CS 663: Assignment 1 Question 3

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1 Introduction

If we consider a (non-discrete) image $I(x)$ with a continuous domain and real-valued intensities within [0, 1]. Let the image histogram be $h(I)$, with mass 1. Consider the histogram $h(I)$ is split into two histograms (i) $h_1(I)$ over the domain [0, a] and (ii) $h_2(I)$ over the domain (a, 1], for some arbitrary a is an element of (0, 1). Assume that the histogram mass within [0, a] is an element of (0, 1).

2 Derivation of Mean Intensity

2.1 Part 1

Suppose we perform histogram equalization over the two intensity invervals [0, a] and (a, 1] separately, in a way that preserved the masses of the two histograms $h_1(I)$ and $h_2(I)$ after the transformation. We have to derive the mean intensity for the resulting histogram (or, equivalently, image) which is listed below.

Mass of histogram 1

$$h_1(i) \equiv \int_0^a h(I)dl$$

Mass of histogram 2

$$h_2(i) \equiv \int_a^1 h(I)dl$$

After histogram equalization on both sides,

$$h_1(I) = \frac{\int_0^a h(I)dl}{a} \forall I \in [0, a] \tag{1}$$

$$h_2(I) = \frac{\int_a^1 h(I)dl}{1-a} \forall I \in (a, 1] \tag{2}$$

Mean Intensity

$$I_\mu \equiv \int_0^a Ih_1(I)dI + \int_a^1 Ih_2(I)dI \tag{3}$$

$$\implies I_\mu = \frac{a}{2}\int_0^a h(I)dI + \frac{1-a}{2}\int_a^1 h(I)dI \tag{4}$$

Since mass of histogram $h(I)$ is 1 , therefore

$$\int_0^a h(I)dl + \int_a^1 h(I)dl = 1$$

Using the above equation in (4),

$$I_\mu = a\int_0^a h(I)dl - \frac{1}{2}\int_0^a h(I)dl - \frac{a}{2} + \frac{1}{2}$$

2.2 Part 2

Let the chosen intensity a be the median intensity for the original histogram $h(I)$. Assume that the mean intensity for the original histogram $h(I)$ is also a. We have to derive the mean intensity for the resulting histogram. The derivation is provided below.

$$\int_0^a h(I)dl = \int_a^1 h(I)dl$$

as $a = \frac{1}{2}$ is the median intensity now.

Also, mean intensity $\equiv a$

$$\implies a = \int_0^a Ih(I)dl + \int_a^1 Ih(I)dl \tag{5}$$

$$\implies \int_0^a h(I)dl = \int_a^1 h(I)dl = \frac{1}{2} \tag{6}$$

Thus, the mean intensity is calculated to be

$$I_\mu = a \times \frac{1}{2} - \frac{1}{4} - \frac{a}{2} + \frac{1}{2}$$

$$\implies I_\mu = \frac{1}{4}$$

2.3 Observation

This can retain more information than histogram equalisation and the whole brightness of the image is also not compromised. The reason is that, if an image contains parts which are very bright and very dark equal in proportion, then this could perform equalisation for both of those parts individually and hence providing good contrast than the entire histogram equalisation.

3 Intensity Transformation

We use a image from the internet and performed the given histogram transformation on it. We observed a better performance on the image and obtained a good result.

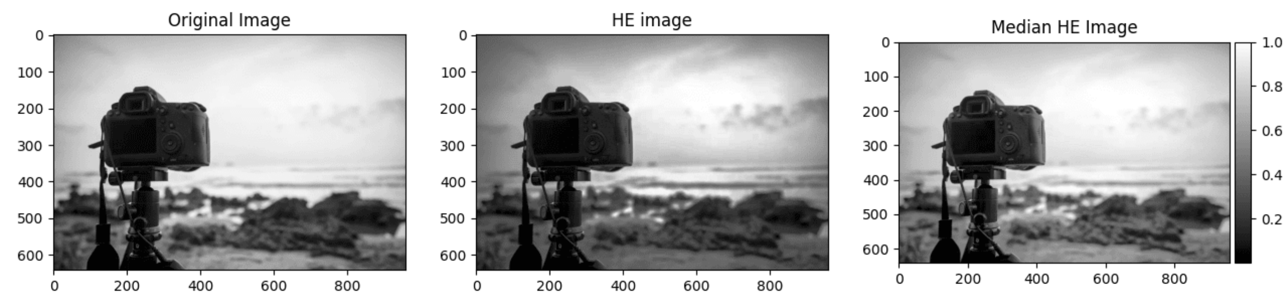

Figure 1: Histogram Equalization and Median Histogram Equalization

4 Conclusion

By doing this question, we get to know about histogram based intensity transformations which does a better job than a simple histogram equalization. We derived the mean intensity for the resulting histogram when some information or constraints are given. It was overall a good experience.