Homogenous coordinate

Consider the effect of general 2 by 2 transformation applied to the origin

So for the origin, Origin is invariant under general 2 by 2 transformation.

This limitation is overcome by homogenous coordinates

It is necessary to be able to modify position of origin i.e. to transform every point in 2 dimension plane.

This can be accomplished by translating origin or any other point in 2 dimension plane

If
$$x^* = ax + by + m$$

 $y^* = bx + dy + n$

In homogenous coordinate representation we add third coordinate to a point.

Instead of representing by a pair of number (x, y) each point is represented by a triple (x, y, h).

We say that 2 sets of homogenous coordinates (x, y, h) and (x*, y*,h*) represent the same point if and only if one is multiple of another i.e. (2,3,6), (4,6,12) are same points represented by different coordinate triples.

In order to transform a point (x, y) into homogenous representation we choose a non zero number 'n' and form a vector [hx, hy, h] and h is called scale factor or homogenous coordinate parameter.

For point [2,3] in 2 dimensional space it's representation in homogenous coordinated will be

[2, 3, 1] for
$$h = 1$$

[4, 6, 2] for $h = 2$
[-2,-3,-1] for $h = -1$

Affine Transformation: a transformation that preserves collinearity (i.e. points lying on a line initially still lie on a line after transformation) and ratios of distances

It is a combination of single transformations such as translation, rotation or reflection on an axis

Hence the general transformation matrix is of the form

$$[T] = \begin{pmatrix} a & b & m \\ c & d & n \\ 0 & 0 & 1 \end{pmatrix}$$

hence for translation

$$P* = T(t_x, t_v)$$
. P

$$= \left[\begin{array}{ccc} x & + & t_x \\ y & + & t_y \\ 1 \end{array} \right]$$

now every point in 2 dimension plane evern origin (x = y = 0) can be transformed. Similarly, rotation transformation equation about coordinate origin EW $P^* = R(0)$. P

$$= \begin{pmatrix} x\cos 0 - y\sin 0 \\ x\sin 0 + y\cos 0 \\ 1 \end{pmatrix}$$

Scaling transformation relative to coordinate origin is

$$P^* = S(s_x, s_y) . P$$

$$\begin{bmatrix} x^* \\ y^* \\ 1 \end{bmatrix} = \quad \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \left[\begin{array}{c} x \cdot s_x \\ y \cdot s_y \\ 1 \end{array} \right]$$