Department of MCA

LECTURE NOTE

ON

SIMULATION AND MODELLING

COURSE CODE: MCA-302

Prepared by :

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MCA-302   **Simulation & Modelling**   **L-T-P: 3-1-0**

Prerequisite: Familiarity with Linear Algebra, Probability and Statistics, Discrete structures, graph theory Object-oriented design and programming.

Module-1 (10 Hours)
Inventory Concept: The technique of Simulation, Major application areas, concept of a System, Environment, Continuous and discrete systems, systems modeling types of models progress of a Simulation Study, Monte Carlo Method, Comparison of Simulation and Analytical Methods. Numerical Computation Technique for discrete and continuous models, Continuous System Simulation.

Module-2 (10 Hours)
Probability Concepts in Simulation : Stochastic variables, Discrete and Continuous Probability Functions, Numerical evaluation of continuous probability functions, continuous uniformly distributed random numbers, Random Number Generators – Linear congruential Generator, Mid Square Method, Multiplicative Congruential generator, rejection Method, Testing of random Numbers, Generation of Stochastic variates, Arrival Patterns Service times.

Module-3 (10 Hours)
Discrete System Simulation and GPSS: Discrete Events, Representation of Time, generation of arrival patterns, fixed time step versus next event simulation, Simulation of a Telephone System, delayed calls. Introduction to GPSS : Creating and moving transactions, queues, facilities and storages, gathering statistics, conditional transfers, program control statements, priorities and parameters, standard numerical attributes, functions, gates, logic switches and tests, Variables, Select and Count.

Module-4 (10 Hours)
Simulation Languages and Practical Systems: Continuous and discrete systems languages, factors in the section of discrete systems simulation language. Computer model of queuing, inventory and scheduling systems. Design and Evaluation of simulation Experiments: Length of simulation runs, validation, variance reduction techniques, experimental layout, analysis of simulation output, Recent trends and developments.

Text Books:

1. Geoffrey Gordon, System Simulation,2/E, PHI Learning.
2. Narsingh Deo, System Simulation with Digital Computer, 2007, PHI Learning.

References:

1. Jerry Banks, John S. Carson II, Barry L. Nelson, David M. Nicol, P. Shahabudeen, Discrete-Event System Simulation, Fourth Edition, Pearson Publication.

SIMULATION & MODELING

CONTENT

SIMULATION AND MODELLING

MODULE – 1

Concept of a system:-

A system is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence .Systems are of two types :-

- Static System
- Dynamic System

In **static systems** there are no changes over time where as in **dynamic systems** the interactions cause changes over time.

Example 1

An aircraft system

Θi

Desiredheading

| Gyroscope | → | Control surface | → | Airframe |

Θo
Actual heading
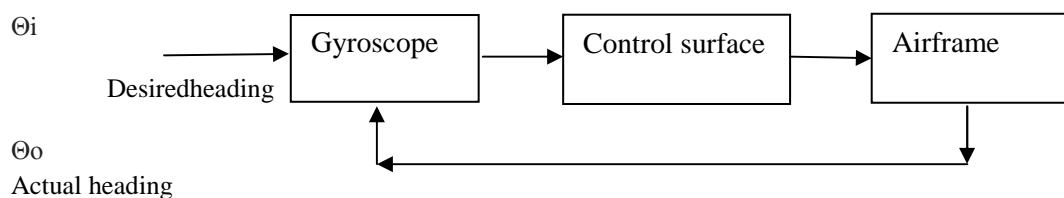
An aircraft under autopilot control

Consider an aircraft flying under the control of an autopilot. A gyroscope in the autopilot detects the difference between the actual heading and discard heading .It sends a signal to move the control surfaces. In response to the control surface movement the airframe steer towards the desired heading to the desired destination.

Example 2

A factory system

Customer order → Production control department

Raw material

| Purchasing department | → | Fabrication department | → | Assembly department | → | Shipping department |

Finishing product
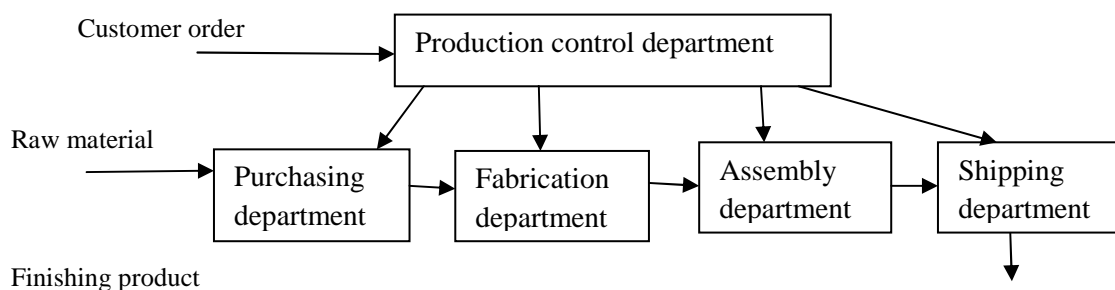
Consider a factory that make assembles parts for a product. Two major components of the factory system are the fabrication department that makes the part & the assembly

department that produces the product. A purchasing department maintains & a shipping department dispatches the finished product. A production control department receives order & assigns work to the other department.

Components of a system

Three basic components are:-

- Entity
- Attributes
- Activities

Entity: - It is used to denote an object of interest in a system.

Attribute: - It denotes a property of an entity.

Activity:-Any process that causes changes in the system is called as an activity.

Example:-

In the factory system the entities are the department, orders, parts and products.Attributes are such factors as the quantities for each order, type of part or number of machines in a department.The activities are the manufacturing process of the departments.

State of the system

It is used to mean a description of all the entities, attributes and activities as they exist at one point of time.
Progress:-The progress of the system is studied by the system following a change in the state of the system.

System Environment

A system is often affected by changes occurring outside the system. Such changes occurring outside the system are said to occur in the system environment. An important step in modelling system is to decide upon the boundary between the system and its environment.

The term *endogenous* is used to describe activities occurring within the system.
Example: sports, cultural functions in a university system.

The term *exogenous* is used to describe the activities in the environment that affect the system.
Example: strikes in a university system.

Based on these activities a system may be classified as *open* or *closed* system. A system for which there is no exogenous activity is said to be a **closed system.** A system that has exogenous activities is called as an **opensystem**.

Deterministic vs.Stochastic Activities

Depending on the manner on which they can be described activities can be classified as deterministic or stochastic.

Deterministic

An activity is said to be deterministic where the outcome of an activity can be described completely in term of its input,
Example: AND, OR, NOT operations.

Stochastic

An activity is said to be stochastic where the effects of the activity vary randomly over various possible outcomes.
Example: Throwing a dice or tossing a coin.

Continuous vs. Discrete system

Continuous system

Systems in which the changes are predominantly smooth are called continuous system.
Example:
The movement of the aircraft occurs smoothly so aircraft system is a continuous system.

Discrete system

Systems in which the changes are predominantly discontinuous are called discrete system.
Example:
Changes in the factory occur discontinuously so factory system is a discrete system.

Sampled data system

There are also systems that are intrinsically continuous but information about them is only available at discrete points of time. These are called **sampled data system.**
Example:
Examination system

System modelling

The model is defined as the body of information about a system gathered for the purpose of studying the system. The tasks of deriving a system model are divided into two subtasks. They are

1. Establishing the model structure
2. Supplying the data

Establishing the model structure

It determines the system boundary and identifies the entities, attributes and activities of the system.

Supplying the data

The data provides the values that the attributes can have and define the relationships involved in the activities.

Types of models

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                          │  MODELS  │
                          └──────────┘
                    ┌───────┘      └────────┐
              ┌──────────┐            ┌──────────────┐
              │ PHYSICAL │            │ MATHEMATICAL │
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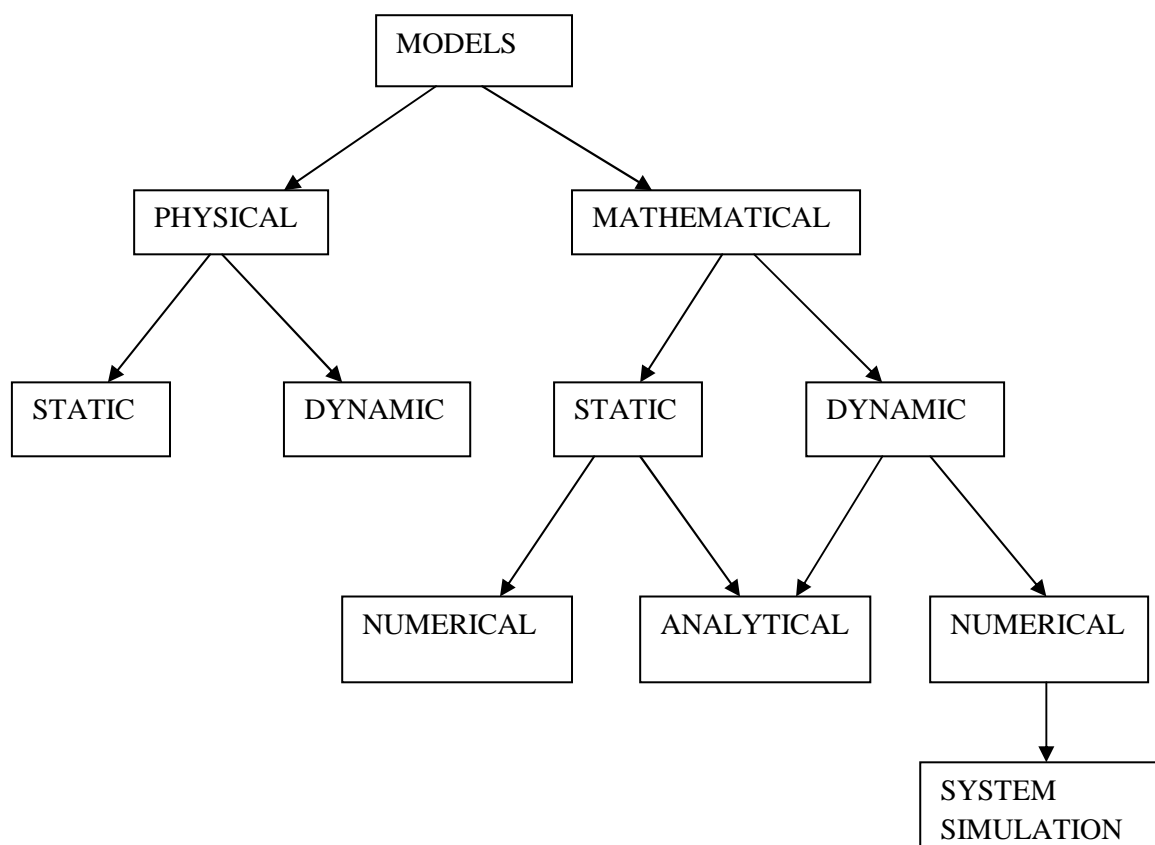
Models are broadly classified into physical models or mathematical models.

Physical models:

Physical models are based on some analogy between such systems as mechanical and electrical or electrical or hydraulic. Here the system attributes are represented by such measurements as voltage or the position of a shaft. The system activities are reflected in the physical laws that derive the models.

Mathematical models:

The mathematical model use symbolic notations and mathematical equations to represent a system. The system attributes are represented by variables and the activities that represented by mathematical functions that interrelate the variables.

A second distinction is between static models and dynamic models. Static models can only show the values that system attributes take where the system is in balanced. Dynamic models follow the changes over time that result from system activities.

In case of mathematical model the third distinction is a technique by which the model is solved that is actual values are assigned to system attributes.

A distinction is made between analytical and numerical method. Applying analytical techniques means using the deductive reasoning of mathematical theory to solve a model.

Example: linear differential method

Numerical methods involve applying computational procedure to solve equations. Any assignment of numerical values that uses mathematical tables involves numerical methods.

System Simulation:

It is considered to be a numerical technique used in conjunction with dynamic models. Models are of two types:

- Deterministic
- Stochastic

Static physical models:

They are used in ship buildings, deciphering of DNA molecules, Wind Tunnels and Water Tanks in the course of designing air craft or ship. They are also used for solving equations with particular boundary condition. Example: Scale models.

Dynamic Physical models:

They rely upon an analogy between the system being studied and some other system of a different nature.

(Mechanical System)

(Electrical System)
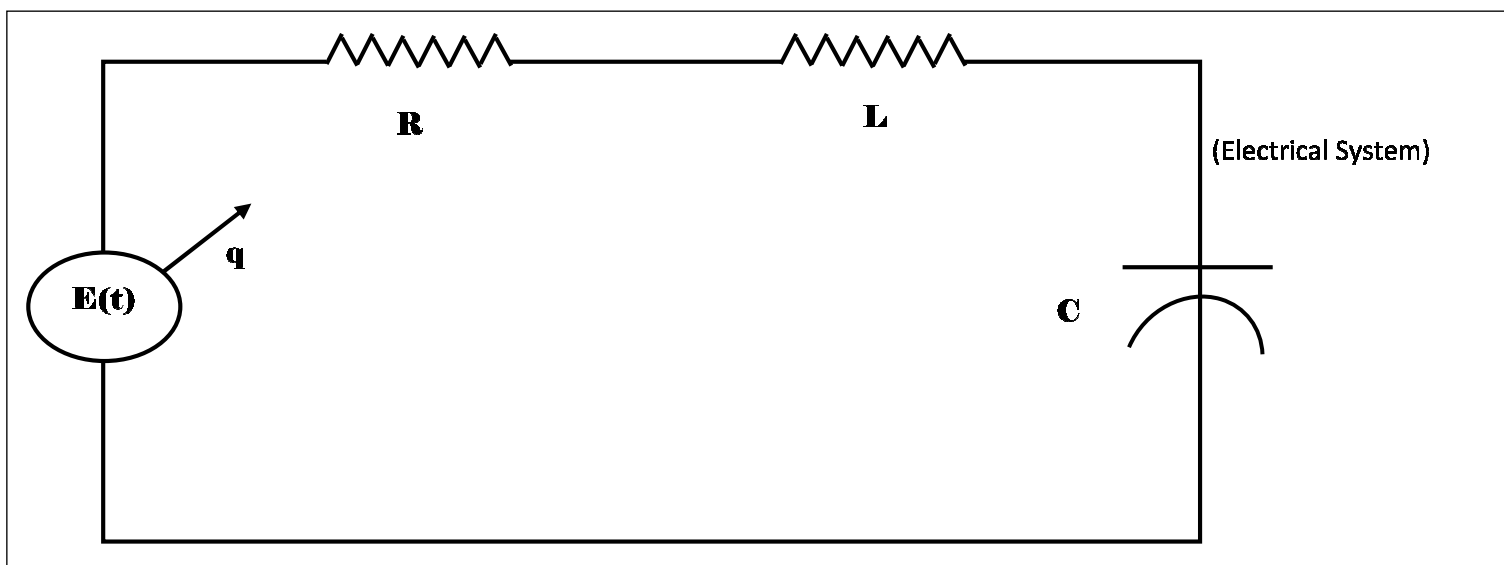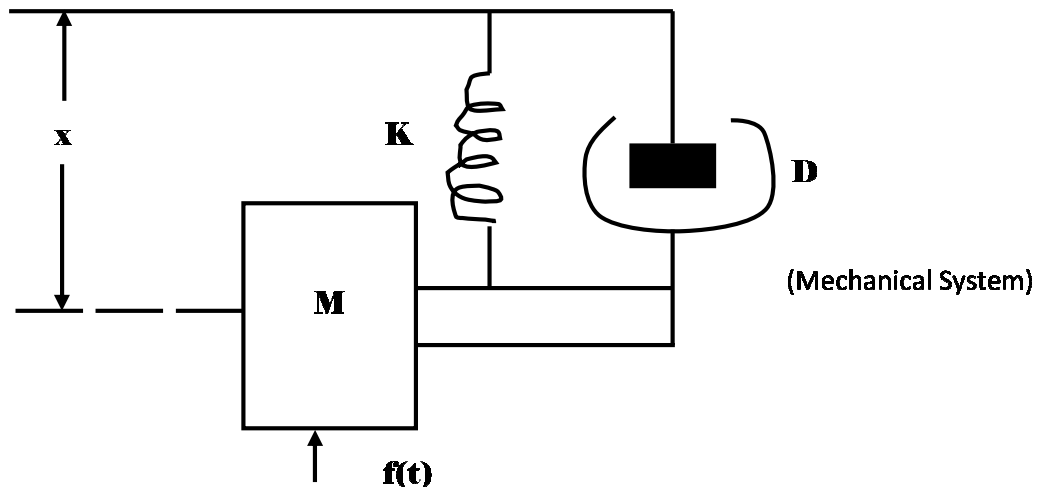
Extension and contraction and shock absorber that absorbs the damping force proportional to the velocity of the mass. The motion of the system can be represented by the following differential equation.

$$\Rightarrow M\ddot{x} + D\dot{x} + kx = \text{KF (t)}$$

$$\Rightarrow \dot{x} = \frac{dx}{dt} = \frac{displacement}{time} = \text{velocity}$$

$\Rightarrow \ddot{x} = \frac{d^2x}{dt^2}$ = Rate of change of velocity = Acceleration

Where, M = Mass

D = Damping factor of shock absorber

K = Stiffness constant of spring

x = Displacement of mass

F (t) = Applied force

The 2nd figure represents an electrical circuit with a resistance "R" and inductance "L" and capacitance "C" connected in series with a voltage source that varies in time according to the function E (t). Let "q" be the charge on the capacitance. This system can be represented by the following equation-

$$L\ddot{q} + R_{\dot{q}} + \frac{q}{c} = \frac{E(t)}{c}$$

$\Rightarrow \dot{q} = \frac{dq}{dt}$ = Rate of change of charge = Current

$\Rightarrow \ddot{q} = \frac{d^2q}{dt^2}$ = Rate of change of current = Voltage

Where, L = Inductance

R = Resistance

q = Charge on Capacitance

C = Capacitance

E (t) = Function on voltage source that varies with time

By comparing two equation of mechanical system and electrical system which are similar to each other given below.

MECHANICAL SYSTEM	ELECTRICAL SYSTEM
Mass (M)	Inductance (L)
Damping Factor (D)	Resistance (R)
Spring Constant (K)	Capacitance (1/C)
Force F(t)	Applied Voltage E(t)
Displacement (x)	Charge (q)
Velocity $(\dot{x})$	Current $(\dot{q})$
Acceleration $(\ddot{x})$	Voltage $(\ddot{q})$

Both the systems are analogues of each other and the performance of either can be studied with the other. It is similar to modify the electrical system than to change the mechanical system. Example: Two credit what effect a change in the shock absorber will have on the performance of the car. It will only necessary to change value of resistance in the electrical circuit and observe the effect on the way the voltage varies.

