Date
Page

(ii) For glb / Inf ($\wedge$) (Downward direc^n)

$\wedge$	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	a	a	b	b
c	a	a	c	a	c	a	c
d	a	a	a	d	d	d	d
e	a	a	c	d	e	d	e
f	a	b	a	d	d	f	f
g	a	b	c	d	e	f	g

a → a
e → edca
b → ba
f → fdab
directly connected so b

c → ca
d → da
e → ecda
f → fdba
g → gefdcba

a
b
glb

Here don't consider upward direc^n

∴ A lattice, as each pair has Unique lub, glb

Unit 3

Fundamental Principle of Counting

- If an operat^n can be performed in 'm' different way and another operat^n can be performed in 'n' different ways then

(i) Sum Rule — if these 2 operat^n can't occur simultaneously then total no. of ways to perform the operat^n is → $m+n$

(ii) Product Rule — If 2 op occur simultaneously, then total no. of ways to perform the op is $m \times n$

Q There are 11 seats vacant in a classroom. In how many ways 5 students seat themselves

11

Date
Page

Qy No. of vacant seats = 11
students = 5

$^{11}C_5$

$^{11}C_6$

$\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6}{5!}$

∵ No. of ways for
- 1st student = 11
- 2nd " = 10
- 3rd = 9
- 4th = 8
- 5th = 7

$^{11}P_5 = \frac{11!}{6!}$

By FPC

Total no. of ways = $11 \times 10 \times 9 \times 8 \times 7$

Factorial

- The continuous product of 1st 'n' natural no is called Factorial.
- denoted by $n!$ or $\underline{|n}$
- $n! = n(n-1)(n-2)(n-3)\ldots 3,2.1$

eg. $\underline{|5} = 5 \times 4 \times 3 \times 2 \times 1 = 120$

- $0! = 1$, $\underline{|-no.}$ = not defined

Q Find n such that $(n+2)! = 2550 \cdot \underline{|n}$

$n+2 \cdot n+1 \cdot \cancel{\underline{|n}} = 2550 \cdot \cancel{\underline{|n}}$

$(n+2)(n+1) = 2550$
$\quad\quad 51 \times 50$

here diff = 1

Trick: so bring diff = 1

$\Rightarrow n+2 = 51$

$n = 49$ Ans

OR

$(n+2)(n+1) = (49+2)(49+1)$

comparing $n = 49$ Ans

Date
Page

Permutation - The different arrangement which can be made out of a given no. of things taking some or all of them at a time is called Permutation.

- The permutatⁿ of n-different things taking $r\ (1 \le r \le n)$ is denoted by ${}^nP_r$ or $P(n, r)$

r ↓ some, $\le n$ ↳ all

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Q In how many ways can the letters of the word TRIANGLE be arranged. / OR How many diff. words can be formed, How many of these :- (i) begin with A
(ii) end with L
(iii). begin with A and end with L
(iv) A, L occupy end places.
(v). All vowels are together
(vi) No 2 vowels are together.
(vii) No All vowels are
(viii) Vowels occupy odd places
(ix) Vowels occupy even places

Ans

Total arrang. = $8!$ or ${}^8P_8$
∵ No. of letters in the word TRIANGLE are all different
The Permutatⁿ of 8 letters taking 8 at a time
$= {}^8P_8 = {}^nP_n = n!$

$= 40320$

(i) Begin with A - $\boxed{A}$ → fix _ _ _ _ _ _ _

7 places, 7 letters
↓ r, ↓ n

- since the words begin with A, ∴ we fix A at 1st positⁿ
- No. of ways to arrange A $= {}^1P_1$
- No. of ways arranging rem. 7 letters among 7 places $= {}^7P_7$
$= 5040$

∴ Total no. of words $= 1 \times 5040$
$= 5040$

Date
Page

(ii) Ends with L → same = 5040

(iii) $\boxed{A}$ - - - - - - $\boxed{L}$ $= {}^6P_6 = 6!$

$\underbrace{\quad\quad\quad}_{6l,\ 6p}$

then by FPC $= 6! \times 1! \times 1!$

$= 720$

(iv) $6! \times 2!$

$= 1440$

i.e. - - - - - - $\overset{2!}{\boxed{A \mid L}}$

$\underbrace{\quad\quad\quad}_{6p,\ 6l}$

720
×2
1440

OR

A - - (OR) (+) - L → 720
+
L - - - - - A → 720

1440

(v). All vowels together (I, A, E)

$= 6! \cdot 3!$
$= 720 \times 6$
$= 4320$

No. of vowels = 3 → ${}^3P_3$
×
No. of conso = 5, +1 → ${}^6P_6$

since all vowels are together we tie them together → (A E I) and considered as 1 object

∴ (AEI) TRNGL

∴ No. of ways of arrang 6 objects $= {}^6P_6 = 720$

and now

(AEI) can be arrang. in ${}^3P_3 = 3! = 6$

∴ By FPC Total $= 6 \times 720 = 4320$ Ans

(vi) No 2 vowels are together.

