Frame buffer = resolution × pixel depth

= $640 \times 480 \times 11$

= 3379200 bits

= 0.4028 MB

Frequency = 50 Hz.

Size of video for 1 sec = size of frame buffer × frequency

= 0.4×50

= 20 MB.

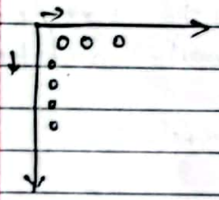
In 5 mins = 300 sec

∴ Size of video = 300×20

= 6000 MB.

Aspect Ratio

Generally = 3:4 or 16:9



Aspect ratio 3:4 means that we need 3 pixels in horizontal axis and 4 pixels in vertical axis so that they become equal.

Numerical:

Q ① How much time is spent scanning across each row of pixels during screen refresh on a raster system with resolution of 1280 * 1024 and a refresh

(If we draw with the help of dot of any picture then it is raster system).

rate of 60 frames per second?

Solⁿ. Here,

Resolution = 1280 * 1024

frequency = 60 fps.

Now,

To scan 1 frame, time required = $\frac{1}{60}$ sec.

1024 rows are scanned in $\frac{1}{60}$ sec.

∴ For 1 row = $\frac{1}{(60 \times 1024)}$ sec.

= 0.0000162 sec

Oct 5, 2020

Q.② Suppose RGB raster system is to be designed using 8*10 inch screen with a resolution of 100 pixels per inch in each direction. If we want to store 6 bits per pixel in the frame buffer, how much storage in bytes do we need for frame buffer?

Solⁿ. Here,

Frame buffer = resolution * pixel depth.

Resolution = $(8 \times 100) * (10 \times 100)$

= 800 * 1000

So,

$$\text{Frame buffer} = 800 \times 1000 \times 6 \text{ bits}$$
$$= \frac{800 \times 1000 \times 6}{8}$$
$$= 6 \times 10^5 \text{ bytes}$$

Q.3) Find the aspect ratio of raster system using 8x10 inches screen & 100pixels/inch

Sol^n. Aspect ratio = width/height

For example 600*800

$$\text{Aspect ratio} = \frac{600}{800} = 3:4$$

I need to connect 3 pixels horizontally & to get the same length of line in vertical direction, I need to connect 4 pixels.

$$\text{Resolution} = (8 \times 100) * (10 \times 1000)$$
$$= 800 * 1000$$

$$\therefore \text{Aspect ratio} = 800/1000 = 4:5$$

Q.4) Consider two raster systems with resolution of 640 by 480 & 1280 by 1024. How many pixels could be accessed per second in each of these systems by a display controller that refreshes the screen at a rate of 60 frames per second? What is the access time per pixel in each system?

Sol^n. Case I: For 640 by 480 resolution system.

Resolution = 640 * 480 ; No. of pixel in 1 frame = 640 * 480

Frequency = 60 fps.

∴ Total no. of pixel in one second
$$= 640 * 480 * 60$$

$$\text{Access time for 1 pixel} = 1/(640 * 480 * 60)$$
$$= 5.4 \times 10^{-8} \text{ sec/pixel.}$$

Case II: For 1280 * 1024 resolution

No. of pixel in one frame = 1280 * 1024

Frequency = 60 fps

Total no. of pixel in one sec = 1280 × 1024 × 60

$$\text{Access time for 1 pixel} = 1/(1280 \times 1024 \times 60)$$
$$= 1.271 \times 10^{-8} \text{ sec/pixel}$$

Q.5) How long would it take to load a 640 by 480 frame buffer with 12 bits per pixel, if 10^5 bits can be transferred per second? How long would it take to load a 24 bits per pixel frame buffer with a resolutions of 1280 by 1024 using the same transfer rate?

solⁿ. Case I: 640* 480.

Frame buffer = resolution * pixel depth
= 640×480 ×12
= 3686400 bits.

Time required = $3686400/10^5$ = 36.864 sec.

Case II: 1280*1024
Frame buffer = resolution * pixel depth
= 1280 * 1024 × 24
= 31457280 bits.

Time required = $31457280/10^5$ = 314.57 sec.

Q.6) A laser printer is capable of printing two pages (size 9*11 inches) per second at a resolution of 600 pixels per inch. How many bits per second such device required.

solⁿ. Storage required per page = (9×600)* (11×600) pixel sec

(600 pixels per inch is same as 600 dpi)
(Printer resolution is expressed in terms of DPI (Dots per inches)

Since laser printer is capable of printing 2 pages per second, bits per second required by the printer
= 2× 9×600× 11×600 pixels per sec

We are assuming the pixel depth to be 'n' bits.

∴ Total no. of bits required = 2 * 9 * 600 * 11 * 600 * n bits/sec

2.3 Raster and Random system architecture.

- Size of CRT monitor was heavy & large due to CRT (electron gun). but in the case of LCD, LED & plasma panel it is slim & light-weight.

- Display technology :- (way to draw on comp. screen)
  - Raster based system (based on dot & pixel) ↳ eg. GIF, jpg, jpeg
  - Vector " " ↳ eg. pdf file

① Raster based system is a display technology where picture is displayed on computer screen with the help of dots & those dots are known as pixel.

Repeat either dy or dx times.

Numerical.

Q. Draw a straight line from (20,10) to (30,18) using DDA algorithm.

Sol^n. First we compute slope of a line i.e. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{18-10}{30-20}$

$= 0.8$

(which has less co-ordinate value that is takes as starting point) i.e. (20,10)

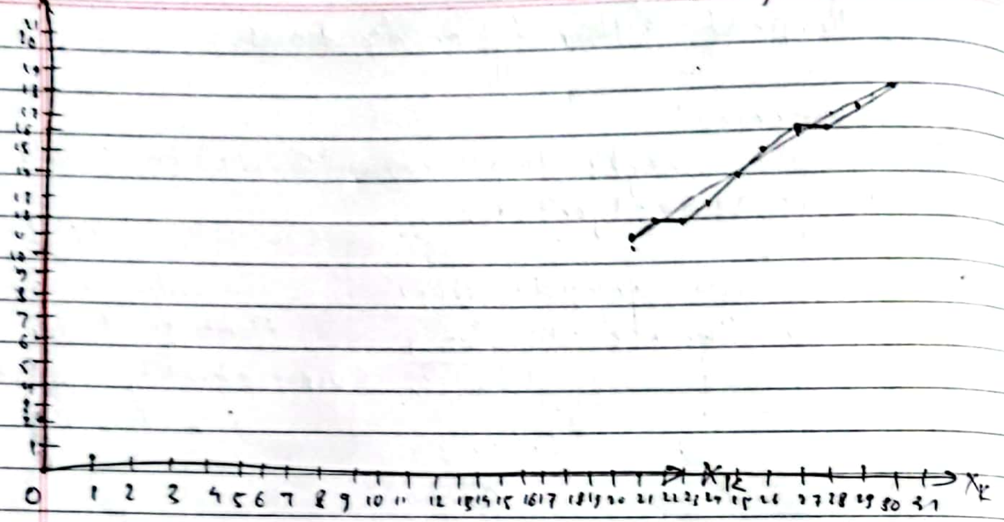
Since $|m| < 1$, we perform sampling along x-direction. So, we repeat 'dx' times i.e. 10 times

$(x_{k+1}, y_{k+1}) = (x_k + 1, y_k + m)$

S.No	x_k	y_k	y_k (Round off value of y_k)
1	21	10.8	11
2	22	11.6	12
3	23	12.4	12
4	24	13.2	13
5	25	14	14
6	26	14.8	15
7	27	15.6	16
8	28	16.4	16
9	29	17.2	17
10	30	18	18

These two columns value are plotted in graph.

⇒ There is jaggies or staircase problem in DDA due to rounding off. which is disadvantage of DDA



Q. Digitize a line having end point (10,20) to (18,24).

Soln.

We compute slope $= \frac{24-20}{18-10}$

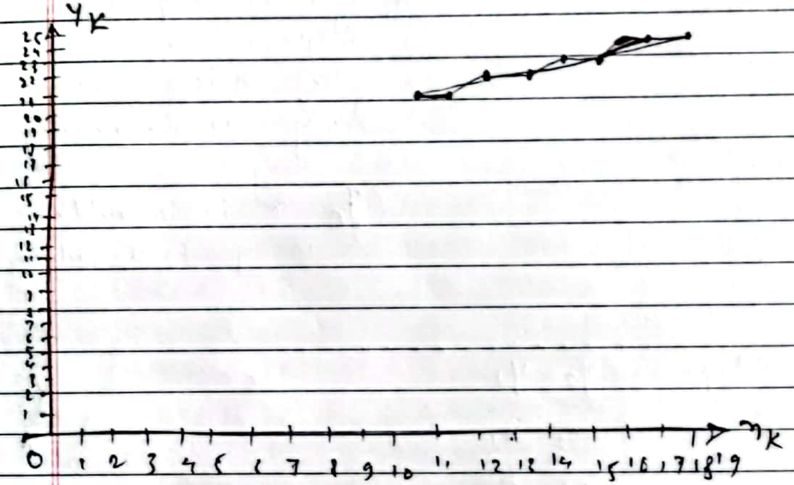
$= 4/8$

$= 0.5$

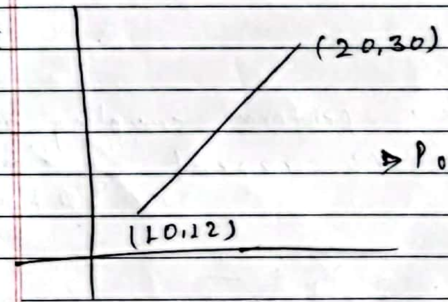
since $|m| < 1$, we perform sampling along x-direction. So, we ~~repe~~ repeat dx times i.e. 8 times.

$(x_{k+1}, y_{k+1}) = (x_k + 1, y_k + m)$

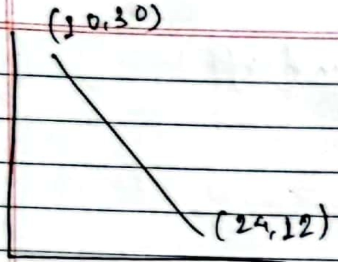
S.N.	x_k	y_k	y_k (round off)
1	11	20.5	21
2	12	21	21
3	13	21.5	22
4	14	22	22
5	15	22.5	23
6	16	23	23
7	17	23.5	24
8	18	24	24



#



⇒ Positively slope line i.e. both value of 'x' & 'y' needs to be increased.



⇒ Negatively slope line

→ Here, 'x' needs to be increased & 'y' needs to be decreased.