Static Mathematical Models:

A static model gives the relationship between the system attributes when the system is in equilibrium. Example: In marketing a commodity there is a balance between the supply and demand for the commodity. Both factor depend upon price. The demand for a commodity will be low when the price is high and it will increase as the price drops.
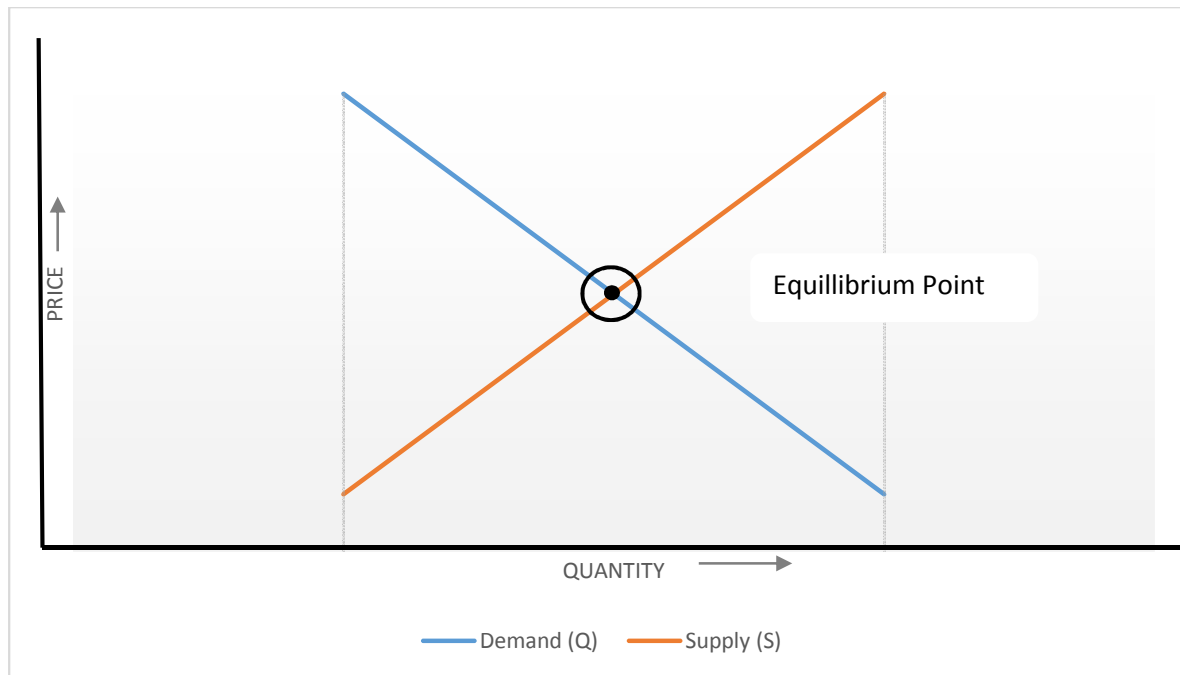
Q = a - bp

S = c + dp

S = Q (Equilibrium)

Q = Demand

P = Price

S = Supply

a, b, c, d are constants.

c + dp = a − bp⇒ (b + p) d = a - c

$$P = \frac{a - c}{b + d}$$

(Equilibrium price where demand = supply)

Dynamic Mathematical Model:

It allows the changes of system attributes to be derived as a function of time. This derivation may be made with analytical solution or with a numerical computation depending upon the complexity of the model.

$M\ddot{x} + D\dot{x} + kx = $ KF (t)

This an equation of a wheel suspension of a vehicle.

Principles used in modelling:

The guiding principle of modelling are:

a. Block Building
b. Relevance
c. Accuracy
d. Aggregation

Block Building:

The description of the system should be organized in a series of block. The aim is to simplify the specification of the interaction with in a system. Each block describes a part of the system that depends upon the few input variables and results in few output variables. The system as a whole can be described in terms of interconnection between the blocks and can be represented graphically as a simple block diagram. Example: The description of a factory is an example of a block diagram where each department has been treated as a separate block where the inputs and outputs being the work passed from department to department.

Relevance:

The model should only include those aspects of the system that are relevant to the study of the objective. A relevant information should be excluded because it increases the complexity of the model and causes more work in solving the model.

Accuracy:

The accuracy of the information gathered for the model should be considered.

Aggregation:

It is the extent to which the number of individual entities can be grouped together into larger entities. In some studies it may be necessary to construct artificial entities through the process of aggregation. Example: An economic or social studies usually treats a population as a number of social classes and conduct the study as each social class as a distinct entity.

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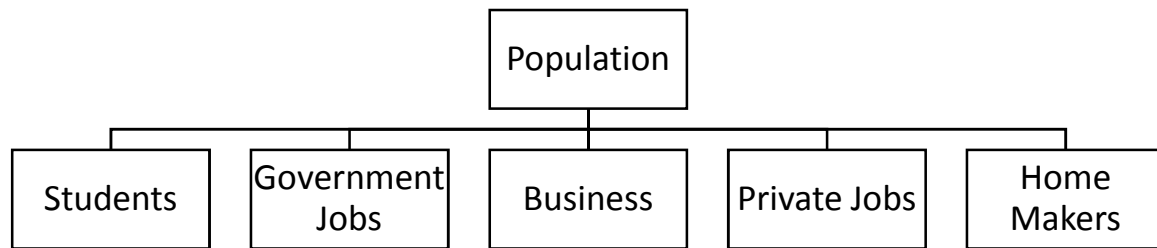
Application Areas of Simulation:

Simulation is an experimental technique. It is a fast and experimental method of doing an experiment under computer. There is no specific unifying theory of computer simulation and no principle guiding the formulation of simulation model. Simulation may be applied for system, there is difficult to predict like whether or public jam where the system itself is complex or the theory is not sufficiently regular. Simulation provides an alternative that is cheap and fast and fills the gap between exact analysis and physical intuition.

Simulation in Science and Engineering Research:

Earlier most experiments were carried out physically in the laboratories. Millions were spent on physical models like network analysers, Aircraft flight, Simulators and expensive experiments.

Today a majority of experiments are simulated in a simulator. Computer experiments are faster cheaper, easier and provide better inside into the system than laboratory experiments.

Simulation in Soft science:

Simulation plays vital role in biology, sociology and economics, medicines etc., where experimenting could be very expensive dangerous or even impossible.

In field such as Biology and economics, the problems are large involving thousands of variables and are complicated due to uncertainty.

Simulation for Business executive:

There are many problems faced by management that cannot be solved by standard operations research tools like linear and dynamic programming inventory and queuing. So a business executive has to make a decision best on his intuition and experiment. But with computer simulation he can make better and meaningful decisions. Simulation is widely used for inventory control, facility planning, production scheduling etc.

System Simulation:

It is defined as the technique of solving problems by the observations of performance over time of a dynamic model of system. This definition includes the use of dynamic physical models where the results are derived from physical measurements rather than numerical computations.

Monte Carlo Technique of Simulation:

Figure - The Monte Carlo Method
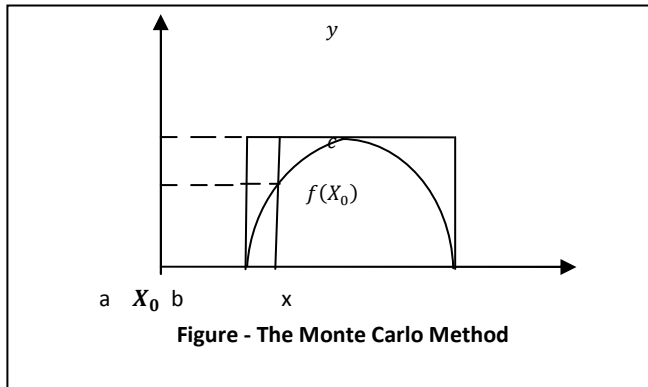
The Monte Carlo method is a numerical computational method which consists of experimental sampling with random number.

For example, the integral of a single variable over a given range corresponds to finding the area under the graph representing the function. Suppose the function $f(x_0)$ is positive and has lower and upper bound a and b respectively and the function is bounded above by the value c. The graph of the function is then contained within a rectangle with sides of length (b-c) and c. If we are pick points at random within the rectangle and determined whether they lie beneath the curve or not, it is apparent that, providing the distribution of selected points is uniformly spread over the rectangle, the fraction of points falling on or below the curve should be approximately the ratio of the area under the curve to the area of the rectangle.

If 'N' points are used and 'n' of them fall under the curve, then approximately

$$n/N = \int_a^b \frac{f(x)dx}{c(b-a)}$$

$$\Rightarrow \int_a^b f(x)dx = n/N * c(b-a)$$

The accuracy improves as the number N increases. When it is decided that sufficient points have been taken, the value of integral is estimated by multiplying n with N by the area of rectangle i.e.)c(b-a).

Computational Technique:-

For each point, a value of 'x' is selected at random between a and b, say x_0. A second random selection is made between 0 and c to give Y. If Y_0 is $<=f(x_0)$, the point is accepted in the count 'n', otherwise it is rejected and the next point is picked. This method often based on integral of many variables by using a random number for each of the variables.

Monte Carlo application are sometimes classified as being simulation. Simulation and Monte Carlo method are both numerical computational technique but the difference is that simulation applies to dynamic model whereas MonteCarlo technique applied to static model.

Comparison of Simulation and Analytical Method:

The main drawback of simulation is:

- It gives specific solution rather than general solution.
  For example, in the study of automobiles wheel, an analytical solution gives all the condition that can cause oscillation. But each execution of a simulation only tells whether a particular set of condition did or did not cause oscillation. To try to find all such condition required that the simulation be repeated under many different condition.
- The step by step nature of the simulation technique means that the amount of computation increases very rapidly as the amount of detail increases. Coupled with the need to make many runs, the simulation model result in extensive amount of computing.
- Many simulation runs may be needed to find a maximum and yet leave undecided the question of whether it is a local or global maximum.

Drawback of Analytical Technique:

- The range of problem that can be solve mathematically is limited.
- Mathematical technique requires that the model be expressed in some particular format. For example, in the form of linear algebraic equation and continuous linear differential equation.
- There are many simple limitation on a system such as physical stock, finite time delays or non-linear forces which makes a soluble mathematical model insoluble. But simulation removes this limitation.

Numerical Computation Technique for Continuous Models:-

To illustrate the general numerical technique of simulation based on a continuous model, consider the following example. A builder observes that the *rate* at which he can sell houses depends directly upon the number of families who do not yet have a house. As the number of people without houses diminishes, the rate at which he sells house drops. Let **H** be the potential number of households, and **y** be the number of families with houses. The situation is represented in Fig. 1.
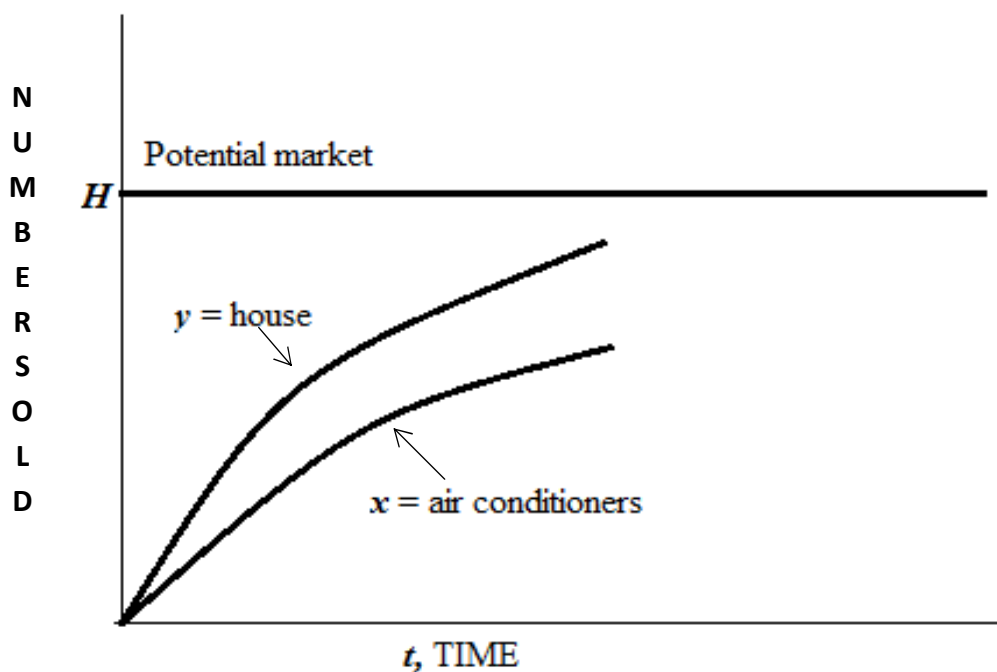
Figure 1. Sale of houses and air conditioners

The horizontal line at **H** is the total market for houses. The curve for y indicates how the number of houses sold increases with time. The slope of the curve (i.e., the rate at which **y** increases) decreases as **H-y** gets less. This reflects the slowdown of sales as the market becomes saturated. Mathematically, the trend can be expressed by the equation

$$\dot{y} = k_1 (H\text{-}y), \qquad y\text{=}0 \text{ at } t\text{=}0$$

Consider now a manufacturer of central air conditioners designed for houses. His rate of sales depends upon the number of houses build. (For simplicity, it is assumed that all houses will install an air conditioner.) As with house sales, the rate of sales diminishes as the unfilled market diminishes.

Let s be the number of installed air conditioners. Then the unfilled market is the difference between the number of houses and the number of installed air conditioners. The sales trend may be expressed mathematically by the equation

$$x` = k_2(y\text{-}x), \quad x\text{=}0 \text{ at } t\text{=}0$$

The change of s with time is also illustrated in Fig. 2. The two equation constitute a mode of the growth of air conditioner sales. Because of its simplicity, it is in fact possible to solve the method analytically. However, it quickly becomes insoluble if it is expanded to become more representative of actual marketing conditions. The market limit, for example, may not be stable. It could grow with population growth or fluctuate with economic conditions. The coefficients that determine the rates of growth could be influences by the amount of money spent on advertising, and there could be competitive influences, such as mobile homes or apartment housing. These influences could also depend upon the population growth or prevailing economic condition, and so further complicate the model.

The simple model, however, will serve to illustrate the general methods applied in suppose that the computation is made at uniform intervals of time and that the calculation has
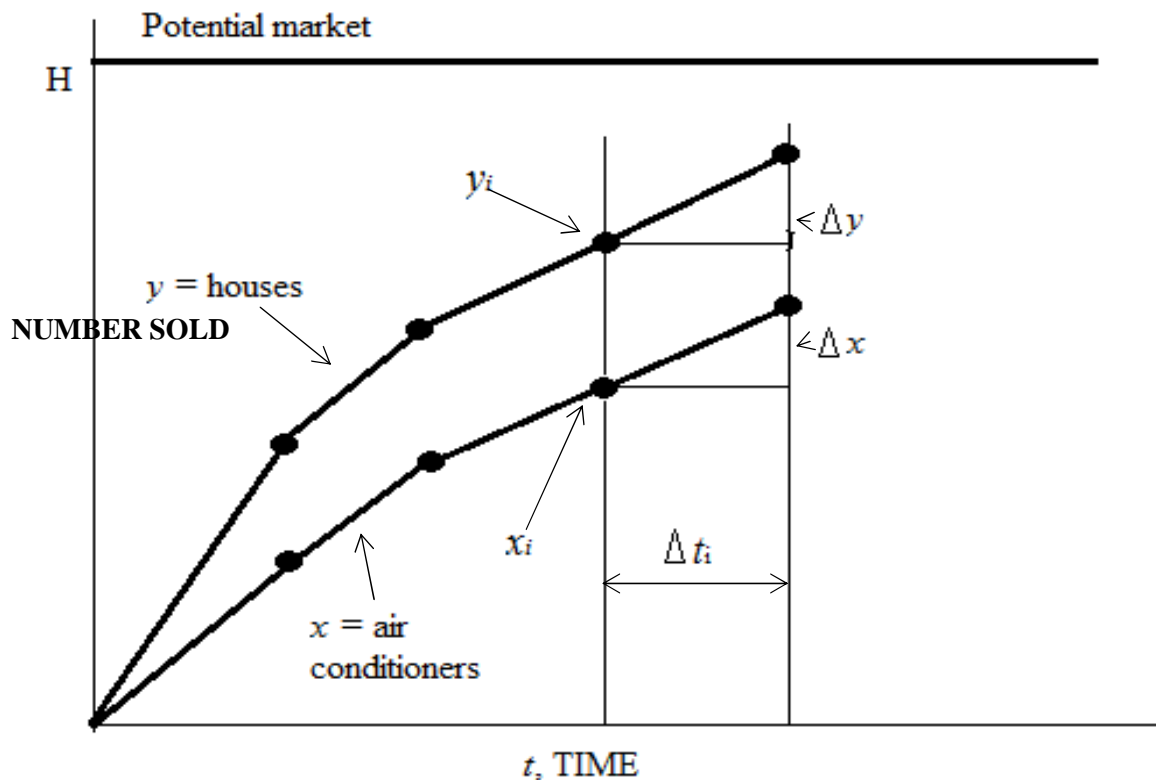

Figure 2. Calculations for air-conditioner sales model

already progressed to the time t_i when the two variables of the problem have the values y_i and s_i. Figure 2 shows the next step in the calculation.

The calculation steps forward an interval Δt to $t_{i+1} = t_i + \Delta t.$ The rates of sales are assumed to be constant over the interval. The rates can be interpreted as the amount of change per unit time. That is,

$$\text{rate of change of } y = \frac{\Delta yi}{\Delta t}$$

$$\text{rate of change of } x = \frac{\Delta yi}{\Delta t}$$

From the equations of the model, these may be written

$$\Delta y_i = k_1 (H\text{-}y_i) \, \Delta t$$

$$\Delta x_i = k_2 (H\text{-}x_i) \, \Delta t$$

Since y_i and x_i are known, it is a simple matter to get the values of y and x at time t_{i+1}. However, it will be noticed that the equation for Δy_i must be solved first to get the value of y_i needed in the equation for x_i. In preparation for the solution of a continuous system model, therefore, there must be a careful sorting of the equations to establish a workable order.

Repetition of the calculation using the new values of y and x produces the output at the end of the next interval. As illustrated in Fig.2, the calculation is equivalent to calculation the slope at each point and projecting a short straight line at that slope. The simulation output is a series of such line segments, approximating the continuous curve that represents the true output of the model.

The method described is a very simple way of integrating differential equations numerically, but it is not a very accurate method, unless small steps are used, compared with the rate at which the variables change. There are other much more accurate, and often more efficient, ways of integrating numerically which do not rely simply upon the last-known value of the variables. Rather, they use several previous values to predict the rate at which the variables are changing. (Special methods are used to supply initial values to start the process.) In addition, the computation interval is often adjusted in size to match the rate at which the variables are changing.

There are many programming systems available that incorporate continuous system simulation languages. They usually include a number of computational methods for the user's selection.

NCT for Discrete Models: -

Document No.	Start Time(Ts)	Work Time(Tw)	Finish Time(Tf)	Cumulative Time(Tc)	Break Flag(Y/N)	Pending Jobs(N)
1	0	45	45	45	N	57
2	45	16	61	61	Y	56
3	66	5	71	5	N	55
4	71	29	100	34	N	54
5	100	33	133	67	Y	53
6	138	25	163	25	N	52
7	163	21	184	46	N	51

NOTE :- $Tf = Ts + Tw,\ Tc = Tc + Tw$

Consider the above example, a clerk begins his days of works with a file of document to be processed. The time taken to process them varies. He works through the file beginning each document, as soon as he finishes the previous one except that he takes that 5min break if at that time, he finishes a document is an hour or more, since he begins work or he left a break. We assume that the time to process the documents are given. We will keep a count of no. of documents left for processing. This count will be initially set to the no. of document at the beginning of the day and we assume that no. of document arrive during the day. The count will be decremented for each completed job and the work will stop when count goes to zero.