Firstly, we will arrange the consonants (T, R, N, G, L, E)

$= {}^5P_5 = 120$

Then we place vowels b/w them

_ T _ R _ N _ G _ L _

Date
Page

since, No 2 vowels should be together, then we place vowels in the blanks and we fill 3 at a time from 6 places

$$\therefore \quad {}^6P_3 = \frac{720}{6} = 120$$

Then by FPC

$$\longrightarrow = 120 \times 120 = 14400$$

(vii) No all vowels are together

$$= \boxed{\text{Total} - \text{All}}$$

$$= 40320 - 4320 \quad \text{do}$$

(viii). Vowels at odd places

(ix) same

No. of odd places = 4

No. of vowels = 3

$\therefore$ No. of ways $= {}^4P_3 = 24$

and consonants (5) $\therefore$ no. of ways $= {}^5P_5$

$= 120$

$\therefore$ By FPC $= 120 \times 24$

Post I In New Notebook

Date.
Page.

MTH → Unit-3 se

7th March

Note

Permutation of 'n' things out of which, 'p' are one kind, 'q' are of 2nd kind, 'r' are of 3rd kind are given by :-

$$\frac{n!}{p!\,.\,q!\,.\,r!} \quad \text{or} \quad \boxed{\frac{\lfloor n}{\lfloor p\,.\,\lfloor q\,.\,\lfloor r}}$$

Ques How many words can be formed from the letters of word MATHEMATICS ?

Ans No. of letters = 11 = n

and A occurs 2 times ∴ p = 2
T " 2 " ∴ q = 2
M " 2 " ∴ r = 2

∴ No. of words = $\frac{\lfloor 11}{\lfloor 2\,.\,\lfloor 2\,.\,\lfloor 2}$ Ans

Circular Permutation

Formula = $\boxed{\lfloor n-1}$

Note

- In case of Necklace with different beads (moti) and garland of different flowers then,

circular permutation = $\frac{1}{2}.\lfloor n-1$ or $\boxed{\frac{\lfloor n-1}{2}}$

Ques In how many ways can 6 men and 4 women be arranged - (i) in a line (ii). in a circle, so that no 2 women sit together

soln

given No. of men = 6
No. of women = 4

(i) In a line $* M_1 * M_2 * M_3 * M_4 * M_5 * M_6 *$

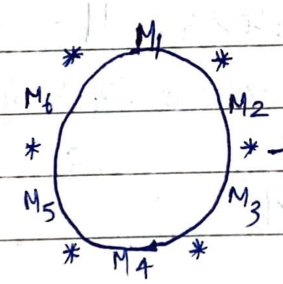
• No. of ways of arranging 6 Men = ${}^6P_6 = \lfloor 6 = 720$

• since, No 2 women are to sit together, so we will place the women at '*' position (7 places)

∴ No. of ways arrang. of 4 Women = ${}^7P_4 = 840$

• Now By FPC,
Total ways = 720 × 840

(ii) In a circle



• Ways of arranging 6 Men in a circle = $\lfloor n-1 = \lfloor 6-1$
= 120

• " " 4 women = ${}^6P_4$ = 360

• ∴ By FPC = 120 × 360 ans

Q If the letter of the word SAMPLE be arranged in a dictionary then what is the rank of the word ?

Date. |
Page. |

sol^n No. of letters = 6

- arrange the letters in order → A, E, L, M, P, S

∴ The 1st word will be starting from A = AELMPS

so,

fix

how

No. of words starting with A = $1 \times {}^5P_5$ = 120

A - - - - -

5 places, 5 letters

∴ $^5P_5$

Similarly,

No. of words starting with E = 120

" " L = 120

M = 120

P = 120

Now,

- Next word will be = SAELMP

fix

S A E - - -

∴ No. of words starting with SAE = $1 \times {}^3P_3$ = 6

- Next word will be = SALEMP

∴ No of words starting with SAL = 6 ↑ same

- Next word will be = SAMELP

∴ No. of words starting with SAME = $^2P_2$ = 2

- Next word will be = SAMLEP

∴ No. of words start. with SAML = $^2P_2$ = 2

- Next word will be = SAMPEL

∴ No. of words start with SAMPE = $^1P_1$ = 1

∴ No. of words before the word SAMPLE = 5(120) + 2(6) + 2(2) + 1 = 617

∴ Rank of SAMPLE = 617 + 1 = 618 Ans

Q Rank → SACHIN = 601

Date. | Page. |

Combination – Combination is a selection or grouping of things taking some or all of them at a time.

