

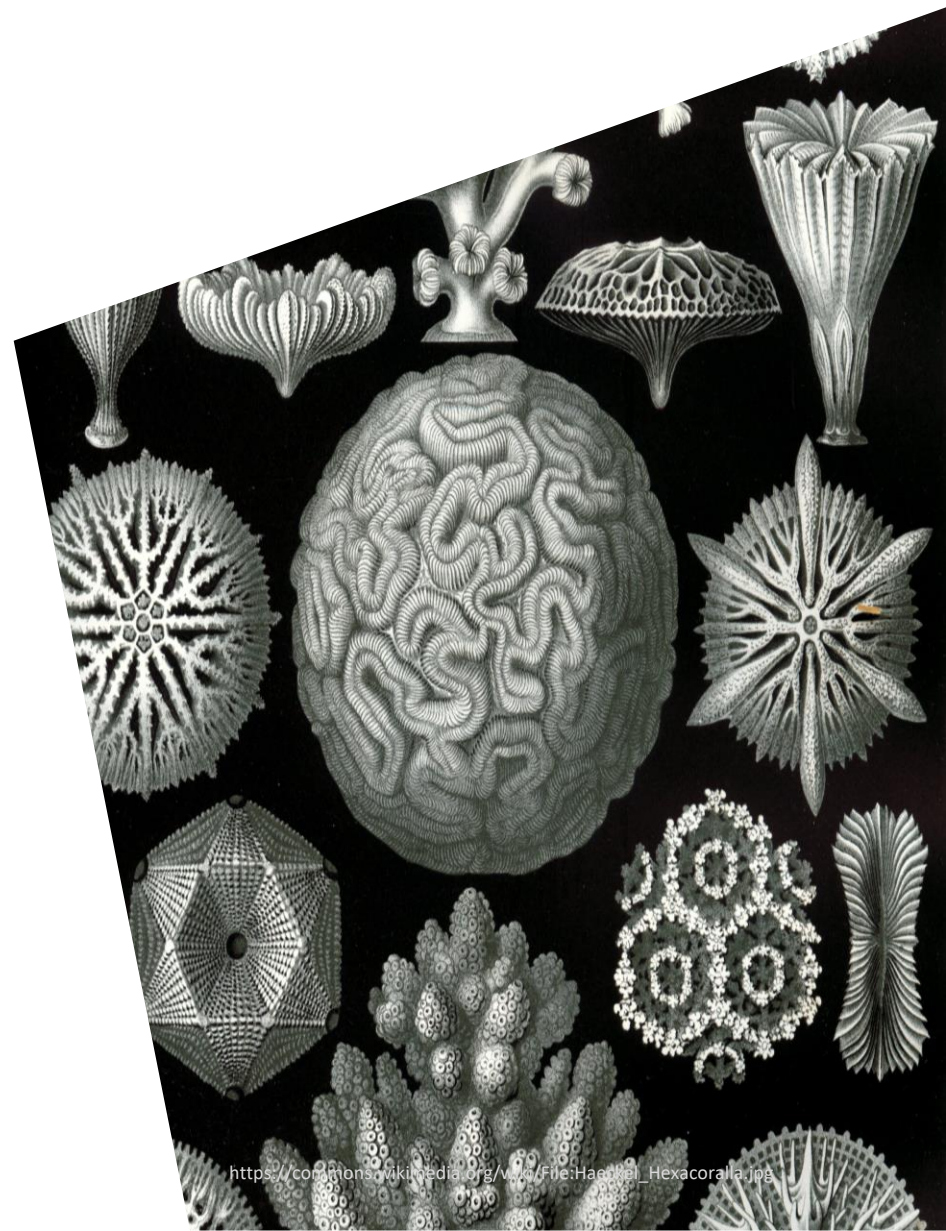


**COMP20003**  
**Algorithms and Data Structures**

# Introduction to Graphs

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University of Melbourne  
Semester 2