The first column numbers the documents, the second column gives the time the clerk begins to work for a document denoted by (Ts), the third column give the time required to work on the document called as the work time(Tw), the fourth column gives the time at which the document is finished processing in time (Tf), the fifth column contains the

cumulative time since work started on since the last break measured at the time each job is completed (Tc), the sixth column contains the break flag, this take the value 'Y' ,if the clerk should take a break after the ith document and the value 'N' if he should not. The clerk works until there are no more documents to be processed. On the time he finishes the document goes beyond some prescribed time limit representing the end of the working period. The computation proceeds row by row and from left to right.
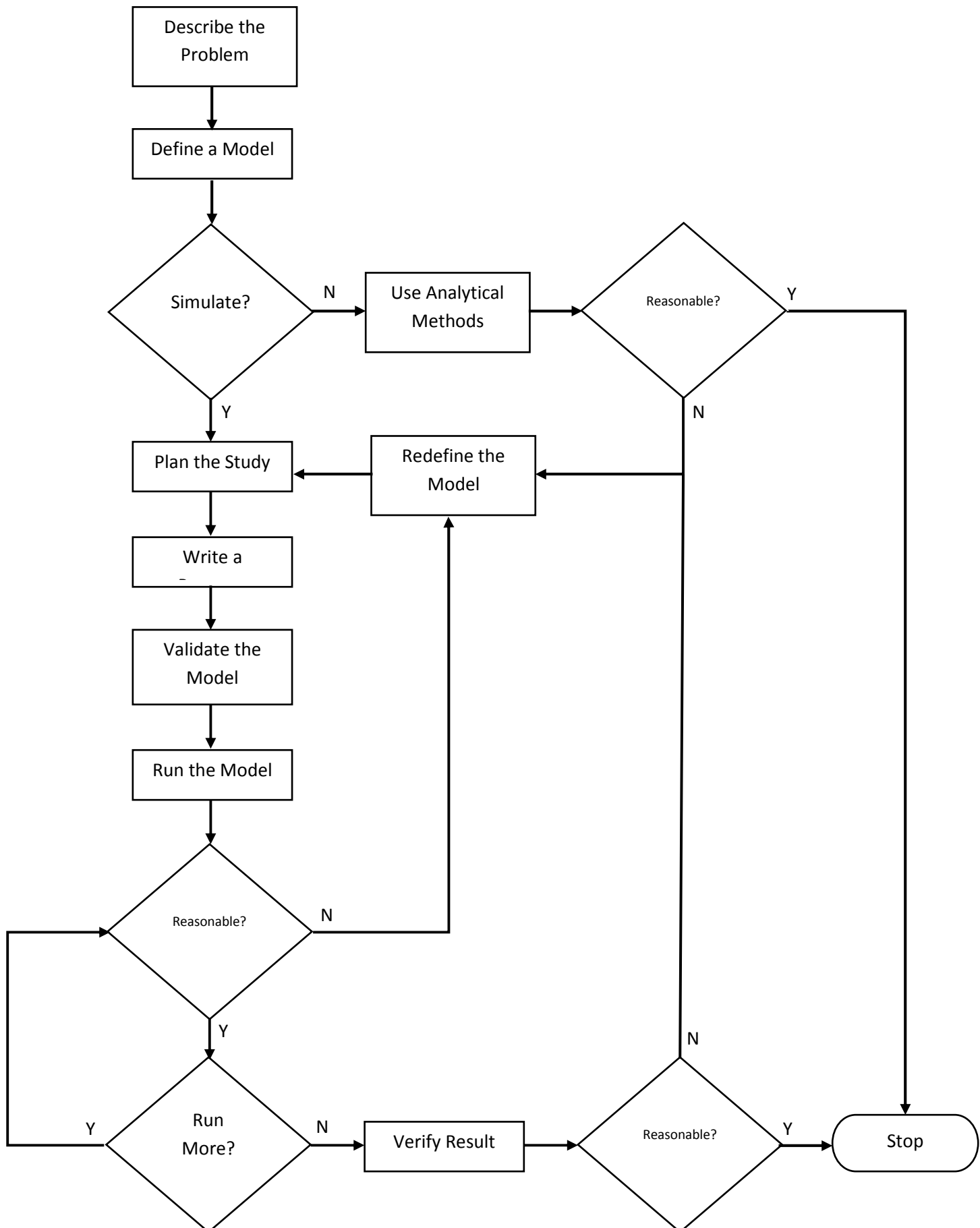
Progress of simulation study:-

An initial state is to describe the problem to be solved in a concise manner so that there is a clear statement of what question are being asked and what measurements need to be taken in order to answer those question. Based on this problem definition a model must be defined.

A possible solution is to explore to pass the model that can be solved analytically. The result will help to guide the simulation technique.

When it is decided to simulate, we must plan the study by deciding upon the major parameters to be varied, the no. of cases to be conducted and order in which runs are to be made.

Given that, the simulation is to be on a digital computer and its program must be written. The next stage is, executing a series of runs according to the study plan. The main value of the early runs in a simulation study is to get insight in to general behaviour of the system and to know which parameters are important for the system. This may lead to reassessment of the model then results are verified and if found reasonable, simulation stops.

PROGRESS OF SIMULATIONSTUDY

An initial state to describe the problem to be solved in a concise manner so that there is a clear statement of what questions to be asked and what measurements needed to be taken answer these questions.

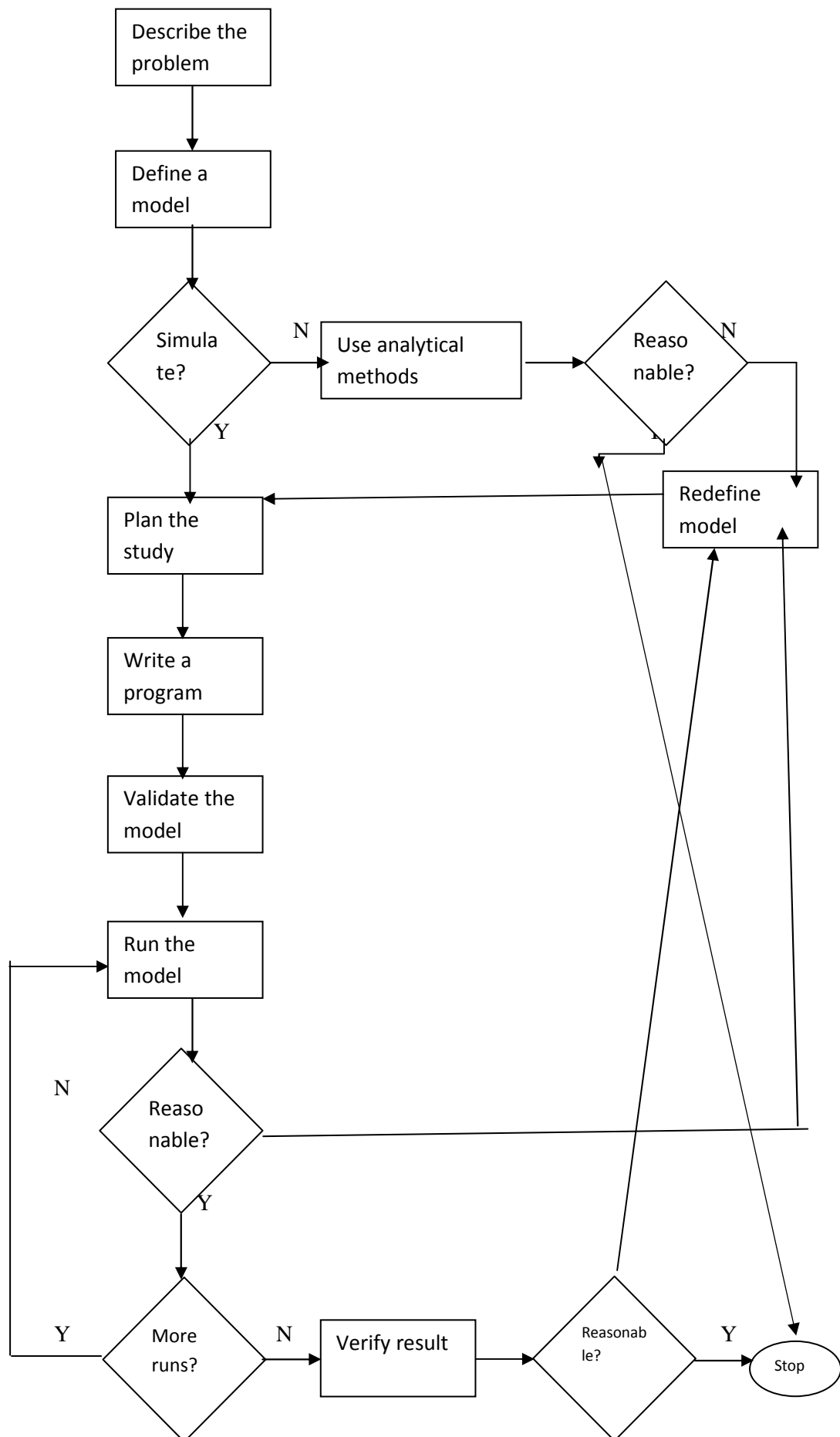- Based on these problem definition a problem must be defines
- A possible part explore the cost of a model that can be solved analytically the results in guiding the simulation study.
- When it is decided to simulate we must plan the study by deciding upon the measure parameter to be varied the number of cases to be conducted and the order in which runs at to be made.
- Given that the simulation is to be on a digital computer a program must be written. The next stage is executing runs to be study plans.
- The main value of the early runs in a simulation study is to get inside into the general behaviour of the system and to know which Parameters are important for the system.
- This may lead to re-assessment of the model, when results are verified and it found reasonable the simulation will be stopped.

Continuous System Simulation

"A Continuous system is one in which the predominant activities of the system cause smooth changes in the attributes of the system entities".

When a system is modelled mathematically the variable of the model representation the attributes are controlled by continuous system. In Continuous system the relationship describes rates at which attributes changes, so changes that the model consists of differentiation equation.

For Example wheel suspension system of an Automobile.
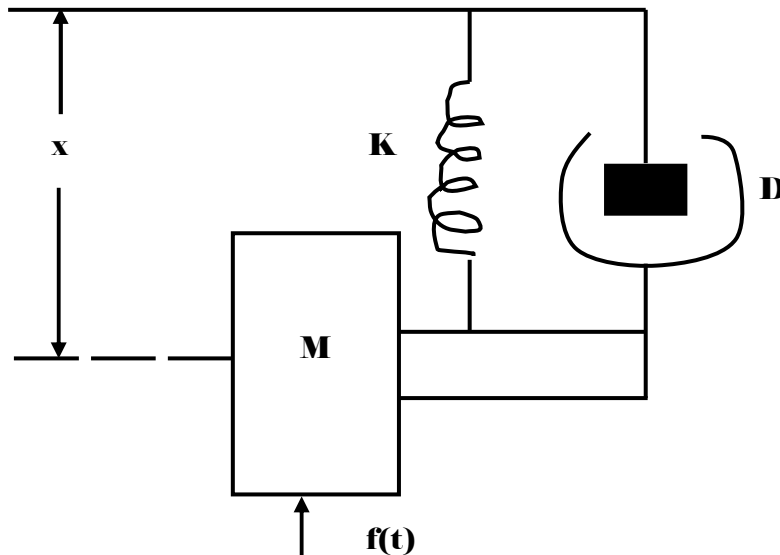
(DIAGRAM WHEEL SUSPENSION SYSTEM OF AN AUTOMOBILE)

This system represents the suspension of an automobile wheel when the automobile body is assured to be immobile in a vertical direction. The figure represents a mass 'M' which is subject to an applied force F(t) varying with time. A spring whose force is proportional to its extension or contraction given by 'K' and a shock absorber which exerts a damping force 'D' proportional to velocity of the mass.

If we pick a point of the wheel as a reference point from which to measure the vertical displacement of the wheel, the variable 'x' represents the displacement of the point taking 'x' to be positive for an upward movement. The velocity of the wheel in the vertical direction is the rate of change of displacement which is first differential.

$$\dot{x} = \frac{dx}{dt}$$

The acceleration of wheel in the vertical direction is the rate of change of velocity which is second differential i.e.$\ddot{x} = \frac{d^2x}{dt^2}$, for mass 'M' and applied force KF (t).The equation of motion in the absence of any other forces should be

$$\boxed{\text{KF(t)}= M\ddot{x}} \quad \text{---(1)}$$

But the shock absorber exerts a resisting force that depends on the velocity of wheel. This force is represented by $D\dot{x}$,where 'D' is the measure of viscosity of the shock absorber. The spring also exerts a resisting force which depends on the extend to which it has been compressed, represented by K(x), where 'K' is a constant defining a stiffness of the spring. Since both these forces oppose the motion of wheel they are to be subtracted from the applied force to given following equation.

KF (t)-$D\dot{x}$-Kx= $M\ddot{x}$

$M\ddot{x}$=KF (t)-$D\dot{x}$-kx --(2)

CSMPIII PROGRAM OF WHEEL SUSPENSION SYSTEM OF AN AUTOMOBILE

TITLE AUTOMOBILE SUSPENSION SYSTEM

*

PARAM D=(5.656,16.968,39.582,56.56)

*

CONST M=2.0, F=1.0, K=400.0

X2DOT=(1.0/M)*(K*F-D*XDOT-K*X)

XDOT=INTGRL (0.0, X2DOT)

X= INTGRL (0.0, XDOT)

*

TIMERDELT =0.005, FINTIM =1.5, PREDEL =0.05, OUTDEL =0.05

PRINT X, XDOT ,X2DOT

PRTPLT X

LABEL DISPLACEMT VERSUS TIME

END

STOP

DIAGRAM FOR CSMPIII PROGRAM OF WHEEL SUSPENSION SYSTEM OF AN AUTOMOBILE
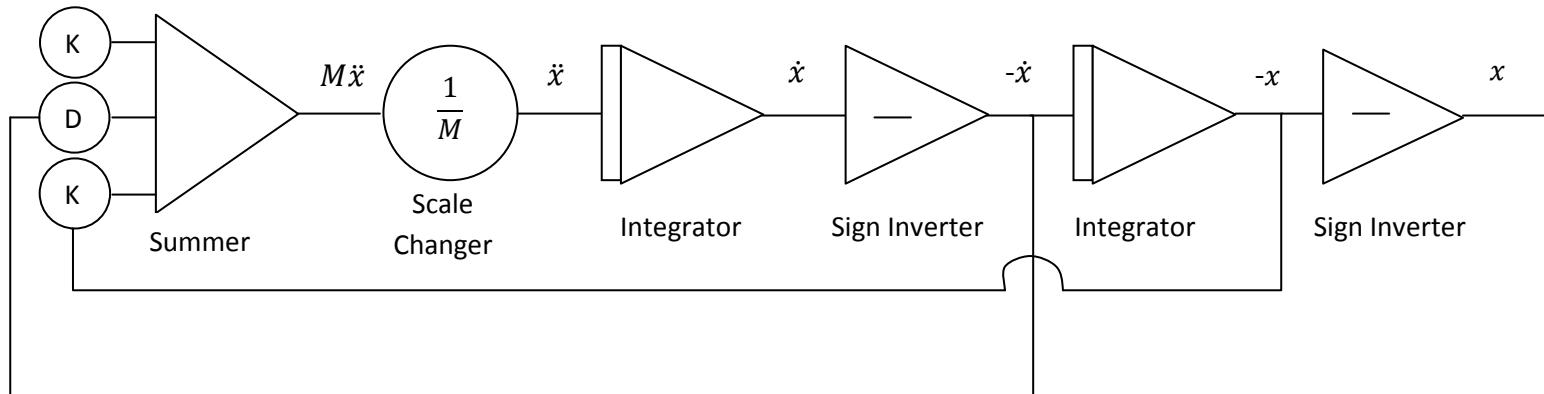
Continuous System Simulation Language (CSSL)

They use statement type of input for digital computer and allow a problem to program directly from an equation of mathematical model.

CSMP III

It consists of three types of statement,

1. Structural Statement
2. Data Statement
3. Control Statement

Structural Statement

It defines the model; it consists of FORTAN like statement and functional blocks.

Data Statement

They assign numerical value to parameters, constraints and initial conditions.

Control Statement

They specify option's in assembly an execution of the program & the choice of output.

Example:

X = 6Y/W+ (Z-2)

Structural statement, X = (6.0*Y)/W+ (Z-2.0) **2.0

Functional blocks

There are various functional block's.

1. INTEGRATOR

$$Y = \int_0^t X dt + Ic$$

Functional block=

Y=INTGRL(IC, X)

Y (0) =IC

2. **LIMITOR**

It is used to limit a function within some range.

Y=LIMIT (P1, P2, X)

P2
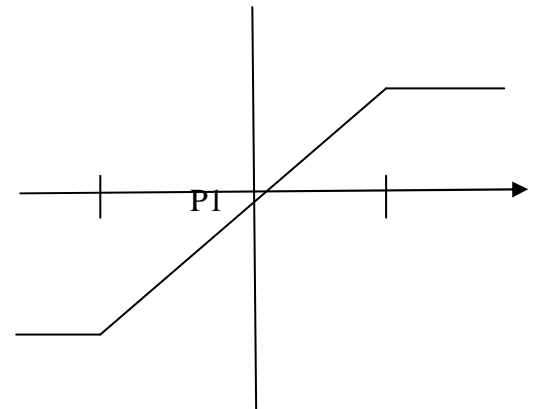
Y=P1, X<P1

Y=P2, X>P2

$Y=Y=X1P1 \leq X \leq P2$

3. **STEP FUNCTION**
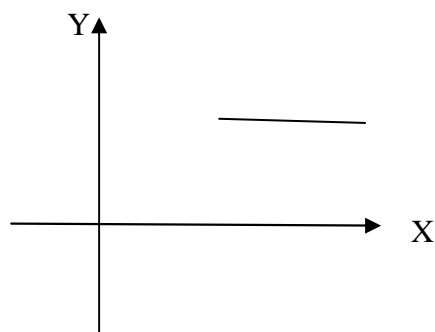
Y=STEP (P)

Y=

Y=SIN(X)

Y=ABS(X)

DATA STATEMENT

INCON

It can be used to set the initial values of the integration function.

CONST A=0.5

PARAM

It is used to specify a series of values in a parameter.

PARAM D= (0.25, 0.50, 0.75, 1.0)

CONTROL STATEMENT

TIMER

It is used to specify certain time interval.

Ex:-

TIMER DELT = 0.05 (Integration interval)

FINTIM = 1.5 (Finish time)

PRDEL = 0.1 (Interval at which to print result)

OUTDEL = 0.1 (Interval at which to print-plot)

- If printed or print-plot output is required, control statement with the word PRINT and PRTLT are follows by the names of variable.
- TITLE, LABLE is used to put heading on the printed and PRTLT output.

Example

Write a CSMP III program of an automobile suspension system.

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

Answer:-

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

$$\ddot{x} = 1/M \{KF(t) - D\dot{x} - Kx\}$$

TITLE AUTOMOBILE SUSPENSION SYSTEM

*

PARAM D= (5.656, 16.968, 39.582, 56.56)

*

CONST M = 2.0, F = 1.0, K = 400.0

X2DOT = (1.0/M) * (K * F – D* XDOT – K * X)

XDOT = INTGRL (0.0, X2DOT)

X = INTGRL (0.0, XDOT)

TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.05, OUTDEL = 0.05

PRINT X, XDOT, X2DOT

PRTPLT X

LABLE DISPLACEMENT VERSUS TIME

END

STOP

WRITE A CSMP-III PROGRAM TO SOLVE THE EQUATION

$3\dddot{X} + 15\ddot{x} + 50\dot{x} + 200x = 10$

$\dddot{x} = \ddot{x} = \dot{x} = x = 0$ at t=0

*

$3\dddot{X} + 15\ddot{x} + 50\dot{x} + 200x = 10$

$3\dddot{X} = 10 - 15\ddot{x} - 50\dot{x} - 200x$

$\dddot{X} = -1/3(15\ddot{x} + 50\dot{x} + 200x - 10)$

*

CONST M=-1/3,F=1.0,K=-10

X3DOT=(1.0/-3.0)*{(15.0*X2DOT)+(50.0*XDOT)+(200.0*X)-10}

X2DOT = INTGRL (0.0, X3DOT)

XDOT = INTGRL (0.0, X2DOT)

X = INTGRL (0.0, XDOT)

TIMER DELT = 0.005, FINTIM = 1.5, PRDEL = 0.05, OUTDEL = 0.05

PRINT X, XDOT, X2DOT

PRTPLT X

END

STOP

Advantages of analog simulation over digital simulation

- Higher speed of solution
- Direct access to an immediate display of the computer results.

MODULE – 2

PROBABILITY CONCEPTS IN SIMULATION

STOCHASTIC VARIABLES:

The variable which is random is called stochastic variables.

Ex: Tossing of a coin

STOCHASTIC PROCESS

A stochastic process is defined as an order set of random variables. The ordering of a set usually being with respect to time. Stochastic activities are those activities where the output is random. They give rise to a stochastic variable represented by a sequence of random numbers occurring over time. These activities may be discrete or, continuous. The stochastic variable might represent the number of days in a month that can take only values {28,29,30,31} or, it might be a variable representing wind velocity which can vary over a continuous range.

DISCRETE PROBABILITY FUNCTIONS

It is represented by a **probability mass function(PMF)**.If a stochastic variable can take **I** different values x_i for (**i=1,2,3,4,5,…….I**)and the probability of the value x_i being taken as **$P(x_i)$** i.e. $x_i \rightarrow P(x_i)$,the set of numbers **$P(x_i)$** is said to be **PMF** and $\sum_{i=1}^{I} P(xi) = 1$.

EXAMPLE 1.0

NO OF ITEMS BOUGHT BY CUSTOMERS

No. of items (x$_i$)	No. of customers (n$_i$)	Probabilitydistribution	Cumulative Distribution
1	25	0.10	0.10
2	128	0.51	0.61
3	47	0.19	0.80
4	38	0.15	0.95
5	12	0.05	1.00

N=250

A **cumulative distribution function** is defined as a function that gives the probability of a random variable being less than or, equal to a given value. It can increased to a maximum of 1.

