

Morphological Image Processing

**Logic Operations involving binary images,
Dilation and Erosion,
Opening and Closing,
The Hit or-Miss Transformation**

Introduction

Morphology: Shape, Form, Structure / Extracting and describing image component regions / Applied on binary image / Based on set theory

- Binary images may contain numerous imperfections.
- In particular, the binary regions produced by simple thresholding are distorted by noise and texture.
- Morphological image processing pursues the goals of removing these imperfections by accounting for the form and structure of the image. These techniques can be extended to greyscale images.

Primitives:

Question

What is Mathematical Morphology ?

An (imprecise) Mathematical Answer

A mathematical tool for investigating geometric structure in **binary** and **grayscale** images.

Shape Processing and Analysis

Visual perception requires transformation of images so as to make explicit particular **shape information**.

Goal: Distinguish meaningful shape information from irrelevant one.

The vast majority of shape processing and analysis techniques are based on designing a **shape operator** which satisfies desirable properties.

Morphological Operators

Erosions and **dilations** are the most elementary operators of mathematical morphology.

More complicated **morphological operators** can be designed by means of combining erosions and dilations.

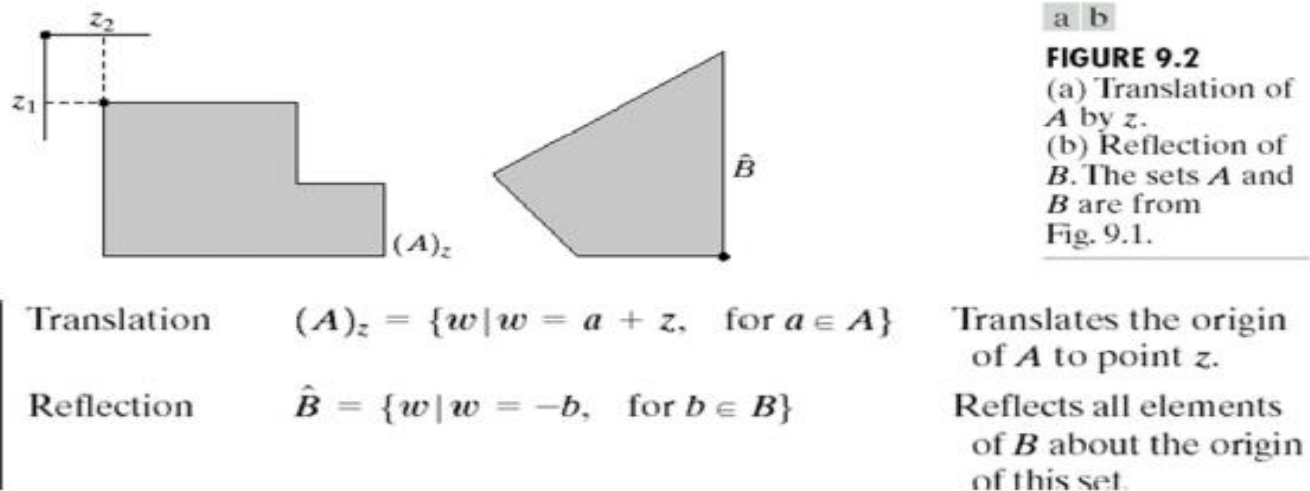
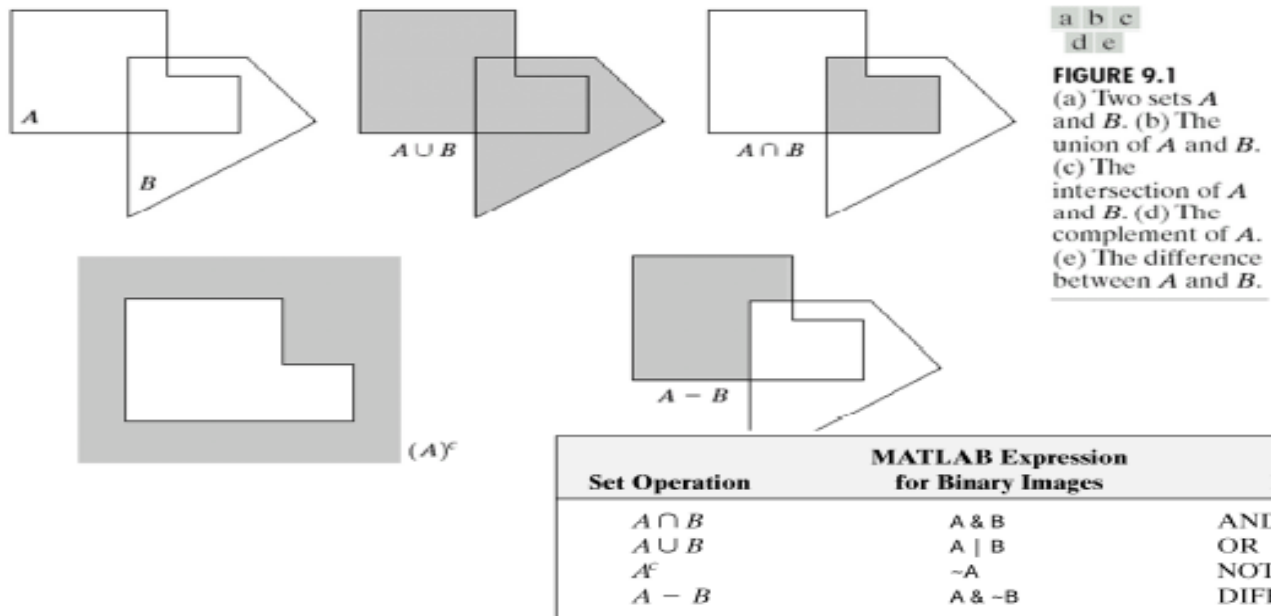
Some History

