

Descriptive Statistics

Unit - 3

CLASSMATE

Date _____

Page _____

2M

- 1) what is a statistics ? Mention its types
Statistics is the science, or a branch of mathematics that involves collecting, classifying, analyzing, interpreting, and presenting numerical facts & data.
i) Descriptive statistics ii) Inferential statistics
- 2) compare Descriptive statistics & Inferential statistics.
Descriptive statistics is used to summarize & describe the data. where as population statistics make a prediction about a population based on a sample data.
- 3) what are the 4 types of data & measurement scale
a) Nominal b) Ordinal c) Interval d) Ratio
- 4) what are nominal & ordinal data? Give ex
Nominal: Nominal scales are used for labeling variables, without any quantitative value.
ex: blood type, eye color, gender
ordinal: It is used to rank or order objects.
ex: education level (highschool, puc, bs, ms, phd)
- 5) what are interval & ratio data? Give ex

Interval: Interval scales are numeric scales in which we know both the order & exact difference b/w the values.

ex: Celsius temperature bcz diff b/w each value is same

Ratio: Ratio have the same properties as interval data, but ratio have an absolute zero, & the ratio of 2 numbers is meaningful.

ex: ratio data are height, weight, time, volume

- g) Define Measure of central Tendency. List 3 types
 - It yields information about the center,
 or middle part, of a group of numbers
 i) Mode ii) Median iii) Mean

h) Define mode. Determine the mode for the following numbers

2 4 8 4 6 8 7 8 4 3 8 9 4 3 5

mode is a most frequently occurring value in a set of data.

2 2 3 3 4 4 4 5 6 7 8 8 8 9

mode: 4

- g) Define median

median is the middle value in an ordered array of numbers.

- b) write the steps to calculate Median

Step 1: Arrange the observations in an ordered data array.

Step 2: For an odd number of terms, find the middle term of the ordered array.
 It is the median.

Median: $\frac{(n+1)}{2}$ th term

Step 3: For an even number of terms, find the avg of middle two terms. This avg is the median.

Median: $\frac{(\frac{n}{2})^{th} \text{ term} + (\frac{n}{2}+1)^{th} \text{ term}}{2}$

- c) Determine the median

2 4 8 4 6 2 7 8 4 3 8 9 4 3 5
 2 2 3 3 4 4 4 5 6 7 8 8 8 9
 $n = 15$

Median: $\frac{(n+1)}{2}$

$= \frac{15+1}{2} = 8^{\text{th}}$ term = $\frac{8+9}{2} = 8.5$

- 10) Determine mode & median for

813 345 609 073 167 243 444 524 199
 682

Mode: There is no repeating numbers so the mode is not applicable

MedPan:

073 167 199 313 243 345 444 524 609 682

$$\text{MedPan} = \frac{243 + 345}{2} = 294$$

- ii) Compute the mean for the following numbers

17.3 44.5 31.6 40.0 52.8 38.8 30.1 78.5

$$\text{Mean} = \frac{\sum x}{N} = \frac{17.3 + 44.5 + 31.6 + 40.0 + 52.8 + 38.8 + 30.1 + 78.5}{8} = 41.7$$

- 10) Define Percentiles. Write the steps to calculate location of percentile.

Percentiles are measures of central tendency that divide a gap of data into 100 parts.

Steps

i) Organize the numbers into an ascending-order array.

- ii) Calculate the percentile location (L) by:

$$L = P(N) \quad \text{where } P = \text{the percentile of interest}$$

100

$P = \text{percentile location}$

$N = \text{number in the data set}$

- iii) determine the location by either (a) or (b)

a) If P is a whole number, the P th percentile is the avg of the value at the P th location & the value at the $(P+1)$ th location

b) If P is not a whole number, the P th percentile value is located at the whole number part of it + 1.

- 13) What is quartile? Determine Q_3 for

14, 12, 19, 23, 5, 13, 28, 17

Quartile are measures of central tendency that divide a group of data into four subgroups or parts. The 3 quartiles are denoted by

Q_1, Q_2, Q_3

$Q_3 = 25$

$$N = 8 \quad i = \frac{75}{100} (8)$$

100

= 6th location $\rightarrow i$ is whole num

Q_3 is found as avg of 6th & 7th term

$$5, 12, 13, 14, 17, 19, 23, 28 = \frac{19 + 23}{2} = 21$$

- 14) determine 30th percentile of the following 8 numbers: 14, 12, 19, 23, 5, 13, 28, 17

$$P = 30 \quad N = 8$$

$$i = \frac{30}{100} (8) = 2.4 \rightarrow \text{odd num}$$

$$5, 12, 13, 14, 17, 19, 23, 28$$

($2.4 + 1$) = 3rd location

= 13

- 15) Define Range. write the range of following numbers. It is difference b/w the largest value of a data set and smallest value of a set.

16, 28, 29, 13, 17, 20, 11, 24, 22, 27, 25

30, 19, 18, 33

11, 13, 16, 17, 18, 19, 20, 25, 27, 28, 29, 30, 32, 33

34

$$= 34 - 11 = \underline{\underline{23}}$$

- 16) Define Interquartile Range. write its formulae. It is the range of values b/w the first & 3rd quartile.

$$IQR = Q_3 - Q_1$$

- 17) Define Standard Deviation. write its formulae.

It is a spread of a group of numbers from the mean. Standard deviation is the square root of the variance.

$$\text{Population standard deviation: } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$$

$$\text{Sample SD: } s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

- 17) Define Mean Absolute Deviation. write its formulae.

The MAD is the avg of absolute values of the deviations around the mean for a set of numbers.

$$MAD = \frac{\sum |x - \bar{x}|}{N}$$

- 18) Define Variance. write its formulae.

Variance is the avg of the squared deviation s about the arithmetic mean for a set of members.

$$\text{Population variance: } \sigma^2 = \frac{\sum (x - \bar{x})^2}{N}$$

$$\text{Sample variance: } s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

- 19) write the formulae of sample variance.

$$\text{sample variance: } s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$\text{sample variance: } s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$\text{sample SD: } s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

21) Define Z score, write its formula.

It gives you an idea of how far from the mean a data point is

$(x_i - \bar{x})$ → population

$$Z = \frac{x_i - \bar{x}}{\sigma}$$

$$Z = \frac{(x_i - \bar{x})}{S} \rightarrow \text{sample}$$

S

22) State Empirical Rule. List the condition.

It is an important rule of thumb that

Is used to state the approximate percentage of value that lie within a given number of SD from the mean of a set of data.

If the data are normally distributed. What is the ratio? Distance from the mean values within Distance

$$\pm 1\sigma$$

68%

$$\pm 2\sigma$$

95%

$$\pm 3\sigma$$

99.7%

23. Define Coefficient of variation. write its formula.

It is a statistic that is the ratio of the standard deviation to the mean expressed in percentage, and is denoted as C_V .

$$C_V = \frac{\sigma}{\bar{x}} \times 100$$

24)

long

1) Explain the types of data & measurement scale with ex:

2M) refer the 5th question

1) Nominal:

- the data can only be categorized
- It is the lowest level of data measurement

2) Ordinal:

- the data can be categorized & ranked
- It is higher than the nominal level.

3) Interval:

- the data can be categorized, ranked & evenly spaced.

