Incorporating Diagnostic Expectations into the New Keynesian Framework

Jean-Paul L'Huillier [*] Sanjay R. Singh [§] Donghoon Yoo [‡]

Barcelona Summer Forum
19 June 2023

[*] Brandeis University; Federal Reserve Bank of Cleveland

[§] Federal Reserve Bank of San Francisco; UC Davis

[‡] Academia Sinica

Introduction

- What are Diagnostic Expectations (DE)?
  - "Representativeness heuristic" (Kahneman & Tversky)
  - Mechanism for **extrapolation**
  - Advantages: Plausible & portable, parsimonious & tractable

- DE can be productively integrated into the NK framework
  First: Solution method. **Then:**
  A) Analytically, address 3 key issues
     1. Amplification
     2. Supply shocks
     3. Fiscal policy
  B) Empirically
     - Show DE improve the fit of medium-scale models
     - Outcompete news/noise shocks

Related Literature

- Departures from Full Information Rational Expectations

  Marcet & Sargent (1989); Evans & Honkapohja (2001); Coibion & Gorodnichenko (2015); Eusepi & Preston (2018); Gabaix (2018); Azeredo da Silveira & Woodford (2019); Farhi & Werning (2019); Garcia-Schmidt & Woodford (2019); Angeletos, Huo & Sastry (2020)

- Diagnostic expectations in cognitive psychology and finance

  Kahneman & Tversky (1972); Kahneman, Slovik & Tversky (1982); Kahana (2012); Bordalo, Gennaioli & Shleifer (2018); Bordalo, Gennaioli, La Porta & Shleifer (2018); Choi & Mertens (2019); Bordalo, Conlon, Gennaioli, Kwon & Shleifer (2022); Afrouzi, Landier, Ma, Kwon & Thesmar (2023)

- Diagnostic expectations in macro

  Bordalo, Gennaioli, Ma & Shleifer (2020); Bordalo, Gennaioli, Shleifer & Terry (2021); Bianchi, Ilut & Saijo (2022); Chodorow-Reich, Guren & McQuade (2022); Maxted (2022)

Outline

1. Introduction
2. **Brief Overview of Diagnostic Expectations**

   Themes: Excess Volatility (on impact)
   and Predictable Reversals
3. Example: Muth (1961)
4. Analytical: 3-Equation NK model
5. Empirical: DSGE models

Diagnostic Expectations

- Consider the process

$$x_t = \rho_x x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- Diagnostic pdf is defined as

$$\log f_t^\theta(x_{t+1}) = \underbrace{\log f(x_{t+1}|G_t)}_{\text{RE}} + \underbrace{\theta \left(\log f(x_{t+1}|G_t) - \log f(x_{t+1}|G_t^r)\right)}_{\text{distortion}} + C, \quad \theta > 0$$

- Information sets:
  - G_t: current state t
  - G_t^r: reference state. Here, Information set at time $t-1$

  θ: degree of diagnosticity

Formula for Univariate Case and AR(1) Example

- Diagnostic expectation is:

$$\mathbb{E}_t^{\theta}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}])$$

(Bordalo, Gennaioli & Shleifer (2018), henceforth BGS)

- We have that:

$\mathbb{E}_t[x_{t+1}] = \rho_x \check{x}_t$ and $\mathbb{E}_{t-1}[x_{t+1}] = \rho_x^2 \check{x}_{t-1}$

- So:

$$\mathbb{E}_t^{\theta}[x_{t+1}] = \rho_x \check{x}_t + \theta(\rho_x \check{x}_t - \rho_x^2 \check{x}_{t-1}) = \rho_x \check{x}_t + \theta \rho_x \check{\varepsilon}_t$$

$\Longrightarrow$ excess volatility in beliefs on impact

- Predictable Forecast errors:

$$x_{t+1} - \mathbb{E}_t^{\theta}[x_{t+1}] = \theta \rho_x \check{\varepsilon}_t$$

$\Longrightarrow$ systematic reversal in beliefs

Outline

Setting the Stage: Muth (1961) with DE

"*Rational Expectations and the Theory of Price Movements*" Muth, J., Econometrica 1961

An isolated market for a commodity:

- Demand:

$$Q_t^d = -\beta P_t, \quad \beta > 0$$

- Supply:

$$Q_t^s = I_{t-1} + (1-\delta) Q_{t-1} + \epsilon_t, \quad \delta \in (0,1)$$

Time-to-build investment:

$$I_t = \gamma \tilde{\mathbb{E}}[P_{t+1}], \quad \gamma > 0$$

- Market Clearing: $Q_t^d = Q_t^s = Q_t$

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- Market Clearing: $Q_t^d = Q_t^s = Q_t$

