Robotics Mini Project

Kinematic Analysis of a Robot Arm

Department of Electronic and Telecommunication

EN4563 - Robotics

INCLUDE

- Denavit-Hartenberg (DH) table
- Forward Kinematics
- Inverse Kinematics
- Pick And Place Demo
- Manipulator Jacobian

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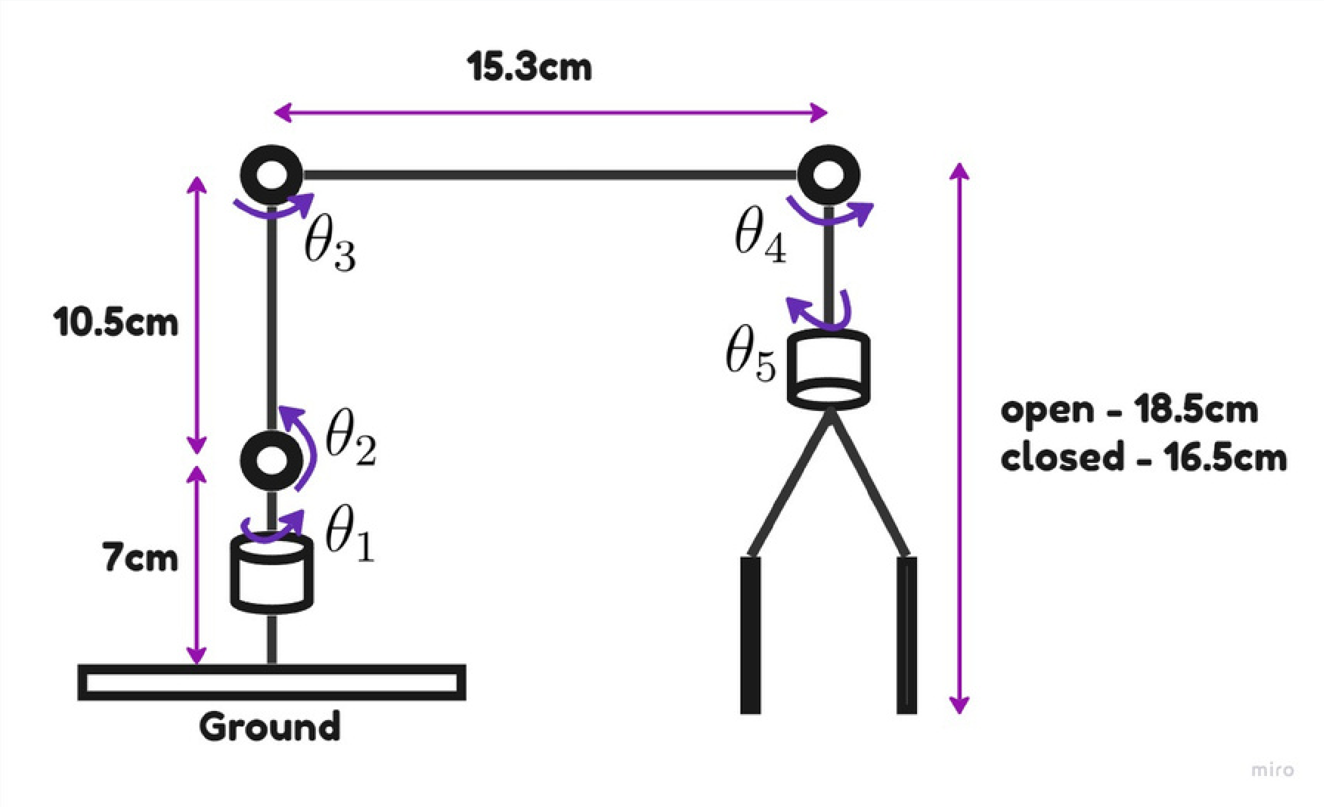
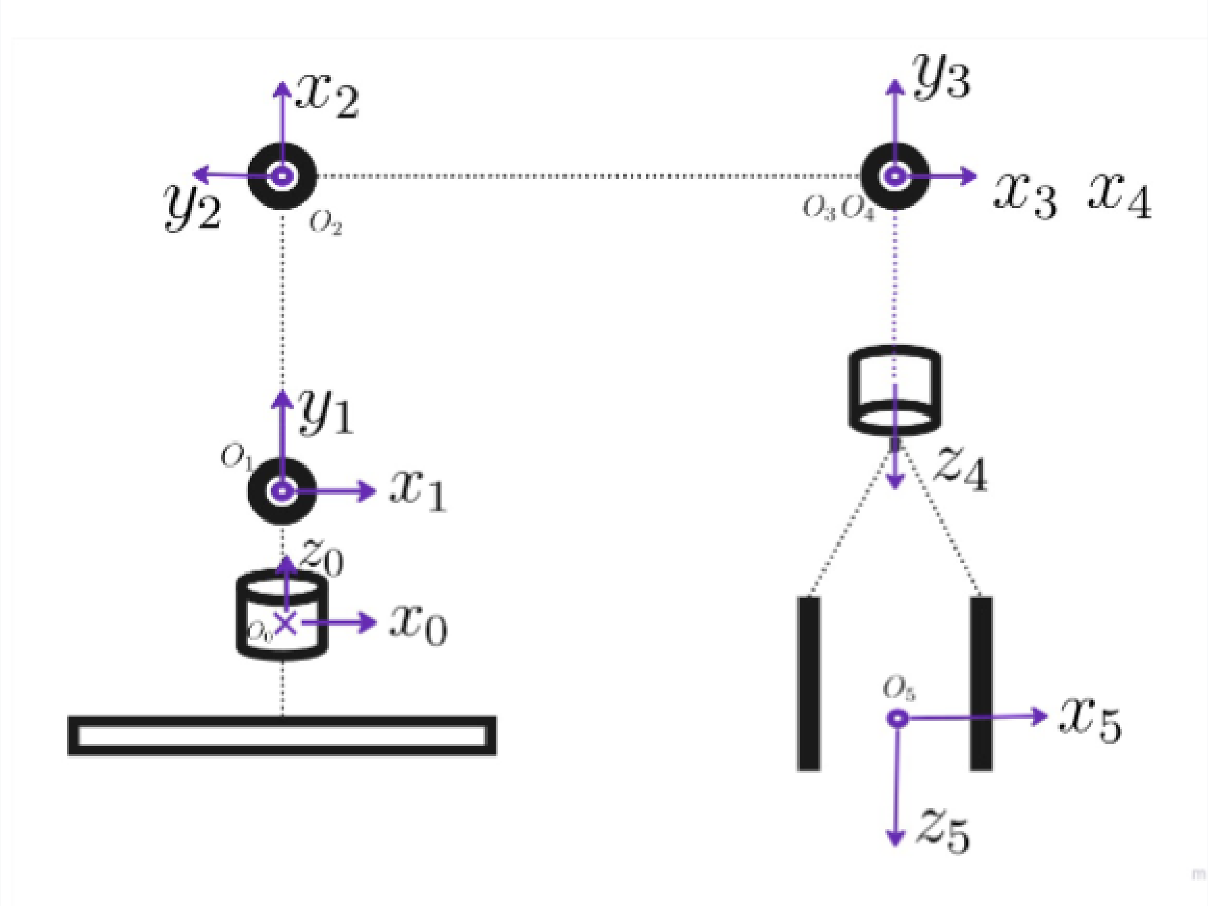
Robotic Manipulator