[https://commons.wikimedia.org/wiki/File:Haeckel\\_Hexacoralla.jpg](https://commons.wikimedia.org/wiki/File:Haeckel_Hexacoralla.jpg)



# Graph definition

Graph:

- a representation of a set of objects
- some pairs of objects are connected by links



# Graph definition

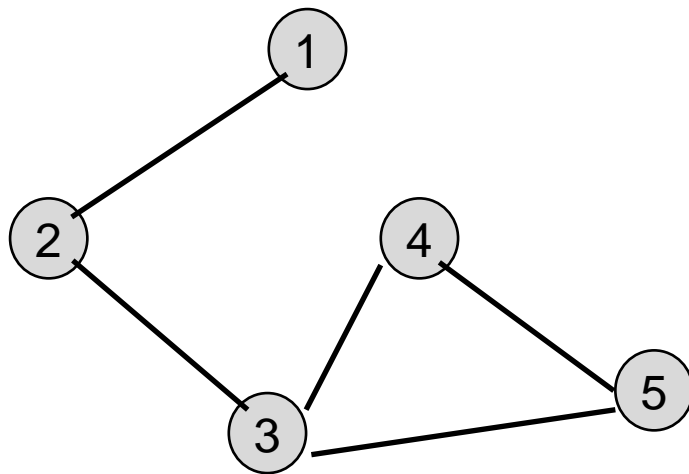
Graph  $G = \{V, E\}$  Set of:

- Vertices  $V$ : can contain information
- Edges  $E$  (links between vertices): can have direction and/or weight

Compared to trees and linked lists:

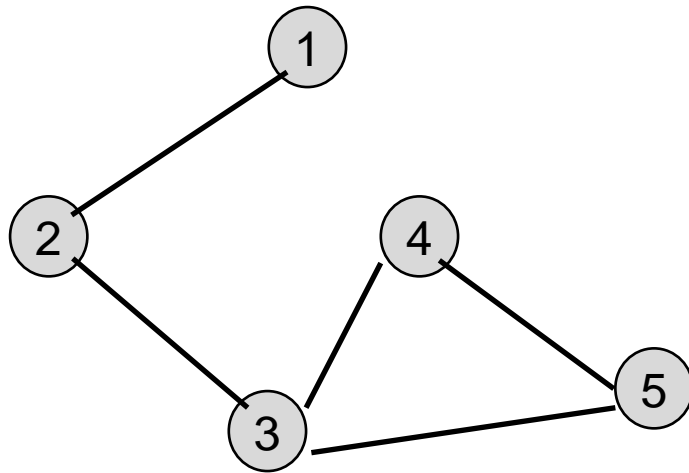
- vertices = nodes
- edges = links

# Undirected graph



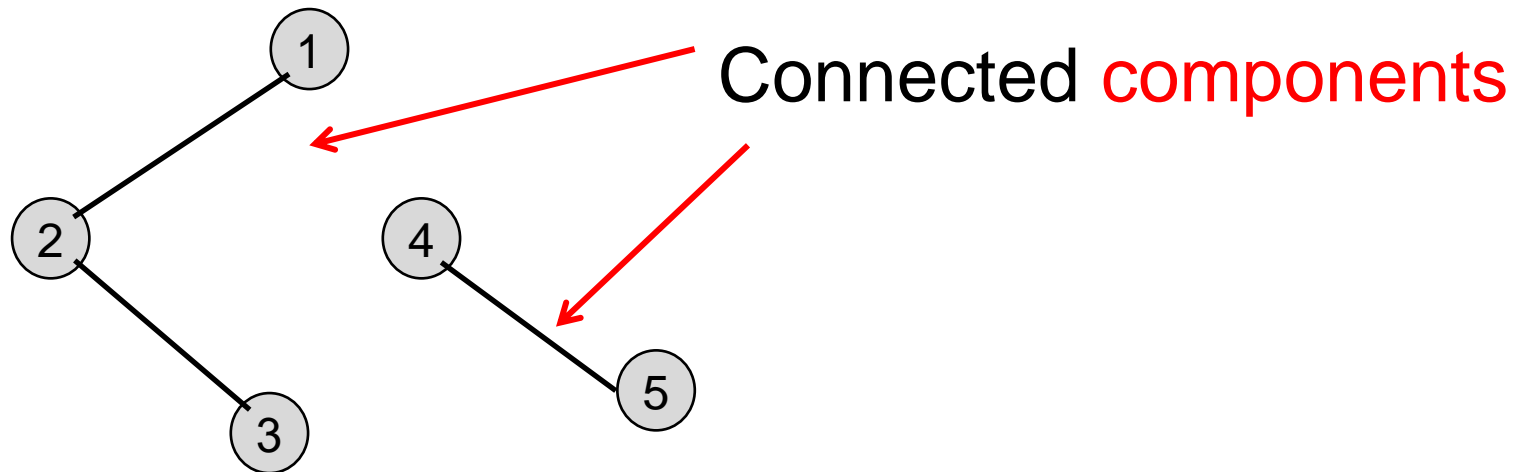