GRAPHICAL REPRESENTATION OF PROBABILITY MASS FUNCTION
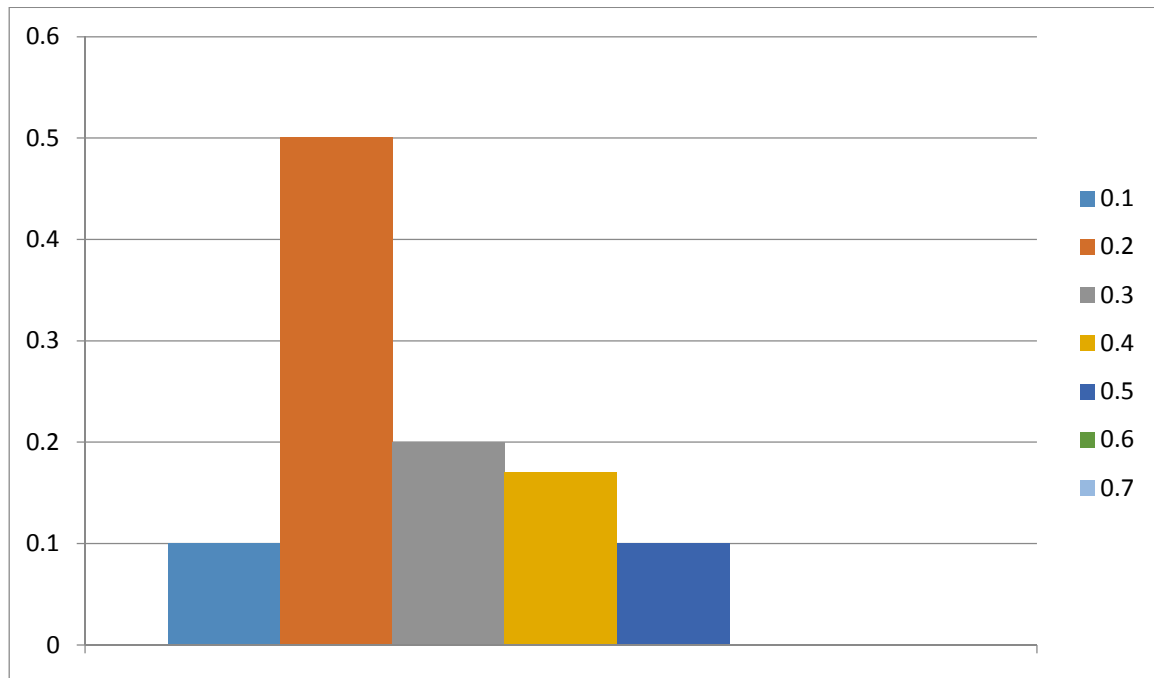
GRAPHICAL REPRESENTATION OF CUMULATIVE DISTRIBUTION FUNCTION
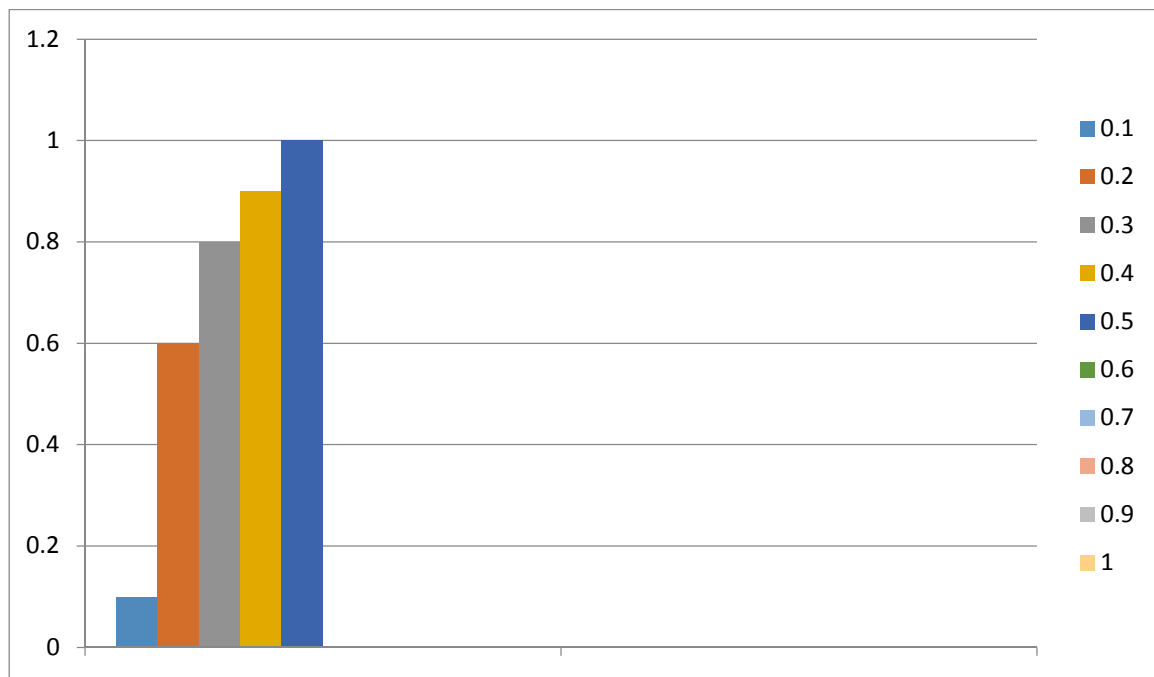
CONTINUOUS PROBABILITY FUNCTION

When a variable being observed is continuous and not limited to discrete value, it can take an infinite number of possible values. To describe these variables probability density function (**PDF**)is defined. The probability that 'x' falls in the range x_1 to x_2 is $\int_{x1}^{x2} f(x)dx, f(x) \geq 0$ where $f(x)$ is a positive function. It has the property that the integral of probability density function taken over all possible values is 1 i.e. $\int_{-\infty}^{\infty} f(x)dx = 1$

CUMULATIVE PROBABILITY FUNCTION

The cumulative distribution function for continuous variables is given by $F(x) = \int_{-\infty}^{x} f(x)dx$ .Probability of 'x' falling in the range x_1 to x_2 is $F(x_1) - F(x_2)$.

EXAMPLE 2.0

Suppose we take probability density function $f(x) = xe^{-x}$

Cumulative distribution function = cdf = F(x) = $\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} xe^{-x}dx$

Let $u = x, v = e^{-x}$ $\qquad$ So, $\int_{-\infty}^{x} xe^{-x}$

$$= x \int_{0}^{x} e^{-x} - \int_{0}^{x} \left(\frac{d}{dx}(x) \int e^{-x}dx \right) dx$$

$$= -xe^{-x} - \int_{0}^{x} -e^{-x}dx$$

$$= -xe^{-x} + \int_{0}^{x} e^{-x}dx$$

$$= -xe^{-x} - [e^{-x}]_{0}^{x}$$

$$= -xe^{-x} - (e^{-x} - e^{0})$$

$$= -xe^{-x} - (e^{-x} - 1)$$

$$= -xe^{-x} - e^{-x} + 1$$

$$= 1 - e^{-x} - xe^{-x}$$

MEASURES OF PROBABILITY FUNCTIONS

The mean value for a discrete distribution of 'N' observation is given by $m = \frac{1}{N}\sum_{r=1}^{N} x^r$

If the observation fall into 'i' group, then the formula is $m = \frac{1}{N}\sum_{i=1}^{I} n_i x_i$ OR, $m = \sum_{i=1}^{I} \frac{n_i}{N} x_i$ OR, $\sum_{i=1}^{I} P(x_i) x_i$ $(\frac{n_i}{N} = P(x_i))$

EXAMPLE 3.0

For the previous **EXAMPLE 1.0**

M=P(1)x_1+P(2)x_2+P(3)x_3+P(4)x_4+P(5)x_5

=(0.10)1+(0.51)2+(0.19)3+(0.15)4+(0.05)5 = **2.54**

The mean value for a continuous variable is given by $m = \int_{-\infty}^{\infty} x f(x) dx$

STANDARD DEVIATION

STANDARD DEVIATION FOR DISCRETE VARIABLE

Standard deviation for a discrete distribution is given by, $\left[S = \frac{1}{N-1} \sum_{r=1}^{N} (m - x_r)^2 \right]^{1/2}$ (**ungrouped data**). For grouped data $S = \sum_{i=1}^{I} P(x_i) x_i^2 - m^2$

STANDARD DEVIATION FOR CONTINUOUS VARIABLE

Standard deviation for a continuous variable is given by,

$$S = \int_{-\infty}^{\infty} f(x) x^2 dx - m^2$$

CO-EFFICIENT OF VARIATION

It is defined as the standard deviation to the mean value, $\frac{standard\ deviation}{mean}$ .It is a more useful measure of data dispersion (**scattered about mean**).Since it is expressed in relative term and does not depend upon the scale in which the data are described.

Computer generation of random numbers:

There are different methods

i)   Congruence methods
- Linear congruence generation (LCG)
- Additive congruence generation
- Multiplicative congruence generation
- Mixed congruence generation

ii)   Mid-square method

iii)   Inverse transformation method

iv)   Rejection method

Linear congruence generation:

It produces the sequence of integer x_1, x2 ..... between 0 and m-1 may following by recursive relationship

X_{i+1}= (ax_i+c) mod m ,i=0,1,2,.......... m-1.

The initial value x_0 is called the seed. a is called as multiplier, c is called increment and m is modulus.

- If $c \neq 0$ then the form is called mixed congruential generator.
- If $c = 0$, then the form is called multiplicative congruential generator.
- If $a = 1$, then the form is called additive congruential generator.

Examples:

Q1: Generate 4 digit random number by LCG with $x_0=21$, $a=34$, $c=7$.

Ans: Given generating 4-digit random number i.e. $m=10000$ and $X_0=21$, $a=34$, $c=7$.

$X_1 = (ax_0+c) \bmod m = (34*21+7) \bmod 10000 = 721$

$R_1 = X_1/m = 721/10000 = 0.0721$

$X_2 = (ax_1+c) \bmod m = (34*721+7) \bmod 10000 = 4521$

$R_2 = X_2/m = 4521/10000 = 0.4521$

$X_3 = (ax_2+c) \bmod m = (34*4521+7) \bmod 10000 = 3721$

$R_3 = X_3/m = 3721/10000 = 0.3721$

$X_4 = (ax_3+c) \bmod m = (34*3721+7) \bmod 10000 = 9311$

$R_4 = X_4/m = 9311/10000 = 0.9311$

$X_5 = (ax_4+c) \bmod m = (34*9311+7) \bmod 10000 = 6581$

$R_5 = X_5/m = 6581/10000 = 0.6581$

Q2: Generate 5 random numbers using additive congruential method with $X_0=5$, $c=7$, $m=64$.

Ans:

Here $a=1$, $x_0=5$, $c=7$, $m=64$

$X_1 = (ax_0+c) \bmod m = (1*5+7) \bmod 64 = 12$

$R_1 = X_1/m = 12/64 = 0.1875$

$X_2 = (ax_1+c) \bmod m = (1*12+7) \bmod 64 = 19$

$R_2 = X_2/m = 19/64 = 0.2968$

$X_3 = (ax_2+c) \bmod m = (1*19+7) \bmod 64 = 26$

$R_3 = X_3/m = 26/64 = 0.4063$

$X_4 = (ax_3+c) \bmod m = (1*26 +7) \bmod 64 = 33$

$R_4 = X_4/m = 33/64 = 0.5156$

$X_5 = (ax_4+c) \bmod m = (1*33+7) \bmod 64 = 40$

$R_5 = X_5/m = 40/64 = 0.625$

Q3: Generate 5 random number using multiplicative congruential method with $X_0=63$, a=19, m=100.

Ans:

Given generating random numbers using multiplicative method i.e. c=0 and given $X_0=63$, a=19 , m=100

So $X_1 = (ax_0+c) \bmod m = (19*63+0) \bmod 100 = 97$

$R_1 = X1/m = 97/100 = 0.97$

$X_2 = (ax_1+c) \bmod m = (19*63+0) \bmod 100 = 43$

$R_2 = X_2/m = 43/100 = 0.43$

$X_3 = (ax_2+c) \bmod m = (19*43+0) \bmod 100 = 17$

$R_3 = X_3/m = 17/100 = 0.17$

$X_4 = (ax_3+c) \bmod m = (19*17+0) \bmod 100 = 23$

$R_4 = X_4/m = 23/100 = 0.23$

$X_5 = (ax_0+c) \bmod m = = (19*23+0) \bmod 100 = 37$

$R_5 = X_5/m = 37/100 = 0.37$

Q4: Use the mixed congruential method to generate a sequence of three 2-digit random number with $X_0=37$, a=7, c=29

Ans:

Given generating 2-digit random number so m=100 and given $x_0=37$, a=7, c=29

So $X_1 = (ax_0+c) \bmod m = (7*37+29) \bmod 100 = 88$

$R_1 = X1/m = 88/100 = 0.88$

$X_2 = (ax_1 + c) \bmod m = (7*88+29) \bmod 100 = 45$

$R_2 = X_2/m = 45/100 = 0.45$

$X_3 = (ax_2 + c) \bmod m = (7*45+29) \bmod 100 = 44$

$R_3 = X_3/m = 44/100 = 0.44$

MID-SQUARE METHOD

This method was proposed by Von Neumann and metropolis. The technique starts with an initial no. or seed. This number is squared and the middle digit of the square becomes the random number after placement of the decimal. Again the middle digits are squared to generate the second random number and so on.

Example:

$X_0 = 5497$

$(X_0)^2 = (5497)^2$

$= 30217009$

$X_1 = 2170$

$R_1 = X_1/m = 2170/10^4 = 0.2170$

$(X_1)^2 = (2170)^2 = 04708900$

$X_2 = 7089$

$R_2 = X_2/m = 0.7089$

Drawbacks:

- Zeros are repeated in further numbers generated.
- It starts degenerating for multiples of 10, 100, ……

REJECTION METHOD :-

This method is applicable when the probability density function f(x) has a lower and upper limit, its range 'a' and 'b' respectively and an upper bound 'c'.

Compute the values of two independent uniformly distributed variate v_1 and v_2.

Compute $X_0 = a + v_1(b-a)$

Compute $Y_0 = cv_2$

If $Y_0 \leq f(X_0)$, accept X_0 as the desired output otherwise repeat the process with two new variants.
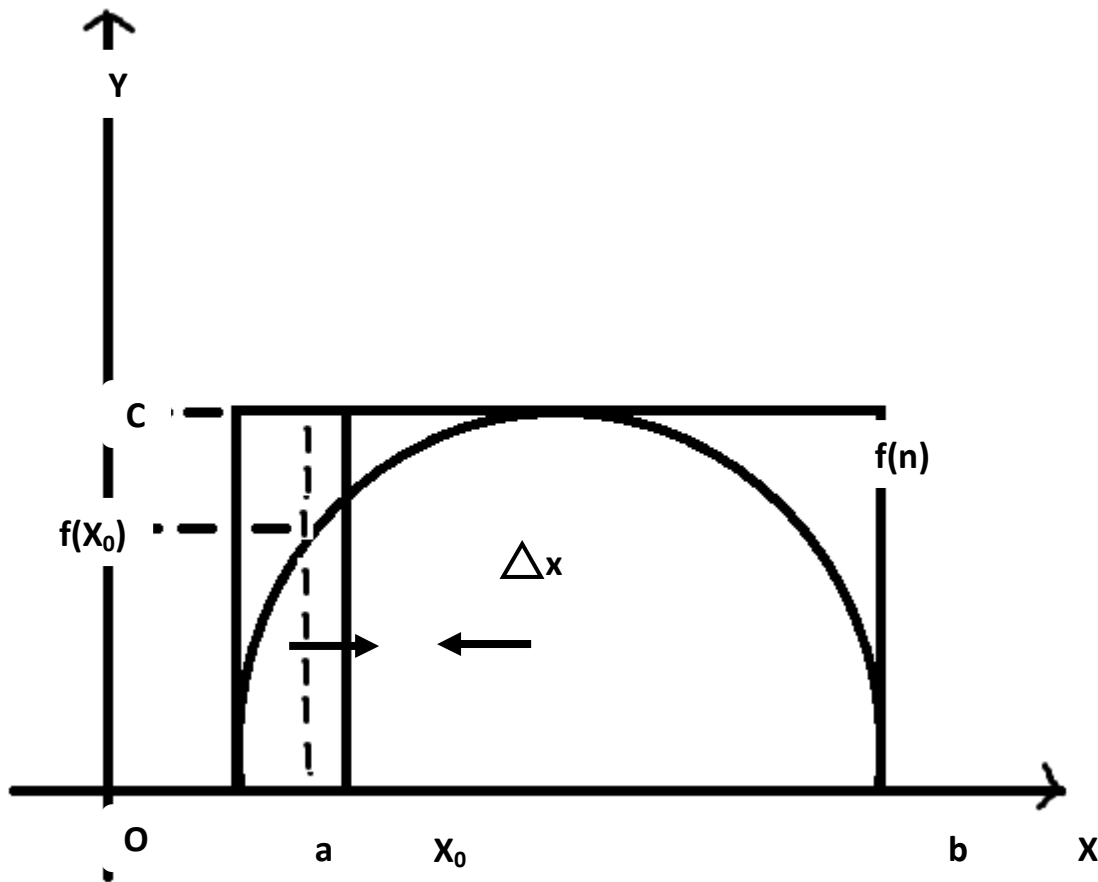
This method is closely related to Monte-Carlo technique. Here the probability density function is enclosed in a rectangle with sides '(b-a)' and 'c'. The first 3 steps of the method just create a random point and the last step relates the point to the curve of the pdf. If the points fall on or below the curve, the value 'X_0' is accepted as a sample from the desired distribution, otherwise the point is rejected and the process is repeated.

As the curve is a pdf, so the area under the curve must be '1' it means that the scale of the graph must be chosen so that the area of the rectangle must be 1. i.e. (b-a)c= 1

Consider a small interval of 'X' axis 'Δx' at the point X_0.

The probability of 'X' being less than or equal to 'X_0' is given by the cdf

$$P(X \leq X_0) = cdf = F(X_0)$$

The probability of 'Y$_0$' falling on or below the curve to the left of 'X$_0$' is the ratio of the area under that part of the curve to the rectangle with sides (X$_0$-a) and 'c'.

Since 'X$_0$' is uniformly distributed between 'a' and 'b' the probability that 'X' will be in the range

$$P(a \le X \le X_0) = X_0\text{-a/b-a}$$

So c.d.f is given by :

$$F(X_0) = \left(\left(\int_a^{x0} f(x)\,dx \right) / c(X_0\text{-a})\right) \times ((X_0\text{ -a})/\text{b-a})$$

$$\Rightarrow F(X_0) = \int_a^{X0} f(x).\,dx \,/ c(\text{y-a})$$

$\Rightarrow$ $\qquad (\because c(b-a) \approx 1)$

$$\int_a^{X0} f(x)dx$$

Which demonstrate that X$_0$ has the desired distribution.

DISADVANTAGES:

- Two uniform variants must be calculated for each trail point and since some points are rejected more than two uniform variants are needed for creation of each output point.
- The correct application of the rejection method requires that the pdf been limited it means the function should be identically '0' below a and above b.

TESTS FOR RANDOM NUMBERS

- Uniformity
- Independence

UNIFORMITY:

Frequency Test:

It uses Kolmogorov-Smirnov or the Chi-square test to compare the distribution of the set of numbers generated to a uniform distribution.

INDEPENDENCE:

Auto-co-relation test:

It tests the co-relation between numbers and compares the sample co-relation to the expected co-relation that should be 0.

There are some more tests for independence-

1. Run test

2. Gap test

The testing for uniformity of the hypothesis are:

- H_0: Ri~U[0,1] called null hypothesis.

The null hypothesis H_0 reads that the random number distributed in between [0,1]

- H_1:Ri≁U[0,1]

Failure to reject the null hypothesis means that evidence of non-uniformity has not been detected by this test.

