0$ to FIs when $p_1 = \underline{p}$, taxes OIs. FIs budget contraint

$$c_1^f + \underline{p}a_1^f + b^f = \underline{p}e^f + T^f$$

large *unanticipated* transfer can prevent fire-sale because FIs can repay without selling trees. What if transfers are anticipated?

***anticipated* bailouts**

If FIs anticipate bailout, they borrow more so their borrowing constraint

$$b^f \leq \underline{p} e^f + T^f$$

holds with equality

HHs hold more 'publicly backed' safe assets – similar to effect of govt debt!

- can interpret as govt guarantees (deposit insurance, MMMF guarantee in Sep 08) (cf. Benigno & Robatto 2019)
- transfers 'pass through' FIs to households
- HH consumption when $p_1 = \underline{p}$ is $\underline{p} + T^f$

If $T^f \geq \underline{p}^{\frac{1}{\gamma}} \left(1 - \chi_0^f\right) - \underline{p}$, then no non-fundamental equilibrium exists

Difference from Farhi & Tirole (2012)

Farhi & Tirole (2012): anticipated bailouts make ex-post intermediaries' leverage decisions *strategic complements*

- if only a few banks lever up, a bailout is unlikely, so it is unprofitable to lever up
- if many banks lever up, policymakers will have to bailout, so profitable to lever up

Here: nonfundamental eqm exists *absent* bailout, large enough anticipated bailout can eliminate them:

- profitability of levering up depends on risk premium (HH demand for safe assets)
- large enough bailout/publicly backed safe asset supply satiates demand for safe assets, reduces risk premium
- making it privately unprofitable to lever up
- This channel's absent in Farhi & Tirole's risk neutral economy

Stand ready to buy any quantity of trees at some price $p^{\diamond} > \underline{p}$

- the ECB's Outright Monetary Transactions
- the Municipal Liquidity Facility and the Secondary Market Corporate Credit Facility
- the Federal Reserve's standing repo facilities

Let $p^{\diamond} < 1$ be the price at which the government stands ready to buy.

$$
\begin{aligned}
p_1 e_1^g &= T && (6)\\
p_1 \geq p^{\diamond}, \qquad e_1^g &\geq 0, \qquad \text{with at least one equality} && (7)\\
a_1^f + a_1^o + e_1^g &= 1 && (8)
\end{aligned}
$$

Govt raises taxes T on OIs to fund purchases; apples from trees they buy are wasted

Market maker of last resort

- fundamental equilibrium unchanged (no intervention)
- non-fundamental equilibrium:
  - price cannot fall below $p^\diamond$
  - when prices fall, govt is the marginal buyer of trees, purchasing $e^g = 1 - v'^{-1}(p^\diamond)$ trees and levying taxes $T = p^\diamond e^g$ on OIs
  - Higher $p^\diamond$ reduces risk premium and HHs' demand for insurance
  - effectively govt provides a certain amount of insurance at zero price

if $p^\diamond \geq \left(1 - \chi_0^f\right)^{\frac{\gamma}{\gamma-1}}$, no non-fundamental equilibrium exists.

Environment with fundamental risk

- 2 dates: 0 and 1
- 3 agents:
  1. risk-averse households (HHs)
  2. risk-neutral financial intermediaries (FIs)
  3. outside investors (OIs) who only trade at date 1
- fixed endowment of cookies (c) at both dates
- unit endowment of trees (e) at date 0
- trees can be traded at date 0

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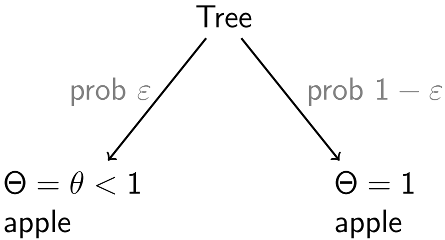

Date 1 price of trees now also depends on fundamental state: $p_1(\Theta)$

Fundamental Equilibria with fundamental risk, only trees traded at date 0

Define $\overline{e}$ s.t. $v'(\overline{e}) = 1$
Assume that $\chi_0^f \geq (1 - \overline{e})\,\mathbb{E}\,[\Theta]$.
Then, the equilibrium date 0 and date 1 price of trees is given by

$$p_0 = \left[\mathbb{E}\Theta^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \qquad p_1(\Theta) = \Theta$$

households retain

$$e^* = 1 - \frac{\chi_0^f}{p_0} \text{ trees.}$$

Households' face consumption risk at date 1:

$$c_1^h(\theta) = \theta e^* < e^* = c_1^h(1)$$

Allowing for trade of more assets

With the introduction of insurance, households' budget constraintz now become

$$c_0^h + p_0 e^h + \sum_{p \in \mathbb{P}} q(p) z^h(p) = \chi_0^h + p_0 \tag{9}$$

$$c_1^h(\Theta) = p_1(\Theta)\, e^h + \Big(1 - p_1(\Theta)\Big) z^h, \qquad \Theta \in \{\theta, 1\} \tag{10}$$

where
$q(p)$ is the date 0 price of the derivative which pays off $1 - p$ cookies at date 1 if the price realized is p,
$z^h(p)$ denotes the quantity of that derivative purchased by households

Allowing for trade of more assets

the date 0 and date 1 budget constraints of FIs can be written as

$$c_0^f + p_0 e^f = \chi_0^f + \sum_{p \in \mathbb{P}} q(p) z^f(p) \tag{11}$$

$$c_1^f(\Theta) + \frac{p_1(\Theta)}{\Theta} a_1^f + \Big(1 - p_1(\Theta)\Big) z^f = p_1(\Theta)\, e^f \qquad \Theta \in \{\theta, 1\} \tag{12}$$

where $z^f(p)$ denotes the quantity of the derivative sold by FIs.

Allowing for trade of more assets

the date 0 and date 1 budget constraints of FIs can be written as

$$\begin{aligned} c_0^f + p_0 e^f &= \chi_0^f + \sum_{p \in \mathbb{P}} q(p) z^f(p) && (11) \\ c_1^f(\Theta) + \frac{p_1(\Theta)}{\Theta} a_1^f + \Big(1 - p_1(\Theta)\Big) z^f &= p_1(\Theta)\, e^f \qquad \Theta \in \{\theta, 1\} && (12) \end{aligned}$$

where $z^f(p)$ denotes the quantity of the derivative sold by FIs.

FIs issuance of derivative z^f is limited by:

$$\Big(1 - p_1(\Theta)\Big) z^f \leq p_1(\Theta)\, e^f, \tag{13}$$

Fundamental Equilibria with fundamental risk and trade in insurance

For large enough χ_0^f, in the economy with insurance,

$\exists$ a unique fundamental equilibrium with perfect hh consumption insurance

$$c^h(1) = c^h(\theta)$$

in which

$$p_0 = \varepsilon\theta + 1 - \varepsilon, \qquad q = \varepsilon(1-\theta)$$
$$p_1(\Theta) = \Theta$$

and $e^h = z^h$.

HHs better off with insurance <u>and</u> no sunspots than without insurance.

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$$p_1(\Theta) = \Theta$$

and $e^h = z^h$.

HHs better off with insurance <u>and</u> no sunspots than without insurance.

but there also exist equilibria with sunspots.