George Matheron (1975) *Random Sets and Integral Geometry*, John Wiley.

Jean Serra (1982) *Image Analysis and Mathematical Morphology*, Academic Press.

Petros Maragos (1985) *A Unified Theory of Translations-Invariant Systems with Applications to Morphological Analysis and Coding of Images*, Doctoral Thesis, Georgia Tech.

Logic Operations involving binary images



Logic operations are fundamental image processing techniques that apply logical operations to binary images. These operations consider the binary images as sets and perform set operations on corresponding pixels. The primary logical operations are AND, OR, NOT, and XOR

AND Operation

- The AND operation is used to find the intersection of two binary images.
- Each pixel in the output image is 1 if both corresponding pixels in the input images are 1, otherwise, it is 0.
- **Application:** This operation is useful for masking, where you want to retain certain parts of an image.

OR Operation

- The OR operation is used to find the union of two binary images.
- Each pixel in the output image is 1 if at least one of the corresponding pixels in the input images is 1, otherwise, it is 0.
- **Application:** This operation is useful for combining multiple binary masks.

NOT Operation

- The NOT operation inverts the binary image. Each 0 becomes 1 and each 1 becomes 0.
- **Application:** This operation is used to find the complement of an image.

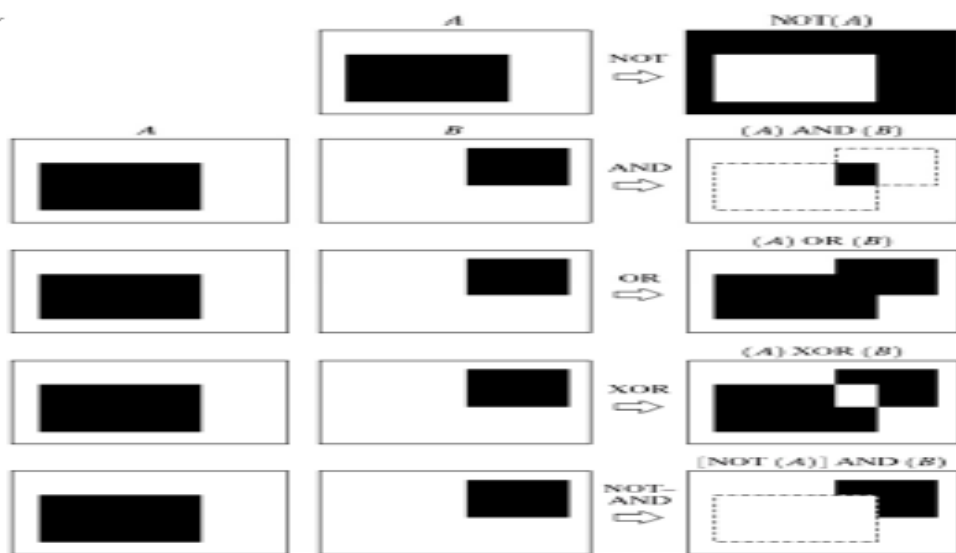


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

XOR Operation

- The XOR operation is used to find the difference between two binary images.
- Each pixel in the output image is 1 if the corresponding pixels in the input images are different, otherwise, it is 0.
- **Application:** This operation is useful for detecting changes between two images.

Morphological image processing

- **It** is a collection of non-linear operations related to the shape or morphology of features in an image.
- Morphological operations rely only on the *relative ordering of pixel values, not on their numerical values*, and therefore are especially suited to the processing of binary images.
- Morphological operations can also be applied to greyscale images such that their light transfer functions are unknown and therefore their absolute pixel values are of no or minor interest.

