Prey & Predator Model

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Deer ← Lion, Wolves

Rabbit ← Fox

Insects ← Bird

- Lotka–Volterra Equation
- 1st order diff^n eq^n
- Assume constant environment, no other factor
- wild life management, host–pathogen, economic–social system

- Let $x(t)$ & $y(t)$ is the pop^n of the Prey & predator species at any time t.

- Conditions

- Now the following for Prey & predator Model.

1) In the absent of predator the pop^n of prey will growth at the natural rate.

$$\frac{dx}{dt} = ax \qquad [a > 0]$$

2) In the absent of prey the ~~pop^n~~ predator pop^n could decrease at the natural rate.

$$\frac{dy}{dx} = -py \qquad [p > 0]$$

3) The presence of both Predators & Prey is beneficial of growth of predator species & is harmful to growth to Prey species

ie, the predator species increases and the prey species decreases at rate proportional to the product of the two population.

These assumptions give the system of non linear first order Ordinary differential equation.

$$\frac{dx}{dt} = ax - bxy = x(a - by) \quad \text{--- (1)}$$

$$\frac{dy}{dt} = -py + qxy = y(qx - p) \quad \text{--- (2)}$$

∴ a, b, p, q are positive constants
∴ a, b > 0 and p, q > 0

a & p are the growth rate of prey & ~~predator~~ death rate of predator respectively.

b & q are measure of effect of the interspecific between two species.

- Analytical Solution

System of non-linear 1st order O.D.E
$$\frac{dx}{dt} = x(a - by), \quad \frac{dy}{dt} = y(qx - p)$$

$m \log n = \log n^m$ | $\log e = \ldots$

To find the sol^n of system of eq^n ① & ② for $x(t) > 0$, $y(t) > 0$ with initial condition

$$x(0) = x_0 \quad , y(0) = y_0$$

So, $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y(qx - p)}{x(a - by)}$$