– The combination of 'n' different things taking 'r' at a time is :-

$$\boxed{{}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}} \quad \therefore \boxed{{}^nC_r = \frac{{}^nP_r}{\lfloor r}}$$

– Note Formulae

${}^nC_r = {}^nC_{n-r}$

if ${}^nC_p = {}^nC_q$ then either $p = q$ or $p+q = n$

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

${}^nC_n = 1$

${}^nC_0 = 1$

Q In how many ways a team of 6 players can be selected from 11 players and how many of them :-
i) always have 2 particular players.
ii) Do not have " " .

Soln

• Total no. of players = 11
since, a team of 6 players is to be selected
∴ No. of ways = ${}^{11}C_6$ = 462

• (i) 2 Alway Selected

check

(i) ∴ ways of slecting 2 particular player = ${}^2C_2$ = 2 (?)

checks

and

No. of ways of selecting remaining 4 players out of 9
= ${}^9C_4$ = 126 ✓ d

(ii) Since 2 particular players are never selected
∴ we have to select 6 players from 9 players only
∴ ways = ${}^9C_6$ = 84 ✓ A

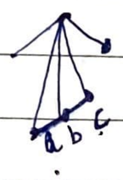
Date. 10-03-2023
Page.

10^{th}

Note-1

If there 'n' points in a plane and out of these 'p' points are collinear (lies on the same line) then,

(i) the total no. of lines formed $= \boxed{{}^{n}C_{2} - {}^{p}C_{2} + 1}$



(ii) Total no. of triangles $= \boxed{{}^{n}C_{3} - {}^{p}C_{3}}$

2 pts = 1 line

(iii). No. of ~~quad~~ 'r' sided fig $= \boxed{{}^{n}C_{r} - {}^{p}C_{r}}$

Note-2

- If 'n' = no. of points | Angular pts. | vertices | sides of a polygon then,

line joining vertices

(i) No. of Diagonals $= \boxed{{}^{n}C_{2} - n}$ OR $= \dfrac{n^2 - 3n}{2}$

Q Find the No. of diagonals of an Octagon $(n=8)$

$\therefore \quad d = \dfrac{64 - 24}{2} = 32 - 12 = 20$ Ans

Q Find the no. of sides of a polygon who has 47 diagonals

$47 = \dfrac{n^2 - 3n}{2} \Rightarrow n^2 - 3n - 94$

$n^2 - ($

$9 - 6$

$\dfrac{b^2 - 4ac}{2a}$

doesn't exist

Q $44 = \frac{n^2 - 3n}{2}$

$n^2 - 3n - 88$

$n^2 - \cancel{3n}(11-8)n - 88$

$n^2 - 11n + 8n - 88$

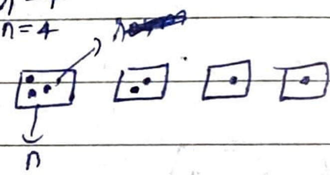
$n(n-11) + 8(n-11)$

$\therefore n = 11$ Ans

11 side Polygon Ans

- What if $n > m$

if $n = 7$
$m = 4$



Pigeonhole Principle

- If 'n' pigeons are assigned to 'm' pigeonhole where $m > n$., then atleast 1 pigeonhole contains 2 or more pigeons.

- If 'n' pigeons " " " " " " , then ~~atleast 1 pigeonhole contain~~ one of the pigeonhole must contain atleast $\left[\frac{n}{m} + 1\right]$ or $\left[\frac{n-1}{m}\right] + 1$ pigeons.

Note → $[x]$ = greatest Integer function

eg

$[25.27] = 25$ $\quad [-1.7] = -2$

25 → integral

.27 → decimal / fractional part → $\frac{27}{100}$

$[2.69] = 2$

Note

$p \to q$ ⟹

p	q	$p \to q$
T	F	F

MCQ Practice

Truth Q

Truth table

take

$p \vee q$ is logically equivalent to

options

(i) $\sim q \to \sim p$

ii) $q \to p$

iii) $\sim p \to \sim q$

(iv) $\sim p \to q$ ✓

Note

but = and = $\wedge$

p	q	$\sim p$	$\sim q$	$\sim p \to q$	$p \vee q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Find which opn has (T) truth Value