4) Ratio:

- the data can be categorized, ranked, evenly spaced & has a natural zero.
- Highest level of data measurement

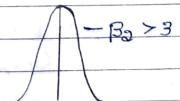
8) Explain kurtosis types with diagram

2m-99] Kurtosis is a numerical method in statistics that measures the sharpness of the peak, in the data distribution.

Types:

1) leptokurtic distribution:

distributions that are high & thin.



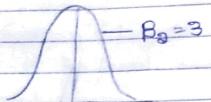
2) platykurtic distribution:

distributions that are flat & spread out.



3) Mesokurtic distribution

Between these two types are distributions that are more "normal" in shape.



$$\beta_2 = 3$$

3) Explain measure of skewness with its types

Skewness is a statistical measure that describes the asymmetry of a probability distribution.

Types:

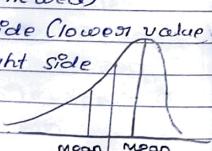
1) Zero skewness

- A distribution is zero skewness when it is perfectly symmetrical with both tails being of equal length & mean, mode, median coinciding at the center.



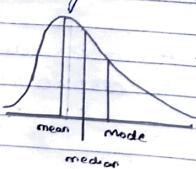
2) Negative skewness (left skewed)

- when the tail of left side (lower value) is longer or fatter than right side (higher value).



3) Positive skewness (right skewed)

- the tail of right side (higher value) is longer or fatter than left side (lower value)



- 4) The number of U.S. cars in service by top car rental companies in a recent year according to Auto Rental News follows. Compute mode, the median, and the mean.

Company	Number of cars in service
Hertz	643,000
National/Alamo	327,000
Avis	233,000
Dollar	204,000
Budget	167,000
Advantage	144,000
U-Save	20,000
Payless	12,000
ACE	10,000
Fox	9,000
Rent-A-Car	9,000
Tripangle	7,000
	6,000

$$\Rightarrow \text{Mode} = 9800 \quad \text{is repeated 2 times}$$

$$\text{Median} = \frac{(n+1)}{2} = \frac{(13+1)}{2} = 7^{\text{th}} \text{ position} = 80,000$$

$$\text{Mean} = \bar{x} = \frac{\sum x}{N} = \frac{1,791,000}{13} = 137,769.23$$

- 5) Compute the 25th percentile, the 55th percentile, ϕ_1 , ϕ_2 , and ϕ_3

$$16 \ 28 \ 29 \ 13 \ 17 \ 20 \ 11 \ 34 \ 32 \ 27 \ 25 \ 30 \ 19 \ 18 \ 33$$

$$\Rightarrow 11 \ 13 \ 16 \ 17 \ 18 \ 19 \ 20 \ 25 \ 27 \ 28 \ 29 \ 30 \ 31 \ 32$$

$$33 \ 34$$

$$P = 35 \quad P = 15 \quad i = P/N = \frac{35}{100} (15) = 5.25$$

5.25 is not a whole number

$$= 5.25 + 1 = 6^{\text{th}} \text{ location}$$

$$= 19$$

$$P=55 \quad i=15 \quad P = \frac{P}{100} (N) \Rightarrow \frac{55}{100} (15) \Rightarrow 8.25$$

8.25 is not a whole $\therefore 8.25 + 1 = 9^{\text{th}}$ location

27

$$\therefore Q_1 = \frac{85}{100} (15) \Rightarrow 3.75 + 1 = 4^{\text{th}} \text{ location} = \underline{\underline{3.75}}$$

$$Q_2 = \frac{50}{100} (15) \Rightarrow 7.5 + 1 = 8^{\text{th}} \text{ location} = \underline{\underline{7.5}}$$

$$Q_3 = \frac{75}{100} (15) \Rightarrow 11.25 + 1 = 12^{\text{th}} \text{ location} = \underline{\underline{11.25}}$$

- 6) The following shows the top 15 global market categories for advertising spending for a recent year according to advertising. Ad spending is given in millions of US dollar. Determine the 1st, 2nd & 3rd quartile of the data.

category Ad spending

Automotive	\$28,195
Personal care	19,586
Entertainment	9,538
Food	7,793
Doug	7,707
Electronics	10,823
Soft drinks	3,910
Metal	8,576
Cleaners	3,571
Per	5,553
Com	5,847
Tele	24,88
Fm	24,33
Gas	5,050
Conduy	11,37
Toys	6,99

$$\Rightarrow N = 16$$

$$\therefore Q_1 = 25$$

$i = 25$ (16) $\Rightarrow 4$ whole num \therefore add 4th & 5th term
 $\therefore 100$

$$(4+5)^{\text{th}} \text{ term} = 2433 + 2488 = \underline{\underline{2460.5}}$$

$$\therefore Q_2 = 50$$

$$i = 50 \quad (16) \Rightarrow 3.571 + 3.576 = \underline{\underline{3.573.5}}$$

$$\therefore Q_3 = 75$$

$$i = 75 \quad (16) \Rightarrow 7707 + 7793 = \underline{\underline{7750}}$$

$$7) \quad 6 \quad 2 \quad 4 \quad 9 \quad 1 \quad 3 \quad 5$$

a) Find range

$$\text{range} = 9 - 1 = \underline{\underline{8}}$$

$$b) \quad 4 \quad 3 \quad 0 \quad 5 \quad 2 \quad 9 \quad 4 \quad 5$$

Some of 7

b) Find the mean absolute deviation

x	x - 4.28	(x - 4.28) ²	x - 4.28
1	-3.28	10.75	3.28
2	-2.28	5.19	2.28
3	-1.28	1.63	1.28
4	-0.28	0.07	0.28
5	0.72	0.52	0.72
6	1.72	2.95	1.72
9	4.72	22.2	4.72

$$MAD = \frac{\sum |x - 4.28|}{N} = \frac{14.28}{7} = \underline{\underline{2.04}}$$

c) Find the population variance

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{N} = \frac{43.3}{7} = \underline{\underline{6.18}}$$

d) Find the population standard deviation

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} = \sqrt{\frac{43.3}{7}} = \underline{\underline{2.48}}$$

e) Find the Interquartile range.

1 2 3 4 5 6 7
Q1 = 2.5 n=7

$$P = \frac{Q_1}{100} (7) = 1.75 + 1 = 2^{\text{nd}} \text{ pos} = \underline{\underline{2}}$$

Q3 = 7.5

$$P = \frac{Q_3}{100} (7) = 5.25 + 1 = 6^{\text{th}} \text{ pos} = \underline{\underline{6}}$$

$$IQR = 6 - 2 = \underline{\underline{4}}$$

f) Find the zscore for each value.

$$z = \frac{x - \bar{x}}{\sigma} \quad \sigma = 2.48$$

$x - \bar{x}$	z
-3.28	$-3.28/2.48 = -1.32$
-2.28	-0.91
-1.28	-0.51
-0.28	-0.11
0.72	0.29
1.72	0.69
4.72	1.90

g) calculate coefficient of variation

$$CV = \frac{\sigma}{\bar{x}} (100)$$

$$= \frac{2.48}{4.72} (100) \Rightarrow \underline{\underline{57.94}}$$

9) Shown here is a sample of 6 of the largest accounting firms in the US and the no of partners associated with each firm as reported by the public Accounting Report. Calculate Sample variance & Sample standard deviation.

