Discussion of: The Subjective Belief Factor

Cui, Delao, and Myers (2025)

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What This Paper Does

An SDF for beliefs: just as the SDF prices many assets with one object M, the SBF explains many subjective forecasts with one probability distortion S

If subjective expectations are coherent, $\exists\, S_{t+1}$ with $\mathbb{E}_t[S_{t+1}] = 1$ such that $\mathbb{E}^*_t[X] = \mathbb{E}_t[S_{t+1}X]$

The general formula. For *any* target Y, given anchor variables $\hat{X}$ with observed survey biases:

$$\widehat{\mathbb{E}}^*_t[Y_{t+1}] = \mathbb{E}_t[Y_{t+1}] + \underbrace{\mathrm{Cov}(\varepsilon_Y, \hat{\varepsilon})'\, \hat{\Sigma}^{-1}}_{\text{objective covariances only}} \cdot \underbrace{\left(\mathbb{E}^*_t[\hat{X}] - \mathbb{E}_t[\hat{X}]\right)}_{\text{anchor survey biases } b_t}$$

- $\varepsilon_Y = Y_{t+1} - \mathbb{E}_t[Y_{t+1}]$: target's objective forecast error
- $\hat{\varepsilon}$: anchor objective forecast errors; $\hat{\Sigma} = \mathrm{Var}(\hat{\varepsilon})$
- $b_t = \mathbb{E}^*_t[\hat{X}] - \mathbb{E}_t[\hat{X}]$: observed anchor biases from survey data

Step by Step: One Anchor (TBILL $\to$ TBOND)

anchor = TBILL survey, target = TBOND

$$\hat{\varepsilon} = \varepsilon^{\mathsf{TBILL}}, \qquad b_{t+h} = b_{t+h}^{\mathsf{TBILL}} = \mathbb{E}_t^*[\mathsf{TBILL}_{t+h}] - \mathbb{E}_t[\mathsf{TBILL}_{t+h}]$$

Substituting:

$$\widehat{\mathbb{E}}_t^*[\mathsf{TBOND}_{t+h}] \;=\; \mathbb{E}_t[\mathsf{TBOND}_{t+h}] \;+\; \frac{\mathsf{Cov}(\varepsilon^{\mathsf{TBOND}},\; \varepsilon^{\mathsf{TBILL}})}{\mathsf{Var}(\varepsilon^{\mathsf{TBILL}})} \;\cdot\; b_{t+h}^{\mathsf{TBILL}}$$

$$\beta \;\equiv\; \text{OLS coefficient of TBOND forecast errors on TBILL forecast errors}$$

So: $\quad \widehat{\mathbb{E}}_t^*[\mathsf{TBOND}_{t+h}] \;=\; \mathbb{E}_t[\mathsf{TBOND}_{t+h}] \;+\; \beta \cdot b_{t+h}^{\mathsf{TBILL}}$

β answers: when the statistical model is surprised by 1 unit about short rates, how much is it *jointly* surprised about long yields?

The Projection in the Data

- In the data: $\beta \approx 0.5$ — half of a short-rate belief distortion transmits to long yields
- $R^2 = 0.85$ of synthetic beliefs
- Also worked for: RGDP + TBILL $\to$ RCONSUM, INDPROD (bias $R^2 \approx 0.7$)

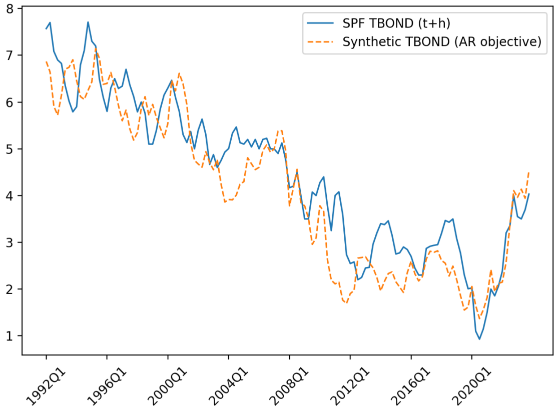
Why this is powerful:

- One main assumption: coherent distortions
- β is pinned by objective shock covariances
- Testable: synthetic forecast should track actual survey when it exists — *and it does*
- Agnostic about belief formation $\Rightarrow$ portable across applications