Q. Digitize the st. line having end point (10,30) to (24,12)

→ (This is negatively slope line i.e. 'x' is increasing & 'y' is decreasing)

(In 'x' it is added & in 'y' it is subtracted)

Solⁿ. Here

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-30}{24-10} = -1.28$$

Since $|m| > 1$, we perform sampling along y-direction. So, we repeat dy times i.e. 18 times

$$\cancel{x_{k+1} = x_k + 1/m = x_k + \frac{1}{1.28}}$$

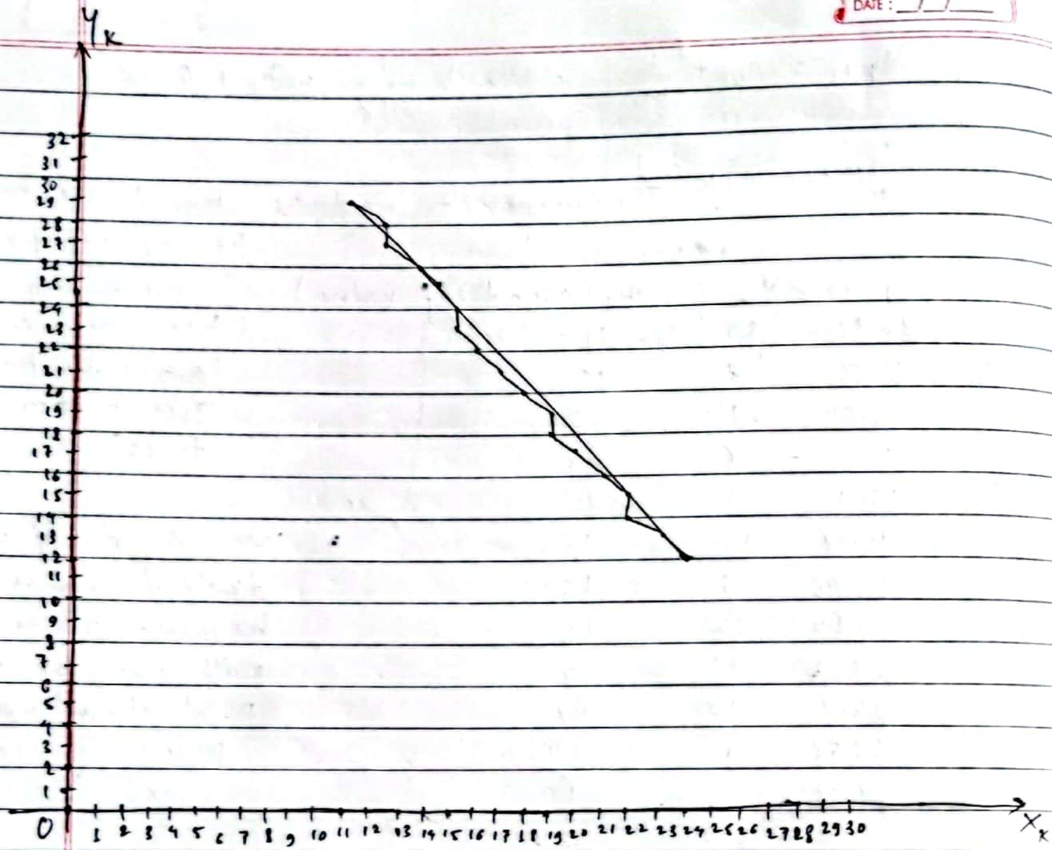
$$\cancel{y_{k+1} = y_k + 1 = y}$$

$$x_{k+1} = x_k + \frac{1}{m} = x_k + \frac{1}{1.28} = x_k + 0.78$$

$$y_{k+1} = y_k \pm 1$$ → (means to update by '1' i.e may be by adding or subtracting (here we subtract because we have to reach from 30 to 12)

S.N.	x_k	y_k	x_k (round off value)
1	10.78	29	11
2	11.56	28	12
3	12.34	27	12
4	13.12	26	13
5	13.9	25	14
6	14.68	24	15
7	15.46	23	15
8	16.24	22	16
9	17.02	21	17
10	17.8	20	18
11	18.58	19	19
12	19.36	18	19
13	20.14	17	20
14	20.92	16	21
15	21.7	15	22
16	22.48	14	22
17	23.26	13	23
18	24.04	12	24

→ (we round off the decimal value because there will not be the p. decimal value point in computer)



Q. Digitize a line segment from (10,12) to (20,18).

Sol^n.

We compute the slope (m) $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{18-12}{20-10}$$

$$= 0.6$$

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$|m| < 1$; so, we perform sampling along x-direction.

i.e. we repeat dx times = 10 times.

$$x_{k+1} = x_k + 1$$
$$y_{k+1} = y_k + m.$$

S.No.	x_k	y_k	y_k (round off)
1	11	12.6	13
2	12	13.2	13
3	13	13.8	14
4	14	14.4	14
5	15	15	15
6	16	15.6	16
7	17	16.2	16
8	18	16.8	17
9	19	17.4	17
10	20	18	18

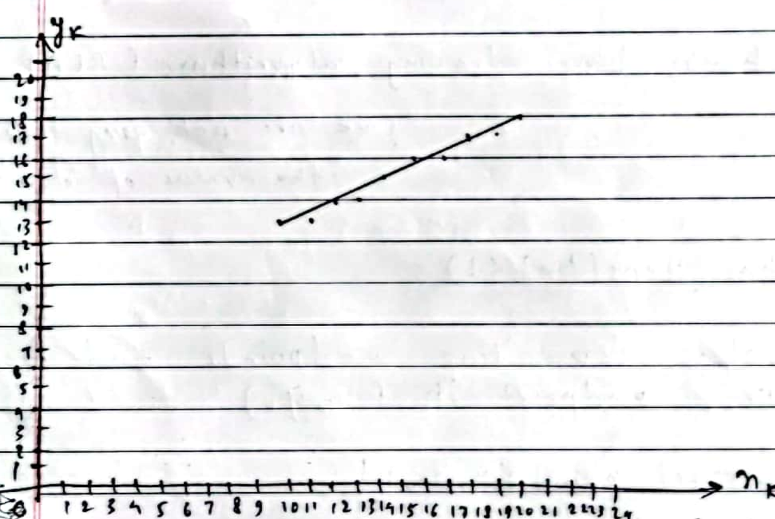


Fig. Digitized line with (10,12) to (20,18)

For recursive calculation, initially,

$p_0 = 2\Delta y - \Delta x$ (substitute $b = y_0 - m.x_0$ and $m = \Delta y / \Delta x$ in (1)

Note: If Δx and Δy is -ve, we discard the sign

Formula for Numerical $|m| < 1$

$p_0 = 2\Delta y - \Delta x$

If $p_k < 0$ $(x_k + 1, y_k)$

$p_{k+1} = p_k + 2\Delta y$

else $(x_k + 1, y_k + 1)$

$p_{k+1} = p_k + 2\Delta y - 2\Delta x$

$\Rightarrow$ Repeat dx times

Q. Digitize the line segment (20,10) to (30,18)

soln.

$(x_0, y_0) = (20, 10)$

$(x_1, y_1) = (20, 10)$

$(x_2, y_2) = (30, 18)$

Now

$dx = x_2 - x_1 = 30 - 20$
$= 10$

$dy = y_2 - y_1 = 18 - 10$
$= 8$

$2dy = 16$

$\Rightarrow 2dy - 2dx = 16 - 20 = -4$

$P_1 = 6 + -4 = 2$ ($\because P_{k+1} = P_k + 2\Delta y - 2\Delta x$)

$P_0 = 2dy - dx = 16 - 10 = 6$

K	P_K	x_{K+1}	y_{K+1}
0	6	21	11
1	2	22	12
2	-2	23	12
3	14	24	13
4	10	25	14
5	6	26	15
6	2	27	16
7	-2	28	16
8	14	29	17
9	10	30	18

($\because P_K = 6 > 0$, matching 'else' case, so we use formula $P_{K+1} = P_K + 2\Delta y - 2\Delta x$ to calculate P_1)

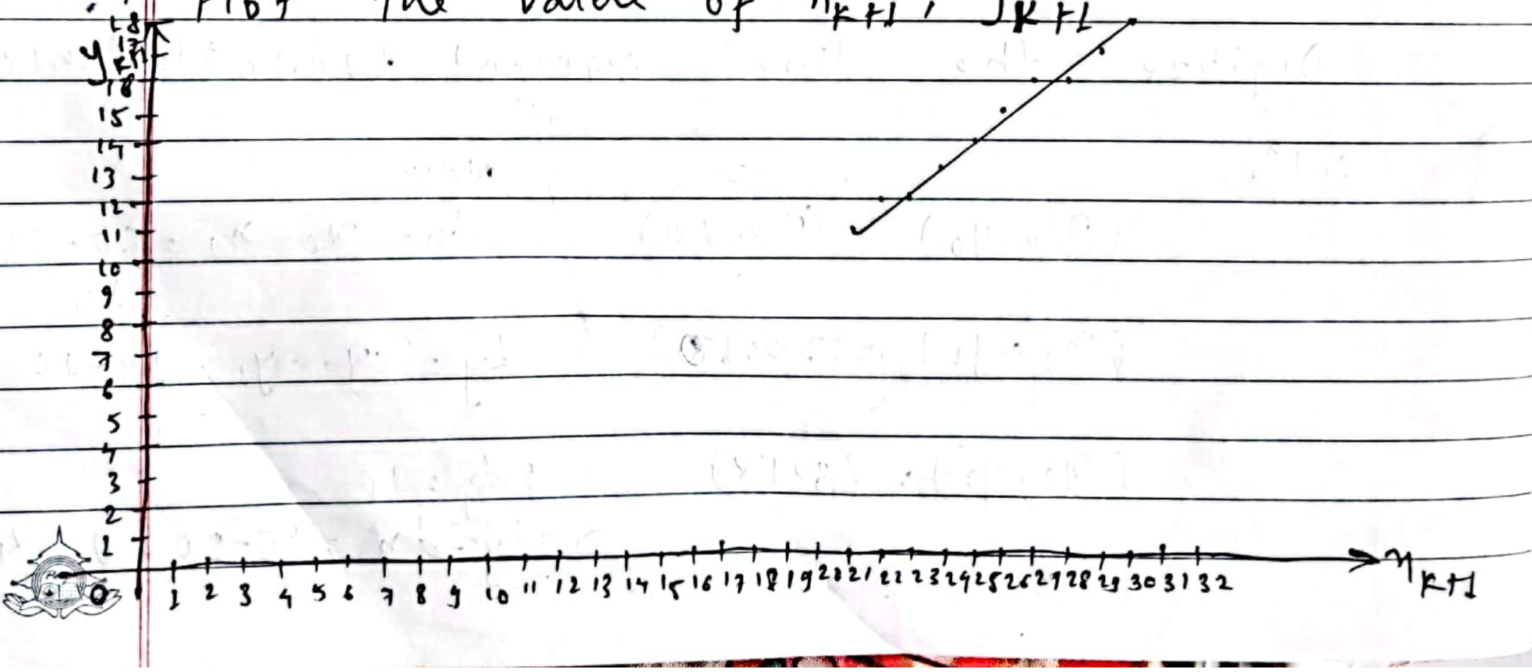
(matching 'if' case ~~ie if~~ i.e. $P_K < 0$)

