

Cobweb Model

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In
Cobweb market situation

the output decision of the producer
depends on the price of the previous period

and

the demand for products depends on
the price of the current period.



Therefore, in Cobweb market model

$$Q_d = a - bP_t \quad [a, b > 0]$$

$$Q_s = -c + dP_{t-1} \quad [c, d > 0]$$

$$Q_d = Q_s$$

Q_d = quantity demand; P_t = price of the current period

Q_s = quantity supply P_{t-1} = price of the previous period

'b' is the slope of the demand function.

'd' is the slope of the supply function.

$$Q_d = a - bP_t \quad [a, b > 0] \quad \dots \dots \dots (1)$$

$$Q_s = -c + dP_{t-1} \quad [c, d > 0] \quad \dots \dots \dots \quad (2)$$

Substituting equation (1) and (2) in equation (3) we get,

$$a - bP_t = -c + dP_{t-1}$$

$$-bP_t - dP_{t-1} = -a - c$$

$$P_t + \frac{d}{b} P_{t-1} = \frac{a+c}{b}$$

Shifting the time subscript ahead of one period we rewrite the equation as,

$$P_{t+1} + \frac{d}{h} P_t = \frac{a+c}{h} \quad \dots \dots \dots (4)$$

$$P_{t+1} + \frac{d}{b} P_t = \frac{a+c}{b} \quad \dots\dots\dots (4)$$

Equation (4) is the first order difference equation.

The solution of this first order difference equation $y_{t+1} + ay_t = c$

$$\text{is } y_t = \left(y_0 - \frac{c}{1+a} \right) (-a)^t + \frac{c}{1+a}$$

Similarly, the solution of equation (4) is

$$P_t = \left(P_0 - \frac{\frac{a+c}{b}}{1+\frac{d}{b}} \right) \left(-\frac{d}{b} \right)^t + \frac{\frac{a+c}{b}}{1+\frac{d}{b}}$$

...

$$P_t = \left(P_0 - \frac{\frac{a+c}{b}}{1+\frac{d}{b}} \right) \left(-\frac{d}{b} \right)^t + \frac{\frac{a+c}{b}}{1+\frac{d}{b}}$$

$$\Rightarrow P_t = \left(P_0 - \frac{\frac{a+c}{b}}{\frac{b+d}{b}} \right) \left(-\frac{d}{b} \right)^t + \frac{\frac{a+c}{b}}{\frac{b+d}{b}} \quad \dots$$

$$\Rightarrow P_t = \left(P_0 - \frac{a+c}{b+d} \right) \left(-\frac{d}{b} \right)^t + \frac{a+c}{b+d} \quad \dots\dots\dots (5)$$

...

$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots\dots\dots (6)$$

Here, P_0 is the initial price and

\bar{P} is the inter-temporal equilibrium price

Now, the nature of time path P_t depends on $\frac{d}{b}$

i.e. the ratio of the slope of supply curve ' d ' and the slope of demand curve ' b '

There are three cases.

$$Q_d = a - bP_t$$

$$Q_s = -c + dP_{t-1}$$

$$P_t = \left(P_0 - \frac{a+c}{b+d} \right) \left(-\frac{d}{b} \right)^t + \frac{a+c}{b+d} \quad \dots\dots\dots (5)$$

Since, $\bar{P} = \frac{a+c}{b+d}$ is the inter-temporal equilibrium price

We write equation (5) as,

$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots\dots\dots (6)$$

Equation (6) is the time path of Cobweb market model.

$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots\dots\dots (6)$$

Case –I: If $b > d$ or the slope of demand curve is more than the slope of supply curve. Then $\frac{d}{b} < 1$ or a fraction.

