

UNIT 2

Problem Solving

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Steps in Problem Solving

- Goal Formulation
- Problem Formulation
- Searching
- Execution

Defining Problems as a State Space Search

- Things that must be done to solve a problem are:
 - ▢ Define the problem: It includes precise specifications of what the initial situation(s) will be as well as what final situations constitute acceptable solutions to the problem
 - ▢ Analyse the problem: Few very important aspects those having immense impact on the appropriateness of various possible techniques for problem solving is to be critically examined
 - ▢ Isolate and Represent the task knowledge: Knowledge necessary to solve the problem must be identified, isolated and represented
 - ▢ Choose and apply the best technique: Among the alternatives identify the best technique to solve the problem and apply it

Defining Problems as a State Space Search

- Problem Statement: Play Chess, Solve Tower of Hanoi
 - ▢ To develop the solution to the problem, problem must be well identified
 - ▢ Starting with specification of states, rules and outcomes
 - ▢ Focusing on Solving the Problem i.e. Identifying the GOAL
- Incomplete and Complete Problem Statement
- Complete Set of Rules is a must
 - ▢ Example: If White pawn at sqr (row e, column 2) is true and White pawn at sqr (row e, column 3) is empty then Move pawn from sqr (row e, column 2) to sqr (row e, column 3) → its specific way of writing rule
 - ▢ Developing such rules is very tedious as chess game alone will have 10e120 rules
 - ▢ So, generalization is to be done

Defining Problems as a State Space Search

- Problem
- State
- State Space: where each state corresponds to a stable situation
 - ▢ Initial State
 - ▢ Rules for transition from one state to another
 - ▢ Final State → Ultimate Goal
- State Space Representation → forms the basis for AI methods
- State Space Structure corresponds to the structure of problem solving in two ways
 - ▢ It allows for a formal definition of the problem
 - ▢ It permits to define the process of solving the particular problem as a combination of known techniques and searching mechanism

Defining Problems as a State Space Search: An Example – A Water Jug Problem

Problem

You are given two jugs, a 4 litre one and a 3 litre one. None of them have any measuring markers on it. There is a pump that can be used to fill the jugs with water. How can you get exactly a 2 litres of water in the 4 litre jug?

Defining Problems as a State Space Search: An Example – A Water Jug Problem

State Space

- Set of ordered pair of integers (x, y) , such that $x = 0, 1, 2, 3, \text{ or } 4$ and $y = 0, 1, 2, \text{ or } 3$, where x represents the quantity of water in 4 litre jug and y represents the quantity of water in 3 litre jug
- The start state is $(0, 0)$
- The final state is $(2, n)$

Defining Problems as a State Space Search: An Example – A Water Jug Problem → Rules

1. (x, y) if $x < 4 \rightarrow (4, y)$
Fill the 4 litre jug
2. (x, y) if $y < 3 \rightarrow (x, 3)$
Fill the 3 litre jug
3. (x, y) if $y > 0 \rightarrow (x, 0)$
Empty the 3 litre jug
4. (x, y) if $x > 0 \rightarrow (0, y)$
Empty the 4 litre jug
5. (x, y) if $x > 0 \rightarrow (x-d, y)$
Put some water out of 4 l jug
6. (x, y) if $y > 0 \rightarrow (x, y-d)$
Put some water out of 3 l jug
7. (x, y) if $x+y \geq 4$ & $y > 0 \rightarrow (4, y-(4-x))$
Put water from 3l jug into 4l jug to fill it
8. (x, y) if $x+y \geq 3$ & $x > 0 \rightarrow (x-(3-y), 3)$
Put water from 4l jug into 3l jug to fill it
9. (x, y) if $x+y < 3$ & $x > 0 \rightarrow (0, x+y)$
Put all the water from 4l jug in to 3l jug
10. (x, y) if $x+y < 4$ & $y > 0 \rightarrow (x+y, 0)$
Put all the water from 3l jug in to 4l jug

Defining Problems as a State Space Search: An Example – A Water Jug Problem → Solution

Water Quantity in 4 l Jug	Water Quantity in 3 l Jug	Rule Applied
0	0	Initial Phase
0	3	2
3	0	10
3	3	2
4	2	7
0	2	4
2	0	10

Defining Problems as a State Space Search

- Solve (Goose, Grain, Fox, Man) problem.
- Solve (Missionary, cannibal) Problem

Defining Problems as a State Space Search

- Operationalization
 - Its the process of creation of a formal and manipulability description of the problem
 - It is the first step toward the design of a program to solve a problem
 - We expect to create such programs which can themselves produce formal descriptions of the problems from the informal ones

Problem Formulation

→ Process of deciding what action and states to consider, given a goal.

For providing a formal description of a problem, the things to be done are:

- Define a state space that contains all the possible configuration of the relevant objects.
- Specify one or more states within the space that describe possible situations from which the problem-solving process may start, known as the initial states.
- Specify one or more states that would be acceptable as solutions to the problem, known as the goal states or final states.
- Specify a set of rules that describe the actions (operators) available. While doing these, the following issues are to be highlighted:
 - What unstated assumptions are present in the informal problem description?
 - How much the rules should be generalized?
 - How much of the work required solving the problem should be pre-computed and represented in the rules.

Problem Characteristics

- What is the possibility in terms of **decomposability**?
- Can **solution be ignored or undone**?
- What about **predictability of universe**?
- Is it **absolute or relative solution**?
- Is the **solution a path or a state**?
- What is the **role of knowledge**?
- Does the task require **interaction with a person**?
- Can problem be **classified**?

