

# An Optimal Labeling Scheme for Workflow Provenance Using Skeleton Labels

Zhuwei Bao, Susan B. Davidson, Sanjeev Khanna, Sudeepa Roy  
Department of Computer and Information Science, University of Pennsylvania  
Philadelphia, PA 19104, USA  
[{zhuwei, susan, sanjeev, sudeepa}@cis.upenn.edu](mailto:{zhuwei, susan, sanjeev, sudeepa}@cis.upenn.edu)

## ABSTRACT

We develop a compact and efficient reachability labeling scheme for answering provenance queries on workflow runs that conform to a given specification. Even though a workflow run can be structurally more complex and can be arbitrarily larger than the specification due to fork (parallel) and loop executions, we show that a compact reachability labeling for a run can be efficiently computed using the fact that it originates from a fixed specification. Our labeling scheme is optimal in the sense that it uses labels of logarithmic length, runs in linear time, and answers any reachability query in constant time. Our approach is based on using the reachability labeling for the specification as an effective *skeleton* for designing the reachability labeling for workflow runs. We also demonstrate empirically the effectiveness of our skeleton-based labeling approach.

## Categories and Subject Descriptors

H.2.8 [Database Management]: Database Applications—*scientific databases*

## General Terms

Algorithms, Performance

## 1. INTRODUCTION

Efficiently maintaining the provenance of data produced by scientific workflow systems is of great current interest. Using provenance information, scientists can examine the data, parameter settings, and analysis tools that were used in an “in-silico” experiment to produce a good data result, or determine which downstream data objects were affected by a bad data result. Tracking such provenance information entails answering reachability queries on a directed acyclic graph (DAG) which encodes the dependencies between data and modules in a workflow execution.

As observed in [8, 11], two immediate approaches to answering reachability queries – using graph traversals and pre-

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

SIGMOD’10, June 6–11, 2010, Indianapolis, Indiana, USA.  
Copyright 2010 ACM 978-1-4503-0032-2/10/06 ...\$10.00.

computing the transitive closure – are prohibitively expensive for large graphs. A better approach is to use *reachability labels*, *i.e.*, to assign each vertex in the graph a label such that by comparing only the labels of any two vertices, we can decide if one can reach the other. However, the effectiveness of this approach crucially depends on the ability to develop a *compact* and *efficient* labeling scheme, where compactness refers to the space used by the labels, and efficiency refers to the time complexity of creating and comparing the labels.

Motivated by applications in XML database systems, several compact and efficient labeling schemes for trees have been developed; see, for example, [15, 13]. These labeling schemes are generally considered to be *optimal* in the sense that they use labels of logarithmic length, run in linear time, and answer queries in constant time. In contrast, any labeling scheme for general DAGs may require labels of linear length (in the number of vertices), even if arbitrary construction and query time are allowed.

However, workflow executions are not arbitrary DAGs: Each execution (a.k.a. *run*) of a workflow follows the same basic structure as the directed graph representing its specification, but may become much larger due to fork (parallel) and loop executions. Building on this observation, we propose a *skeleton-based* labeling scheme, that uses the reachability labeling for the specification for designing the reachability labeling for workflow runs. Our approach labels a run in two phases: (1) label its specification using any labeling scheme for directed graphs, and (2) extend the reachability labels on the specification (called the *skeleton labels*) with additional run-time information about the forking and looping behavior to label the run.

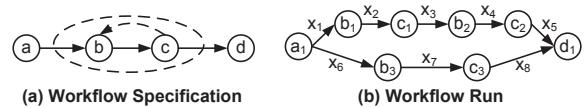


Figure 1: Workflow Example

As an example, consider the workflow specification and one of its runs shown in Figure 1, in which vertices represent modules (a.k.a. tasks) and edges indicate potential data flow between the modules. In the specification, the dotted oval around *b* and the dotted backarrow from *c* to *b* a *loop*. In the run, the fork is executed twice in parallel; in one fork copy, the loop is executed twice, while in the other fork copy it is executed only once.<sup>1</sup> Edges in the run are also annotated with *x<sub>i</sub>*’s, indicating data that was created by the first module and passed to the second.

<sup>1</sup> A realistic run may execute the fork and loop hundreds of times.

To illustrate the approach, consider the following three provenance queries: (1) *Does  $x_8$  (output of  $c_3$ ) depend on  $x_1$  (input to  $b_1$ )?* Note that we need to check if  $c_3$  is reachable from  $b_1$ . The answer is ‘no’, since  $b_1$  and  $c_3$  are in parallel fork copies. This would be encoded by the extended labels, and skeleton labels would not be checked; (2) *Does  $x_4$  (output of  $b_2$ ) depend on  $x_2$  (input to  $c_1$ )?* The answer is ‘yes’ since, despite the fact that in the specification  $b$  is not reachable from  $c$ ,  $c_1$  and  $b_2$  are in successive loop iterations. This would again be encoded by the extended labels; and (3) *Does  $x_3$  (output of  $c_1$ ) depend on  $x_1$  (input to  $b_1$ )?* Given that  $b_1$  and  $c_1$  are in the same fork and loop copy,  $c_1$  is reachable from  $b_1$  iff  $c$  is reachable from  $b$ . This query can therefore be answered using the skeleton labels.

At first glance it would appear that extending skeleton labels to workflow runs is straightforward as long as unique process ids are given for different executions of the same module. But the example above shows that the reasoning must take into account the hierarchy of nested forks and loops within which the module execution occurs. Note that it is non-trivial to encode the interaction between nested fork and loop executions using only labels of logarithmic length; even a unique process id takes that much space. Our primary technical contribution is to design logarithmic length labels for the run in linear time, and, using these labels along with the given skeleton labels, answer reachability queries in constant time. Furthermore, we extract the hierarchy of fork and loop executions from the run graph in linear time, so our approach works even if no additional information is available to identify each fork and loop copy in the run.

The effectiveness of our skeleton-based labeling approach follows from three observations: First, the size of the specification is typically much smaller than the size of a run. Therefore, the specification can be efficiently labeled using short labels. Second, once a workflow is created it will be executed repeatedly (using different parameter settings or data inputs). Therefore, the cost (both space and construction time) of labeling a specification can be amortized over a large number of runs. Third, using our algorithm, reachability queries on the run may frequently be answered using only the extended labels without comparing the skeleton labels. Therefore, our approach is effective even when reachability queries on the specification are slow.

**Contributions.** The contributions of this paper are as follows. First, we propose a two-phase labeling scheme for workflows with well-nested forks and loops, which extends skeleton labels on the specification to labels on a run by capturing fork and loop executions. Second, we show that the scheme is compact and efficient, yielding logarithmic size labels that are created in linear time and which can be used to answer reachability queries in constant time. The scheme is thus *optimal* for labeling runs that conform to a given specification; it in fact matches the bounds achievable for trees even though runs can have an arbitrarily more complex network structure. Third, we extend the reachability labels to data, and show how those labels can be used to answer provenance queries which test dependencies between data or between data and modules. Finally, we perform extensive experimental evaluations on both real and synthetic datasets. The results empirically validate our complexity analysis, and show that our skeleton-based labeling scheme is robust against the approach used for labeling the underlying specification.

**Paper Organization.** Related work is discussed in Section 2. Section 3 introduces the workflow model and formulates the reachability labeling problem. Section 4 presents and analyzes the skeleton-based labeling scheme. Section 5 outlines a linear time algorithm that is used by our labeling scheme to calculate the execution plan and context. Section 6 extends the labeling techniques to answer the data provenance. Issues of labeling the specification are discussed in Section 7, followed by experimental results in Section 8.

## 2. RELATED WORK

The reachability labeling problem for trees has been extensively studied. The well-known *interval* scheme was first proposed by [15], which uses labels of length at most  $2 \log n$  bits, where  $n$  is the number of nodes in the tree. A considerable amount of work [1, 16, 3] has been devoted to reduce the label length. [13] also proposed a *prefix* scheme which, though not theoretically best, is likely to be the winner for labeling XML files in practice. Note that an obvious lower bound for maximum label length is  $\log n$ , however, the tight lower bound for this problem remains open.

The reachability labeling problem for general graphs has also attracted a lot of attention. Existing approaches fall into three categories: Chain Decomposition [10], Tree Cover [2] and 2-Hop [6]. Other variants of Tree Cover include Label+SSPI [5], Dual Labeling [18] and GRIPP [17]. The most recent work hybridizes three main labeling techniques: Path-Tree Cover [12] combines Chain Decomposition with Tree Cover; and 3-Hop [11] combines Chain Decomposition with 2-Hop. A detailed comparison is available in [11]. Thus existing research tries to find a good balance among label length (*i.e.*, index size), construction time and query time. Note that encoding general directed graphs may require labels of length  $\Omega(n)$  bits, where  $n$  is the number of vertices, even if we allow arbitrary construction and query time.

To provide efficient provenance storage and queries in workflow systems, [4] identifies several factorization and inheritance techniques. [8] proposes a reachability labeling scheme to encode the dependency graph of a workflow execution by transforming a general graph into a tree and then applying the interval scheme [15]. The problem with this approach is that the size of the transformed tree may be exponential in the size of the original graph.

## 3. MODEL AND PROBLEM STATEMENT

We start with an informal description of our workflow model in Section 3.1 before formalizing in Section 3.2. In Section 3.3 we formulate the reachability labeling problem, and summarize the main result of this paper.

### 3.1 Workflow Model Description

Our workflow model has two components: A *workflow specification* defines the control and data flow between a set of modules, and serves as a template for executions; an execution of a workflow is called a *workflow run*. Both are represented by directed graphs, in which each vertex denotes a *module*, and each edge denotes a *data channel* associated with a set of *data items*. For the purpose of this paper, we treat the data item as a logical data unit used by the workflow, and the module as a black box that takes a set of data items as input and produces a set of data items as output. A data channel is directed from a module  $u$  to another

module  $v$  if the associated data items are generated by  $u$  as output and then used by  $v$  as input. This graph captures the dependency between modules and data items.

To present our main labeling techniques, we first study a simplified workflow model which considers only the control flow between modules (i.e., vertices) and ignores the data flow over data channels (i.e., edges). Consequently, only vertices in the graph are labeled with their module names. We will extend the model to include data flow in Section 6.

