Computer Number Systems

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1 Number Systems

It seems pretty intuitive that $26 + 10 = 36$ in base 10, but how about $26_8 + 10_8$? If we break down on how to solve 26_10, it's just 2*10 + 6. This same concept can be extended to numbers in other bases. For instance 26_8 would be 2*8 + 6. By evaluating this expression, you arrive at 22 in base 10. Essentially, you sum the product of the digit * $base^a$ where a is the number of the digits from the right.

To change from base 10 to another base, the desired base is divided into the base 10 number repeatedly. The remainders at each step of the divisions are the digit of the number in the desired base. For instance, by using the 2107 to change from base 10 to base 5:

$$2107/5 = 421\ R2$$

$$421/5 = 84\ R1$$

$$84/5 = 16\ R4$$

$$16/5 = 3\ R1$$

$$3/5 = 0\ R3$$

$$31412_5$$

2 Binary, Octal, Hexadecimal

The three most commonly seen bases besides decimal are binary, octal, and hexadecimal. Binary, or base 2, is just ones and zeros. Since eight is 2^3 one digit in octal can represent 3 digits in binary. The largest 3 digit number in binary 111_2 and the largest one digit number in octal 7_8 both represent 7 in base 10. Hexadecimal works the same way except one digit represents four binary digits. This concept makes changing between the three bases very fast.
(Credits: 2016-2017 ICT Officers)

Example 1. Convert 25E from base 16 to base 2.
This is just 001001011110 or eliminating the zeroes in the front 1001011110.

Example 2. Convert 11011111 from base 2 to base 8.
Separate the digits into 3-digit groups: 11 011 111. Then convert each group into base 8: 337.

3 Tips to Solving ACSL CNS Problems

1. When dealing with any number not in base 10, convert it into base 10. If a problem involved multiplying numbers in base 8, converting the numbers into base 10 would make your life much easier.

2. When a problem involves converting from base 16 to base 8, convert it first into binary and then binary into base 8.

4 Easy to Medium Problems

1. Convert 8765_{16} to binary form.

2. Evaluate in base 16 $FEED_16 - 6ACE_16$.

3. What is the base 16 representation for $FEDCBA_{16}$ - $ABCDEF_{16}$?

4. What is the value of $A98_{16}/23_8$ in base 10 (leave it in simplified fraction form if necessary)?

5. In the ACSL computer, each "word" of memory contains 20 bits representing 3 pieces of information. The most significant 6 bits represent Field A; the next 11 bits, Field B; and the last 3 bits represent Field C. For example, the 20 bits comprising the "word" 18149_{16} has fields with values of 6_{16} , 29_{16} and 1_{16}. What is Field B in $E1B7D_{16}$? (Express your answer as a base 16 number.)

6. $X37_8 = 1XF_{16}$

7. Which of the following 5 numbers is the largest? $F1_{16}$, 375_8, $10F_{16}$, 264_{10}, 11111000_2

(Problems from Official ACSL Website)

5 Hard Problems

1. How many numbers from 300 to 500, inclusive, have 8 1's in their binary representation?

2. Let n be any positive base 10 integer from 1 to 2^12 inclusive. Let S(n) be the number of 1's in the binary representation of n. Find the number of n's such that S(n) - S(n+1) = 3.

3. Find the number of carries when summing 2345_{10} and 3459_{10}. (Check out Kummer's Theorem).