

Cobweb Model

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In
Cobweb market situation

the **output decision** of the producer
depends on the price of the previous period

and

the **demand** for products depends on
the price of the current period.



Therefore, in Cobweb market model

$$Q_d = a - bP_t \quad [a, b > 0]$$

$$Q_s = -c + dP_{t-1} \quad [c, d > 0]$$

$$Q_d = Q_s$$

Q_d = quantity demand; P_t = price of the current period

Q_s = quantity supply P_{t-1} = price of the previous period

'b' is the slope of the demand function.

'd' is the slope of the supply function.

$$Q_d = a - bP_t \quad [a, b > 0] \quad \dots\dots\dots (1)$$

$$Q_s = -c + dP_{t-1} \quad [c, d > 0] \quad \dots\dots\dots (2)$$

$$Q_d = Q_s \quad \dots\dots\dots (3)$$

Substituting equation (1) and (2) in equation (3) we get,

$$a - bP_t = -c + dP_{t-1}$$

$$-bP_t - dP_{t-1} = -a - c$$

$$P_t + \frac{d}{b}P_{t-1} = \frac{a+c}{b}$$

Shifting the time subscript ahead of one period we rewrite the equation as,

$$P_{t+1} + \frac{d}{b}P_t = \frac{a+c}{b} \quad \dots\dots\dots (4)$$

$$P_{t+1} + \frac{d}{b}P_t = \frac{a+c}{b} \quad \text{..... (4)}$$

Equation (4) is the first order difference equation.

The solution of this first order difference equation $y_{t+1} + ay_t = c$

is
$$y_t = \left(y_0 - \frac{c}{1+a}\right)(-a)^t + \frac{c}{1+a}$$

Similarly, the solution of equation (4) is

$$P_t = \left(P_0 - \frac{\frac{a+c}{b}}{1 + \frac{d}{b}}\right)\left(-\frac{d}{b}\right)^t + \frac{\frac{a+c}{b}}{1 + \frac{d}{b}}$$

$$P_t = \left(P_0 - \frac{\frac{a+c}{b}}{1 + \frac{d}{b}}\right)\left(-\frac{d}{b}\right)^t + \frac{\frac{a+c}{b}}{1 + \frac{d}{b}}$$

$$\Rightarrow P_t = \left(P_0 - \frac{\frac{a+c}{b}}{\frac{b+d}{b}}\right)\left(-\frac{d}{b}\right)^t + \frac{\frac{a+c}{b}}{\frac{b+d}{b}}$$

$$\Rightarrow P_t = \left(P_0 - \frac{a+c}{b+d}\right)\left(-\frac{d}{b}\right)^t + \frac{a+c}{b+d} \quad \text{..... (5)}$$

$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \text{..... (6)}$$

Here, P_0 is the initial price and

\bar{P} is the inter-temporal equilibrium price

Now, the nature of time path P_t depends on $\frac{d}{b}$

i.e. the ratio of the slope of supply curve
'd' and the slope of demand curve 'b'

There are three cases.

$$Q_d = a - bP_t$$

$$Q_s = -c + dP_{t-1}$$

$$P_t = \left(P_0 - \frac{a+c}{b+d} \right) \left(-\frac{d}{b} \right)^t + \frac{a+c}{b+d} \quad \text{..... (5)}$$

Since, $\bar{P} = \frac{a+c}{b+d}$ is the inter-temporal equilibrium price

We write equation (5) as,

$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \text{..... (6)}$$

Equation (6) is the time path of Cobweb market model.

$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots\dots\dots (6)$$

Case -I: If $b > d$ or the slope of demand curve is more than the slope of supply curve. Then $\frac{d}{b} < 1$ or a fraction.