Equilibrium:

$$Q_t = -\frac{\gamma}{\beta} \tilde{\mathbb{E}}_{t-1}[Q_t] + (1-\delta) Q_{t-1} + \epsilon_t$$

Rational Expectations: $\tilde{\mathbb{E}}_t = \mathbb{E}_t$
Diagnostic Expectations: $\tilde{\mathbb{E}}_t = \mathbb{E}_t^\theta$

Implications with Muth

Solution:

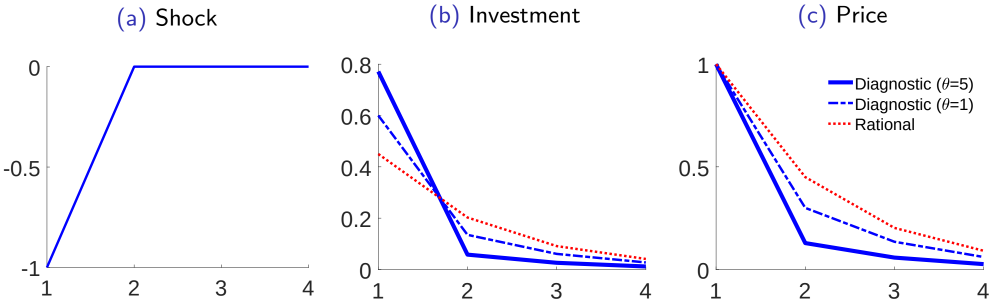
$$I_t = -\frac{\gamma}{\beta+\gamma}(1-\delta)Q_t - \frac{\beta}{\beta+\gamma}\frac{\theta\gamma(1-\delta)}{\beta+\gamma(1+\theta)}\epsilon_t$$

Consider a supply contraction: $\epsilon_t < 0$

- lower-than-expected inventories $\implies$ selective memory forecasts low Q_{t+1}
- expect higher prices to prevail at $t+1$ because of supply contraction
- Investment goes up.
- Under DE, expectations about prices overreact. $I_t^{DE} > I_t^{RE}$.
- Reversal, ex- post. Market glutted with commodity
- Price rises by less than in the RE economy

Implications with Muth

Figure: Implications of a Negative Commodity Supply Shock



Two key themes from DE: Over-reaction in beliefs and systematic reversals

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1. Introduction
2. Diagnostic Expectations
3. Example: Muth (1961)
4. **Analytical: 3-Equation NK model**
5. Empirical: DSGE models

Representative Household Problem

Consider household optimization under diagnostic expectations:

$$\max_{\{C_t, L_t, B_{t+1}\}} \log C_t - \frac{\omega}{1+\nu} L_t^{1+\nu} + \mathbb{E}_t^{\theta}\left[\Sigma_{s=t+1}^{\infty} \beta^{s-t}\left(\log(C_s) - \frac{\omega}{1+\nu} L_s^{1+\nu}\right)\right]$$

subject to a budget constraint:

$$P_t C_t + \frac{B_{t+1}}{(1+i_t)} = B_t + W_t L_t + D_t + T_t$$

First-order condition:

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta}\left[\frac{u'(C_{t+1})}{P_{t+1}}\right]$$

Obtaining Log-Linear Approximation

- Inter-temporal Consumption Euler Equation

$$\frac{u'(C_t)}{P_t} = \beta(1 + i_t)\mathbb{E}_t^{\theta}\left[\frac{u'(C_{t+1})}{P_{t+1}}\right]$$

- Notice!

$$\mathbb{E}_t^{\theta}[X_{t+1}Y_t] \neq \mathbb{E}_t^{\theta}[X_{t+1}]Y_t$$

- Hence, use conditioning on $t-1$:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1 + i_t)\mathbb{E}_t^{\theta}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

and approximate

Obtaining Log-Linear Approximation

- We have:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

- Resulting diagnostic Fisher equation:

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}] - \underbrace{\theta(\mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_{t-1}[\pi_{t+1}])}_{\frac{P_t}{P_{t+1}}} - \underbrace{\theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])}_{\frac{P_{t-1}}{P_t}\text{(momentum)}}$$

- Appendix presents loglinearization steps of medium-scale DSGE

Obtaining Diagnostic Fisher Equation

- We have:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^\theta\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