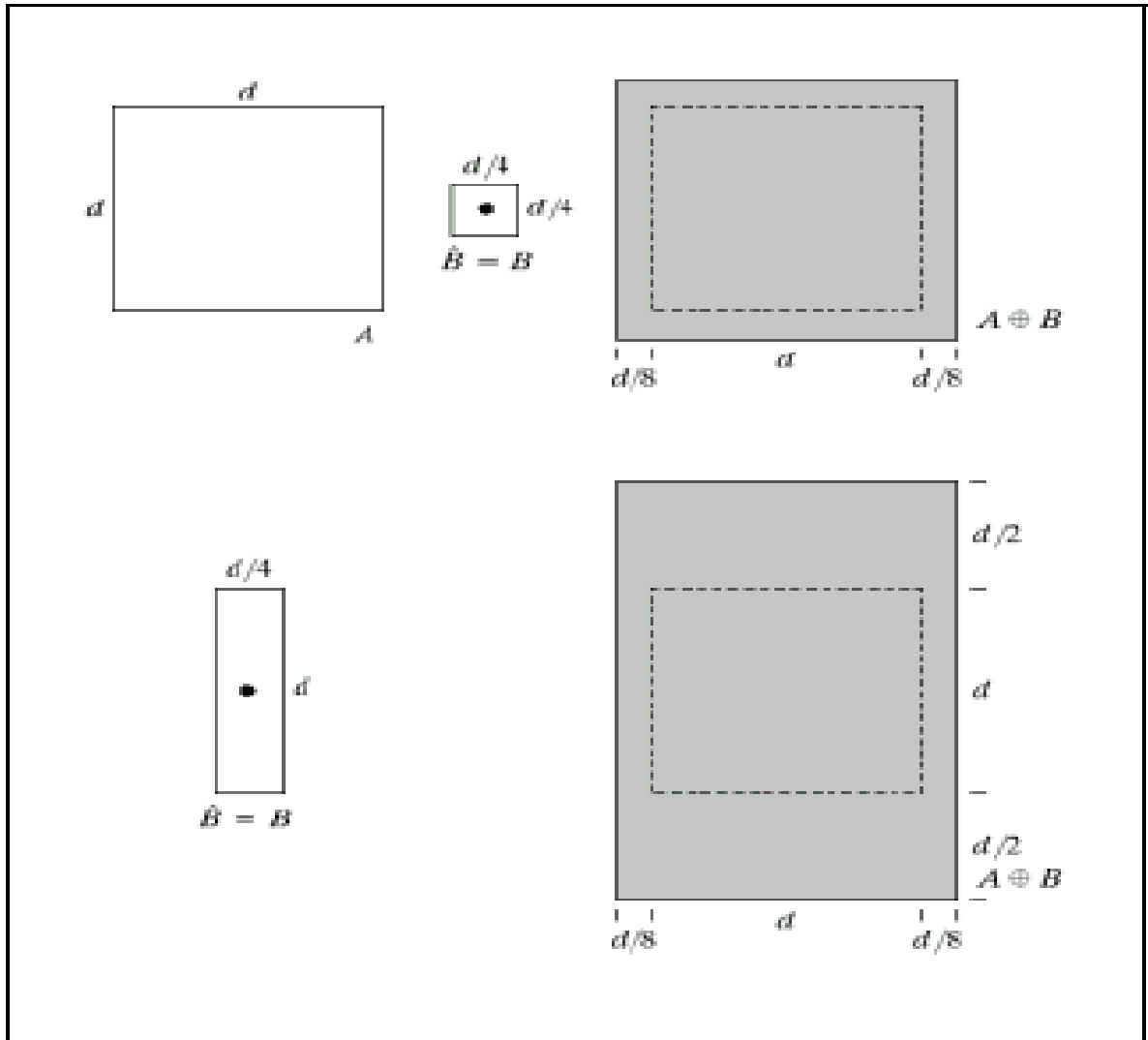
Dilation and Erosion

- **Definition:** **Dilation** is a morphological operation that adds pixels to the boundaries of objects in an image.
- **Purpose:**
 - Expands the shape of objects.
 - Fills in small holes and gaps in objects.
 - Connects nearby objects.
- **Operation:** The dilation of a binary image A by a structuring element B is defined as:

$$(A \oplus B) = \{ z \mid (B)_z \cap A \neq \emptyset \}$$

where $(B)_z$ denotes the translation of B by z .

In other words, the structuring element B is slid over the image, and wherever it intersects with the foreground pixels (1s), the output pixel is set to 1.



➤ Example:

- To perform dilation there are two methods
 - Substitution
 - Vector addition

@ Substitution method

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

original image Structure element
 ↓ no value (Initial 1 size 2x2)
 ↓

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

yes
replace by
s.e.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

yes

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

yes

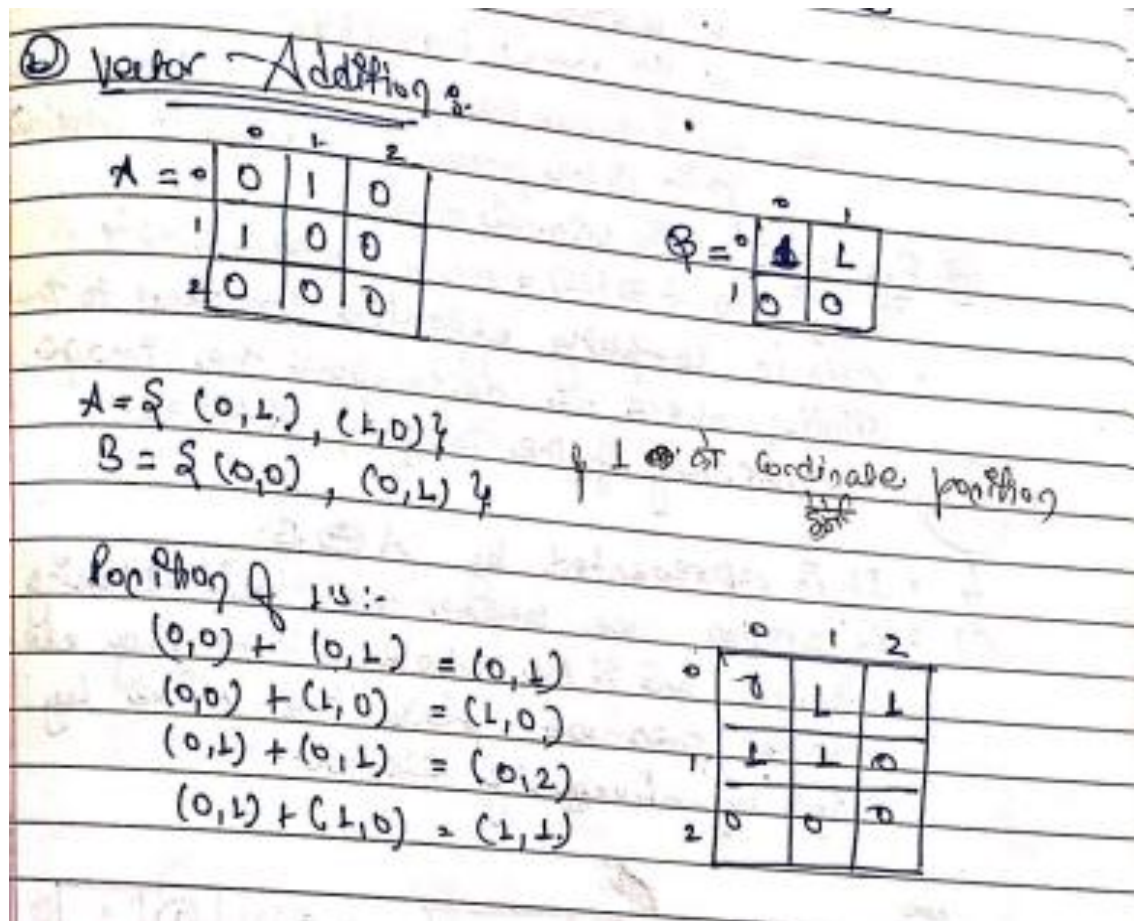
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↓

yes

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{yes}} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

