Unit 5

Probabilistic Methods

Contents

- Introduction to Probabilistic Reasoning
- Bayes and Markov Network, DBN’s and HMN’s

Introduction to Probabilistic Reasoning

- One of the most common characteristics of the human information available is its imperfection due to partial observability, non deterministic or combination of both
- An agent may not know what state it is in or will be after certain sequence of actions
- Agent can cope with these defects and make rational judgments and rational decisions to handle such uncertainty and draw valid conclusions

Introduction to Probabilistic Reasoning

What is uncertainty?

- The lack of the exact knowledge that would enable us to reach a perfectly reliable conclusion
- Classical Logic permits only exact reasoning i.e. perfect knowledge always exists

IF A is true THEN A is not false **and** **IF B is true THEN B is not false**

- In Real world such clear cut knowledge could not be provided to systems

Introduction to Probabilistic Reasoning

Sources of Uncertain Knowledge

- **Weak Implication**: Domain experts and knowledge engineer have rather painful or hopeless task of establishing concrete correlation between IF(Condition) and THEN(action) part of rules. Vague Data.
- **Imprecise Language** : NLP is ambiguous and imprecise. We define facts in terms of **often, sometimes, frequently, hardly ever**. Such can affect IF-THEN implication
- **Unknown Data**: incomplete and missing data should be processes to an approx. reasoning with this values
- **Combining the views of different experts**: Large system uses data from many experts

Introduction to Probabilistic Reasoning

- The basic Concept of probability plays significant role in our life like we try to determine the probability of rain, prospect of promotion, likely hood of winning in Black Jack
- The probability of an event is the proportion of cases in which the event occurs (Good, 1959)
- Probability, mathematically, is indexed between 0 and 1
- Most events have probability index strictly between 0 and 1, which means that each event has at lease two possible outcomes: favourable outcome or success and unfavourable outcomes or failure

$$\mathrm{P}(success) = \frac{The\ number\ of\ successes}{The\ number\ of\ possible\ outcomes}$$

$$\mathrm{P}(failure) = \frac{The\ number\ of\ failure}{The\ number\ of\ possible\ outcomes}$$

Introduction to Probabilistic Reasoning

- If *s* is the number of success and *f* is the number of failure then:

- If *s* is the number of success and *f* is the number of failure then:

$$P(success) = \frac{s}{s+f}$$

$$P(failure) = \frac{f}{s+f}$$

and

and

$$p + q = 1$$

$$p + q = 1$$

Introduction to probabilistic Reasoning

- Let us consider classical examples with a coin and a die. If we throw a coin, the probability of getting a head will be equal to the probability of getting a tail. In a single throw, s = f = 1, and therefore the probability of getting a head (or a tail) is 0.5.
- Consider now a dice and determine the probability of getting a 6 from a single throw. If we assume a 6 as the only success, then s = 1 and f = 5, since there is just one way of getting a 6, and there are five ways of not getting a 6 in a single throw. Therefore, the probability of getting a 6 is

- Let us consider classical examples with a coin and a die. If we throw a coin, the probability of getting a head will be equal to the probability of getting a tail. In a single throw, s = f = 1, and therefore the probability of getting a head (or a tail) is 0.5.
- Consider now a dice and determine the probability of getting a 6 from a single throw. If we assume a 6 as the only success, then s = 1 and f = 5, since there is just one way of getting a 6, and there are five ways of not getting a 6 in a single throw. Therefore, the probability of getting a 6 is

$$P = \frac{1}{1+5} = 0.1666$$

Likewise, the probability of not getting 6 is

Likewise, the probability of not getting 6 is

$$q = \frac{5}{1+5} = 0.8333$$

Introduction to Probabilistic Reasoning

- Above instances are for independent events i.e. mutually exclusive events which can not happen simultaneously
- In the dice experiment, the two events of obtaining a 6 and, for example, a 1 are mutually exclusive because we cannot obtain a 6 and a 1 simultaneously in a single throw. However, events that are not independent may affect the likelihood of one or the other occurring. Consider, for instance, the probability of getting a 6 in a single throw, knowing this time that a 1 has not come up. There are still five
  ways of not getting a 6, but one of them can be eliminated as we know that a 1 has not been obtained. Thus,

