Math 305-01: Partial Differential Equations (Fall, 2001)

MWF 3.30: 4.20 pm, King 227

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- **Office Hours:**

  Monday: 4.30-6.30 pm
  Tuesday: 11:00 to noon
  Thursday: 2:30-4.30 pm
  (and by appointment)

  You may also stop by my office at any other time (though appointments are strongly encouraged).

- **Text:**
  Nakhlé Asmar, *Partial Differential Equations and Boundary Value Problems*, Prentice-Hall (1999).

- **Textbook's Website:**
  http://math.missouri.edu/~nakhle/pdebvp/notebooks.html

  (Contains a collection of Mathematica Notebooks which can be run to support your comprehension of the material in the course.)

- **Homework:**

  This is the most important assessment tool of the course. Fairly large homework sets will be typically due every fortnight. On the due date, instead of a typical "lecture," I will ask students to individually present solutions to selected homework problems.

- ## Student-Taught Class:

Towards the end of the semester, each student will be asked to select a topic of interest, which illustrates an application or some aspect not covered in detail in the regular class. You will be in charge of a class-period during which you will teach that topic to your classmates and the professor. You are responsible for preparing handouts for this class, in addition to leading the class in any way you think fit. I will be available for any help you need in this endeavour (such as choosing a topic, finding source material, deciding how much to cover, etc).

- ## Examinations:

There will be only two examinations:

Midterm (take-home, closed-book): due **Wednesday, October 17**
Final (take-home, open-book): due **Friday, December 21**

- ## Grading:

Homework (solutions + presentation)	40%
Student-taught class	10%
Midterm examination	25%
Final examination	25%

- ## Late Assignments:

Barring exceptional circumstances, late assignments will **not** be accepted.

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SYLLABUS

Preamble

Partial Differential Equations (PDEs) arise frequently in all fields of physics and engineering. These are often associated with *boundary conditions* and/or *initial conditions.* PDEs are substantially more difficult to solve than Ordinary Differential Equations, and in this course we will explore some of the standard analytical techniques that are known classically. A fundamental tool in this regard is *Fourier Analysis*, in which general functions are represented as linear combinations of classes of simpler functions (for example, periodic functions can be represented as sums of sines and cosines). We will spend some time on basic ideas of Fourier Analysis, since they are of importance in their own right. However, the focus of the course will be on using Fourier Analysis, and other techniques, to help us solve PDEs.

The breakdown of the syllabus follows. The numbers in parentheses indicate the sections from your textbook which cover the relevant material. We will cover sections A through F completely. Depending on student interests and time constraints, we will choose some additional topics from Section G (Miscellaneous Topics).

A: Introduction

- Partial Differential Equations (*PDEs*): order, linearity, boundary conditions, initial conditions, examples (1.1)
- Ordinary Differential Equations (*ODEs*): reminder!

B: Fourier Series

- Periodic functions and Fourier series (2.1, 2.2, 2.3)
- Fourier sine and cosine series (2.4)
- Parseval's identity (2.5)
- Integrating and differentiating Fourier series
- Complex form of Fourier series (2.6)

C: PDEs in Rectangular Coordinates

• Examples	(3.1)
• Separation of variables for 1-D heat equation	(3.5, 3.6)
• 1-D Wave Equation	(3.2, 3.3)
• 2-D Laplace's Equation	(3.8)
• 3 independent variables...	(3.7)
• Poisson's Equation and eigenfunction expansions	(3.9)

D: PDEs in Polar/Cylindrical/Spherical Coordinates

• Laplacian in various coordinate systems	(4.1)
• Problems in 2-D polar coordinates	(4.4, 4.2, 4.3)
• Problems in spherical coordinates	(5.1, 5.5, 5.7)

E: Sturm-Liouville Theory

• Generalised Fourier Series	(6.1)
• Second-order [Regular] Sturm-Liouville Theory	(6.2)
• A fully solved example; eigenfunction expansions	

F: Fourier Transforms

• Fourier representation in infinite domains	(7.1)
• Fourier transform and properties	(7.2)
• Solving PDEs using Fourier transforms	(7.3)
• Wave equation and D'Alembert's solution	(3.4)
• Heat kernel and fundamental solutions	(7.2, 7.4)
• Fourier sine and cosine transforms	(7.6)

G: Miscellaneous Topics

• Divergence theorem applications: Maximum principle for Laplace's Equation Deriving PDEs from integral conservation laws	
• Nonlinear PDEs: characteristic curves and shock waves	
• Numerical solution of PDEs: finite difference method	(Chapter 9)
• Green's functions	
• Heisenberg's Uncertainty Principle through Fourier transforms	(11.3)