In such case, as the value of t is increasing then the value of $\left(-\frac{d}{b} \right)^t$ is falling or as $t \rightarrow \infty$ then $\left(-\frac{d}{b} \right)^t \rightarrow 0$

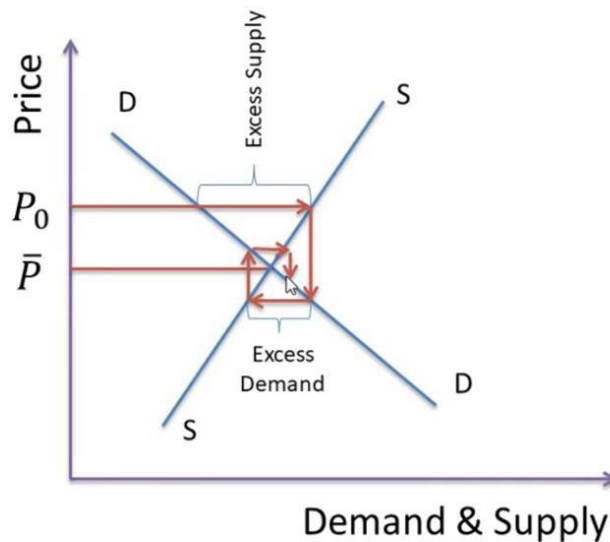
In equation (6), $P_t = \bar{P}$ i.e. with the passes of time

$$P_t \rightarrow \bar{P}$$

The time path is convergent and the market is dynamically stable.

Case -I: If $b > d$ or the slope of demand curve is more than the slope of supply curve. Then $\frac{d}{b} < 1$ or a fraction.

Convergent time path
and
dynamically stable
market.



$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots\dots\dots (6)$$

Case –II: If $b < d$ or the slope of demand curve is less than the slope of supply curve. Then $\frac{d}{b} > 1$ or more than one.

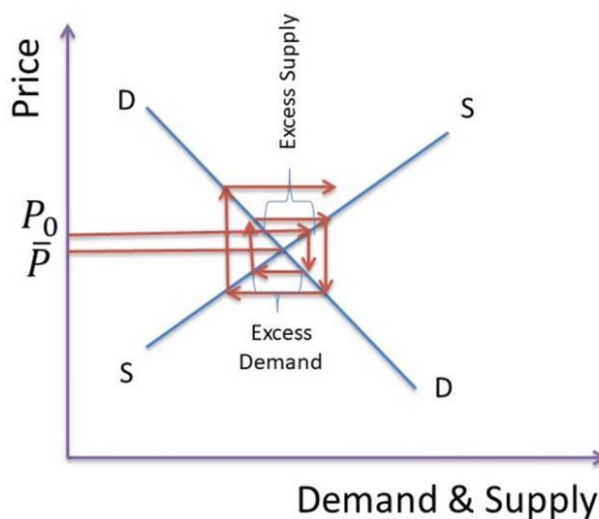
In such case, as the value of t is increasing then the value of $\left(-\frac{d}{b} \right)^t$ is increasing or as $t \rightarrow \infty$ then $\left(-\frac{d}{b} \right)^t \rightarrow \infty$

In equation (6), the current price P_t will divert more and more from the equilibrium price \bar{P}

The time path is divergent and the market is not dynamically stable.

Case –II: If $b < d$ or the slope of demand curve is less than the slope of supply curve. Then $\frac{d}{b} > 1$ or more than one.

Divergent time path
and
explosive market
price.



$$P_t = (P_0 - \bar{P}) \left(-\frac{d}{b} \right)^t + \bar{P} \quad \dots\dots\dots (6)$$

Case –III: If $b = d$ or the slope of demand curve is equal to the slope of supply curve. Then $\frac{d}{b} = 1$

