Practice Problems (PPHY503L)

Dr. Sanjit Das

06 August 2025

Q1. **Point charge in the field of a magnetic monopole.**

The equation of motion of a point electric charge e, of mass m, in the field of a magnetic monopole of strength g at the origin is

$$m\ddot{\mathbf{r}} = -ge\frac{\dot{\mathbf{r}} \times \mathbf{r}}{r^3}.$$

The monopole may be taken as infinitely heavy.

(a) Show that the kinetic energy $T = m\dot{r}^2/2$ is a constant of the motion.

(b) Show that

$$\mathbf{J} = \mathbf{L} + eg\frac{\mathbf{r}}{r}$$

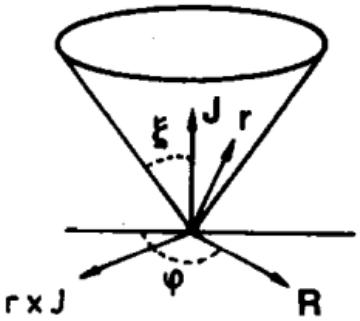
is also a constant of the motion, where

$$\mathbf{L} = m\mathbf{r} \times \dot{\mathbf{r}}.$$

(c) Use part (b) to show that the charged particle moves on the surface of a right circular cone of opening angle ξ given by

$$\cos\xi = -\frac{eg}{|\mathbf{J}|},$$

with $\mathbf{J}$ as its symmetry axis (see Fig). *[Hint: Consider* $\mathbf{r} \cdot \mathbf{J}$*]*



Figure

Define a new variable $\mathbf{R}$ by

$$\mathbf{R} = \frac{1}{\sin\xi}\hat{\xi} \times (\mathbf{r} \times \hat{\mathbf{J}}) = \frac{1}{\sin\xi}\left[\mathbf{r} - \hat{\mathbf{J}}(\mathbf{r} \cdot \hat{\mathbf{J}})\right],$$

where $\hat{\mathbf{J}} = \mathbf{J}/|\mathbf{J}|$. $\mathbf{R}$ lies in the plane perpendicular to $\mathbf{J}$, but with $|\mathbf{R}| = R = |\mathbf{r}|$ so that $\mathbf{R}$ may be obtained by rotating $\mathbf{r}$ as shown in the figure. You may use the fact that $m\mathbf{R} \times \dot{\mathbf{R}} = \mathbf{J}$.

(d) Find the equation of motion for $\mathbf{R}$.

(e) Solve the equation of motion part (d) by finding an effective potential $V_{\text{eff}}(R)$, and describe all possible motions in $\mathbf{R}$.