- Frame with five revolute joints.
- Motor: MG995 metal gear servos.
- Powered by 5V 20A supply(adequate supply considering the stall current of 1.2A each.)



Denavit-Hartenberg (DH) table



Frame Assignment

Link	Lenght (a)	Twist (α)	Offset (d)	Angle (θ)
1	0	90	7	θ_1^*
2	10.5	0	0	θ_2^*
3	15	0	0	θ_3^*
4	0	90	0	θ_4^*
5	0	0	17.5	θ_5^*

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 10.5C_2 \\ S_2 & C_2 & 0 & 10.5S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 15C_3 \\ S_3 & C_3 & 0 & 15S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 17.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

$$T_1^0 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & 10.5C_1C_2 \\ C_2S_1 & -S_1S_2 & -C_1 & 10.5C_2S_1 \\ S_2 & C_2 & 0 & 10.5S_2 + 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} C_1C_2C_3 - C_1S_2S_3 & -C_1C_2S_3 - C_1C_3S_2 & S_1 & 10.5C_1C_2 - 15C_1S_2S_3 + 15C_1C_2C_3 \\ S_1C_2C_3 - S_1S_2S_3 & -C_2S_1S_3 - S_1C_3S_2 & -C_1 & 10.5S_1C_2 - 15S_1S_2S_3 + 15C_2C_3S_1 \\ C_2S_3 + C_3S_2 & C_2C_3 - S_2S_3 & 0 & 10.5S_2 + 15C_2S_3 + 15C_3S_2 + 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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H_0_4 =
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$$\begin{pmatrix} -\cos(\theta_4)\,\sigma_3 - \sin(\theta_4)\,\sigma_4 & \sin(\theta_1) & \cos(\theta_4)\,\sigma_4 - \sin(\theta_4)\,\sigma_3 & \frac{21\cos(\theta_1)\cos(\theta_2)}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ -\cos(\theta_4)\,\sigma_1 - \sin(\theta_4)\,\sigma_2 & -\cos(\theta_1) & \cos(\theta_4)\,\sigma_2 - \sin(\theta_4)\,\sigma_1 & \frac{21\cos(\theta_2)\sin(\theta_1)}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \cos(\theta_4)\,\sigma_6 + \sin(\theta_4)\,\sigma_5 & 0 & \sin(\theta_4)\,\sigma_6 - \cos(\theta_4)\,\sigma_5 & \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) + 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)$$