- Resulting diagnostic Fisher equation:

$$\hat{r}_t = \hat{i}_t \underbrace{-\mathbb{E}_t[\pi_{t+1}] - \theta(\mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_{t-1}[\pi_{t+1}])}_{\mathbb{E}_t^\theta[\pi_{t+1}]} - \underbrace{\theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])}_{\frac{P_{t-1}}{P_t}\text{(momentum)}}$$

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- We have:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

- Resulting Diagnostic Fisher equation :

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t^{\theta}[\pi_{t+1}] - \underbrace{\theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])}_{\frac{P_{t-1}}{P_t}\text{(momentum)}}$$

Firm Price-Setting

Monopolistically competitive intermediate firms; demand is given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t; \quad \epsilon_p > 1$$

Production technology

$$Y_t(j) = A_t L_t(j)$$

Firms' per period profits (Rotemberg adjustment costs)

$$D_t \equiv P_t(j)Y_t(j) - W_t L_t(j) - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$$

The firm's profit maximization problem is

$$\max_{P_t(j)} \left\{ P_t(j)Y_t(j) - W_t L_t(j) - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t + \mathbb{E}_t^{\theta}\left[\sum_{s=1}^{\infty} \beta^s Q_{t,t+s} D_{t+s}\right]\right\}$$

where $Q_{t,t+s}$ is the household's nominal stochastic discount factor.

Rest of the Model

1. Central bank follows Taylor Principle (no ZLB constraint)
2. Government runs balanced budget
3. Goods market clears

Log-linearize around the non-stochastic steady state.

Textbook new Keynesian Model with DE

- Model

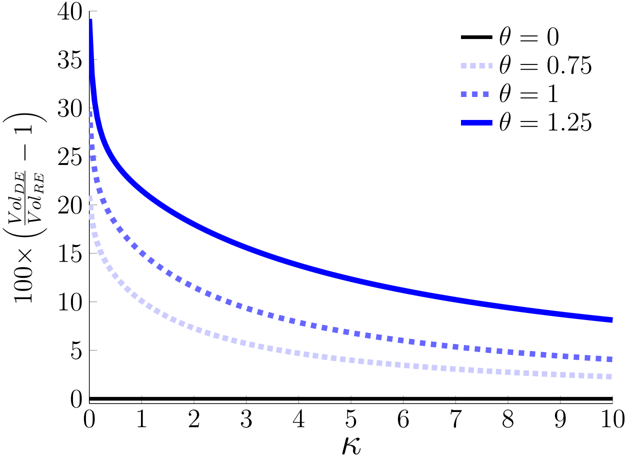
$$\hat{y}_t = \mathbb{E}_t^{\theta}[\hat{y}_{t+1}] - (\hat{i}_t - \mathbb{E}_t^{\theta}[\pi_{t+1}]) + \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$$
$$\pi_t = \beta\mathbb{E}_t^{\theta}[\pi_{t+1}] + \kappa(\hat{y}_t - \hat{a}_t)$$
$$\hat{i}_t = \phi_\pi \pi_t + \phi_x(\hat{y}_t - \hat{a}_t)$$

- Euler equation and NK Phillips curve with DE

- Notice diagnostic real rate

(i) Amplification: Interaction with Price Stickiness (κ^{-1})

Figure: Excess Output Volatility under DE rel to RE



$\kappa \to \infty$: Excess output volatility is 0 with flexible prices

(i) Amplification: NK vs. RBC ($\theta = 1$)

- New Keynesian Model

Variable	RE	DE	Percentage Increase
Output	0.0048	0.0085	77%

Volatility of output **increases**

- (Frictionless) Real Business Cycle Model

Variable	RE	DE	Percentage Increase
Output	0.0064	0.0059	-7%
Consumption	0.0015	0.0030	100%
Investment	0.0533	0.0503	-6%

Volatility of output **falls**

(i) NK Versus RBC: Understanding the Differences

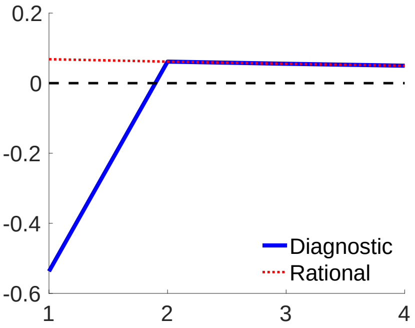
- DE generate errors in expectations
  - After + TFP shock, agents extrapolate
    and believe they will get unrealistically richer: Income effect

- NK and nominal rigidities
  - Errors in expectations are able to propagate to output
  - Output higher than output under DE

- RBC and aggregate supply
  - Consumption increases by more, but so does leisure
  - Labor supply falls, and so does output and investment
  - Output similar to output under RE
    Related to Barro & King (1984); Beaudry & Portier (2006); Jaimovich & Rebelo (2009)

(ii) "Covid" Shock: Fall of Output Gap After Negative TFP Shock

Figure: Output Gap Response to a Negative TFP Shock, Baseline NK Model



Intuition: DE agent expects TFP to fall by a lot
(in excess of reality)
$\Longrightarrow$ Sharp drop in consumption

(iii) Fiscal Policy

Proposition

Consider i.i.d. government spending shocks.