(so we use $P_{K+1} = P_K + 2\Delta y$ formula to calculate P_3)

$\because P_3 = P_2 + 2\Delta y$
$= -2 + 16$
$= 14$

→ (It is better to show the computation part of $P_0 \cdots P_9$)

→ Plot the value of x_{K+1}, y_{K+1}



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Q. Digitize the line segment (20,10) to ~~(12,22)~~ (8,20) using BLA.

soln. $(x_0, y_0) = (20,10)$

$(x_1, y_1) = (20,10)$
$(x_2, y_2) = (8,20)$

$m = y_2 - y_1 / x_2 - x_1 = \frac{20-10}{8-20} = -0.83 < 1.$

$dx = x_2 - x_1 = 8-20 = 12$ (we take only value, doesnot consider ~~sign~~ sign in dx, dy)

$dy = y_2 - y_1 = 20-10 = 10$

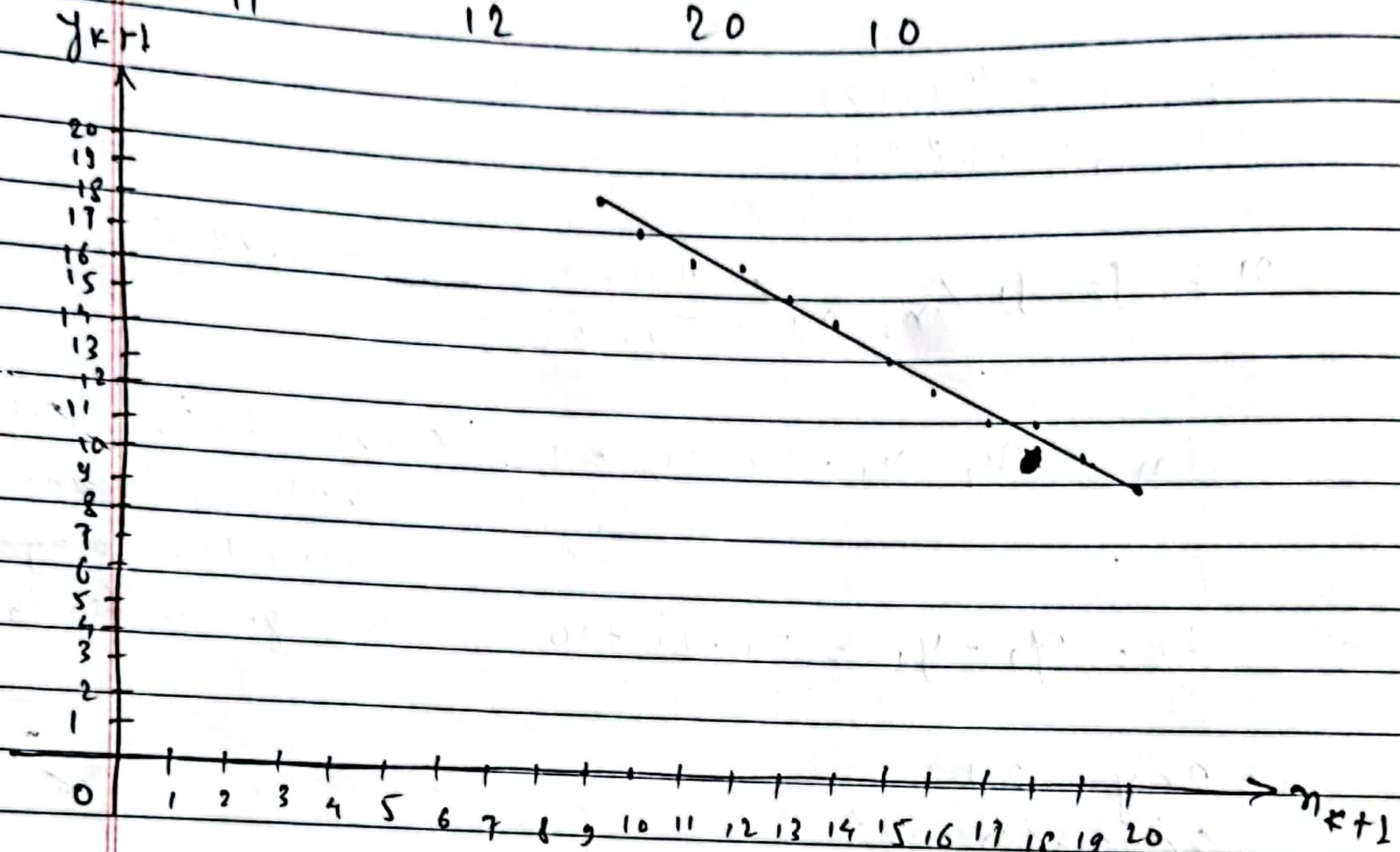
$2dy = 20$
$2dy - 2dx = -4$

$P_0 = 2dy - dx = 20 - 12 = 8$

K	P_K	x_{K+1}	y_{K+1}
0	8	9	19
1	4	10	18
2	0	11	17
3	-4	12	17
4	16	13	16
5	12	14	15
6	8	15	14

(we take (8, 20) as start point) in (8,20) is small so, we take it first.

7	4	16	13
8	0	17	12
9	-4	18	12
10	16	19	11
11	12	20	10



Case II : if $|m| > 1$

$P_0 = 2\Delta x - \Delta y$

if $P_k \geq 0$ $(x_k + 1, y_k + 1)$

$P_{k+1} = P_k + 2\Delta x - 2\Delta y$

else $(x_k, y_k + 1)$

$P_{k+1} = P_k + 2\Delta x$

Q. Digitize the line segment (10,12) to (20,24)

Solⁿ.

$(x_0, y_0) = (10,12)$

$(x_1, y_1) = (10,12)$

$(x_2, y_2) = (20,24)$

$m = y_2 - y_1 / x_2 - x_1 = \frac{24-12}{20-10} = 1.2 > 1$

$dy = y_2 - y_1 = 24 - 12 = 12$

$2dy = 24$, $2dx = 20$

$2dx - 2dy = -4$

$P_0 = 2dx - dy = 20 - 12 = 8$

Repeat dy times. i.e. 12 times. $(dy > dx)$

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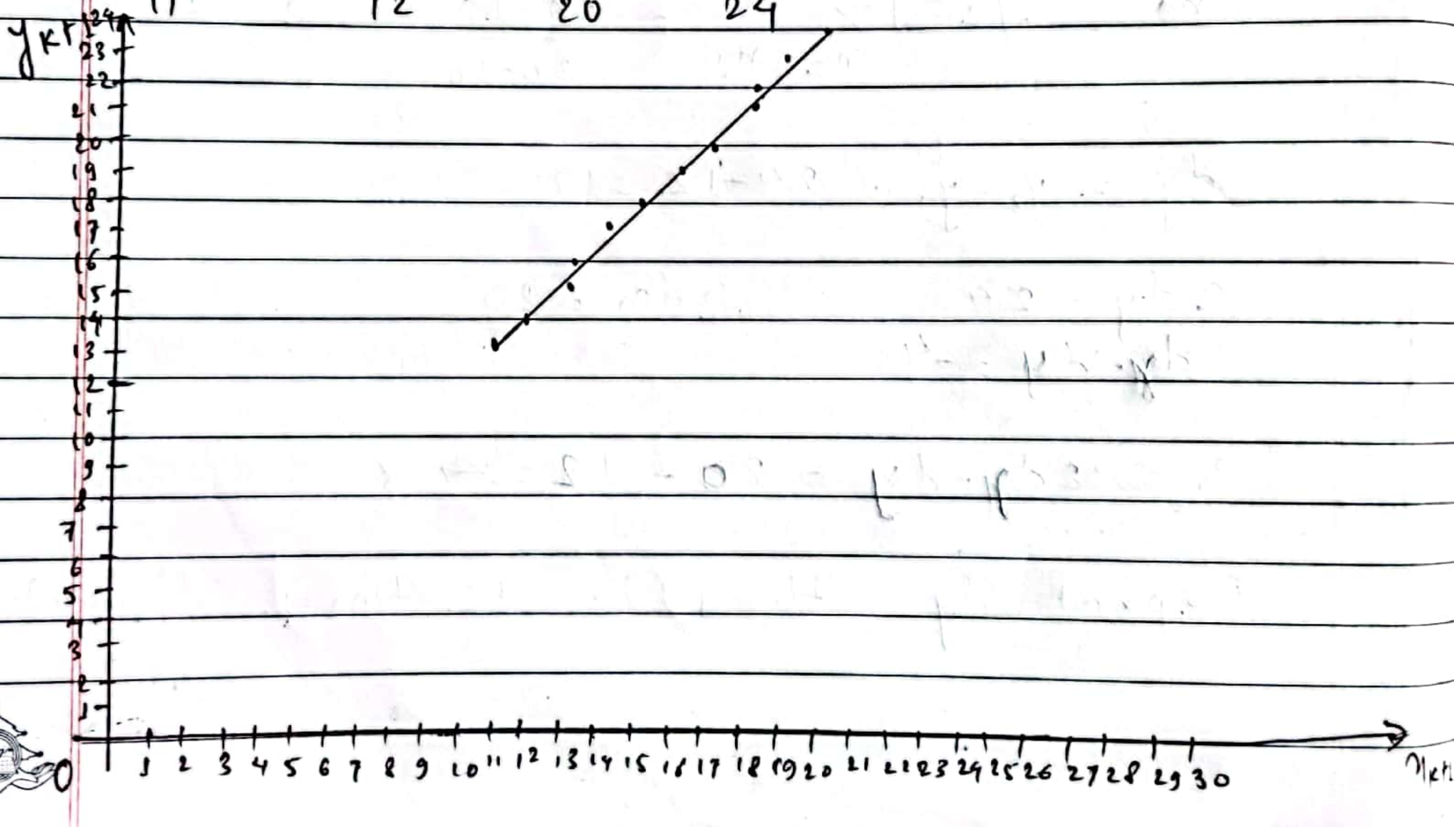
k	P_k	x_{k+1}	y_{k+1}
0	14	11	13
1	18	12	14
2	22		