In test for independence the hypothesis are-

- H_0:Ri~independently
- H_1:Ri≁independently

KOLMOGOROV SMIRNOV

This test compares the continuous cdf F(x) of the uniform distribution with the empirical cdf $S_N(x)$ of the sample of N observation.

By definition,

F(x)=x, 0<=x<=1

If the sample from the random number generator is-

R_1,R_2,R_3……R_N

$S_N(n)$=no. of $R_1,R_2,......R_N$ which are <=x

This test is based on the statistical D=max | f(x)=$S_N(x)$ |

Step-1

Rank the data from smallest to largest. Let R_i denotes the smallest i^{th} observation-

$R_{(1)}$<=$R_{(2)}$<=……<=$R_{(N)}$

Step-2

Compute –

$$D^+=\max_{1<=i<N}\{i/N=R_{(i)}\}$$

$$D^-=\max_{1<=i<N}\{R_i-(i-1)/N\}$$

Step-3

Compute $D=\max(D^+,D^-\}$

Step-4

Locate in table the critical value $D\alpha$ for the specified significant table 'α' & the given sample size 'N'.

Step-5

If the sample statistic (D) > the critical value (D_α) the null hypothesis that the data are sample from a uniform distribution is rejected.

If $D<= D_\alpha$ conclude that no difference has been detected between the true distribution & the uniform distribution.

For each test a table of significance 'α' is tested. This table is the probability of rejecting the null hypothesis , when the null hypothesis is true.

α =p(reject H_0| H_0 true)

The decision marker sets the value of 'α' for any tests frequently α=0.01 or 0.05.

Ex

Suppose 5 numbers 0.44, 0.81, 0.14, 0.05, 0.93.

Test these no. of uniformity using Kolmogorov Smirnov . (N=5, α=0.05)

i	1	2	3	4	5
R_i	0.05	0.14	0.44	0.81	0.93
i/N	0.20	0.40	0.60	0.80	1.0
i/N-R_i	0.15	0.26	0.16	_	0.07
(i-1)/N	0.00	0.20	0.40	0.60	0.80
R_i-(i-1)/N	0.05	_	0.04	0.21	0.13

Step-1

$D^+=0.26$

Step-2

D⁻=0.21

Step-3

D=max(0.26,0.21)=0.26

Step-4

α=0.05 , N=5

D α=0.565

Step-5

As D<D α, the sequence of random no. is uniformly distributed & the null hypothesis is not rejected.

3.Run Test (Test for independence)

This test examines the arrangement of numbers in a sequence to test the hypothesis of independence.

Run:

Run is defined as a succession of similar events proceeded & followed by a different event.

HT TH HT TTHT

1   2   3   4   5 6

Total run=6

Length of run:

Length of the run is the no. of the event occur in the event.

HTTHHTTTHT

1 2   2   3   1 1

UP RUN:

An up run is a sequence of number each of which is succeeded by a larger number- 0.07 0.12 0.53 0.85 0.92

Down Run:

A down run is a sequence of numbers each of which is succeeded by a smaller no.

0.92   0.87   0.75   0.53   0.21   0.13

-        -   -   -   -

If 'N' is the no. of numbers in a sequence the maximum no. of runs is N-1 & minimum no. of runs is 1.

If 'a' is the total no. of runs in a truly random sequence the mean & variance of 'a' is given by-

Mean

$\mu_a = (2N-1)/3$

Variance

$\sigma_a^2 = (16N-29)/90$

$\Rightarrow \sqrt{(16N-29)/90}$

Test Statistic

$Z_0 = $(total no. of runs-mean)/standard deviation

$\Rightarrow (a-\mu_a)/\sigma_a$

where Z_0 is normally distributed variable with mean 0 & variance 1.

Failure to reject hypothesis of independence occurs when $-Z_\alpha <= Z_0 <= Z_{\alpha/2}$ where α is the label of significance.

|
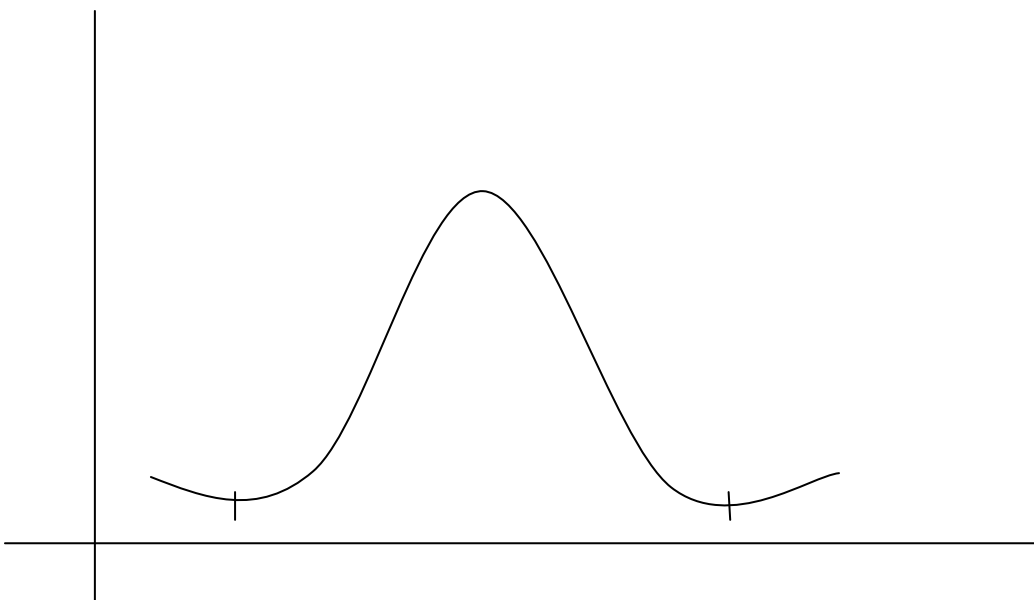
$$Z\alpha_{/2} \qquad\qquad\qquad -Z\alpha_{/2}$$

Steps

1. Find the sequence of runs up & runs down.
2. Find total no. of runs denoted by 'a'.
3. Find mean & standard deviation.
4. Compute the test statistic Z_0.
5. Compare it with the theoretical value.
6. If $-Z_{\alpha/2} < Z_0 <= Z_{\alpha/2}$, then null hypothesis of independence cannot be rejected on the basis of test and the sequence of random no. is independent. Hence it passes the runs test.

Ex

Perform runs test on the following sequence of random numbers :

0.08,0.09,0.43,0.29,0.42,0.53,0.68,0.10,0.76,0.98

<u>0.08   0.090.430.29   0.42   0.53   0.680.100.76   0.98</u>

      1   2   3   4   5

No. of runs a=5,N=10

Mean, $\mu_a = (2N-1)/3 = ((2*10)-1)/3 = 19/3 = 6.33$

Variance $= \sigma_a^2 = (16N-29)/90 = ((16*10-29)/90 = 1.455$

Standard deviation $\sigma_a = \sqrt{1.455}$

Test statistic $Z_0 = $(no. of runs-mean)/standard deviation

$$= (5-6.33)/\sqrt{1.455}$$

$$= -1.099$$

<u>Critical value</u>

Suppose $\alpha = 0.05$

    ⇨ $\alpha/2 = 0.025$
    ⇨ $-Z_{\alpha/2} = -1.96$, $Z_{\alpha/2} = 1.96$

So,

-1.96<=Z_0<=1.96, so we cannot reject the null hypothesis of independence on the basis of this test & this sequence of numbers passes the runs test.

Chi-Square Test:

The test uses the sample statistics

$$X_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Where, O_i = The observed number in the ith class

E_i = Expected number in the ith class

n = Number of classes

For uniform distribution E_i is given by,

$$E_i = \frac{N}{n}$$

Where, N = Number of Observation

n = classes

➔ It is generally done when N≥0.

Q: Test the following number for uniformity by Chi-square test.

0.15, 0.22, 0.11, 0.25, 0.09, 0.01, 0.28, 0.07, 0.13, 0.03

A: Here, N = 10

n = 10

So, $E_i = \frac{N}{n} = \frac{10}{10} = 1$

Interval Range	Interval Number	O_i	E_i	O_i- E_i	$(O_i$- $E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
0.00 – 0.10	1	4	1	3	9	9
0.10 – 0.20	2	3	1	2	4	4
0.20 - 0.30	3	3	1	2	4	4
0.30 - 0.40	4	0	1	-1	1	1
0.40 - 0.50	5	0	1	-1	1	1
0.50 - 0.60	6	0	1	-1	1	1

0.60 - 0.70	7	0	1	-1	1	1
0.70 - 0.80	8	0	1	-1	1	1
0.80 - 0.90	9	0	1	-1	1	1
0.90 – 1.00	10	0	1	-1	1	1

$$\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = 24$$

U = n-1 = 10-1 = 9

$D_\alpha = 0.05$

$$X_{9,0.05}^2 = 16.9$$

Here, $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = \; > X_0^2$ then it is not a uniform distribution and the random number is rejected.

(ans)

Generation of Stochastic Variables (Inverse Transformation Technique):-

This technique can be used for any distribution but it is most useful when the cumulative distribution system (CDF) is of a form so simple that's it inverse can be computed easily. It consists of following steps:-

I.   Compute the CDF of the desired random variable X.
II.   Set   f(x)=R, on the range of X.
III.   Solve the equation f(x) =R for X in terms of R.
IV.   Generate uniform random numbers R1,R2…………..etc. and compute the desired random variates by $X_i = f^{-1}(R_i)$ for i=1,2,3…………………………….

Example:-

Given the PDF, f(x) $= x^2/9$ on   0≤x≤3.Develope a random variable generation for this distribution using Inverse Transformation Technique.

Ans:-

Step-1:-

CDF   f(x)=∫f(x) dx

$$= \int \frac{x^2}{9} dx = \frac{1}{9} * \frac{x^3}{3} = \frac{x^3}{27}$$

$$f(x) = \frac{x^3}{27}$$

Step-2

$$f(x) = R$$

$$\frac{x^3}{27} = R$$

Step-3

$$\frac{x^3}{27} = R$$

$\Rightarrow X^3 = 27R$

$\Rightarrow X = \sqrt[3]{27R}$

$\Rightarrow X = 3\sqrt[3]{R} \ , \ 0 \leq R \leq 1$

Step-4

$X_{i=} X = 3\sqrt[3]{R} \ , \ 0 \leq R \leq 1 \ , \ i=1,2,3\ldots\ldots\ldots\ldots$

Exponential Distribution:-

It is given by the PDF $f(x) = \begin{cases} \lambda e^{-\lambda x} \ , x \geq 0 \\ 0 \qquad , x < 0 \end{cases}$

Where 'λ' is the mean no. of the occurrences per time unit.

Step-1

CDF, $\quad f(x) = \int_{-\infty}^{x} f(t) dt = \begin{cases} 1 - e^{-\lambda x} \ , x \geq 0 \\ 0 \qquad , x \end{cases}$

Step-2

Set $f(x) = R$

Step-3

$1 - e^{-\lambda x} = R$

$\Rightarrow e^{-\lambda x} = 1\text{-}R$

$\Rightarrow -\lambda x = \ln(1\text{-}R)$

$\Rightarrow x = \frac{-1}{\lambda} \ln(1\text{-}R)$

Step-4

$X_i = \frac{-1}{\lambda} \ln(1\text{-R})$,i=1,2,3……………

If R_i=0.05

$X_i = \frac{-1}{\lambda} \ln R_i$

In case of Uniform Distribution where range on R_i[0,1]

Uniform Distribution:-

Consider a random variable x,that is uniformly distributed on the interval$[a, b]$.

➢ The PDF of x is given by:-

$$F(x) = \begin{cases} \frac{1}{b-a} & , a \le x \le b \\ 0 & , otherwise \end{cases}$$

Step-1

CDF, $f(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$

Step-2

Set $f(x) = \frac{x-a}{b-a} = R$

Step-3

$\frac{x-a}{b-a} = R$

=> x-a=R(b-a)

=> -λx=ln(1-R)

=> $x = a + (b-a)R$,0≤R≤1

Step-4

$X_i = a + (b-a)R$,0≤R_i≤1

I=1,2,3……………………….

Triangular Distribution:-

Let x bethe random variable with PDF given by

$$F(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \\ 0, & otherwise \end{cases}$$

Step-1

CDF, $f(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{x^2}{2}, & 0 \le x \le 1 \\ 1 - \dfrac{(2-x)^2}{2}, & 1 < x \le 2 \\ 1, & x > 2 \end{cases}$

Step-2

Set $f(x) = R$

Step-3

$\dfrac{x^2}{2} = R$

$\Rightarrow$ $X = \sqrt{2R}, 0 \le R \le 1$

Step-4

$X_i = \sqrt{2R}, 0 \le R_i \le 1$, for i=1,2,3……………………

Arrival Patterns & Service Times

Congestion in systems

Most systems in simulation study contain processes in which there is a demand for service that causes congestion.

There are random fluctuations either in the rate of arrival, the rate of service or both.

As a result, there are times when more entities arrive than can be served at one time and some entity must wait for service.

These entities then join a waiting line.

The combination of all entities in the system those being served and those waiting for service will be called a queue.

Congestion can be described in terms of 3 main characteristics.

Arrival Pattern:-

It describes the statistical property of the arrivals.

The Service Process:-

The service process describes how the entities are served.

Queuing discipline:-

It describes how the next entity to be served is selected.

The service process is described by 3 factors.

1. Service time

2. Capacity

3. Availability of Service

Service time

The Service time is the time required to serve an individual entity.

Capacity

The service capacity is the no. of entities that can be solved simultaneously.

Availability of Service

The service may not be available at odd time.

For e.g.:- A machine can breakdown or be periodically remove from service for inspection.

Stationary and Time Variant System:-

When the statistical properties of the arrivals and the service are independent of time, they are said to be stationary.

When these factors vary with time the process is said to be non-stationary or time variant.

Arrival Patterns:-

They are described in terms of the inter-arrival time defined as the interval between successive arrivals.

For an arrival pattern, that has no variability, the inter-arrival time is a constant where the arrival varies stochastically.

It is necessary to define the probability function of the inter-arrival time.

Let T_a =Mean inter arrival time

λ =Mean arrival rate

They are related by the equation,

$\lambda = 1/T_a$ ----------------- (1)

We define $A_0(t)$ as the arrival distribution so that $A_0(t)$ is the probability that an inter-arrival time is greater than 't'.

Cumulative distribution function F (t) is the probability that the inter-arrival time is less than 't'.

$A_0(t)=1-F(t)$

At t=0, F(t)=0 => $A_0(t)=1$(maximum value)

Poisson Arrival Pattern

It assumes that the time of the next arrival is independent of the previous arrival and the probability of an arrival in an interval □t is proportional 't'.

The probability density function of the inter-arrival time is given by , $f(t)=\lambda e^{-\lambda t}$(t>=0).

The arrival distribution is given by,

$A_0(t)=e^{-\lambda t}$, where λ=the mean no. of arrivals per unit time.

The probability of 'n' arrival occurring in a period,

$P(n)=(\lambda t)^n e^{-\lambda t}/n!$

This distribution is called Poisson distribution and it is a discrete distribution.

Exponential Distribution

The cumulative distribution of exponential distribution is given by,

Cdf: $y=1-e^{-\lambda t}$

$e^{-\lambda t}=1-y$

$=>-\lambda t=\ln(1-y)$

$=>\lambda=(-1/t)\ln(1-y)$

As the numbers in 'y' are uniformly distributed in the range [0, 1], so, the numbers in (1-y) are also in the range [0,1].

So, we can change the equation,

i.e. $\lambda = (-1/t) \ln(y)$

t= - $\ln(y) / \lambda$

t= $(-1/\lambda) \ln(y)$

=> t= $-T_a \ln(y)$ $(1/\lambda = T_a)$

Co-efficient of Variation

Co-efficient of variation is the ratio of the standard deviation of the inter-arrival time to the mean inter-arrival time .

   i.e. T_a is given by,

Co-efficient of variation=S_a/T_a

It is a dimensionless measure of the degree to which data are dispersed about the mean.

A co-efficient of variation means there is no variation as the co-efficient increases, the data becomes dispersed.

Erlang Distribution

There are a class of distribution function named after A. K. Erlang who found this distribution to be representative of separate times of telephone traffics.

An arrival pattern governed by this type of functions has the probability density function given by,

$F(t) = (k\lambda)^k [e^{-k\lambda t}/(k-1)!] t^{k-1}$

An arrival distribution is given by,

$A_0(t) = e^{-k\lambda t} \sum_{n=0}^{k=1} (k\lambda t) \, n/n!$

   Where k>0.

   If k=1,then it will reduce to the standard deviation of Erlang distribution is given by , $S_a = T_a/k^{1/2}$

   And co-efficient of variation is given by,

   Co-efficient of variation=$S_a/T_a = (T_a/k^{1/2})/T_a = 1/k^{1/2} = k^{-1/2}$

Hyper-Exponential Distribution

It defines a class of distribution that has a standard deviation greater than their mean value.

So, they can be used to match data found who have a co-efficient of variation greater than 1`.

They represent data low and high value occur more frequently than in an exponential function .

The arrival distribution is given by,

$$A_0(t)=S\ e^{-2s\lambda t}+(1-s)\ e^{-2(1-sk\lambda t}\quad (0<s<=1/2)$$

Where S = Service time.

The variance is given by,

$$Variance=KT_a^2$$

So, co-efficient of variation$=S_a/T_a=\sqrt{KT_a^2}/T_a=\sqrt{K}.T_a/T_a=K^{1/2}$

Normal Discipline

The normal probability density function is symmetric about its mean value 'µ' and standard deviation (σ).

If the transformation $z = (x - µ) / σ$ is made the distribution is transformed to form in which mean=0 and standard deviation=1.

So, in this form, probability density function (pdf) is given by,

$$Pdf=f\ (t) =1/\ (2\pi)^{1/2}e^{-z2/2}$$

X - µ = z*σ

X=z*σ + µ

Where z = Standard normal variate.

Queuing Discipline

FIFO (First-In-First-Out) Discipline:-

It occurs when the arriving entities assemble in the time order in which they arrive.

Here service is offered next to the entity that has waited longest.

e.g. Railway reservation queue.

2. LIFO (Last-In-First-Out) Discipline:-

It occurs when service is next offered to the entity that arrived most recently.

e.g. the discipline followed by passengers getting in and out of a crowded train or elevator.

3. Random Discipline:-

It means that a random choice is made between all waiting entities at the time service is to be offered.

Here all waiting entities have an equal opportunity have been selected.

MODULE – 3

Discrete Event Simulation:

Discrete system is a system in which the changes are discontinuous. Each change in the state of system is called as an event. For Example, arrival or departure of a customer in a queue is an event. The simulation of a discrete system is referred to as discrete event simulation.

Applications of discrete event simulation:

- It is used by operation research workers to study large complex systems which cannot be studied by analytical method.
- In inventory control.
- Study of sea and airport.
- Steel melting stuffs.
- Telephone exchanges.
- Production line.
- Project scheduling etc.

Representation of time:

There are two different models for moving a discrete system through time.

- Fixed time step model or interval oriented model
- Event to event(next event) model or event oriented model

Fixed time step model v/s next event model:

- In a fixed time step model a timer or clock is simulated by the computer.
- This clock is updated by a fixed time interval □ and the system is examine to see if any event has taken place during this time interval.
- All events that place during this period are treated as if they occur simultaneously at the tail end of this interval.
- In the next event simulation model, the computer advances time to the occurrence of the next event. It shifts from event to event.

- The system does not change in between only those points in time are kept track when something of interest happens to the system.

Ex: To illustrate the difference between the two modules, let us assume that we are simulating the dynamics of population in a fish bowl staring with 10 fishes. If the fixed time step model is used with the interval " τ= 1 day" then the fish bowl would see once every 24 hrs and any births and deaths that take place are presume to be during the last moment of this period. But the next event model is used, then we first find out when the next event birth and death is to be take place then advance the clock exactly to that time. The next event model is preferred because it does not waste any computed time is scanning those points in time when nothing take place. The only drawback of the next event model is that its implementation is more complicated than the fixed time step model.