$$\sigma_2 = \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$\sigma_4 = \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_5 = \cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)$$

$$\sigma_6 = \cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2)$$

$$\sigma_1 = \cos(\theta_4)\,\sigma_4 + \sin(\theta_4)\,\sigma_5$$

$$\sigma_2 = \cos(\theta_4)\,\sigma_6 + \sin(\theta_4)\,\sigma_7$$

$$\sigma_3 = \cos(\theta_4)\,\sigma_9 + \sin(\theta_4)\,\sigma_8$$

$$\sigma_4 = \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$\sigma_5 = \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_6 = \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)$$

$$\sigma_7 = \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2)$$

$$\sigma_8 = \cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)$$

$$\sigma_9 = \cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2)$$

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H_0_5 =
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$$\begin{pmatrix} \sin(\theta_1)\sin(\theta_5) - \cos(\theta_5)\,\sigma_1 & \cos(\theta_5)\sin(\theta_1) + \sin(\theta_5)\,\sigma_1 & \cos(\theta_4)\,\sigma_5 - \sin(\theta_4)\,\sigma_4 & \frac{21\cos(\theta_1)\cos(\theta_2)}{2} + \frac{35\cos(\theta_4)\,\sigma_5}{2} - \frac{35\sin(\theta_4)\,\sigma_4}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ -\cos(\theta_1)\sin(\theta_5) - \cos(\theta_5)\,\sigma_2 & \sin(\theta_5)\,\sigma_2 - \cos(\theta_1)\cos(\theta_5) & \cos(\theta_4)\,\sigma_7 - \sin(\theta_4)\,\sigma_6 & \frac{21\cos(\theta_2)\sin(\theta_1)}{2} + \frac{35\cos(\theta_4)\,\sigma_7}{2} - \frac{35\sin(\theta_4)\,\sigma_6}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \cos(\theta_5)\,\sigma_3 & -\sin(\theta_5)\,\sigma_3 & \sin(\theta_4)\,\sigma_9 - \cos(\theta_4)\,\sigma_8 & \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) - \frac{35\cos(\theta_4)\,\sigma_8}{2} + \frac{35\sin(\theta_4)\,\sigma_9}{2} + 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