In such case, as the value of t is increasing then the

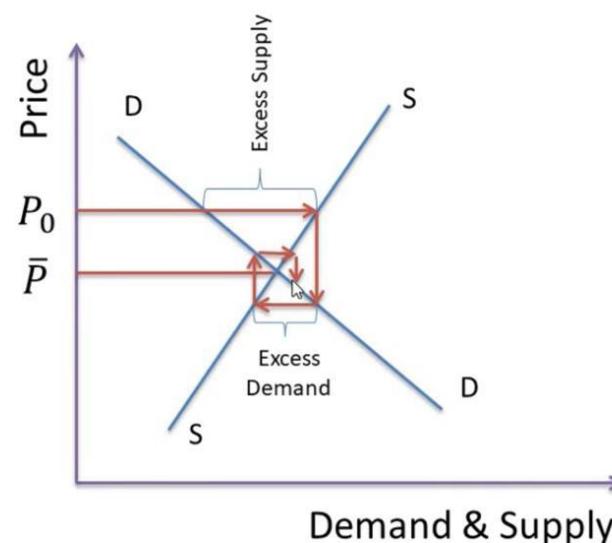
value of $\left(-\frac{d}{b} \right)^t$ is falling or as $t \rightarrow \infty$ then $\left(-\frac{d}{b} \right)^t \rightarrow 0$

In equation (6), $P_t = \bar{P}$ i.e. with the passes of time

$$P_t \rightarrow \bar{P}$$

The time path is convergent and the market is dynamically stable.

Convergent time path
and
dynamically stable
market.



$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots \dots \dots \quad (6)$$

Case –II: If $b < d$ or the slope of demand curve is less than the slope of supply curve. Then $\frac{d}{b} > 1$ or more than one.

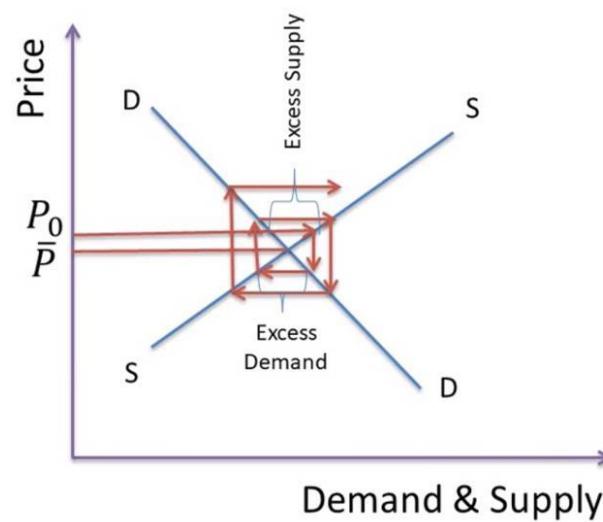
In such case, as the value of t is increasing then the value of $\left(-\frac{d}{b} \right)^t$ is increasing or as $t \rightarrow \infty$ then $\left(-\frac{d}{b} \right)^t \rightarrow \infty$

In equation (6), the current price P_t will divert more and more from the equilibrium price \bar{P}

The time path is divergent and the market is not dynamically stable.

Case –II: If $b < d$ or the slope of demand curve is less than the slope of supply curve. Then $\frac{d}{b} > 1$ or more than one.

Divergent time path
and
explosive market
price.



$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots \dots \dots \quad (6)$$

Case –III: If $b = d$ or the slope of demand curve is equal to the slope of supply curve. Then $\frac{d}{b} = 1$