Firm	Number of partners
Deloitte & Touche	2654
Ernst & Young	2108
Pricewater	2069
KPMG	1664
RSM	720
Giant	309

$$\bar{x} = \frac{\sum x_i}{N} = \frac{9524}{6} = 1587.33 \quad N=6$$

$$(x - \bar{x})^2 \quad 1137784.88$$

$$2108 \quad 520.67 \quad 271097.84$$

$$2069 \quad 481.67 \quad 232005.98$$

$$1664 \quad 76.67 \quad 5878.28$$

$$720 \quad -867.33 \quad 452261.32$$

$$309 \quad -1278.33 \quad 1634187.58$$

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{N-1} = \frac{4033155.88}{5} = \underline{\underline{806631.056}}$$

Sample variance

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{N-1} = \frac{4033155.88}{5} = \underline{\underline{806631.056}}$$

Sample SD

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N-1}} = \sqrt{806631.056} = \underline{\underline{898.12}}$$

- 10) Use your calculator or comp
find population variance & population sd
 $(x - \bar{x})^2$

x	$x - \bar{x}$	$(x - \bar{x})^2$
123	-307.37	944476.31
392	-38.37	1473.85
572	141.63	20059.05
303	-127.37	16203.11
090	-340.37	115851.73
280	-150.37	62500.13
953	582.63	273400.11
468	37.63	1416.01
526	115.63	13370.09
179	-251.37	63186.87
749	318.63	101535.07
531	100.63	10136.39
378	-52.37	2744.61
601	140.63	89111.18
075	355.37	126387.83
646	215.63	46496.89
$\Sigma x = 6886$	$= 430.375$	938098.23
$N = 16$		

Same as 4th q - c & d

- 11) On a certain day the avg closing price of a gap of stocks on the New York Stock Exchange is \$25 (to the nearest dollar). If the median value is \$33 and the mode is \$21, is the distribution of these stock skewed? If so, how?

yes, based on the information provided, it suggests that the distribution of these stock prices is likely positively skewed (or right skewed)

bcz \rightarrow The mean (avg) closing price is higher than both the median and the mode

2) In very skewed distribution, the mean is typically greater than the median

- 12) A local offers ballroom dancing on Friday night. A researcher observes the customers and estimates their ages. Discuss the skewness of the distribution of ages if the mean age is 57, the median age is 54, and the mode age is 59.

The distribution of ages appears -vely skewed. Since the mean (57) is less than the median (54), and the median is less than the mode (59).

This indicates that there are more young individuals pulling the distribution to the left.

- 13) Suppose the following data are the ages of Internet users obtained from a sample. Use these data to compute a Pearsonian coefficient of skewness. What is the meaning of the coefficient?

41	15	31	35	24	formula
23	21	29	22	18	$= \frac{3(\bar{x} - M)}{S}$
30	20	19	19	16	σ
23	27	38	34	24	
19	20	29	17	23	

$$\bar{x} = \frac{\Sigma x}{n} = \frac{600}{25} = 24$$

$$M = \text{Median: } 16, 17, 18, 19, 19, 19, 30, 20, 21, 22, 22$$

$$23, 23, 23, 24, 24, 25, 27, 29, 30, 31, 34, 33$$

$$21, \quad \frac{x_1 + x_2}{2} = \frac{25 + 21}{2} = 23, \quad 13^{\text{th}} \text{ locat} \Rightarrow 23$$



a) To check the outliers, we can calculate the IQR and identify any datapoints outside the range.

$$\text{IQR} = Q_3 - Q_1 \Rightarrow 589 - 500 = 89$$

$$\text{lower bound} : Q_1 - 1.5 * \text{IQR}$$

$$500 - 1.5 * 89 = 366.5$$

$$\text{upper bound} : Q_3 + 1.5 * \text{IQR}$$

$$589 + 1.5 * 89 = 722.5$$

Conclusion: No. since there are no values below 366.5 & above 722.5.

b) Distribution of data is left skewness (negatively skewed), bcz the Median (Q_2) is closer to Q_3 .

The left whisker is longer than the right whisker.

- 15) Shown here is a list of the top five industrial and farm equipment companies in the United States, along with their annual sales (\$ million). Construct a pie chart & a bar graph to represent these data, and label the slices with the approximate percentages.

Firm Revenue (\$ million)

Caterpillar	30,251
Deere	19,986
Illinoian Tool Works	11,731
Eaton	9,817
American Standard	9,509

- 14) Construct a box-and-whisker plot on the following data. Do the data contain any outliers? Is the distribution of data skewed?

510 690 623 559 558 590 588 580
379 527 570 477 495 490 602 497

\Rightarrow 510 601 503 495 574 563 609 571
527 540 541 558 559 568 570 574
580 588 590 601 602 609 623 690

$$Q_1 = 25 \quad i = \frac{25}{100} (24) = 6 \Rightarrow (6+7)^{\text{th}} \text{ term} \\ = 497 + 503 = 500$$

$$Q_2 = 50 \quad N = 24 \quad i = \frac{50}{100} (24) = 12 \Rightarrow (12+13)^{\text{th}} \text{ term} \\ = 558 + 559 = 558.5$$

$$Q_3 = 75 \quad i = \frac{75}{100} (24) = 18 \Rightarrow (18+19)^{\text{th}} \text{ term} \\ = 588 + 590 = 589$$

$$\max = 690 \quad \min = 379$$

a) pie chart

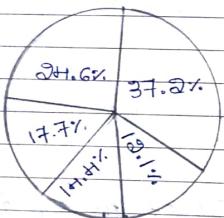
firm	Revenue(\$)	Proportion	degree
a) Caterpillar	30851	0.372	133.92
b) Deere	19986	0.246	88.56
c) Illinois W	11731	0.144	51.84
d) Eaton	9817	0.121	43.56
e) Americanst	9509	0.117	42.12
	81894	1.000	360

$$\text{proportion} = \frac{30851}{81894} = 0.372$$

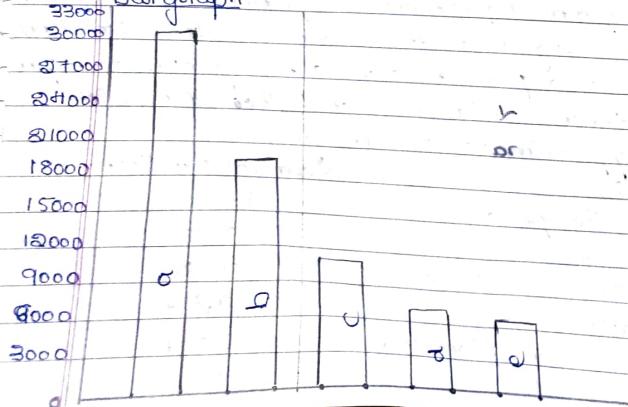
$$\text{degree} = 0.372 \times 360^\circ = 133.92$$

- To construct a pie chart

$$\text{proportion} * 100 \Rightarrow 0.372 * 100 = 37.2\%$$
$$0.246 * 100 = 24.6\%$$
$$0.144 * 100 = 14.4\%$$
$$0.121 * 100 = 12.1\%$$
$$0.117 * 100 = 11.7\%$$



b) Bar graph



16) The following list shows the top 6 pharmaceutical company. construct pie & bar graph.

pharm	sales
Phizer	50,981
John & Johnson	47,348
Merck	33,939
Bristol	31,886
Abbot	30,473
Wyeth	17,353

Same as 15th question

17) The following data represent the costs (in dollar) of a sample 30 postal mailings by a company.

