Department of Electronic and Telecommunication

EN4563 - Robotics

Robotic Mini Project

Kinematic Analysis of a Robotic Arm

Team : Count-3

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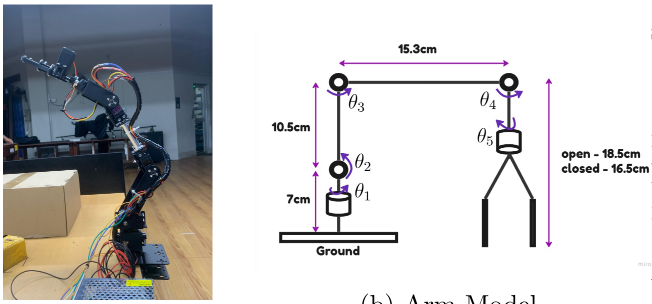
All the related executable codes can be found here.

1 Introduction

In this report, we have kinematically analyzed a robotic arm with five degrees of freedom together



(a) Robot Manipulator

(b) Arm Model

with a grip. The frame was brought from outside the country and six MG995 servo motors are used to actuate the joints. All the joints are re-volute and kinematic analysis is performed accordingly. In the later stages of the kinematic analysis considering the complexity of the arm, the last two actuation joints are constrained to be either horizontal or vertical based on the requirement. All the kinematic analyses are performed using Matlab after taking proper physical measurements.

2 DH-Table

Consider the angles marked in the Fig 1b. Frame assignment and the DH table are as follows,

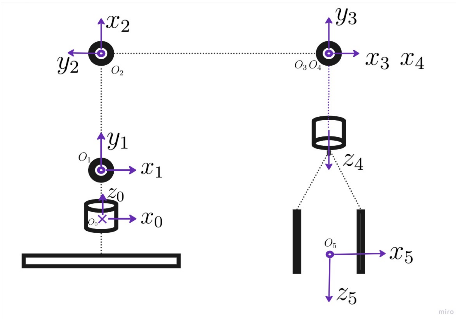


Figure 2: Frame Assignment

Link	Lenght (a)	Twist (α)	Offset (d)	Angle (θ)
1	0	90	7	θ_1^*
2	10.5	0	0	θ_2^*
3	15	0	0	θ_3^*
4	0	90	0	θ_4^*
5	0	0	17.5	θ_5^*

Table 1: DH Parameters

3 Forward kinematics

Individual transformation matrices associated with each joint are indicated as A_i where $i = 1, 2, 3, 4, 5$. The forward kinematics matrix T_5^0 is obtained by multiplying all these A_i matrices, representing the position and orientation of the end-effector with respect to the base frame.

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 10.5C_2 \\ S_2 & C_2 & 0 & 10.5S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 15C_3 \\ S_3 & C_3 & 0 & 15S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 17.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matlab is used in the simplification of forward kinematics considering the complexity of the transformations to be obtained. The transformation of each joints w.r.t $o_0x_0y_0z_0$ frame is as follows,

$$T_1^0 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_2^0 = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & 10.5C_1C_2 \\ C_2S_1 & -S_1S-2 & -C_1 & 10.5C_2S_1 \\ S_2 & C_2 & 0 & 10.5S_2+7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} C_1C_2C_3 - S_1S_3 & -C_1S_2 & C_1C_2S_3 + S_1C_3 & 10.5C_1C_2 + 15C_1C_{C3} \\ S_1C_2C_3 + C_1S_3 & -S_1S_2 & S_1C_2S_3 - C_1C_3 & 10.5S_1C_2 + 15S_1C_2C_3 + 7 \\ -C_2S_3 & -C_2 & -S_2S_3 & -15S_2 - 10.5S_2S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} C_1C_2C_3C_4 - S_1S_3S_4 & -C_1S_2 & C_1C_2S_3S_4 + S_1C_3C_4 & 10.5C_1C_2 + 15C_1C_2C_3 + 10S_1C_2C_3C_4 \\ S_1C_2C_3C_4 + C_1S_3S_4 & -S_1S_2 & S_1C_2S_3S_4 - C_1C_3C_4 & 10.5S_1C_2 + 15S_1C_2C_3 + 10S_1C_3C_4 + 7 \\ -C_2S_3C_4 & -C_2C_4 & -S_2S_3S_4 & -15S_2 - 10.5S_2S_3 + 17.5S_2S_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^0 = \begin{bmatrix} C1C2C3C4C5{-}S1S3S4S5 & -C1S2 & C1C2S3S4C5{+}S1C3C4C5 & 10.5C1C2{+}15C1C2C3{+}10S1C2C3C4{+}10S1C2C3C4C5 \\ S1C2C3C4C5{+}C1S3S4S5 & -S1S2 & S1C2S3S4C5{-}C1C3C4C5 & 10.5S1C2{+}15S1C2C3{+}10S1C3C4{+}7{+}10S1S3C4S5 \\ -C2S3C4C5 & -C2C4C5 & -S2S3S4C5 & -15S2{-}10.5S2S3{+}17.5S2S4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

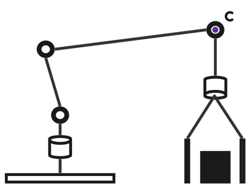
4 Inverse kinematics

The aim of the inverse kinematic is to find joint angles required to reach the specific position with a certain orientation.