Q a) If the sun is a planet (F), elephants will fly (F) ⟹ $F \to F = $ (T) ✓

b) $3+2=8$ if $5-2=7$ ✓

$3+2=8 \to q$ (F), $5-2=7 \to p$ (F) $\to$ (T)

c) $1>3$ and 3 is a positive (F)

$1>3 \to$ F, and $\to \wedge$, 3 is a positive $\to$ F

d) $-2>3$ or 3 is a negative integer $=$ (F)

$-2>3 \to$ F, $\wedge$, 3 is a negative integer $\to$ F

Note

⇓

$\boxed{p \wedge \sim p \to \text{always } (F)}$

i.e is a Contradiction

Q ✓

p	$\sim p$	$\sim(p \wedge \sim p)$
T	F	T
F	T	T

H.P

$\therefore \sim(p \wedge \sim p) \to$ Tautology

Q ✓ $P \leftarrow\!\!\to Q$ or $P \Leftrightarrow Q$ is logically equivalent to

$= \sim p \Leftrightarrow \sim q$

Prove via Truth table

Trick

also a property of $P \Leftrightarrow Q$ check notes

$- \leq n$

Date. |
Page. |

~~p∧q~~

Q It is cloudy and Drizzling. Therefore it is cloudy

Find which Law of Inference is used here

$$\frac{p \wedge q}{\therefore p}$$

Simplication

Q $\frac{p}{\therefore p \vee q}$ Bhavika has pen. Therefore Bhavika has pen or Kanika has paper

Addition

Q If it's holiday, the local office will be closed. The local office will be not be closed. Thus, it not holiday today

$$p \to q$$
$$\frac{\sim q}{\therefore \sim p}$$

Modus Tollens

Q Vimp It is either colder than Himalaya today or the pollutⁿ is harmful, It is hotter than Himalaya today. Therefore the pollutⁿ is harmful

not colder means hot

$$p \vee q$$
$$\frac{\sim p}{\therefore q}$$

Disjunctive syllogism

Vimp solve Q $(p \wedge q) \vee r$, $r \to s$

by simplificatⁿ

$\frac{p \wedge q}{\therefore p}$

$p \vee r$, $r \to s$

can be replaced with s

If r then s

mtlb r hai toh s hai

∴ r ki jagah pe s

Vimp

a) $p \vee s$

b) $p \vee s$

c) $p \vee q$

d) NOTA

Date.
Page.

$\therefore$ $p \vee s$ be

Q. Jay is an awesome student. Jay is also a good dancer.
Therefore, Jay is an awesome student and a good dancer

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$ be Conjunction

Q. $P(x)$ $x > 7$ for which has Truth value true

i) $P(2)$
ii) $P(5)$
iii) $P(10)$
iv) $P(4)$

at $P(10) \rightarrow$ has (T)
as $10 > 7$ ✓

Q. "Parul is out for trip or it is not snowing" and
"it is snowing or Raju is playing chess"

$$\begin{array}{l} p \vee \sim q \\ q \vee r \\ \hline \therefore p \vee r \end{array}$$ "Resolution"

Parul is out for trip or Raju is playing chess.

Date. |
Page. |

Q Let A and B two non-empty relat^n on set S. which is false

i) A and B are transitive $\Rightarrow$ $A \cap B$ is transitive

ii) A and B are symmetry $\Rightarrow$ $A \cup B$

~~iii~~ A and B are transitive $\Rightarrow$ $A \cup B$ is not transitive

iv)

Q How many Binary relat^n with 10 elements

$$2^{10 \cdot 10} \quad (2^{mn})$$

$$= 2^{100}$$

eg

Q all L.B of 10 and 15 on $D_{30} = \{1, 2, 3, 4, 5, 6, 10, 15, 30\}$

~~a)~~ 1, 5

Date.
Page.

Relation (Formula)

(4) No. of Reflexive Relatⁿ on a set with n elements $= 2^{n(n-1)}$

(3) No. of diff relatⁿ (Binary relatⁿ) from a set with n element to a set m is $= 2^{mn}$ if $m=n \rightarrow 2^{m^2}$

(5) No. of symmetric Relatⁿ on a set with n elements $= 2^{\frac{n(n+1)}{2}}$

(6) No. of Antisymmetric . . . . . $= 2^n 3^{\frac{n(n-1)}{2}}$

(7) No. of Assymmetric . . . $= 3^{\frac{n(n-1)}{2}}$

(8) Irreflexive relat $= 2^{n(n-1)}$

(9) Reflexive and symmetric on a set . . . $= 2^{\frac{n(n-1)}{2}}$

else

eg

0,15,30