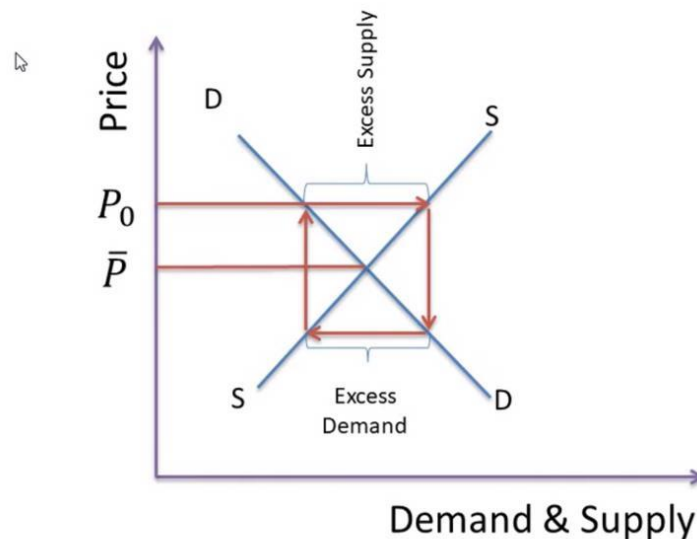
In such case, as the value of t is increasing then the value of $\left(-\frac{d}{b} \right)^t$ is alternatively +1 and -1 depending whether t is even or odd

In equation (6), the difference between P_t and \bar{P} is remain the same.

The time path is said to be regular.

Case –III: If $b = d$ or the slope of demand curve is equal to the slope of supply curve. Then $\frac{d}{b} = 1$

Regular time path price in Cobweb market model.





Question 1

In a local market for rice:

- The **demand equation** is
 $Q_d = 120 - 2P \dots\dots (D)$
- The **supply equation** is
 $Q_s = 20 + 3P \dots\dots (S)$

where

Q_d = quantity demanded (in kg),

Q_s = quantity supplied (in kg),

P = price per kg (in rupees).

1. Find the **equilibrium price** and **equilibrium quantity**.
2. Check whether the market is in **surplus or shortage** if price is fixed at $P = 10$ rupees.
Also calculate the amount of surplus or shortage.

Solution

1. Equilibrium price and quantity

At **equilibrium**:

$$Q_d = Q_s \dots\dots (E0)$$

Substitute (D) and (S) into (E0):

$$120 - 2P = 20 + 3P \dots\dots (E1)$$

Bring all P -terms to one side:

$$\begin{aligned}
 120 - 20 &= 3P + 2P \\
 100 &= 5P \\
 P^* &= \frac{100}{5} = 20 \text{ rupees (Equilibrium Price)}
 \end{aligned}$$

Now substitute $P^* = 20$ into either (D) or (S):

Using demand equation (D):

$$Q_d = 120 - 2(20) = 120 - 40 = 80 \text{ kg}$$

Using supply equation (S):

$$Q_s = 20 + 3(20) = 20 + 60 = 80 \text{ kg}$$

Both give same value, so:

- **Equilibrium price:** $P^* = 20$ rupees
- **Equilibrium quantity:** $Q^* = 80$ kg

2. Market position when $P = 10$ rupees

Demand at $P = 10$:

$$Q_d = 120 - 2(10) = 120 - 20 = 100 \text{ kg}$$

Supply at $P = 10$:

$$Q_s = 20 + 3(10) = 20 + 30 = 50 \text{ kg}$$

Compare:

- $Q_d = 100$ kg
- $Q_s = 50$ kg

Since $Q_d > Q_s$:

$$\text{Shortage} = Q_d - Q_s = 100 - 50 = 50 \text{ kg}$$

Answer for part (2): At $P = 10$ rupees, there is a **shortage** of **50 kg** in the market.

Question 2:

In an agricultural market, the **demand** and **supply** for a crop in year t are given by:

Demand (current price): $Q_d(t) = 120 - 2P_t$

Supply (lagged price – farmers look at last year's price): $Q_s(t) = 20 + 0.5P_{t-1}$

Each year the market clears, so: $Q_d(t) = Q_s(t)$

- Find the **equilibrium price** and **equilibrium quantity**.
- If the initial price in year 0 is $P_0 = 10$, compute prices P_1, P_2, P_3, P_4, P_5 .
- Show that this market is **convergent** (prices move toward equilibrium over time).