Inverse kinematics

Naive Approach

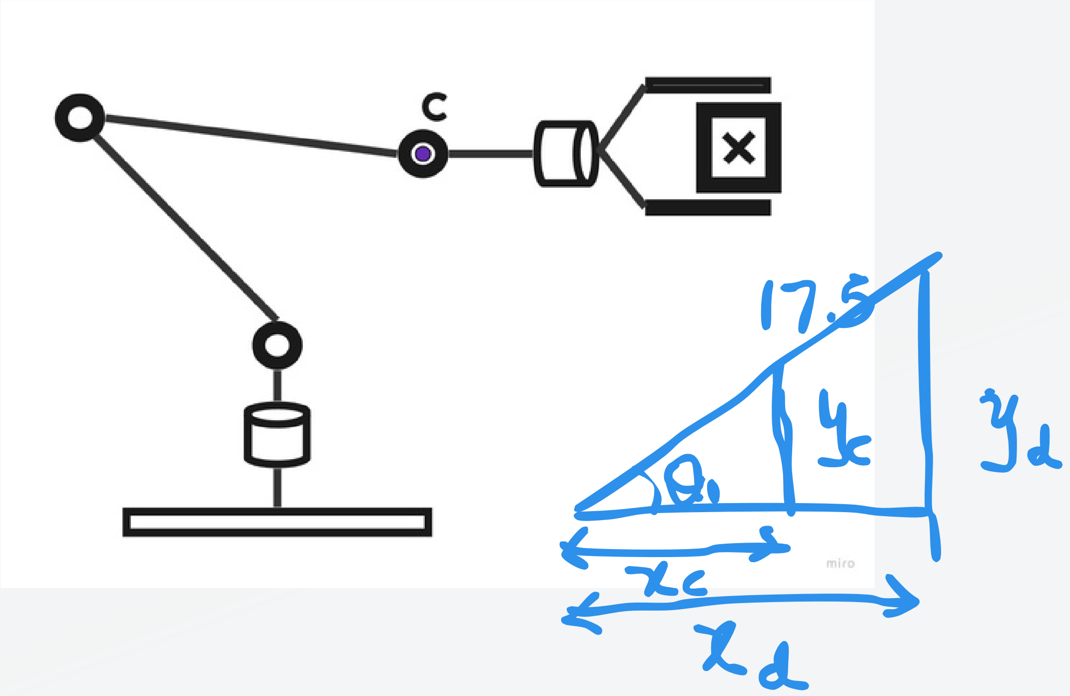
Position & Orientation

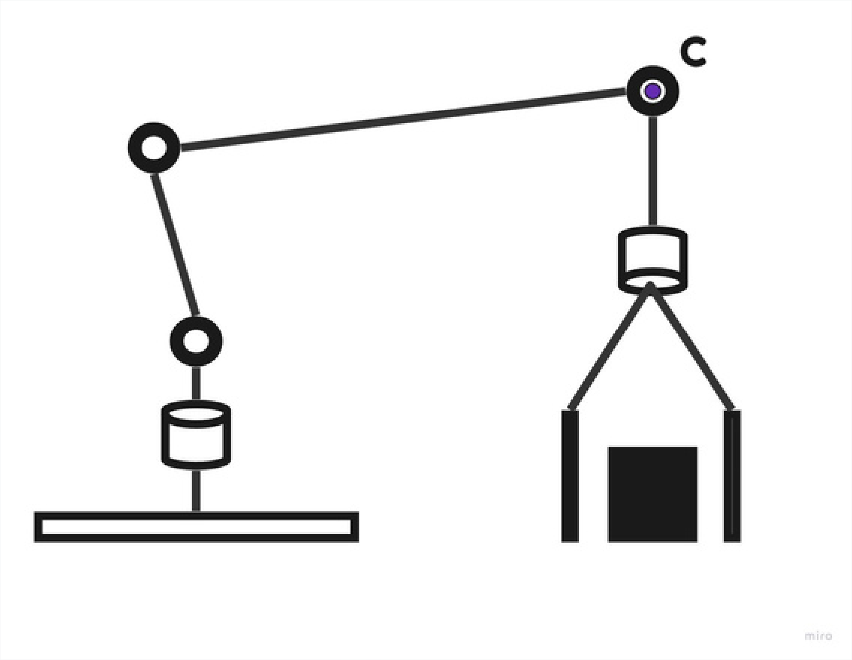
$$
H_0^5 = \begin{pmatrix}
\sin(\theta_1)\sin(\theta_5) - \cos(\theta_5)\,\sigma_1 & \cos(\theta_5)\sin(\theta_1) + \sin(\theta_5)\,\sigma_1 & \cos(\theta_4)\,\sigma_5 - \sin(\theta_4)\,\sigma_4 & \frac{21\cos(\theta_1)\cos(\theta_2)}{2} + \frac{35\cos(\theta_4)\,\sigma_5}{2} - \frac{35\sin(\theta_4)\,\sigma_4}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\
-\cos(\theta_1)\sin(\theta_5) - \cos(\theta_5)\,\sigma_2 & \sin(\theta_5)\,\sigma_2 - \cos(\theta_1)\cos(\theta_5) & \cos(\theta_4)\,\sigma_7 - \sin(\theta_4)\,\sigma_6 & \frac{21\cos(\theta_2)\sin(\theta_1)}{2} + \frac{35\cos(\theta_4)\,\sigma_7}{2} - \frac{35\sin(\theta_4)\,\sigma_6}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\
\cos(\theta_5)\,\sigma_3 & -\sin(\theta_5)\,\sigma_3 & \sin(\theta_4)\,\sigma_9 - \cos(\theta_4)\,\sigma_8 & \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) - \frac{35\cos(\theta_4)\,\sigma_8}{2} + \frac{35\sin(\theta_4)\,\sigma_9}{2} + 7 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

From 12 equations solve for 5 angles

- No closed form solutions
- Iterative solutions may not converge
- Singularities

Our Approach





Zc = Zd + 17.5

- Additional constraint -

  Link 5 is either horizontal or vertical
- Achieve position, orientation decoupling
- Expectation - simple tasks can be performed with constraint

