Bolzano's Theorem:

Let $h : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$, and suppose $h(a)$ and $h(b)$ have opposite signs. Then, there exists $c \in (a, b)$ such that $h(c) = 0$.

Proof: (*Complete the proof by filling in the blanks*)

Without loss of generality, assume $h(a) < 0$ and $h(b) > 0$. So $h(a) < 0 < h(b)$. Define three sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ as follows. First, let

$$x_1 = a \quad , \quad y_1 = b \quad , \quad \text{and} \quad z_1 = \frac{1}{2}(x_1 + y_1) \ .$$

For $n > 1$,

(i) If $h(z_n) < 0$, define $x_{n+1} = z_n$ and $y_{n+1} = y_n$,

(ii) If $h(z_n) \geq 0$, define $x_{n+1} = x_n$ and $y_{n+1} = z_n$,

(iii) For either situation, define $z_{n+1} = \frac{1}{2}(x_{n+1} + y_{n+1})$.

Define the interval I_n by $I_n = [x_n, y_n]$.

Since I_n forms a [______________________________], by the Nested Interval Property, there exists c such that [______________________].

However, [length$(I_n) =$ __________________], and thus [$x_n \to$ ____________] and [$y_n \to$ ____________].

Now, since h is continuous at c, [$\lim h(x_n) =$ ____________] where [$h(x_n)$ ______ 0] (insert a comparative here, such as $\geq$, $=$ or $<$) . Thus, [$h(c)$ ______ 0].

On the other hand, [$\lim h(y_n) =$ ____________] where [$h(y_n)$ ______ 0]. Thus, [$h(c)$ ______ 0].

Therefore, [$h(c)$ ____________ 0].

Now, $c \in [a, b]$. But $c \neq a$ since $h(a) \neq 0$. Similarly, [____].

Therefore, $c \in (a, b)$.