Edges have **no direction** specified

# Connected Undirected graph



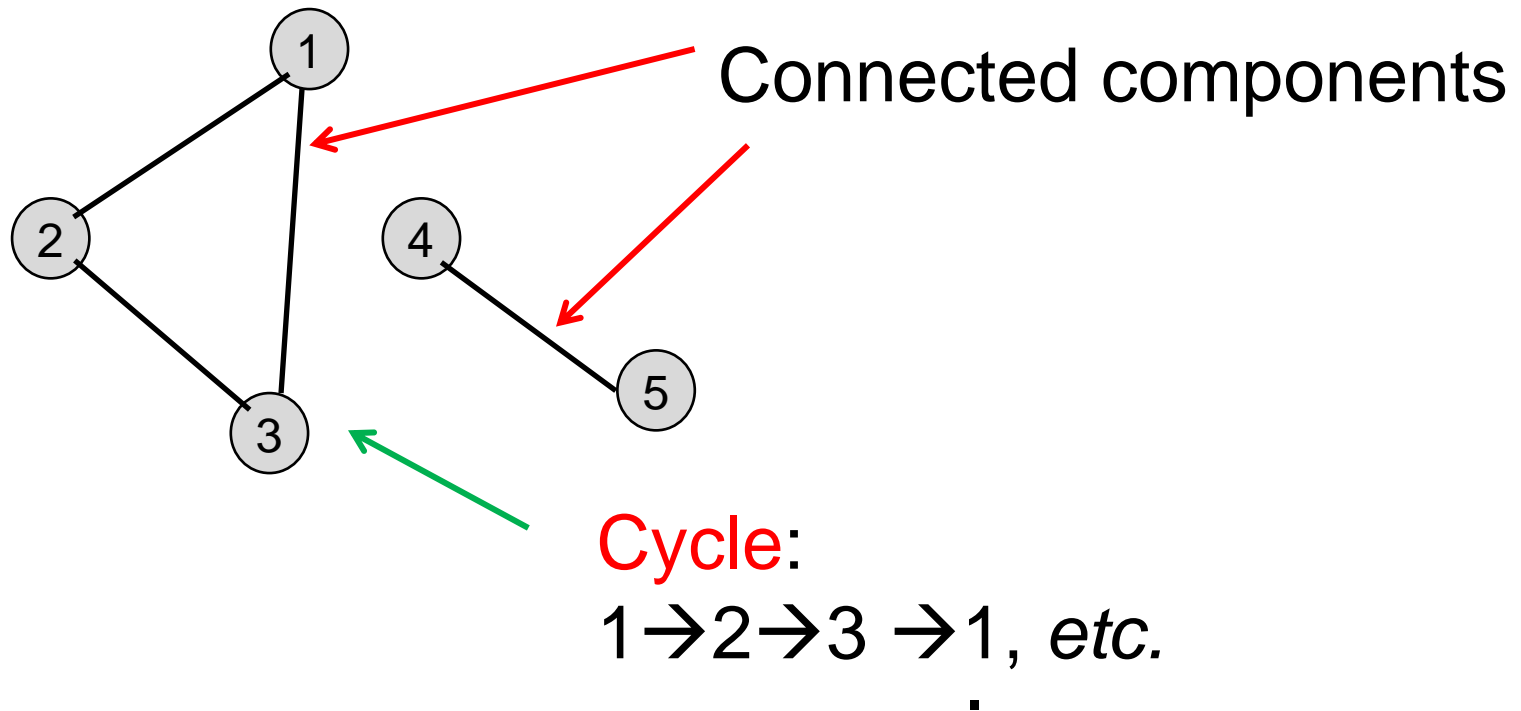
Every pair of vertices is connected  
(possibly indirectly)

