AB IdeaLab, Competitive Programming Team, Fall 18–Spring 19 February 8, 2019

Lecture 3: FSA/Regular Expressions

American Computer Science League, February Contest

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1 Fun Facts

- Developed in 1951 by mathematician Stephen Cole Kleene.
- Ken Thompson (one of the guys who developed UNIX) used regular expressions on an early Unix editor. This eventually lead to its use in the famous UNIX tool grep.
- Applications include string searching algorithms, input verification, and search engines.
- You can even use it inside your programming editor to find where you've put stuff.

2 Background

What are regular expressions? According to Wikipedia, regular expressions (regex) are "a sequence of characters that define a search pattern". In other words, a regex defines a set of possible strings in a concise manner for some later purpose. For example, reali[sz]e defines the set {realize, realise} of possible strings. These set can be later used for cross-referencing American English spellings with British English spellings.

All the regex syntax you need to know. As mentioned above, regex includes metacharacters that define more complex types of string matching. The following is a list of all the regex metacharacters you need to know:

1. **|, or,** $\cup$ These are booleans that tell the processor to take the set union of the left and right regexes.
2. λ The null or empty string.
3. **Quantification** Defining the number of something allowed to occur.
   (a) **?** Zero or one. E.g., colou?r = {color, colour}.
   (b) ***** Zero or more.
   (c) **+** One or more.
4. **.** Wildcard, or any character. Combine . and * for a.*b, which means any string with a and b as the left and right characters with anything inbetween.
5. **[...]** Set of possible character matches. Think reali[sz]e example above. This can get slightly more complex by using hyphens to define ranges of possible characters. E.g., [a-z] means every *lowercase* char from a to z; [abcx-z] means a, b, c, and x, y, z; and [a-cx-z] means a, b, c and x, y, z.
6. **[^...]** Set of characters not contained withing the brackets. E.g., [^a-z] matches any characcter that is not a lowercase letter from a to z.

7. **()** Just like in math, parentheses imply grouping. E.g., if we wanted the set {gray, grey}, gra|ey would give us {gra, ey}. Instead, using parentheses we can get gr(a|e)y, which gives us the correct regex. A more complex example is H(ä|ae?)ndel, which matches {Handel, Händel, Haendel}.

Order of operations: Kleene Star (*), concatenation (ab), and union(∪).

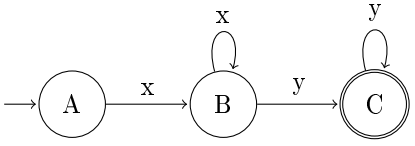
Practicing the syntax via identity proofs. To make sure you understand the syntax, see if you can prove the following identities:

1. (a*)* = a*
2. aa* = a*a
3. aa* ∪λ = a*
4. a(b ∪ c) = ab ∪ ac
5. a(ba)* = (ab)*a
6. (a ∪ b)* = (a* ∪ b*)*
7. (a ∪ b)* =(a*b*)*
8. (a ∪ b)* = a*(ba*)*

How are regex interpreted by the computer? In a regex, there are two types of chars: literals and metacharacters. Literals define regular characters, while metacharacters indicate more nuanced behaviors. After creating a regex, a regex processor transforms the characters into an internal representation that computers can process. For our purposes, we can consider this representation to be a Finite State Automata (FSA). FSAs are an abstract concept in theoretical computer science consisting of the following:

1. A finite number of states, of which exactly one is active at any given time
2. Transition rules to change the active state
3. An initial state
4. One or more final states

We can draw an FSA by representing each state as a circle, the final state as a double circle, the start state as the only state with an incoming arrow, and the transition rules as labeled-edges connecting the states. For instance, the following is an FSA diagram for the regex x+y+:



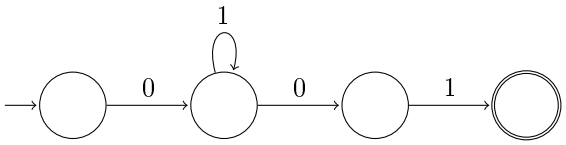
If you would like to learn more about FSAs, I recommend the Wikipedia page. Outside the ACSL bubble, automata and finiteness are an important field of research in theoretical CS. They connect back to problems such as P vs. NP and whether a program will stop in a reasonable amount of time or even in an infinite amount of time. Alan Turing, the godfather of CS, wrote a paper related to FSAs (LINK PAPER) in the 1940s.