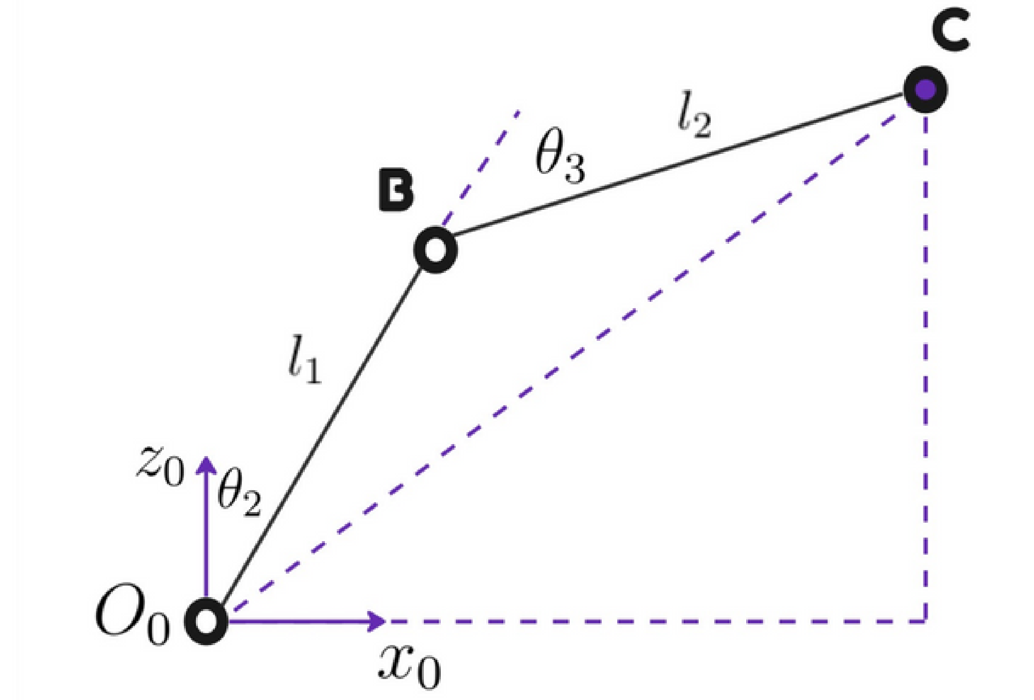
- Price - Narrowing the workspace
- Limiting the possible orientations

Joint Angle Calculation

- Case selection
- Coordinate of the point Oc using the enforced coordinate transformation
- θ_1 by considering Oc lies in X-Z plane.