# Unconnected Undirected graph

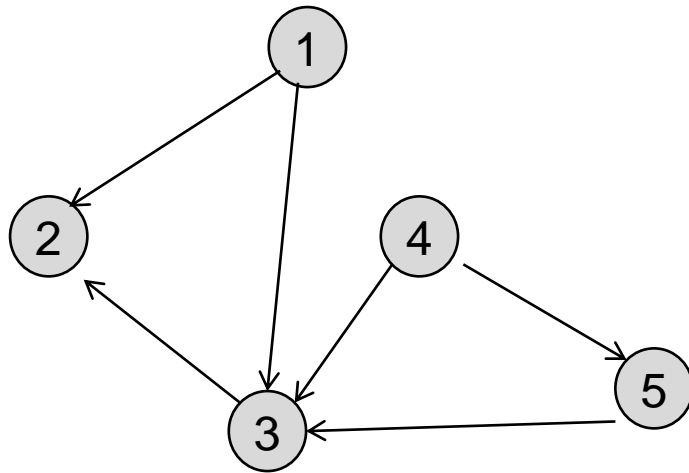


■

# Unconnected Undirected graph with cycle



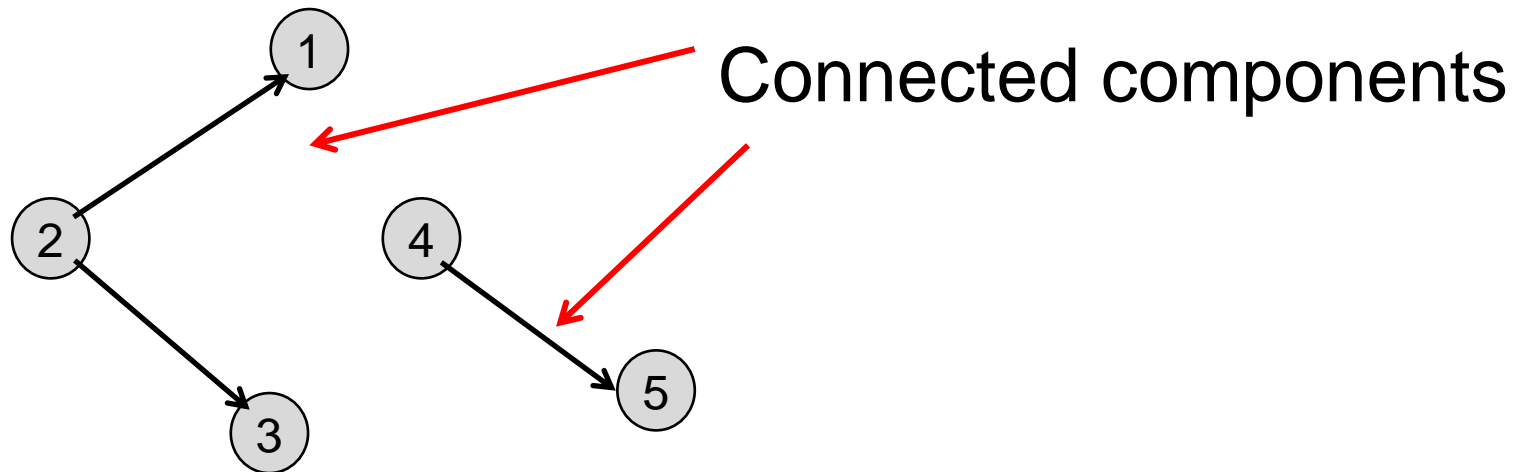
# Directed graph



- Edge **direction** is specified
- Links are **not symmetrical**

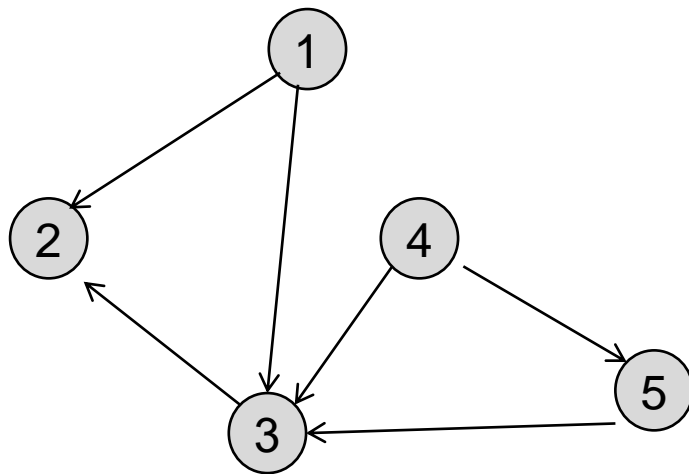


# Acyclic, unconnected directed graph



■

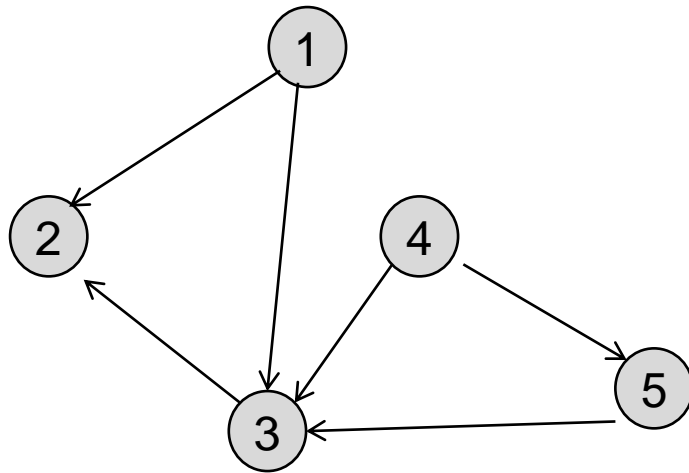
# Directed graph



Reachability:

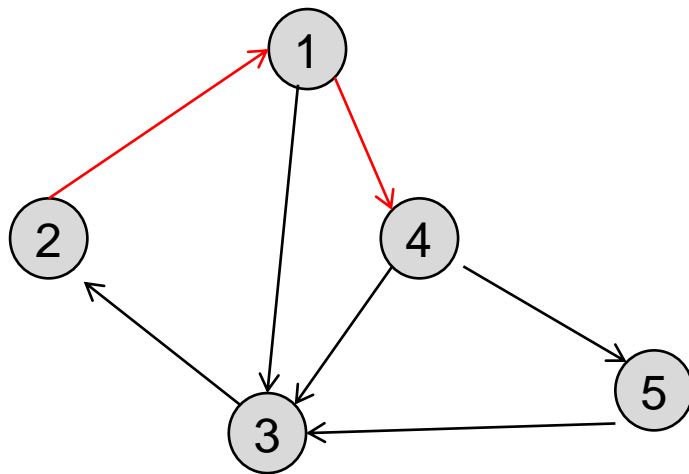
Can you get **from** Vertex 1 **to** Vertex 5?