3.67	5.10	9.15	5.11	3.32	2.09	
1.83	2.75	1.93	3.89	7.20	3.78	
6.72	7.80	5.47	4.15	3.55	3.53	
3.34	4.95	5.43	8.64	4.84	4.10	
10.94	6.45	4.65	1.97	3.84	3.51	
Stem Leaf						
=>	1	8	3	9	3	97
	2	0	9	7	5	78
	3	2	3	2	34	53
	4	1	0	15	65	84
	5	1	0	11	42	47
	6	4	5	7	3	
	7	3	0	8	0	
	8	6	4			
	9	1	5			
	10	9	4			

Q18) Define stem & leaf plot.

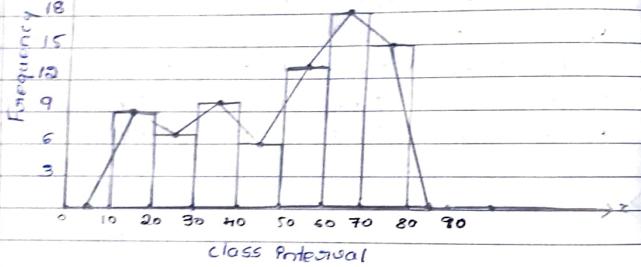
A stem & leaf plot is constructed by separating the digits from each number of the data into two parts, a stem & leaf. left most digit is stem & rightmost digit contains lower value.

18) construct a histogram and a frequency polygon

Class Interval	Frequency
10 - under 20	19
20 - under 30	7
30 - under 40	10
40 - under 50	6
50 - under 60	13
60 - under 70	18
70 - under 80	15

$$\text{For polygon} = 10 + 80 = 15$$

$$+ 80 + 30 = 35$$



19) Class Interval

Class Interval	Frequency
30 under 32	5
32 under 34	7
34 under 36	15
36 under 38	21
38 under 40	34
40 under 42	34
42 under 44	17
44 under 46	8

construct a histogram and a frequency polygon; Same as 18) g

20) construct the stem and leaf plot using 2 digits for stem

21 239 240 233 249 265 234
etc...

consider 2 digit as stem and remaining 1 digit as leaf. stem leaf
Same as 17 9 21 3

Probability

23) what is probability? Mention its types

probability is a numerical measure of chance of occurrence of an event

- Types:
- classical probability
 - empirical probability
 - subjective probability

24) what is a experiment give ex

It is a process that produce outcome

ex: Rolling a die and observing outcomes

event: It is an outcome of an experiment

ex: experiment is rolling a die

event is the roll a number 5

25) long! Explain general Methods of assigning probabilities with ex

- the classical Method
- the relative frequency of occurrence method
- subjective probability

25-QM: what is the classical method of assigning probability. Give ex.

classical method:

when probabilities are assigned based on laws and rules, the method is called classical method.

Formula: $P(E) = N_e/N$

N : total no of possible outcomes of an experiment.

N_e : the no of outcomes in which the event occurs out of N outcomes.

Ex: A die is rolled, find the probability that even no is obtained.

$$S = \{1, 2, 3, 4, 5, 6\} \quad E = \{2, 4, 6\}$$

$$P(E) = N_e/E = 3/6 = \underline{\underline{1/2}}$$

36-8M) what is relative frequency of occurrence method assigning of a probability. Give ex.

Relative: It is based on cumulated historical data.

Formula:

Prob: $\frac{\text{Number of Times an Event Occurred}}{\text{total no of opportunities for the event to occur.}}$

Ex: A jar contains 4 red marble & 10 white marble. If the marble is drawn from the jar is white. What is the probability that the marble is white?
 $\Rightarrow \frac{10}{14} \Rightarrow \frac{5}{7} \Rightarrow 0.71$

37-8M) subjective probability with ex

It is based on the feelings or insights of the person determining the probability. It comes from person's intuition or reasoning.

ex: If someone claims that their favorite sports team has an 80% chance of winning the next game

Q) Define elementary events. Give an ex elements that cannot be decomposed or broken down into other events.

Ex: In the experiment of rolling a die. there are 6 elementary events $\{1, 2, 3, 4, 5, 6\}$.

39) Define Sample space. Give an ex

It is a complete listing of all elementary events for an experiment.

Ex: Tossing a coin $S = \{H, T\}$

40) Give an example for

a) Union : ex : $A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$
 $A \cup B = \{1, 2, 3, 4, 5, 6\}$

b) Intersection : ex : $A = \{1, 2, 3\}$ $B = \{3, 4\}$
 $A \cap B = \{3\}$

41) Define with ex

a) Mutually exclusive events

Two or more events are mutually exclusive if the occurrence of one event precludes the occurrence of other event.

Ex: When a coin is tossed then the result will be either head or tail, but we cannot get both the results.

b) Independent Events.

If the occurrence or nonoccurrence of one of the event does not affect the occurrence or nonoccurrence of other events.

ex: If we roll a die twice, the outcome of the first roll and second roll have no effect on each other.

4.2) Define collectively exhaustive events. ex
A list of collectively exhaustive events contains all possible elementary events for an experiment.

ex: rolling a die, 1, 2, 3, 4, 5, 6 outcomes are collectively exhaustive.

4.3) Define Complementary events. ex

The complement of event A is denoted A' , pronounced "not A".

ex: rolling one die

event x: getting even no

complement of x: odd num

4.4) If a population consists of the positive even numbers through 80 and if $A = \{2, 6, 12, 24\}$, what is A' ?

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{4}{36} = \frac{32}{36} = \frac{8}{9} \approx 0.89$$

$$A' = \{4, 8, 10, 14, 16, 18, 20, 22, 26, 28, 30\}$$

4.5) what are 3 types of counting the possibilities:

1) nn counting rule

2) Sampling from a population with replacement

3) combinations: sampling from a population without replacement

4.6) write the general Addition & special Addition laws.

General law of addition

If P s used to find the union of 2 events
 $P(x \cup y) = P(x) + P(y) - P(x \cap y)$

Special law of addition

If the 2 events are mutually exclusive, the probability of the union of the 2 events is the probability of the first event + prob of 2nd event
 $P(x \cup y) = P(x) + P(y)$

4.7) write) General law of multiplication

If P s used to find joint probability.

$$P(x \cap y) = P(x) \cdot P(y/x) = P(y) \cdot P(x/y)$$

2) special law of multiplication.

If x & y are independent, then this law can be used to find the intersection of x & y .

$$P(x \cap y) = P(x) \cdot P(y)$$

4.8) write the conditional probability. ex (long)

$$P(x|y) = \frac{P(x \cap y)}{P(y)} = \frac{P(x) \cdot P(y/x)}{P(y)}$$

• If P is a probability that E_1 will occur, given that E_2 is known to have occurred.

• It involves knowledge of some prior information
 ex: $P(y) = 0.70$ $P(x \cap y) = 0.56$

$$P(x|y) = \frac{0.56}{0.70} = 0.80$$

- 49) what are discrete random variables?
 A random variable is discrete random variable if the set of all possible values is at most a finite or a countable infinite number of possible values.
 ex: determining the num of defects in a batch of 50 items

- 50) what are continuous random variables?
 Give an ex

continuous random variables are generated from experiments in which things are "measured" not "counted".
 ex: measuring the length of newly designed automobile.

