Least Squares

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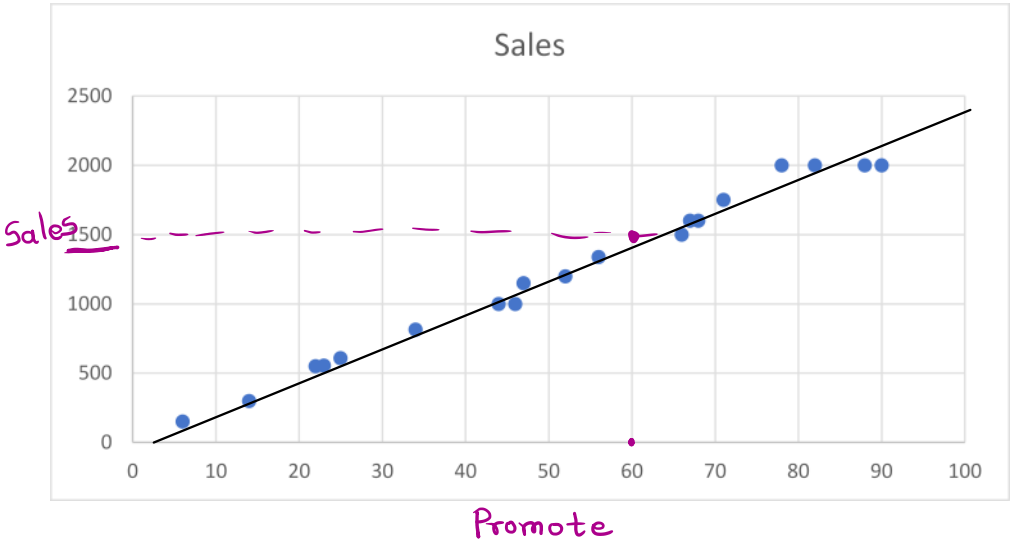
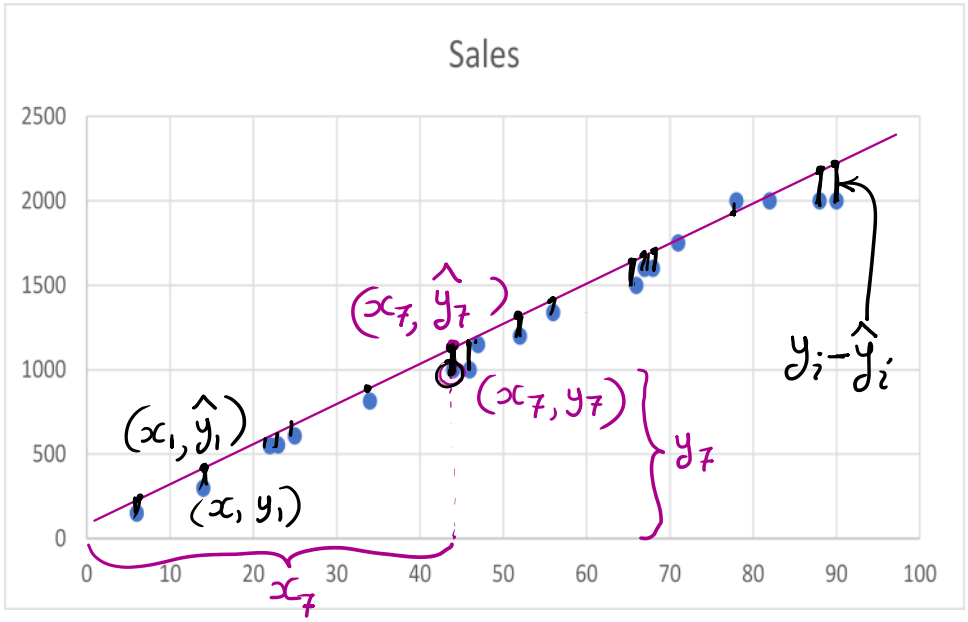
$$\text{Sales} = b_0 + b_1 \text{ Promote}$$





