

# Mathematics-III

## Vector Integration

### Tutorial 2

1. Find the workdone in a force field

$\bar{F} = (y^2 \cos x + z^2)\bar{i} + (2y \sin x - 4)\bar{j} + (3xz^2 + 2)\bar{k}$   
moving in a particle from  $(0, 1, -1)$  to  $(\pi/2, -1, 2)$ . Is the field conservative?

2. Verify Green's theorem for  $\bar{F} = x\bar{i} + y^2\bar{j}$  over the first quadrant of the circle  $x^2 + y^2 = a^2$ .

3. Evaluate  $\iint_S (2xy\bar{i} + yz^2\bar{j} + xz\bar{k}) \cdot d\bar{s}$  over the surface of the region bounded by  $x = 0, y = 0, y = 3, z = 0$  and  $x + 2z = 6$

4. Evaluate  $\iint_S (\nabla \times \bar{F}) \cdot \hat{n} \, ds$  where S is the curved surface of paraboloid  $x^2 + y^2 = 2z$  bounded by the plane  $z = 2$ , where

$$\bar{F} = 3(x - y)\bar{i} + 2xz\bar{j} + xy\bar{k}$$

5. Use Stoke's theorem to evaluate  $\int_C (4y\bar{i} + 2z\bar{j} + 6y\bar{k}) \cdot d\bar{r}$ , where C is the curve of intersection of  $x^2 + y^2 + z^2 = 2z$  and  $x = z - 1$ .

6. Evaluate  $\int_C \bar{F} \cdot d\bar{r}$  for  $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$  along the straight line joining the points  $(0, 0, 0)$  and  $(2, 1, 3)$

7. Evaluate  $\iiint_S [xz^2 dydz + (x^2y - z^2) dzdx + (2xy + y^2z) dxdy]$ , where S is the surface enclosing a region bounded by hemisphere  $x^2 + y^2 + z^2 = 4$  above the XOY plane.