# Weakly connected Directed graph



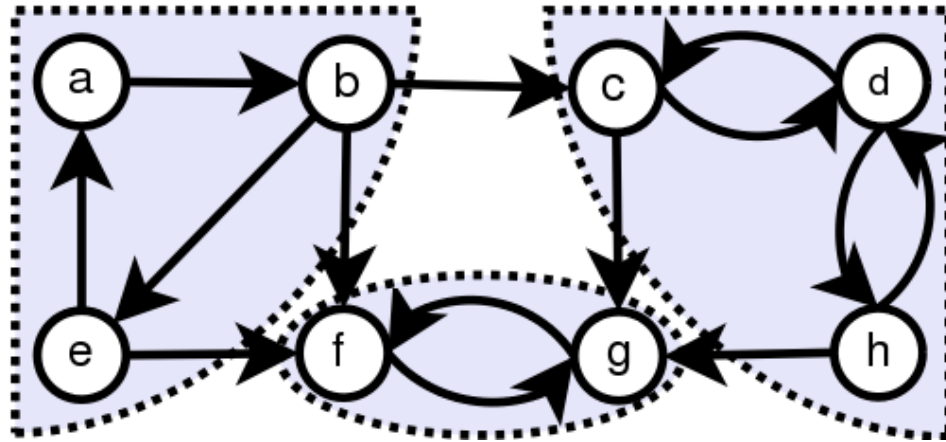
Replace all directed edges with undirected edges,  
to obtain a connected (undirected) graph

# Strongly connected Directed graph



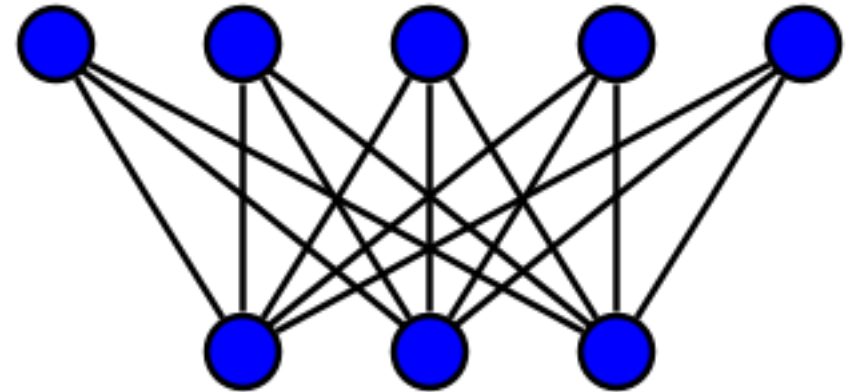
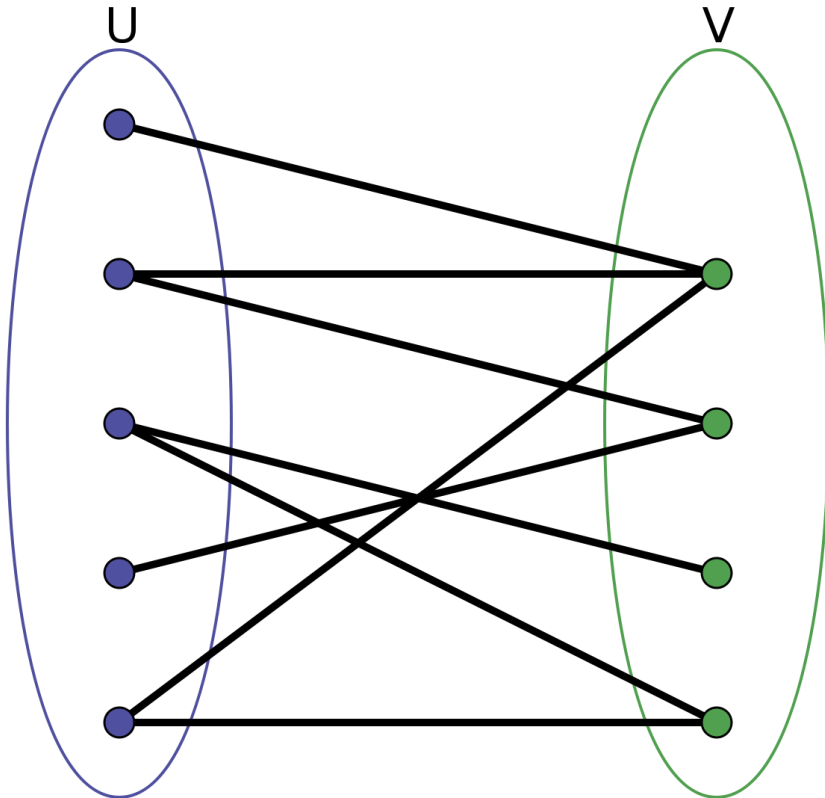
If every vertex is reachable  
from every other vertex