Output: synthetic subjective beliefs for variables where no survey exists (earnings growth, equity risk premium, credit spreads)

Replication: TBILL → Synthetic TBOND

Exercise: anchor = SPF TBILL, target = TBOND, $h = 4$ quarters



Benchmark	β	Level R^2	Bias R^2
AR(h)	0.479	0.853	0.428
VAR(1)	0.498	0.840	0.508

Bias R^2: regress $\mathbb{E}^*_t[Y] - \mathbb{E}_t[Y]$ on $\hat{\mathbb{E}}^*_t[Y] - \mathbb{E}_t[Y]$

Replication: RGDP Growth + TBILL $\rightarrow$ Macro Targets

Multivariate rotation: anchors = RGDP growth + TBILL wedges; targets = CPI, RCONSUM, INDPROD

Target	β_{RGDP}	β_{TBILL}	Level R^2	Bias R^2
CPI inflation	0.063	0.267	0.344	0.087
RCONSUM growth	0.929	−1.198	0.780	0.776
INDPROD growth	1.783	1.056	0.721	0.677

- Replicates paper's Table 2: RCONSUM and INDPROD bias $R^2 \approx 0.7$
- CPI: spanning condition binds — RGDP + TBILL forecast errors do not capture inflation surprise variation

Why This Paper Is Useful and Ambitious

Useful:

- Produces belief wedges for variables with no surveys (earnings growth, equity risk premium, credit spreads)
- Portable across applications (term structure, macro-to-finance, cross-country)
- Agnostic about belief formation — works for DE, sticky info, robust control, learning

Ambitious:

- A general-purpose methodology for belief measurement

Why This Paper Is Surprising

- Surprising result 1: heterogeneous CG coefficients (-0.29 to 1.26 across 24 variables) replicated by a *single* two-factor SBF
  - $\Rightarrow$ Correlation between synthetic and actual CG coefficients: 0.96
  - $\Rightarrow$ Observing heterogeneous behavioral patterns does *not* imply heterogeneous mechanisms

- Surprising result 2: variable-by-variable models rejected (Proposition 5)
  - $\Rightarrow$ 17 of 22 variables exceed the upper bound on factor structure from independent distortions

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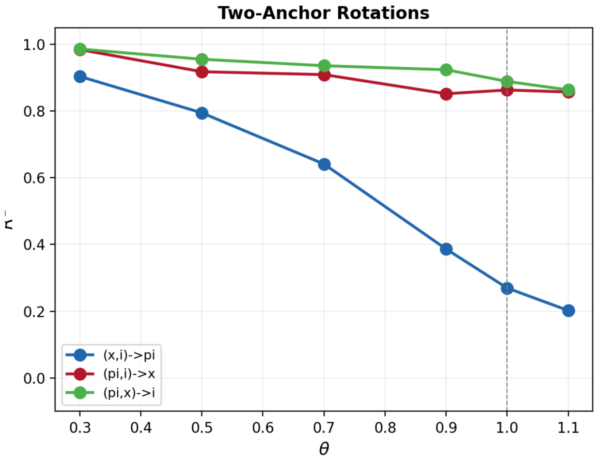
Main implication: Biases have factor structure too strong to arise from independent distortions ⇒ you need a model with *joint* probability distortions

Simulation Validation: NK with Diagnostic Expectations

Exercise: 3-equation NK with diagnostic expectations (BGS):
$\mathbb{E}_t^\theta[y'] = \mathbb{E}_t[y'] + \theta(\mathbb{E}_t[y'] - \mathbb{E}_{t-1}[y'])$

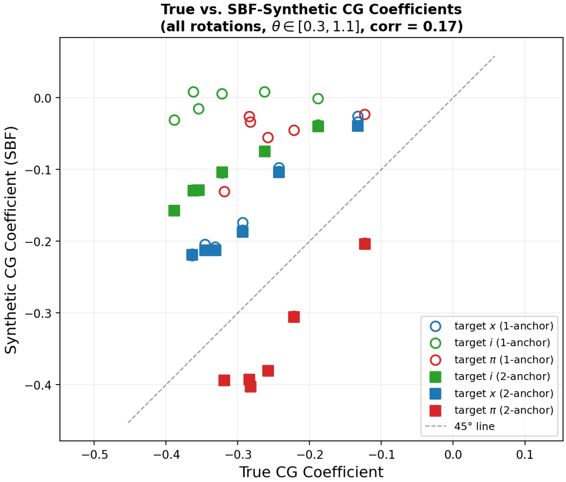


Two-anchor rotations, varying θ:

- $(\pi, x) \to i$ and $(\pi, i) \to x$: $R^2 > 0.85$ across all θ
- $(x, i) \to \pi$ collapses as θ rises $(0.90 \to 0.20)$
- Cost-push shock partially unspanned by (x, i) under DE

Simulation: How Well Does SBF Recover CG Coefficients?

Exercise: simulate NK-DE model across $\theta \in [0.3, 1.1]$; compare true vs. SBF-synthetic CG coefficients (all rotations)



Open = single-anchor, filled = two-anchor; color = target variable

- Most rotations: synthetic CG attenuated toward zero — SBF projection strips overreaction signal
- Correlation: 0.17 in 3-variable model vs. paper's 0.96 across 24 variables with wide CG spread

Suggestion: Perceived Policy Reaction Functions?

Motivation: Bauer–Pflueger–Sunderam estimate perceived Taylor rules from survey expectations and find that perceived coefficients vary over time

The SBF connection: the SBF implies *specific differential biases* across Taylor rule inputs

- T-bill expectations underreact strongly (CG ≈ 0.94)
- CPI expectations have a much smaller CG coefficient
- This differential bias across inputs could generate spurious time variation in perceived policy coefficients?

The horse race:

- Estimate perceived Taylor rule from *raw survey* expectations
- Re-estimate from *SBF-synthetic* expectations (which strip out the common distortion)
- How much time variation in perceived coefficients disappears?

The SBF provides two distinct things:

1. An empirical fact: biases are low-rank (approximately rank 2)
   - Model-free — constrains the architecture of any belief specification
   - Models with many belief parameters must produce rank-2 biases
   - Single-θ models (standard DE): constraint automatically satisfied
2. A spanning-dependent tool for extending biases to unobserved variables
   - Reliability depends on whether anchor shocks span the target's shock space
   - Spanning quality is endogenous to θ — degrades where distortion is largest
3. An identification caveat: data identifies $\theta_k \rho_k$, not the components
   - Single θ + heterogeneous persistence $\equiv$ shock-specific θ_k + common persistence
   - Prop. 5 rejects variable-by-variable models but cannot separate these

CG validates beliefs; R^2 validates shocks. Use SBF as *diagnostic*, not calibration target

Summary

- The SBF is an SDF for beliefs — one probability distortion S_{t+1} that "prices" many biases through objective shock covariances
- Key empirical finding: low-dimensional $\widehat{S}$ from two anchors explains biases across 24 variables; variable-by-variable models rejected; beliefs explain $\sim$80% of anomaly returns
- My simulations confirm: SBF works well structurally, but spanning quality is endogenous
- Interpreting the evidence: R^2 validates shock structure; Prop. 5 and CG correlation validate beliefs — these are distinct tests
- Suggestions: horizon dependence, model discrimination via β_t, time-varying coefficients, conditional asset pricing
- For structural modelers: the rank constraint is portable; CG validates beliefs; R^2 validates shocks; use SBF as diagnostic, not calibration target
- This is a very cool paper!

Backup: Individual-Level SBFs and Disagreement

Observation: each SPF panelist i has their own $S_{i,t+1}$ with loadings $\beta_{i,t}$

Key question: is cross-forecaster disagreement about the *level* or the *composition* of the distortion?

- Level disagreement: everyone distorts the same shocks but by different amounts — single-factor model of disagreement may suffice
- Composition disagreement: some forecasters overweight demand shocks, others overweight supply shocks — requires multi-factor model of disagreement

- SPF has individual-level data $\Rightarrow$ estimate $\hat{\beta}_{i,t}$ for each panelist
- Study the *distribution* of $\hat{\beta}_{i,t}$ across forecasters
- Connects to Patton–Timmermann (2010), Bordalo et al. (2020) disagreement literature