Testing regex syntax. If you would like to practice regex and have your code actually matched against strings, I recommend this website.

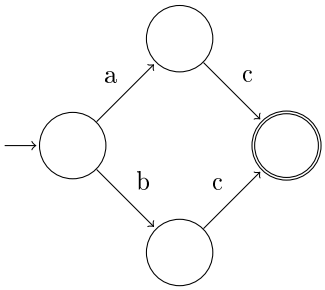
3 Exercises

3.1 Translate an FSA to a Regular Expression

1. Find a simplified Regular Expression for the following FSA:

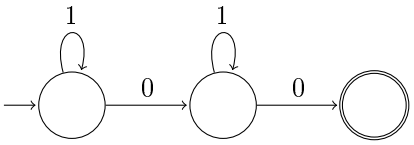


2. Find a simplified Regular Expression for the following FSA:

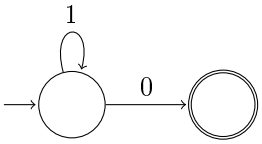


3. List all of the following FSAs which represent 1*01*0:

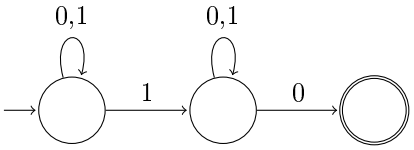
(a)



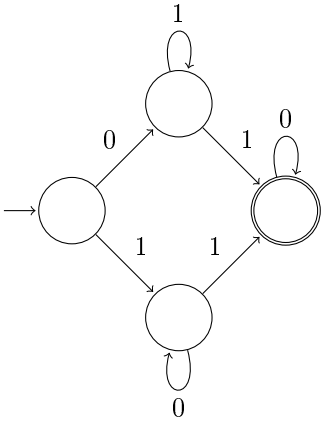
(b)



(c)



(d)



4. Random website with diagrams.

3.2 Simplify a Regular Expression

3.3 Determine which Regular Expressions or FSAs are equivalent

1. Which, if any, of the following Regular Expressions are equivalent?
   (a) (a∪b)(ab*)(b*∪a)
   (b) (aab*∪bab*)a
   (c) aab*∪bab*∪aaba∪bab*a
   (d) aab*∪bab*∪aab*a∪bab*a
   (e) a*∪b*

3.4 Determine which strings are accepted by either an FSA or a Regular Expression

1. Which of the following strings are accepted by the following Regular Expression "00*1*1U11*0*0"?
   (a) 0000001111111
   (b) 1010101010
   (c) 1111111
   (d) 0110
   (e) 10
2. Which of the following strings match the regular expression pattern "[A-D]*[a-d]*[0-9]"?
   (a) ABCD8
   (b) abcd5
   (c) ABcd9
   (d) AbCd7
   (e) X
   (f) abCD7
   (g) DCCBBBaaaa5
3. Which of the following strings match the regular expression pattern "Hi?g+h+[^a-ceiou]"?
   (a) Highb
   (b) HiiighS
   (c) HigghhhC
   (d) Hih
   (e) Hghe
   (f) Highd
   (g) HgggggghX

4 Solutions

4.1 Answers for Section 3.1

1. 01*01
2. (a|b)c or ac ∪ bc
3. a. The other choices correspond to 1*0, (0∪1)*1(0∪1)*0, and 01*10*∪10*10*

4.2 Answers for Section 3.3

1. B is different from the rest because it requires an ending 'a'. E is different from the rest because it doesn't allow for alternating a's and b's. C and D are different because of the third 'or' condition. Upon very close inspection, A and D are equivalent (check this carefully yourself). Therefore, A and D are the answers.

4.3 Answers for Section 3.4

1. 0000001111111 and 10
2. ABCD8, abcd5, ABcd9, and DCCBBBaaaa5
3. HigghhhC, Highd, and HgggggghX