Unit 4
Image Restoration

Image Restoration

The Image Degradation / Restoration Process,

Noise Model based Restoration,

Spatial filtering,

Periodic Noise Reduction by Frequency Domain Filtering,

Inverse filtering,

Wiener filtering,

Geometric Mean Filter

Previously in unit 3:

Image Transformation

An image is obtained in spatial coordinates **(x, y) or (x, y, z).** There are many advantages if the spatial domain image is transformed into another domain. In which solution of any problem can be found easily.

Following are two types of transformations:

Fourier Transform

Discrete Cosine Transformation (DCT)

1. Fourier Transform

- Fourier transform is mainly used for image processing.
- In the Fourier transform, **the intensity of the image is transformed into frequency variation** and then to the frequency domain.

- It is used for slow varying intensity images such as the background of a **passport size photo can be represented as low-frequency components** and the **edges can be represented as high-frequency components**.

- Low-frequency components can be **removed** using filters of FT domain. When an image is filtered in the FT domain, it contains only the edges of the image. And if we do inverse FT domain to spatial domain then also an image contains only edges.

- Fourier transform is the **simplest technique in which edges** of the image can be fined.

Two-Dimensional Fourier Transform

$$v(k,l)=\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}u(m,n)W_N^{km}W_N^{ln}, \quad W_N \triangleq \exp\left(\frac{-j2\pi}{N}\right)$$

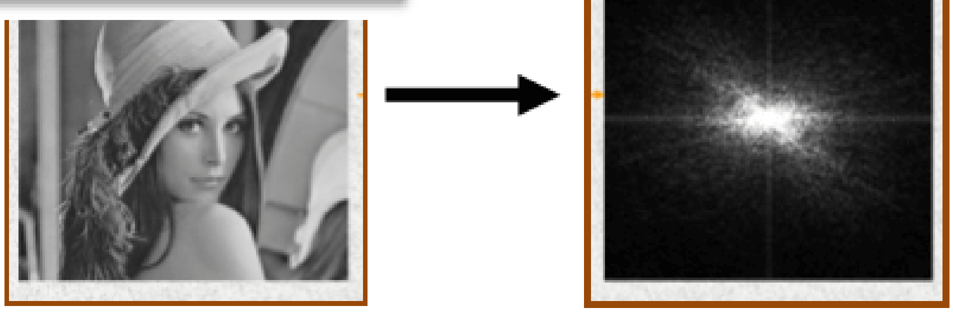
$$u(m,n)=\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}v(k,l)W_N^{-km}W_N^{-ln}$$

$$\left.\begin{aligned} v(k,l)&=\frac{1}{N}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}u(m,n)W_N^{km}W_N^{ln} \\ u(m,n)&=\frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}v(k,l)W_N^{-km}W_N^{-ln} \end{aligned}\right\}\text{Unitary DFT Pair}$$

Matrix Notation:

$$\mathbf{V}=\mathbf{FUF} \Leftrightarrow \mathbf{U}=\mathbf{F}^{*}\mathbf{VF}^{*}$$

$$\mathbf{F}=\left\{\frac{1}{\sqrt{N}}W_N^{kn}\right\}_{k,n=0}^{N-1}$$



Fourier transformation of the image

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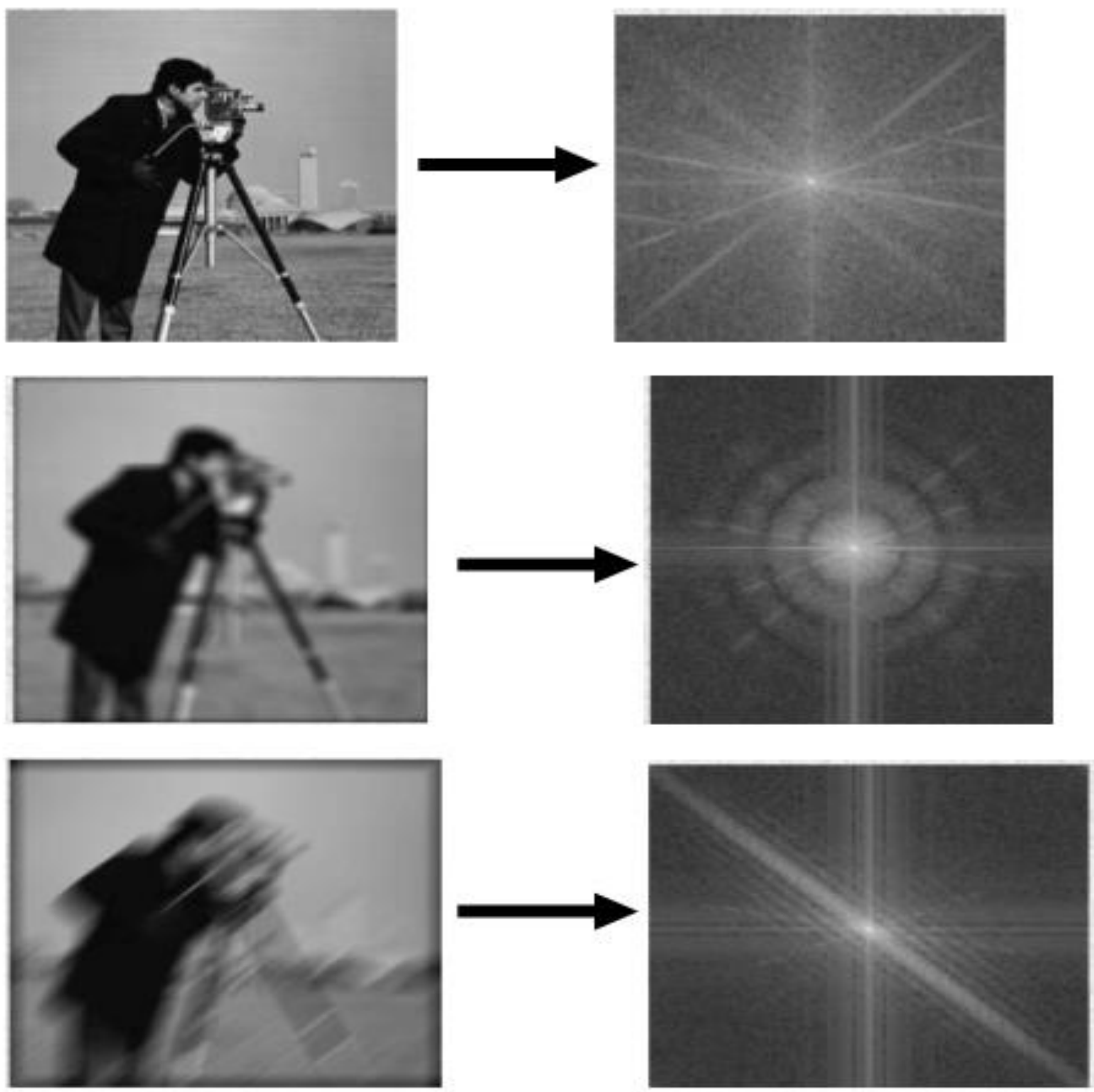
Properties of Fourier transformation

- Linearity
- Time Shifting
- Frequency Shifting
- Scaling
- **Convolution**
- Multiplication
- Duality
- Parseval’s Theorem
- Differentiation
- Integration



Example of Blurred image and its Fourier transformation

2. Discrete Cosine Transformation (DCT)

- In Discrete Cosine Transformation, **coefficients carry information about the pixels of the image**.

