Nonlinear Dynamics

(Semester 1, 2005)

Lecturer: Sanjeeva Balasuriya

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Classes: Tuesday 3.05 to 3.55 pm, Thursday 11.05 to 11.55 (Carslaw 353)

Consultations: Tuesday 11.00 am to 12.00 noon *and* by appointment

Unit Webpage: http://www.maths.usyd.edu.au/u/sanjeeva/nd.html

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General Description:
This unit of study addresses nonlinear dynamics at an advanced undergraduate / beginning postgraduate level. The theory of dynamical systems shall be presented mainly from the viewpoint of geometrical ideas as popularised in the 1970s. The focus will be on continuous dynamical systems (ordinary differential equations, or "ODEs"), with discrete dynamical systems (difference equations, or mappings) taking a lesser role. An intuitive development is emphasised, coupled with some theoretical underpinnings. We will prove many of the theorems, but not all.

Prerequisites: Basic ideas on solution techniques of ordinary differential equations, and of phase space (as one would obtain from MATH2005, say). Some linear algebra, specifically eigenvalues and eigenvectors (MATH2002, say). Some background in analysis is assumed (MATH2907, say). This unit will complement some of the ideas in MATH3003, but will be from a different viewpoint.

Assessment:

Final examination (take-home, open-book, cumulative)	40 %
Midterm quiz (in-class)	20 %
Homework assignments (approximately fortnightly)	30 %
Presentation	10 %

Homework Policy:
Problem Sets will be assigned roughly every fortnight. You should think of these as something of a cross between "assignments" and "tutorials" from your past experience. These problem sets are designed to help you understand the concepts through *doing* mathematics. You are *strongly encouraged* and *expected* to work with classmates in attempting these assignments. The only rule concerning this is that you may not see anyone else's written homework solutions, neither are you permitted to show your written solutions to any other student. The final write-up must be your own. You are also welcome to see me with any questions on the homework assignments, even before you hand them in. Remember – the main role of homework is fostering understanding, not assigning marks.

Presentation:
Towards the end of the semester, each student will be giving a 15-20 minute presentation to the rest of the class, based on an original research article, chosen from a seminal collection of dynamical systems articles [2]. More details on this will be given later.

Syllabus

1. Theoretical preliminaries

(a) Implicit Function Theorem
(b) Gronwall's inequality
(c) ODEs: Existence
(d) ODEs: Uniqueness
(e) ODEs: Smoothness in initial conditions

2. Phase space

(a) Critical points and periodic orbits
(b) α and ω-limit sets
(c) Invariant sets
(d) Flow-box theorem
(e) Hamiltonian systems
(f) Lyapunov functions and the Lasalle invariance principle

3. Critical point analysis

(a) Stability
(b) Review of linear systems
(c) Stable and unstable subspaces
(d) Hartman-Grobman Theorem
(e) Stable/unstable manifold theorems
(f) Centre manifolds

4. Maps, Poincaré maps & Poincaré-Bendixson

(a) Maps: critical points, stability, manifolds
(b) Poincaré maps
(c) Poincaré-Bendixson Theorem [for 2-D flows]
(d) van der Pol oscillator

5. Local bifurcations

(a) Necessity of a centre manifold
(b) Saddle-node bifurcation
(c) Transcritical bifurcation
(d) Pitchfork bifurcation
(e) Hopf bifurcation
(f) Bifurcations for maps (period-doubling, saddle-node, transcritical, pitchfork)

6. Chaos

(a) Smale horse-shoe map
(b) Symbolic dynamics
(c) Smale-Birkhoff Theorem
(d) Melnikov theory [for 2-D flows]

Some Books on Nonlinear Dynamics

Much of what we will do in this unit cannot be obtained from any one textbook. Indeed, some material possibly does not appear in any *textbooks. Some textbooks which have a relationship to what we shall cover, are listed below. A brief description of relevance is also given.*

1. http://www-chaos.umd.edu/publications/references.html
   Maintained by the University of Maryland, this page has a list of references with particular regard to chaos. Several of the textbooks listed below are on this list. Has descriptions of each book.

