1) Solve the following recurrence relations

a) $x(n) = x(n-1) + 5$ for $n > 1$ $\quad x(1) = 0$

$x(n) = x(n-1) + 5 \Rightarrow (1)$

Let $(n = n-1)$ in (1)

$x(n-1) = x(n-1-1) + 5 = x(n-2) + 5 \Rightarrow (2)$

let $(n = n-2)$ in (1)

$x(n-2) = x(n-3) + 5 \Rightarrow (3)$

Now,

$x(n) = x((n-2)+5) + 5$

$x(n) = x(n-2) + 10 \Rightarrow (4)$

put the value of $(x-2)$ in (3), (4)

$x(n) = x(n-3) + 5 + 5 + 5 = x(n-3) + 15 \Rightarrow (5)$

From eqn no (1), (4) & (5)

$x(n) = x(n-i) + 5i$ for $i < n$

put $i = n-1$,

$x(n) = x(n-(n-1)) + 5(n-1)$

$= x(1) + 5(n-1)$

$x(n) = x(1) + 5(n-1)$

$(x(1) = 0)$ then,

$x(n) = 5(n-1)$

b) $x(n) = 3x(n-1) \Rightarrow (1)$ $\quad (x(1) = 4)$

let $(n = n-1)$; $\quad n > 1$

$x(n-1) = 3x(n-1-1)$

$x(n-1) = 3x(n-2) \Rightarrow (2)$

let $(n = n-2)$; in (1)

$x(n-2) = 3x(n-2-1)$

$x(n-2) = 3x(n-3) \Rightarrow (3)$

Now $x(n-1)$ value in (1),

$x(n) = 3.3x(n-2) = 3^2 x(n-2) \Rightarrow (4)$

put $x(n-2)$ in (4)

$x(n) = 3^3 x(n-3) \Rightarrow (5)$

From (1), (4) & (5)

$x(n) = 3^i x(n-i)$ for $i < n$

Now, $i = n-1$;

$x(n) = 3^{n-1} x(n-(n-1))$ [$\because x(n) = 3^{n-1} x(1)$]

$x(n) = 3^{n-1} x(1)$

but $x(1) = 4$ then, from equation

$x(n) = 3^{n-1} 4$

c) $x(n) = x(n/2) + n$ for $n > 0$, $x(1) = 1$

(for $n = 2^k$)

$x(n) = n(n/2) + n \Rightarrow (1)$

Sub n,

$x(1) = 1$

Sub $(n = 2^k)$ then;

$x(2^k) = x(2^{k-1}) + 2^k \Rightarrow (2)$

Sub $(2^k = 2^{k-1})$

$x(2^{k-1}) = x(2^{k-2}) + 2^{k-1} \Rightarrow (3)$

Sub $(2^k = 2^{k-1})$

$x(2^{k-2}) = x(2^{k-3}) + 2^{k-2} \Rightarrow (4)$

$x(2^k) = x(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k \Rightarrow (5)$

When $(2^k = 0)$

$x(2^0) = x(1) = 1$

From (5)

The sum of the first;

$2^0 + 2^1 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$

So we have

$x(2^k) = 2^{k+1} - 1$

sub $(2^k = n/2)$

$\therefore x(n/2) = 2^n - 1 = 2^{n/2} - 1$

d) $x(n) = n(n/3) + 1$ for $n \geq 1$ $x(1) = 1$ (Solve for $n = 3^k$)

$x(n) = x(n/3) + 1 \rightarrow D$

But $(x(1) = 1)$;

sub $(n = 3^k)$ in ①

$x(3^k) = x\left(\frac{3^k}{3}\right) + 1$

$x(3^2) = x(3^{k-1}) + 1 \Rightarrow E$

2 Evaluate the following

i) $T(n) = T(n/2) + 1$ where $n = 2^k$ for all $k \geq 0$

assume

$n = 2^k \; ; \; k = \log n$

$T(2^k) = T\left[\frac{2^k}{2}\right] + 1$

$T(2^k) = T(2^{k-1}) + 1$

$= (T(2^{k-2}) + 1) + 1$

$= [T(2^{k-3}) + 1] + 2$

$= T(2^{k-3}) + 3$

$T(2^k) = T(2^{k-k}) + k$

$= T(2^0) + k$

$= T(1) + k$

$T(1) = 1$ then,

$T(2k) = 1 + k$

$T(n) = \log n + 1$

We got $T(n) = O(\log n)$

2) $T(n) = T(\lceil n/3 \rceil) + T(\lceil 2n/3 \rceil) + cn$

$T(n) \leq T(\lceil n/3 \rceil) + T(\lceil n/3 \rceil) + cn$

$\leq d\lceil n/3 \rceil \log(\lceil n/3 \rceil) + d\lceil 2n/3 \rceil \log(\lceil 2n/3 \rceil)$

$= (d\lceil n/3 \rceil \log n - d\lceil n/3 \rceil \log 3) + (d\lceil 2n/3 \rceil \log n - d\lceil 2n/3 \rceil \log(\lceil 3/2 \rceil) + cn$

$= dn \log n - d(\lceil n/3 \rceil \log 3 + \lceil 2n/3 \rceil \log(\lceil 3/2 \rceil) + cn$

$= dn \log n - dn(\log 3 - 2/3) + cn$

$\leq dn \log n$

$[\because d \geq c/(\log(3) - 2/3)]$

$\therefore$ Order $= O(n \log n)$

b

3 consider the following recussion algorithm

```
min1(A[0...n-1]).
if n=1 return A[0]
else temp = min1[A[0...n-2]]
    if temp <= A[n-1] return temp
    else
    Return A[n-1]
```

a) The recursive algorithm computes the min value in array A of size n it does this by comparing the cost element of the array A[n-1] with the minimum value of the rest of the array A[0...n-2] & returning smaller value

b The algorithm makes call to min1(A[0..n]) which is n-1, then,

$$T(n) = T(n-1) + c$$

c is the constant representing the time taken for the operation outside the call

```
def min1(A, n):
    if n == 1:
        return A[0]
    else:
        temp = min1(A, n-1)
        if temp <= A[n-1]:
            return temp
        else:
            return A[n-1]
```