K	P_K	x_{k+1}	y_{k+1}
0	8	11	13
1	4	12	14
2	0	13	15
3	-4	13	16
4	16	14	17
5	12	15	18
6	+8	16	19
7	4	17	20
8	0	18	21
9	-4	18	22
10	16	19	23
11	12	20	24



Formula

$P_0 = 1 - r$ $\qquad\qquad$ $P_0 = 5/4 - r$

if $P_k < 0$: $(x_k + 1, y_k)$

$$P_{k+1} = P_k + 2x_{k+1} + 1$$

else $(x_k + 1, y_k - 1)$

$$P_{k+1} = P_k + 2x_{k+1} + 1 - 2y_{k+1}$$

Repeat $(x \geq y)$

need to perform 8-way symmetry
no need " " " "

Q. Digitize octant/circle with radius $r = 8$ $(x^2 + y^2 = 64)$

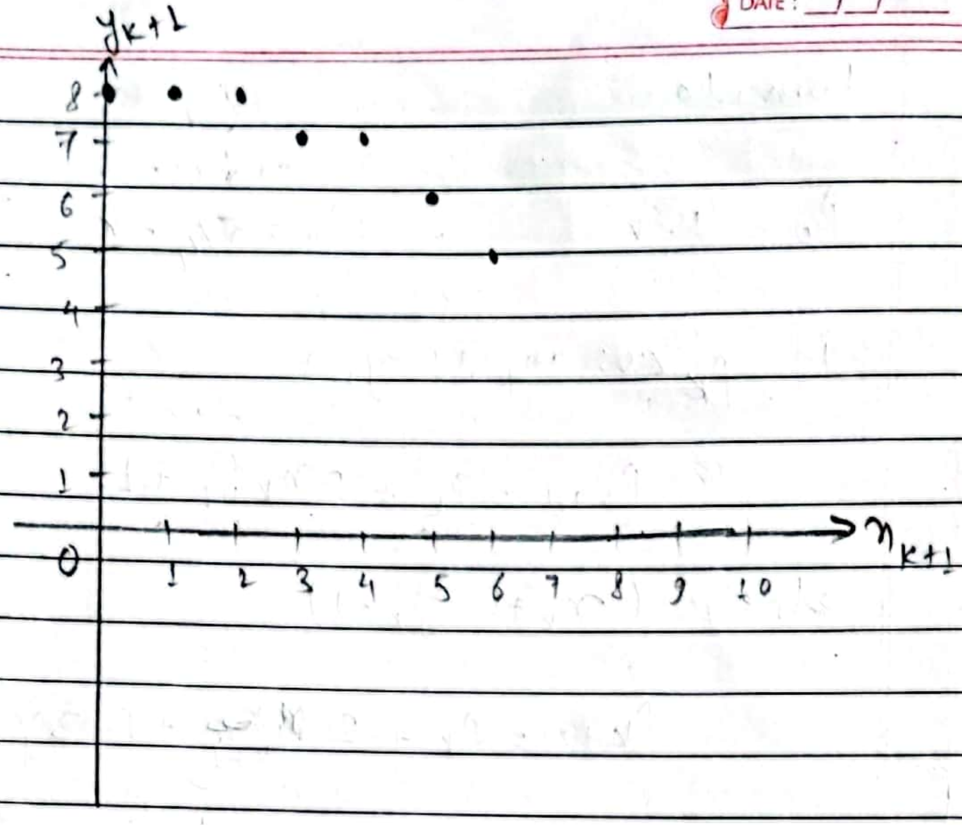
Sol^n. $P_0 = 1 - r = 1 - 8 = -7$

$(x_0, y_0) = (0, r) = (0, 8)$

k	P_k	x_{k+1}	y_{k+1}	$2x_{k+1}$	$2y_{k+1}$	
0	-7	1	8	2	16	$P_0 = -7$
1	-4	2	8	4	16	$P_1 = -7 + 2 + 1 = -4$
2	1	3	7	6	14	$P_2 = -4 + 4 + 1 = 1$
3	-6	4	7	8	14	$P_3 = 1 + 6 + 1 - 14 = -6$
4	3	5	6	10	12	$P_4 = -6 + 8 + 1 = 3$
5	2	6	5	12	10	$P_5 = 3 + 10 + 1 - 12 = 2$
~~6~~	~~6~~	~~7~~	~~4~~			~~$P_6 = 3 + 12 + 1 - 10 = 6$~~

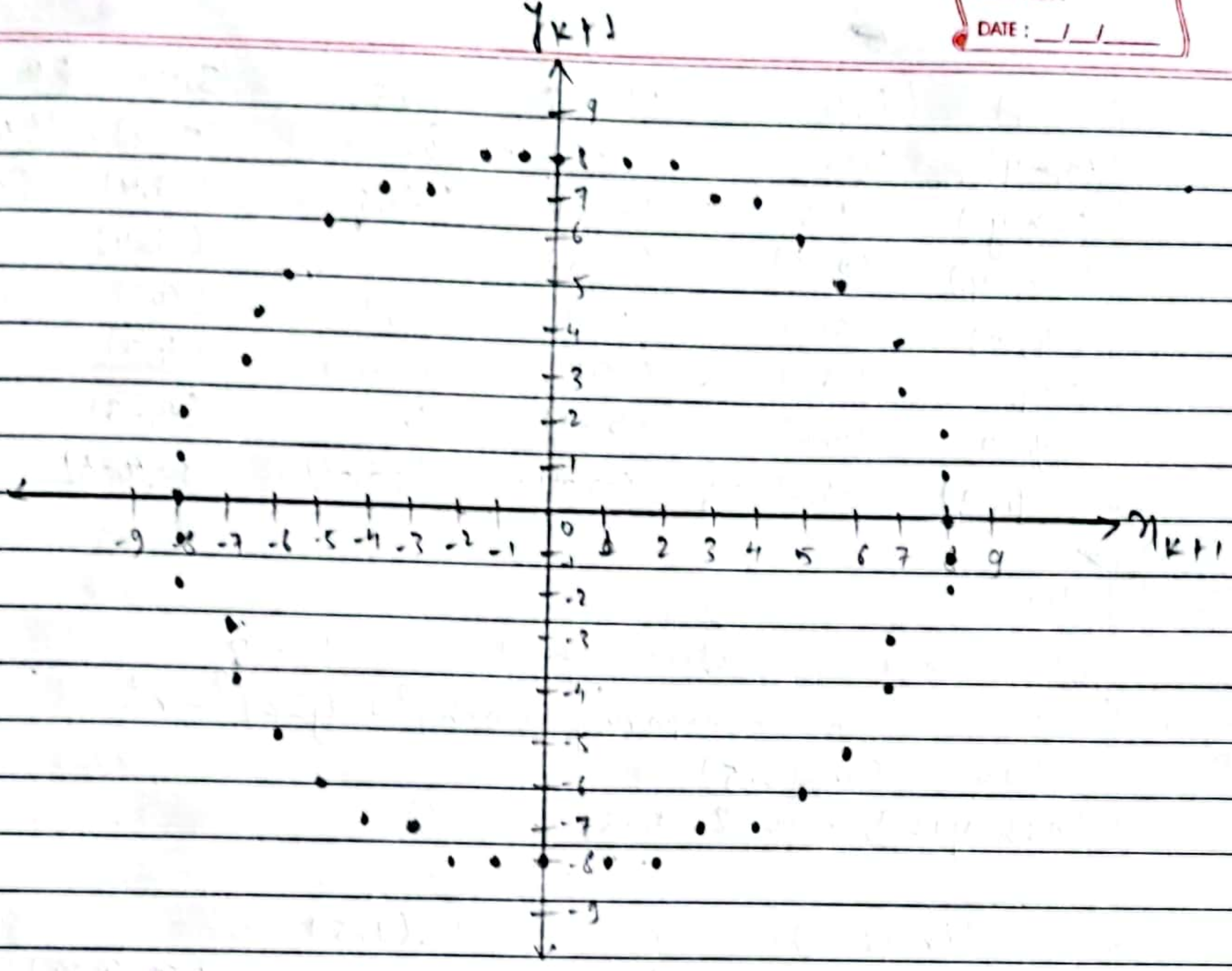
Since $x \geq 6$ i.e. $6 \geq 5$, we stop the iteration here.

(Note: Don't forget to plot (0, 8) also.)



To draw circle :

(x,y)	(0,8)	(1,8)	(2,8)	(3,7)	(4,7)	(5,6)	(6,5)
(x,-y)	(0,-8)	(1,-8)	(2,-8)	(3,-7)	(4,-7)	(5,-6)	(6,-5)
(-x,y)	(0,8)	(-1,8)	(-2,8)	(-3,7)	(-4,7)	(-5,6)	(-6,5)
(-x,-y)	(0,-8)	(-1,-8)	(-2,-8)	(-3,-7)	(-4,-7)	(-5,-6)	(-6,-5
(y,x)	(8,0)	(8,1)	(8,2)	(7,3)	(7,4)	(6,5)	(5,6)
(-y,x)	(-8,0)	(-8,1)	(-8,2)	(-7,3)	(-7,4)	(-6,5)	(-5,6)
(y,-x)	(8,0)	(8,-1)	(8,-2)	(7,-3)	(7,-4)	(6,-5)	(5,-
(-y,-x)	(-8,0)	(-8,-1)	(-8,-2)	(-7,-3)	(-7,-4)	(-6,-5)	(-5,-



Q. Digitize the circle $(x-2)^2 + (y-3)^2 = 25$

soln. $p_0 = 1 - r = 1 - 5 = -4$

$(x_0, y_0) = (0, r) = (0, 5)$ (here we don't matter which is center it is always $(0, r)$)

k	p_k	x_{k+1}	y_{k+1}	$2x_{k+1}$	$2y_{k+1}$	
						$p_0 = -4$
0	-4	1	5	2	10	$p_1 = -4+2+1 = -1$
1	-1	2	5	4	10	$p_2 = -1+4+1 = 4$
2	4	3	4	6	8	$p_3 = 4+6+1-8 = 3$
3	3	4	3			

since $4 \geq 3$, i.e. $x \geq y$, we stop here.