- Also, much information is contained using very few coefficients, and the remaining coefficient contains minimal information. These coefficients **can be removed without losing information**. By doing this, the file size is reduced in the DCT domain. DCT is used for lossy compression.

$$\mathbf{C} = \{c(k,n)\}$$

$$c(k,n) = \begin{cases} \dfrac{1}{\sqrt{N}} & k = 0, 0 \le n \le N-1 \\ \dfrac{2}{\sqrt{N}} \cos\left(\dfrac{\pi(2n+1)k}{2N}\right) & 1 \le k \le N-1, 0 \le n \le N-1 \end{cases}$$

$$v(k) = \alpha(k) \sum_{n=0}^{N-1} u(n) \cos\left(\frac{\pi(2n+1)k}{2N}\right), \quad 0 \le k \le N-1$$

$$\alpha(0) \triangleq \frac{1}{\sqrt{N}}, \quad \alpha(k) \triangleq \frac{2}{\sqrt{N}}, \quad 1 \le k \le N-1$$

$$u(n) = \sum_{k=0}^{N-1} \alpha(k) v(k) \cos\left(\frac{\pi(2n+1)k}{2N}\right), \quad 0 \le n \le N-1$$

One Dimension Discrete cosine transformation:

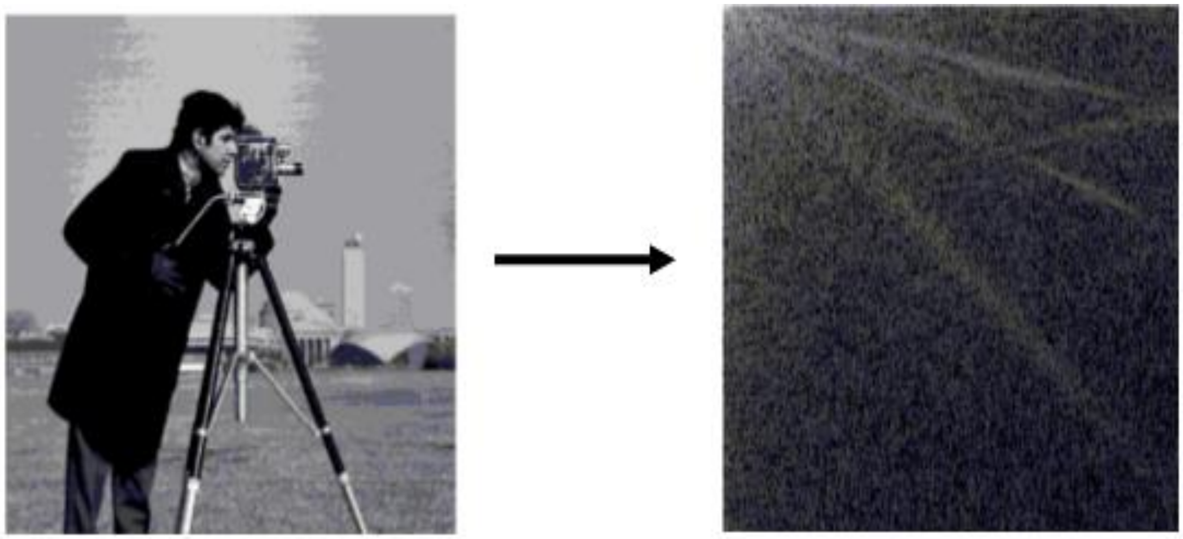
Two Dimension Discrete cosine transformations:

$$\boxed{-\ \mathbf{A} = \mathbf{A}^* = \mathbf{C}}$$

Properties of Discrete cosine transformation are as following:

- Real and Orthogonal: C = C* → C-1=CT
- Fast Transform
- Excellent Energy Compaction (Highly Correlated Data)

Example:



Applications of image transforms are as follows:

- Fourier transform is used for Edge Detection.
- Discrete Cosine Transform is used for image compression.

Before starting let's look this image and block diagram.

