→ **Quantum indeterminism**: In quantum mechanics says that we can't predict exact outcomes but only probabilities. This randomness is not due to lack of knowledge its part of reality itself according quantum theory.

* Albert Einstein who belived in determinism where as Niles Bohr who embrassed quantum randomness and said nature is fundamentally probabilistic

(or) which we can not predict

* **Observer effect/rule**:

* In quantum mechanics the acts of measuring some thing changs the state this is known as observer effect They can't observe a quantum system without describing it. Not necessary a human but anything that interact with the system in a way that a causes a measurement. Before measurement a system can be in superposition. But once you observe the wave function collapses to a single outcome.

fn

... superposition ... it can be coolant.

→ Quantum Gate:-

Quantum gate is a basic unit of a quantum computation that changes the state of qubit, just like classical logic gates [AND or NOT] operate on classical bit[0]. In classical computers logic gates are used to process bits into meaningful information. In quantum computing gates manipulates qubit to perform computation entangle them interfer between paths.

→ Basic Single qubit Gates:-

* These gates operates on a single qubit

i) Pauli X – Gate (NOT Gate)

* It is equivalent to the classical not gate. It flips the qubit from $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$ (or)

$$|0\rangle \longrightarrow |1\rangle$$

$$|1\rangle \longrightarrow |0\rangle$$

* NOT Gate

A	$\bar{A}$
0	1
1	0

In matrix form: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+1 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

ii) Pauli Y-Gate:

* It introduces imaginary phases and flips. It is used to rotate the qubit around the y-axis on the block sphere.

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \rightarrow \text{imaginary}$$

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ i+0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = \underbrace{i\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{i|1\rangle}$$

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-i \\ 0+0 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = \underbrace{-i\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{-i|0\rangle}$$

iii) Pauli Z-Gate:

* It introduces phase flip only. No fliping of 0 or 1, but applies -1 phase to $|1\rangle$.

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \underbrace{-1\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{-1|1\rangle}$$

iv) Hadamard gate (H gate):

* It creates superposition. It turns $|0\rangle$ into equal mix of $|0\rangle$ and $|1\rangle$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

1) $H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1+0 \\ 1-0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \frac{1}{1.414} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{bmatrix}$

$H|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0+1 \\ 0-1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

→ w.r.t. $= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, prove that

$\cancel{\frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{bmatrix}} + \cancel{\frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{bmatrix}}$

$= \cancel{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} \quad \cancel{\frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}} + \cancel{\frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}}$

$= \cancel{\frac{2}{2}}$

$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1+0 \\ 1+0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1+0 \\ 0+1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

→ we have find that $|0\rangle$ & $|1\rangle$ are mix or not, we find out through probabilites

→ $\underbrace{\frac{1}{\sqrt{2}}}_{\alpha} |0\rangle + \underbrace{\frac{1}{\sqrt{2}}}_{\beta} |1\rangle$ } → abstraction

$\alpha^2 |0\rangle + \beta^2 |1\rangle$ } → super position

→

$p(0) = \alpha^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 0.5$

$p(1) = \beta^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 0.5$

} 1 → superposition
∴ both are mix equally

→ Phase Gate (S-gate):-

* It applies a phase of i to the $|1\rangle$ state

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 0+i \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix}$$