**Workflow Specification.** A workflow specification is a directed acyclic graph whose vertices are labeled with *unique* module names. Without any loss of generality, we assume that there exist a single source (a vertex with no incoming edges) and a single sink (a vertex with no outgoing edges). The source denotes a virtual start module that sends out all source data and initializes the execution; similarly, the sink denotes a virtual finish module that collects all final data products and stops the execution. Due to the presence of these two terminals, every module in the workflow lies on some path from the source to the sink. In addition, we are given two sets of subgraphs of the specification, called *forks* and *loops*, which are allowed to be executed repeatedly in a parallel and serial manner respectively. These fork and loop subgraphs must be (1) *self-contained*: each subgraph has a single source and a single sink, and there are no edges coming into or going out from this subgraph through any internal vertices (see Definition 1); and (2) *well-nested*: for any two subgraphs, either one contains another or they are disjoint (see Definition 2).

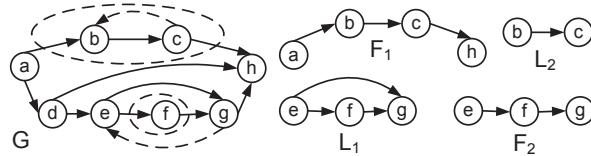


Figure 2: Workflow Specification  $(G, \mathcal{F}, \mathcal{L})$

**Example 1.** A simple workflow specification  $(G, \mathcal{F}, \mathcal{L})$  is shown in Figure 2, where  $G$  denotes the specification graph, and  $\mathcal{F} = \{F_1, F_2\}$  and  $\mathcal{L} = \{L_1, L_2\}$  denote two sets of well-nested fork and loop subgraphs respectively. The letter inside each vertex indicates its module name. We also use the name as a unique identifier for each vertex in  $G$ . We denote a fork subgraph by a dotted oblong which includes only the internal vertices, and a loop subgraph by a dotted backward edge directed from the sink to the source.

**Workflow Run.** Each run of a workflow must follow all the paths in the specification, and may repeatedly execute the given fork and loop subgraphs. A *fork* execution replicates one or more copies of a fork subgraph, and combines them in *parallel*; a *loop* execution replicates one or more copies of a loop subgraph, and combines them in *series* (see Definition 4). Two copies of the same fork or loop may differ due to the presence of some inner forks and loops.

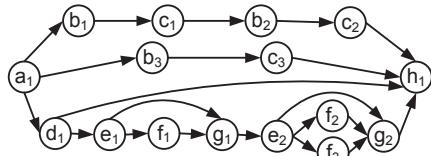


Figure 3: Workflow Run  $R$

**Example 2.** Figure 3 shows a run  $R$  of the specification  $(G, \mathcal{F}, \mathcal{L})$  given in Figure 2. The module names in  $R$  are not unique due to the fork and loop executions. We thus add a subscript to the module name to obtain a unique identifier for each vertex in  $R$ . Comparing Figure 3 with Figure 2, we can see that  $F_1$  is executed twice: in one copy ( $a_1 \rightarrow b_1 \rightarrow c_1 \rightarrow b_2 \rightarrow c_2 \rightarrow h_1$ ),  $L_2$  is executed twice, while in the other copy ( $a_1 \rightarrow b_3 \rightarrow c_3 \rightarrow h_1$ ),  $L_2$  is executed only once.

### 3.2 Workflow Model Formalization

Given a directed graph  $G$ , we denote the vertex set of  $G$  by  $V(G)$ , and the edge set of  $G$  by  $E(G)$ . We use a pair  $(u, v) \in E(G)$  to denote an edge directed from a vertex  $u \in V(G)$  to another vertex  $v \in V(G)$ . We say a directed acyclic graph  $G$  is an *acyclic flow network* if it has a single source  $s(G)$  and a single sink  $t(G)$ . Let  $V^*(G)$  be the set of internal vertices of  $G$ . That is,  $V^*(G) = V(G) \setminus \{s(G), t(G)\}$ .

**Definition 1. (Self-Contained Subgraph)** Given an acyclic flow network  $G$ , we say a subgraph  $H$  of  $G$  is *self-contained* if (1)  $H$  has a single source  $s(H)$  and a single sink  $t(H)$  ( $s(H) \neq t(H)$ ); (2) For any  $u \in V^*(H)$  and  $v \in V(G) \setminus V(H)$ ,  $(u, v) \notin E(G)$  and  $(v, u) \notin E(G)$ ; and (3) For any  $u, v \in V(H)$ ,  $(u, v) \in E(G)$  and  $(u, v) \notin E(H)$  only if  $u = s(H)$  and  $v = t(H)$ .

A self-contained subgraph  $H$  of an acyclic flow network  $G$  is said to be (1) *atomic*, if there is no self-contained subgraph  $H'$  of  $G$  such that  $s(H') = s(H)$  and  $t(H) = t(H')$  and  $E(H') \subset E(H)$ ; and (2) *complete*, if for any  $v \in V(G) \setminus V(H)$ ,  $(s(H), v) \notin E(G)$  and  $(v, t(H)) \notin E(G)$ .

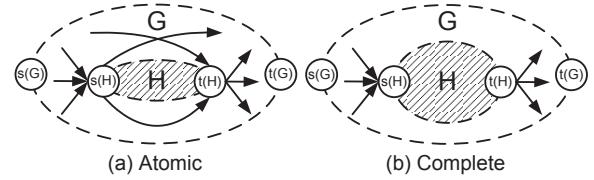


Figure 4: Self-Contained Subgraphs

Figure 4 illustrates the above definitions. Intuitively, a self-contained subgraph connects to other portions of the graph through only its source and sink. It is (1) *atomic*, if it forms a single branch between its source and sink (i.e., it cannot be split into two parallel self-contained subgraphs) (see Figure 4(a)); and (2) *complete*, if it contains all branches between its source and sink. In addition, any incoming edge must go through the source and any outgoing edge must go through the sink (see Figure 4(b)).

**Definition 2. (Well-Nested Fork and Loop System)** Given an acyclic flow network  $G$ , a set  $\mathcal{F}$  of atomic self-contained subgraphs of  $G$ , and a set  $\mathcal{L}$  of complete self-contained subgraphs of  $G$ , we say a pair  $(\mathcal{F}, \mathcal{L})$  forms a *well-nested fork and loop system* for  $G$  if (i)  $\mathcal{F} \cap \mathcal{L} = \emptyset$  and (ii) for any  $H_1, H_2 \in \mathcal{F} \cup \mathcal{L}$ , exactly one of the following is true:

1.  $\text{DomSet}(H_1) \subseteq \text{DomSet}(H_2)$  and  $E(H_1) \subset E(H_2)$ ; or
2.  $\text{DomSet}(H_1) \supseteq \text{DomSet}(H_2)$  and  $E(H_1) \supset E(H_2)$ ; or
3.  $\text{DomSet}(H_1) \cap \text{DomSet}(H_2) = \emptyset$  and  $E(H_1) \cap E(H_2) = \emptyset$ .

where  $\text{DomSet}(H)$  denotes the set of vertices *dominated* by a subgraph  $H \in \mathcal{F} \cup \mathcal{L}$ , and is defined as

$$\text{DomSet}(H) = \begin{cases} V^*(H) & \text{if } H \in \mathcal{F} \\ V(H) & \text{if } H \in \mathcal{L} \end{cases}$$

Note that a fork subgraph dominates only the internal vertices, since its source and sink may be shared by other edge-disjoint fork or loop subgraphs.

**Definition 3. (Workflow Specification)** A *workflow specification* is denoted by a triple  $(G, \mathcal{F}, \mathcal{L})$ , where  $G$  is a uniquely labeled acyclic flow network, and  $(\mathcal{F}, \mathcal{L})$  forms a well-nested fork and loop system for  $G$ .

**Definition 4. (Parallel and Serial Composition)** Given two acyclic flow networks  $G_1$  and  $G_2$ , a *parallel composition* of  $G_1$  and  $G_2$  forms a new acyclic flow network  $G$  by identifying  $s(G_1)$  with  $s(G_2)$  and  $t(G_1)$  with  $t(G_2)$ ; and a *serial composition* of  $G_1$  and  $G_2$  forms a new acyclic flow network  $G$  by adding a new edge directed from  $t(G_1)$  to  $s(G_2)$ .

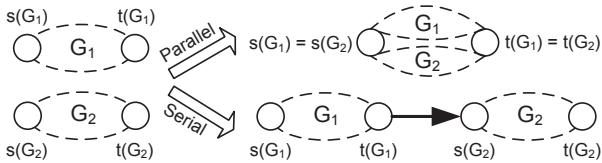


Figure 5: Parallel and Serial Composition

Figure 5 illustrates series and parallel compositions. Note that this definition can be extended to an arbitrary number of acyclic flow networks combined in a single step.

**Definition 5. (Replacement)** Given an acyclic flow network  $G$ , *replacing* a self-contained subgraph  $H_1$  of  $G$  with another acyclic flow network  $H_2$  will identify  $s(H_1)$  with  $s(H_2)$  and  $t(H_1)$  with  $t(H_2)$ .

**Definition 6. (Workflow Run)** Given a workflow specification  $(G, \mathcal{F}, \mathcal{L})$ , a *workflow run* of this specification is denoted by a labeled graph  $R$  that can be derived from  $G$  by applying a sequence of the following operations:

- **Fork Execution:** Replace a fork subgraph  $H \in \mathcal{F}$  with the parallel composition of two copies of  $H$ .
- **Loop Execution:** Replace a loop subgraph  $H \in \mathcal{L}$  with the serial composition of two copies of  $H$ .

**LEMMA 3.1.** For any workflow run  $R$ , let  $\mathcal{F}'$  and  $\mathcal{L}'$  be the sets of all fork and loop copies in  $R$  respectively. Then  $(\mathcal{F}', \mathcal{L}')$  forms a well-nested fork and loop system for  $R$ .

### 3.3 Problem Statement and Main Result

In this paper, we develop a compact and efficient reachability labeling scheme for workflow runs. More precisely, we will assign each vertex in the run with a *reachability label* such that, using only the labels of any two vertices, we can quickly decide if one can reach the other.

