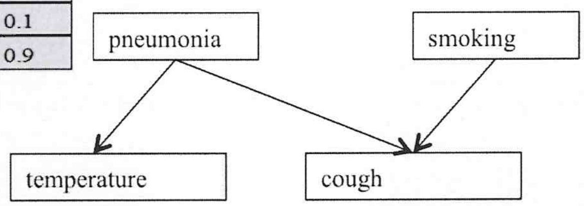
Bayesian Networks In-Class Exercises

Consider the following Bayesian network:

pneumonia	
true	0.1
false	0.9

smoking	
yes	0.2
no	0.8



	temperature	
pneumonia	yes	no
yes	0.9	0.1
no	0.2	0.8

		cough	
pneumonia	smoking	true	false
true	yes	0.95	0.05
true	no	0.8	0.2
false	yes	0.6	0.4
false	no	0.05	0.95

Let $C = [\underset{= true}{cough}]$, $P_n = [\underset{= true}{pneumonia}]$, $S = [\underset{= true}{smoking}]$.

(a) What is P(cough | smoking and pneumonia)?

$P(C \mid S \,\&\, P_n) \overset{\text{From table}}{=} \boxed{.95}$

(b) What is P(smoking | cough)?

$P(s \mid c) \overset{\text{Bayes rule}}{=} \dfrac{P(c \mid s)\,P(s)}{P(c)}$

Numerator:

$P(c \mid s)\,p(s) = [P(c \mid s \wedge P_n)P(P_n) + P(c \mid s \wedge \neg P_n)\,P(\neg P_n)]\,P(s)$
$= [(.95)(.1) + (.6)(.9)](.2) = .127$

Denominator

$P(c) = p(c \mid P_n \wedge S)P(P_n)p(S) + P(c \mid P_n \wedge \neg S)p(P_n)P(\neg S) + P(c \mid \neg P_n \wedge \neg S)\,p(\neg P_n)p(\neg S)$
$+ p(c \mid \neg P_n \wedge S)\,p(\neg P_n)\,p(\neg S)$

$= (.95)(.1)(.2) + (.8)(.1)(.8)$
$+ (.6)(.9)(.2) + (.05)(.9)(.8) = .227$

So: $P(s \mid c) = .127 / .227 = \boxed{.56}$

(c) What is P(pneumonia | cough)?

$$P(pn \mid cough) = \frac{P(cough \mid pn)\,P(Pn)}{P(cough)}$$

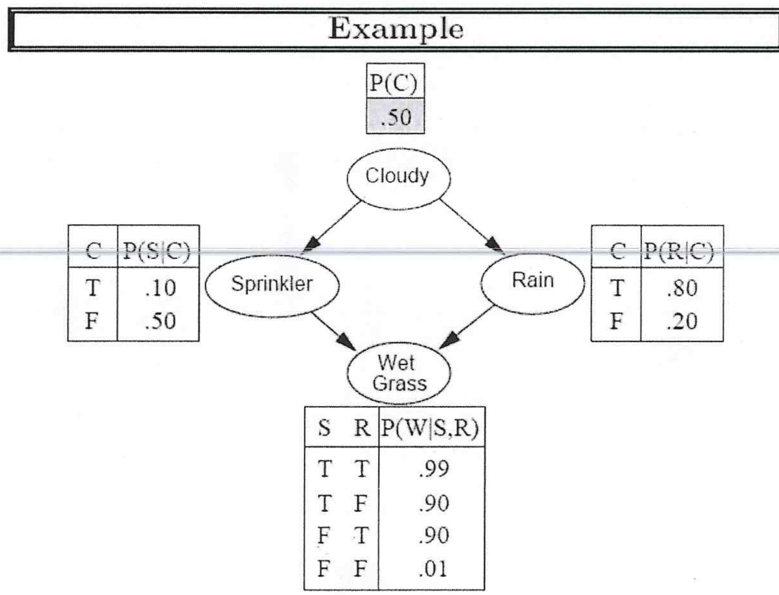
Numerator: $p(c \mid pn)\,p(Pn) = [p(c \mid pn \wedge s)\,p(s) + p(c \mid Pn \wedge \neg s)\,p(\neg s)]\,p(Pn)$

$= [(.95)(.2) + (.8)(.8)]\,(.1) = .083$

Denominator $= .227$ (from problem (b)). So:

$$P(Pn \mid c) = \frac{.083}{.227} = \boxed{.37}$$

2. Consider the following Bayesian network:



C = [Cloudy = true] S = [Sprinkler = true]
R = [Rain = true] W = [Wet Grass = true]

(a) What is P(C,R,¬S,W)?

$P(C, R, \neg S, W) =$

$P(C)\,P(R \mid C)\,P(\neg S \mid C)\,P(W \mid R, \neg S)$

$= (.5)(.8)(.9)(.9) = \boxed{.324}$

(b) Suppose you observe it is cloudy and raining. What is the probability that the grass is wet? Since "Wet Grass" is conditionally independent of "Cloudy" given "Rain" and "Sprinkler", we have

$P(W|C,R) = P(W|R,S)P(S|C) + P(W|R,\neg S)P(\neg S|C)$

$P(S|C) = .1$

$P(\neg S|C) = .9$

So

$P(W|C,R) = (.99)(.1) + (.9)(.9)$

$= .909$

(c) Suppose you observe the sprinkler to be on and the grass is wet. What is the probability that it is raining?

$$P(R|S,W) = \frac{P(R \wedge S \wedge W)}{P(S \wedge W)}$$

Numerator: $P(R \wedge S \wedge W) = P(R \wedge S \wedge W \wedge C)$
$+ P(R \wedge S \wedge W \wedge \neg C)$
$= P(R|C)P(S|C)P(W|R,S)P(C)$
$+ P(R|\neg C)P(S|\neg C)P(W|R,S)P(\neg C)$
$= (.8)(.1)(.99)(.5) + (.2)(.5)(.99)(.5) = .089$

(see next page for denominator)

(d) Suppose you observe that the grass is wet and it is raining. What is the probability that it is cloudy?

$P(C|W,R) = P(C|R)$ since "Cloudy" is conditionally independent of W given "Rain".

$$P(C|R) = \frac{P(R|C)P(C)}{P(R)} = \frac{(.8)(.5)}{P(R|C)P(C) + P(R|\neg C)P(\neg C)}$$

$$= \frac{.4}{(.8)(.5) + (.2)(.5)} = \boxed{.8}$$

2 (c) Continued:

Denominator:

$P(S \wedge W) =$

$P(S \wedge W \wedge R \wedge C)$
$+ P(S \wedge W \wedge R \wedge \neg C)$
$+ P(S \wedge W \wedge \neg R \wedge C)$
$+ P(S \wedge W \wedge \neg R \wedge \neg C)$

$= P(S|C) P(W|S,R) P(R|C) P(C)$
$+ P(S|\neg C) P(W|S,R) P(R|\neg C) P(\neg C)$
$+ P(S|C) P(W|S,\neg R) P(\neg R|C) P(C)$
$+ P(S|\neg C) P(W|S,\neg R) P(\neg R|\neg C) P(\neg C)$

$= (.1)(.99)(.8)(.5)$
$+ (.5)(.99)(.2)(.5)$
$+ (.1)(.9)(.2)(.5)$
$+ (.5)(.9)(.8)(.5)$

$= .278$

So: $P(R|S,W) = \frac{.089}{.278} = \boxed{.320}$