discuss

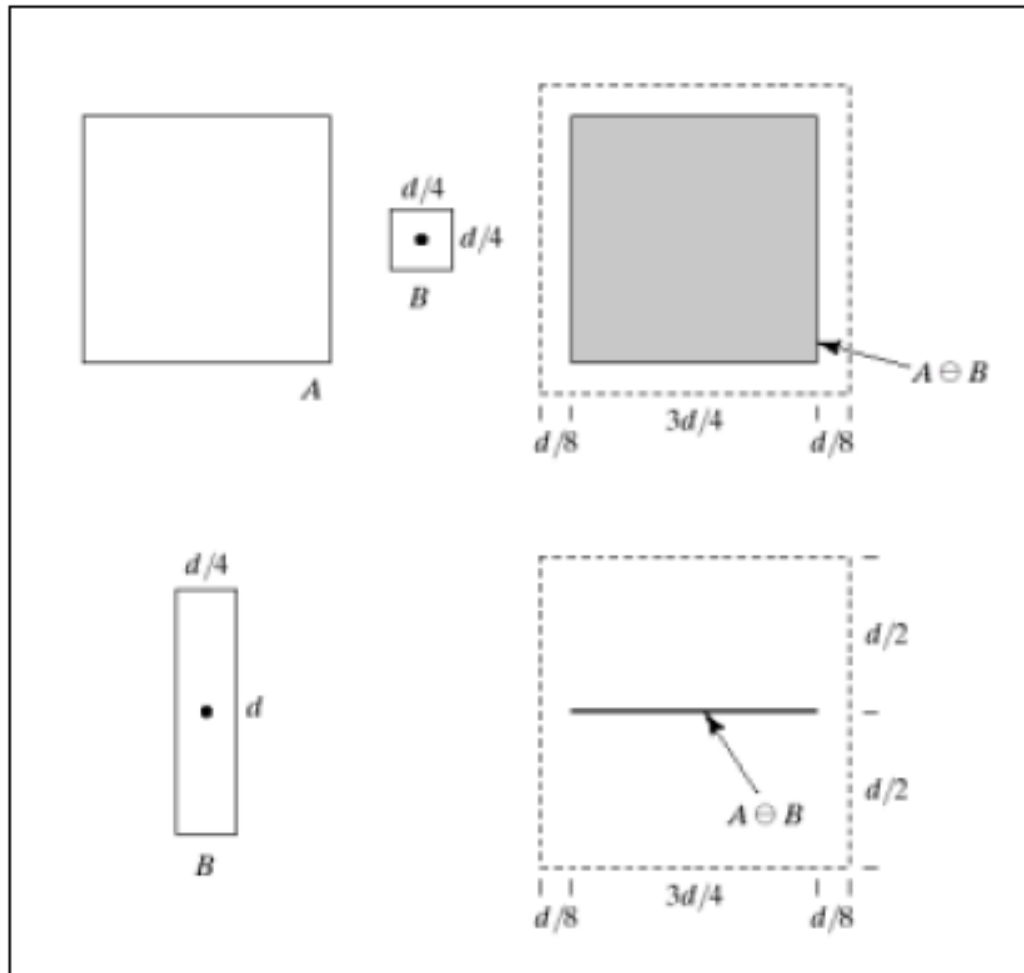


- **Definition:** **Erosion** is a morphological operation that removes pixels from the boundaries of objects in an image.
- **Purpose:**
 - Shrinks the shape of objects.
 - Removes small noise.
 - Detaches objects that are connected by thin bridges.
- **Operation:** The erosion of a binary image A by a structuring element B is defined as:

$$(A \ominus B) = \{z \mid (B)_z \subseteq A\}$$

where $(B)_z$ denotes the translation of B by z.

In other words, the structuring element B is slid over the image, and only if B fits entirely within the foreground pixels (1s) of A, the output pixel is set to 1.



➤ **Example:** If A is a binary image and B is a 2 x 2 square structuring element:

ex

$A =$	1	1	1	1
	1	1	0	1
	0	1	1	1
	0	1	1	0

original element

$B =$	1	1
	1	0

structuring element

for each replace 1st, not only 1

$A \ominus B =$	0	1	0	0
	0	0	0	0
	0	1	0	0
	0	0	0	0

Applications of Dilation and Erosion

- **Noise Removal:** Erosion removes small noise, and dilation restores the original object size.
- **Shape Enhancement:** Dilation and erosion can enhance or suppress specific shapes within an image.
- **Bridge Gaps:** Dilation can bridge small gaps or connect close objects.
- **Segmentation:** These operations are essential in object segmentation and shape analysis.



Greyscale image



Binary image
thresholding



by Erosion: a 2×2 square structuring element

Erosion with small (e.g. 2×2 - 5×5) square structuring elements shrinks an image by stripping away a layer of pixels from both the inner and outer boundaries of regions. The holes and gaps between different regions become larger, and small details are eliminated:

Morphological Erosion + Dilation

The effect of erosion followed by dilation is illustrated below.



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Fingerprint Image Cleanup

The use of ERODE + DILATE is illustrated by this example

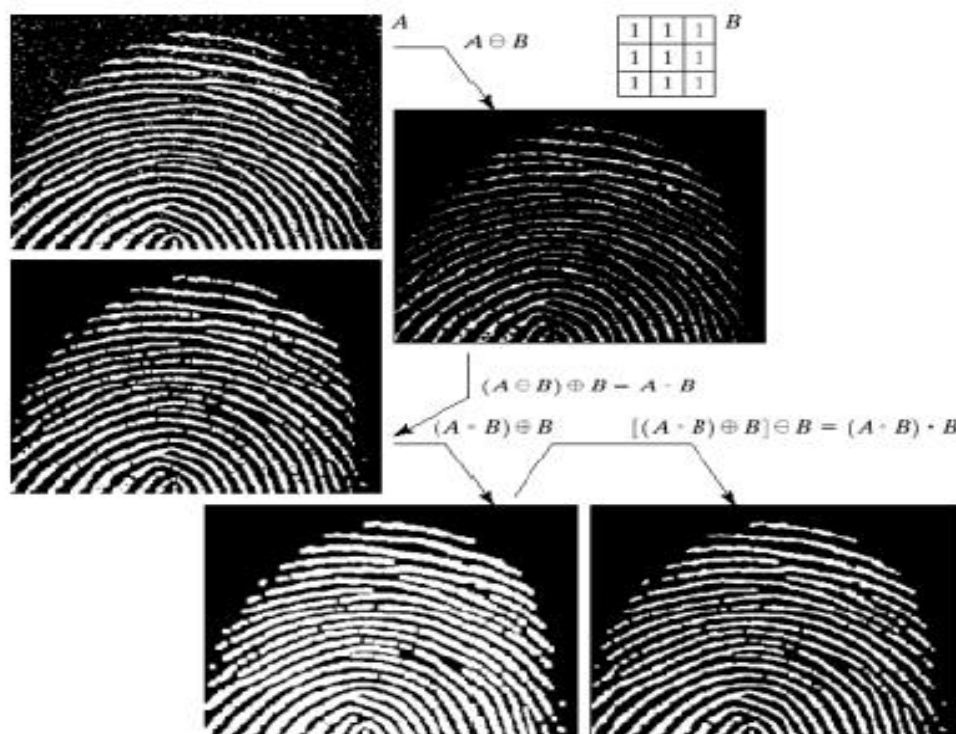


FIGURE 9.11

(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Structuring Element:

- A **structuring element** is a fundamental component used in morphological image processing.
- It is a small, predefined shape or matrix that is used to probe and interact with the binary image to perform operations like dilation, erosion, opening, and closing.