or $$\frac{(a - by)}{y} \cdot dy = \frac{(qx - p)}{x} \cdot dx$$

or $$\left(\frac{a}{y} - b\right) dy = \left(q - \frac{p}{x}\right) \cdot dx$$

Integrating both side

$$a \log y - by = qx - p \log x + \log k_1$$

or $$a \log y - by - qx + p \log x = \log k_1$$

where $\log k_1$ is constant

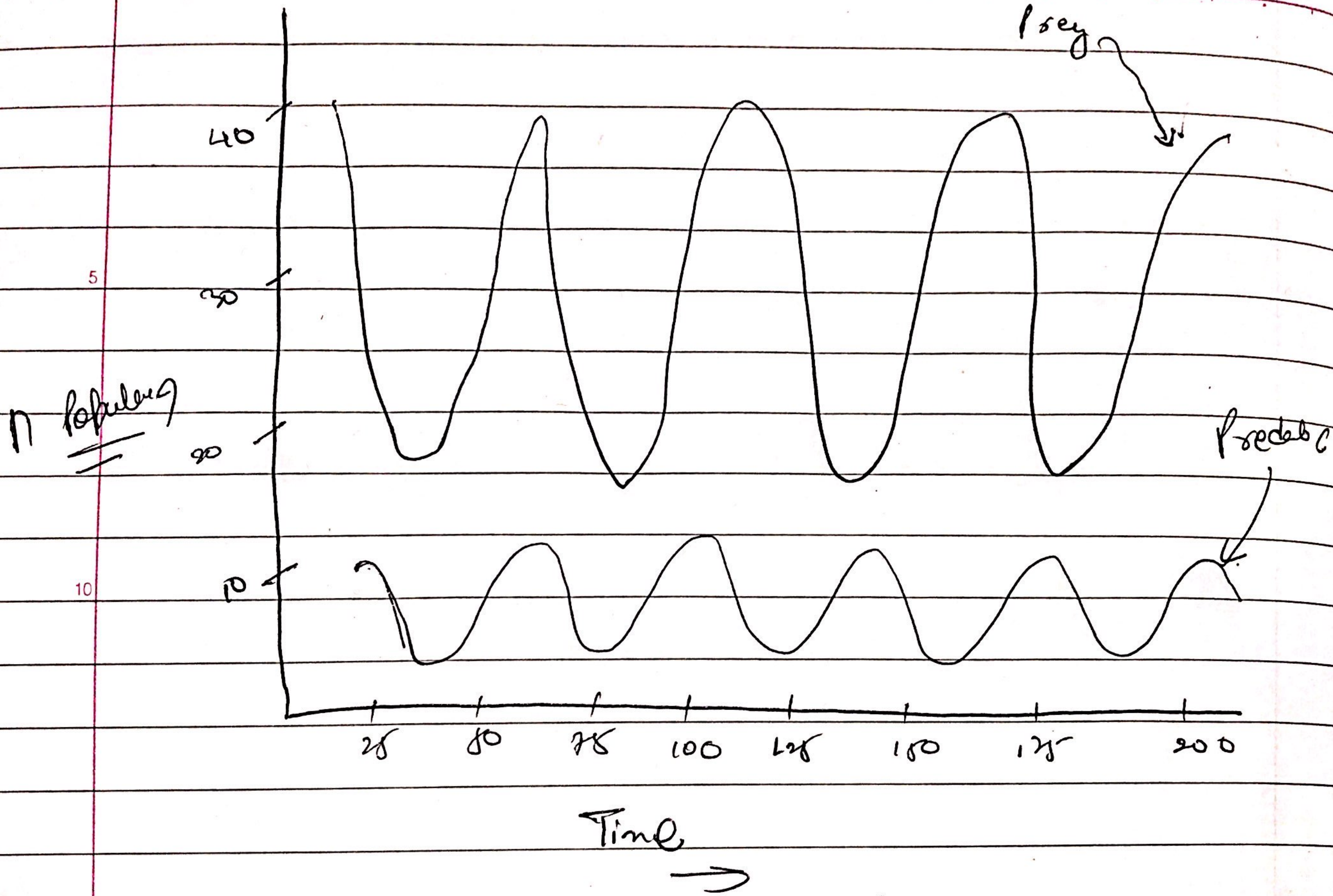
or

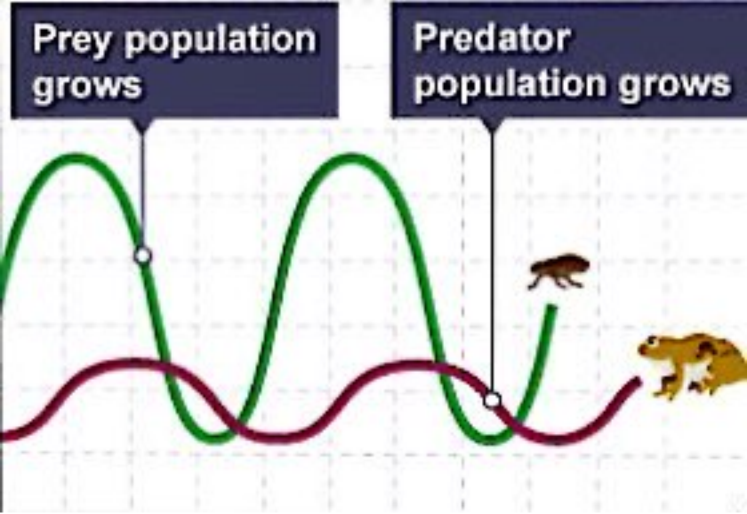
or $$a \log y - b \log e^{y} - q \log e^{x} + p \log x = \log k_1$$

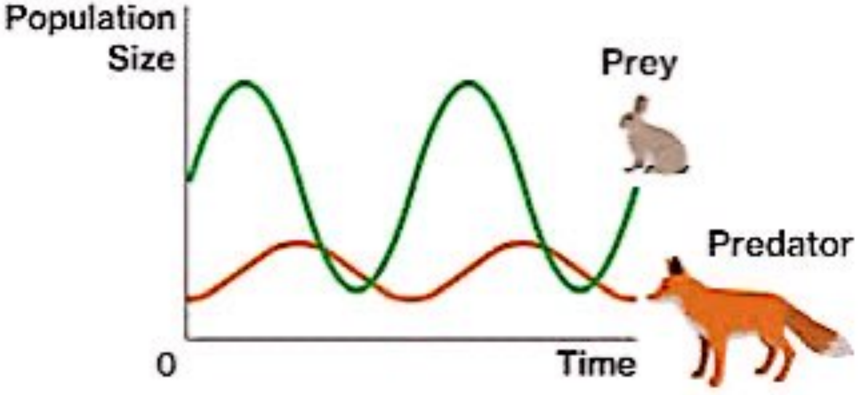
or $$\log y^{a} - \log e^{by} - \log e^{qx} + \log x^{p} = \log k_1$$

or

or $$\log\left(\frac{y^{a} \cdot x^{p}}{e^{by} \cdot e^{qx}}\right) = \log k_1$$


Prey
40
30
n Population
20
Predator
10
25
50
75
100
125
150
175
200
Time


Prey population grows
Predator population grows


Population
Size
Prey
Predator
0
Time