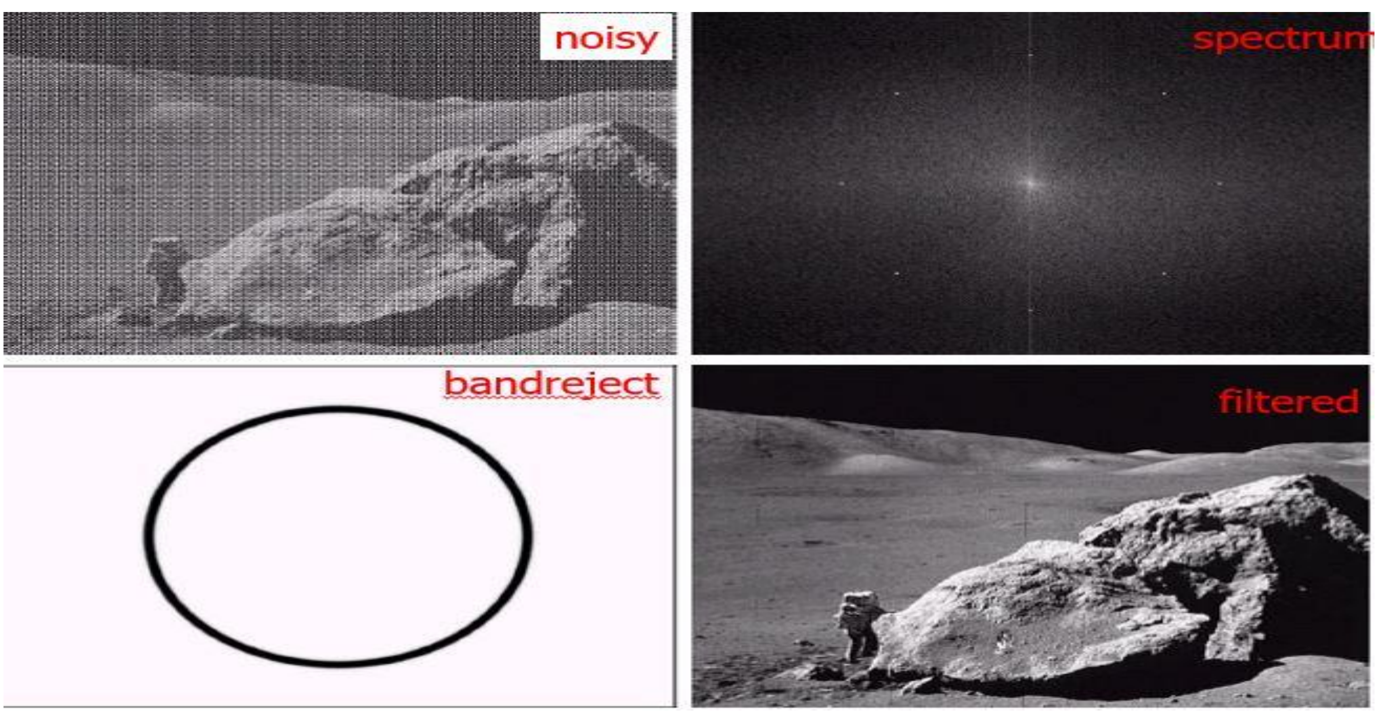
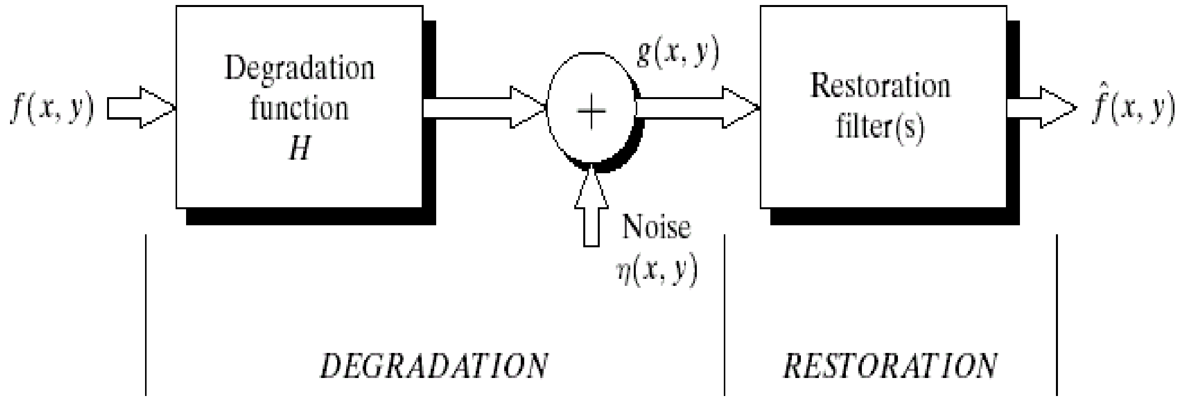


Fig. A model of the image degradation/restoration process

In contrast to image enhancement, in image restoration the degradation is modelled. This enables the effects of the degradation to be (largely) removed.

$$g(x,y)= f(x,y)*h(x,y)+h(x,y)$$



- So now we discussed about the images in frequency domain, we are going to define a ***relationship between frequency domain and the images (spatial domain).***
- **And some terminologies**

Concept of Convolution

- Convolution is used for many things like calculating derivatives, **detect edges, apply blurs etc**. and all this is done using a "**convolution kernel**".
- A convolution kernel is a very small matrix and, in this matrix, each cell has a number and also **an anchor point**.

0	**-2**	**0**
-1	**0** anchor	**-1**
0	**-2**	**0**

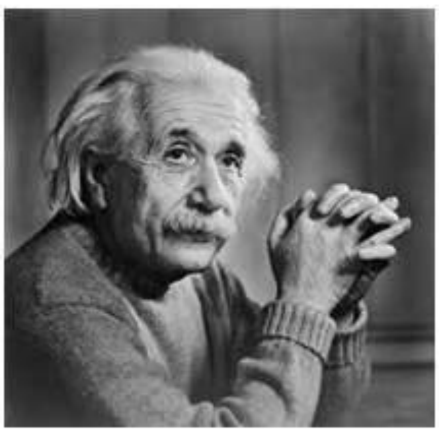
- The anchor point is used to know the **position of the kernel** with respect to the image. It starts at the top left corner of the image and moves on each pixel **sequentially**.

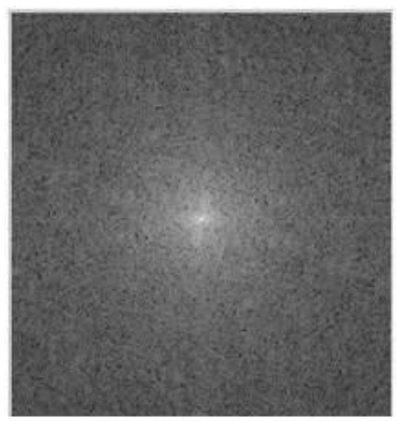
- Kernel overlaps few pixels at each position on the image. Each pixel which is overlapped is multiplied and then added. And the sum is set as the value of the current position.

- Convolution is the process in which each element of the image is added to its **local neighbors**, and then it is **weighted by the kernel**. It is related to a form of mathematical convolution.
- In Convolution, the matrix does not perform **traditional matrix multiplication** but it is denoted by *.

For example

Consider this example. The same image in can be represented as the frequency domain





Now what's the relationship between image or **spatial domain and frequency domain**. This relationship can be explained by a theorem which is called as ***Convolution theorem.***

Convolution Theorem

- The Convolution Theorem states that convolution in the **time domain is equivalent to multiplication in the frequency domain.**

- This theorem is extremely useful in signal processing because it allows the use of the Fourier Transform to simplify convolution operations.

- The **relationship between the spatial domain and the frequency domain** can be established by convolution theorem.

The convolution theorem can be represented as.

$$f(x,y)*h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y) \Leftrightarrow F(u,v)*H(u,v)$$

$$h(x,y) \Leftrightarrow H(u,v)$$

It can be stated as the **convolution in spatial domain is equal to filtering in frequency domain and vice versa.**

The filtering in frequency domain can be represented as following:

The steps in filtering are given below.

1. At first step we have to do some pre – processing an image in spatial domain, means increase **its contrast or brightness**

2. Then we will take **discrete Fourier transform** of the image

3. Then we will center the discrete Fourier transform, as we will bring the discrete Fourier transform in center from corners

4. Then we will apply filtering, means we will multiply the Fourier transform by a filter function

5. Then we will again shift the DFT from center to the corners
6. Last step would be taken to **inverse discrete Fourier** transform, to bring the result back **from frequency domain to spatial domain**
7. And this step of post processing is optional, just like preprocessing, in which we just increase the appearance of image.