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(x, y)	$(0,5)$	$(1,5)$	$(2,5)$	$(3,4)$	$(4,3)$
$(x, -y)$	$(0,-5)$	$(1,-5)$	$(2,-5)$	$(3,-4)$	$(4,-3)$
$(-x, y)$	$(0,5)$	$(-1,5)$	$(-2,5)$	$(-3,4)$	$(-4,3)$
$(-x, -y)$	$(0,-5)$	$(-1,-5)$	$(-2,-5)$	$(-3,-4)$	$(-4,-3)$
(y, x)	$(5,0)$	$(5,1)$	$(5,2)$	$(4,3)$	$(3,4)$
$(-y, x)$	$(-5,0)$	$(-5,1)$	$(-5,2)$	$(-4,3)$	$(-3,4)$
$(y, -x)$	$(5,0)$	$(5,-1)$	$(5,-2)$	$(4,-3)$	$(3,-4)$
$(-y, -x)$	$(-5,0)$	$(-5,-1)$	$(-5,-2)$	$(-4,-3)$	$(-3,-4)$

~~To find octet) doing here~~

Now,

(To find circle) Since value of $h = 2$, $k = 3$

→ on comparing $(x-h)^2 + (y-k)^2 = r^2$

Then, For $(0,5)$

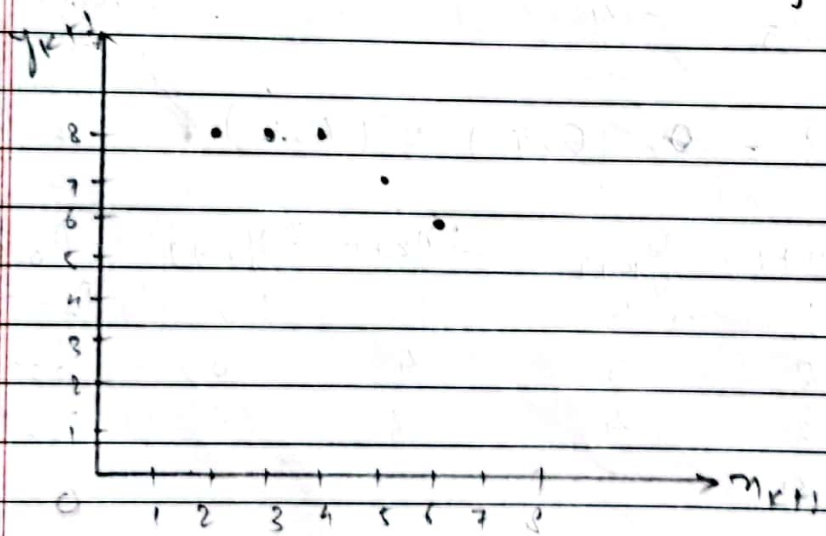
$(x+h, y+k) = (0+2, 5+3) = (2,8)$

For $(1,5)$
$(x+h, y+k) = (3,8)$

For $(2,5)$
$(x+h, y+k) = (4,8)$

For $(3,4)$
$(x+h, y+k) = (5,7)$

For $(4,3)$
$(x+h, y+k) = (6,6)$



For octant

⑦ Move each calculated pixel position (x, y) onto the elliptical path centered on (x_c, y_c) & plot the co-ordinate values:

$$x = x + x_c, \quad y = y + y_c$$

⑧ Repeat the steps for region 1 until

$$2r_y^2 x \geq 2r_x^2 y$$

Q. Digitize an ellipse with $r_x = 8$ & $r_y = 6$

Soln. given,

$r_x = 8$ & $r_y = 6$

$(x_0, y_0) = (0, r_y) = (0, 6)$

Now,

Decision parameter,

$\cancel{P_1 0}$. $P1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$

$= 6^2 - 8^2 \times 6 + \frac{1}{4} \times 8^2$

$= -332$

$(P_k < 0)$

$p1_{k+1} = p1_k + 2r_y^2 x + r_y^2$

K	P_K	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-332	(1, 6)	72	768
1	-224	(2, 6)	144	768
2	-44	(3, 6)	216	768
3	208	(4, 5)	288	768

$P1_{k+1} = P1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$

4	-236	(5, 5)	360	640
5	160	(6, 4)	432	512
6	-36	(7, 3)	~~504~~ 936	896

We now move out of region 1, since $2r_y^2 x > 2r_x^2 y$

<u>For region 2</u>

Region 1 को last point

The initial point is $(x_0, y_0) = (7, 3)$ and initial distance parameter is,

$$p2_0 = r_y^2 \left(x_0 + \frac{1}{2}\right)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$$

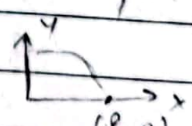
$$= 6^2 \left(7 + \frac{1}{2}\right)^2 + 8^2 (3-1)^2 - 8^2 \times 6^2$$

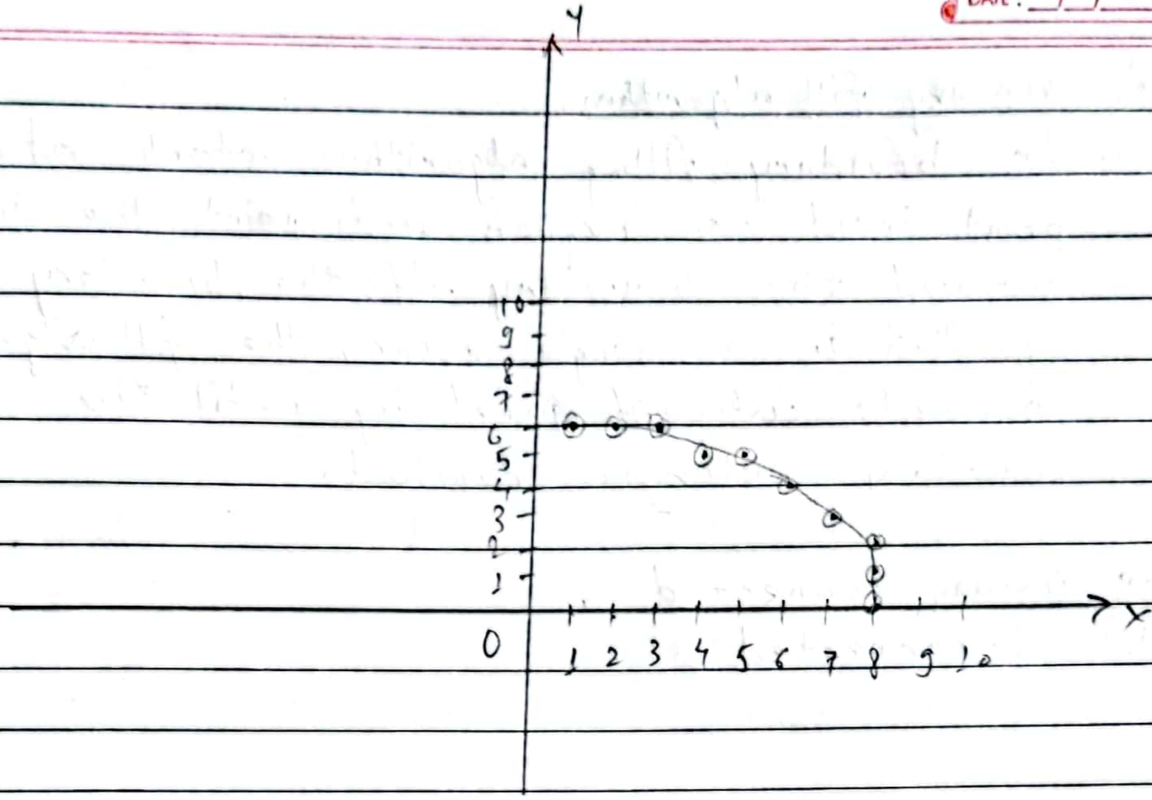
$$= -23$$

Now,

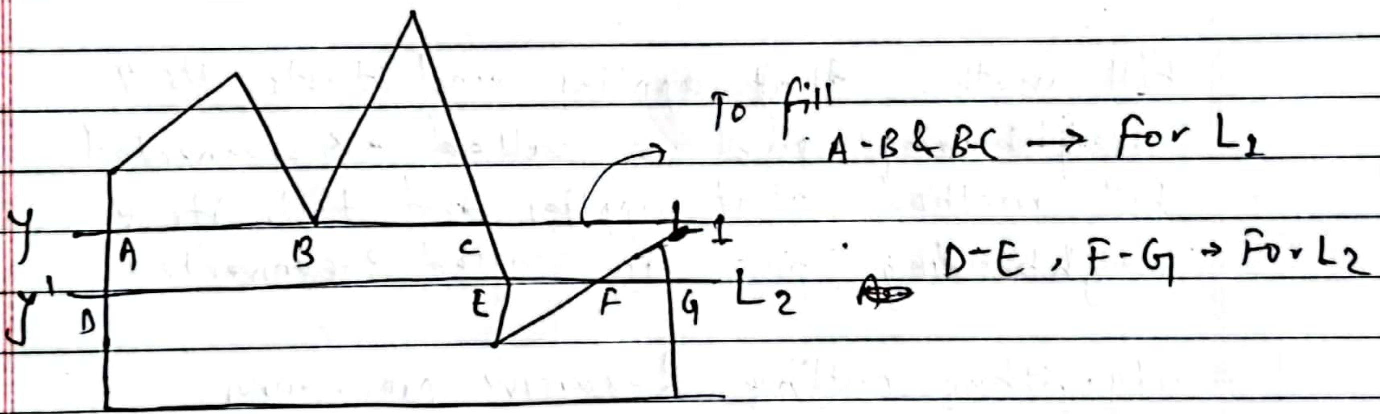
K	P_K	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-3	(8, 2)	576	256
1	381	(8, 1)	576	128
2	317	(8, 0)		