- 51) write formula for Mean, Variance & SD of discrete Distribution.

$$\text{Mean : } \mu = E(x) = \sum [x_i \cdot P(x_i)]$$

$$\text{variance : } \sigma^2 = \sum [(x_i - \mu)^2 \cdot P(x_i)]$$

$$\text{SD : } \sigma = \sqrt{\sum [(x_i - \mu)^2 \cdot P(x_i)]}$$

- 52) list the assumption of binomial distribution
- The experiment involves n identical trials
 - Each trial has only 2 possible outcomes denoted as success or a failure
 - Each trial is independent of the previous trials
 - The p & q remain constant throughout the experiment.

p is the prob of getting success.
 $q = (1-p)$ "failure

- 53) write the formulae of binomial distribution.

$$P(x) = {}^n C_x \cdot P^x \cdot q^{n-x} = \frac{n!}{x!(n-x)!} \cdot P^x \cdot q^{n-x}$$

- 54) A company places a seven-digit serial number on each part that is made. Each digit of the serial number can be any number from 0 through 9. Digits can be repeated in the serial num. how many different serial numbers are possible?

since Each digit of the seven-digit serial number can be any num from 0 through 9, there are 10 options for each digit
 $\therefore 10^7 = 10,000,000$

- 55) A small company has 20 employees. Six of these employees will be selected randomly to be interviewed as part of an employee satisfaction program. how many different groups of 6 can be selected?

$$\begin{aligned} {}^{20}C_6 &= \frac{{}^{20}P_6}{6!(20-6)!} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{6!} \\ &= \frac{10}{3} \times \frac{19}{2} \times \frac{18}{1} \times \frac{17}{0} \times \frac{16}{-1} \times \frac{15}{-2} \\ &= 38,760 \end{aligned}$$

- 56) list the characteristics of poisson distribution

- It is a discrete distribution
- It describes rare events.
- Each occurrence is independent of other occurrences.
- It describes discrete occurrences over a continuum or interval.

long

- 22) A supplier shipped a lot of 6 parts to a company. The lot contained 3 defective parts. Suppose the customer decided to randomly select 2 parts and test them for defects. How large a sample space is the customer potentially working with? List the sample space. Using the sample space listed, determine the probability that the customer will select a sample with exactly one defect if the customer is selecting 2 parts out of 6 parts.

$$S = {}^6C_2 = \frac{6!}{2!(6-2)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 4 \times 3 \times 2 \times 1} = 15$$

The sample space is 15. Now let's list the possible outcomes:

$$S = \{(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), \\ (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), \\ (4,6), (5,6)\}$$

To find the probability of selecting a sample with exactly one defective part,

defective (9%)
Da Ab Ac
Ba Bb Bc
Ca Ch Cc
Ab AB Ac
Ba Bb Cc

- 23) Given $x = \{1, 3, 5, 7, 8, 9\}$ $y = \{2, 4, 7, 9\}$
 $z = \{1, 2, 3, 4, 5, 7, 8, 9\}$
- a) $x \cup z = \{1, 2, 3, 4, 5, 7, 8, 9\}$
b) $x \cap z = \{1, 3, 7\}$
c) $x \cap y \cap z = (x \cap y) = \{7, 9\}$
 $(x \cap y \cap z) = \{7, 9\}$
d) $x \cap y = \{4, 9\}$
e) $x \cup y \cup z = (x \cup y) = \{1, 2, 3, 4, 5, 7, 8, 9\}$
 $(x \cup y \cup z) = \{1, 2, 3, 4, 5, 7, 8, 9\}$
f) $(x \cup y) \cap z = (x \cup y) = \{1, 2, 3, 4, 5, 7, 8, 9\}$
 $(x \cup y) \cap z = \{1, 2, 3, 4, 7, 8\}$
g) $(y \cap z) \cup (x \cap y) = (y \cap z) = \{2, 4, 7\}$
 $(x \cap y) = \{7, 9\}$
 $(y \cap z) \cup (x \cap y) = \{3, 4, 7, 9\}$

H) $x \cap z = \{1, 2, 3, 4, 5, 7, 8, 9\}$
I) y and $x = \{1, 3, 7\}$

- 24) A company's customer service 800 telephone system is set up so that the caller has six options. Each of six options leads to menu with 4 options. For each of these 4 options, 3 more options available. For each of these 3 options, another 3 options are presented. If a person calls the 800 number for assistance, how many total options are possible?

• When a person calls 800, he has six options, the caller then chooses 1 option out of 6, $= {}^6C_1$.

• After choosing one of these options, he gets another 4 options & he chooses one out of 4, $= {}^4C_1$.

• And next menus are again chosen in 3C_1 and 3C_1 ways each.

Thus the total num of options available when on call 800 =

$$6C_1 \times 4C_1 \times 3C_1 \times 3C_1 = 6 \times 4 \times 3 \times 3 = 216$$

hence there are total of 216 options available.

- 25) A bin contains six parts. Two of parts are defective & 4 are acceptable. If 3 of 6 parts are selected from the bin, how large is the sample space?
- which counting rule did you use, & why?
 - For this sample space, what is the probability that exactly one of the three sampled parts is defective?

• 3 of 6 parts are selected.

$$6C_3 = 6!$$

$$3!(6-3)!$$

$$\begin{aligned} &= \frac{6 \times 5 \times 4 \times 3!}{3 \times 2 \times 1 \times 3!} = 20 \\ &\text{Ans: } 20 \end{aligned}$$

- Combinations are used to count the sample space bcz Sampling is done (with or without) replacement.

$$\bullet \text{ Probability } 4C_2 \times 2C_1 = 12 - 1 \text{ defective}$$

let E be the event that exactly one of 3 sampled part is defective

$$P(E) = \frac{12}{20} = 0.6$$

Q5) Explain Marginal, Union, Joint & Conditional probabilities with ex.

Conditional probability: $P(M \mid H)$

a) Marginal:

• It is computed by dividing some subtotal by the whole

• denoted by $P(E)$ & E is some event

ex: a persons own a BMW car

Marginal Prob: no. of BMW owners

total num of car owners

b) Union:

• It is second type of probability denoted by $P(E_1 \cup E_2)$, where E_1 & E_2 are 2 events

• $P(E_1 \cup E_2)$ is the probability that E_1 will occur or E_2 will occur or both E_1 & E_2 occur.

$$\text{ex: } A = \{1, 2, 3, 4\} \quad B = \{5, 6, 7\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

c) Joint:

• It is denoted by $P(E_1 \cap E_2)$, where E_1 & E_2 are 2 events.