$y = b_0 + b_1 x$

$b_0 = ?$

$b_1 = ?$

(x_i, y_i) are data points

$(x_i, \hat{y}_i)$ are corresponding points on the line

Residual $\underbrace{y_i - \hat{y}_i}_{\text{error}}$; $(x_i, \hat{y}_i)$ is on line $y = b_0 + b_1 x$

$$\Rightarrow \hat{y}_i = b_0 + b_1 x_i$$

$$y_i - (b_0 + b_1 x_i)$$

Z = Loss $f^n = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$ = Residual Sums of Squares

To minimize

$$\therefore \quad Z = \sum_{i=1}^{n} \left[y_i - (b_0 + b_1 x_i) \right]^2$$

$$\frac{\partial Z}{\partial b_0} = 0 \Rightarrow -2 \left[\sum_i (y_i - b_0 - b_1 x_i) \right] = 0$$

$$\frac{\partial Z}{\partial b_1} = 0 \Rightarrow -2 \sum x_i (y_i - b_0 - b_1 x_i) = 0$$

$$\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} b_0 - b_1 \sum_{i=1}^{n} x_i = 0$$

$$\sum y_i - n b_0 - b_1 \sum x_i = 0$$

$$\bar{y} - b_1 \bar{x} = b_0$$

$$b_0 = \bar{y} - b_1 \bar{x} \quad \text{---(1)}$$

$$\sum x_i y_i - b_0 \sum x_i - b_1 \sum x_i^2 = 0$$

$$\frac{\sum x_i y_i}{n} = b_0 \frac{\sum x_i}{n} + b_1 \frac{\sum x_i^2}{n} \quad \text{---(2)}$$

$$\bar{x} \times \bar{y} = b_0 + b_1 \bar{x}$$

$$\bar{x}\bar{y} = b_0 \bar{x} + b_1 \bar{x}^2$$

$$\frac{\sum x_i y_i}{n} - \bar{x}\bar{y} = b_1 \frac{\sum x_i^2}{n} - b_1 \bar{x}^2$$

$$\frac{\frac{\sum x_i y_i}{n} - \bar{x}\bar{y}}{\frac{\sum x_i^2}{n} - \bar{x}^2} = b_1$$

x_i : Promote

y_i : Sales

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$y_i = b_0 + b_1 x_{1i} + b_2 x_{2i}$$

$$z = \sum \left[y_i - (b_0 + b_1 x_{1i} + b_2 x_{2i}) \right]^2$$

$$\frac{\partial z}{\partial b_0} = -2 \sum (y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}) = 0$$

$$\frac{\partial z}{\partial b_1} = -2 \sum x_{1i} (y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}) = 0$$

$$\frac{\partial z}{\partial b_2} = -2 \sum x_{2i} (y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}) = 0$$

$$\overline{y} - n b_0 - b_1 \overline{x_1} - b_2 \overline{x_2} = 0$$

$$\therefore b_0 = \overline{y} - b_1 \overline{x_1} - b_2 \overline{x_2}$$

$$(2) \longrightarrow \sum x_{1i} y_i - b_0 \sum x_{1i} - b_1 \sum x_{1i}^2 - b_2 \sum x_{2i} x_{1i} = 0$$

$$(3) \longrightarrow \sum x_{2i} y_i - b_0 \sum x_{2i} - b_1 \sum x_{1i} x_{2i} - b_2 \sum x_{2i}^2 = 0$$

$$(2) \longrightarrow b_0 \sum x_{1i} + b_1 \sum x_{1i}^2 + b_2 \sum x_{2i} x_{1i} = \sum x_{1i} y_i$$

$$(3) \longrightarrow b_0 \sum x_{2i} + b_1 \sum x_{1i} x_{2i} + b_2 \sum x_{2i}^2 = \sum x_{2i} y_i$$

$$(1) \longrightarrow b_0 + b_1 \overline{x_1} + b_2 \overline{x_2} = \overline{y}$$