→ This is in x-axis so, first quadrant completes & no need to compute from here





3.4 # Scan-line polygon fill Algorithm



- For scan line L1 we count the point of intersection as twice whereas for scan Line L2 we count the point of intersection only once

$$\begin{bmatrix} A & D \\ C & B \end{bmatrix} \xrightarrow{\text{Inverse}} \begin{bmatrix} B & -D \\ -C & A \end{bmatrix}$$

sign change

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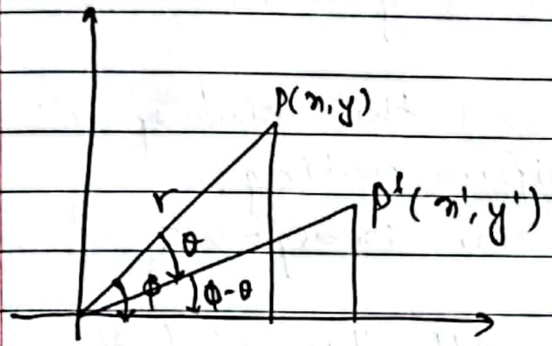
If co-ordinates represented as row vector, then,

$$p'^T = (R \cdot p)^T$$
$$= p^T \cdot R^T.$$

* Derive for clockwise direction. (Negative rotation)



$$x = r\cos\phi$$
$$y = r\sin\phi$$

Now,

$$x' = r\cos(\phi - \theta)$$
$$= r\cos\phi \cdot \cos\theta + r\sin\phi\sin\theta$$

$$y' = r\sin(\phi - \theta)$$
$$= r\sin\phi\cos\theta - r\cos\phi\sin\theta$$

$$\therefore x' = x\cos\theta + y\sin\theta$$
$$y' = y\cos\theta - x\sin\theta$$

Now,

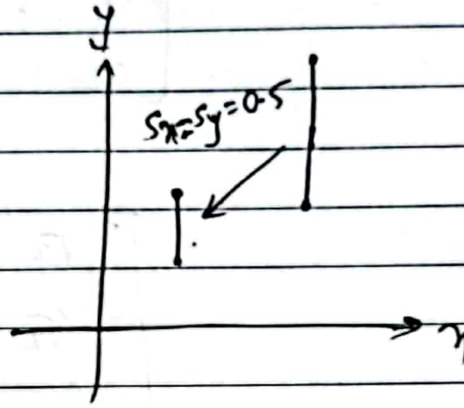
$$p' = R \cdot p$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

In homogeneous co-ordinate.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Scale to half of its original size i.e. $S(s_x, s_y) : (0.5, 0.5)$



$\Rightarrow$ $x' = x \cdot s_x, \quad y' = y \cdot s_y$

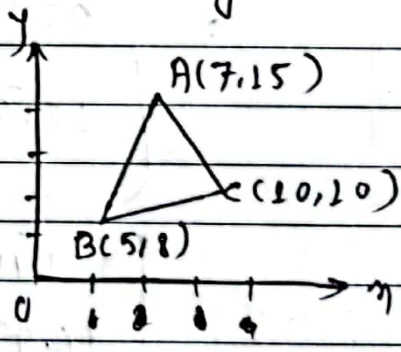
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = S \cdot P$$

In homogeneous co-ordinate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow P' = S(s_x, s_y) \cdot P$$

Q. Rotate the ΔABC by $45°$ clockwise about origin and scale it by $(2,3)$ about origin.



Sol^n: Step I: Rotation by $45°$ (clockwise)
Step-II: Scaling by $(2,3)$
$S(2,3)$

Now,

For clockwise

Net transformation $= S(2,3) \cdot R(-45)$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{2} & 2/\sqrt{2} & 0 \\ -3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ The transformation points are,

$A' = T A$

$$= \begin{bmatrix} 2/\sqrt{2} & 2/\sqrt{2} & 0 \\ -3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 15 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22\sqrt{2} \\ 12\sqrt{2} \\ 1 \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} 31.11 \\ 16.97 \\ 1 \end{bmatrix}$$

$B' = T \cdot B$

$$= \begin{bmatrix} 2/\sqrt{2} & 2/\sqrt{2} & 0 \\ -3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 13\sqrt{2} \\ 9/\sqrt{2} \\ 1 \end{bmatrix}$$

$$\therefore B' = \begin{bmatrix} 18.38 \\ 6.36 \\ 1 \end{bmatrix}$$

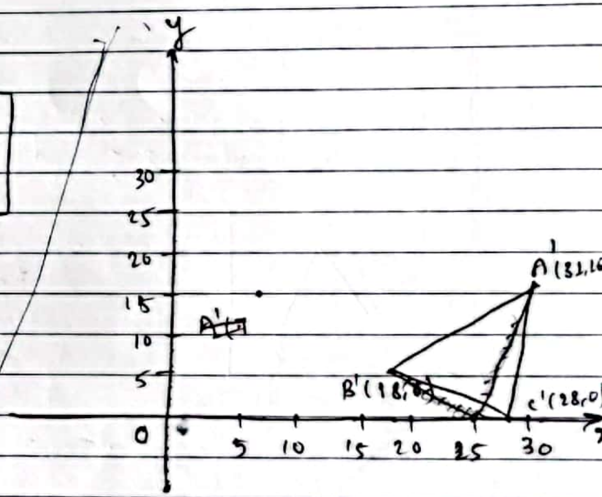
&

$C' = T \cdot C$

$$= \begin{bmatrix} 2/\sqrt{2} & 2/\sqrt{2} & 0 \\ -3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix}$$

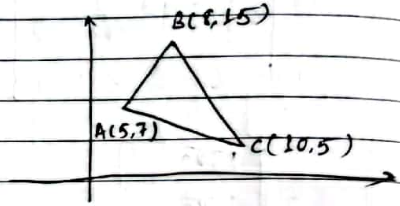
$$= \begin{bmatrix} 20\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore C' = \begin{bmatrix} 28.28 \\ 0 \\ 1 \end{bmatrix}$$



Q. Rotate a triangle ABC by 45° in CW about origin and scale it to twice its original size about origin.

Soln. T1 : Rotate by 45° in CW
i.e. R(45°) CW



T2 : Scale to double
i.e. S(2,2)

Net Transformation (T) = $T_2 \times T_1$ (Starting from rightmost)

$$T = S(2,2) \times R(45)\ CW$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Final Transformed points

$$\begin{bmatrix} A' & B' & C' \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 5 & 8 & 10 \\ 7 & 15 & 5 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 12\sqrt{2} & 23\sqrt{2} & 15\sqrt{2} \\ 2\sqrt{2} & 7\sqrt{2} & -5\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

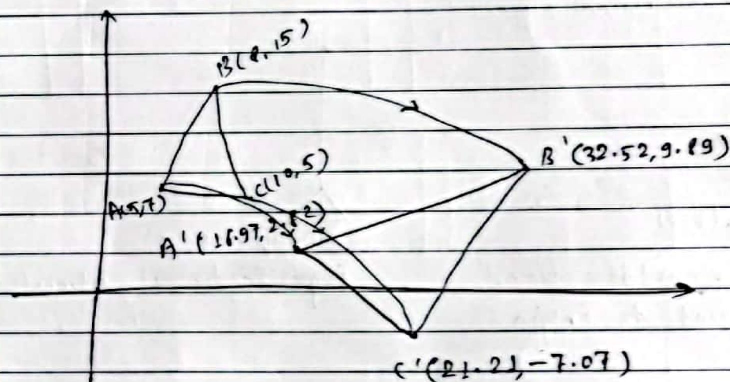


fig. Original and transformed object.

T_1 : Translate fixed point to origin i.e. $T(-x_r, -y_r)$
T_2 : Rotate about origin i.e. $R(\theta)$ ACW
T_3 : Inverse translation of fixed point i.e. $T(x_r, y_r)$

Net Transformation $(T) = T_3 \times T_2 \times T_1$

$$= T(x_r, y_r) \times R(\theta).ACW.T(-x_r, -y_r)$$

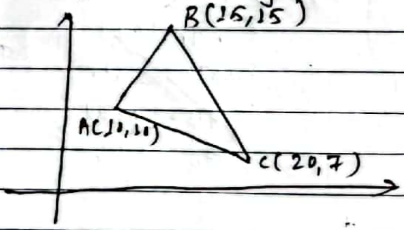
$$= \begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & -x_r\cos\theta + y_r\sin\theta \\ \sin\theta & \cos\theta & -x_r\sin\theta - y_r\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r\sin\theta \\ \sin\theta & \cos\theta & -x_r\sin\theta + y_r(1-\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}$$

For CW

Q. Rotate $\triangle ABC$ by 45° in CW about $(10,10)$ (x_r, y_r) & scale it to double of its original size about $(5,8)$ (x_f, y_f).



Sol^n.
Fixed point rotation:
T_1 : Translate $(10,10)$ to origin i.e. $T(-10,-10)$
T_2 : Rotate by 45° in CW i.e. $R(45^\circ)$ CW
T_3 : Inverse translation of fixed point i.e. $T(10,10)$

Fixed point scaling:
T_4 : Translate $(5,8)$ to origin i.e. $T(-5,-8)$
T_5 : Scale to double i.e. $S(2,2)$
T_6 : Inverse translation of fixed point i.e. $T(5,8)$

Net Transformation $(T) = T_6 \times T_5 \times T_4 \times T_3 \times T_2 \times T_1$

$$= T(5,8) \times S(2,2) \times \underbrace{T(-5,-8) \times T(10,10)}_{} \times R(45^\circ)\text{CW} \times T(-10,-10)$$

If there is successive Translation (T) then we can add two translations

$$= T(5,8) \times S(2,2) \times T(5,2) \times R(45^\circ)\text{CW} \times T(-10,-10)$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & 0.707 & -14.1 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & 0.707 & -9.192 \\ -0.707 & 0.707 & -2.121 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 20 \\ 0 & 2 & 8 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & 0.707 & -9.142 \\ -0.707 & 0.707 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

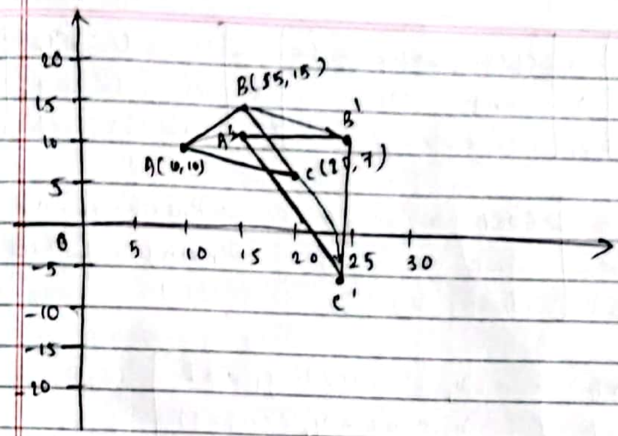
$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1.414 & 1.414 & -18.28 \\ -1.414 & 1.414 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1.414 & 1.414 & -13.28 \\ -1.414 & 1.414 & 12 \\ 0 & 0 & 1 \end{bmatrix}$$

Final tranceformed points

$$\begin{bmatrix} A' & B' & C' \end{bmatrix} = \begin{bmatrix} 1.414 & 1.414 & -13.28 \\ -1.414 & 1.414 & 12 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 10 & 15 & 20 \\ 10 & 15 & 7 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} A' & B' & C' \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 15 & 25.142 & 24.899 \\ 12 & 12 & -6.384 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$



Fixed point Rotation (CW)

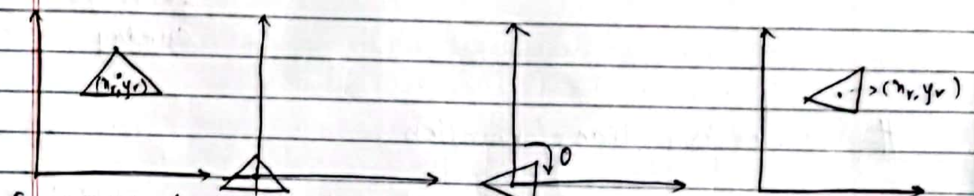


Fig. a) Original object

Fig. b) Translate fixed point to origin.

