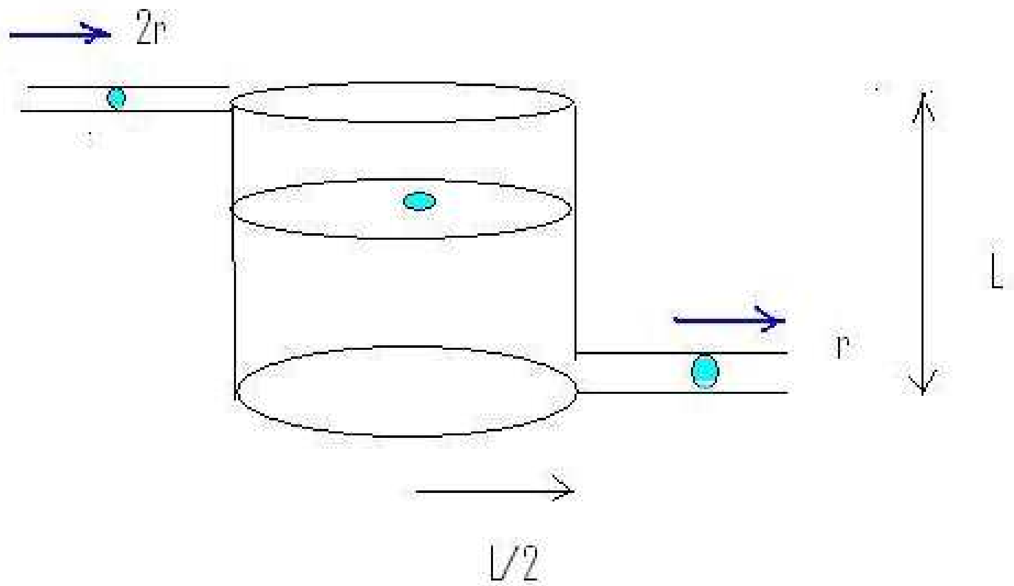
MAT 112 (*Fall, 2007*)
Calculus I
Group Project 1
Due: October 24 (Wednesday), at 4 pm

This project is to be completed in your assigned group of students. All students should participate in finding a solution, and preparing the report. **One** *carefully written report is to be submitted by each group. The report will be graded on presentation in addition to content. Please list all group members' names on your report. You may* **not** *discuss this project with anyone other than your group members or myself. If you use any reference material, make sure you give appropriate credit.*

A recent Connecticut College graduate is entrusted the task of designing a process which is to reduce an incoming flow rate of pipe-borne water by one-half, while providing a water tank for emergency storage purposes. She decides to use the following design, which uses a cylindrical tank of height L and radius $L/2$.



There is a constant inflow of water at a rate of $2r$ along the inflow pipe, while it is desired that the outflow rate be a constant r (each rate is expressed as a volume per unit time). To control this process, a bob equipped with a sensor floats on the surface of the water inside the tank. When the bob reaches the exact top of the tank, the inflow valve automatically (and instantaneously) closes. If, however, the bob drops below a height of $L/4$, the inflow valve opens, permitting water to once again flow into the talk at a constant rate $2r$. Meanwhile, the outflow valve ensures that the outflow rate is *always* a constant rate r, independent of what is happening elsewhere in the apparatus.

Suppose the process begins operation at time $t = 0$, with there being no water in the tank initially. Let

$$\begin{aligned} W(t) &= \text{rate of increase of volume (of water in the tank) at time } t \\ V(t) &= \text{volume of water in the tank at time } t \\ h(t) &= \text{height (measured from the bottom of the tank) of bob at time } t \\ u(t) &= \text{velocity (in upwards direction) of bob at time } t \end{aligned}$$

(A) Sketch the graphs of each of the above functions of t in the domain $0 < t < 13\alpha$, where α is the constant defined through $\alpha = \dfrac{\pi L^3}{16r}$. Make sure all important values on the graph are identified. (You may find it convenient to label your t-axis in units of α.)

(B) Explain the behavior of each of the functions for t values beyond 13α.

(C) Discuss the continuity of each function.

(D) Discuss the differentiability of each function.

(E) What is the relationship between each of the following pairs of functions? (Explain!)

i. $W(t)$ and $V(t)$.
ii. $u(t)$ and $h(t)$.
iii. $W(t)$ and $u(t)$.
iv. $V(t)$ and $h(t)$.

(F) Find the following limits, if they exist. (If they don't, explain why.)

i. $\displaystyle\lim_{t\to 131313313\alpha} h(t)$
ii. $\displaystyle\lim_{t\to 13\alpha} W(t)$
iii. $\displaystyle\lim_{t\to 13\alpha^+} u(t)$

(G) Suppose the incoming water has a constant lead concentration of λ (in units of weight per unit volume). In order to reduce the lead from the water, a chemical will be placed inside the tank, which neutralizes lead at a constant rate of μ units of weight per unit time. If $g(t)$ is the amount (weight) of lead within the tank at time t, give a *careful* explanation for why the following equation is valid:

$$g'(t) = 2\,r\,\lambda - \mu - \frac{r\,g(t)}{V(t)} \qquad \text{if} \quad 0 < t < 4\alpha\,.$$

(You need to fully justify all terms appearing in this expression, and why they appear in this particular form, to obtain full credit.) What is a similar expression for $g'(t)$ valid in the domain $4\alpha < t < 7\alpha$?