• Both events must occur.

ex: A person owning both car & bike

$$\text{ex: } A = \{1, 2, 3\} \quad B = \{2, 3\} \quad A \cap B = \{2, 3\}$$

- 27) The client company data form the decision file *rrma* reveal that 155 employees worked one of four types of positions. Show here again the raw value matrix (also called a contingency table) with a frequency count for each category and for subtotals and totals containing a breakdown of these employees by type of position and by sex. If an employee of the company is selected randomly,

what is the probability that the employee
is female or a professional worker
Company human resource Data

Sex

	Male	Female	
Type of position	Managerial	8	11
	Professional	31	13
	Technical	52	17
	Clerical	9	22
	100	55	155

Let F : denote the event of female
P : denote the event of professional worker

$$P(F \cap P) = ?$$

By the general law of addition.

$$P(F \cup P) = P(F) + P(P) - P(F \cap P)$$

- of the 155 employees, 55 are women

Therefore $55/155 = 0.355 \rightarrow P(F)$

- The 155 employees include 44 professional

$$\therefore P(P) = 44/155 = 0.284 \rightarrow P(P)$$

- 13 employees are both female &

$$P(F \cap P) = 13/155 = 0.084 \rightarrow P(F \cap P)$$

$$P(F \cup P) = P(F) + P(P) - P(F \cap P)$$

$$= 0.355 + 0.284 - 0.084$$

$$= \underline{\underline{0.555}}$$

Given $P(A) = 0.10$ $P(B) = 0.12$ $P(C) = 0.21$

$P(A \cap C) = 0.05$ and $P(B \cap C) = 0.03$ solve
the following

$$a) P(A \cup C) = P(A) + P(C) - P(A \cap C)$$

$$= 0.10 + 0.21 - 0.05$$

$$= \underline{\underline{0.26}}$$

$$b) P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

$$= 0.12 + 0.21 - 0.03$$

$$= \underline{\underline{0.30}}$$

c) If A & B are mutually exclusive, $P(A \cup B) =$

If A & B are mutually exclusive $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B) = 0.10 + 0.12 = 0.22$$

29) According to the U.S Bureau of Labor Statistics, 75% of the women 25 through 49 years of age participate in the labour force. Suppose 48% of the women in that age group are married. Suppose also that 61% of all women 25 through 49 years of age are married and are participating in the labour force.

a) What is the probability that a randomly selected woman in that age group is married or is participating in a labour force?

b) What is the probability that a randomly selected woman in that age group is married or is participating in the labour force but not both?

c) What is the probability that a randomly selected woman in that age group is neither married nor participating in the labour force?

Given that

$$P(\text{Married}) = 48\% = 0.48 = P(M)$$

$$P(\text{Labour force}) = 75\% = 0.75 = P(L) = P(B)$$

$$P(\text{Married} \cap \text{Labour force}) = 61\% = 0.61 = P(M \cap L)$$

$$a) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.48 + 0.75 - 0.61$$

$$= \underline{\underline{0.62}}$$

b) $P(A \cap B) \cup P(B \cap A') = (0.78 - 0.61) + (0.75 - 0.61)$
 $= 0.17 + 0.14 = 0.31$

c) $1 - P(A \cup B) = 1 - 0.92 = 0.08$

30) A company has 140 employees, of which 90 are supervisors. Eighty of the employees are married, and 20% of the married employees are supervisors. If a company employee is randomly selected, what is the probability that the employee is married and is a supervisor?

M : denote married

S : denote supervisor

$$P(M \cap S) = ?$$

calculate marginal probability

$$P(M) = \frac{80}{140} = 0.5714$$

• 20% of the married employees are supervisors, which is the conditional probability $P(S|M) = 0.20$

Applying general law of multiplication

$$P(M \cap S) = P(M) \cdot P(S|M) = (0.5714)(0.20) = 0.1143$$

Hence 11.43% of the employees are married and are supervisors.

31) Use the values in the contingency table to solve the equations given.

	C	D	E	F	
A	5	11	16	8	40
B	2	3	5	7	17
	7	14	21	15	

a) $P(A \cap E) = P(A) \cdot P(E|A)$ $P(A) = \frac{40}{140}$
 $= \frac{16}{140}$ $P(E|A) = \frac{16}{40}$
 $= \frac{4}{14}$

b) $P(D \cap B) = P(D) \cdot P(B|D)$ $P(D) = \frac{14}{140}$
 $= \frac{1}{10}$ $P(B|D) = \frac{3}{14}$
 $= \frac{3}{140}$

c) $P(C \cap E) = P(C) \cdot P(E|C)$ $P(C) = \frac{14}{140}$
 $= \frac{1}{10}$ $P(E|C) = \frac{5}{14}$
 $= \frac{5}{140}$

d) $P(A \cap B)$

- 32) A study by Peter D. Hart Research Association for the Nasdaq Stock Market revealed that 43% of all American adults are stockholders. In addition, the study determined that 75% of all American adults have some college education. An American adult is randomly selected.
- a) What is the probability that the adult does not own stock?

- b) What is the probability that the adult owns stock and has some college education?
- c) What is the probability that the adult owns stock or has some college education?

d) what is the probability that the adult has neither some college education nor owns stock?

e) what is the probability that the adult does not own stock or has no clg education

f) what is the probability that the adult has some college education & owns no stock?
A : the event that an American Adult is a stockholder

B : the event that an American adult has some clg education

The given probabilities are:

$$P(A) = 43\% = 0.43 \text{ (prob of owning stock)}$$

$$P(B) = 37\% = 0.37 \text{ (having some clg education)}$$

$$P(ANB) = 75\% = 0.75 \text{ (owning stock \& having some clg education)}$$

$$a) P(A') = 1 - P(A) \Rightarrow 1 - 0.43 = \underline{\underline{0.57}}$$

$$b) P(ANB) = \underline{\underline{0.75}} \quad \text{- doubt}$$

$$c) P(A \cup B) = P(A) + P(B) - P(ANB) \\ \Rightarrow 0.43 + 0.37 - 0.75 = \underline{\underline{0.05}}$$

$$d) P(A' \cap B') = 1 - (A \cup B) \Rightarrow 1 - 0.05 = \underline{\underline{0.95}}$$

$$e) P(A' \cap B') = P(A') \cdot P(B') \quad \text{- doubt}$$

$$f) P(B \cap A') = P(B) - P(ANB) \\ = 0.37 - 0.75 = \underline{\underline{-0.38}}$$

33) Determine the mean, the variance and SD of the following discrete distribution.

x	P(x)	$x \cdot P(x)$	$(x - \bar{x})^2$	$[(x - \bar{x})^2 \cdot P(x)]$
1	0.238	0.238	2.775	0.660
2	0.390	0.58	0.443	0.128
3	0.177	0.31	0.11	0.0196
4	0.158	0.632	1.779	0.381
5	0.137	0.685	5.447	0.746

$$\text{Mean} = \underline{\underline{2.666}}$$

$$\text{Variance} = \sigma^2 = \sum [(x - \bar{x})^2 \cdot P(x)] = 1.8346$$

$$\sigma^2 = \sqrt{\sum [(x - \bar{x})^2 \cdot P(x)]} = \sqrt{1.8346} = \underline{\underline{1.354}}$$

34) Determine the mean, the variance & SD of the following discrete distribution

Discrete Distribution of occurrence of daily corpses

Number of corpses Probability

0	0.37
1	0.31
2	0.18
3	0.09
4	0.04
5	0.01

x	P(x)	$x \cdot P(x)$	$(x - \bar{x})^2$	$[(x - \bar{x})^2 \cdot P(x)]$
0	0.37	0	$(0 - 1.15)^2 = 1.32$	0.49
1	0.31	0.31	$(1 - 1.15)^2 = 0.09$	0.01
2	0.18	0.36	0.72	0.13
3	0.09	0.27	3.45	0.31
4	0.04	0.16	8.12	0.32
5	0.01	0.05	14.82	1.15

$$\text{Mean} = \sum [x \cdot P(x)] = \underline{\underline{1.15}} \quad \bar{x} = \underline{\underline{1.15}}$$

$$\text{Variance} = \sigma^2 = \sum [(x - \bar{x})^2 \cdot P(x)] = 1.41$$

$$\text{SD} = \sqrt{\sum [(x - \bar{x})^2 \cdot P(x)]} = \sqrt{1.41} = 1.19$$

- 35) A Gallup survey found that 65% of all financial consumers were very satisfied with their primary financial institution. Suppose that 25 financial consumers are sampled and if the Gallup survey result still holds true today, what is the probability that exactly 19 are very satisfied with their primary financial institution? (Using binomial distribution formula).