Fig. c) Rotate about origin

Fig. d) Inverse translation of fixed point.

T_1: Translate fixed point to origin i.e. $T(-x_r, -y_r)$

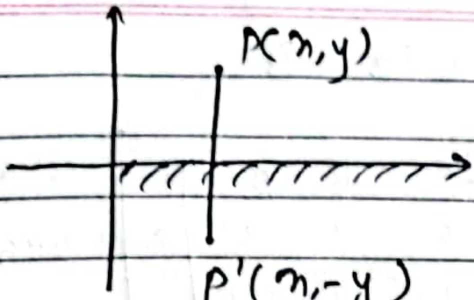
T_2: Rotate about origin i.e. $R(-\theta)$ CW

T_3: Inverse translation of fixed point i.e. $T(x_r, y_r)$

Net transformation $(T) = T_3 \times T_2 \times T_1$

$= T(x_r, y_r) \times R(-\theta)\,CW \times T(-x_r, -y_r)$

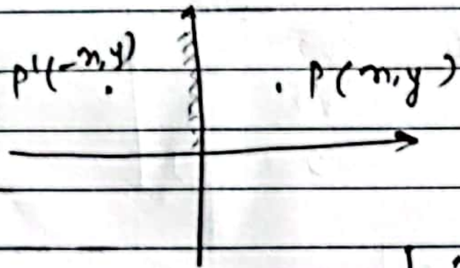
(We can only reflect either x or y-axis. If random reflection in either axis, coincide the axis)



$$x' = x$$
$$y' = -y$$
$$1 = 1$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

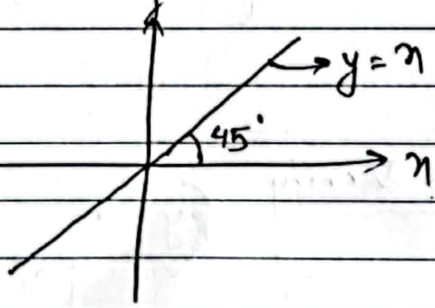
1.b Reflection about y-axis



$$x' = -x$$
$$y' = y$$
$$1 = 1$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q. Find the transformation matrix for reflection about the line $y = x$.



T1: Rotate reflection axis by 45° in ACW i.e. R(45°) ACW.

T2: Reflection about y-axis i.e. R_{fy}.

T3: Rotate back reflection axis by 45° in CW ~~i.e. R(45°)~~ i.e. R(45°) CW.

Net Transformation (T) = $T_3 \times T_2 \times T_1$.

$$= \begin{bmatrix} \cos45 & \sin45 & 0 \\ -\sin45 & \cos45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos45 & -\sin45 & 0 \\ \sin45 & \cos45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 & 0 \\ -1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt2 & -1/\sqrt2 & 0 \\ 1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 & 0 \\ -1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1/\sqrt2 & 1/\sqrt2 & 0 \\ 1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2+1/2 & 1/2+1/2 & 0 \\ 1/2+1/2 & -1/2+1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

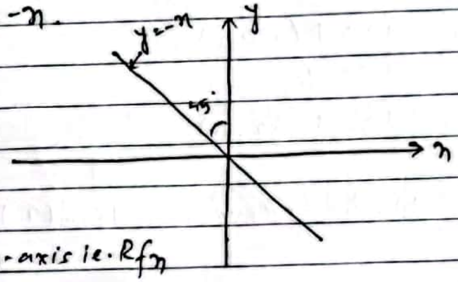
To Verify

$y = x$

$x' = y$
$y' = x$
$1 = 1$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q. Find the transformation matrix for reflection about the line $y = -x$.



Soln. T1: Rotate reflection axis by 45° in ACW i.e. R(45°) ACW

T2: Reflection about x-axis i.e. Rf_x

T3: Rotate back reflection axis by 45° in CW i.e. R(45°) CW.

Net Transformation (T) = $T_3 \times T_2 \times T_1$

$$= \begin{bmatrix} \cos45 & \sin45 & 0 \\ -\sin45 & \cos45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos45 & -\sin45 & 0 \\ \sin45 & \cos45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 & 0 \\ -1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt2 & -1/\sqrt2 & 0 \\ 1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt2 & 1/\sqrt2 & 0 \\ -1/\sqrt2 & 1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt2 & -1/\sqrt2 & 0 \\ -1/\sqrt2 & -1/\sqrt2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac12 - \frac12 & -\frac12 - \frac12 & 0 \\ -1/2 - 1/2 & \frac12 - 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To verify

$y = -x$

$x' = -y$
$y' = -x$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

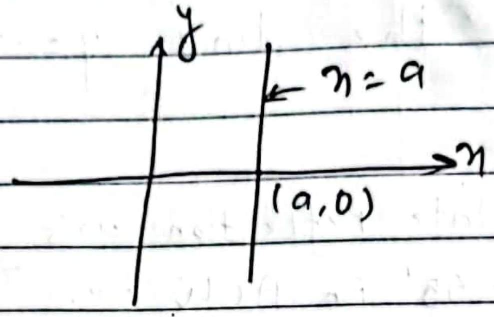
Q. Reflect about the line $x = a$.

Sol^n.

$T1 : T(-a, 0)$

$T2 : Rf_y$

$T3 : T(a, 0)$



Net transformation $(T) = T3 \times T2 \times T1$

$$= \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T = \begin{bmatrix} -1 & 0 & 2a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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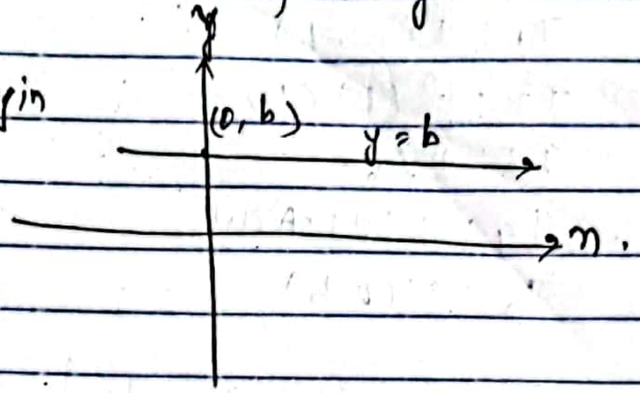
Q. Find the transformation matrix for $y = b$.

T_1: Translate $(0, +b)$ to origin
i.e. $T(0, -b)$



T_2: Reflect about x-axis
Inverse the Rf_x.

T_3: Translate to original place.
i.e. $T(0, b)$

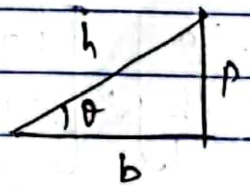
Net transformation(T) = $T_3 \times T_2 \times T_1$.

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2b \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Find the transformation matrix for reflect about the line $y = mx + b$.



$\tan\theta = p/b$

$m < 1$
$m > 1$
$m = 1$

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Sol^n.

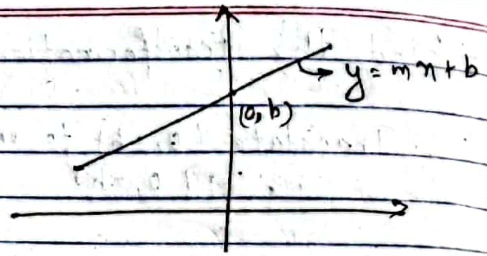
$T_1 : T(0,-b)$

$T_2 : R(\theta)\ CW$

$T_3 : Rf_x$

$T_4 : R(\theta)\cdot ACW$

$T_5 : T(0,b)$



Net transformation (T) = $T_5 \times T_4 \times T_3 \times T_2 \times T_1$

$$T = T(0,b) \times R(\theta)\cdot ACW \times Rf_x \times R(\theta) CW \times T(0,-b)$$

or, $$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta & -b\sin\theta \\ -\sin\theta & \cos\theta & -b\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta & -b\sin\theta \\ \sin\theta & -\cos\theta & b\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos^2\theta-\sin^2\theta & \cos\theta\cdot\sin\theta+\sin\theta\cos\theta & \cos\theta(-b)\sin\theta - \sin\theta\cdot b\cos\theta \\ \sin\theta\cdot\cos\theta+\cos\theta\cdot\sin\theta & \sin^2\theta-\cos^2\theta & -b\sin^2\theta+b\cos^2\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$b\left(\frac{-m^2}{1+m^2} \ldots\right)$ $\tan\theta = m/1 = P/b$ $\sin\theta = m/\sqrt{1+m^2}$, $\cos\theta = \frac{1}{\sqrt{1+m^2}}$

$-m^2 \quad +1+m^2$

$$= \begin{bmatrix} \cos^2\theta-\sin^2\theta & \cos\theta\cdot\sin\theta+\sin\theta\cdot\cos\theta & -b\cos\theta\cdot\sin\theta - b\sin\theta\cdot\cos\theta \\ \sin\theta\cdot\cos\theta+\cos\theta\cdot\sin\theta & \sin^2\theta-\cos^2\theta & -b\sin^2\theta+b\cos^2\theta+b \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{1}{\sqrt{1+m^2}}\right)^2-\left(\frac{m}{\sqrt{1+m^2}}\right)^2 & 2\left(\frac{1}{\sqrt{1+m^2}}\cdot\frac{m}{\sqrt{1+m^2}}\right) & -2b\left(\frac{1}{\sqrt{1+m^2}}\cdot\frac{m}{\sqrt{1+m^2}}\right) \\ 2\left(\frac{m}{\sqrt{1+m^2}}\right)\cdot\left(\frac{1}{\sqrt{1+m^2}}\right) & \left(\frac{m}{\sqrt{1+m^2}}\right)^2-\left(\frac{1}{\sqrt{1+m^2}}\right)^2 & b-b\left(\frac{m}{\sqrt{1+m^2}}\right)^2+b\left(\frac{1}{\sqrt{1+m^2}}\right)^2 \\ 0 & 0 & 1 \end{bmatrix}$$

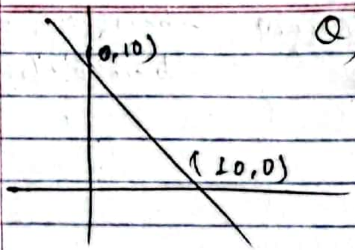
$$= \begin{bmatrix} \frac{1-m^2}{1+m^2} & 2\cdot\frac{m}{1+m^2} & \cancel{2b\frac{m}{1+m^2}}\ -2b\frac{m}{1+m^2} \\ +\frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \cancel{b\left(\frac{1-m^2}{1+m^2}\right)}\ \frac{2b}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$b\left(1-\frac{m^2}{1+m^2}+\frac{1}{1+m^2}\right)$$

$$= b\left(\frac{1+m^2-m^2+1}{1+m^2}\right)$$

$$= 2b/1+m^2$$

Q) Reflect about the line A(10,0) and B(0,10)



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

② Shearing

- Transformation that cause deformation in the original object by assuming as if the object is composed of internal layers and these layers are caused to slide over each other.