2. ⋆Cvitanović, Predrag (Editor). *Universality in Chaos*, Adam Hilger, Bristol (1984).
   A collection of seminal articles in the field of chaos. Students will be asked to select an article from this book, for presentation to the rest of the class, towards the end of the semester.

3. ⋆Alligood, K.T., Sauer, T. & Yorke, J.A. *CHAOS: An Introduction to Dynamical Systems*, Springer (1997).
   Undergraduate text. Much of it is on discrete systems, but there's a lot of continuous stuff as well. A good reference, since it gives many examples of the sorts of things that we cover, at a lower level.

4. ⋆Wiggins, Stephen. *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Springer-Verlag, New York (1990).
   Beginning postgraduate text – somewhat difficult. Has a large proportion of what we study in this unit – and a lot more!

5. ⋆Strogatz, Steven. *Nonlinear Dynamics and Chaos*, Perseus Publishing, Cambridge, Massachusetts (1994).
   A beautifully written introduction at around the third year level. No theorems or proofs. Very good intuition, and a large collection of applications.

6. ⋆Robinson, Clark. *Dynamical Systems: Stability, Symbolic Dynamics and Chaos*, CRC Press, Boca Raton, Florida (1995).
   A postgraduate textbook, with a style occasionally similar to that we'll be adopting in this unit. Has a mathematical perspective.

7. ⋆Perko, L. *Differential Equations and Dynamical Systems*, Springer, New York (1993)
   Postgraduate text. Has a good selection of the topics that we will cover.

8. ⋆Drazin, P.G. *Nonlinear Systems*, Cambridge University Press, Cambridge (1992)
   Good undergraduate text, but with more emphasis on discrete systems than this unit of study. Chapters 1, 4 and ∮8.3 are related to us.

9. ⋆Jordan, D.W. & Smith, P. *Nonlinear Ordinary Differential Equations*, Clarendon Press, Oxford (1987).
   This text is often used in units similar to this offered in the U.K. and Australia. Somewhat traditional approach.

10. ⋆Arrowsmith, D.K. & Place, C.M. *An Introduction to Dynamical Systems*, Cambridge University Press, Cambridge (1990).

A hard postgraduate text, but there are some sections which are relevant to what we will do. Theorem 1.5.1 is the 'flow-box theorem,' $\oint$3.5 has an advanced discussion on the horse-shoe map,$\oint$3.7 describes Smale-Birkhoff, $\oint$3.8 the Melnikov function, and Chapter 4 describes bifurcations (with some nice diagrams).

11. $\star$Guckenheimer, John & Holmes, Philip. *Nonlinear Oscillation, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York (1990).
A classic reference for nonlinear dynamicists (if there is such a word). Not too easy to learn from. At times quite terse. Some good examples. Theorems are stated (often in most general form), but rarely proven.

12. Lichtenberg, A.J. & Lieberman, M.A. *Regular and Chaotic Dynamics*, John Wiley (1992).
Another classic reference like the one above, but from a much more applied perspective.

13. Arnol'd, V.I. *Ordinary Differential Equations*, MIT Press, Cambridge, Massachusetts (1973).
The textbook used in MATH3003. Comes from a more 'pure' viewpoint than we will follow. Has basic existence, uniqueness, etc proofs, and also an extensive discussion on linear systems.

14. Ott, Edward. *Chaos in Dynamical Systems*, John Wiley (1992).
A postgraduate text, whose emphasis is on chaos, with particular regard to discrete systems. Well written.

15. Iooss, Gérard & Joseph, Daniel. *Elementary Stability and Bifurcation Theory*, Springer-Verlag, New York (1990).
Don't be fooled by the 'elementary.' Tough text. Its Chapter 2 offers a slightly differing viewpoint on bifurcation than what we do in this unit.

16. Nicolis, G. *Introduction to Nonlinear Science*, Cambridge University Press (1995).
An undergraduate text which is quite descriptive, and gives many examples from the sciences.

17. Pugh, Charles Chapman. *Real Mathematical Analysis*, Springer, New York (2000).
An undergraduate text in mathematical analysis. Has a proof of the Implicit Function Theorem (page 286). However, you should also be able to find this in your favourite analysis text.

$\star$The books denoted with a star symbol$\star$ have been placed on closed reserve in the Mathematics Library (Carslaw 8th floor).