$$\theta_1 = \tan^{-1}\left(\frac{Y_c}{X_c}\right)$$

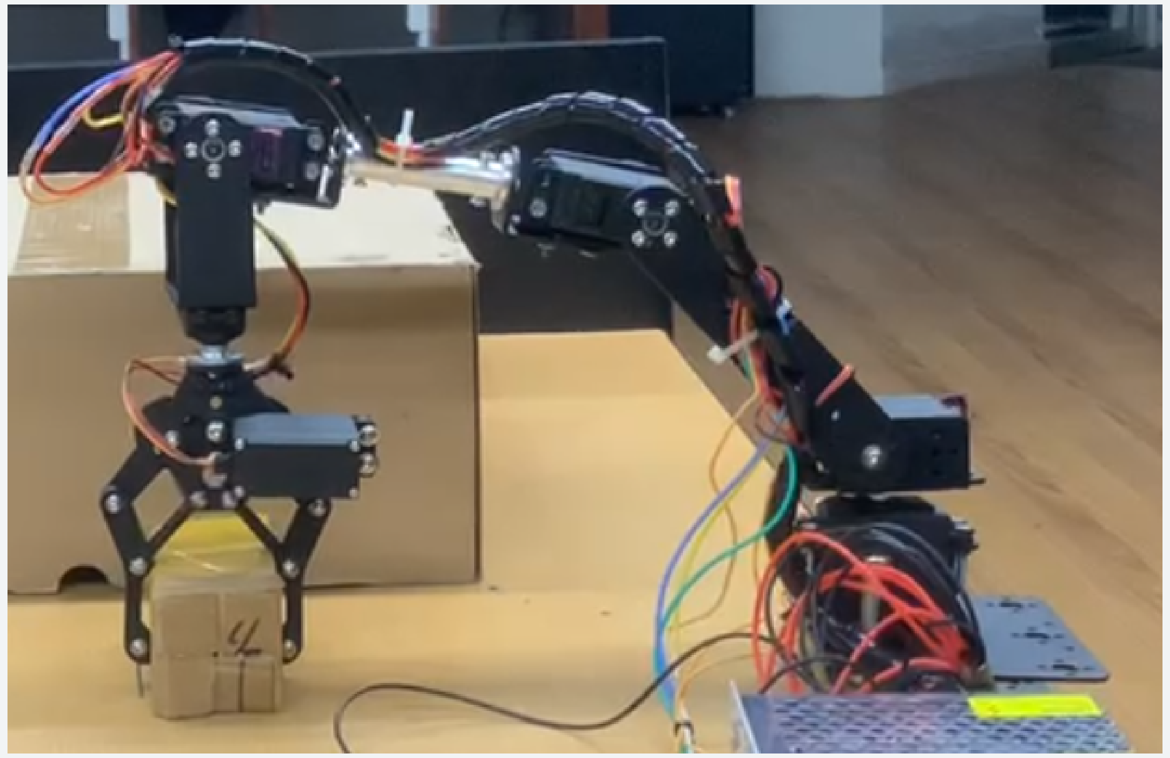
- θ_3 using cosine rule

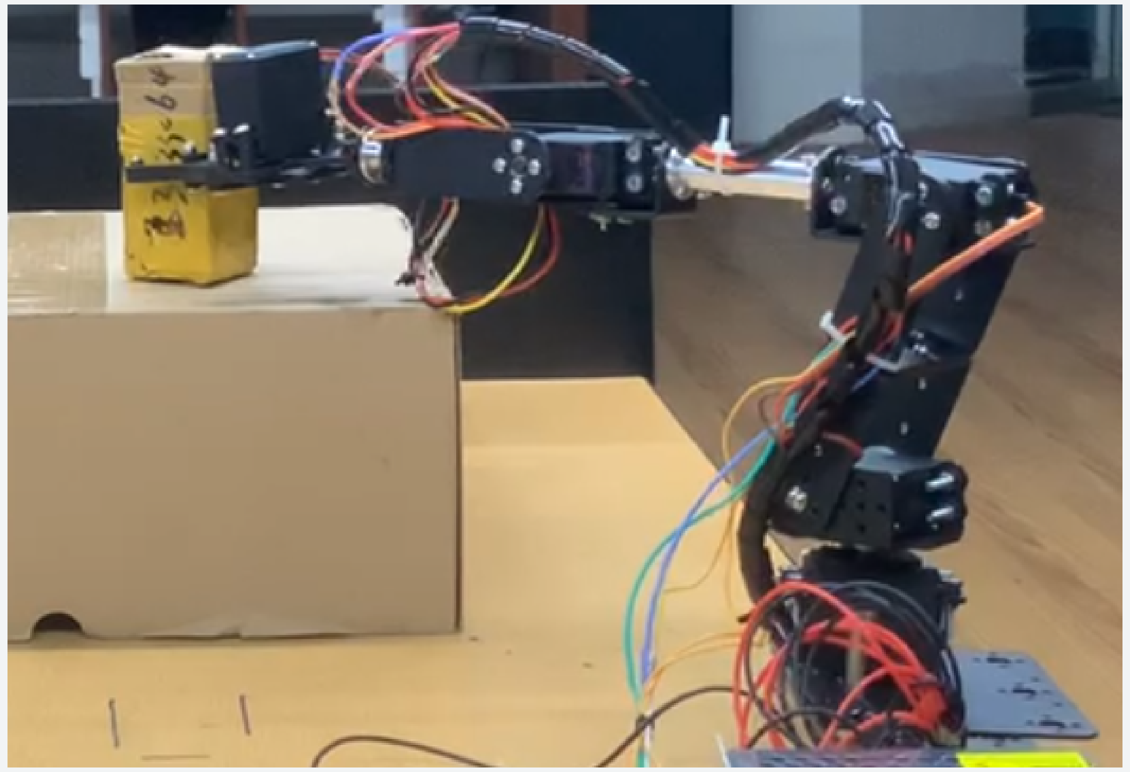


$$\theta_3 = \cos^{-1}\left(\frac{x_c^2 + y_c^2 + z_c^2 - l_1^2 - l_2^2}{2 \cdot l_1 l_2}\right)$$

$$\theta_4 = \begin{cases} \pi - \theta_3 - \theta_4 & \texttt{Link 5 - vertical} \\ \frac{\pi}{2} - \theta_3 - \theta_4 & \texttt{Link 5 - horizontal} \end{cases}$$

$$\theta_2 = 90^\circ - \tan^{-1}\left(\frac{z_c - 7}{\sqrt{x_c^2 + y_c^2}}\right) - \tan^{-1}\left(\frac{l_2 \sin(\theta_3)}{l_1 + l_2 \cos(\theta_3)}\right)$$





Initial Position

1. Coordinates - [22, 0, 0]
2. Link 5 - Vertical

Final Position

1. Coordinates - [18.7,18.7, 8.6]
2. Link 5 - Horizontal

Demo Calculation

case I - Link 5 Vertical

$$t_5^0 \equiv [22, 0, 0]^T \implies O_c \equiv [22, 0, 10.5]^T$$

$$\theta_1 = \tan^{-1}\left(\frac{0}{12}\right) = 0^\circ$$

$$\theta_3 = \cos^{-1}\left(\frac{22^2 + 10.5^2 - 10.5^2 - 15^2}{2 \cdot 10.5 \cdot 15}\right) \approx 32^\circ$$

$$\theta_2 = 90^\circ - \tan^{-1}\left(\frac{10.5}{22}\right) - \tan^{-1}\left(\frac{15\sin(\theta_3)}{10.5 + 15\cos(\theta_3)}\right) \approx 45^\circ$$

