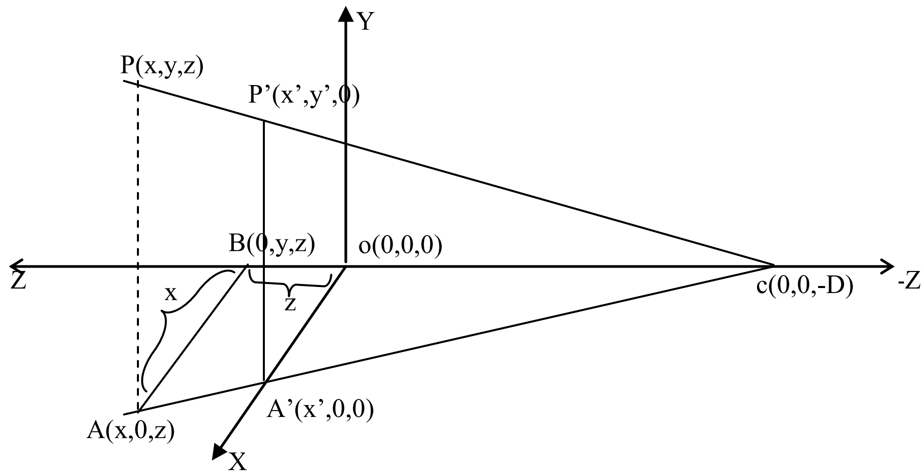
Perspective Projection



Here center of Projection is c (0,0,-D) along the direction of Z axis so the reference point is taken of world coordinate space W_c and the normal vector N is aligned with the y axis.

So now the view plane vp is the xy plane and center of projection is c (0,0,-D) now from similar triangles ABC and A'OC

$$\frac{x}{x'} = \frac{z+D}{D} = \frac{AC}{A'C}$$

or $\frac{xD}{z+D} = x'$ or $x' = \frac{Dx}{z+D}$

similarly from triangles APC and A'P'C

$$\frac{y}{y'} = \frac{z+D}{D} = \frac{AC}{A'C}$$

or $y' = \frac{yD}{z+D}$

and z' =0

now in homogenous coordinates

$$\text{Perk} = \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \frac{1}{z+D} \begin{pmatrix} Dx \\ Dy \\ 0 \\ z+D \end{pmatrix} = \frac{1}{z+D} \begin{pmatrix} D & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & D \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

A unit cube is projected into xy plane . Draw the projected image using standard perspective transformation where center of projection is (0,0,-20)

Here,

Center of projection = (0,0,-20) i.e. d = 20

$$\text{Persp} = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{pmatrix}$$

The cube represented in Homogenous coordinate is

$$V(A,B,C,D,E,F,G,H) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$V' = \text{Persp} * V \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= 1/z+D \begin{pmatrix} 0 & 20 & 20 & 0 & 0 & 0 & 20 & 20 \\ 0 & 0 & 20 & 20 & 20 & 0 & 0 & 20 \\ 0 & 0 & 0 & 0 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 21 & 21 & 21 & 21 \end{pmatrix}$$

Hence,

1/(z+D) = 1/ (0 +20) == 1/ 20

A' =(0,0,0) B' =(1,0,0) C' =(1,1,0) D' =(0,1,0)

1/(z+D) = 1/ (1 +20) = 1/ 21

E' =(0,20/21,0) F' =(0,0,0) G' =(20/21,0,0) H' =(20/21, 20/21,0)