Filters

- The concept of filter in frequency domain is same as the concept of a **mask in convolution**.
- Filters in image processing are used to ***enhance images or extract useful information by emphasizing or suppressing certain features***.
- Various types of filters exist, each with specific applications and characteristics.
- After converting an image to frequency domain, some filters are applied in filtering process to perform different kind of processing on an image. The processing includes **blurring an image, sharpening an image** etc.
- The common type of filters for these purposes are:
  - **Ideal high pass filter**
  - **Ideal low pass filter**
  - **Gaussian high pass filter**
  - **Gaussian low pass filter**
  - **Wiener filter**

Here are some key filters and concepts used in image processing:

Spatial Filtering

Spatial filtering involves directly manipulating the pixel values of an image in the spatial domain. Common spatial filters include:

- **Averaging Filter (Mean Filter)**: Reduces noise by averaging the pixel values in a neighborhood.
- **Median Filter**: Reduces noise by replacing each pixel value with the median value of the neighboring pixels.
- **Gaussian Filter**: Applies a Gaussian function to blur the image, useful for noise reduction.
- **Sobel Filter**: Detects edges by calculating the gradient of the image intensity.

- **Laplacian Filter**: Highlights regions of rapid intensity change, useful for edge detection.

2. Frequency Domain Filtering

Frequency domain filtering involves transforming the image into the frequency domain using the Fourier Transform, manipulating the frequency components, and transforming back to the spatial domain. Common techniques include:

- **Low-Pass Filter**: Allows low-frequency components to pass through, reducing high-frequency noise and blurring the image.
- **High-Pass Filter**: Allows high-frequency components to pass through, enhancing edges and fine details.
- **Band-Pass Filter**: Allows a specific range of frequencies to pass through, useful for texture analysis.

Original image

ideal lowpass

Original image

Gaussian highpass

Butterworth highpass

3. Noise Model Based Restoration

Noise model-based restoration techniques aim to reconstruct the original image by modeling and removing the noise. Common approaches include:

- **Additive Noise Model**: Assumes noise is added to the original image.
- **Multiplicative Noise Model**: Assumes noise multiplies with the original image.
- **Impulse Noise Model**: Models noise as random spikes in pixel values (e.g., salt-and-pepper noise).

4. Periodic Noise Reduction by Frequency Domain Filtering

Periodic noise, such as electrical interference, appears as periodic patterns in the frequency domain. It can be reduced by:

- **Notch Filters**: Attenuate specific frequency components associated with the periodic noise.
- **Band-Stop Filters**: Attenuate a range of frequencies around the noise frequencies.

5. Inverse Filtering

Inverse filtering attempts to reverse the effects of blurring by applying the inverse of the blurring function. It is sensitive to noise, making it less effective in practical scenarios.

6. Wiener Filtering

Wiener filtering is an optimal filter that minimizes the mean square error between the estimated and the original image. It considers both the degradation function and the statistical properties of the noise.

7. Geometric Mean Filter

The geometric mean filter is used to reduce multiplicative noise. It calculates the geometric mean of the pixel values in a neighborhood, providing a balance between noise reduction and edge preservation.

Numericals

Q. Consider the following image what will be the new value of the pixel (2,2) if smoothing is done using a 3x3 neighborhood using
- (a) mean filter
- (b) weighted average filter
- (c) median "
- (d) min "
- (e) max "

0	1	0	2	7
2	7	7	4	0
5	6	4	3	3
1	1	0	7	5
5	4	2	2	5

Sol^n

Pixel (2,2) = 4

(a) 3x3 neighborhood is

7	7	4
6	4	3
1	0	7

3x3

(a) mean filter, $= \frac{7+7+4+6+4+3+1+0+7}{9}$

$= 39/9 = 4.33 \approx 4$

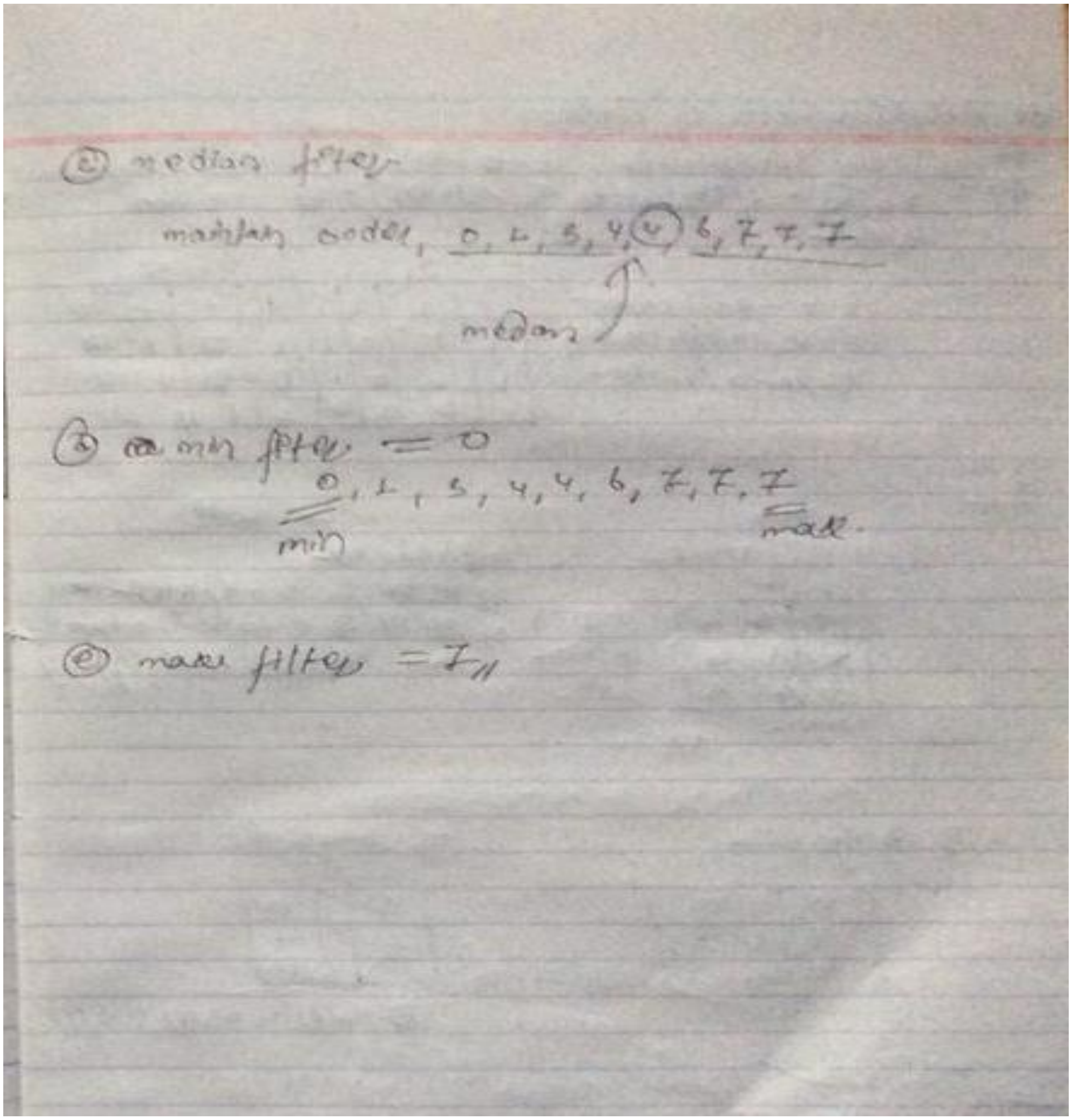
(b) weighted average filter.