$$\theta_4 = 180^\circ - \theta_2 - \theta_3 \approx 105^\circ$$

case II - Link 5 Horizontal

$$t_5^0 \equiv [18.7, 18.7, 8.6]^T \implies O_c \equiv [6.3, 6.3, 10.5]^T$$

$$\theta_1 \approx 45^\circ$$

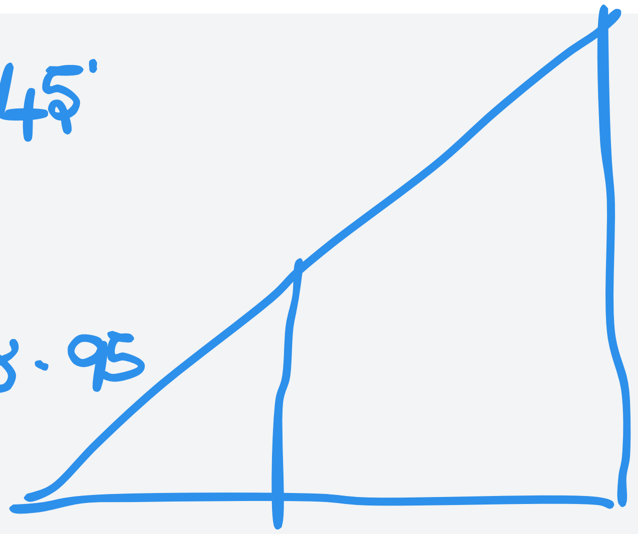
$$\theta_3 \approx 125^\circ$$

$$\theta_2 \approx -35^\circ$$

$$\theta_4 \approx 0^\circ$$

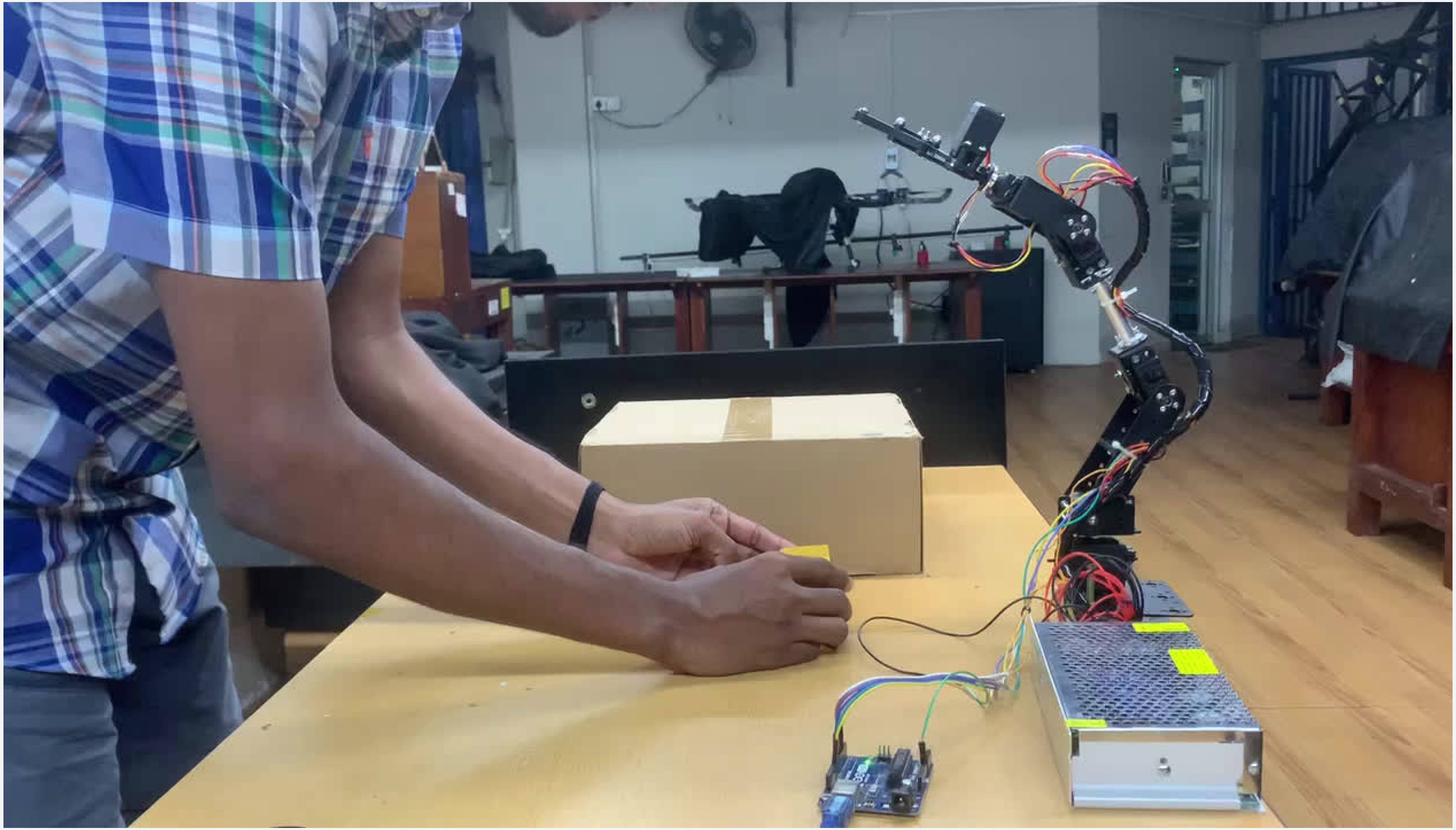
$\theta_1 = 45^\circ$

$26.45 - 17.5 = 8.95$



18.7

Demo: Pick and Place



Manipulator Jacobian

$$J = \begin{bmatrix} z_0^0 \times (t_5^0 - t_0^0) & z_1^0 \times (t_5^0 - t_1^0) & z_2^0 \times (t_5^0 - t_2^0) & z_3^0 \times (t_5^0 - t_3^0) & z_4^0 \times (t_5^0 - t_4^0) \\ z_0^0 & z_1^0 & z_2^0 & z_3^0 & z_4^0 \end{bmatrix}$$

$$z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1^0 = \begin{bmatrix} S_1 \\ -C_1 \\ 0 \end{bmatrix}, \quad z_2^0 = \begin{bmatrix} S_1 \\ -C_1 \\ 0 \end{bmatrix}, \quad z_3^0 = \begin{bmatrix} S_1 \\ -C_1 \\ 0 \end{bmatrix}$$

$$t_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad t_1^0 = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}, \quad t_2^0 = \begin{bmatrix} 10.5C_1C_2 \\ 10.5C_2S_1 \\ 10.5S_2 + 7 \end{bmatrix}$$

$$z_4^0 = \begin{bmatrix} C_4\left(C_1C_2S_3 + C_1C_3S_2\right) - S_4\left(C_1S_2S_3 - C_1C_2C_3\right) \\ C_4\left(C_2S_1S_3 + C_3S_1S_2\right) - S_4\left(S_1S_2S_3 - C_2C_3S_1\right) \\ S_4\left(C_2S_3 + C_3S_2\right) - C_4\left(C_2C_3 - S_2S_3\right) \end{bmatrix}$$

$$t_3^0 = \begin{bmatrix} 10.5C_1C_2 - 15C_1S_2S_3 + 15C_1C_2C_3 \\ 10.5C_2S_1 - 15S_1S_2S_3 + 15C_2C_3S_1 \\ 10.5S_2 + 15C_2S_3 + 15C_3S_2 + 7 \end{bmatrix}$$

$$t_4^0 = \begin{bmatrix} 10.5C_1C_2 - 15C_1S_2S_3 + 15C_1C_2C_3 \\ 10.521C_2S_1 - 15S_1S_2S_3 + 15C_2C_3S_1 \\ 10.5S_2 + 15C_2S_3 + 15C_3S_2 + 7 \end{bmatrix}$$