- Another approach to represent the passage of time is called **significant event simulation.**
- This method is applicable to continuous system in which there are quiescent periods. The interval between the events in an event oriented approach is a quiescent period.
- This approach assumes that simple analytical function can be used to protect the span of a quiescent period.
- The significant event is one with the least span.
- Determining this event by simple comparisons of the projections allows the clock to be updated by an extended period of time.

Generation of Arrival Patterns

Trace driven simulation:

➢ Here the sequence of inputs are generated from observations of a running system.
➢ Programs monitors are attached to the running system to extract the data with little or no disturbances of the system operation.

Bootstrapping:

Here the arrival time of the next entity is immediately calculated from the inter-arrival time distribution.

The term bootstrapping is used to describe this process of making one entity create its successor.

The method requires keeping only the arrival time of the next entity.

It is the most preferred method of generating arrival through computer simulation program.

Simulation of Telephone System

The System has a number of Telephone connecting to a switch board by lines. A switch has a number of links which can be used to connect any two lines subject to the condition that only one connecting at a time can be made to each line. It assume that it lost call system i.e. any call that can be connected at time immediately it abandon. A call may be lost because no link is available it is called as a lost call.

The object of the simulation is to process a given number of calls and determine what proportion are successfully completed block or busy. The current state of the system is that lne-2 is connected to line-5 and a line-4 is connected to line-7.Each line is treated as an entity and it's availability is an attribute a table of numbers is establish which show the current status of each line. A zero in the table means the line is free and where 1 means it is busy.

FROM TO LENGTH

1	7	20

LINES

1	0
2	1
3	0
4	1
5	1
6	0
7	1
8	0

LINKS

| Maximum | 3 |
| In Use | 2 |

CLOCK

| 1027 |

aRRIVAL TIME

| 1057 |

CALLS IN PROGRESS

| 1 | 7 | 1075 |
| 2 | 5 | 1053 |
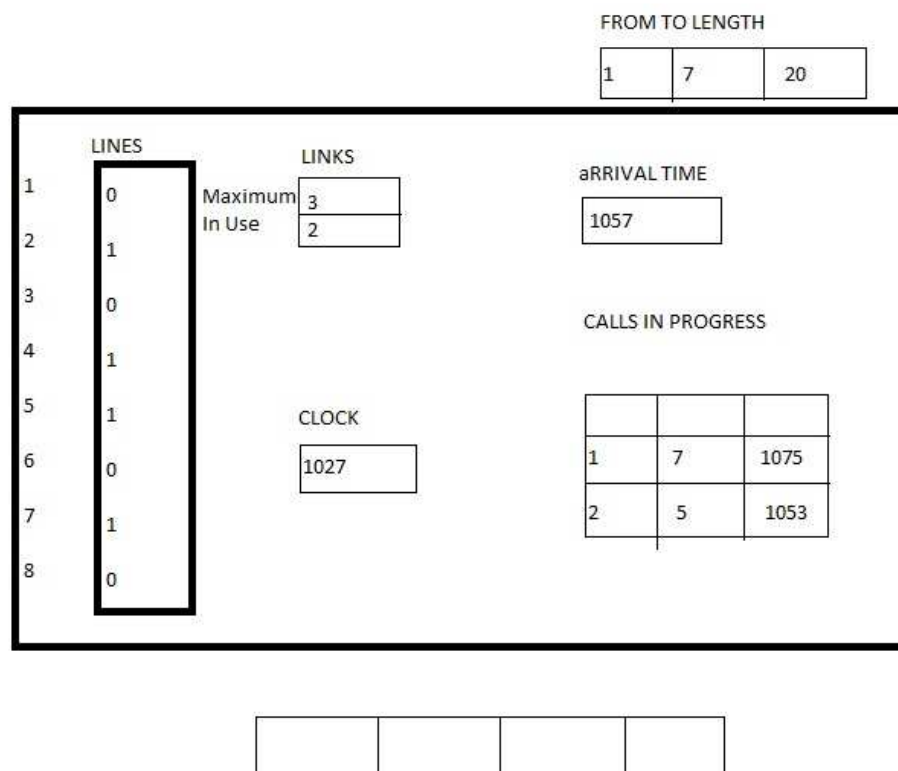
To keep track of event the number of representing currently the clock time 5 1027.The unit of time is taken to be in second. There is a list of call in progress showing which line call is connected and the time each call finishes. The arrival of call is generated by the boot strapping technique.

There are 2 activities causing event.

1. New calls can arrive
2. Existing calls can finish

There are 3-future events :

- The call between lines 2 and 5 is due to finish at time 1053.
- The call between line 4 and 7 is due to finish at time 1075.
- A new call is due to arrive at time 1057.

The simulation proceeds by exhibiting a cycle of state to simulate each event.

Steps:-

1. Find the next potential event.
2. Select an activity.
3. Test if the event can be executed.
4. Change the system event.
5. Gather statistics.

The simulation will be run until a given number of calls have been processed or until the simulation time has elapsed.

Delayed Call

Suppose the telephone system is modified so that cannot be connected are not loosed instead the wait until they can be connected, they have called as delayed calls. To keep records of these delayed calls a delayed call list is made. When a call completely ends, it is necessary to check the delayed call list to see if a waiting call can be connected.

Introduction to GPSS

GPSS (General Purpose Simulation System) language is developed by IBM Corporation. The system to be simulated in GPSS is described as a block diagram in which the blocks represent the activities and lines joining the blocks indicate the sequence in which the activities can be executed.

The approach taken in GPSS is to define a set of 48 specific block types each of which represents a characteristics action of system. The program user must draw a block diagram of the system. Using only these 48 block diagram.

e.g. - A communication system is concerned with movement of messages. A rood transparent system with motor vehicle or a data processing system with records.

In simulation these entities is called transaction. The sequence of event is in real time is reflected in the movement of transaction from block to block in simulated time. Transactions are created at one or more generated block and are removed from simulation at terminate blocks.

Transaction have a priority level and that can carry item of data called parameter. This parameter exists in 4 format. They can be signed integers of

- Full words (F)
- Half words (H)
- Byte size (B)
- Floating point (L)

Simulation of a manufacturing shop

A machine tool in a manufacturing shop is a turning out parts at the rate of 1 every 5 minute. As they have finished the part go to an inspector who takes 4±3 to examine each one & rejects about 10% of the parts. Each part will represented by a transaction and the time unit selected for the problem will be 1 min.

Block diagram
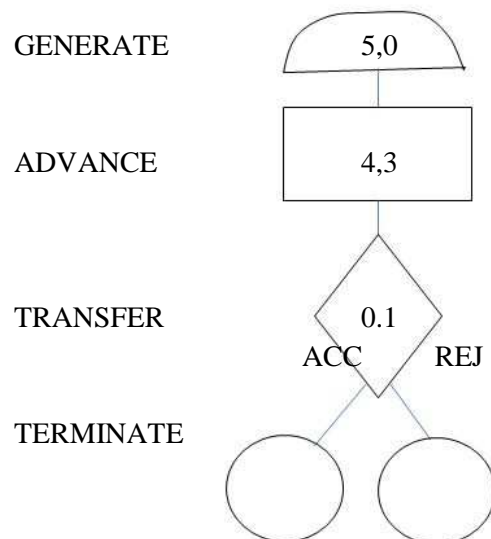
GPSS program

Rules

- Column 1 only used for a comment kind.
- Column 2 to 6 contain the location of the block to be specified.
- Column 8 to 18 contain the block type name which must begin in column 8.
- Begin at column 19, a series of fields each separated by commas and having no embedded blanks.

GENERATE	5,0	create parts
ADVANCE	4,3	inspect parts
TRANSFER	0.1,AAC,REJ	select or reject
TERMINATE	1	accepted path
TERMINATE	1	Rejected path
START	1000	Run for 1000 Parts

Statistics

Relative Block :	5005	Absolute Clock :	5005

Block Count

Block	Current		Total
1	0	1001	
2	1	1001	
3	0	1000	
4	0	900	
5	0	100	

FACILITIES AND STORAGES

There are two types of permanent entities defining GPSS to represent system equipment. Facilities and& storages.
A facility is defined as an entity that can be engaged by a single transaction at a time like tollgate.
A storage is defined as an entity that can be occupied by many transactions at a time up to predetermined limit.
e.g.- in transportation system vehicles are transaction a tollbooth maybe be considerable as a facility and a road as a storage.
e.g.- In a communication system messages are transaction switch is a facility and a trunk is a storage. A switch is assumed to pass only one message at time while a trunk consists of many wires which can carry several messages simultaneously.

Facilities {
SEIZE

RELEASE
}

Storage {
ENTER
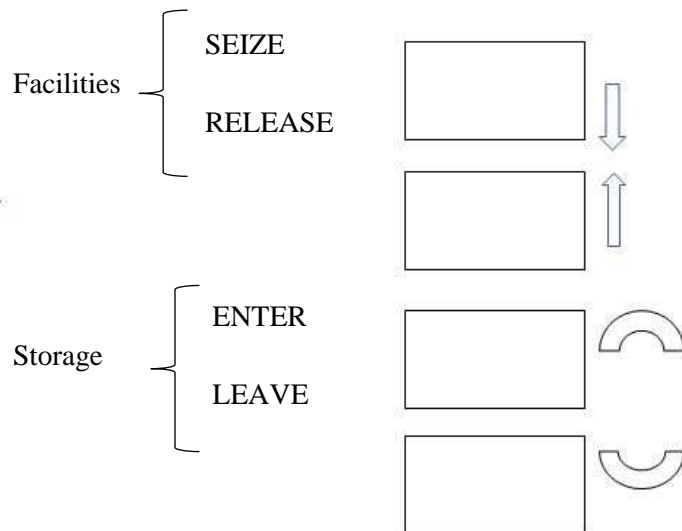
LEAVE
}

The seize block allows a transaction engage a facility if it is available. The RELEASE block allows a transaction to disengage the facility.
An ENTER block allow a transaction to occupy a space in a storage if it is available and the leave blocks allows it to give up the space (free the space).
In previous model inspection is a facility.

GPSS diagram with facility

GENERATE   5,0

SEIZE

ADVANCE   4,3

RELEASE

TRANSFER   0.1

ACE   REJ
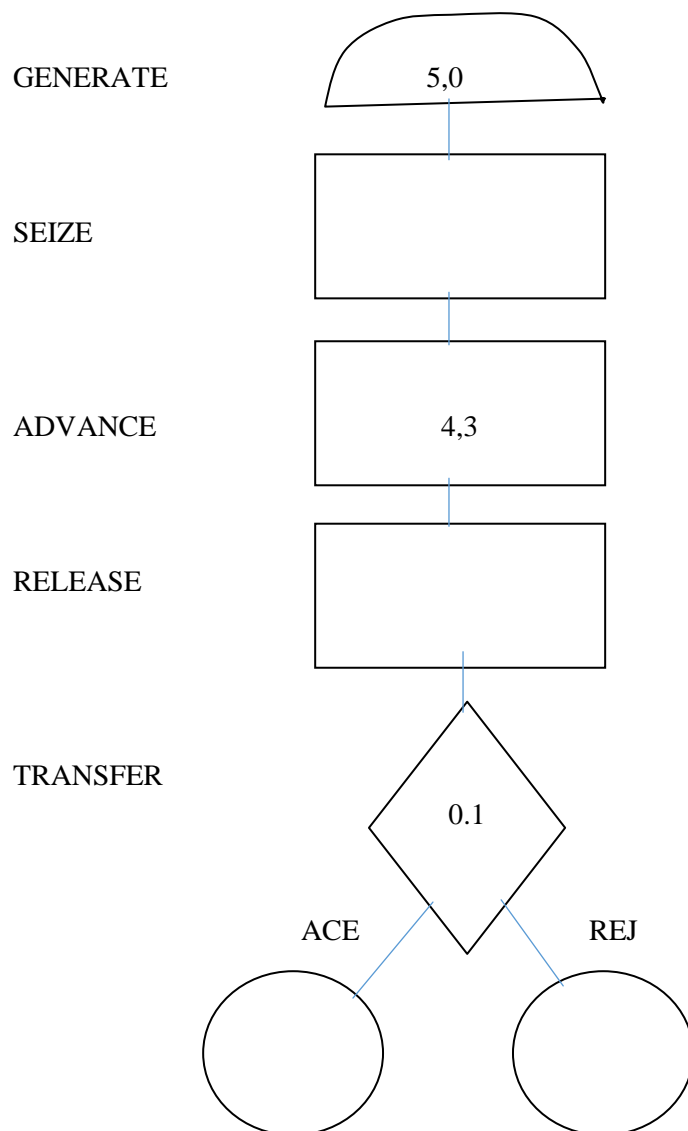
TERMINATE

PROGRAM CONTROL STATEMENT

1. Simulate

2. Reset

3. Clear

4. Job

5. End

Simulate

The first statement of GPS system is the control statement in the word simulate in the operation field without the statement the problem will be assembled but not exhibited.

Reset

The statement with the single word reset in the operation field with wipe out all the statistics gathered so far but will leave the system loaded with the transaction. The purpose is to gather statistics in the second run initial build up period excluded.

The output of the first run is not of interest and is suppressed by portions the rest or NP in the start statement.

The reset statement also saves to zero.

The absolute glow gives the time since the run began and the relations gives the time since the since the reset statement.

Clear

Clear is a control statement that not only wipe out the statistics of the perceives run but also wipe out the transaction the system. Here system does not reset the random number generator seeds.

Job

It is a control statement that instructs the program wipe out the entire models preceding the statement, proceed with a new problem. It resets the random number seeds.

End

The control statement with the word end in the operation field must be placed in the end of all programs to terminate all simulation.

Priorities and parameters

Each transaction has on of 128 levels of priority indicated by the no 0 to 127 with 0 being the lowest priority. When there is completion between transactions to occupy a block the transaction are advanced according to priority and FIFO within the same priority cases.

Parameters: These are items of data that represent attributes of the transaction.

Example: A transaction representing a message in a communication system can have parameters to represent the message length, message type of the destination.

SNA – (Standard Numerical Attributes)

There are the attributes of the other entities of system such as number of transaction in storage of length of a queue which are made available to the program user. Collectively they are called SNA.

Each type of SNA is identified by a one or two letter code and a number.

Example: The content of storage no 5 is denoted by S5 and length of queue 12 is denoted by 12.

Variable statement

Computation are carried in GPSS by defining variable statements in which simple SNAs are combined mathematically by operators +, -, *, /, @

Example: 5 VARIABLE S6+5*(Q12+Q17)

It define variable 5 as the sum of the current contents of storage no 6 + 5 times the sum of the length of queue 12 and 17.

In floating point, 5 FVARIABLE S6+5*(Q12+Q17)

The values of SNA change as simulation proceeds.

A block type can SAVEVALUE is used to save the value of SNA computed one time for use at some later time.

SAVEVALUE			
XF full word	XH half word	XB byte sized	XL floating point
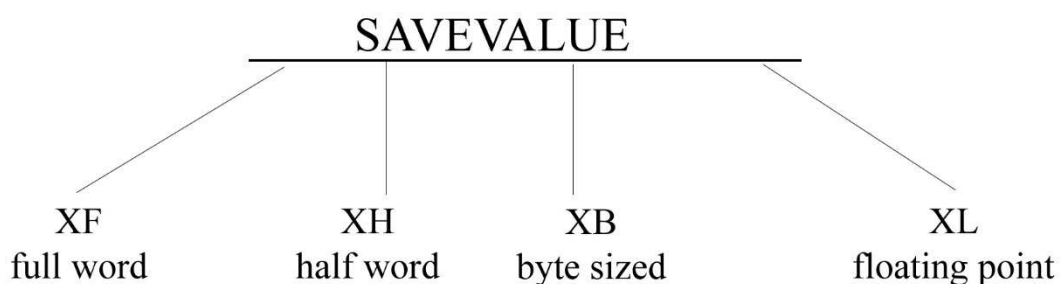
Example: SAVEVALUE 10, S6, XH

This block saves the current content of storage no 6 in half word SAVEVALUE no 10.

A control statement called INITIAL can set the value of a SAVEVALUE at the beginning of simulation.

S$BSKT can be a storage.

Functions in GPSS

Each function is defined by giving two or more pairs of number that relate and an input X to an output Y. The function mode can be in continuous or discrete mode.

To define a function, a function statement is written followed by one or more function data structure.

LOCATION	OPERATION	A	B
1	FUNCTION	S4	C5
		S4	D3

C- Continuous

D- District

C5- Continuous 5 times

The function data statement carries the value of x and y that defines the function. $X_1, Y_1 / X_2, Y_2 / X_3, Y_3$

Transfer modes:

- Unconditional
- Random
- Conditional
- Parameter
- Function

Example: TRANSFER FN, 1

1 FUNCTION PH3, L4

1, LOCA|2, LOCB|3, LOCC|4, LOCD

A list mode function assumes that an input is an integer n and it returns the nth listed value of the function.

In above, transactions are to be sent to one out of 4 locations LOCA, LOCB, LOCC or LOCD depending on the value of parameter PH3. The character L4 signifies that the function is in list mode and has 4 listed values.

Logic Switches

It is an entry used to represent 2 state condition in a system. Each switch is either ON or OFF and a block type called logic is used to change the status of the switch.

A transaction entering the block can either set the switch ON, reset the switch OFF or invert the switch fromits current state.

The letters S, R or I indicate set, reset or invert operations.

LOGIC R PH1

LOGIC S PH2

LOGIC I PH3

S→Set

R→Reset

I→Invert

A model representing factory might uses a logic switch to indicate a machine is in working order or not.

Testing condition

1. Gate block
2. Test block

1. Gate Block

Gate block is used to test the condition of any facility, storage or logic switch in the block diagram.

LS n logic switch n set
LR n logic switch n Reset
U n facility n in use
NU n facility n not in use
SF n storage n full
SNF n storage n not full
SE n storage n empty
SNE n storage n not empty

GATE LR PH2, Busy

Busy

A transaction will enter the GATE block if the condition being tested is true. When the condition is not true there is a choice of action. If an alternative block is satisfied, the transaction will be wait until the tested condition becomes true & then enter the GATE block.

2. Test block

This block is used to test a variety of relationship between any 2 SNAs (Standard Numerical Aptitude)

G - Greater than

NE - not equal

GE - greater than or equal

LE - less than or equal

E – Equal

L - Less than

TEST L S6, XHIZ, ABND

ABND TERMINATE 1

This block test whether the content of storage 6 is less than the no recorded in half word save value location 12. A transaction will enter the test block if the condition is true & will either go to an alternative block if 1 is specified or wait to enter when the condition changes.

MODULE – 4

Simulation of a Single Server Queue

Let us simulate the arrival and servicing of 'N' customers by a single server. Let these customers be marked 1 to N. Let the inter-arrival time AT_k denote the time gap between the arrival of $(k-1)^{th}$ and k^{th} customer into the system.

Let ST_k denote the service time of kth Customer from k=1, 2, 3, 4….N. Let CAT_k denote the cumulative arrival time of kth customer.

We assume that initially there is no queue and the server is free. As the first customer arrives at time 0 it goes directly to the service counter. After the service the customer departs at time ST_1. The second customer arrives at time $CAT_2 = AT_2$.

When $ST_1 > CAT_2$, $WT_2 = ST_1 - CAT_2$

When $ST_1 < CAT_2$, $IDT_2 = CAT_2 - ST_1$

When $ST_1 > CAT_2$ the second customer has to wait forming a queue of length 1 for a period $WT_2 = ST_1 - CAT_2$ where WT_k denotes the waiting time of kth customer in the queue.

If $ST_1 < CAT_2$ then the departure of the first customer takes place before the arrival of the second. Thus the service counter remains idle waiting for the arrival of second customer for a period $IDT_2 = CAT_2 - ST1$ where IDT_k denotes the idle time of the server awaiting the arrival of the kth customer. Let us assume that so far (i-1) customers have arrived into the system and j-1 customers have left, i.e. the next customer due to arrive is i^{th} and the next customer due to depart is the jth. Suppose $1 <= j <= i <= n$.

Queue length = i – j – 1, if i > j

Next Arrival Time (NAT) = CAT_i

Next Departure Time (NDT) = CDT_j

$$= CAT_j + WT_j + ST_j$$

We use the next event simulation procedure for this simple queuing situation.

We must have to determine which event to take place first i.e. i^{th} arrives first or j^{th} departs first. This is decided by comparing NAT with NDT.