$= \frac{1}{9}\left[(7\times1)+(7\times1)+(4\times1)+(6\times1)+(4\times2)+(3\times1)+(1\times1)+(0\times1)+(1\times7)\right]$

let's structure element

1	1	1
1	2	1
1	1	1

$= \frac{1}{9}\times 43 = 4.77 \approx 5$



(c) median filter-

arranging order, 0, 1, 3, 4, (4) 6, 7, 7, 7

median

(d) min filter = 0

0, 1, 3, 4, 4, 6, 7, 7, 7

min max

(e) max filter = 7

Consider a 3x3 image matrix for simplicity: [Incomplete]

$$I = \begin{bmatrix} 10 & 10 & 80 \\ 20 & 50 & 60 \\ 30 & 40 & 70 \end{bmatrix}$$

Averaging Filter (Mean Filter)

An averaging filter (mean filter) with a 3x3 kernel:

$$K = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Convolution with Averaging Filter:

$$I * K = \left(\frac{1}{9}\right) * \begin{bmatrix} 10 & 10 & 80 \\ 20 & 50 & 60 \\ 30 & 40 & 70 \end{bmatrix}$$

For the center pixel (50):

Icenter = (1 / 9) * (10+10+80+20+50+60+30+40+70)

= (1 / 9) * 370 = 41.1

Median Filter

Apply a 3x3 median filter to the same image matrix. The filter sorts the neighborhood pixel values and selects the median.

- For the center pixel (50):

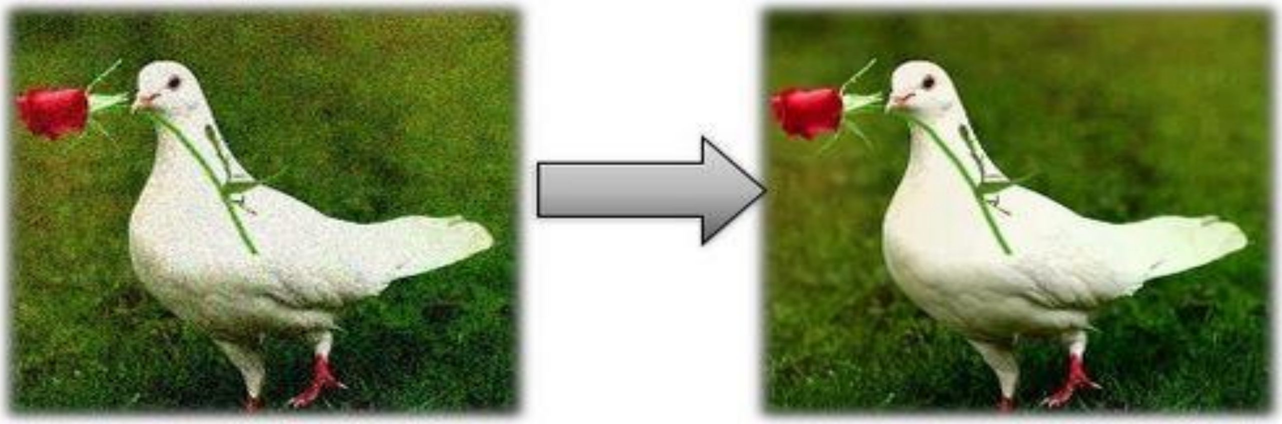
  Neighborhood: {10,10,80,20,50,60,30,40,70}

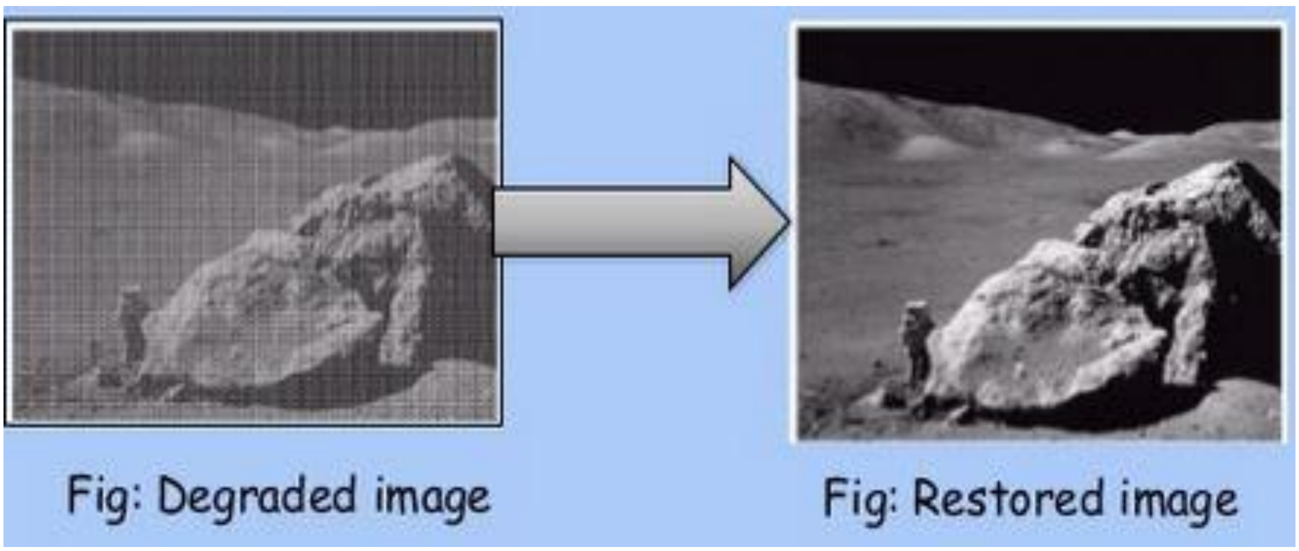
  Sorted: {10,10,20,30,40,50,60,70,80}

  Median: 50

What is image restoration?