Characteristics of a Structuring Element

- **Shape and Size:**
 - The structuring element can be of various shapes, such as a square, rectangle, circle, line, or custom shape.
 - Its size and shape are chosen based on the specific requirements of the image processing task.
- **Origin:**

- The structuring element has an origin (or anchor point), usually its center, which is used as a reference point for interaction with the image.
- **Binary or Grayscale:**
 - In binary morphological operations, the structuring element contains binary values (0s and 1s).
 - In grayscale morphological operations, the structuring element contains integer values.

Common Shapes of Structuring Elements	
Square or Rectangular: These are the most common shapes used for simplicity and efficiency.	1 1 1 1 1 1 1 1 1
Cross: Useful for detecting and enhancing specific features like thin lines.	0 1 0 1 1 1 0 1 0
Circle or Disk: Used for operations requiring isotropic properties.	0 0 1 0 0 0 1 1 1 0 1 1 1 1 1 0 1 1 1 0 0 0 1 0 0
Line: Used for detecting linear features or aligning elements in a particular direction. Example of a horizontal line structuring element	1 1 1 1 1

Opening and Closing

Opening and closing

OPENING is erosion followed by dilation

CLOSING is dilation followed by erosion

Opening	$A \circ B = (A \ominus B) \oplus B$	Smooths contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smooths contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Opening

Definition: Opening is a morphological operation that is used to remove small objects from the foreground of an image, while preserving the shape and size of larger objects in the image. It is defined as the erosion of an image followed by the dilation of the eroded image.

Purpose:

- To smooth the contour of an object.
- To break narrow isthmuses (thin connections between objects).
- To eliminate thin protrusions.

Process:

1. **Erosion:** Apply erosion to the input image using a structuring element. This step removes small objects and erodes boundaries of larger objects.
2. **Dilation:** Apply dilation to the eroded image using the same structuring element. This step restores the size of the remaining objects but does not restore the small objects removed during erosion.

Mathematical Representation:

$$A \circ B = (A \ominus B) \oplus B$$

where A is the input image and B is the structuring element.

Explanation:

- $A \ominus B$: Erosion of A by B .
- $(A \ominus B) \oplus B$: Dilation of the eroded image by B .

A different formulation:

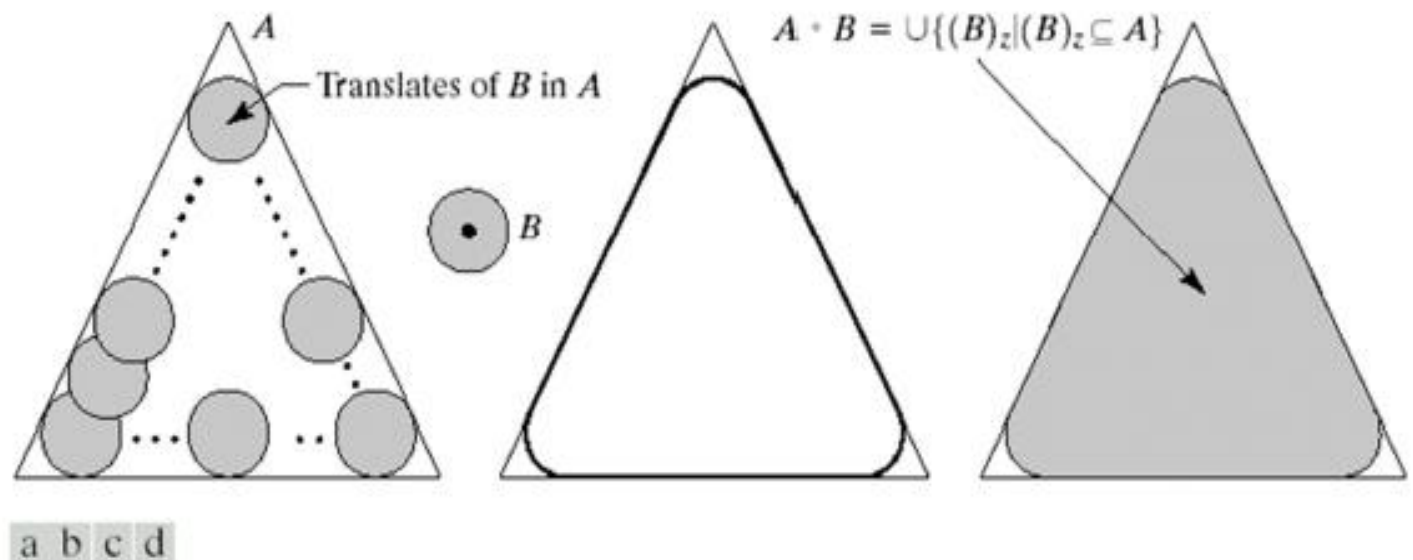


FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Effects:

- Removes small objects from the foreground.
- Smooths the contour of larger objects, making their shapes more regular.
- Breaks thin connections between objects, separating them.

