WEIGHTED VOTING SYSTEMS

A weighted voting system with n voters is described by a set of numbers, which is listed in the following format:

[quota: weight of voter 1, weight of voter 2, ... , weight of voter n]

The quota is the number of votes required to get a resolution passed. The weight of each voter is the number of votes controlled by that voter.

Explain each of the following voting systems, and list any special characteristics. Assume the voters are labelled A, B, C, D, etc, for convenience.

1. [51: 26, 26, 12, 12, 12, 12]

   There are 6 voters. A and B each control 26 votes, and C, D, E and F control 12 votes each. 51 votes are required to pass a resolution (a simple majority of the 100 total votes).

2. [4: 1, 1, 1, 1, 1, 1, 1]

3. [14: 15, 2, 3, 3, 5]

4. [10: 4, 3, 2, 1]

5. [12: 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

6. [39: 7, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

Coalition: Collection of voters who decide to vote the same way.
Weight of a coalition: Sum of the weights of the voters of a coalition.
Winning coalition: A coalition whose weight exceeds the quota.
Critical voter (in a winning coalition only): A voter is called *critical* if it is the case that if he or she were to leave the coalition, it would no longer be winning.

A town has two large political parties, (R)epublican and (D)emocrat, and one small party, (I)ndependent. Suppose the town council consists of 9 members from R, 7 members from D, and 2 members from I. Suppose moreover that the members of the political parties vote as a bloc, and that a simple majority is required to pass a resolution. We can think of this as a three-member weighted voting system where the (bloc) voters are R, D and I, having weights 9, 7 and 2 respectively.

- Express this as a weighted voting system using the notation of the previous page.

- Fill in the following table.

Coalition	Weight	Winning?	Critical voters
{R}	9	No	--
{R, D}	16	Yes	Both R and D

- What is the number of winning coalitions in which R is critical? Similarly, find the number of coalitions in which D is critical, and the number of coalitions in which I is critical.

[**Discussion**] At the outset, we may have thought that the relative "power" of R, D and I in this voting system is given by R=9/18, D=7/18 and I=2/18 (or, using alternative notation, R:D:I=9:7:2). Do you think this accurately reflects the comparative power distribution? Can you think of another way of measuring their relative "power"?