DIF = NAT – NDT

If the difference is negative, an arrival is takes place first and the length of the queue is increase by one.

If the difference is positive and queue length is also positive then departure will take place first reducing the length of queue by one.

If the both these cases there is no idle time for the service counter. But DIF is positive and the queue length is zero then the service counter will lied idle waiting for the ith customer for the duration.

$IDT_i = DIF$

Finally DIF $= 0$ then both event and next arrival and departure will takes place simultaneously leaving the queue length unaltered.

Let N = 8,

AT_i 's = (0 , 10 , 15 , 35 , 30 , 10 , 5 , 5)

ST_i 's = (20 , 15 , 10 , 5 , 15 , 15 , 10 , 10)

CAT_i 's = (0 , 10 , 25 , 60 , 90 , 100 , 105 , 110)

CDT_i 's = (20 , 35 , 45 , 65 , 105 , 120 , 130 , 140)

WAT_i 's = (0 , 10 , 10,……..)

Queue Length's = (0 , 0 , 0 , 15 , 25 , 0 , 0 , 0)

IDT_i 's = (0 , 0 , 0 , 15 , 25 , 0 , 0 , 0)

TWO-SERVER QUEUE

Since many queuing system in real life has more than on server the study of multi server queues is of practical importance. Let us consider the following situation. Ships arrive at a harbour according to given probability distribution for inter-arrival time. There are two docking facilities 1 and 2 at the harbour. When the ship arrives at the system both facilities are checked to determine of either of them free. The ship directly goes to whichever facility is free. If both are free then the ship goes to the facility that is being idle for a longer period. If both are occupied then the ship joins the common queue causing a waiting time for itself and increase in the queue length by one (1). The ship in the queue are serviced - unloaded ,loaded and dispatched on first come first serve basis. We are interesting in determining the operating characteristics such as queue length, waiting tine through simulation.

Suppose we have six (6) ships arrives at time 0, 10, 48,55,65,85 hours. The simulation starts at time zero. At that time there are no ships in the system.

K	At	CAT_K	FACILITIES 1			FACILITES 2			WT_K	OL_K
			$ST_{K,1}$	$CDK,1$	IDT_K	$ST_{K,2}$	$CD_{K,2}$	$IDT_{K,2}$		
1	0	0	25	25	0	-	-	-	0	0
2	10	10	-	-	-	30	-	10		0
3	38	48	22	22	23	-	-	-	0	0
4	7	55	-	-	-	50	105	15	0	0
5	10	65	45	45	-	-	-	-	5	1
6	20	85	-	-	-	15	102	0	0	1

$CAT_K = CAT_{K-1} + AT_K$

$CADT_{1,1} = CAT_2 + WT_2 + ST_{2,2}$

$CADT_{1,1} \qquad = 0 + 25 = 25$

The first column in the table is given serial no 'k'. The second given the interval arrival time i.e. AT_k = timegap between the arrival $(k-1)^{th}$ ship and the kth ship. $AT_K = 0$. The third column gives the cumulative arrival time of the ship. It is given by $CAT_k = CAT_{k-1} + AT_k$ and $CAT_1 = AT_1 = 0$. The first ship arrives at the time zero it goes directly to facilities-1. The service time ship one at facilities one is suppose 25 hours. So on 25 hours ship one leaves the system. The column 4 gives the services time for the kth ship a t facility-1. The column 5 contains cumulative departure time there is a time elapset. Simulation times at the departure of the kth ship & it is given by the formula $CDT_{k,1} = AT_k + WT_k + ST_{k,1}$. The 6th column contains the ideal time of faclitts-1 while waiting for ship 'k' to arrive. The next 3-column gives corresponding 3 number for facility-2. The 10th column gives the waiting time in the queue for the kth ship (WT_k). The queue length immediately after the arrival of the ship 'k'. When the 3rd ship arrives which had to see which of the two facilities is valent or will fall.

This is done by comparing the latest values of the cumulative departure time $CAT_{1,1}$ & $CAT_{2,2}$. The smaller of the two indicates the time when the earliest departure from the system would take place. It is given by the variable MNDT (minimum next departure time) . MNDT = min $\{CDT_{1,1}, CDT2,2\}$ = MIN $\{25, 40\}$ = 25. So the next departure time is 25, which gives the earliest time when the ship number 3 can be started. Since $CAT_3 >$ MNDT shipnumber-3 has not arrived at time 25 and therefore

$$IDT_{3,1} = CAT_3 - MNDT$$

$IDT_{3,1}=48-25=23$

This is the amount of ideal time of facility -1 spent by waiting for ship number 3 to arrive. When ship 5 number 3 to arrive. When ship number 5 arrives at time 65, both facilities one and MNDT $=MIN\{CAT_{3,1},CAT_{4,2}\}$ i.e MIN $\{70,105\}=70$, $CAT_5=65$. So ship number 5 will form a queue and wait there for a period given by $WT_5=MNDT-CAT_5=70-65=5$.

The figure (1) gives the GPSS diagram for a single server queue simulation and figure (2) shows for a two-server queue simulation.
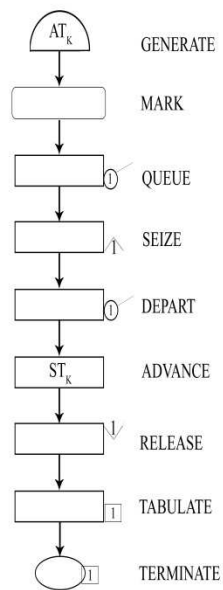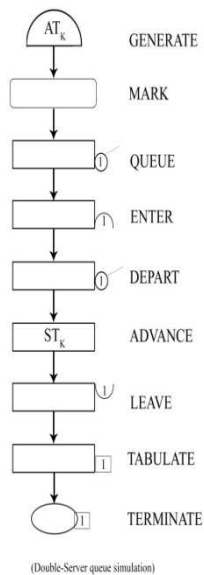
Figure (1)	Figure(2)

SIMULATION OF AN INVENTORY SYSTEM

Suppose a person works in a retail store. And it is His responsibility is to keep replenishing a certain item.

Ex: - Automobile tyres in the store by ordering it from the whole seller. We adopt a simple policy for ordering new supplies - When the stock goes down to 'P' items called reorder point then order 'q' more item called reorder quantity from the wholeseller.

If the demand on any day exceeds the amount of inventory on hand then excess amount represents lost sell and loss of good will. On the other hand over stocking implies increase carrying cost (cost of storage, insurance, interest, deterioration)

Ordering too frequently will result in excess in reorder cost.

Let us consider the following assumption.

There is a three days lag between the orders and arrival i.e. merchandise/ goods that ordered in the evening of the ith day is received in the morning of the i+3rd day.

For each unit of inventory, the carrying cost for each time is Rs. 0.75.

Each unit out of stock when order is results in to a loss of good will worth Rs. 2 per unit plus the loss of Rs. 16 income that could have resulted on a total loss of Rs. 18 per unit.

Placement of each order cost should be Rs. 75 regardless of the number of units ordered (Reorder Cost)

The demand in a day can be for any no. of units between 0 and 99, both equi-probable.

There is never more than one replenishment order outstanding.

Initially we are 115 units on hand and no reorder outstanding.

We will compare the following 5 replenishment policies and select the one that has the minimum total cost.

Total Cost = Re-order cost + carrying cost + lost sales cost

	P (Reorder Point)	**Q** (Reorder Quantity)
Policy I	125	150
Policy II	125	250
Policy III	150	250
Policy IV	175	250
Policy V	175	300

Let us simulate the running of the store for about 6 month (180 days) under each of the 5 policies and the compared with their cost.

A simulation model of this inventory system can be easily constructed by stepping time forward in the fixed increment in a day starting with day 1 and continuing up to day 180.

On a typical day day1 first we check to see whether the merchandise is due to arrive today. If yes the existence stock is increased by Q.

$$S = S + Q$$

If DEM is demand for today and DEM <= S, new stock = (S - DEM) units
If DEM > S, and the new stock will be zero.

In either case we calculate the total cost resulting from 2 days transaction and add it to the total cost incite till yesterday, then determine the inventory on hand plus units on order is greater than the reorder point. If not place an order by starting the

amount ordered and the day it is due to be received. We repeat the procedure for 180 days.

Initially we set   i= 1, Stock S = 115 units, Number of units due UD = 0 and

Due date DD = 0

FLOW CHART

```
        ┌──────────────────────────────┐
       /    read P, Q, initialize stock /
      /    due date, units due starting /
     /              day                 /
    /           total cost             /
   └──────────────────────────────┘
```

```
        ┌─────────────┐        (Merchandise received)
       /   Is today    \       ┌──────────────────────────┐
      /   the due        \ Yes │ Add Q to stock units     │
      \   date?          / ───▶│                          │
       \               /       │ Due = 0                  │
        └─────────────┘        └──────────────────────────┘
              │
              ▼  No
   ┌──────────────────────────────┐
   │ Generate random demand for   │
   │ today                        │
   └──────────────────────────────┘
```

```
                              (Shortage)
        ┌─────────────┐       ┌──────────────────────────┐
       /    Is         \      │ Add storage cost to the  │
      /   demand         \ No │ total cost               │
      \   <= stock?      / ──▶│                          │
       \               /      │ Reset stock to zero      │
        └─────────────┘       └──────────────────────────┘
              │
            Yes
              ▼
```

```
┌─────────────────────────────┐
│ Compute new stock           │
│                             │
│ Add carrying cost           │
└─────────────────────────────┘
              │
              ▼
          ◇ Is stock
            units due        ──Yes──►  ┌──────────────────────┐
            <= P?                      │ Units due = Q        │
              │                        │                      │
             No                        │ Add reorder cost     │
              │                        │                      │
              ▼                        │ Due date = today + 3 │
┌─────────────────────────────┐       └──────────────────────┘
│ Increase the day by 1       │
│                             │
│                             │
└─────────────────────────────┘
              │
              ▼
   No  ◄──  ◇ Have 180        ──Yes──►  ┌──────────────────┐  ──►  ┌────────┐
             days                       │ Print total cost │       │ Stop   │
             passed?                    └──────────────────┘       └────────┘
```

Length of simulation run

A simulation run is an uninterrupted recording of a system behavior under a specified combination of controllable variable.

One of the important question is how long to run a simulation experiment so that we have reasonable label degree of confident.

Suppose the probability density function is stationary and we take n independent sample $x_1, x_2, \dots x_n$

$X = 1/n \sum_{i=1}^{n/2} xi$

The number of required sample is given by:-

$N = (y_{j-\alpha/2}) \square^2 / t^2$

Where t=tolerence limit

$\square^2$ is variance of population parent

$(y_{j-\alpha/2})$ is two tailed standard normal static for the probability 1-α

Variance Reduction Techniques:-

These are the techniques used to reduced the variance without increasing the sample size

1. Antithetic sampling

2. Co-related sampling

3. Importance sampling

4. Control variates

5. Stratified sampling

6. Russian roulette and splitting

1.Antithetic sampling:-

The method of purposely introducing a negative co-relation between pairs of observation for obtaining a more statistically reliable answer is called antithetic sampling.

Suppose we have made two simulation runs each of length n/2 in order to estimate the mean value of some random variable. Let the value of the observation in the two runs be :

$y_1, y_2, \dots y_{n/2}$ and $z_1, z_2, \dots z_{n/2}$

The average of these two replications are

$y = y_1 + y_2 + \ldots\ldots + y_{n/2} / n/2 = z/n \sum_{i=0}^{n/2} y_1$

$\overline{z} = z_1 + z_2 + \ldots + z_{n/2} / n/2 = z/n \sum_{i=0}^{n/2} z_1$

$\square^2 \, y = \square^2 = \square^2 /_{n/2} = 2 \, \square^2 / n$

Where sigma square is the variance of the parent population. the grand mean of two raplication is

$X = 1/n \sum_{i=1}^{n/2} y_i + z_i$

$= \left\{ \begin{array}{l} 1/n \sum_{i=1}^{n/2} y_i + \sum_{i=1}^{n/2} z_i \\ \\ \end{array} \right\}$

$= 1/n \{ n y/2 + n z/2 \} = 1/2n.n(y+z)$

$Y + z/2$

i.e it is equal to avg. of two operations.

The variance of the grand mean x is given by

$\square^2 \, y = 1/n \sum_{i=1}^{n/2} [y_i - y)^2 + (z_i - z)^2]$

$\square^2 \, y + \square^2 \, z/4 + 1/2(y - z)$

Cov(u,0) denotes the co-variance of random variables (u,adv).if the two replications are independent the co-variance term will be 0. If we introduce a negative co-relation between each of the n/2 pairs of observations the variance should be reduced.

The most commonly used procedure for generating negatively co-related pair by sample is to perform two identical simulation runs but using different sets of uniformly distributed random number . if in the first run random number used the $u_1, u_2, \ldots\ldots u_{n/2}$ then the second run will be $(1-u_1), (1-u_2), \ldots\ldots (1-u_{n/2})$ thus if u_1 is greater than the avg value $(1-u_1)$ will be less. so the two set of random number are negative co-relation.

Example:-in a single server queue the inter arrival time and the services time have opposite effect on the time the customer spend in the server.

So we can reduce the variance by performing the queuing simulation twice under identical stating condition as follows .use the random number sequence that's generate the inter arrival time on the first application to generate the service time on the second.

2.Co- related sampling

This method is opposite to the antithetic sampling technique and is useful when we are estimating difference in average characteristics of two alternate designs.

Suppose a and b are sample which estimate the average waiting time pen customer using 2 different queue disciplines – FIFOand LIFO. Then the difference is given by

$\overline{D} = \overline{A} - \overline{B}$ (performance).

Statistically independent

But if the same set of random no are used in the two experiment a and b are likely to be positively co-related .In that case the variance of the difference is

$\sigma^2{}_D = \sigma^2{}_A + \sigma^2{}_B - 2\ cov(A,B)$

Thus we can reduce the variance with no increase in computing time.

3.Importance sampling:-

Age group	%
Below 15	10
15-45	50
45-60	30
Above 60	10

Here the idea is to use the sampling procedure which deliberately distorts the original distribution, so that a larger proportion of sample is drawn from the interval of importance.

Then an appropriate weighting function is used to multiply the sample data which makes the correction for distortion introduced.

4. Control variates

Here we simulate the simpler model using a sequence of Random number. Let x is estimate of the parameter of interest obtained through simulation.

Let x is the value with same parameter obtain analytically then sampling error E=x-x1.

This error is estimate if the error due to sampling by using the random number sequence u_1, u2, .u_n.

At last we simulate the complex system using same number sequence and obtain the value. Next we correct the object value y by subtracting the estimated sampling error from it.

Z=Y-E

=y-(x-x1)

Z=y-x+x1

The variance of this newest z is smaller than variance of y because x and y have strong positive co-relation.

5. Stratified sampling

Here the parent population is divided into classes (of strata) such that there is a minimum variation within each class. Then we take small number of samples from each strata. These samples have been combined to obtain the overall estimate of the mean.

6. Russian roulette and splitting

Suppose we wish to determine the probability of getting number three as the sum of two dice by rolling two dies at a time. Instead of making two rolls and observing the sum, we can roll one die and proceed with the second by observing the result of the first roll.

If the first roll produces number other than 1 or 2 we do not need to proceed to thenext roll.

This technique is used by classifying experiments with partial results at each stage as interesting or un-interesting and then proceeding only with the interesting ones.

Continuous and Discrete Simulation Language

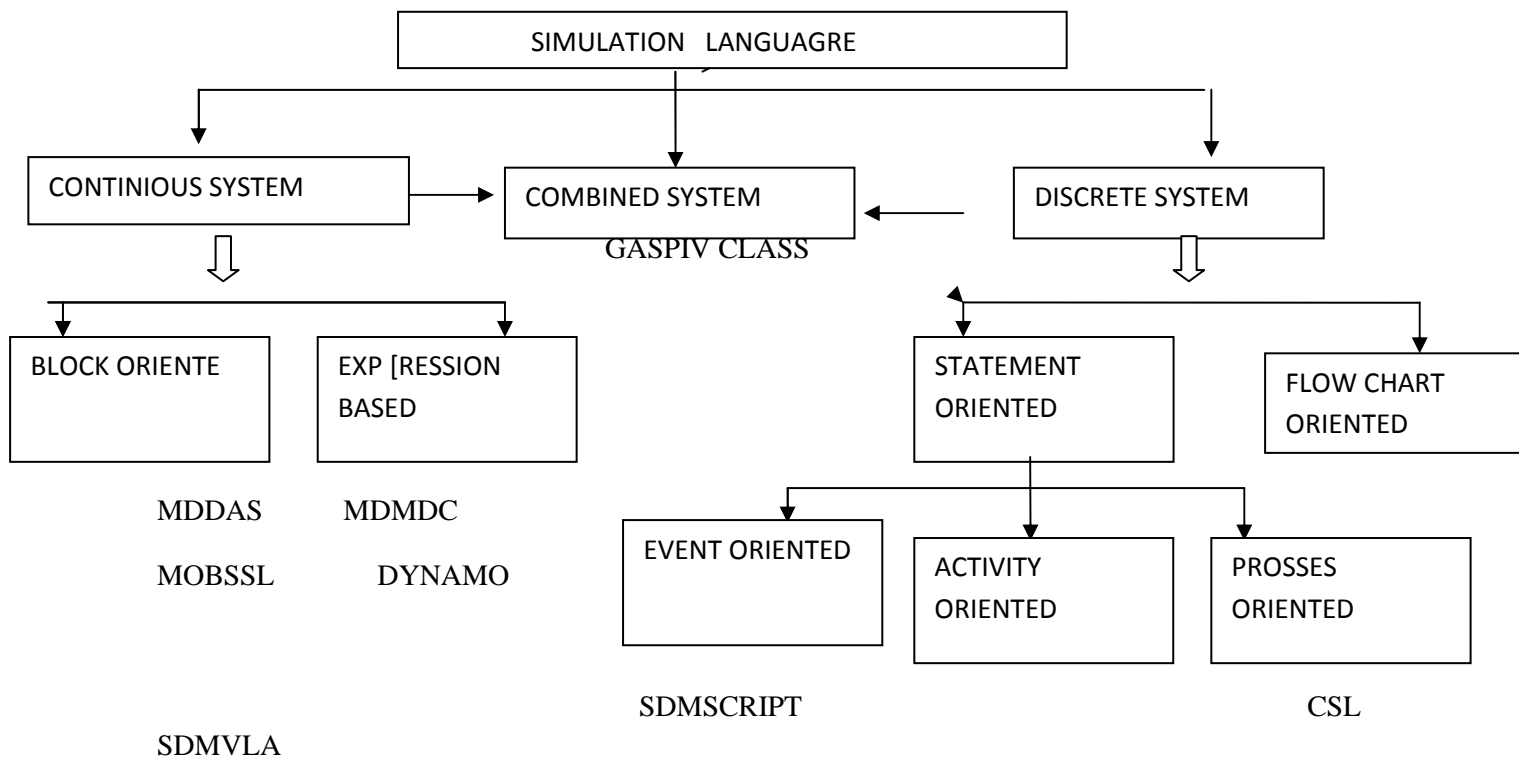Simulation languages are divided into 2 types

> ➤ Discrete
> ➤ Continuous

Continuous simulation languages are designed for simulating continuous models and discrete simulation languages for discrete models.

A language called a combined simulation language is written for system models in which some of the variables change continuously and other variables change discretely. Both variables - discrete and continuous interact with each other.

e.g.: No. Of passenger in a lift change discretely whereas the lift distance from the ground floor is continuous.

Classification of simulation language:

```
                        ┌─────────────────────────────────────┐
                        │     SIMULATION   LANGUAGRE          │
                        └─────────────────────────────────────┘
```

CONTINIOUS SYSTEM →	COMBINED SYSTEM	DISCRETE SYSTEM
	GASPIV CLASS	

BLOCK ORIENTE	EXP [RESSION BASED		STATEMENT ORIENTED	FLOW CHART ORIENTED

MDDAS   MDMDC

MOBSSL   DYNAMO

EVENT ORIENTED	ACTIVITY ORIENTED	PROSSES ORIENTED

SDMSCRIPT   CSL

SDMVLA

Block oriented continuous simulation language:

A block structure language required a user to first prepare a block diagram represent the system to be simulated.

