NAME:

MAT 212 (Multivariable Calculus)

Final Examination

Spring, 2013

Instructor: Sanjeeva Balasuriya

Answer all questions in the space provided. The points awarded for each problem are indicated. No notes or books are permitted.

You may use a calculator in this exam. If you do so, you are honor-bound not to use its graphing, programable, differentiating or integrating capabilities.

Please write and sign the Connecticut College Honor Pledge in the space below.
(I promise neither to give nor receive any aid in this examination.)

1. Calculate each of the following. [4 points each]

(a) The rate of change of the function $T(x, y, z) = \cos x \sin y \cos z$ if going in the direction $\underline{j} - 2\underline{k}$ from the origin.

(b) A normal vector to the surface $x^2 - z + y = z^2 + xy - 1$ at the point $(1, 1, 1)$.

(c) div $\left[xyz\underline{i} + y^2z\underline{j} - z^2x\underline{k}\right]$

(d) grad $\left[x^2 + y^2 + z^2 - xyz\right]$

(e) curl $\left[e^{xy}\underline{i} + 3e^{xy}\underline{j} + ze^{xy}\underline{k}\right]$

(f) $\displaystyle\int_0^7 \int_0^2 \int_0^y \frac{y}{z+1}\, dx\, dy\, dz$

(g) The curvature at a general location t of the curve given parametrically by $\underline{r}(t) = 3\underline{i} + t\underline{j} + \ln(\sin t)\,\underline{k}$.

2. Compute the following integrals. [6 points each]

(a) The double integral of the function $f(x,y) = \left(x^2+y^2\right)^2 y$ over the region bounded by the unit circle (circle of radius 1 centered at the origin) in the xy-plane.

(b) The line integral $\int_C x\,ds$ where s represents the arclength along the curve C defined parametrically by $\underline{r}(t) = \sin(2t)\,\underline{i} - \cos(2t)\,\underline{j} + 3t\underline{k}$, where $0 \leq t \leq \pi$.

(c) The double integral $\iint_R \left(2x^2 - xy - y^2\right) \, dx \, dy$, where R is the region in the first quadrant of the xy-plane which is bounded by the straight lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$ and $y = x + 1$. [Hint: choose a transformation of coordinates from (x, y) to appropriately chosen (u, v).]

(d) The triple integral of the function $h(x, y, z) = z\left(x^2 + y^2 + z^2\right)$ over the "ice-cream" cone region defined in terms of spherical polar coordinates by $0 \leq \rho \leq 2$, $0 \leq \phi \leq \pi/4$ and $0 \leq \theta < 2\pi$.

3. Consider the function

$$p(x, y) = x^2 + kxy + y^2$$

in which k is a constant. You are to figure out values for the constant k in order to make $(0, 0)$ one of the following: (i) a saddle point, (ii) a local maximum, or (iii) a local minimum. For each of these possibilities, find conditions on k which guarantee the relevant behavior, *or* explain why it is impossible to do so. [6 points]

4. Show that the integral

$$I_C = \oint_C \left[\left(2x + y^2 \right) \, dx + \left(2xy + 3y \right) \, dy \right]$$

takes the same value for any simple closed curve C in the xy-plane. [6 points]

5. Recall that a general position in three-dimensional space can be represented by $\underline{r}(x, y, z) = x\underline{i}+y\underline{j}+z\underline{k}$. Consider the vector field defined by $\underline{g}(\underline{r}) = \dfrac{\underline{r}}{|\underline{r}|}$, that is, the position vector divided by its magnitude. Express $\underline{g}$ as a function of (x, y, z) and hence calculate its divergence. Simplify as much as possible, and convert your final answer to depend on $\underline{r}$ as opposed to x, y and z independently. [6 points]