Sanjivani College of Engineering, Kopargaon

Subject- Mathematics-III

Tutorial 1

Unit-1 Vector Differentiation

1) If $\bar{r}(t) = t^2\bar{\imath} + t\bar{\jmath} - 2t^3\bar{k}$ then evaluate $\int_1^2 \left(\bar{r} \times \frac{d^2\bar{r}}{dt^2}\right) dt$
2) If $\bar{r}(t) = sinht\bar{a} + cosht\bar{b}$, then prove that $\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} =$ constant.
3) If directional derivative of $\emptyset = ax^2y + by^2z + cz^2x$ at point (1,1,1) has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$, hence find the values of a, b, c.
4) For a solenoidal vector field $\bar{E}$, show that $curlcurlcurlcurl\bar{E} = \nabla^4\bar{E}$.
5) If $\bar{F} = (x^2 - y^2 + 2xz)\bar{\imath} + (xz - xy + yz)\bar{\jmath} + (z^2 + x^2)\bar{k}$ then show that $curl\bar{E}$ at point (1, 2, -3) and (2, 3, 12) are orthogonal.
6) Verify whether the following field is irrotational, if so find corresponding scalar point function $\emptyset$ such that $\bar{F} = \nabla\emptyset$
   $\bar{F} = (ysinz - sinx)\bar{\imath} + (xsinz + 2yz)\bar{\jmath} + (xycosz + y^2)\bar{k}$.
7) Show that the vector field $f(r)\bar{r}$ is always irrotational and find $f(r)$ such that $\nabla^2 f(r) = 0$.
8) Show that
   i) $\nabla^4(r^2logr) = \frac{6}{r^2}$
   ii) $\nabla \cdot \left[r\nabla\left(\frac{1}{r^n}\right)\right] = \frac{n(n-2)}{r^{n+2}}$
   iii) $\nabla^2\left(\frac{\bar{a}\cdot\bar{b}}{r}\right) = 0$