1. *Under DE, the multiplier is greater than 1 if $\theta > \phi_\pi$.*
2. *The multiplier is greater under DE than under RE.*
3. *The multiplier is increasing in θ, and tends to ∞ as $\theta \to \phi_\pi + \kappa^{-1}$.*

- Diagnostic Fisher equation:
  $\hat{r}_t = \hat{i}_t - \mathbb{E}^\theta_t[\pi_{t+1}] - \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$

- Role of **endogenous** extrapolation of inflation

- Dominates effect of monetary policy if $\theta > \phi_\pi$

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- Diagnostic Fisher equation $(+\ \hat{i}_t = \phi_\pi \pi_t)$:
  $\hat{r}_t = \phi_\pi \pi_t - \mathbb{E}_t^\theta[\pi_{t+1}] - \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$
- Role of **endogenous** extrapolation of inflation
- Dominates effect of monetary policy if $\theta > \phi_\pi$

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1. Introduction
2. Brief Overview of Diagnostic Expectations
3. Solution Method for general class of linear models
4. Analytical: 3-Equation NK model
5. **Empirical: DSGE models**

Bayesian Estimation

Is there evidence in favor of diagnosticity?

Subquestions:

i What is the estimated value of θ?

ii Is it away from 0?

What changes in the interpretation of the data?

Estimations:

1. Model with news and noise
2. Off-The-Shelf Models (for Robustness)
   - Smets & Wouters (2007)
   - Justiniano, Primiceri & Tambalotti (2010)

News & Noise Model (Blanchard, L'Huillier and Lorenzoni AER 2013)

- **Rich** model with host of frictions, shocks and alternative expectations channel
  News & Noise: shocks to rational expectations

  Question: Do DE improve the fit to the data, even in the presence of all these other ingredients?

- Include consensus forecast data (SPF) (1q ahead, 5 vars)
  - Notice: Diagnostic Kalman filter
    Connects to Coibion & Gorodnichenko (AER 2015), Bordalo, Gennaioli, Ma & Shleifer (AER 2020) and Miyamoto & Nguyen (JME 2020)

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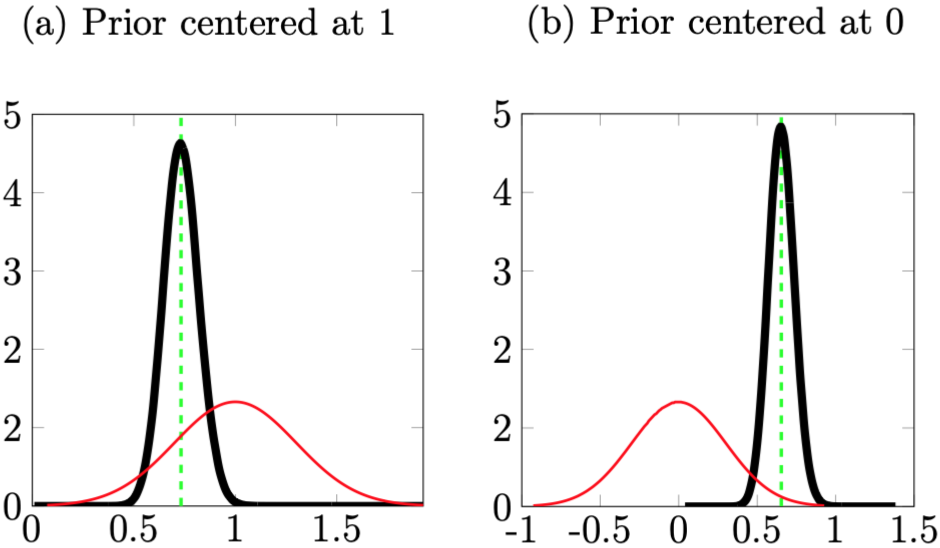
$2\times$ Bayes Factor $= 2 \times 34.67 = 69.34$
Kass & Raftery (1995) classification: "very strong" evidence in favor of DE