**Definition 7. (Reachability Labeling Scheme)** A *reachability labeling scheme* (in short, labeling scheme) is denoted by a triple  $(\mathcal{D}, \phi, \pi)$ , where  $\mathcal{D}$  is the label domain,  $\phi : V(G) \rightarrow \mathcal{D}$  is a labeling function (i.e., encoding algorithm) that, given a directed graph  $G$ , assigns each vertex  $v \in V(G)$  a reachability label  $\phi(v) \in \mathcal{D}$ , and  $\pi : \mathcal{D} \times \mathcal{D} \rightarrow \{0, 1\}$  is a binary predicate (i.e., decoding algorithm), such that for any two vertices  $v, v' \in V(G)$ ,  $\pi(\phi(v), \phi(v')) = 1$  if and only if there exists a path from  $v$  to  $v'$  in  $G$ .

The quality of a labeling scheme  $(\mathcal{D}, \phi, \pi)$  is measured by three criteria: (1) *label length*: the length of reachability labels used from the domain  $\mathcal{D}$ ; (2) *construction time*: the time taken to compute the labeling function  $\phi$ ; and (3) *query time*: the time taken to evaluate the binary predicate  $\pi$ .

**Problem Statement.** This paper addresses the problem that whether there is a labeling scheme for workflow runs that optimizes the above three parameters simultaneously.

**Main Result.** We develop a *skeleton-based* labeling scheme for workflow runs which uses reachability labels from the underlying specification (called the *skeleton labels*). The following theorem summarizes our main result.

**THEOREM 1. (Main Theorem)** Given any fixed labeled specification, there exists a reachability labeling scheme for any run conforming to the specification that uses logarithmic length labels, takes linear construction time, and answers queries in constant time.

Note that just to index  $n$  distinct nodes we need  $\log n$  bits per vertex, to read the whole graph we need linear time and to answer any query we need at least constant time; in this sense our labeling scheme is *optimal*. However, in general it is impossible to obtain an optimal labeling scheme for acyclic flow networks: We can construct a family of networks such that encoding the reachability for these graphs requires labels of length  $\Omega(n)$ , even if we allow arbitrary construction and query time. However, our workflow runs originate from a *fixed* specification, and thus have some inherent structure that we can exploit to obtain an optimal labeling scheme.

In the rest of this paper, we will use  $(G, \mathcal{F}, \mathcal{L})$  to denote a specification and  $R$  to denote a run of this specification. When  $\mathcal{F}$  and  $\mathcal{L}$  are clear from the context, we may simply refer to the specification as  $G$ . To illustrate our labeling techniques, we will use the running examples of specification and run given in Figure 2 and 3 respectively.

## 4. SKELETON-BASED LABELING SCHEME

We now present a skeleton-based labeling scheme for workflow runs. We assume that the run to be labeled is given along with its underlying specification, that is already labeled by some other labeling scheme. The main idea is described as follows. Recall that each run follows the same basic structure as the specification, but becomes much larger due to the fork and loop behavior. We will design a compact labeling scheme that encodes the basic structure of a run using the given reachability labels from its specification (i.e., skeleton labels), and encode the well-nested fork and loop structure using some extended labels. Combining these two labels, we are able to efficiently answer reachability queries.

The rest of this section is organized as follows. Section 4.1 introduces some preliminary notions, namely origin, execution plan and context, which play a central role in our skeleton-based labeling scheme. Section 4.2 describes how to answer reachability queries using context and skeleton labels. A compact three-dimensional encoding for the context is presented in Section 4.3. We complete the description of our skeleton-based labeling scheme in Section 4.4, and finally present the correctness and quality analysis in Section 4.5. Due to space constraints, proofs of most claims are omitted.

## 4.1 Preliminaries

**Origin.** We start with the notion of *origin*, which establishes the correspondence between the vertices of a specification and a run based on their module names. Let  $\text{MODULE}(v)$  denote the module name of a vertex  $v \in V(G)$  or  $V(R)$ .

**Definition 8. (Origin)** Given a specification  $G$  and a run  $R$  of  $G$ , a vertex  $u \in V(G)$  is said to be the *origin* of vertex  $v \in V(R)$ , denoted by  $u = \text{Orig}(v)$ , if  $\text{MODULE}(u) = \text{MODULE}(v)$ .

Note that the origin is well-defined for each vertex in  $R$ , since the module names in  $G$  are unique. The *skeleton* label for a vertex  $v \in V(R)$  then refers to the given reachability label on its origin  $\text{Orig}(v) \in V(G)$ .

**Fork and Loop Hierarchy.** All fork and loop subgraphs given a specification  $(G, \mathcal{F}, \mathcal{L})$  are well-nested, and therefore can be captured by an unordered tree  $T_G$ , called the *fork and loop hierarchy*, where the root of  $T_G$  corresponds to the entire specification graph  $G$ , and any other node in  $T_G$  corresponds to a unique fork or loop subgraph  $H \in \mathcal{F} \cup \mathcal{L}$ .

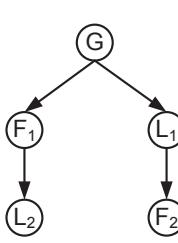


Figure 6: Fork and Loop Hierarchy  $T_G$

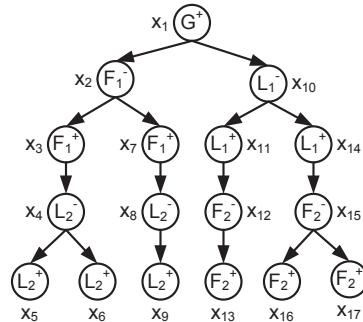


Figure 7: Execution Plan  $T_R$

**Example 3.** Figure 6 shows the fork and loop hierarchy  $T_G$  for the running example, where the label inside each node indicates its corresponding subgraph.

**Execution Plan.** By Lemma 3.1, all fork and loop copies generated in a run  $R$  are also well-nested. We can thus similarly describe them by a tree  $T_R$ , called the *execution plan*, where the root of  $T_R$  corresponds to the entire run graph  $R$  (called a  $G^+$  node), and any other node in  $T_R$  either corresponds to a single fork or loop copy (called an  $\mathcal{F}^+$  or an  $\mathcal{L}^+$  node respectively) or corresponds to all copies of the same fork or loop (called an  $\mathcal{F}^-$  or an  $\mathcal{L}^-$  node respectively). Note that  $T_R$  is a *semi-ordered* tree in the sense that the children of an  $\mathcal{L}^-$  node are ordered while the children of any other node are unordered. In the following, we may simply refer to an  $\mathcal{F}^+$  node as a  $+$  or  $\mathcal{F}$  node, and follow a similar convention for other types of nodes.

**Example 4.** Figure 7 shows the execution plan  $T_R$  for the running example, where the label inside each node indicates its type, and the label beside each node indicates a unique identifier. For instance,  $x_3$  is an  $\mathcal{F}^+$  node, corresponding to a single fork copy of  $F_1$ . Similarly, its sibling  $x_7$  corresponds to a second copy of  $F_1$ . Their parent  $x_2$  is then an  $\mathcal{F}^-$  node, corresponding to the parallel composition of these two fork copies. Comparing Figure 7 with Figure 6, we can see that an execution plan essentially describes how many times each fork and loop subgraph is executed in the corresponding run.

For any node  $x \in V(T_R)$ , let  $\text{Sub}_R(x)$  denote the corresponding fork or loop copy for  $x$  (i.e., a subgraph of  $R$ ), and let  $\text{Sub}_G(x)$  denote the corresponding fork or loop subgraph for  $x$  (i.e., a subgraph of  $G$ ) from which  $\text{Sub}_R(x)$  is derived.

**LEMMA 4.1.** For any node  $x \in V(T_R)$ , let  $y_1, \dots, y_k$  be the children of  $x$  in  $T_R$ , then

- If  $x$  is an  $\mathcal{F}^-$  node, then  $\text{Sub}_R(x)$  is obtained by a parallel composition of all  $\text{Sub}_R(y_i)$ 's.
- If  $x$  is an  $\mathcal{L}^-$  node, then  $\text{Sub}_R(x)$  is obtained by a serial composition of all  $\text{Sub}_R(y_i)$ 's.
- If  $x$  is a  $+$  node, then  $\text{Sub}_R(x)$  is obtained from  $\text{Sub}_G(x)$  by replacing each  $\text{Sub}_G(y_i)$  with  $\text{Sub}_R(y_i)$ .

**Context.** Intuitively, the context of a vertex in  $R$  refers to the smallest fork or loop copy that dominates this vertex. Recall that each  $+$  node in  $T_R$  corresponds to a single fork or loop copy. We therefore assign each vertex in  $R$  a unique  $+$  node in  $T_R$  as its context. Formally,

**Definition 9. (Context)** Given a run  $R$  and its execution plan  $T_R$ , a node  $x \in V(T_R)$  is said to be the *context* of a vertex  $v \in V(R)$ , denoted by  $x = \mathcal{C}(v)$ , if  $x$  is the deepest  $+$  node in  $T_R$  such that  $v \in \text{DomSet}(x)$ , where

$$\text{DomSet}(x) = \begin{cases} V^*(\text{Sub}_R(x)) & \text{if } x \text{ is an } \mathcal{F} \text{ node} \\ V(\text{Sub}_R(x)) & \text{otherwise} \end{cases}$$

In Definition 9,  $\text{DomSet}(x)$  denotes the set of vertices in  $R$  dominated by a node  $x$  in  $T_R$ . This extends the notion of domination<sup>2</sup> introduced in Definition 2. Note that the context is well-defined for each vertex in  $R$ , since Lemma 3.1 ensures that the dominating sets are well-nested in  $T_R$ .

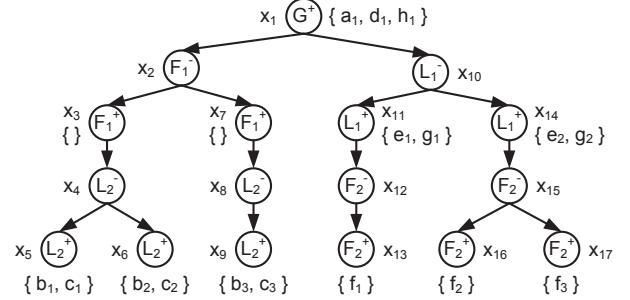


Figure 8: Context Assignment  $\mathcal{C} : V(R) \rightarrow V(T_R)$

**Example 5.** Figure 8 shows the context assignment for the running example. Note that by Definition 9, each fork copy dominates only the internal vertices, since its source and sink may be shared by other edge-disjoint fork or loop copies. For instance,  $a_1$  and  $h_1$  are shared by two fork copies represented by  $x_3$  and  $x_7$  respectively. To ensure that the context for  $a_1$  and  $h_1$  is well-defined, we will assign the least common  $+$  ancestor  $x_1$  of  $x_3$  and  $x_7$  as their context.

