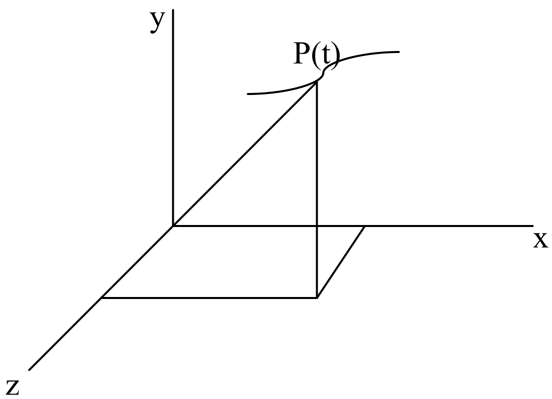
Parametric Cubic Curve

A parametric cubic curve is defined as $P(t) = \sum_{i=0}^{3} a_i t^i \qquad 0 <= t <= 1$ ------ (i)

Where, $P(t)$ is a point on the curve



Expanding equation (i) yields

$P(t) = a_3t^3 + a_2t^2 + a_1t + a_0$ -----------------------------(ii)

This equation is separated into three components of P(t)

$x(t) = a_{3x}t^3 + a_{2x}t^2 + a_{1x}t + a_{0x}$

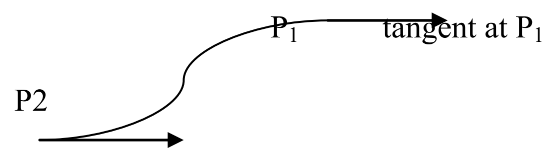
$y(t) = a_{3y}t^3 + a_{2y}t^2 + a_{1y}t + a_{0y}$

$z(t) = a_{3z}t^3 + a_{2z}t^2 + a_{1z}t + a_{0z}$ -----------------------------(iii)

To be able to solve (iii) the twelve unknown coefficients a_{ij} (algebraic coefficients) must be specified

From the known end point coordinates of each segment, six of the twelve needed equations are obtained. The other six are found by using tangent vectors at the two ends of each segment

The direction of the tangent vectors establishes the slopes(direction cosines) of the curve at the end points



This procedure for defining a cubic curve using end points and tangent vector is one form of *hermite* interpolation

Each cubic curve segment is parameterized from 0 to 1 so that known end points correspond to the limit values of the parametric variable *t*, that is P(0) and P(1)

Substituting $t = 0$ and $t = 1$ the relation ship between two end point vectors and the algebraic coefficients are found

$P(0) = a_0 \qquad P(1) = a_3 + a_2 + a_1 + a_0$

To find the tangent vectors equation ii must be differentiated with respect to t

$P'(t) = 3a_3t^2 + 2a_2t + a_1$

The tangent vectors at the two end points are found by substituting t = 0 and t = 1 in this equation

$P'(0) = a_1 \qquad P'(1) = 3a_3 + 2a_2 + a_1$

The algebraic coefficients 'a_i' in equation (ii) can now be written explicitly in terms of boundary conditions – endpoints and tangent vectors are

$a_0 = P(0) \qquad a_1 = P'(0)$

$a_2 = -3\,P(0) + 3\,P(1) - 2\,P'(0) - P'(1) \qquad a3 = 2\,P(0) - 2\,P(1) + P'(0) + P'(1)$

substituting these values of 'a_i' in equation (ii) and rearranging the terms yields

$P(t) = (2t^3 - 3t^2 + 1)\,P(0) + (-2t^3 + 3t^2)\,P(1) + (t^3 - 2t^2 + t)\,P'(0) + (t^3 - t^2)\,P'(1)$

The values of P(0), P(1), P'(0), P'(1) are called *geometric coefficients* and represent the known vector quantities in the above equation

The polynomial coefficients of these vector quantities are commonly known as *blending functions*

By varying parameter t in these blending function from 0 to 1 several points on curve segments can be found