Image Restoration is the operation of taking a **corrupt/noisy image** and estimating the clean, original image.





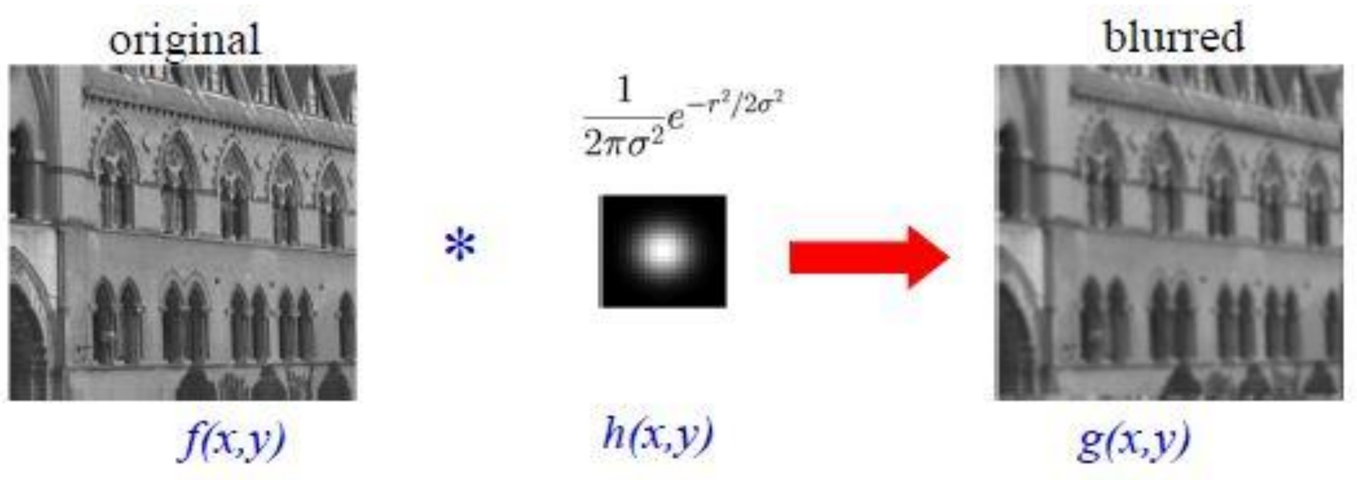
- Corruption may come in many forms such as **motion blur, noise and camera mis-focus**.
- Image restoration is performed by **reversing the process** that blurred the image and such is performed by imaging a point source and use the point source image, which is called the **Point Spread Function (PSF)** to restore the image information lost to the blurring process.
- Identify the degradation process and attempt to reserve it.
- It's similar to image enhancement, but more objective.

Image enhancement vs Image restoration:

- Image restoration assumes a degradation model that is known or can be estimated.
- Original content and quality do not mean good looking appearance.
- Image enhancement is subjective process, whereas restoration is objective process.
- Image restoration try to recover original image from degraded with prior knowledge of degradation process.
- Restoration involves modeling of degradation and applying the inverse process to recover the original image.
- Although the restore image is not the original image, its approximation of actual image.

Image Degradation Model

Model degradation as a convolution with a linear, shift invariant, filter *h(x,y)*

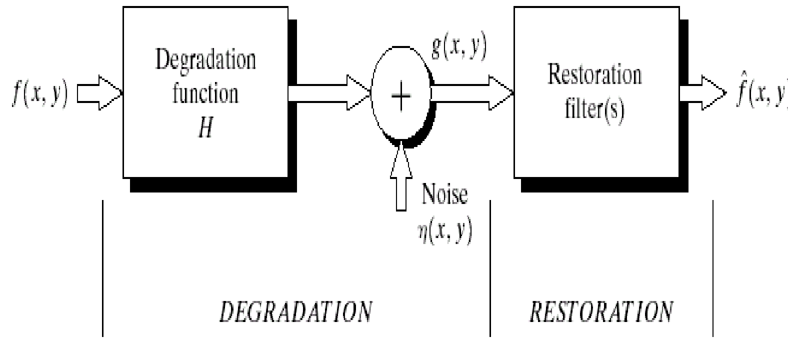


Example: for out of focus blurring, model *h(x,y)* as a Gaussian

$$\text{i.e.: } g(x,y) = h(x,y) * f(x,y)$$

h(x,y) is the impulse response or point spread function of the imaging system

Derivation:



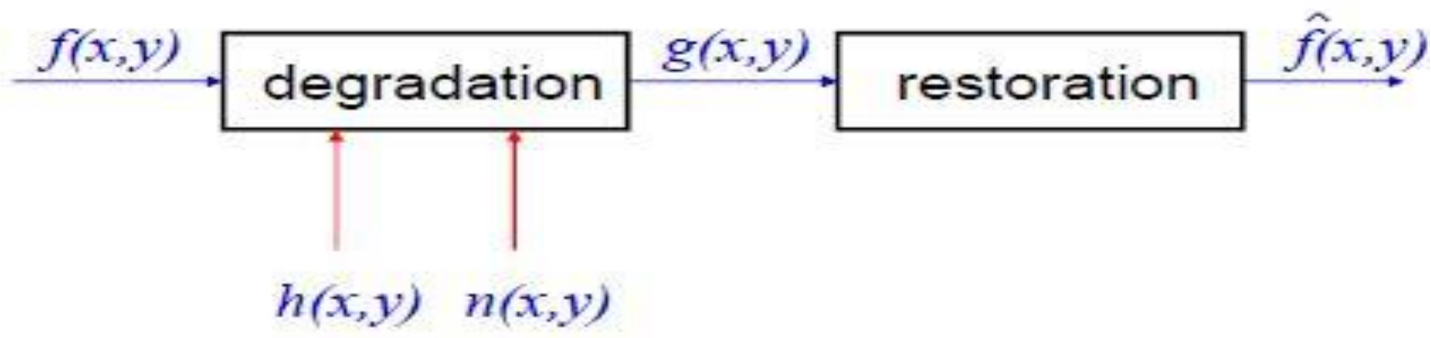
f(x,y) – image before degradation, 'true image'

g(x,y) – image after degradation, 'observed image'