We say a  $+$  node  $x \in V(T_R)$  is *empty* if there does not exist any vertex  $v \in V(R)$  such that  $\mathcal{C}(v) = x$ . For example, as shown in Figure 8,  $x_3$  and  $x_7$  are two empty  $+$  nodes. Obviously, the context of a vertex  $v \in V(R)$  always refers to a nonempty  $+$  node  $x \in V(T_R)$ .

<sup>2</sup>Note that our definition of dominating set differs from the one typically used in graph theory.

## 4.2 Answering Reachability Queries

We now show that the reachability between any two vertices in the run can be determined using only their context and given skeleton labels. The main idea is to check the *least common ancestor* of their context in the execution plan. If this ancestor is an  $\mathcal{F}^-$  or an  $\mathcal{L}^-$  node, then we can immediately determine the reachability by showing that they are dominated by two distinct copies of the same fork (unreachable) or loop (reachable). Otherwise (this ancestor is a  $+$  node), we show that the reachability between these two vertices in the run is the same as the reachability between their origins in the specification, and therefore can be determined using the given skeleton labels. Recall that for any vertex  $v \in V(R)$ ,  $\mathcal{C}(v)$  denotes the context of  $v$  and  $\text{Orig}(v)$  denotes the origin of  $v$ . The lemma below formalizes this idea.

LEMMA 4.2. *For any two vertices  $v, v' \in V(R)$ , if the least common ancestor of  $\mathcal{C}(v)$  and  $\mathcal{C}(v')$  in  $T_R$  is*

- *an  $\mathcal{F}^-$  node, then there does not exist a path from  $v$  to  $v'$  or from  $v'$  to  $v$  in  $R$ .*
- *an  $\mathcal{L}^-$  node, then there exists a path either from  $v$  to  $v'$  or from  $v'$  to  $v$  in  $R$ .*
- *a  $+$  node, then there exists a path from  $v$  to  $v'$  in  $R$  iff there exists a path from  $\text{Orig}(v)$  to  $\text{Orig}(v')$  in  $G$ .*

Example 6. We can easily verify above three rules using Figure 3. First, consider two vertices  $b_1$  and  $c_3$  in  $R$ . We can see from Figure 8 that the least common ancestor of their context (i.e.,  $x_5$  and  $x_9$ ) is an  $\mathcal{F}^-$  node (i.e.,  $x_2$ ). Therefore, by Lemma 4.2, there does not exist a path between  $b_1$  and  $c_3$ , which is confirmed by Figure 3. Similarly, consider  $f_1$  and  $e_2$ . The least common ancestor of their context (i.e.,  $x_{13}$  and  $x_{14}$ ) is an  $\mathcal{L}^-$  node (i.e.,  $x_{10}$ ), and there exists a path from  $f_1$  to  $e_2$ , which again verifies our Lemma 4.2. Finally, consider  $c_1$  and  $d_1$ . The least common ancestor of their context (i.e.,  $x_5$  and  $x_1$ ) is a  $+$  node (i.e.,  $x_1$ ). Moreover, we can see from Figure 2 that there does not exist a path between their origins (i.e.,  $c$  and  $d$ ) in  $G$ . Consequently, by Lemma 4.2, there also does not exist a path between  $c_1$  and  $d_1$  in  $R$ , which is again confirmed by Figure 3.

## 4.3 Encoding Context

As seen in Section 4.2, the reachability query against two vertices in the run can be reduced to a related query against their context in the execution plan. Recall that the context always refers to a nonempty  $+$  node. The related query is therefore stated as follows. Given two nonempty  $+$  nodes in the execution plan, we want to decide if their least common ancestor is an  $\mathcal{F}^-$  or an  $\mathcal{L}^-$  or a  $+$  node. In this section, we will show that we can efficiently answer this related query using a three-dimensional encoding for the context.

**Generate Three Total Orders.** We first present a simple algorithm to generate three total orders  $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$  on all nonempty  $+$  nodes in the execution plan  $T_R$ . The algorithm performs three *preorder* traversals<sup>3</sup> on  $T_R$ , and records the order in which all nonempty  $+$  nodes are visited in the  $i$ th traversal as the total order  $\mathcal{O}_i$  ( $i = 1, 2, 3$ ). Recall that only the children of an  $\mathcal{L}^-$  node are ordered in  $T_R$ , for any other

<sup>3</sup>A *preorder* traversal on a tree starts at the root of the tree, and recursively visits the root of a subtree before visiting its children.

---

### Algorithm 1 GenerateThreeOrders

---

**Input:**  $T_R$  : an execution plan

**Output:**  $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$  : three total orders

/\* Generate  $\mathcal{O}_1$  \*/

Do a preorder traversal on  $T_R$ : for each node  $x \in V(T_R)$ , visit the children of  $x$  from left to right.

/\* Generate  $\mathcal{O}_2$  \*/

Do a preorder traversal on  $T_R$ : for each node  $x \in V(T_R)$ , if  $x$  is an  $\mathcal{F}^-$  node, visit the children of  $x$  from right to left; otherwise, visit the children of  $x$  from left to right.

/\* Generate  $\mathcal{O}_3$  \*/

Do a preorder traversal on  $T_R$ : for each node  $x \in V(T_R)$ , if  $x$  is an  $\mathcal{L}^-$  node, visit the children of  $x$  from right to left; otherwise, visit the children of  $x$  from left to right.

return  $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$

---

node, we fix an arbitrary ordering of its children. The details of three preorder traversals are described in Algorithm 1. Note that these traversals differ only in the order in which the children of an  $\mathcal{F}^-$  or an  $\mathcal{L}^-$  node are visited.

The above three total orders lend themselves to a compact three-dimensional encoding for the context: Each nonempty  $+$  node in  $T_R$  is simply encoded by three natural numbers indicating its positions in  $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$  respectively.

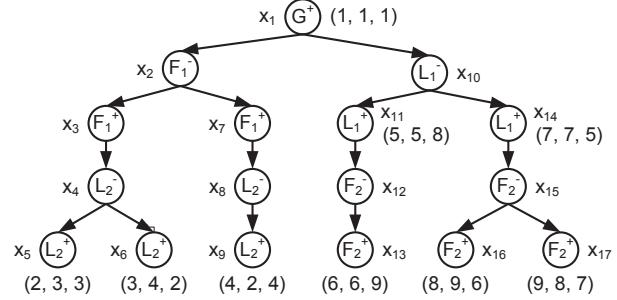


Figure 9: Context Encoding

Example 7. Figure 9 shows the context encoding for the running example. Note that Algorithm 1 will traverse every node in the execution plan, but records only the ordering of nonempty  $+$  nodes. In particular,  $x_3$  and  $x_7$  are empty  $+$  nodes (as shown in Figure 8) and are therefore skipped.

**Compare Three Total Orders.** The lemma below shows how to answer the related query using the context encoding. Let  $x <_{\mathcal{O}} x'$  denote that  $x$  precedes  $x'$  in the total order  $\mathcal{O}$ .

LEMMA 4.3. *For any two nonempty  $+$  nodes  $x, x' \in V(T_R)$ ,*

- *If  $x <_{\mathcal{O}_1} x'$  and  $x' <_{\mathcal{O}_3} x$ , then (1a) the least common ancestor of  $x$  and  $x'$  is an  $\mathcal{F}^-$  node and (1b)  $x <_{\mathcal{O}_3} x'$ .*
- *If  $x <_{\mathcal{O}_1} x'$  and  $x' <_{\mathcal{O}_2} x$ , then (2a) the least common ancestor of  $x$  and  $x'$  is an  $\mathcal{L}^-$  node and (2b)  $x <_{\mathcal{O}_2} x'$ .*
- *If  $x <_{\mathcal{O}_1} x', x <_{\mathcal{O}_2} x'$  and  $x <_{\mathcal{O}_3} x'$ , then (3) the least common ancestor of  $x$  and  $x'$  is a  $+$  node.*

Example 8. We can easily verify the above three rules using Figure 9. For instance, consider two nonempty  $+$  nodes  $x_5$  and  $x_9$  in  $T_R$ . Observe that  $x_5 <_{\mathcal{O}_1} x_9$  and  $x_9 <_{\mathcal{O}_2} x_5$ . Hence, by Lemma 4.3, the least common ancestor of  $x_5$  and  $x_9$  (i.e.,  $x_2$ ) is an  $\mathcal{F}^-$  node and  $x_5 <_{\mathcal{O}_3} x_9$ , which is confirmed by Figure 9. Using  $(x_{13}, x_{14})$  and  $(x_5, x_1)$ , we can verify the other two rules in a similar way.

## 4.4 Labeling Scheme Description

We are now ready to present the skeleton-based labeling scheme. Recall that a labeling scheme is denoted by a triple  $(\mathcal{D}, \phi, \pi)$ , where  $\mathcal{D}$  is the label domain,  $\phi$  is the labeling function (*i.e.*, encoding algorithm) and  $\pi$  is the binary predicate (*i.e.*, decoding algorithm). In the following, let  $(\mathcal{D}_g, \phi_g, \pi_g)$  denote the given labeling scheme used to label the specification, and  $(\mathcal{D}_r, \phi_r, \pi_r)$  denote our proposed skeleton-based labeling scheme used to label the run.

**Label Domain ( $\mathcal{D}_r$ ).** Each label assigned by our skeleton-based labeling scheme consists of a three-dimensional encoding for the context (*i.e.*, three natural numbers) and a skeleton label from the specification, *i.e.*,  $\mathcal{D}_r = \mathbb{N}^3 \times \mathcal{D}_g$ .