$$P = \frac{1}{1 + (5 - 1)}$$

Introduction to Probabilistic Reasoning

- Let A and B be two not mutually exclusive events, but occur conditionally on the occurrence of other.
- The probability of event A will occur if event B occurs is called conditional Probability

$$p(A|B) = \frac{the\ number\ of\ times\ A\ and\ B\ can\ occur}{the\ number\ of\ times\ B\ can\ occur}$$

The probability of both A and B will occur is called joint probability $(\boldsymbol{A} \cap \boldsymbol{B})$

$p(A|B) = \frac{p(A \cap B)}{p(B)}$, the probability of A occurring given B has occurred

$p(B|A) = \frac{p(B \cap A)}{p(A)}$, the probability of B occurring given A has occurred

Introduction to Probabilistic Reasoning

- Joint probability is commutative, thus

- Joint probability is commutative, thus

$$p(A \cap B) = p(B \cap A)$$

Therefore,

Therefore,

$$p(A \cap B) = p(B|A) * p(A)$$

Now the final equation becomes:

Now the final equation becomes:

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B)} \text{ --(a)}$$

--(a)

Where:

Where:

$p(A|B)$ is the conditional probability that event A occurs given event B has occurred

$p(B|A)$ is the conditional probability that event B occurs given event A has occurred

$p(A)$ is the probability of event A occurring

$p(B)$ is the probability of event B occurring

The above equation (a) is known as Bayesian Rule

The above equation (a) is known as **Bayesian Rule**

Introduction to Probabilistic Reasoning

- For *n* number of mutually exclusive event B we have

$$p(A \cap B_1) = p(A|B_1) \times p(B_1)$$
$$p(A \cap B_2) = p(A|B_2) \times p(B_2)$$
$$\vdots$$
$$p(A \cap B_n) = p(A|B_n) \times p(B_n)$$

or when combined:

$$\sum_{i=1}^{n} p(A \cap B_i) = \sum_{i=1}^{n} p(A|B_i) \times p(B_i)$$

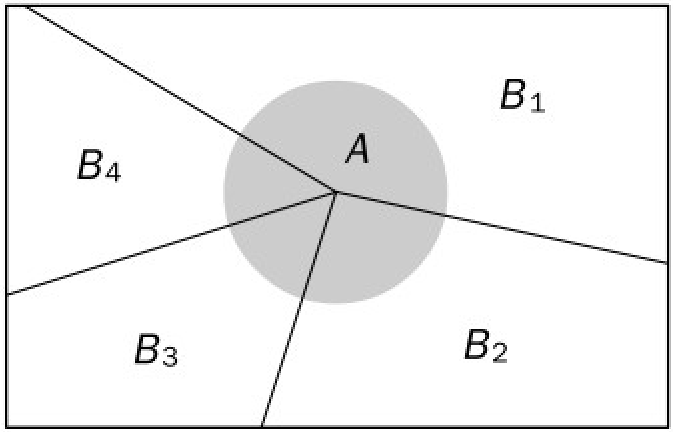
Introduction to Probabilistic Reasoning

- Summed over an exhaustive list of events for Bi, we get :

$$\sum_{i=1}^{n} p(A \cap B_i) = p(A)$$

- Which reduces to:

$$p(A) = \sum_{i=1}^{n} p(A|B_i) \times p(B_i)$$



The joint probability

Introduction to Probabilistic Reasoning

- If the occurrence of A depends on only two mutually exclusive events, i.e. B and NOT B, then above equation

$$p(A) = p(A|B) \times p(B) + p(A|\neg B) \times p(\neg B)$$

$$p(B) = p(B|A) \times p(A) + p(B|\neg A) \times p(\neg A)$$

- ayesian Equation, We

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B|A) \times p(A) + p(B|\neg A) \times p(\neg A)}$$

THANK YOU