$$= i\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= i\,|1\rangle$$

-> **Double Qubit**

i) ~~Go Gate~~

i) **CNOT Gate (controlled NOT)**

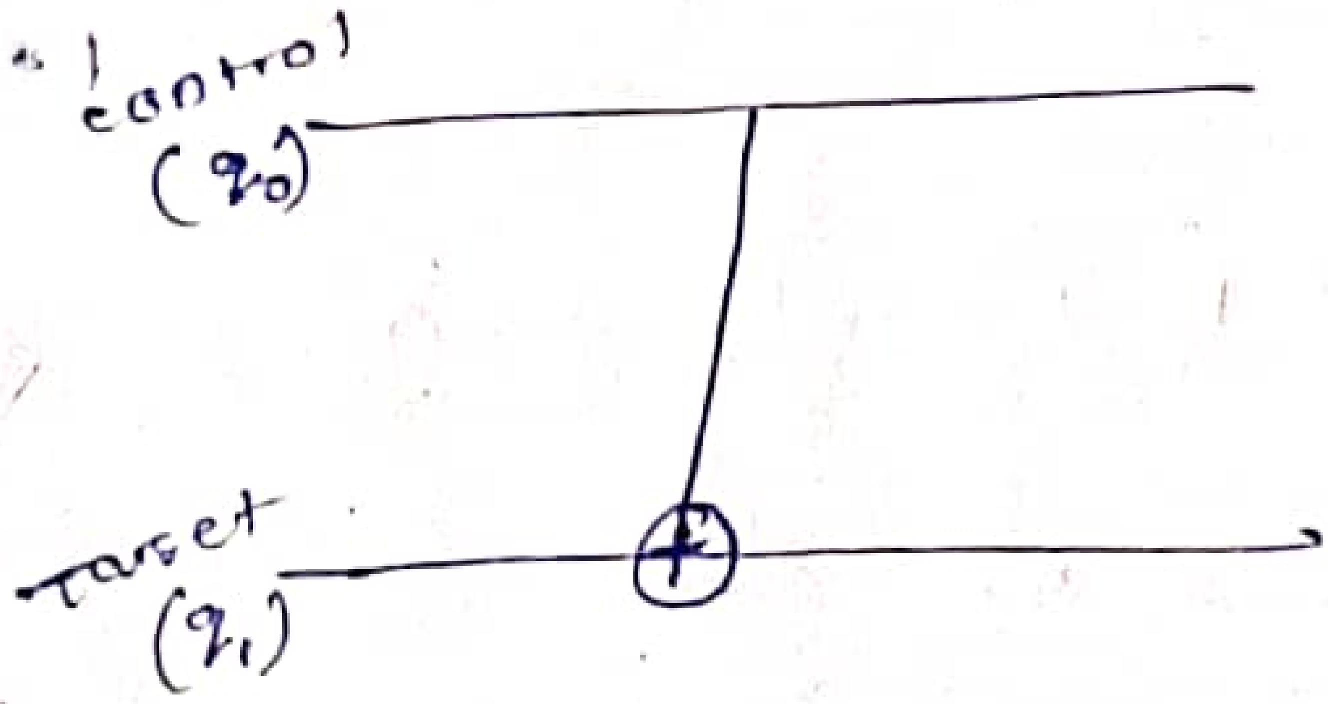
XOR:-

A	B	XOR
0	1	1
0	0	0
1	1	0
1	0	1

-> If the controlled qubit $|1\rangle$ apply not gate to the target qubit.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Diagram:-



$$CNOT|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+1+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \boxed{0} \\ \boxed{1} \end{bmatrix} \rightarrow \text{Target} \rightarrow |11\rangle$$

If input is $|10\rangle$, where the control is one(1) and the target flips the output to $|11\rangle$.

Unitary operators:-

* A unitary operator is like special rule for action it does'nt change the overall amount of quantum state, ~~this~~ just moves it around.

* Let 'U' be the square matrix then, U is unitary if $\boxed{U^T U = U U^T = I}$ Here U^T is conjugate transpose of U (also called hermitain adjoint and 'I' is the identity matrix.

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Ex:- $X \rightarrow X^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ x^T = row become column

from formula:-

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0+1 & 0+0 \\ 1+0 & 0+0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore$ X is unitary

Ex! 2: H Gate is unitary or not

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H^T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

→ w.k.T →

$H H^T = I$ (prove)

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1+1 & 1-1 \\ 1-1 & 1+1 \end{bmatrix}$$

$$\frac{1}{\cancel{2}} \begin{bmatrix} \cancel{2} & 0 \\ 0 & \cancel{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

∴ It is unitary.

→ (i) Quantum & classical & diffe information and principles & differences:

i) What is information: Information refers to the data (or) message that makes or helps us reduce uncertainity (are make decisions.

* In classical (computers) the data is store in bits.

(or)

In classical computers that information is store is using bit (0 or 1) where as in quantum computing the information is store in qubits ($|0\rangle$ & $|1\rangle$).

→ Classical Information:

① A bit is a smallest unit of data in classical computing. A bit can have only one value at a time either 0 or 1.

② Bits are represented using electrical circuits, magnetic charges (or) transisters.

③ We use Logic gates like AND, OR (or) NOT, NAND, NOR, XOR, XNOR. All operations are deterministic same input always give same output.

④ This has definite state always 0 or 1 copying is allowed, we can make copies of classical information. Measuring a classical Bit, tells us exactly what it is.

→ Quantum information:

① A qubit is a unit of quantum information a qubit can be in a State |0⟩ (or) |1⟩ (or) both States as a same time which we called as superposition.

② It is represent as a vecter in a hellbert space (all the States pure and mixed) and a kind of all mathematicle state) and it is implement using atoms, photons, electrons and superconducters.

③ They are performed using quantum gates Hadarmard Gate, x-gate, y-gate, s-gate etc. These are reversable and unitary operations.

④ Classical info can be hacked using classical algorithms, we as quantum info uses quantum increption [like QKD]) is unhackable due to no cloing and observer effect

→ Operators:

The state of a quantum system is described by a vector $|\psi\rangle$ in a hilbert space observables (Things you can measure ~~like~~ position, momentum, energy) are not the same as the state they are represented by operators.