Forecast Errors: Explained 'Internally' by DE Model

Variable	Structural Shocks	Measurement Errors
Consumption		
DE	0.44	0.56
RE	0.31	0.69
Investment		
DE	0.33	0.67
RE	0.17	0.83
Output		
DE	0.44	0.56
RE	0.30	0.70
Price Inflation		
DE	0.56	0.44
RE	0.33	0.67
Nominal Rate		
DE	0.91	0.09
RE	0.76	0.24

Posterior Distribution of θ



Summary

"*Subjective probabilities play an important role in our lives. The decisions we make, the conclusions we reach, and the explanations we offer are usually based on our judgements of the likelihood of uncertain events*" Kahneman & Tversky (1972)

- How to integrate diagnostic expectations into linear models
- Rich insights in the context of NK models
- Better fit to business cycle data

Data and Sample

Sample: 1954:III-2004:IV (consistent with Smets and Wouters 2007)

1. Real GDP
2. Real Non-durable consumption
3. Investment = Real Gross Investment + Real Personal Durable Consumption
4. Employment
5. Inflation: GDP Deflator
6. Real Wage
7. Effective Federal Funds Rate

Introducing Imperfect Information

- Follow Blanchard, L'Huillier & Lorenzoni (AER 2013)
  - Agents try to gauge path of future TFP
  - Noisy signals about permanent component of TFP
  - Here: add diagnosticity

  - Obtain a 'diagnostic Kalman filter'
    (Bordalo, Gennaioli, Ma & Shleifer AER 2020)

Estimation: News & Noise Model

Parameter	Description	Diagnostic Mean	Diagnostic [05, 95]	Rational Mean	Rational [05, 95]
θ	diagnosticity	0.7325	[0.5917, 0.8746]		
α	cap. share	0.1340	[0.1226, 0.1453]	0.1390	[0.1278, 0.1505]
h	habits	0.7211	[0.6922, 0.7502]	0.5803	[0.5424, 0.6178]
$\frac{\chi''(1)}{\chi'(1)}$	cap. util. costs	5.0666	[3.4432, 6.6709]	5.5929	[3.9095, 7.2242]
ψ_p	Rotemberg prices	125.58	[98.710, 152.17]	181.84	[126.66, 188.88]
ϕ_w	Rotemberg wages	582.13	[256.01, 897.76]	9710.9	[4510.5, 14712.]
ν	inv. Frisch elas.	3.8520	[2.4474, 5.2254]	1.2832	[0.5012, 1.9475]
$S''(1)$	inv. adj. costs	6.9588	[5.8400, 8.0723]	7.0701	[6.0111, 8.1332]
ρ_R	m.p. rule	0.5818	[0.5429, 0.6209]	0.5563	[0.4380, 0.6806]
ϕ_π	m.p. rule	1.5363	[1.4173, 1.6537]	1.0682	[1.0001, 1.2046]
ϕ_x	m.p. rule	0.0061	[0.0001, 0.0109]	0.0013	[0.0001, 0.0030]
Technology Shocks					
ρ	persist.	0.8573	[0.8368, 0.8780]	0.9535	[0.9352, 0.9716]
σ_a	tech. shock s.d.	1.3772	[1.2603, 1.4947]	1.5258	[1.3896, 1.6601]
σ_s	noise shock s.d.	0.5400	[0.3196, 0.7531]	1.0594	[0.3781, 1.7574]
Investment-Specific Shocks					
ρ_μ	persist.	0.3027	[0.2474, 0.3575]	0.3310	[0.2631, 0.4003]
σ_μ	s.d.	18.905	[15.017, 22.716]	20.212	[16.369, 23.989]
Markup Shocks					
ρ_p	persist.	0.8749	[0.8303, 0.9209]	0.8205	[0.7663, 0.8769]
ϕ_p	ma. comp.	0.5858	[0.4728, 0.7022]	0.5563	[0.4380, 0.6806]
σ_p	s.d.	0.1591	[0.1306, 0.1877]	0.1988	[0.1700, 0.2271]
ρ_w	persist.	0.9969	[0.9939, 0.9999]	0.6543	[0.5146, 0.7978]
ϕ_w	ma. comp.	0.5765	[0.3942, 0.7630]	0.5142	[0.2882, 0.7444]
σ_w	s.d.	0.4383	[0.3434, 0.5300]	0.4490	[0.3836, 0.5142]
Policy Shocks					
ρ_{mp}	persist.	0.0295	[0.0100, 0.0514]	0.0197	[0.0009, 0.0383]