Solution:

1. Equilibrium price and quantity

In equilibrium, price is the same in consecutive periods, so:

$$P_t = P_{t-1} = P_e$$

Set demand = supply at equilibrium:

$$120 - 2P_e = 20 + 0.5P_e$$

Bring terms together:

$$120 - 20 = 2P_e + 0.5P_e$$

$$100 = 2.5P_e$$

So:

$$P_e = 100/2.5 = 40$$

Now find equilibrium quantity:

$$Q_e = 120 - 2P_e = 120 - 2(40) = 120 - 80 = 40$$

So:

- Equilibrium price: $P_e = 40$**
- Equilibrium quantity: $Q_e = 40$**

Each year:

$$Q_d(t) = Q_s(t)$$

So:

$$120 - 2P_t = 20 + 0.5P_{t-1}$$

Rearrange:

$$-2P_t = 20 + 0.5P_{t-1} - 120$$

$$-2P_t = -100 + 0.5P_{t-1}$$

Multiply by -1 :

$$2P_t = 100 - 0.5P_{t-1}$$

Divide by 2:

$$P_t = 50 - 0.25P_{t-1}$$

This is the lag (cobweb) equation for price.

3. Compute price path starting from $P_0 = 10$

Use: $P_t = 50 - 0.25P_{t-1}$

• $t = 1$:

$$P_1 = 50 - 0.25(10) = 50 - 2.5 = 47.5$$

• $t = 2$:

$$P_2 = 50 - 0.25(47.5) = 50 - 11.875 = 38.125$$

• $t = 3$:

$$P_3 = 50 - 0.25(38.125) = 50 - 9.53125 = 40.46875$$

• $t = 4$:

$$P_4 = 50 - 0.25(40.46875) = 50 - 10.1171875 \approx 39.8828$$

• $t = 5$:

$$P_5 = 50 - 0.25(39.8828) \approx 50 - 9.9707 = 40.0293$$

You can see:

$$P_0 = 10$$

$$P_1 = 47.5$$

$$P_2 \approx 38.13$$

$$P_3 \approx 40.47$$

$$P_4 \approx 39.88$$

$$P_5 \approx 40.03$$

Prices are **oscillating**, but getting closer and closer to **40**.

(Optionally, quantities each period: $Q_t = 120 - 2P_t$, also converge to 40.)

4. Show convergence (why is this a convergent market?)

From the recursion:

$$P_t = 50 - 0.25P_{t-1}$$

Write it in deviation-from-equilibrium form.

We know $P_e = 40$. Consider $(P_t - 40)$:

Start from:

$$P_t = 50 - 0.25P_{t-1}$$

Subtract 40 from both sides:

$$P_t - 40 = 50 - 0.25P_{t-1} - 40$$

$$P_t - 40 = 10 - 0.25P_{t-1}$$

But $10 = 0.25 \times 40$, so:

$$10 - 0.25P_{t-1} = 0.25 \cdot 40 - 0.25P_{t-1} = -0.25(P_{t-1} - 40)$$

Thus:

$$P_t - 40 = -0.25(P_{t-1} - 40)$$

The factor on the deviation is **-0.25**.

- Absolute value $|-0.25| = 0.25 < 1 \rightarrow$ deviations **shrink** over time.
- Negative sign \rightarrow path oscillates around equilibrium (above–below–above–below), but because $0.25 < 1$, oscillations become **smaller and smaller**.

Therefore, this is a **convergent cobweb market**:

- Prices move toward equilibrium $P_e = 40$.
- Quantities move toward $Q_e = 40$.
- Each period, the market gets closer to equilibrium.

$$P_{t+1} + \frac{d}{b}P_t = \frac{a+c}{b} \quad \dots\dots\dots (4)$$

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