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Teaching Time Savers: Reacting to Analysis

By Sanjeeva Balasuriya

When teaching real analysis to junior and senior undergraduates last semester, it struck me that the textbook that I was using provided an opportunity for me to try something different in a math course. Four times during the semester, I had the students read introductory "discussion" sections from their textbook, "Understanding Analysis" by Stephen Abbott, published by Springer. These sections contained intuition on the concepts that were to be covered in the upcoming chapter, introduced some strange functions and sets, and were cleverly written to highlight "surprising" ideas which were apparently contrary to what students might have guessed from their previous calculus experience. The students were entrusted with the task of writing "reaction papers" (of roughly a page in length) regarding the material in those sections, and had to hand in their papers before I began the corresponding chapter.

Being students at a liberal arts institute, they accepted this challenge with gusto. The response papers ranged from ones which simply paraphrased the sections, to those which tried to give their own examples, to those who expressed disbelief in the statements and attempted to back up their complaints. Even the paraphrased reaction papers (not usually the desired outcome when reaction papers are assigned) had their uses, as I will outline. I graded the papers quickly, using a "check," "check-plus" scheme, and the papers were in total worth 10% of the student grade.

In deference to the title of this column, I will only focus on how this process bought me time. Firstly, the discussion sections introduced some functions and entities, such as Thomae's function and the Cantor set, in straightforward language. I did not have to reintroduce their construction when needed in class. Secondly, the readings provided interesting motivation, providing intuitive explanations of why, say, Thomae's function was continuous at all irrational numbers but discontinuous on the rationals, and the Cantor set has zero length but dimension $\ln 2 / \ln 3$. Could there, for example, be a function with the opposite property of Thomae's functon? Such ideas whetted the appetite of students, who were curious to know formal definitions (of continuity, say), and correct theorems (establishing that there was no function on the reals which was continuous only on the rationals), enabling me to proceed to these quickly in class with very little additional motivation. I was amazed at the speed at which I was able to get through the nitty gritty definitions and proofs, having as my audience students who had already invested time battling with the ideas needed, and possessing at the very least a vague awareness of the technical issues that might need careful attention. Thirdly, I was able to assign homework problems which either extended the ideas introduced in the readings, or requested rigorous proofs of some of the claims. Fourthly, I did not have to spend time preparing for topics which students managed to learn themselves through this process.

Getting student to do readings before coming to class is of course a standard model in humanities classes. Professors in such courses can then easily incite discussions on the material, or insist on students handing in reaction papers which (among other things) makes sure that students have done the readings, and are able to make contributions in class. This model, as we know, does not work as easily in mathematics courses, in which students usually gain more by reading the relevant section after they have been covered in class. Among the difficulties is the fact that students usually require some guidance and motivation before being able to digest dense mathematical formalism. Yet – if it were possible to get the students to have some understanding of the topics before coming to class – great time gains could be achieved. My assigning the reaction papers in my analysis class was such an attempt; to adapt an established liberal arts pedagogy to mathematics.

The main difficulty in assigning reaction papers in this way is finding readings which possess certain criteria. Unlike in a standard mathematics textbook section, the formalism should be minimal. Explanations should be simple and intuitive. Examples which come out and grab the interest of students should be available. There should be evidence as to why "calculus intuition" is not sufficient, thereby motivating the need for rigorous definitions. (For example, the inadequacy of using the idea of "drawing the graph of the curve without lifting the pencil off the paper" in understanding continuity is easily highlighted through Thomae's example, and the example x^n sin(1/x) demonstrates that derivatives of differentiable functions need not be continuous, but perhaps satisfy a weaker "intermediate value property.") Some instructors might like readings with a historical context. Finding readings which have such properties is clearly difficult. With my choice of textbook, I was able to poach these readings with no effort whatsoever, since such "discussion" sections were provided at the beginning of each chapter. Here's a partial list of possible sources, including the one I used: Stephen Abbott's "Understanding Analysis" (Springer, http://community.middlebury.edu/~abbott/UA/UA.html), the Interactive Real Analysis online textbook (http://web01.shu.edu/projects/reals/reals.html), Drexel University's Math Forum (http://mathforum.org/advanced/analysis.html), Robert Brabenec's "Resources for the Study of Real Analysis" (http://maa.org/reviews/RealResources.html), A.B. Kharazishvili's "Strange Functions in Real Analysis" (CRC Press).

Time spent: In my case, no time was spent on seeking good readings. Grading reaction papers took about a minute per student each time.

Time saved: I probably saved about 6 hours in preparation time, since students learnt certain topics by themselves. I also estimate a gain of about 4-5 hours of class time (which I would have had to spend on specific examples, and motivating definitions and theorems), which can also be construed as time saved.

Sanjeeva Balasuriya teaches mathematics at Connecticut College. He has previously taught at the University of Sydney (Australia), the University of Peradeniya (Sri Lanka) and Oberlin College (Ohio).