$$p = 0.65 \quad n = 25 \quad x = 19 \quad q = 1 - p = 1 - 0.65 = 0.35$$

$$\begin{aligned} P(x) &= \frac{n!}{x!(n-x)!} \cdot p^x \cdot q^{n-x} \\ &= \frac{25!}{19!(25-19)!} \cdot (0.65)^{19} \cdot (0.35)^{25-19} \\ &= \frac{25 \times 24 \times 23 \times 22 \times 21 \times 20}{19!} \cdot (0.65)^{19} \cdot (0.35)^6 \\ &= (17,100) (0.000978) \cdot (0.00183) = 0.0908 \end{aligned}$$

- 36) According to the U.S. Census Bureau, approximately 12% of all workers in Jackson, Mississippi, are unemployed. In conducting a random telephone survey in Jackson, what is the probability of getting 2 or fewer unemployed workers in a sample of 20? (Using binomial Distribution formulae?)

The problem must be worked as the union of 3 problems

$$a) x=0 \quad p=0.06 \quad q=1-0.06=0.94 \quad n=20$$

$$= \frac{20!}{0!} \cdot (0.06)^0 \cdot (0.94)^{20} = 0.3901$$

$$b) x=1$$

$$= \frac{20!}{1!(19)!} \cdot (0.06)^1 \cdot (0.94)^{19} = 0.3703$$

$$c) x=2$$

$$= \frac{20!}{2!(18)!} \cdot (0.06)^2 \cdot (0.94)^{18} = 0.2446$$

$$\text{total} = 0.3901 + 0.3703 + 0.2446 = 0.9050$$

- 27) Suppose bank customers arrive randomly on weekday afternoons at avg of 3.2 customers every 4 minutes. What is the probability of exactly 5 customers arriving in a 4-minute interval on a weekday afternoon? The lambda for this problem is 3.2 customers per 4 minutes. The value of x is 5 customers per 4 minutes. (Using Poisson formula)

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x=5 \quad \lambda=3.2$$

$$= \frac{(3.2)^5 (e^{-3.2})}{5!} = \frac{(33554)(0.0408)}{5!} = 0.1141$$

$$e = 0.418282$$

- 38) Bank customers arrive randomly on weekday afternoons at an avg of 3.2 customers every 4 minutes. what is the probability of having more than 7 customers in a 4-minute interval on a weekday afternoon? (poisson formula)
- $\lambda = 3.2$ customers / minute $e^{-3.2} = 0.18282$
- $x > 7$ customers / 4 minutes

$$P(x=8 | \lambda = 3.2) = \frac{(3.2)^8}{8!} (e^{-3.2}) = 0.0111$$

$$P(x=9 | \lambda = 3.2) = \frac{(3.2)^9}{9!} (e^{-3.2}) = 0.00410$$

$$P(x=10 | \lambda = 3.2) = \frac{(3.2)^{10}}{10!} (e^{-3.2}) = 0.0013$$

$$P(x=11 | \lambda = 3.2) = \frac{(3.2)^{11}}{11!} (e^{-3.2}) = 0.0004$$

$$P(x=12 | \lambda = 3.2) = \frac{(3.2)^{12}}{12!} (e^{-3.2}) = 0.0001$$

$$P(x=13 | \lambda = 3.2) = \frac{(3.2)^{13}}{13!} (e^{-3.2}) = 0.0000$$

$$P(x > 7) = P(x \geq 8) = 0.0169$$

- 39) A bank has an avg random arrival rate of 3.2 customers every 4 minutes. what is the probability of getting exactly 10 customers during an 8 minutes interval? (poisson formula)

$\lambda = 3.2$ customer / 4 minutes

$x = 10$ customer / 8 minutes

- 40) Find the following values by using Poisson formula

$$a) P(x=5 | \lambda = 2.3) = \frac{(2.3)^5}{5!} (e^{-2.3})$$

$$\Rightarrow \frac{6.3634}{120} \cdot 0.100359 = 0.05378$$

$$b) P(x=8 | \lambda = 3.9) = \frac{(3.9)^8}{8!} (e^{-3.9}) \Rightarrow 0.15394$$

$$c) P(x \leq 8 | \lambda = 4.1) = P(x=1 | \lambda = 4.1) + P(x=2 | \lambda = 4.1) + P(x=3 | \lambda = 4.1)$$

$$\Rightarrow \frac{(4.1)^1}{1!} (e^{-4.1}) + \frac{(4.1)^2}{2!} (e^{-4.1}) + \frac{(4.1)^3}{3!} (e^{-4.1}) +$$

$$\frac{(4.1)^0}{0!} (e^{-4.1}) = 0.06806 + 0.1395 + 0.22881 + 0.0166 \Rightarrow 0.415497$$

$$d) P(x=0 | \lambda = 2.7) = \frac{(2.7)^0}{0!} (e^{-2.7}) = 0.0672055$$

$$e) P(1 < x \leq 8 | \lambda = 4.4) = P(x=5 | \lambda = 4.4) + P(x=6 | \lambda = 4.4) + P(x=7 | \lambda = 4.4)$$

$$\Rightarrow P(x=1 | \lambda = 5.4) = \frac{(5.4)^1}{1!} (e^{-5.4})$$

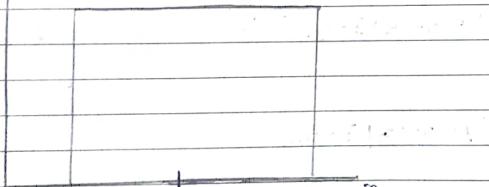
4.) Suppose the amount of time it takes to assemble a plastic module ranges from 27 to 39 seconds and that assembly times are uniformly distributed. Describe the distribution. What is the probability that a given assembly will take less than 30 & 35 seconds? Fewer than 30 seconds.

$$f(x) = \text{Height} = \frac{1}{b-a} = \frac{1}{39-27} = \frac{1}{12}$$

$$\text{Mean} = a + b / 2 = \frac{39+27}{2} = 33$$

$$\sigma = \sqrt{\frac{b-a}{12}} = \sqrt{\frac{39-27}{12}} = \sqrt{\frac{12}{12}} = \sqrt{1} = 1$$

Height



$$P(30 \leq x \leq 35) = \frac{x_2 - x_1}{b-a} = \frac{35-30}{39-27} = \frac{5}{12} = 0.4167$$

$$P(x < 30) = \frac{x_1}{b-a} = \frac{30-27}{39-27} = \frac{3}{12} = 0.2500$$

There is a 0.2500 probability that it will take less than 30 seconds to assemble the module.