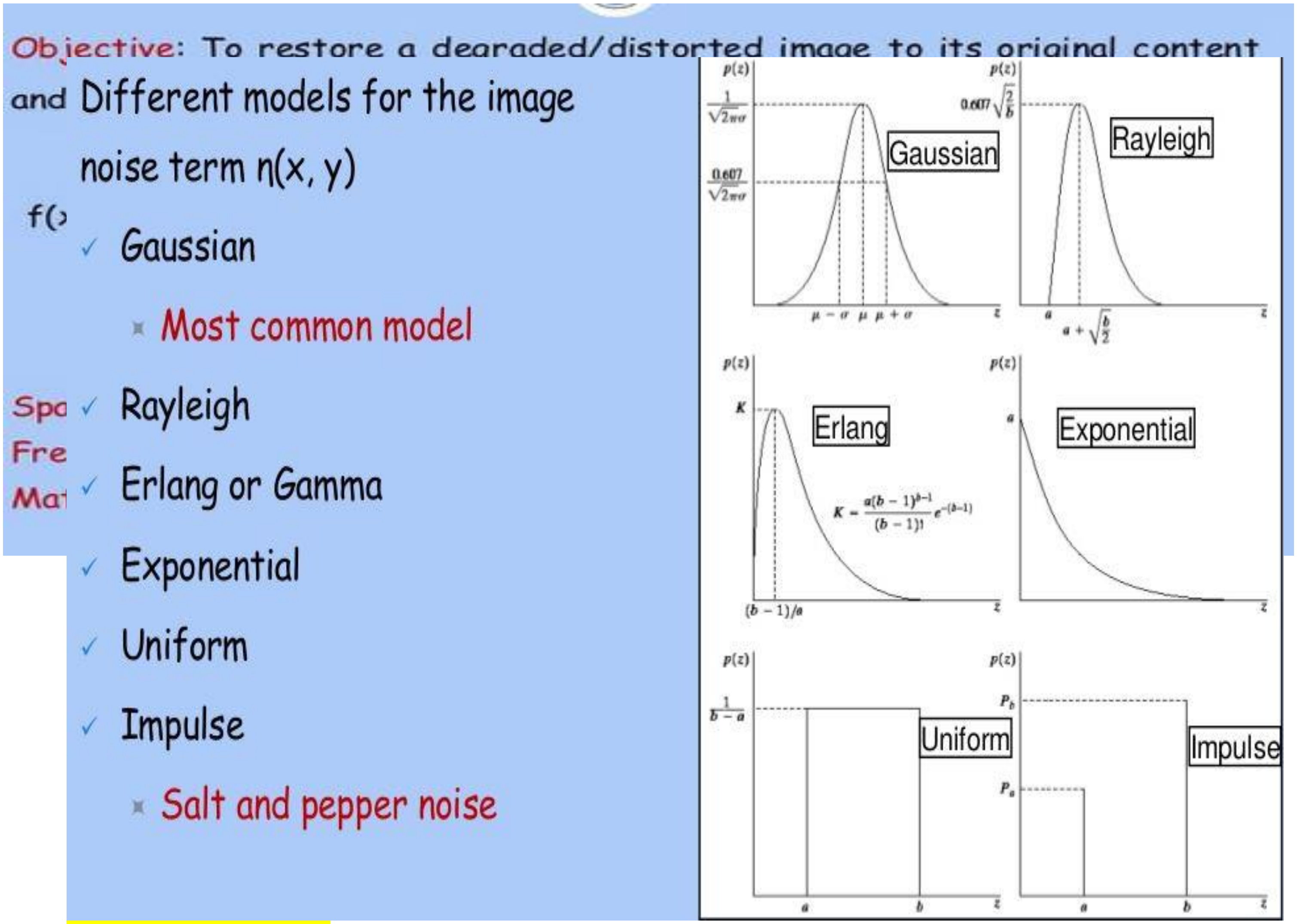
h(x,y) – degradation filter

f(x,y) – estimate of *f(x,y)* computed from *g(x,y)*

n(x,y) – additive noise



$$g(x,y) = h(x,y) * f(x,y) + n(x,y) = G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$



Noise Modes:

- The principal sources of noise in digital images arise during image acquisition and transmission.
- Most types of noise are modeled as probability density functions (PDFs) represented as p(z) for gray levels z.
- Parameters can be estimated based on histogram on small flat area of an image.

Estimation of Degradation Model:

Weather the spatial or frequency domain or Matrix, in all cases knowledge of degradation function is important.

Estimation of **H** is important in image restoration.

There are mainly three ways to estimate the **H** as follows:

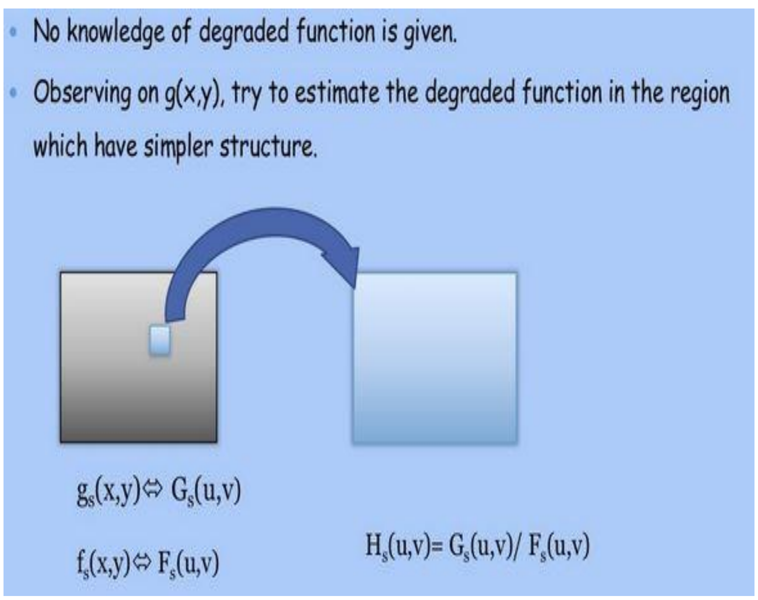
By Observation

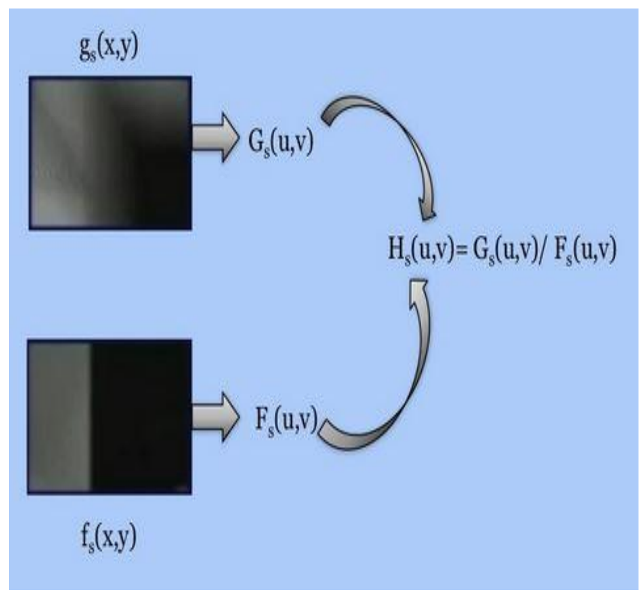
By Experimentation

Mathematical Modeling

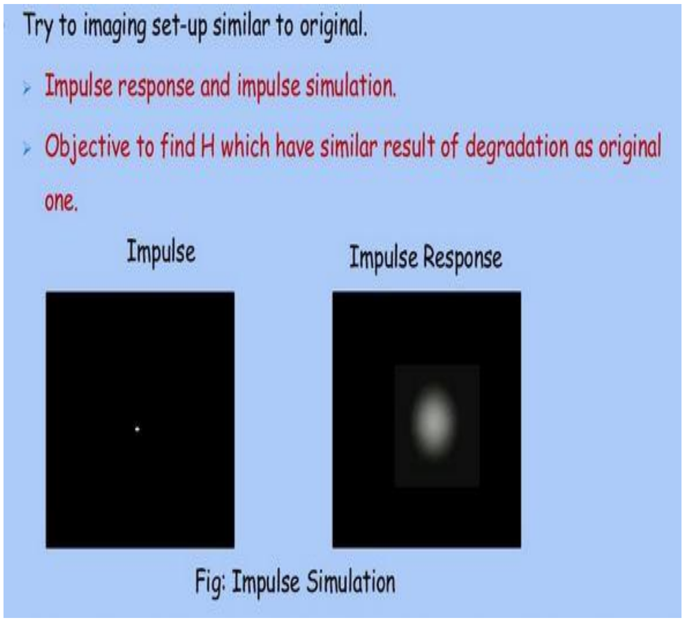
After the approximate the degradation function, we apply the **BLIND CONVULOTION** to restore the image.

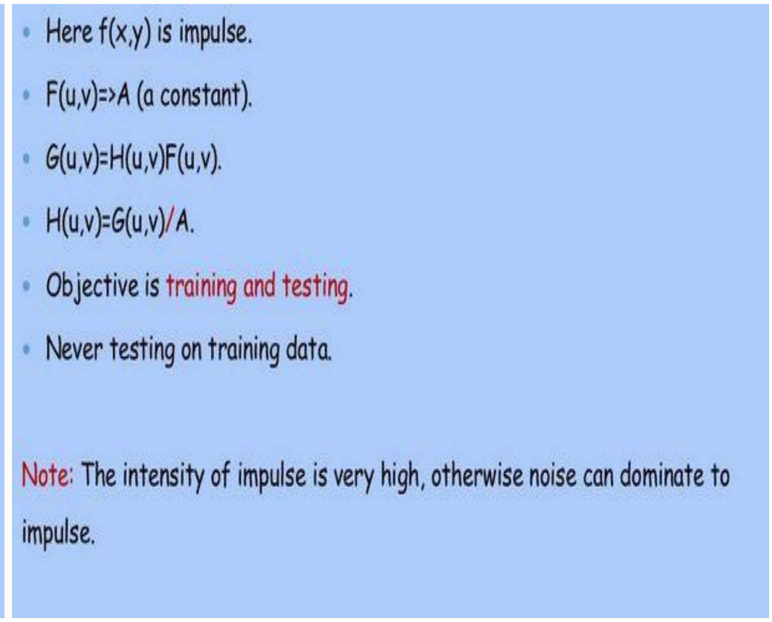
Observation …………………………………….



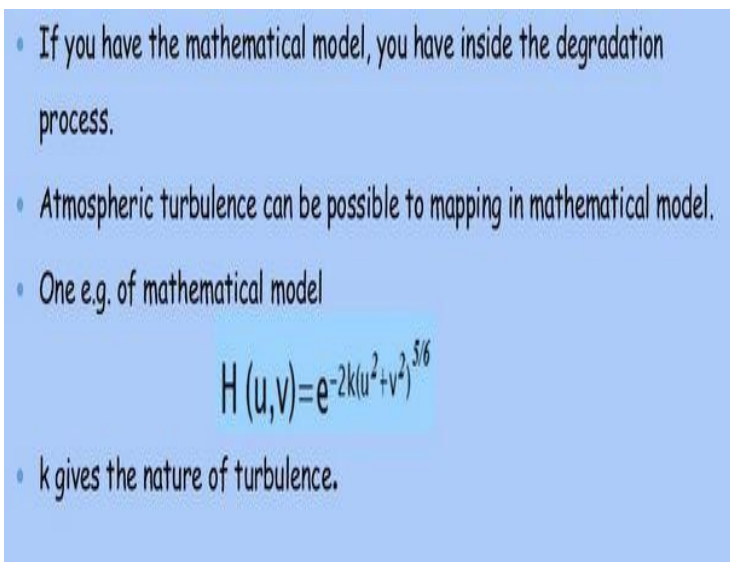


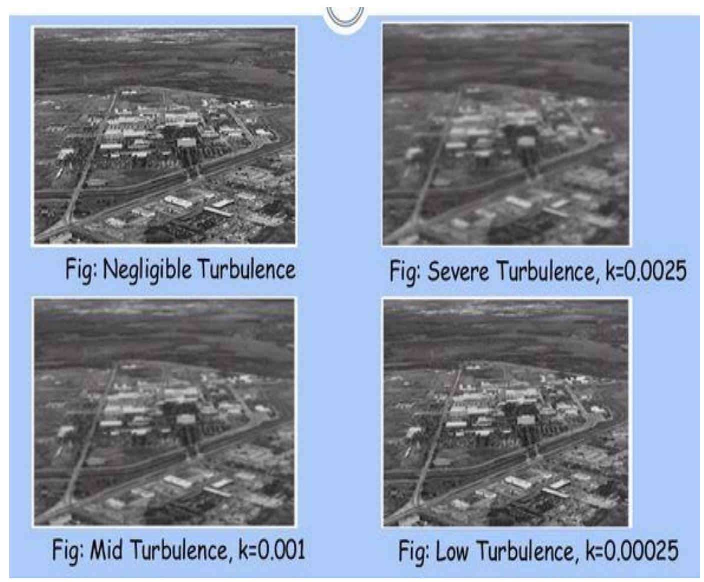
Experimentation……………………………….





Mathematical Modeling……………………………………..





Restoration Techniques:

1. Inverse Filtering
2. Minimum Mean Square Errors
   - Weiner Filtering
3. Constrained Least Square Filter
4. Non Linear filtering
5. Advance Restoration Technique