Example: Consider a binary image containing several small noise dots and a larger object with a rough boundary. When opening is applied:

1. **Erosion** removes the small noise dots and slightly reduces the size of the larger object.

2. **Dilation** restores the size of the larger object but does not bring back the removed noise dots, resulting in a cleaner image with smoother boundaries for the larger object.

Applications:

- **Noise Removal:** Opening can be used to eliminate small noise particles from an image while preserving the main objects.
- **Shape Smoothing:** It is useful in preprocessing steps where the goal is to smooth the shapes of objects without altering their overall structure significantly.
- **Image Analysis:** Enhances the accuracy of subsequent image analysis tasks by removing irrelevant small objects and noise.

A different formulation:

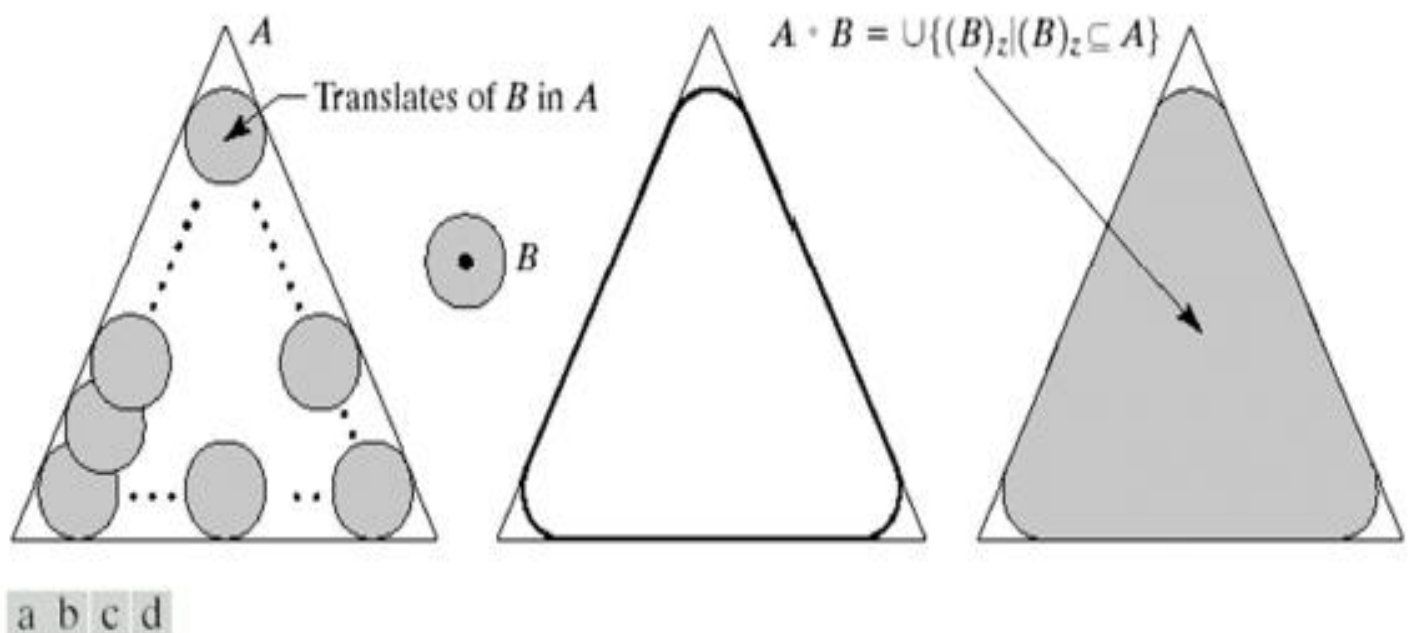


FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Closing

Definition: Closing is a morphological operation that is used to close small holes and gaps in the foreground of an image, while preserving the shape and size of the objects in the image. It is defined as the dilation of an image followed by the erosion of the dilated image.

Purpose:

- To smooth sections of contours.
- To fuse narrow breaks and long thin gulfs.
- To eliminate small holes and fill gaps in the objects.

Process:

1. **Dilation:** Apply dilation to the input image using a structuring element. This step expands the boundaries of objects and fills small holes and gaps.
2. **Erosion:** Apply erosion to the dilated image using the same structuring element. This step reduces the size of the expanded objects but does not re-open the small holes and gaps that were filled during dilation.

Mathematical Representation:

$$A \cdot B = (A \oplus B) \ominus B$$

where A is the input image and B is the structuring element.

Explanation:

- $A \oplus B$: Dilation of A by B.
- $(A \oplus B) \ominus B$: Erosion of the dilated image by B.