4.) According to the National Association of Insurance Commissioners, the avg annual cost for automobile insurance in the United States in a recent year was \$691. Suppose automobile insurance costs are uniformly distributed in the United States with range of from \$400 to \$1,189. what is SD of this uniform distribution? what is the height of the distribution? what is the probability that a person's annual cost for automobile insurance in the United States is below \$410 and \$825?

$$\text{Mean} = \$691 \quad a = \$400 \quad b = \$1,189$$

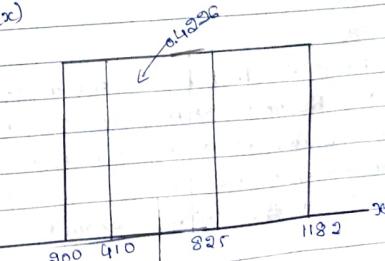
$$\sigma = \sqrt{\frac{b-a}{12}} = \sqrt{\frac{1,189-400}{12}} = \sqrt{\frac{789}{12}} = \sqrt{65.75} = 8.035$$

$$\text{Height} = \frac{1}{b-a} = \frac{1}{1,189-400} = \frac{1}{789} = 0.001$$

$$x_1 = 410 \quad x_2 = 825 \quad p(410 \leq x \leq 825)$$

$$= \frac{x_2 - x_1}{b-a} = \frac{825-410}{1,189-400} = \frac{415}{789} = 0.4126$$

The probability that a randomly selected person pays between \$410 & \$825 annually for automobile insurance in the United States is 0.4126. $f(x)$



$$\text{Mean} = \$691 \quad \sigma = \$8.035$$

H3) What is the probability of obtaining a score greater than 700 on a GMAT test that has a mean of 494 & SD of 100? Assume GMAT scores are normally distributed.

$p(x > 700 | \mu = 494 \text{ and } \sigma = 100) = ?$

$\Rightarrow z = \frac{x - \mu}{\sigma} = \frac{700 - 494}{100} = 2.06$

z value in z distribution table is
 -0.4803 [2.06 $\rightarrow 0.06$]

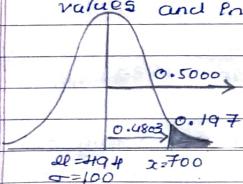
Finding the probability of getting a score > 700 , which is a tail distribution, requires subtracting prob value 0.4803 from 0.5000, bcz each half of the distribution contains 0.5000 of the area.

0.5000

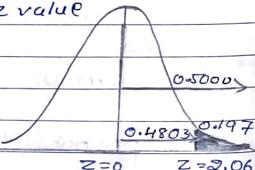
-0.4803

0.0197

The soln is depicted graphically in (a) for x values and in (b) for z value



(a)



(b)

H4) GMAT test that has a mean of 494 & SD of 100. Assume GMAT scores are normally distributed. what is the probability of randomly obtaining a score b/w 300 & 600 on the GMAT exam?

$$p(300 < x < 600 | \mu = 494 \text{ and } \sigma = 100) = ?$$

$$\therefore z = \frac{x - \mu}{\sigma} = \frac{600 - 494}{100} = 1.06 \quad \frac{106}{100} = 1.06$$

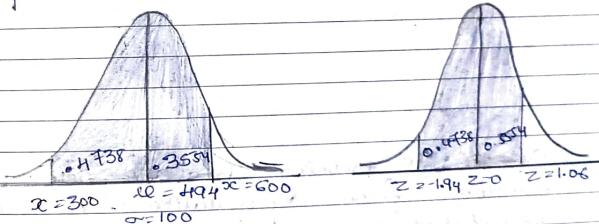
z value in z distribution table is
 $= 0.3554$ [1.06 $\rightarrow 0.06$]

$$\therefore z = \frac{x - \mu}{\sigma} = \frac{300 - 494}{100} = -1.94 \quad \frac{-194}{100} = -1.94$$

z value in z distribution table is
 $= 0.4738$ [1.94 $\rightarrow 0.04$]

(Note: $z(-1.94)$ is -ve. A -ve z value indicates that the x value is below the mean. z value is left side of distribution.

$\therefore p(300 < x < 600)$ is obtained by summing the probabilities. $0.3554 + 0.4738 = 0.8292$



$$H5) p(350 < x < 450 | \mu = 494 \text{ and } \sigma = 100) = ?$$

Same as dt.

H5) GMAT test that has a mean of 494 and SD of 100. Assume GMAT scores are normally distributed. what is the probability of getting a score b/w 350 & 450 on the GMAT by randomly drawing a score that is 350 or less?

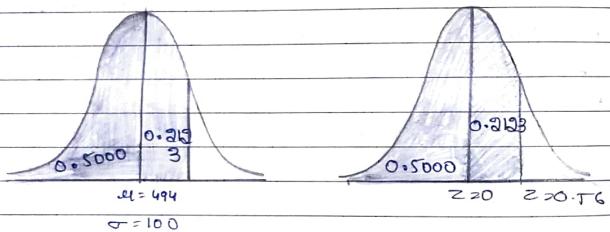
$$P(Z \leq 550 | \mu = 494 \text{ and } \sigma = 100) = ?$$

$$Z = \frac{x - \mu}{\sigma} = \frac{550 - 494}{100} = \frac{56}{100} = 0.56$$

The area under the curve for $Z = 0.56$
 $= 0.2123$ $[0.5! \rightarrow 0.0607]$

obtaining the probability for all values less than or equal to 550 also requires including the value less than the mean. Because 0.5000 of the value are less than the mean.

$$0.5000 + 0.2123 = \underline{\underline{0.7123}}$$



- 47) Suppose the following data are selected randomly from a population of normally distributed values. Construct a confidence interval to estimate the population mean. And 90% confidence level. Using 7 statistics. The sample mean is 13.56 & the sample SD is 7.8

6 21 17 20 7 0 8 16 29
 3 8 12 11 9 21 25 15 10

$$\bar{x} = 13.56 \quad \text{SD} = 7.8 \quad \sqrt{s^2} = 0.05 \quad df = n - 1 = 17$$

$$\Rightarrow \bar{x} \pm t_{\frac{\alpha}{2}, n-1, \sqrt{s^2}} \cdot \frac{s}{\sqrt{n}}$$

$$t_{0.05, 17} = 1.7410$$

$$\Rightarrow \bar{x} - t_{\frac{\alpha}{2}, n-1, \sqrt{s^2}} \leq \bar{x} \leq \bar{x} + t_{\frac{\alpha}{2}, n-1, \sqrt{s^2}}$$

$$13.56 - t_{0.05, 17} \left(\frac{7.8}{\sqrt{17}} \right) \leq \bar{x} \leq 13.56 + t_{0.05, 17} \left(\frac{7.8}{\sqrt{17}} \right)$$

$$13.56 - 1.7410 \left(\frac{7.8}{\sqrt{17}} \right) \leq \bar{x} \leq 13.56 + 1.7410 \left(\frac{7.8}{\sqrt{17}} \right)$$

$$13.56 - 1.7410 \left(\frac{7.8}{\sqrt{17}} \right) \leq \bar{x} \leq 13.56 + 1.7410 \left(\frac{7.8}{\sqrt{17}} \right)$$

$$(4.1231) \leq \bar{x} \leq (22.8917)$$

$$13.56 - 3.2917 \leq \bar{x} \leq 13.56 + 3.2917$$

$$10.36 \leq \bar{x} \leq 16.76$$

48) 3 1 3 2 5 1 2 1 4 2 1 3 1 1

$$\bar{x} = 2.14 \text{ (sample mean)} \quad \text{SD} = 1.29$$

Same as above.