Estimation: News & Noise Model

Parameter	Description	Diagnostic		Rational	
		Mean	[05, 95]	Mean	[05, 95]
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$\frac{\chi''(1)}{\chi'(1)}$	cap. util. costs	5.0666	[3.4432, 6.6709]	5.5929	[3.9095, 7.2242]
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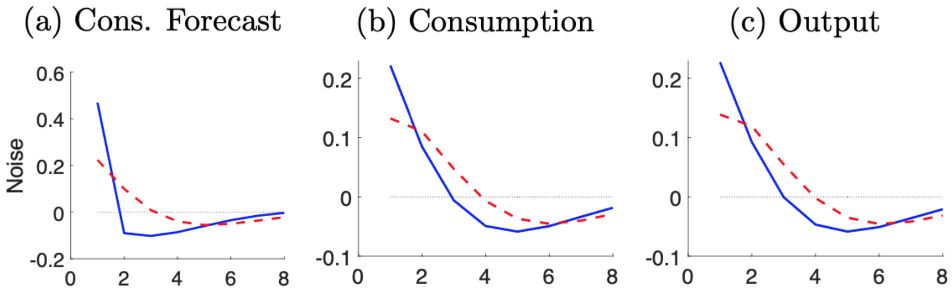
Impulse Response Functions: boom-bust in DE beliefs



Note: DE in blue, RE in red

noise shock raises expectations of future income– acts like an aggregate demand shock

1Q-ahead Variance Decomposition: DE Outcompetes News & Noise

Variable	Noise	Perm. TFP	Temp. TFP	Invest.	Price Markup	Wage Markup	Monet.	Fiscal
Consumption								
DE	0.1158	0.0432	0.2976	0.0013	0.0313	0.3010	0.1814	0.0283
RE	0.4310	0.0039	0.1509	0.0006	0.0334	0.0121	0.3680	0.0001
Investment								
DE	0.0020	0.0018	0.0279	0.9347	0.0102	0.0187	0.0035	0.0012
RE	0.0156	0.0002	0.0104	0.9585	0.0050	0.0014	0.0089	0.0001
Output								
DE	0.0707	0.0262	0.1776	0.2842	0.0373	0.1942	0.1093	0.1005
RE	0.2493	0.0021	0.0716	0.2867	0.0278	0.0059	0.2017	0.1547

Smets & Wouters (2007)

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h	habits	0.7100	[0.6385, 0.7839]	0.7027	[0.6334, 0.7725]
$\frac{\chi''(1)}{\chi'(1)}$	cap. util costs	0.6241	[0.4539, 0.8013]	0.5785	[0.4016, 0.7549]
ψ_p	Rotemberg prices	399.36	[292.11, 506.07]	383.13	[272.30, 490.97]
ψ_w	Rotemberg wages	2266.5	[1083.3, 3407.9]	2265.1	[1092.8, 3375.0]
ν	inv. Frisch elas.	1.9577	[1.0626, 2.7971]	2.0293	[1.1717, 2.8701]
$S''(1)$	inv. adj. costs	5.6924	[3.9384, 7.3949]	5.7666	[4.0637, 7.4253]
ρ_R	m.p. rule	0.7962	[0.7560, 0.8381]	0.8132	[0.7754, 0.8515]
ϕ_π	m.p. rule	2.0801	[1.7974, 2.3631]	2.0199	[1.7277, 2.3092]
ϕ_x	m.p. rule	0.0836	[0.0450, 0.1220]	0.0839	[0.0478, 0.1199]
ϕ_{dx}	m.p. rule	0.2412	[0.1943, 0.2886]	0.2327	[0.1862, 0.2790]
ι_p	index. prices	0.3075	[0.1491, 0.4647]	0.2268	[0.0905, 0.3584]
ι_w	index. wages	0.6287	[0.4343, 0.8238]	0.5712	[0.3695, 0.7756]
$100G_a$	s.s. growth rate	0.4206	[0.3950, 0.4467]	0.4226	[0.3982, 0.4465]
$\log L$	s.s. hours	0.6699	[-1.169, 2.5050]	0.6560	[-1.147, 2.4377]
$100(\pi - 1)$	s.s. infl.	0.7775	[0.6156, 0.9427]	0.7543	[0.5932, 0.9219]
$100(\beta^{-1} - 1)$	disc. factor	0.1640	[0.0708, 0.2523]	0.1671	[0.0731, 0.2576]

Justiniano, Primiceri & Tambalotti (2010)