**Labeling Function ( $\phi_r$ ).** A high level description of the labeling function  $\phi_r$  is shown in Algorithm 2. Given a specification  $G$  labeled by  $\phi_g$ , we label a run  $R$  of  $G$  in four steps: First, we compute the origin function  $\text{Orig} : V(R) \rightarrow V(G)$  by matching their module names (Line 2). Second, we construct the execution plan  $T_R$  for  $R$ , and compute the context function  $\mathcal{C} : V(R) \rightarrow V(T_R)$  (Line 5). Since this part of computation may be of independent interest, we defer the description of the algorithm `ConstructPlan` to Section 5. Third, we build three-dimensional context encoding by generating three total orders  $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$  using Algorithm 1 (Line 8). Finally, we assign each vertex  $v \in V(R)$  a reachability label  $\phi_r(v) = (q_1, q_2, q_3, \phi_g(u))$ , where  $(q_1, q_2, q_3)$  denotes its context encoding (*i.e.*, the positions of  $x = \mathcal{C}(v)$  in  $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$  respectively) and  $\phi_g(u)$  denotes its skeleton label (*i.e.*, the reachability label on  $u = \text{Orig}(v)$  given by  $\phi_g$ ). (Line 12 to Line 15).

**Binary Predicate ( $\pi_r$ ).** Given the above labels, we can answer any reachability query on  $R$  by a simple binary predicate  $\pi_r$  described in Algorithm 3. Observe that evaluating the predicate  $\pi_r$  for a pair of labels from  $\mathcal{D}_r$  only requires comparing three pairs of natural numbers plus one potential evaluation of the predicate  $\pi_g$  for a pair of labels from  $\mathcal{D}_g$ .

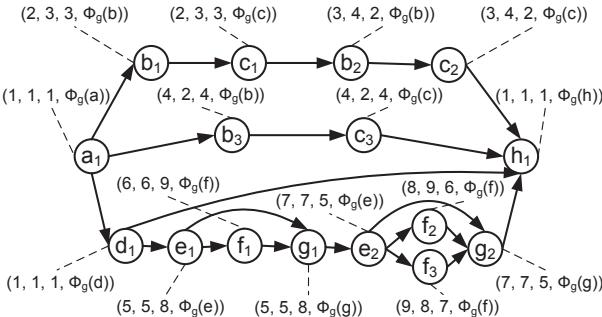


Figure 10: Skeleton-Based Labeling Scheme

*Example 9.* The reachability labels assigned by  $\phi_r$  for the running example are shown in Figure 10, where  $\phi_g$  denotes the given skeleton labels. To illustrate how to answer reachability queries using these labels, let us consider two vertices  $c_1$  and  $d_1$  in  $R$ . We start by comparing their reachability labels  $(2, 3, 3, \phi_g(c))$  and  $(1, 1, 1, \phi_g(d))$  using the predicate  $\pi_r$  described in Algorithm 3. Since  $(3 - 1) \times (3 - 1) > 0$ , we need to further compare their skeleton labels  $\phi_g(c)$  and  $\phi_g(d)$  using the given predicate  $\pi_g$ . Observe from Figure 2 that there is no path from  $c$  to  $d$  (*i.e.*,  $\pi_g(\phi_g(c), \phi_g(d)) = 0$ ). Hence, we can conclude that there is also no path from  $c_1$  to  $d_1$  in  $R$ , which is confirmed by Figure 10.

---

### Algorithm 2 Labeling Function $\phi_r : V(R) \rightarrow \mathcal{D}_r$

---

**Input:**  $G$  : a specification  
 $\phi_g : V(G) \rightarrow \mathcal{D}_g$  : a labeling function for  $G$   
 $R$  : a run of  $G$

**Output:**  $\phi_r : V(R) \rightarrow \mathcal{D}_r$  : a labeling function for  $R$

```

1: /* Step 1: Compute Origin */
2:  $\text{Orig} \leftarrow \text{ComputeOrigin}(G, R)$ 
3:
4: /* Step 2: Construct Execution Plan and Context */
5:  $\langle T_R, \mathcal{C} \rangle \leftarrow \text{ConstructPlan}(G, R)$ 
6:
7: /* Step 3: Build Context Encoding */
8:  $\langle \mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3 \rangle \leftarrow \text{GenerateThreeOrders}(T_R)$ 
9:
10: /* Step 4: Assign Reachability Labels */
11: for each vertex  $v \in V(R)$  do
12:    $x \leftarrow \mathcal{C}(v)$  : the context of  $v$ 
13:    $q_i \leftarrow$  the position of  $x$  in  $\mathcal{O}_i$  ( $1 \leq i \leq 3$ )
14:    $u \leftarrow \text{Orig}(v)$  : the origin of  $v$ 
15:    $\phi_r(v) \leftarrow (q_1, q_2, q_3, \phi_g(u))$ .
16: end for
17:
18: return  $\phi_r$ 

```

---



---

### Algorithm 3 Binary Predicate $\pi_r : \mathcal{D}_r \times \mathcal{D}_r \rightarrow \{0, 1\}$

---

**Input:**  $d_r = (q_1, q_2, q_3, d_g) \in \mathcal{D}_r$   
 $d'_r = (q'_1, q'_2, q'_3, d'_g) \in \mathcal{D}_r$   
 $\pi_g : \mathcal{D}_g \times \mathcal{D}_g \rightarrow \{0, 1\}$

**Output:**  $\pi_r(d_r, d'_r)$

```

1: if  $(q_2 - q'_2) \times (q_3 - q'_3) < 0$  then
2:   if  $q_1 < q'_1$  and  $q_3 > q'_3$  then
3:      $\pi_r(d_r, d'_r) \leftarrow 1$ 
4:   else
5:      $\pi_r(d_r, d'_r) \leftarrow 0$ 
6:   end if
7: else
8:    $\pi_r(d_r, d'_r) \leftarrow \pi_g(d_g, d'_g)$ 
9: end if
10: return  $\pi_r(d_r, d'_r)$ 

```

---

## 4.5 Correctness and Quality Analysis

The following lemma shows the correctness of our skeleton-based labeling scheme.

LEMMA 4.4. *Given a run  $R$  labeled by our skeleton-based labeling scheme  $(\mathcal{D}_r, \phi_r, \pi_r)$ , for any two vertices  $v, v' \in V(R)$ ,  $\pi_r(\phi_r(v), \phi_r(v')) = 1$  if and only if there exists a path from  $v$  to  $v'$  in  $R$ .*

PROOF. (Sketch) Recall that  $T_R$  denotes the execution plan for  $R$ , and  $\mathcal{C}(v)$  denotes the context of a vertex  $v \in V(R)$ . Let  $\phi_r(v) = (q_1, q_2, q_3, d_g)$  and  $\phi_r(v') = (q'_1, q'_2, q'_3, d'_g)$ . We can prove the theorem by doing a case analysis based on their context encodings. For example, if  $q_1 < q'_1, q_2 > q'_2, q_3 < q'_3$ , then by Lemma 4.3, the least common ancestor of  $\mathcal{C}(v)$  and  $\mathcal{C}(v')$  in  $T_R$  is an  $\mathcal{F}^-$  node. Therefore, by Lemma 4.2, there does not exist a path from  $v$  to  $v'$  in  $R$ . This is correctly encoded by Algorithm 3. The other cases can be proved in a similar way using Lemma 4.2 and 4.3. Note that by Lemma 4.3, two cases  $(q_1 > q'_1, q_2 < q'_2, q_3 < q'_3)$  and  $(q_1 < q'_1, q_2 > q'_2, q_3 > q'_3)$  are impossible.  $\square$

The following lemma analyzes the quality of our skeleton-based labeling scheme in terms of label length, construction time and query time. We do not count the cost of labeling the specification, since it is fixed for all runs.

**LEMMA 4.5.** *Given a fixed labeled specification  $G$ , for any run  $R$  of  $G$ , our skeleton-based labeling scheme for  $R$  guarantees (1) the label length  $\leq 3 \log n_R + \log n_G$ , (2) the construction time is  $O(m_R + n_R)$ , and (3) the query time is  $O(1) + t_G$ , where  $n_R = |V(R)|$ ,  $n_G = |V(G)|$ ,  $m_R = |E(R)|$  and  $t_G$  is the query time for  $G$  using the given skeleton labels.*

**PROOF.** Let  $T_R$  be the execution plan of  $R$ , and  $n_T^+$  be the number of nonempty + nodes in  $T_R$ . Then each label in  $R$  consists of three natural numbers between 1 and  $n_T^+$  and a skeleton label from  $G$ . Note that there are at most  $n_G$  distinct skeleton labels in total, so we only need  $\log n_G$  bits to encode each skeleton label. Therefore, the total label length  $\leq 3 \log n_T^+ + \log n_G$ . On the other hand, it is easy to see that  $n_T^+ \leq n_R$ , because each nonempty + node in  $T_R$  serves as the context for at least one distinct vertex in  $R$ . Consequently, the label length  $\leq 3 \log n_R + \log n_G$ .

According to Algorithm 2, labeling a run  $R$  involves four steps: (1) computing the origin. Since the module names in  $G$  are unique, finding the origin for each vertex in  $R$  needs only  $O(n_R)$  time; (2) constructing the execution plan and context, for which we will present in Section 5 a linear time algorithm. So by Lemma 5.1, this step takes  $O(m_R + n_R)$  time; (3) building context encoding (Algorithm 1), which uses three preorder traversals on  $T_R$ . So it takes a total of  $O(n_T)$  time, where  $n_T = |V(T_R)|$ ; and (4) assigning the reachability labels, which requires only one scan on  $V(R)$  and therefore takes  $O(n_R)$  time. It can be shown that  $n_T = O(m_R)$ . So the overall construction time is  $O(m_R + n_R)$ .

Finally, according to Algorithm 3, evaluating the predicate  $\pi_r$  for a pair of labels in  $R$  requires only comparing three pairs of natural numbers plus one potential evaluation of the predicate  $\pi_g$  for a pair of skeleton labels in  $G$ . Hence, the query time is  $O(1) + t_G$ .  $\square$

Note that for a fixed specification  $G$ , we can treat both  $n_G$  and  $t_G$  in Lemma 4.5 as constants. Consequently, our skeleton-based labeling scheme guarantees logarithmic label length, linear construction time, and constant query time (in terms of the size of the run) and therefore is optimal. This proves Theorem 1 claimed in Section 3.3.

