MAT 301
Real Analysis I
(Fall, 2012)

Classes: Tuesday and Thursday, 10.25 am–11.40 am (Blaustein 203)

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Office Hours: Tues 1.45–2.45 pm, Thur 9.30–10.15 am & 3.00–4.00 pm, and by appointment.

Textbook (Required): *Understanding Analysis*, by Stephen Abbott (Springer, 2000).

Course Webpage: Accessible via Moodle at `http://moodle.conncoll.edu`.

Joint Homework: Homework is a very important part of this course, since the only way to learn Analysis is by battling with concepts, going down false paths, and eventually finding logically precise arguments. To help with this process, all homework will be *joint* – you will work with a different partner each time to jointly prepare homework solutions. Joint homework will normally be assigned every two or three classes.

Exams: Both the mid-term and the final examination will be take-home, untimed, and open-book. This is in keeping with the philosophy that Analysis is not about memorization or time-pressure, but about understanding concepts and definitions, and being able to present carefully reasoned arguments. The instructor reserves the right to request a short meeting with individual students after their exam has been handed in, to ask for clarification on the student's written exam. The mid-term is due on **Tues, Oct 23** (tentative date), while the final will be due during exams week.

Reaction/Response Papers: There will be 4 reaction papers requested over the course of the semester. These will be based on readings from the textbook, and will be due in at the beginning of class on the due date. Your target will be to write a one page reaction paper with your responses to the assigned reading. A typed paper is requested (although you may handwrite in the mathematical symbols if it is difficult to type these).

Class Contributions: There will be frequent group work in class, with discussions within groups and also within the class as a whole. Students will also be asked to make (very) short presentations of their group's conclusions on the board. Your contribution in these matters, and in joint homework, will be useful to both yourself and your classmates. Your "class contribution" grade will be computed jointly by yourself, your classmates, and your instructor.

Final Grades: Your final grade will be based on the following weighting.

Final Exam (Open-book, take-home):	30 %
Mid-term Exam (Open-book, take-home):	20 %
Joint Homework:	30 %
4 Reaction/Response Papers:	10 %
Class Contribution:	10 %

Tuesday, November 5: There is no class on this day – a day off in lieu of the mid-term being take-home.

Late Assignment Policy: Late assignments will *not* be accepted. Any assignment not submitted by the due time will earn an automatic zero.

Honor Code: The Connecticut College Honor Code will apply. Its impact on individual assessment tasks will be made specific when those tasks are handed out. Any violations will result in referrals to the Honor Council. A guilty verdict will result in an F for this course, plus any additional penalties imposed by the council.

Special Accommodations: Students with a physical or mental disability (either hidden or visible) must inform the instructor as soon as possible if they require classroom, test-taking, or other reasonable modifications. Such students *must* register with the Office of Student Disability Services (Crozier Williams Room 221, 439-5428 or 439-5240, `barbara.mcllarky@conncoll.edu` or `lillian.liebenthal@conncoll.edu`), who will provide the instructor with a list of required/permissible special accommodations.

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Course Summary

Understanding how the real numbers work has fascinated mathematicians for years. While our intuition works well in simple situations, its breakdown in others (such as Thomæ's example of a function which is continuous at every irrational number while being discontinuous at every rational) made mathematicians realize the necessity of developing an *axiomatic* approach to the subject. In Real Analysis I, we will begin such a study, strengthening the foundations of much of what we have previously encountered in calculus. We will list some of the properties of the real numbers, and gain an understanding of how these entities are distributed on the seemingly innocuous "number line." We will battle with the concept of infinity, and come to terms with its many surprising ramifications. Based on these new insights, we will rigorously re-examine the ideas of functions, limits, continuity and differentiability. We will put into place many definitions which will help later study of advanced analysis, topology, logic, and higher mathematics.

A major goal of Analysis I is also to obtain an introduction into how mathematicians think and write mathematics. We will spend a lot of time in honing our skills in writing *proofs*: carefully reasoned arguments, based on axioms and definitions, which are meant to be water-tight. Achieving such a final proof will not be a linear process; we will formulate possible theorems, be creative with definitions and counter-examples, and poke holes in our classmates' suggestions. We will learn to think and write logically, critically, and precisely (valuable skills even in a non-mathematical context!).

Course Outline

1. **The Real Numbers**
   Basic properties of the real numbers $\mathbb{R}$: the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{I}$, upper bounds and least upper bounds, the axiom of completeness, the nested interval property, density of $\mathbb{Q}$ and $\mathbb{I}$, countable and uncountable sets, cardinality, Cantor's Theorem.

2. **Sequences and Series**
   Sequences as an infinite set, their limits, convergence and divergence of sequences, properties of limits of sequences, Bolzano-Weierstrass Theorem, Cauchy sequences, infinite series, their limits, convergence and divergence of series, tests for convergence, rearrangements of series.

3. **Topology of $\mathbb{R}$**
   Subsets of $\mathbb{R}$, open and closed sets, characterizing compact sets (sequential and topological definitions), Heine-Borel Theorem.

4. **Functional Limits and Continuity**
   Functions, definition of a functional limit ($\epsilon - \delta$ definition, formulation in terms of neighborhoods), connection with the sequential limit, $\epsilon - \delta$ definition of continuity, uniform continuity, Extreme-Value Theorem, Intermediate Value Theorem, intermediate value property.

5. **The Derivative**
   Definition of the derivative, chain rule, Interior Extremum Theorem, Darboux's Theorem, Mean-Value Theorem, a continuous nowhere differentiable function.