→ An operator $\hat{A}$ acts on $|\psi\rangle$ to produce another ket state

$$\hat{A}|\psi\rangle = |\phi\rangle.$$

Problem:- If $|\psi\rangle = (1, 0)$

$$\hat{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\hat{A}|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 \\ 1+0 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{|1\rangle}$$

→ Linear operator:

→ A condition for a linear operator is

$$\text{"} \hat{A}\left[c_1|\psi_1\rangle + c_2|\psi_2\rangle\right] = c_1\hat{A}|\psi_1\rangle + c_2\hat{A}|\psi_2\rangle \text{"}$$

* This means we can understand their actions on any state by knowing how they act on basis vectors.

* Identity operators :-

It does nothing to the state

$$\hat{I}|\psi\rangle = |\psi\rangle$$

Ex :- $\hat{I}|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1+0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

* Zero operators :-

It turns every state into the zero vector

$$\hat{O}|\psi\rangle = 0$$

Ex :- $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0.$

* Unitary operator :-

$$UU^T = U^T U = I$$

* Hermitain operator / Adjoint :-

→ For an operator $\hat{A}$ its Hermitain adjoint

$= \hat{A}^T$

$\hat{H} = \hat{H}^T$ [conjugate Transpose]

Problem :- Is H Gate is unitary or Hermitain or Both

$$\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$H^{\wedge T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$H^{\wedge}H^{\wedge T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1+1 \\ 1+1 \end{bmatrix}$ $\quad \frac{1}{(\sqrt{2})^2}\begin{bmatrix} 1+1 & 1-1 \\ 1-1 & 1+1 \end{bmatrix}$

$= \frac{1}{(\sqrt{2})^2}\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $\quad \frac{1}{(\sqrt{2})^2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \frac{1}{(\sqrt{2})^2} 2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ✓

∴ They are both unitary and Hermitian, & Identity

<u>check:</u>

unitary $UU^T = U^T U = I$ ✓

$HH^T = H^T H = I \to$ Find

$\frac{1}{(\sqrt{2})}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\frac{1}{(\sqrt{2})^2} 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} \to \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ✓

→ <u>Projection operator:</u>

Applying the projections twice gives the same result as applying it once.

$\boxed{P^{\wedge} = P^{\wedge 2}}$

→ A projection operator projected on $|0\rangle$ the $|0\rangle$ in ... mathematically represent as ...

$P_0 = |0\rangle\langle 0|$

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\langle 0| = [1 \; 0]$

Properties

i) $\hat{P}^2 = P^1$

ii) $P^1 = P^{1T}$ (Hermitian)

$P_0 = |0\rangle\langle 0|$ } → Right side add 0   2D

$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1 \; 0]$ 1D $\begin{matrix} 1+0 \\ 0+0 \end{matrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|0\rangle$, $|1\rangle$

$\boxed{P_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\langle 1| = [$

2D

$|0\rangle$ $|1\rangle$

$P_1 = \underbrace{|1\rangle\langle 1|}_{} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ [in case of projection we add $\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}$]

Right add $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
Left

2D

* Hamiltonain operator (H operator);

• It is fundamental concept in quantum mechanics does represents the total energy of a system. It is a mathematical operator that acts on a quantum system wave function to determine its energy level and time evolution.

$$\hat{H} = \underbrace{\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}}_{\text{operator associated with kinetic energy}} + \overbrace{V}^{\text{potential energy}}$$

→ matrices for parctice;

① matrix Transpose;

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

② $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$A \times B = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

③ $|A| = A - \lambda I$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \rightarrow \begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$$

$$|A| = \begin{bmatrix} 1-T & 2 \\ 3 & 4-T \end{bmatrix}$$

$$|A| = (1-T)(4-T) - 6 = 0$$

$$= 4 - T - 4T + T^2 - 6 = 0$$

$$= T^2 - 5T - 2 = 0$$

→ (5) Determinant

(1)

$$m = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\det |m| = ad - cb$

(2)

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$|A| = 12 - 2$

$|A| = 10.$

(3)

$$\rightarrow A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \end{array}$$

$$1(45 - 48) - 2(36 - 42) + 3(32 - 35)$$

$$= (-3) - 2(-6) + 3(-3)$$

$$= -3 + 12 - 3$$

$$= 12 - 6$$

$$= 6.$$

(4). $A_{m\times n} \; B_{n\times p} = ?\ (m \times p)$

1×2

1×2

(5) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = [3, 4]_{1\times 2}$, $B[3, 4]$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2×2

~~A x B~~ = $\begin{matrix} 3+0 & 4+0 \\ 9+0 & 12 \end{matrix}$

$B \times A = [3 + 12 \quad 6 + 16]$

$B \times A = [15, 28].$