		Diagnostic		Rational	
Parameter	Description	Mean	[05, 95]	Mean	[05, 95]
θ	diagnosticity	0.4336	[0.1894, 0.6745]		
α	cap. share	0.1702	[0.1603, 0.1800]	0.1700	[0.1602, 0.1800]
h	habits	0.8788	[0.8443, 0.9142]	0.8270	[0.7655, 0.8902]
$\frac{\chi''(1)}{\chi'(1)}$	cap. util. costs	5.3160	[3.6696, 6.9322]	5.2978	[3.6521, 6.9145]
ψ_p	Rotemberg prices	123.01	[91.513, 154.15]	116.43	[84.65, 147.570]
ψ_w	Rotemberg wages	2863.31	[594.68, 5275.6]	3204.29	[720.56, 5835.5]
ν	inv. Frisch elas.	4.3961	[2.9554, 5.7777]	4.2917	[2.8854, 5.6762]
$S''(1)$	inv. adj. costs	2.9689	[2.0722, 3.8461]	2.7528	[1.8821, 3.6124]
ρ_R	m.p. rule	0.8064	[0.7681, 0.8445]	0.8193	[0.7822, 0.8567]
ϕ_π	m.p. rule	2.1751	[1.8764, 2.4631]	2.0782	[1.7792, 2.3655]
ϕ_x	m.p. rule	0.0559	[0.0269, 0.0847]	0.0600	[0.0306, 0.0887]
ϕ_{dx}	m.p. rule	0.2425	[0.1983, 0.2860]	0.2389	[0.1974, 0.2801]
ι_p	index. prices	0.2589	[0.1266, 0.3888]	0.1964	[0.0821, 0.3062]
ι_w	index. wages	0.1477	[0.0862, 0.2085]	0.1127	[0.0595, 0.1655]
$100G_a$	s.s. growth rate	0.4675	[0.4237, 0.5108]	0.4695	[0.4256, 0.5139]
λ_p	s.s. markup prices	0.2340	[0.1791, 0.2890]	0.2419	[0.1847, 0.2982]
λ_w	s.s. markup wages	0.1347	[0.0525, 0.2127]	0.1360	[0.0543, 0.2130]
$\log L$	s.s. log hours	0.1827	[-0.600, 0.9579]	0.2032	[-0.571, 0.9877]

General Model and Solution Method

- Exogenous process

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{v}_t$$

- Recursive model:

$$\mathbb{E}_t^{\theta}[\mathbf{F}\mathbf{y}_{t+1} + \mathbf{G_1}\mathbf{y}_t + \mathbf{M}\mathbf{x}_{t+1} + \mathbf{N_1}\mathbf{x}_t] + \mathbf{G_2}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{N_2}\mathbf{x}_t = 0$$

- **Question:** How to compute the equilibrium $\mathbb{E}_t^{\theta}[\mathbf{F}\mathbf{y}_{t+1} + \ldots]$?

Solution Method: Rational Expectations Representation

Proposition (Multivariate RE Representation)

The model admits the following RE representation:

$$
\begin{aligned}
\mathbf{F}\mathbb{E}_t[\mathbf{y}_{t+1}] + \mathbf{G}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbb{E}_t[\mathbf{x}_{t+1}] + \mathbf{N}\mathbf{x}_t \\
+\mathbf{F}\theta\big(\mathbb{E}_t[\mathbf{y}_{t+1}] - \mathbb{E}_{t-1}[\mathbf{y}_{t+1}]\big) \\
+\mathbf{M}\theta\big(\mathbb{E}_t[\mathbf{x}_{t+1}] - \mathbb{E}_{t-1}[\mathbf{x}_{t+1}]\big) \\
+\mathbf{G}_1\theta\big(\mathbf{y}_t - \mathbb{E}_{t-1}[\mathbf{y}_t]\big) \\
+\mathbf{N}_1\theta\big(\mathbf{x}_t - \mathbb{E}_{t-1}[\mathbf{x}_t]\big) = 0
\end{aligned}
$$

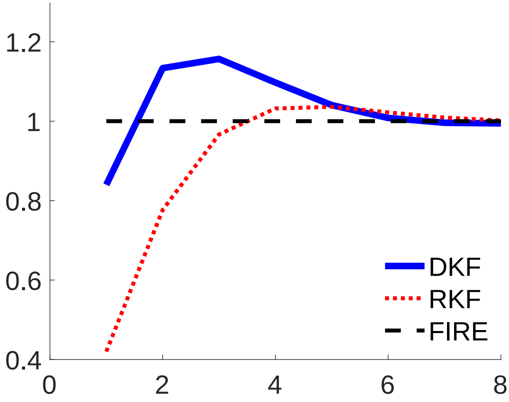
- From here, we can take standard steps to find solution.
- Paper also studies stability and boundedness properties.

