CLASSICAL MECHANICS PROBLEMS (MAY2025)

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May 2025

Problem 1

A charged particle with mass m, charge e, and velocity $\mathbf{v}$ moves in an electromagnetic field described by the electric scalar potential ϕ and magnetic vector potential $\mathbf{A}$. The speed of light is denoted by c. The Lagrangian of the system is given by:

$$L = T - U = \frac{mv^2}{2} - e\phi + \frac{e}{c}\mathbf{v} \cdot \mathbf{A}$$

where T is the kinetic energy, U is the potential energy, and $\mathbf{v} \cdot \mathbf{A}$ represents the dot product between the velocity and the magnetic vector potential. prove this statement.

Problem 2

Huygens' cycloidal pendulum consists of a particle which oscillates under gravity in the vertical plane along a frictionless cycloidal track with parametric equations where the vertical y-axis points downward. Calculate that a Lagrangian for this system.

$$x = R(\theta - \sin\theta), \quad y = R(1 - \cos\theta),$$

Problem 3 Compute the EOM for this Lagrangian

$$L = e^{\lambda t}\left(\frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2\right)$$

Problem 4

Show that the Lagrangian

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta) - \frac{eg}{c}\dot{\phi}\cos\theta$$

describes a charged particle in the magnetic field $\mathbf{B} = g\mathbf{r}/r^3$ of a magnetic monopole and find Lagrange's equations. Hint: check that the vector potential for a magnetic monopole in spherical coordinates has the components $A_r = 0$, $A_\theta = 0$, $A_\phi = g(1 - \cos\theta)/r\sin\theta$.

Problem 5 For this Lagrangian $\mathcal{L}$ find the conserved momenta.

$$\mathcal{L} = \frac{1}{2}\left[-f(r)\dot{t}^2 - 4nf(r)(\cos\theta - 1)\dot{t}\dot{\phi} + \frac{\dot{r}^2}{f(r)} + (r^2 + n^2)\dot{\theta}^2 + \left((r^2 + n^2)\sin^2\theta - 4n^2 f(r)(\cos\theta - 1)^2\right)\dot{\phi}^2\right]. \tag{1}$$

Problem 6 A particle P of mass m moves in a central force field described by the force:

$$\mathbf{F} = -\frac{\gamma m}{r^2}\hat{r},$$

where γ is a positive constant, and $\hat{r}$ is the radial unit vector. The force is attractive and follows an inverse-square law, similar to gravity. We need to determine the conditions for bound and unbound orbits based on the total mechanical energy E.

An asteroid approaches the Sun from a great distance with constant speed u in a straight line, where the perpendicular distance from the Sun is p. We need to find the equation satisfied by the subsequent orbit. For the special case where $u^2 = \frac{4M_\odot G}{3p}$ (where $M_\odot$ is the mass of the Sun, and G is the gravitational constant), find:

(i) the distance of closest approach of the asteroid to the Sun, and

(ii) the speed of the asteroid at the time of closest approach.

Problem 7 Find the force law for a central-force field that allows a particle to move in a logarithmic spiral orbit given by $r = ke^{\alpha\theta}$, where k and α are constants.

Problem 8 Determine $r(t)$ and $\theta(t)$ for the problem in previous problem.

Problem 9 What is the total energy of the orbit of the previous two examples?

Problem 10
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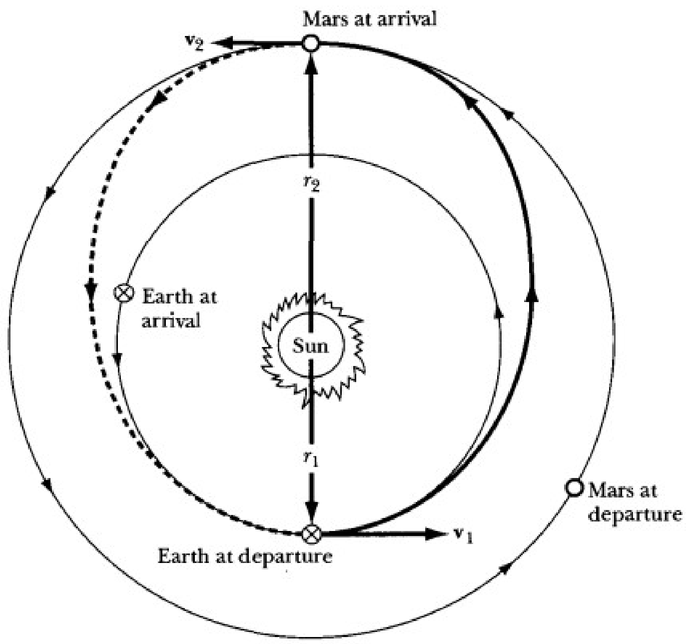

Problem 12

Calculate the time it takes for a spacecraft to make a Hohmann transfer from Earth to Mars and the heliocentric transfer speed required assuming that both planets are in coplanar orbits.