

Tutorial 4

1) Find the Fourier sine and cosine transform of the following functions

i) $f(x) = e^{-x} + e^{-2x}$

ii) $f(x) = 2e^{-5x} + 5e^{-2x}$

iii) $f(x) = e^{-2x} + 4e^{-3x}$

2) Find the Fourier sine and cosine transform of the following functions

i) $f(x) = \begin{cases} x & 0 \leq x < 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & x > 2 \end{cases}$

ii) $f(x) = \begin{cases} x & 0 \leq x < 1/2 \\ 1 - x & 1/2 \leq x < 1 \\ 0 & x > 1 \end{cases}$

3) Represent the following functions in the Fourier integral form

i) $f(x) = \begin{cases} \frac{\pi}{2} \sin x & |x| \leq \pi \\ 0 & |x| > \pi \end{cases}$

ii) $f(x) = \begin{cases} 0 & x < -a \\ 1 & -a \leq x \leq a \\ 0 & x > a \end{cases}$

4) Find the Fourier sine transform of

$$f(x) = \begin{cases} 0 & 0 \leq x < a \\ x & a \leq x \leq b \\ 0 & x > b \end{cases}$$

5) Find the Fourier transform of

$$f(x) = \begin{cases} \sin x + \cos x & |x| \leq \pi \\ 0 & |x| > \pi \end{cases}$$

6) Find $f(x)$ if $f_s(\lambda) = \frac{\lambda}{\lambda^2 + k^2}$

7) Find $f(x)$ if $f_s(\lambda) = \frac{\lambda}{\lambda^2 + k^2}$

8) Using inverse Fourier cosine transform find $f(x)$ if

$$f_c(\lambda) = \begin{cases} \sqrt{\frac{2}{\pi}} \left(a - \frac{\lambda}{2} \right) & \lambda \leq 2a \\ 0 & \lambda > 2a \end{cases}$$

9) Find the Fourier sine and cosine transform of the function $f(x) = e^{-x}$ and hence show

$$\text{that } \int_0^\infty \frac{\cos mx}{1+x^2} dx = \frac{\pi}{2} e^{-m} \text{ and } \int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$$