Derivatives of trigonometric functions

A couple of useful limits:

(a) $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1,$ (b) $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta} = 0.$

We have already proven (a). The limit (b) can be proven by multiplying top and bottom by the "conjugate" $\cos\theta + 1$, and then utilizing (a). Try it!

Differentiation Formulas

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

1. Prove that the derivative of the tangent is the secant squared. [Hint: assume you know the derivatives of the sine and the cosine, and use the quotient rule.]

2. Find $\dfrac{d}{dx}\left[x^3 \sin x - \cos x \cot x\right]$.

3. Differentiate $f(t) = \dfrac{3 \sin t}{2 \cos t - \tan t}$