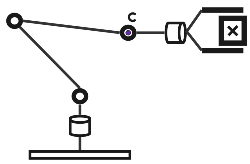
4.1 Naive Approach

The basic approach for the problem is equating the transformation matrix T_5^0 to the transformation formed by expected position and orientation and solving for the five angles using the twelve complex equations. Considering the singularities and complexity this is an impossible problem to tackle.

4.2 Our Approach



(a) Case I



(b) Case II

Considering the complexity of the solution, it is decided to put an external constraint on the final link such that it is either horizontal or vertical as shown in the figures on the side. Although this will narrow down the workspace, the decision will allow us to decouple the position from orientation. The joint angle θ_5 is chosen based on orientation requirement whereas joint angle θ_4 is utilized to satisfy the constraint. Let us derive the formulas for joint angles by considering a general condition after defining the task.

Pick and Place task: A small box that could fit in the arm properly is chosen as the object. Initial position and orientation of the object is chosen in such a way to make it convenient for the arm to pick it with link 5 vertical whereas final position forces link 5 to be horizontal under the constraint. The video attached with the report visualizes the arm in action.

Calculation of Joint Angles

1. Based on the convenient reachability of the arm, the decision on whether to keep the final joint horizontal or vertical is made.

2. The coordinates of point O_c is calculated simply by considering the co-ordinate transformation enforced by the final joint.

3. By considering that the point O_c lies on the X-Z plane, the angle θ_1 is determined.
$$\theta_1 = \tan^{-1}\left(\frac{Y_c}{X_c}\right)$$

4. Cosine rule in ΔOBC will give us the angle θ_3.
$$\theta_3 = \cos^{-1}\left(\frac{x_c^2 + y_c^2 + z_c^2 - l_1^2 - l_2^2}{2 \cdot l_1 l_2}\right)$$

5. calculating $\angle$COB allow us to find θ_2
$$\theta_2 = 90° - \tan^{-1}\left(\frac{z_c - 7}{\sqrt{x_c^2 + y_c^2}}\right) - \tan^{-1}\left(\frac{l_2 \sin(\theta_3)}{l_1 + l_2 \cos(\theta_3)}\right)$$

6. The angle θ_4 is determined to satisfy the external constraint.
$$\theta_4 = \begin{cases} \pi - \theta_3 - \theta_4 & \texttt{Link 5 - vertical} \\ \frac{\pi}{2} - \theta_3 - \theta_4 & \texttt{Link 5 - horizontal} \end{cases}$$

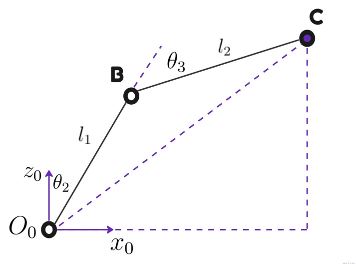


Figure 4: X-Z Plane

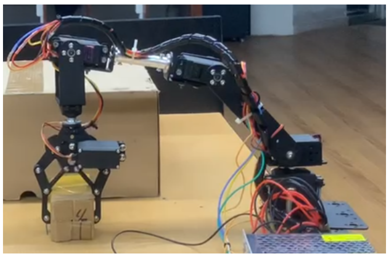


Figure 5: Picking Object

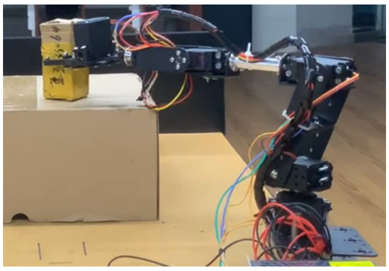


Figure 6: Placing Object

5 Manipulator Jacobian

Jacobian together with joint angular velocities could be used to find the velocity of the end effector in the world coordinate frame. Considering the complexity of the results, the formulae together with the necessary expressions are indicated in the report. Further for completeness one can find the matlab livescript file here that will give the expanded results for the velocity analysis.

$$J = \begin{bmatrix} z_0^0 \times (t_5^0 - t_0^0) & z_1^0 \times (t_5^0 - t_1^0) & z_2^0 \times (t_5^0 - t_2^0) & z_3^0 \times (t_5^0 - t_3^0) & z_4^0 \times (t_5^0 - t_4^0) \\ z_0^0 & z_1^0 & z_2^0 & z_3^0 & z_4^0 \end{bmatrix}$$

Where,

$$z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad z_1^0 = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} \quad z_2^0 = \begin{bmatrix} 1 \end{bmatrix}$$