Then he must provide patching instruction which specify now these block to be connected together.

e.g.: MDDAS, MOBSSL

Expression Based:

It doesnot need to follow block structure of the analog patch box but use different routine to implement the differential equation to the model.

S/360 CSMP

Event Oriented:

In an event oriented language each event is represented by an instantaneous occurrence in simulated time and must be schedule to occur when a proper set of condition exist.

Activity Oriented:

In an activity oriented language the discrete occurrence are not schedule advance. They are created by a program which contain description of the condition under which an activity combination taken. These condition are come between each simulation time advance and he tall necessary condition are met proper action are taken.

e.g.: CLS

Process Oriented:

In consist of a single process routine compressed of a no. of tangents describing a sequence of activity. Each segment behave as an independent control program.

Flowchart Oriented:

Here the flow of the activities passes through specially defined blocks. The system model is represented by a flow chart consisting of the language block the program creates transaction executed them in the block and moves them along the flow chart.

FACTORS FOR THE SELECTION OF A DISCRETE SYSTEM SIMULATION LANGUAGE:

a) Language generality and Program power:

The language should be able to handle a wide area of applications. It should offer strong programming power.

b) Easy to use:

It includes the natural translation of the system under study in corresponding computer program reduction in debugging program convenient   document.

c) Machine efficiency:

The cost of developing a simulation model and coding it forms a measure part of simulation   cost run time efficiency   become important in those cases where   a engage no of multiply runs are required or simulation program is used over an extended period of time. The two measures of machine efficiency are execution time and memory requirement.

d) Availability:

The language must be available and supported on the most important type of computer in order to be useful . It is also describe that the language is machine independent.
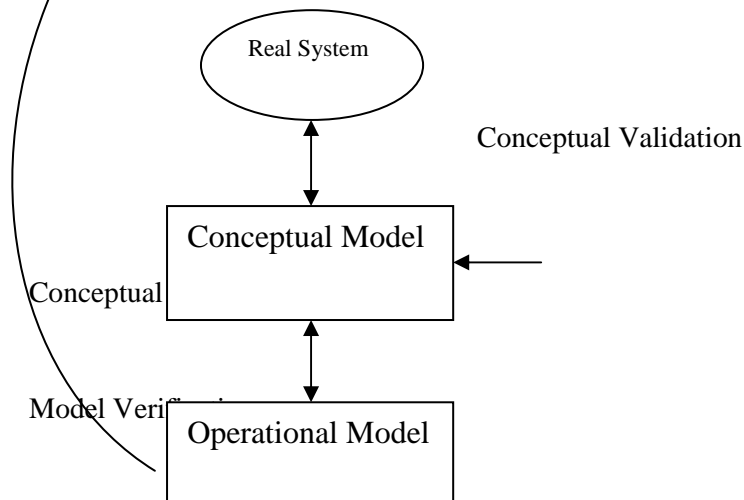

Advantages of GPSS:

- Easy to learn and use.
- Good debugging facilities.
-

Disadvantages of GPSS:

- Restricted to simple query program.
- Poor communication facilities.
- Inflexible I/O.
- No language extension possible.
- Machine efficiency lowd since GPSS is an interpretative system.

Verification of Simulation Models

(Model Building Verification and Validation)

Guidelines for Validation Process:

1. Get the operational model check by someone other than its developer preferably an expert in the simulation being used.
2. Make a flow diagram that includes each logically possibly action a system can take when an event occur and follow the model logic for each action for each event type.
3. Closely examined the model output for reasonable less under a variety of setting of the input parameters.
4. Get the operation model, print the input the input parameter at the end of the simulation to be sure that these parameters values have not changed in between.
5. Make the operational model as self-documentary as possible.
6. If the operational model is animated, variety that what is scene in the animation immediate the actual animation.
7. The interactive run controller (IRC) or debugger helps in finding and correcting errors. If monitors the simulation as it progress.
8. Graphical interfaces are recommended for accomplishing model verification and validation. It simulates the tax of understanding the model.

Validation of Simulation Models:

It ensures that the model used is a reasonable appropriation of the real system. This validation efforts developed in to two parts-

1. Validation of Abstract Model
2. Validation of its Implementation

Validation of Abstract Models:

- It consists of assuming all assumptions which transforms the real world system in to the conceptual model.
- This is highly subjective.
- Testing the validating of an implementation is a more objective and easier task. It consists of checking the logic and the flowchart and the computer program to ensure that the model has been correctly implemented.
- The easiest method of validation and implementation is the use of sample problems. The sample inputs should be chosen such that the correcting output can be obtained analytically.
  - a. Validating Existing Systems
  - b. Validity First Time Models

Validating Existing Systems:

- When the simulated system exists in real life then the best approach is to use the real world inputs to the model and compare its output to their of the real system.
- This process is straight forward but presents some difficulties
  - i. It may not always be easy to obtain input and output data from a real life system without disturbing it.
  - ii. Even if we get actual input/output of an existing model it would not be for very long periods.
  - iii. What usually is done is to simulate this model, several times with different sequences of random number and obtain the range of variation among these.
  - iv. Then if the model is valid, the real output should lie somewhere in the middle of the range of the model output.

Validating First Time Model:

- Here the model describes a hypothetical system and historical data are available to compare its performance.
-
  - a) **SubSystem Validity:**
    - ✓ A model itself may not have any existing system to compare it with but may consists of known subsystems each of whose validity can be tested separately.

  - b) **Internal Validity:**
    - ✓ One trends to reject a model if it has a high degree of internal variability (High variance).
    - ✓ The test can be perform by replicating a simulation run with several different random no sequence and then computing the variance of the output.
    - ✓ It the variance is too high we reject the model.

c) **Sensitivity Analysis:**
  - ✓ It consists of systematically varying the values the parameters or input variable one at a time while keeping all other constant over some range of interest and observing the effect upon the model response.
  - ✓ This analysis tells to which parameter, the system is more sensitive, so that we may ignore the insensitive parameters. It helps building our confidence in the model.

d) **Face Validity:**
  - ✓ If the model goes against the common sense and logic, it should be rejected. If those with experience and inside in to similar system do not judge the models as reasonable, it has to be rejected.

Experimental Layout

It discusses the problems of selection of operating conditions. An independent design variable in an experiment is called a factor and dependent variable is called a response. The number of different choices we have for a factor is called its level. If there are 'k' different factors involved in an experiment and each has 'L' different levels then the total number of different input combinations or design points (or treatments) is L^k.

Full Fractional Design: If we conduct identical experiments with all these L^k design points it is called full fractional design. It is expensive in terms of in computer time.

Fractional Factorial Design: Here we investigate model with less than L^k design points. We estimate the functional relationship between the factors and response using a reduced treatment combination and it is less expensive.

Search for optional design points:
A common purpose of many simulation studies is to determine the combination of input parameters that produce a minimum or a maximum in output.

Ex: The main purpose of simulating an inventory system is to find that policy (P, Q) which gives the minimum total inventory cost.

ANALYSIS OF SIMULATION OUTPUT

REGENERATION TECHNIQUE:-

- It is a technique used in discrete system.
- Discrete System is described by a number of state descriptor (system state-1, 2, 3) they are value at specific point in time.
- Most discrete system are such that exactly the same set of values will occur sometime later.
- That set of values will continue to recur at certain random intervals of time. A system with this property is said to be regenerative.
- A particular set of values is chosen as a reference and the times at which the same set of values recurred are called Regeneration Points.
- Example:- In   M/M/1   (Single Server Queue)
- Between successive regeneration point, a system is set to execute a tour and the time spent system tours is called Epoch. This Epochs are also independent.

Explanation:-

- Consider the measuring the waiting time in M/M/1system.
- Suppose the system starts from an empty state and measurement begin from the time the first entity arrive for service. We use the return to the empty state to define the regeneration points.
- Initially the system becomes busy with the first entity if a second entity arrive before the first complete service, the server will again remain busy and this will continue till the system server becomes idle.
- The point at which the system becomes busy and it will mark as a regeneration point.
- The number of entities served in a tour varies.
- As each busy period begins the system regenerates itself i.e. its future behaviour is independent of past behaviour.
- As a result samples from any statics taken from the tour are independently identically distributed.
- Epochs are also independently identical distributed. As a result the system behaviour in a tour is characteristics of steady state behaviour and is independent of the initial bias.

- There are two types of analysis of Simulation Output such as:-
    - Time Series Analysis
    - Spectral Analysis

Time Series Analysis:-

- At Time Series may be regarded as the summation of oscillations of different frequencies.
- It accepts the presence of Auto Correlation and estimates the variance of sample mean.
- Auto Correlation is measured by a series of auto co-variances that show the extent to which values separated by 'S' time units affect each other.
- The auto covariance between values of a variables made at X_t and X_u is defined as-

$R_{tu}=[(X_t-\mu) (X_u-\mu)]$

Where μ = Population Mean

Spectral Analysis:-

- A Spectral Analysis provides more information than an   estimate of a mean value.
- Comparing two system on the basis of mean values like the mean waiting time or, mean queue length is a gross comparison.
- Two systems may show no significant difference in that mean values but their transient behaviour may be significantly different.
- One system may respond slowly to deviation from its mean values while the other may respond rapidly.
- Spectral analysis can distinguish this difference by showing whether spectrum emphasises low or high frequencies.
- It is also make use of the covariance.
- Another use of   Spectral Analysis is to test a Simulation Model by comparing the spectral pattern of the simulation output against the corresponding pattern derive from system itself. This can be done when analysing an existing system.

SIMULATION OF A SCHEDULING SYSTEM

- CPM
- PERT

CPM (Critical path method) and PERT (program evaluation and techniques) are two methods that depicts the interrelationship between various activities of a project and identifies critical activities which need to be paid most attention.

- CPM is used for project in which the duration of each activity is determines.

- PERT is used when there is an uncertain regarding the time taken by various activities in the project. The analysis of a project through PERT is based on 3 times estimates for each activities. i.e.

    1. Most optimistic time (shortest)

    2. Most pessimistic time(longest)

    3. Most likely time for completion of each activity

NETWORK MODEL OF A PROJECT

Suppose we have a project consisting of 6 well defined non-overlapping individual jobs A, B, C, D, E & F with the restriction that A must precedes C & D, B & D must E and C must precedes F.

Dummy Activity

- It is in which represent only a procedure relationship and not any job in the project.
- Dummy activity becomes necessary when the existing activities are not enough to describe all procedure relationship separately. All dummy activities are of 0 duration and are shown by broken lines.
- The no of edges entering the node is called the in degree of a node and no of edges going out of node is called out degree of a node.
- In an activity network, the source node represent the start of the project and has 0 in degree. The sink node represent the completion of the project and has 0 out degree.

ANALYSIS OF AN ACTIVITY NETWORK

PATH: A sequence adjacent edges from the source node to sink node is called a path in a network.

1. Length of ADE= 5.1+4.5+15.8=25.4 days

2. Length of ACF=5.1+6.0+15.8= 13.6 days

3. Length of BF= 7.2+15.8= 23 days

4. Length of ADGF= 5.1+4.5+0+2.5=12.1 days

5. Length of BGF= 7.2+0+2.5=9.7 days

- The longest path is called the **critical path** of network. The activities along the critical path are called **critical activity**.

- The length of a critical path is the time required to complete a project.

- An important part of analysing network is the identification of critical activities determination of project completion time.

TOPOLOGICAL ORDER

A list of activities from a project is said to be in topological order if no activity appears in the list before all of its immediate precedes are appeared.

Activity No. (k)	S(k)	F(k)	T(k)
1	1	2	5.1
2	1	3	7.2
3	2	4	6.0
4	2	3	4.5
5	3	5	15.8
6	3	4	0
7	4	5	2.5

S (k): is the node at which the activities originates.

F(k): is the node at which the kth activity terminates.

And *T(k):* is the duration of the kth activity for k= 1, 2, 3,….7

CRITICAL PATH COMPUTATION

Where T(k) denotes the duration of the activity. A mile stone is to be achieved where all activities terminating in node I are completed.

ENT(i): denotes the earliest time when node I is achieved when I is achieved.

ENT(i) = max{EFT (all activities terminating in i)}

The earliest start time is represented by EST(k)

EST(k) = ENT(S(k))

EST(2) = ENT(1)

1. Forward Pass:

We traverse a network in forward direction from the source node to the sink node passing through every edge once. The purpose of this forward path is to compute the earliest start time, earliest finish time of each activity and earliest time to achieve each of the node. We start at node (1) of the source node and initially we set all with 0.

ENT(1)=0, ENT(2)=0, ENT(3)=0, ENT(4)=0, ENT(5)=0

EST(1) =0

EFT(1) =EST(1)+T(1)

    =0+5.1=5.1 DAYS

ENT(2) =5.1 days

EST(2) =ENT(S(2)) =ENT(1) =0

EFT(2) =EST(2)+T(2) = 0+7.2 =7.2 days

ENT(3) =7.2 days

EST(3)=ENT(S(3)) =ENT(2) =5.1 days

EFT(3) =EST(3)+T(3) =5.1+6 =11.1 days

ENT(4) =11.1 days

EST(4) =ENT(S(4)) =ENT(2) =5.1 days

TMIN=ENT(5) =25.4 days
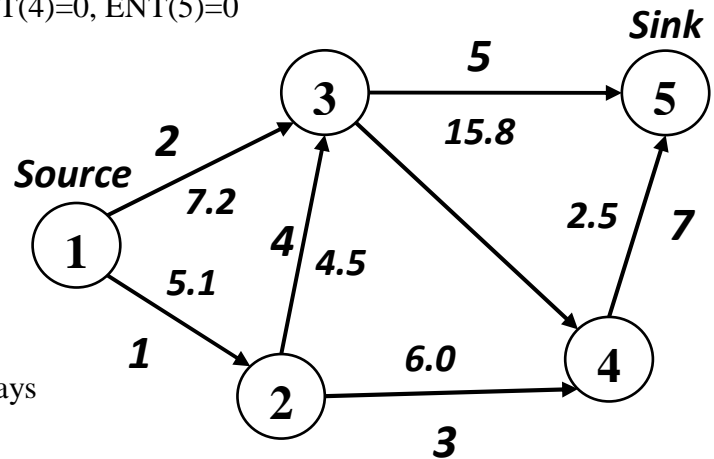
2. Backward Pass:

Here we start from the sink vertex and sets its latest node time equal to the project completion time.

    LNT(5) =TMIN=ENT(5)

LFT(I): denotes the latest time by which activity must be finished.

LST(J): denotes the latest start time of activity J.

Example:

    LFT(J)=LST(J)-T(J)

    LST(J)=LFT(J)-T(J)

    LNT(I)=min{LST(all activity originating in I)}

    LFT(every activity terminating in I)=LNT(I)

    LNT(5)=TMIN=25.4 days

LFT(7)=LNT(5)=25.4days

LST(7)=LFT(7)-T(7)=25.4-2.5=22.9days

LNT(4)=22.9days

LFT(6)=LNT(4)=22.9days

LST(6)=LFT(6)-T(6)=22.9-0=22.9days

LNT(3)=22.9days

LFT(5)=LNT(5)=25days

After doing backward path identify the activities or which the earliest start time (EST) =latest start time (LST).these activities have slack of 0. (Slack=LST-EST=0) and they have the critical activities.

Q. *Find the length of critical path taking node 1 as source and node 6 as sink.*
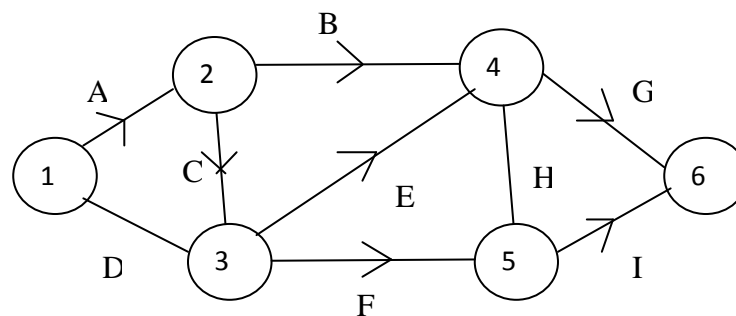
Activity duration is given by

A-7 UNITS   E=5 UNITS

B=8 UNITS   F=7 UNITS

C=2 UNITS   G=13 UNITS

D=8 UNITS   H=2 UNITS

           I=12 UNITS

The objective is to specify parametesr like run length, initial condition, no of replication etc. so has to get the maximum useful inf. about the system performance while minimizing computational cost.

LENGTH OF SIMULATION RUNS

A simulation run is an uninterrupted recording of a system behavior under a specified combination of controllable variables. One of the important question is how long to run a simulation experiment so that we have a reasonable degree of confidence in the numerical results of the experiment.

Suppose the probability density function is stationery and we take an independent samples $x_1, x_2,\ldots\ldots,x_n$. The sample is given by

$$\overline{X} = 1/N \sum_{I=1}^{n} Xi$$

The no of required sample is given by

$$[\ n = (Y1-\alpha/2)^2\alpha^2)/T^2\ ]$$

where, t is the tolerance limit

A^2 is the variance of the population

$Y1-\alpha/2$ is the two tailed standardized normal static for the probability $1-\alpha$

A is the label of significance

If the confidence label is $(1-\alpha)$ is 90%

$Y_1-\alpha/2 = 1.65$

95 %-> 1.96   99 %-> 2.58

EXAMPLE: - Suppose we have a diary for which diary milk yield randomly. We want to estimate the average value of its daily yield within +-40 □ of bits 2 average yield with a confidence level of 95%.the standard deviation of the daily yield is estimated to be 200 h. how many days we must reassure the daily yield of the firm.

Solution:-

$$n= (y1-\alpha/2)^2\alpha^2)/t^2$$

$$= (1.96)^2 *(200)^2 / (40)^2 = 96 \text{ days}$$

RUN LENGTH OF A STATIC SIMULATION:-

The two condition under which the above equation were derived are:-

1. The distribution is stationary i.e. the simulation has reached a steady state before we observing $x_1, x2,.x_n$

2. The sample $x_1, x2,.x_n$ are not correlated i.e. they are statistically independent.

Variance $\alpha^2_{est}=1/n-1\sum_{I=1}^{n}(xi - x)2$ ------------------------------------- (1)

$$(x) \quad = \sum_{i=0}^{n} xi$$

RUN LENGTH OF A DYNAMIC STOCHASTIC SIMULATION

Equation (1) is not valid for determining sample size if

(1) The distribution is not stationary

(2) If the sample x_1, x2, xn not created.

In most dynamic system simulation both of these problems arise & their output data are strongly co-related. *Ex:* In queuing system the waiting time of the kth custom depends on the customer ahead of him.

ELIMINATION OF TRANSIENT

Transient means unsteady behaviour. Since the transient are due to initial bias, initial condition will produce transients of different length and magnitude.

1. The most commonly used approach is to ignore an initial section of the simulation run. The run is started for an empty state and stopped after a certain period. Again the run is restarted and statistics gathered from the 2nd start, the statistics gathered from the first run are simple wipe out.

2. Second we had of reducing the effect of transient is to start the system ion an initial state which is close to the steady state. This method requires a prior knowledge of the system steady state which may not always be available.

3. A third strategy is to reduce the effect of the initial bias is to ensure that the runs have been long enough who have made the initial bias.

SAMPLE SIZE OF AUTOCORELATED OBSERVATION

In a queue the waiting time of the customer depends on the waiting time of the customer ahead of him. If w_k is having livelihood of w_{k+1} being long is also high. A sequence of data that has this property of one value affecting the subsequent value is called serially correlated or auto-correlated. In a sequence of observation $x_1, x_2, ...x_n$ they content to which value separated by 'n' units affects other can be reassured by

$$R_M = 1/N - M \sum_{i=1}^{n-m} \quad (x_i - \bar{x})(x_{i+m} - x)$$

$\bar{x}$ = mean value

x_i = ith observation

$r_{m=}$ auto covariance coefficient orautocorrelation coefficient with log 'm'

The long run of a dynamic stochastic variable is

$$[n = (y_1 - \alpha/2)^2 \alpha 2 [i + 2 \sum_{k=1}^{m} \quad (1 - k/m + 1)r_k]/t^2]$$

The only difficulty in using this approach is the amount of computation time required in evaluating the auto-correlation coefficient.

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