Problem Types

- Well Defined Problems
- Constraint Satisfaction Problems

Problem Types: Well Defined Problem

- Major Components of a Problem
 - **Initial State**
 - **Actions** available to the agent in the **state space**
 - **Goal Test** determining the given state as a goal state
 - **Path Cost** function that assigns numeric value to each path considering the **step cost** as well
- If all these components are completely defined along with the optimal solution for the problem, then such problem is a well defined problem

Problem Types: Well Defined Problem

- Generalization of actions and rules
- Specialization of actions and rules

Problem Types: Constraint Satisfaction Problem

- A search procedure that operates in a space of constraint sets
- Constraints are discovered and propagated as far as possible through out the system
- It (Search) continues until the solution is achieved
- An assumption is made about something and added as a new constraint

Problem Types: Constraint Satisfaction Problem

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- Constraint Satisfaction Problem consists of variables with constraints on them
- Many important real world problems can be described as CSPs
- The structure of a CSP can be represented by constraint graph
- CSP has states and goal test conformed to a standard, structured and very simple representation
- Simply, CSP is such that the goal is to discover some problem state that satisfies a given set of constraints

Problem Types: Constraint Satisfaction Problem

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- A CSP is defined by a set of **variables**, X_1, X_2, \dots, X_n , and a set of constraints, C_1, C_2, \dots, C_m . Each variable X_i has a non empty domain D_i of possible values. Each constraint C_i involves some subset of the variables and specifies the allowable combinations of values for that subset. A state of the problem is defined by an assignment of values to some or all of the variables, $\{X_i=V_i, X_j=V_j, \dots\}$.

Problem Types: Constraint Satisfaction Problem

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- Constraint propagation terminates for one of the two reasons:
 - ▢ Contradiction detected: No solution consistent with known constraints
 - ▢ Propagation has run of stream and there are no further changes that can be made on the basis of current knowledge
- Variables: Discrete and Finite Domain
- Constraints: Linear and Non Linear

Problem Types: Constraint Satisfaction Problem

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- Types of Constraints
 - ▢ Unary: Restricts the value of single variable
 - ▢ Binary: Two variables
 - ▢ Higher order constraints: More than two variables

Production System

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- A production system consists of a set of rules, each consisting of a left side (a pattern), that determines the applicability of the rule and a right side that describes the operation to be performed if the rule is applied.
- Example: $[A, \text{clean}] \rightarrow \text{move right}$

Pattern Action
- Have one or more knowledge base (database) that contain whatever information is appropriate for the particular task

Production System

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- A **Control Strategy** that specifies the order in which the rules will be selected and a way of resolving the conflicts that arise when several rules matched at once
 - ▢ The first requirement of a good control strategy is that it cause motion
 - No motion action: filling the jug each time
 - ▢ The second requirement of a good control strategy is that it should be systematic
 - No systematic action: random selection action
 - ▢ The systematic control strategy is the good search technique
- Have a rule applier

Game Playing

- Multi agent environments
- Games and adversarial search [Unit3 → 3.1]
- Game theory: a branch of economics that views any multi agent environment as a game provided that the impact of each agent on the others is significant, regardless of whether the agents are cooperative and competitive

Constraint Satisfaction Problem: Crypto-arithmetic Problem

- SEND+MORE=MONEY
 - DONALD+HERALD=ROBERT
 - CROSS+ROADS=DANGER
- $$\begin{array}{r}
 96233 \\
 +62513 \\
 \hline
 158746
 \end{array}$$
- WRONG+WRONG=RIGHT

SEND+MORE=MONEY : Solution

- $$\begin{array}{l}
 D+E=Y+10.x1 \\
 x1+N+R=E+10.x2 \\
 x2+E+O=N+10.x3 \\
 x3+S+M=O+10.x4 \\
 x5=M
 \end{array}$$
1. $M=1$, since carry from two digits and another carry can't total more than 19
 2. S can be 8 or 9 since $S+M+x3 > 9$ to generate carry
 $M=1 : S+1+x3 > 9 \rightarrow S+x3 > 8$
 3. $O=0$, since $S+M+x3$ must be at least 10 and at most 11
 But $M=1$ so, $O=0$
 4. $N=E$ or $N=E+1$
 Since, $N < > E$, $N=E+1$

SEND+MORE=MONEY : Solution

- $$\begin{array}{l}
 N=E \text{ or } N=E+1 \\
 \text{as } N < > E, N=E+1 \\
 \text{Let } S=9 \rightarrow x3=0 \\
 \text{Choose } E \text{ as first choice for} \\
 \text{variable selection since it} \\
 \text{appears most.} \\
 x1+N+R=E+10.x2 \\
 \text{Let } E=6 \text{ and } x1=1, N=7 \\
 1+7+R=16
 \end{array}$$
- Hence, $R=8$
 Also, $D+E=Y+10$
 $D+6=Y+10$
 Hence,
 $S=9, E=6, N=7, D=7,$
 $M=1, O=0, R=8, Y=2$

References

- Russell, S. and Norvig, P., 2011, Artificial Intelligence: A Modern Approach, Pearson, India.
- Rich, E. and Knight, K., 2004, Artificial Intelligence, Tata McGraw hill, India.

Thank You

Any Queries?

Now Try to Solve your Problems.