(iv) Introducing Imperfect Information

- Straightforward to obtain a 'diagnostic Kalman filter' (Bordalo, Gennaioli, Ma & Shleifer 2020)
- Investigate in Blanchard, L'Huillier & Lorenzoni (2013)
  - Agents try to gauge path of future TFP
  - Noisy signals about permanent component of TFP
  - Here: add diagnosticity
- Precision of the signal is crucial
  - Can obtain a gradual build-up of overreaction

(iv) Introducing Imperfect Info: Diagnostic Kalman Filter

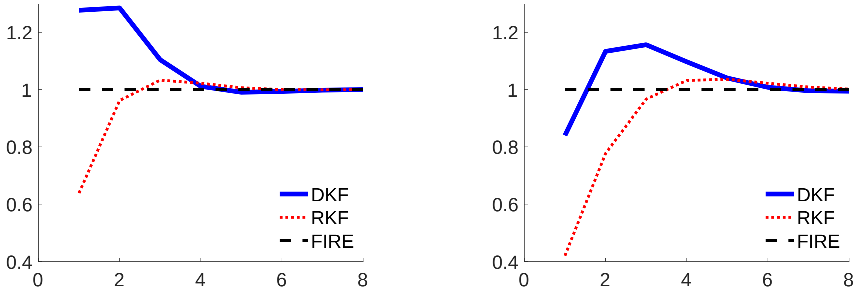
Figure: Beliefs about Long-run income



Short-run underreaction, delayed overreaction, and humps.

Over-reaction

(a) Precise Signal (b) Imprecise Signal

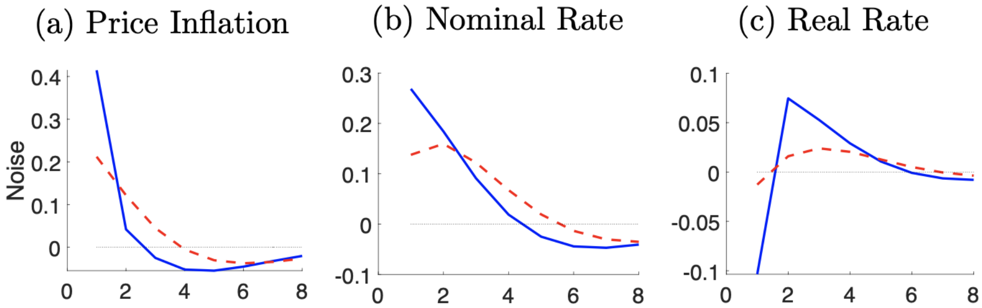


DKF: Diagnostic Kalman Filter
RKF: Rational Kalman Filter
FIRE: Full-information Rational Expectations

1Q-ahead Variance Decomposition

Variable	Noise	Perm. TFP	Temp. TFP	Invest.	Price Markup	Wage Markup	Monet.	Fiscal
Consumption								
DE	0.1158	0.0432	0.2976	0.0013	0.0313	0.3010	0.1814	0.0283
RE	0.4310	0.0039	0.1509	0.0006	0.0334	0.0121	0.3680	0.0001
Investment								
DE	0.0020	0.0018	0.0279	0.9347	0.0102	0.0187	0.0035	0.0012
RE	0.0156	0.0002	0.0104	0.9585	0.0050	0.0014	0.0089	0.0001
Output								
DE	0.0707	0.0262	0.1776	0.2842	0.0373	0.1942	0.1093	0.1005
RE	0.2493	0.0021	0.0716	0.2867	0.0278	0.0059	0.2017	0.1547
Price Inflation								
DE	0.0658	0.0000	0.4055	0.0880	0.3259	0.0314	0.0656	0.0179
RE	0.0175	0.0003	0.2859	0.0025	0.5902	0.1023	0.0007	0.0006
Wage Inflation								
DE	0.1285	0.0216	0.0115	0.1120	0.4138	0.2210	0.0814	0.0101
RE	0.0046	0.0003	0.0835	0.0004	0.2449	0.6662	0.0000	0.0000
Nominal Rate								
DE	0.0279	0.0000	0.1737	0.0378	0.1350	0.0125	0.6053	0.0077
RE	0.0026	0.0000	0.0413	0.0003	0.0840	0.0146	0.8571	0.0001
Real Rate								
DE	0.0319	0.0000	0.1647	0.0431	0.0360	0.0147	0.7006	0.0090
RE	0.0077	0.0001	0.0848	0.0019	0.0006	0.0391	0.8656	0.0002

Impulse Response Functions: boom-bust in DE beliefs



Note: DE in blue, RE in red