MAT 112-01,02 (*Fall, 2007*)
CALCULUS I
Group Project 2
Due: Deember 12 (Wednesday) at 4 pm

This project is to be completed in your assigned group. All students should participate in finding a solution, and preparing the report. **One** *carefully written report is to be submitted by each group. The report will be graded on presentation in addition to content. Please list all group members' names on your report. The report is due by the end of class on the due date. You may* **not** *discuss this project with anyone other than your group members or myself. If you use any reference material, make sure you give appropriate credit.*

MAT 112 students have been hired as consultants by the New London City Council to advice them on three different issues which have come up recently. Each group is requested to prepare a report for the City Council addressing all three of these issues.

(1) 400 meter track

The 400 meter track at the local arena currently has the standard international structure of having two parallel sides of length 100 meters each, connected by two semi-circular segments of 100 meters each on the two sides. The space in the middle of the track is to be used for other sports. The City Council feels that it would be good to design the 400 meter track such that the *rectangular portion* enclosed by the track is maximum (a council member has pointed out that the semi-circular segments are useless for other sports). The City Council of course understands that the basic structure of the 400 meter track (having two parallel sides, connected together by two semi-circular segments), and the fact that the track should be 400 meters in length, are necessary evils. However, it would like to know whether there is a better choice of length for the long straight segments (currently chosen to be 100 meters) such that the relevant "sports area" is maximized?

(2) Why Pi?

In a lively debate in the City Council regarding the areas of the semi-circular segments of the arena track, one council member[1] raised objections about the futile usage of a Greek symbol π in the calculation, rather than 3.2, which at least people understand. To appease this council member, you are asked to *prove* that the area of a circle is πr^2, by the following "method of exhaustion" credited variously to Eudoxus (~408-355 BC) and Archimedes (~287-212 BC), which has many of the elements of the Riemann sum idea used to define integrals. In a circle of radius r, imagine inscribing a n-sided polygon of equal sides. By dividing the polygon into n congruent triangles with central angle $2\pi/n$, show first that each triangle has area $r^2 \sin(2\pi/n)/2$. Find the area, A_n, of the n-sided polygon. Then take an appropriate limit such that a polygon of this sort becomes a better and better approximator for the circle, to show that the area of the circle is πr^2. [Wish yourselves the best of luck in convincing the undectagenarian council member.]

(3) Dam!

A new dam is to be constructed across the River Thames. At this location, the Thames has a width of 200 meters, while its maximum depth (at the center of the river) is 20 meters. It has been established that the river bed slopes linearly away from the bank of the river, and that the profile of the river bed is symmetric. Thus, the dam that will be built across the Thames will have the cross-sectional shape of an inverted isosceles triangle with base 200 m and height 20 m. The City Council wants to use the cheapest possible material to build the dam, and therefore wants to know exactly how much force the dam should be designed to withstand. Find the total hydrostatic force caused by the pressure of the water which builds up behind the dam.

<u>Hints</u>: You'll need some physics for this. The pressure (which is the force per unit area) at a point at depth h under water is given by $h\rho g$, where ρ is the density of water ($1000\,\mathrm{kgm}^{-3}$), and g is the gravitational acceleration ($9.8\,\mathrm{ms}^{-2}$). Imagine slicing the dam into very thin horizontal strips of thickness (you guessed it!) Δh. Then, one can say that the pressure at all points on a given strip is essentially constant, since all points on it are at the same depth below the water level. Determine an approximate expression for the hydrostatic force on this strip resulting from the pressure (note: hydrostatic force = pressure $\times$ area), such that the approximation is known to get very accurate in the limit $\Delta h \to 0$. When you add together the forces of all the strips, do you get something which looks familiar?

[1] A 115 year old hailing from Indiana; see
http://www.cs.uwaterloo.ca/~alopez-o/math-faq/mathtext/node18.html