## 5. EXECUTION PLAN AND CONTEXT

We now describe a linear time algorithm to generate the execution plan  $T_R$  and the context function  $\mathcal{C}$  given a run  $R$  and the specification  $(G, \mathcal{F}, \mathcal{L})$ . We assume that a well-nested fork and loop system  $(\mathcal{F}, \mathcal{L})$  is given in terms of the fork and loop hierarchy  $T_G$ <sup>4</sup>.

Let  $T_G(i)$  and  $T_R(i)$  denote the  $i$ -th level of  $T_G$  and  $T_R$  respectively. Let  $d$  be the depth of  $T_G$ . Recall that, for each level  $T_G(i)$  except  $T_G(1)$ , there are two levels  $T_R(2i-2)$  and  $T_R(2i-1)$ . Our algorithm, namely **ConstructPlan**, generates  $T_R$  in  $d$  iterations in a bottom-up manner; in iteration  $i$  ( $i = d$  to 1),  $T_R(2i-1)$  and  $T_R(2i-2)$  are constructed. Note that since there is a one-one correspondence between the nodes of  $T_G$  and the subgraphs in  $\mathcal{F} \cup \mathcal{L}$ , we directly refer

<sup>4</sup> $T_G$  serves as a compact description of  $(\mathcal{F}, \mathcal{L})$ , and can be efficiently constructed from any other explicit description of  $(\mathcal{F}, \mathcal{L})$ .

to any subgraph  $H \in \mathcal{F} \cup \mathcal{L}$  as a node  $H \in T_G$ . Next we give an overview of the algorithm **ConstructPlan**, highlighting the challenges in obtaining the linear time complexity. (Details can be found in the full version of this paper.)

**(1) Identifying all copies of fork and loop subgraphs:** If  $H \in T_G$  is a leaf node, then we can count the number of copies of any edge  $e \in E(H)$  in  $R$  to count the number of copies of  $H$  in  $R$ . But if  $H \in T_G$  is a non-leaf node, we may not have a unique edge in  $E(H)$  which does not belong to any other fork or loop subgraph (for example, consider a line-graph of length 2,  $u \rightarrow v \rightarrow w$ , and define three forks on  $\{u, v\}$ ,  $\{v, w\}$  and  $\{u, v, w\}$ ). For each leaf node  $H \in T_G$ , we assign any edge  $e \in E(H)$  as the *leader* of  $H$  and call it **leader**( $H$ ). If  $H \in T_G$  is a non-leaf node, we assign any arbitrary child  $H_c$  of  $H$  as a *candidate* for **leader**( $H$ ); as the algorithm **ConstructPlan** executes in the bottom-up fashion, it constructs an edge from a copy of  $H_c$  during its execution. By traversing  $R$  only once we can compute all copies of **leader**( $H$ ) for each leaf node  $H \in T_G(d)$  and subsequently before the  $i$ -th iteration starts ( $i = d-1$  to 1), we have all copies of **leader**( $H$ ) for each node  $H \in T_G(i)$ .

**(2) Avoiding visiting any edge and node in  $R$  more than once:** In iteration  $i$ , we identify and visit all copies of  $H \in T_G(i)$  in  $R$ . To avoid visiting the vertices and edges in these copies in future iterations, after  $T_R(2i-1)$  is constructed, we replace each *copy* of  $H \in T_G(i)$  by a directed edge from the source to the sink of this copy preserving only these two vertices. Similarly, after  $T_R(2i-2)$  is constructed, we replace all *copies* of  $H$  created due to a fork or loop execution by a directed edge (this preserves the source and sink of a fork execution and the source of the first copy and the sink of the last copy of a loop execution). We refer to the new edges added in this process as *special edges*.

Let  $H'$  be a copy of a subgraph  $H \in T_G(i)$  in  $R$ . In iteration  $i$ , we start with an endpoint of a copy of **leader**( $H$ ) in  $H'$  as a seed and explore the vertices and edges in  $H'$  using a modified depth first search. In this search procedure, we use the fact that in the modified run-graph  $R$  at each iteration  $i$  (after deleting and adding some edges in the previous iterations), (1) the induced subgraph on the vertices in  $H'$  is a connected subgraph if  $H \in \mathcal{L}$ ; and (2) the induced subgraph on the vertices in  $H'$  excluding  $s(H'), t(H')$  is a connected subgraph if  $H \in \mathcal{F}$ . We can construct the execution plan  $T_R$  and compute the context function  $\mathcal{C}(v)$  for each vertex  $v \in V(R)$  as we execute the above search procedure.

Since we do not traverse any edge more than once, the time complexity of the algorithm **ConstructPlan** can be bounded by  $O(m_R + n_R + m')$ , where  $m'$  is the number of special edges added in all the iterations. It can also be shown that  $m' = O(|V(T_R)|) = O(m_R)$ , therefore the time complexity is  $O(m_R + n_R)$  (details are omitted).

**LEMMA 5.1.** *Given a fixed specification  $(G, \mathcal{F}, \mathcal{L})$  and a run  $R$ , the execution plan  $T_R$  and the context function  $\mathcal{C}$  can be computed in  $O(m_R + n_R)$  time.*

## 6. ANSWERING DATA PROVENANCE

To efficiently query dependency relations between data or between data and modules, we extend module labels to the data that flow over the data channels (edges) in a run.

We start by associating with each edge  $e = (u, v)$  the set of data items that flow from module  $u$  to module  $v$  in the run, *i.e.*, that are produced (written) by  $u$  and consumed (read) by  $v$ . We call this set  $\text{DATA}(e)$ . Note that each data item must be created by a unique module, *i.e.*, if  $x \in \text{DATA}(u, v)$  and  $x \in \text{DATA}(u', v')$  then  $u = u'$ . We say a data item  $x$  *depends on* another data item  $x'$  if there exists a sequence of modules  $v_0, \dots, v_k$  and a set of data items  $x_1, \dots, x_k$  such that  $x_1 = x'$ ,  $x_k = x$ , and for all  $1 \leq i \leq k$ ,  $x_i \in \text{DATA}(v_{i-1}, v_i)$ .

Given a run whose modules (vertices) are labeled by a labeling scheme  $(\mathcal{D}, \phi, \pi)$ , we label the data as follows. For any data item  $x$ , let  $\text{OUTPUT}(x)$  be the (unique) module that writes  $x$  as output and  $\text{INPUTS}(x)$  the set of all modules that read  $x$  as input. In other words, if  $x \in \text{DATA}(u, v)$  then  $u = \text{OUTPUT}(x)$  and  $v \in \text{INPUTS}(x)$ . We label  $x$  by

$$(\phi(\text{OUTPUT}(x)), \{\phi(v) \mid v \in \text{INPUTS}(x)\})$$

where  $\phi(\text{OUTPUT}(x))$  denotes the reachability label for the output module of  $x$ , and  $\{\phi(v) \mid v \in \text{INPUTS}(x)\}$  denotes the set of reachability labels for all input modules of  $x$ .

Using these labels, we can quickly determine the dependency between data as follows. A data item  $x$  depends on another data item  $x'$  if and only if there exists a module  $v \in \text{INPUTS}(x')$  such that  $\pi(\phi(v), \phi(\text{OUTPUT}(x))) = 1$  (*i.e.*, there is a path from  $v$  to  $\text{OUTPUT}(x)$ ). Similarly, we can determine the dependency between data and modules. For example, to decide if a data item  $x$  depends on a module  $v$ , we only need to check if  $\pi(\phi(v), \phi(\text{OUTPUT}(x))) = 1$ .

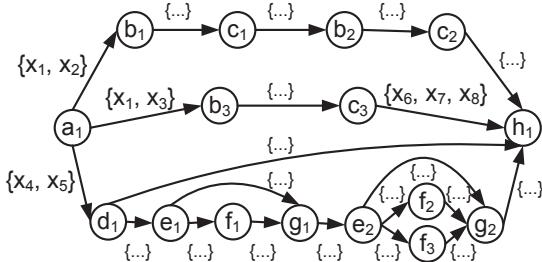


Figure 11: Query Data Dependency

*Example 10.* Figure 11 shows the running example of  $R$ , where each edge is associated with a set of data items. Note that one data item may be read by multiple modules. For instance,  $x_1$  appears on both  $(a_1, b_1)$  and  $(a_1, b_3)$ . Therefore, by the above labeling scheme,  $x_1$  will be labeled by  $(\phi(a_1), \{\phi(b_1), \phi(b_3)\})$ , where  $\phi$  denotes the given module labels. Similarly,  $x_6$  will be labeled by  $(\phi(c_3), \{\phi(h_1)\})$ . To tell if  $x_6$  depends on  $x_1$ , we need to check if at least one of  $\text{INPUTS}(x_1) = \{b_1, b_3\}$  can reach  $\text{OUTPUT}(x_6) = c_3$ . That is, if  $\pi(\phi(b_1), \phi(c_3)) = 1$  or  $\pi(\phi(b_3), \phi(c_3)) = 1$ .

While the construction time for data labels remains linear in the size of the input ( $\sum_e |\text{DATA}(e)|$ ), the label length will increase by a factor of  $k + 1$  and the query time by a factor of  $k$ , where  $k$  is the maximum number of input modules of a data item (*i.e.*,  $k = \max_x (|\text{INPUTS}(x)|)$ ). Note that  $k$  is bounded by the maximum outdegree of a vertex in the run.

## 7. LABELING THE SPECIFICATION

In order to apply our skeleton-based approach, one question remains to be answered: How to label the specification? Since any labeling scheme for acyclic flow networks may require labels of linear length, obtaining an optimal labeling scheme for specifications is impossible in general.

Fortunately, within the skeleton-based labeling framework, we do not need to worry about the quality of the scheme used to label the specification, due to the following three observations. *First, the size of the specification is expected to be much smaller than the size of the run.* For instance, the largest workflow specification we have collected has fewer than 150 vertices, while due to the fork and loop behavior, a run of this workflow may have more than  $10K$  vertices. *Second, the cost (both storage space and construction time) of labeling a specification can be amortized over a large number of runs.* Since workflows tend to be repeatedly executed, we need to label each specification only once and the resulting skeleton labels can be reused by all runs of this specification. We thus assume in Lemma 4.5 that each skeleton label, when used to label the run, can be encoded by  $\log n_G$  bits regardless of its actual size ( $\Omega(n_G)$  bits). *Third, reachability queries on the run may be frequently answered using only the context encodings without comparing the skeleton labels.* Therefore, our approach is effective even when reachability queries on the specification are slow. Interestingly, our experimental results show that given a specification, queries on large runs may even be faster than queries on small runs. A more detailed explanation is given in Section 8.2.