2.1 Shearing along x-axis.

$y' = y$

$x' = x + Sh_x y$

where Sh_x is shearing constant

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2.2 Shearing along y-axis

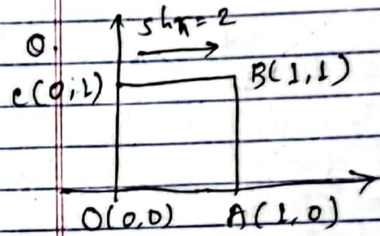
$x' = x$

$y' = y + sh_y x$

$$\cancel{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q.



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The value of S_x and S_y is found by calculating the position of (x,y) in the window to the corresponding position of (u,v) in the viewport.

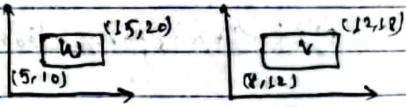
$$S_x = \frac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}}$$

$$S_y = \frac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}}$$

$$T_{wv} = \begin{bmatrix} 1 & 0 & x_{vmin} \\ 0 & 1 & y_{vmin} \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -x_{wmin} \\ 0 & 1 & -y_{wmin} \\ 0 & 0 & 1 \end{bmatrix}$$

Q Determine the window to viewport transformation for the following dimension of window and viewport.

	Window	Viewport
Lower left corner	(5,10)	(8,12)
Upper right corner	(15,20)	(12,18)



Soln. Here,

$$S_x = \frac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}} = \frac{12-8}{15-5} = 0.4$$

$$T_{wv} = T(8,12) \cdot S(S_x, S_y) \cdot T(-5,-10)$$

$$S_y = \frac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}} = \frac{18-12}{20-10} = 0.6$$

Now

$$T_{wv} = T(8,12) \cdot S(S_x, S_y) \cdot T(-5,-10)$$

or,

$$T_{wv} = \begin{bmatrix} 1 & 0 & x_{vmin} \\ 0 & 1 & y_{vmin} \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -x_{wmin} \\ 0 & 1 & -y_{wmin} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0 & -2 \\ 0 & 0.6 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T_{wv} = \begin{bmatrix} 0.4 & 0 & 6 \\ 0 & 0.6 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

Viewing pipeline

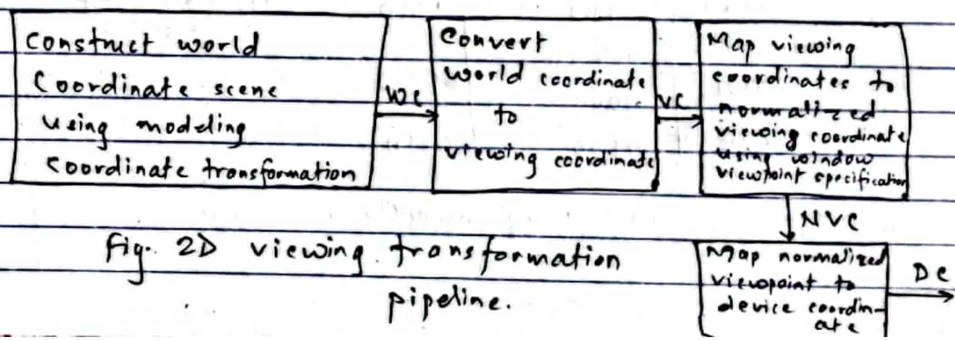


Fig. 2D viewing transformation pipeline.

$\frac{y-y_1}{m} + x_1 = x$

where $x = x_{wmin}$ or x_{wmax}.

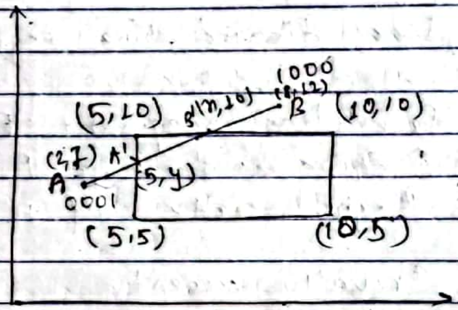
Q. Apply Cohen-Sutherland line clipping algorithm for calculating saved portion of a line from (2,7) to (8,12) in a window. ($x_{wmin} = y_{wmin} = 5$ & $x_{wmax} = y_{wmax} = 10$)

Sol^n.

Region code of A: 0001

Region code of B: 1000

Logical AND of A&B is 0000



Thus, this is non-trival case. so, we need further processing.

We start from point outside the window boundary and find intersection with window boundary. Thus line segment AB is divided into AA', A'B' & B'B. We discard line AA' & B'B. For A'B' R.C. of A' = 0000, R.C. of B' = 0000, so trivally accept A'B'.

Slope of AB

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{12-7}{8-2} = 5/6$

For line AA'

$\cancel{y-y_1 = m(x-x_1)}$

$m = \frac{7-y}{2-5}$

or, $5/6 = \frac{7-y}{-3}$

or, $\frac{-15}{6} - 7 = -y$

$\therefore y = \frac{19}{2}$

or, $y -$

For line BB'

$m = \frac{12-10}{8-x}$

or, $\frac{5}{6} = \frac{2}{8-x}$

or, $8-x = \frac{12}{5}$

$\therefore x = 28/5$

Alternatively

for line AA'

$y - y_1 = m(x - x_1)$

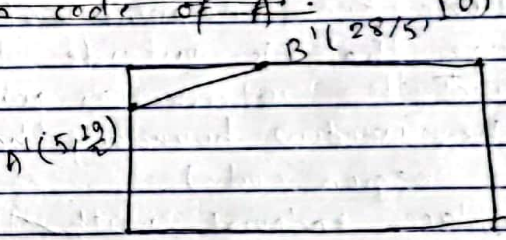
$y = y_1 + m(x - x_1)$

$y = 7 + \frac{5}{6}(5-2)$

$y = 7 + \frac{15}{6}$

$y = 19/2$

~~Region code of A'.~~

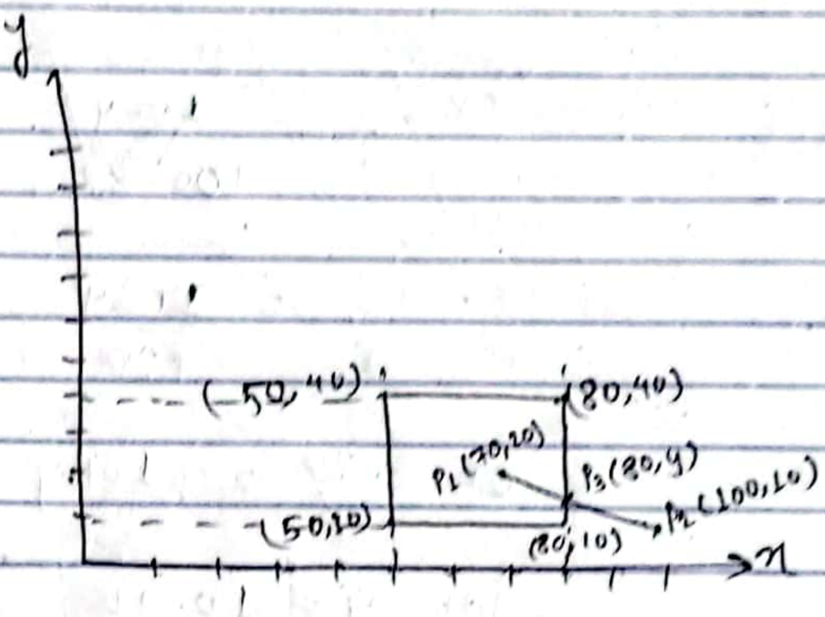


$10-12 = \frac{5}{6}(x-8)$

$-\frac{12}{5} + 8 = x$

$x = 28/5$

Q. Using Cohen Sutherland line clipping algorithm clip the line $P_1(70,20)$ and $P_2(100,10)$ against a window lower left hand corner (50,10) and upper right hand corner (80,40).



soln.

Region code for P_1: 0000

Region code for P_2: 0010

On logical AND operation for both region code, the output is 0000.

Since result = 0000, we need clipping.

Choose the end point of the line ie. $P_3(80,y)$ so, the line P_1P_2 is broken down into P_1P_3 & P_3P_2.

Now finding intersection point $P_3(80,y)$

Slope of line P_1P_2 is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-20}{100-70} = \frac{-1}{3}$$

Now

Slope of line P_2P_3 is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{or, } -\frac{1}{3} = \frac{10 - y}{100 - 80}$$

$$\text{or, } -\frac{1}{3} = \frac{10 - y}{+20}$$

$$\text{or, } -\frac{20}{3} = +10 + y$$

$$\text{or } y = 10 - \frac{20}{3}$$

$$\therefore y = 50/3$$

$$\text{or, } -\frac{20}{3} - 10 = -y$$

$$\text{or, } -\frac{50}{3} = -y$$

$$\therefore y = 50/3$$

∴ Intersection point is $(80, 50/3)$

Now,

Region code of P_3 : 0000

Then, Logical AND of P_1 & P_3 is 0000
Hence trivially accepted.

&

~~P... Logical AND of~~

After clipping

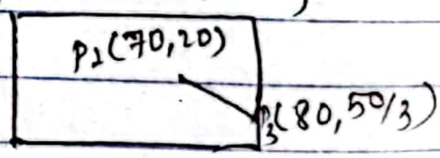


Fig. After Clipping.