$$t_5^0 = \begin{bmatrix} 10.5C_1C_2 + 17.5C_4(C_1C_2S_3 + C_1C_3S_2) - 17.5S_4(C_1S_2S_3 - C_1C_2C_3) - 15C_1S_2S_3 + 15C_1C_2C_3 \\ 10.5C_2S_1 + 17.5C_4(C_2S_1S_3 + C_3S_1S_2) - 17.5S_4(S_1S_2S_3 - C_2C_3S_1) - 15S_1S_2S_3 + 15C_2C_3S_1 \\ 10.5S_2 + 15C_2S_3 + 15C_3S_2 - 17.5C_4(C_2C_3 - S_2S_3) + 17.5S_4(C_2S_3 + C_3S_2) + 7 \end{bmatrix}$$

J1 =

$$
\begin{pmatrix}
\frac{35\sin(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))}{2} - \frac{35\cos(\theta_4)\ (\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2))}{2} - \frac{21\cos(\theta_2)\sin(\theta_1)}{2} + 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\
\frac{21\cos(\theta_1)\cos(\theta_2)}{2} + \frac{35\cos(\theta_4)\ (\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2))}{2} - \frac{35\sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\
0 \\
0 \\
0 \\
1
\end{pmatrix}
$$

J2 =

$$
\begin{pmatrix}
\sin(\theta_1)\left(\frac{21\cos(\theta_2)\sin(\theta_1)}{2} + \frac{35\cos(\theta_4)\ (\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2))}{2} - \frac{35\sin(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1)\right) + \cos(\theta_1)\left(\frac{21\cos(\theta_1)\cos(\theta_2)}{2} + \frac{35\cos(\theta_4)\ (\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2))}{2} - \frac{35\sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cdots}{2} \right. \\
-\cos(\theta_1)\,\sigma_1 \\
-\sin(\theta_1)\,\sigma_1 \\
\sin(\theta_1) \\
-\cos(\theta_1) \\
0
\end{pmatrix}
$$

where

$$
\sigma_1 = \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) - \frac{35\cos(\theta_4)\ (\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3))}{2} + \frac{35\sin(\theta_4)\ (\cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2))}{2}
$$

Thank You