Hence any reasonable labeling scheme can be used to label the specification. Here we briefly describe two extreme and well-known approaches, which entail an expensive encoding and decoding step respectively.

**TCM.** The first approach, called **TCM**, precomputes the transitive closure matrix  $M$  of the specification graph  $G$  (where  $M[i][j] = 1$  if the  $i$ -th vertex can reach the  $j$ -th vertex and 0 otherwise) and assigns the  $i$ -th row  $M[i]$  as the reachability label of the  $i$ -th vertex. While this approach achieves a constant query time over the specification (*i.e.*,  $t_G = O(1)$ ), the label length is  $n_G$  and the construction time is  $O(\min\{m_G \times n_G, n_G^{2.376} \times \log n_G\})$ <sup>5</sup>.

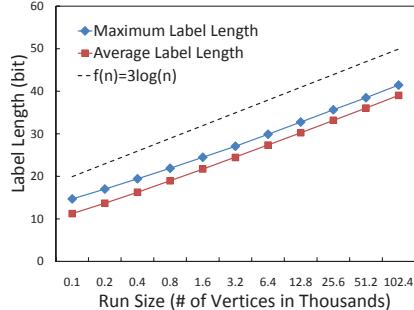
**BFS/DFS.** The other approach, called **BFS** or **DFS**, answers a reachability query in  $G$  by standard graph traversal in a **BFS** or **DFS** manner. Note that this approach may or may not be used as a labeling scheme. If it is used as a labeling scheme, then the entire graph  $G$  has to be coded in each label. Since no extra index structure is used, we can treat the label length and construction time to be zero, but the query time using **BFS** or **DFS** will be linear in terms of the size of the specification (*i.e.*,  $t_G = O(m_G + n_G)$ ).

The experimental results that follow show that even when the above two simple schemes are used, our skeleton-based labeling scheme is scalable to label a run with up to  $100K$  vertices. Links to more sophisticated labeling schemes for general graphs are given in Section 2. These schemes generally provide a better tradeoff between the efficiency of encoding and decoding, and therefore, if used to label the specification, can further improve the overall labeling performance.

<sup>5</sup>Using matrix multiplication [7], we can compute the transitive closure matrix in  $O(n_G^{2.376} \times \log n_G)$  time.

**Table 1: Characteristics of Real-life Scientific Workflows**

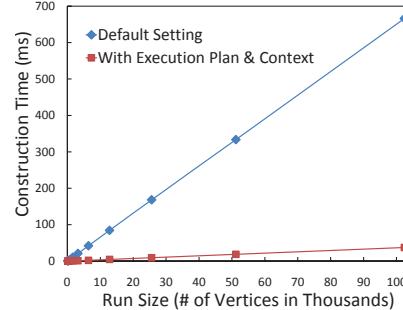
	$n_G$	$m_G$	$ T_G $	$[T_G]$
EBI	29	31	4	2
PubMed	35	45	3	3
QBLAST	58	72	6	3
BioAID	71	87	10	4
ProScan	89	119	9	4
ProDisc	111	158	9	3



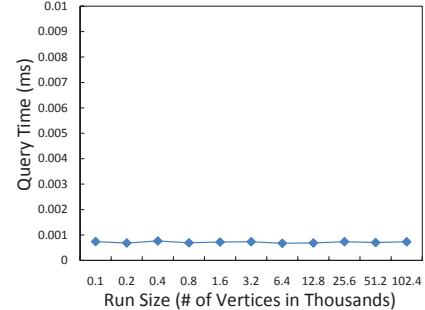
**Figure 12: Real Scientific Workflow (Label Length for QBLAST)**

**Table 2: Complexity Comparison (with Amortized Cost)**

	Label Length	Construction Time	Query Time
TCM+SKL	$3 \log n_R + \log n_G + \frac{n_G^2}{k \times n_R}$	$O(m_R + n_R + \frac{m_G \times n_G}{k})$	$O(1)$
BFS+SKL	$3 \log n_R + \log n_G$	$O(m_R + n_R)$	$O(m_G + n_G)$
TCM	$n_R$	$O(m_R \times n_R)$	$O(1)$
BFS	0	0	$O(m_R + n_R)$



**Figure 13: Real Scientific Workflow (Construction Time for QBLAST)**



**Figure 14: Real Scientific Workflow (Query Time for QBLAST)**

## 8. EXPERIMENTAL EVALUATION

All experiments were performed on a local PC with Intel Pentium 2.80GHz CPU and 2GB memory running Windows XP. All labeling schemes are implemented in Java. Our experiments are conducted on both real and synthetic datasets.

**Real Dataset.** We collected real-life scientific workflows from the myExperiment repository [14], which contains a large public collection of workflows across multiple workflow systems, including Taverna, Kepler and Triana.

**Synthetic Dataset.** We randomly generated synthetic workflows using the following parameters: (1)  $n_G$ : the number of vertices; (2)  $m_G$ : the number of edges; (3)  $|T_G|$ : the size of the fork and loop hierarchy (*i.e.*, the total number of forks and loops plus one); and (4)  $[T_G]$ : the depth of the fork and loop hierarchy.

To simulate the execution of a workflow, we randomly replicated each fork or loop one or more times. For each specification, we generated a set of runs, varying their size (*i.e.*, the number of vertices) from 0.1K to 102.4K by a factor of 2. Both the specification and runs are stored as XML files. In all experiments, the time to parse the XML file is omitted.

We apply the three labeling schemes (and their different combinations) discussed earlier in the paper: (1) TCM, which precomputes the transitive closure matrix; (2) BFS, which traverses the graph by BFS; and (3) SKL, which denotes our proposed skeleton-based labeling scheme. To evaluate their performance, we measured three parameters: label length, construction time and query time. In all results reported in this section, each point for label length and construction time is an average over  $10^3$  sample runs, and each point for query time is an average over  $10^6$  sample queries.

### 8.1 SKL Performance

In the first set of experiments, we measure the performance of SKL using six selected, real-life scientific workflows. Characteristics of these specifications are listed in Table 1. We first label the specification using TCM, and then label the

runs using SKL. Note that the cost of labeling the specification is omitted in this experiment. We only report the results for the QBLAST workflow. Results for other workflows are similar.

Figure 12 reports the maximum and average label length<sup>6</sup> for the QBLAST workflow. As expected, both increase logarithmically with the size of the run, and the average label length is always within a small constant of the maximum. More interestingly, as shown in the proof of Lemma 4.5, the tight upper bound of label length is  $3 \log n_T^+ + \log n_G$  (rather than  $3 \log n_R + \log n_G$ ), where  $n_T^+$  is the number of nonempty + nodes in the execution plan and  $n_T^+ \leq n_R$ . This is confirmed by Figure 12: The actual maximum label length is less than the asymptotic upper bound  $3 \log n_R$  (*i.e.*, the dotted line) by a small constant.

Figure 13 reports the construction time for the QBLAST workflow under two experiment settings. In the default setting, the run is given as a graph. In the other setting, we assume that the run is given along with its execution plan and context. Figure 13 shows that the construction time increases proportionally with the size of the run, confirming our linear time complexity result in Lemma 4.5. Comparing the two settings, we also observe that the main time cost of skeleton-based labeling lies in the computation of the execution plan and context. Hence, we can significantly speed up the labeling process if this run-time information is readily available. For example, in Taverna [9], the execution plan and context can be directly extracted from the system log.

Figure 14 reports the query time for the QBLAST workflow. Since the specification is labeled by TCM, it achieves a constant query time over the specification (*i.e.*,  $t_G = O(1)$  in Lemma 4.5). Hence, the overall query time over the run is also constant, which is confirmed by Figure 14.

To conclude, this experiment empirically validates that SKL guarantees logarithmic label length, linear construction time, and constant query time.

<sup>6</sup>The average length is measured only for the variable-size labels.

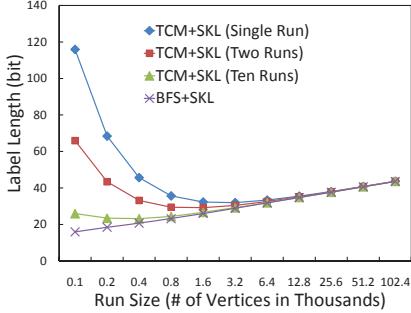


Figure 15: TCM+SKL vs BFS+SKL (Label Length)

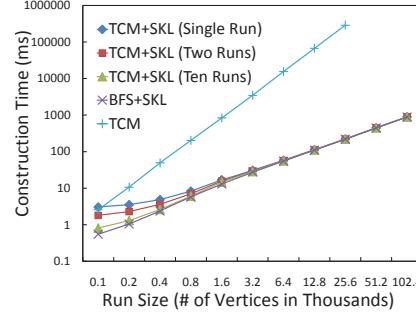


Figure 16: TCM+SKL vs BFS+SKL (Construction Time)

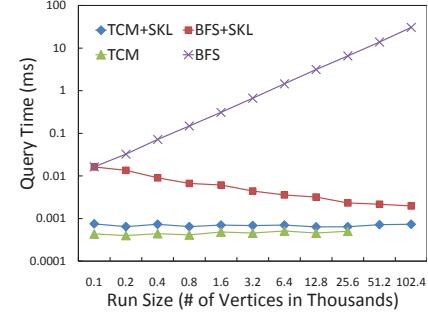


Figure 17: TCM+SKL vs BFS+SKL (Query Time)

## 8.2 TCM + SKL vs BFS + SKL

In the second set of experiments, we compare different combinations with **SKL**: (1) TCM+SKL and (2) BFS+SKL, which first label the specification using **TCM** or **BFS** and then label the run using **SKL**. To demonstrate the scalability, we also compare them with two extreme approaches: (3) TCM and (4) BFS, which label the run using TCM or BFS directly. Note that to obtain a fair comparison, we count the initial cost (both storage space and construction time) of labeling a specification, and amortize this cost over all the runs. We analyze the complexity of these four labeling schemes (with amortized cost) in Table 2, where  $k$  is the number of runs. This experiment was conducted on a synthetic workflow with  $n_G = 100$ ,  $m_G = 200$ ,  $|T_G| = 10$  and  $[T_G] = 4$ . We measured the amortized cost over a single run, two runs and ten runs respectively.