A different formulation:

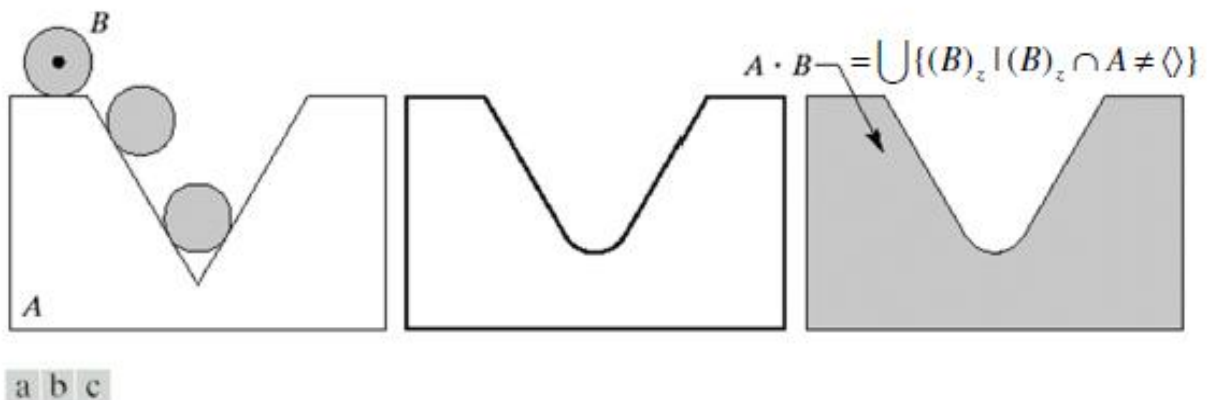


FIGURE 9.9 (a) Structuring element B “rolling” on the outer boundary of set A . (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Effects:

- **Fills small holes** and gaps in the foreground objects.
- Connects nearby objects and smooths their contours.
- Preserves the overall shape and size of the objects while removing imperfections.

Example: Consider a binary image containing several objects with small holes and gaps. When closing is applied:

1. **Dilation** expands the objects and fills the small holes and gaps.
2. **Erosion** reduces the size of the expanded objects back to their original size but retains the filled holes and gaps, resulting in smoother and more solid objects.

Applications:

- **Noise Removal:** Closing can be used to fill small holes and gaps in an image, making the objects more solid and reducing noise.
- **Shape Enhancement:** It is useful in preprocessing steps where the goal is to enhance the shapes of objects by filling in gaps and smoothing contours.
- **Image Analysis:** Enhances the accuracy of subsequent image analysis tasks by providing more solid and connected objects.

eg

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$A \oplus B$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$A \cdot B = (A \oplus B) \ominus B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

The Hit or (Fit)Miss Transformation:

- The Hit-or-Miss Transformation is a specialized morphological operation used in image processing for pattern recognition and shape detection.
- It is particularly useful for detecting specific patterns or structures within an image that match predefined templates.

How It Works:

1. Template Definition:

- Define a template or structuring element that consists of foreground (object) pixels represented by 1s, background (non-object) pixels represented by 0s, and don't-care pixels represented by * (wildcards).

2. Matching Process:

- Slide the template over the image pixel by pixel.
- Check if the template matches the local neighborhood of pixels in the image.
- A match occurs when:
 - For a foreground pixel in the template (1), the corresponding pixel in the image must be foreground (1).
 - For a background pixel in the template (0), the corresponding pixel in the image must be background (0).
 - Wildcard pixels (*) in the template can match either foreground or background pixels in the image.

3. Output:

- Create a new binary image where pixels that match the template are set to 1 (foreground), and non-matching pixels are set to 0 (background).

Applications:

- **Pattern Recognition:** Used to detect specific patterns or shapes in images, such as edges, corners, or specific configurations of pixels.
- **Shape Detection:** Identifies objects based on their structural characteristics defined by the template.
- **Image Filtering:** Can be used as a filter to enhance or suppress specific features in an image based on the template used.

Example:

Consider a binary image where you want to detect a specific pattern, such as a corner formed by three foreground pixels in an L shape. You can define a template that represents this L shape

and use the Hit-or-Miss Transformation to locate all occurrences of this pattern within the image.

1	1	0
1	*	0
*	0	0

 $+$

0	0	0	0	0
0	0	1	0	0
0	1	1	0	0
0	0	0	0	0

 $=$

0	0	0	0	0
0	0	0	0	0
0	1	0	0	0
0	0	0	0	0

In this example, the template matches the L-shaped pattern of three foreground pixels in the image, and the resulting output highlights the locations where this pattern occurs.

The Hit-or-Miss Transformation is a powerful tool in morphological image processing, enabling precise detection and localization of specific patterns or structures within images.

<https://homepages.inf.ed.ac.uk/rbf/HIPR2/hitmiss.htm#:~:text=The%20hit%2Dand%2Dmiss%20transform,can%20be%20derived%20from%20it.>

https://uotechnology.edu.iq/ce/Lectures/Image_Processing_4th/DIP_Lecture11.pdf

Morphological image processing involves applying a series of operations that manipulate the structure of objects in binary and grayscale images. These operations include:

- **Logic Operations:** Basic pixel-wise operations like AND, OR, NOT, and XOR.
- **Dilation and Erosion:** Fundamental operations to add or remove pixels from object boundaries.
- **Opening and Closing:** Derived operations to smooth contours and remove noise or fill holes.
- **Hit-or-Miss Transformation:** A specialized operation for pattern recognition and shape detection.