In such case, as the value of t is increasing then the

value of

$$\left(-\frac{d}{b} \right)^t$$

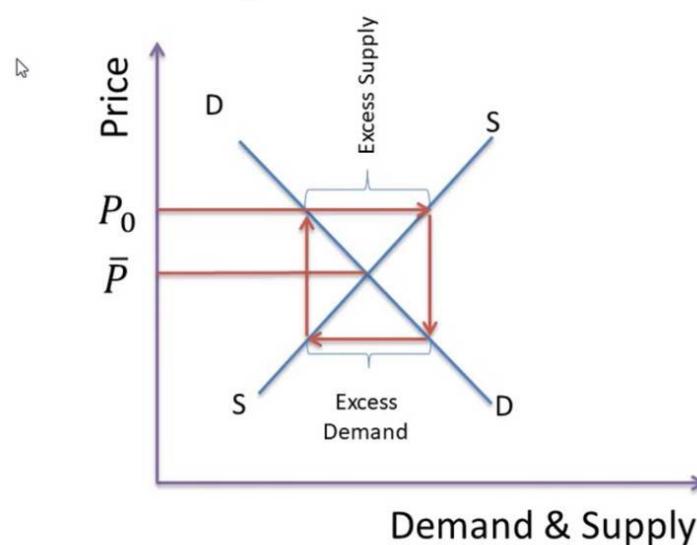
is alternatively +1 and -1
depending whether t is even or odd

In equation (6), the difference between P_t and \bar{P} is remain the same.

→ The time path is said to be regular.

Case –III: If $b = d$ or the slope of demand curve is equal to the slope of supply curve. Then $\frac{d}{b} = 1$

Regular time path
price in Cobweb
market model.





Question 1

In a local market for rice:

- The **demand equation** is

$$Q_d = 120 - 2P \dots\dots (D)$$
- The **supply equation** is

$$Q_s = 20 + 3P \dots\dots (S)$$

where

Q_d = quantity demanded (in kg),

Q_s = quantity supplied (in kg),

P = price per kg (in rupees).

1. Find the **equilibrium price** and **equilibrium quantity**.
2. Check whether the market is in **surplus or shortage** if price is fixed at $P = 10$ rupees.
Also calculate the amount of surplus or shortage.

Solution

1. Equilibrium price and quantity

At **equilibrium**:

$$Q_d = Q_s \dots\dots (E0)$$

Substitute (D) and (S) into (E0):

$$120 - 2P = 20 + 3P \dots\dots (E1)$$

Bring all P -terms to one side:

$$\begin{aligned}
 120 - 20 &= 3P + 2P \\
 100 &= 5P \\
 P^* &= \frac{100}{5} = 20 \text{ rupees} \dots \dots (\text{Equilibrium Price})
 \end{aligned}$$

Now substitute $P^* = 20$ into either (D) or (S):

Using demand equation (D):

$$Q_d = 120 - 2(20) = 120 - 40 = 80 \text{ kg}$$

Using supply equation (S):

$$Q_s = 20 + 3(20) = 20 + 60 = 80 \text{ kg}$$

Both give same value, so:

- **Equilibrium price:** $P^* = 20$ rupees
 - **Equilibrium quantity:** $Q^* = 80$ kg
-

2. Market position when $P = 10$ rupees

Demand at $P = 10$:

$$Q_d = 120 - 2(10) = 120 - 20 = 100 \text{ kg}$$

Supply at $P = 10$:

$$Q_s = 20 + 3(10) = 20 + 30 = 50 \text{ kg}$$

Compare:

- $Q_d = 100$ kg
- $Q_s = 50$ kg

Since $Q_d > Q_s$:

$$\text{Shortage} = Q_d - Q_s = 100 - 50 = 50 \text{ kg}$$

Answer for part (2): At $P = 10$ rupees, there is a **shortage of 50 kg** in the market.

Question 2:

In an agricultural market, the **demand** and **supply** for a crop in year t are given by:

Demand (current price): $Q_d(t) = 120 - 2P_t$

Supply (lagged price – farmers look at last year's price): $Q_s(t) = 20 + 0.5P_{t-1}$

Each year the market clears, so: $Q_d(t) = Q_s(t)$

- Find the **equilibrium price** and **equilibrium quantity**.
 - If the initial price in year 0 is $P_0 = 10$, compute prices P_1, P_2, P_3, P_4, P_5 .
 - Show that this market is **convergent** (prices move toward equilibrium over time).
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Solution:

1. Equilibrium price and quantity

In equilibrium, price is the same in consecutive periods, so:

$$P_t = P_{t-1} = P_e$$

Set demand = supply at equilibrium:

$$120 - 2P_e = 20 + 0.5P_e$$

Bring terms together:

$$120 - 20 = 2P_e + 0.5P_e$$

$$100 = 2.5P_e$$

So:

$$P_e = 100 / 2.5 = 40$$

Now find equilibrium quantity:

$$Q_e = 120 - 2P_e = 120 - 2(40) = 120 - 80 = 40$$

So:

- **Equilibrium price: $P_e = 40$**
 - **Equilibrium quantity: $Q_e = 40$**
-

Each year:

$$Q_d(t) = Q_s(t)$$

So:

$$120 - 2P_t = 20 + 0.5P_{t-1}$$

Rearrange:

$$-2P_t = 20 + 0.5P_{t-1} - 120$$

$$-2P_t = -100 + 0.5P_{t-1}$$

Multiply by -1:

$$2P_t = 100 - 0.5P_{t-1}$$

Divide by 2:

$$P_t = 50 - 0.25P_{t-1}$$

This is the lag (cobweb) equation for price.

3. Compute price path starting from $P_0 = 10$

Use: $P_t = 50 - 0.25P_{t-1}$

• $t = 1$:

$$P_1 = 50 - 0.25(10) = 50 - 2.5 = 47.5$$

• $t = 2$:

$$P_2 = 50 - 0.25(47.5) = 50 - 11.875 = 38.125$$

• $t = 3$:

$$P_3 = 50 - 0.25(38.125) = 50 - 9.53125 = 40.46875$$

• $t = 4$:

$$P_4 = 50 - 0.25(40.46875) = 50 - 10.1171875 \approx 39.8828$$

• $t = 5$:

$$P_5 = 50 - 0.25(39.8828) \approx 50 - 9.9707 = 40.0293$$

You can see:

$$P_0 = 10$$

$$P_1 = 47.5$$

$P_2 \approx 38.13$ $P_3 \approx 40.47$ $P_4 \approx 39.88$ $P_5 \approx 40.03$

Prices are **oscillating**, but getting closer and closer to **40**.

(Optionally, quantities each period: $Q_t = 120 - 2P_t$, also converge to 40.)

4. Show convergence (why is this a convergent market?)

From the recursion:

$$P_t = 50 - 0.25P_{t-1}$$

Write it in deviation-from-equilibrium form.

We know $P_e = 40$. Consider $(P_t - 40)$:

Start from:

$$P_t = 50 - 0.25P_{t-1}$$

Subtract 40 from both sides:

$$P_t - 40 = 50 - 0.25P_{t-1} - 40$$

$$P_t - 40 = 10 - 0.25P_{t-1}$$

But $10 = 0.25 \times 40$, so:

$$10 - 0.25P_{t-1} = 0.25 \cdot 40 - 0.25P_{t-1} = -0.25(P_{t-1} - 40)$$

Thus:

$$P_t - 40 = -0.25(P_{t-1} - 40)$$

The factor on the deviation is **-0.25**.

- Absolute value $|-0.25| = 0.25 < 1 \rightarrow$ deviations **shrink** over time.
- Negative sign \rightarrow path oscillates around equilibrium (above–below–above–below), but because $0.25 < 1$, oscillations become **smaller and smaller**.

Therefore, this is a **convergent cobweb market**:

- Prices move toward equilibrium $P_e = 40$.
- Quantities move toward $Q_e = 40$.
- Each period, the market gets closer to equilibrium.

$$P_{t+1} + \frac{d}{b} P_t = \frac{a+c}{b} \quad \dots \dots \dots \quad (4)$$

Equation (4) is the first order difference equation.

The solution of this first order difference equation $y_{t+1} + ay_t = c$

is $y_t = \left(y_0 - \frac{c}{1+a} \right) (-a)^t + \frac{c}{1+a}$

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