Figure 15 reports the maximum label length (with amortized cost) for TCM+SKL and BFS+SKL. As expected, BFS+SKL builds much shorter labels than TCM+SKL for small runs, but when the run becomes large, they perform equally well. This is because, as shown in Table 2, the amortized storage cost for TCM+SKL is  $n_G^2/(k \times n_R)$ . It dominates the total label length for small runs, but as the run becomes large, quickly decreases to zero. Comparing the three curves for TCM+SKL, we also observe that this amortized cost can be significantly reduced, if the skeleton labels are reused by many runs. Note that the label length for BFS+SKL is not affected by the number of runs. We see a simple logarithmic increase for BFS+SKL, confirming our complexity result in Table 2.

Figure 16 reports the construction time (with amortized cost) for TCM+SKL and BFS+SKL, and shows a similar result to Figure 15. Also observe that both TCM+SKL and BFS+SKL label the runs faster than TCM<sup>7</sup> by several orders of magnitude. Note that TCM has a polynomial time complexity (X and Y axes use a logarithmic scale).

Figure 17 reports the query time for all four labeling schemes. Not surprisingly, both TCM+SKL and TCM achieve a truly constant query time. Note that TCM+SKL is slightly slower than TCM, due to a more complex decoding step. In contrast, BFS+SKL is slower than TCM+SKL, especially for small runs; BFS suffers from a linear query time, and is slower than the other three approaches by several orders of magnitude. More interestingly, we can observe a slightly decreasing query time for BFS+SKL. This counter-intuitive behavior is explained as follows: As discussed in Section 7, we may

avoid the expensive graph search on the specification if the reachability between two vertices can be immediately determined by their context encodings, *i.e.*, if the vertices are dominated by two distinct copies of the same fork or loop. Since large runs may have more fork and loop copies, the chance to avoid the expensive graph search increases with the size of the run. In particular, when  $n_R = n_G$ , we have to search the specification graph for each query (there is no chance). This results in exactly the same performance as BFS, which is confirmed by Figure 17.

To conclude, **SKL** (combined with either TCM or BFS) is scalable to large runs: For a run with 102.4K vertices, it builds labels shorter than 50 bits in 1s, and answers a query in 0.01ms<sup>8</sup>. The comparison between TCM+SKL and BFS+SKL shows that there is no clear winner, and only a small tradeoff between the efficiency of encoding and decoding. In particular, when labeling large runs, **SKL** is insensitive to the quality of the labeling scheme used to label the specification.

## 8.3 Influence of Specification

In the last set of experiments, we evaluate the influence of specifications on the performance of **SKL**. Among all parameters of a specification, we focus only on  $n_G$ . We generated three synthetic workflows by setting  $n_G = 50$ , 100 and 200 respectively and fixing  $m_G/n_G = 2$ ,  $|T_G| = 10$  and  $[T_G] = 4$ . Note that we only measured the label length and construction time (both with amortized cost over 2 runs) for TCM+SKL and the query time for BFS+SKL; As shown in Table 2, only those parameters are significantly affected by  $n_G$ .

Figure 18 reports the maximum label length (with amortized cost) for TCM+SKL. We can observe that the smaller specification results in much shorter labels for small runs, but slightly longer labels for large runs. This is because for small runs, the amortized storage cost for skeleton labels (*i.e.*,  $n_G^2/(k \times n_R)$ ) dominates the total label length. Clearly, the smaller specification uses shorter skeleton labels. In contrast, for large runs, the context encoding (*i.e.*,  $3 \log n_R$ ) dominates the total label length. Since the smaller specification may contain smaller forks and loops, to obtain a run with the same size, we must replicate more fork and loop copies. Hence, the run of a smaller specification may have a larger execution plan, and therefore is labeled by larger numbers as the context encoding.

Figure 19 reports the construction time (with amortized cost) for TCM+SKL, and shows a similar result to Figure 18.

<sup>7</sup>In our experiments, TCM is only scalable to runs with no more than 25.6K vertices, due to memory constraints.

<sup>8</sup>In real database systems, the query time may be much longer due to the slow data loading from the database.

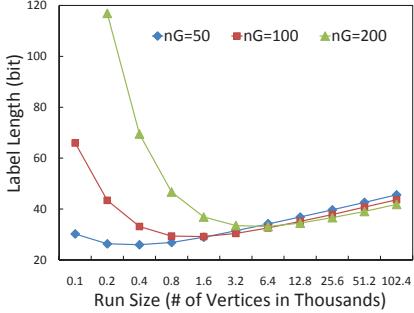


Figure 18: Influence of Specification (Label Length for TCM+SKL)

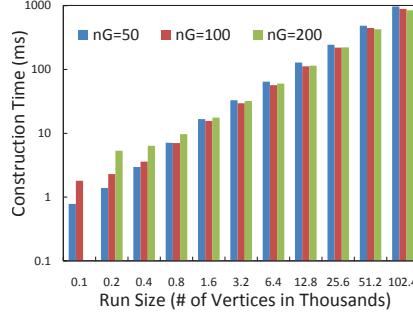


Figure 19: Influence of Specification (Construction Time for TCM+SKL)

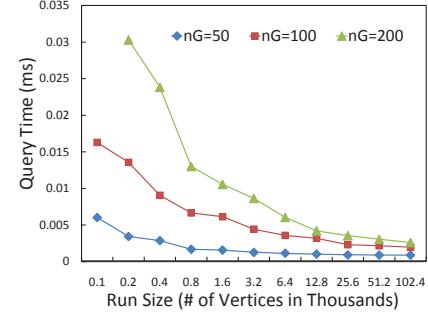


Figure 20: Influence of Specification (Query Time for BFS+SKL)

Figure 20 reports the query time for BFS+SKL. As expected, the query time increases with the size of the specification, since the main query cost lies in the graph search over the specification. Clearly, a smaller specification allows a faster graph search. We can also observe that the query time decreases with increasing size of the run (similar to Figure 17). When the run becomes large, the three specifications achieve very close query times.

To conclude, when the size of the run is close to the size of the specification, SKL works significantly better with smaller specifications; but for large runs, the size of specification has a very weak influence on the overall performance of SKL.

## 9. CONCLUSIONS

In this paper, we present a skeleton-based labeling scheme for workflows with well-nested forks and loops, which uses a labeled specification to efficiently create compact reachability labels for large runs. For any fixed specification, our scheme for labeling runs is optimal in the sense that it uses logarithmic length labels, takes linear construction time, and answers queries in constant time, even though runs can have arbitrarily complex network structure. The experimental results demonstrate the effectiveness of our approach.

An interesting direction for future work is to design efficient and compact *dynamic* or *online* labeling schemes, so that data can be labeled and stored in a database along with its label as soon as it is generated. Given that scientific workflows can take a long time to execute, this would enable efficient provenance queries on intermediate data results even before the workflow completes.

## 10. ACKNOWLEDGMENT

We thank the anonymous reviewers for their helpful comments. This work was supported in part by US National Science Foundation grants IIS-0803524 and IIS-0629846.

## 11. REFERENCES

- [1] S. Abiteboul, H. Kaplan, and T. Milo. Compact labeling schemes for ancestor queries. In *SODA*, pages 547–556, 2001.
- [2] R. Agrawal, A. Borgida, and H. V. Jagadish. Efficient management of transitive relationships in large data and knowledge bases. In *SIGMOD Conference*, pages 253–262, 1989.
- [3] S. Alstrup and T. Rauhe. Improved labeling scheme for ancestor queries. In *SODA*, pages 947–953, 2002.
- [4] A. Chapman, H. V. Jagadish, and P. Ramanan. Efficient provenance storage. In *SIGMOD Conference*, pages 993–1006, 2008.
- [5] L. Chen, A. Gupta, and M. E. Kurul. Stack-based algorithms for pattern matching on dags. In *VLDB*, pages 493–504, 2005.
- [6] E. Cohen, E. Halperin, H. Kaplan, and U. Zwick. Reachability and distance queries via 2-hop labels. In *SODA*, pages 937–946, 2002.
- [7] D. Coppersmith and S. Winograd. Matrix multiplication via arithmetic progressions. In *STOC*, pages 1–6, 1987.
- [8] T. Heinis and G. Alonso. Efficient lineage tracking for scientific workflows. In *SIGMOD Conference*, pages 1007–1018, 2008.
- [9] D. Hull, K. Wolstencroft, R. Stevens, C. A. Goble, M. R. Pocock, P. Li, and T. Oinn. Taverna: a tool for building and running workflows of services. *Nucleic Acids Research*, 34(Web-Server-Issue):729–732, 2006.
- [10] H. V. Jagadish. A compression technique to materialize transitive closure. *ACM Trans. Database Syst.*, 15(4):558–598, 1990.
- [11] R. Jin, Y. Xiang, N. Ruan, and D. Fuhr. 3-hop: a high-compression indexing scheme for reachability query. In *SIGMOD Conference*, pages 813–826, 2009.
- [12] R. Jin, Y. Xiang, N. Ruan, and H. Wang. Efficiently answering reachability queries on very large directed graphs. In *SIGMOD Conference*, pages 595–608, 2008.
- [13] H. Kaplan, T. Milo, and R. Shabo. A comparison of labeling schemes for ancestor queries. In *SODA*, pages 954–963, 2002.
- [14] D. D. Roure, C. A. Goble, and R. Stevens. The design and realisation of the myexperiment virtual research environment for social sharing of workflows. *Future Generation Comp. Syst.*, 25(5):561–567, 2009.
- [15] N. Santoro and R. Khatib. Labelling and implicit routing in networks. *Comput. J.*, 28(1):5–8, 1985.
- [16] M. Thorup and U. Zwick. Compact routing schemes. In *SPAA*, pages 1–10, 2001.
- [17] S. Trißl and U. Leser. Fast and practical indexing and querying of very large graphs. In *SIGMOD Conference*, pages 845–856, 2007.
- [18] H. Wang, H. He, J. Yang, P. S. Yu, and J. X. Yu. Dual labeling: Answering graph reachability queries in constant time. In *ICDE*, page 75, 2006.