# Strongly connected Components in a Digraph



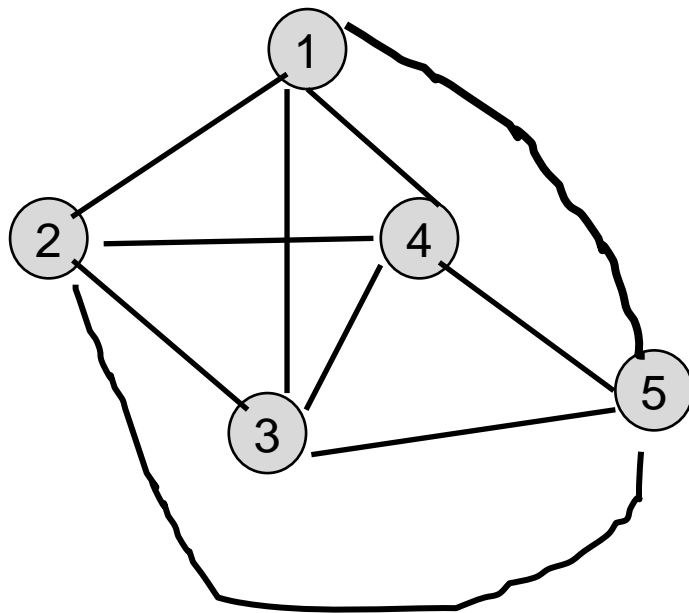
If **every vertex is reachable**  
from every other vertex  
within the **same component**

# Bipartite Graph



- $U$  and  $V$  are **disjoint sets** of vertexes
- Every **vertex in  $U$**  **connects** to a vertex in  **$V$** , and viceversa

# Complete graph



$V(V-1)/2$  edges



# Trees and Graphs

Tree, undirected graph that is:

- Connected
- Acyclic

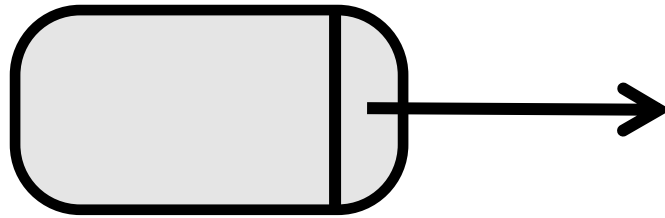
*n.b.* Any two vertices are connected by exactly one simple path

*n.b.* All vertices are connected

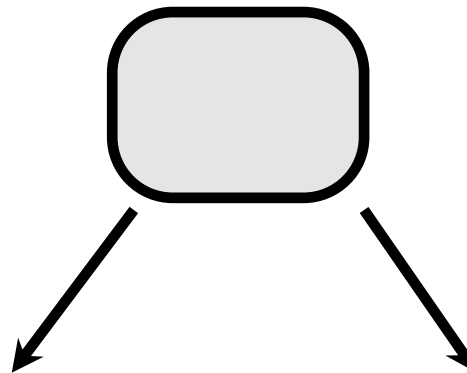


# Representing graph vertices

Linked **list** nodes:

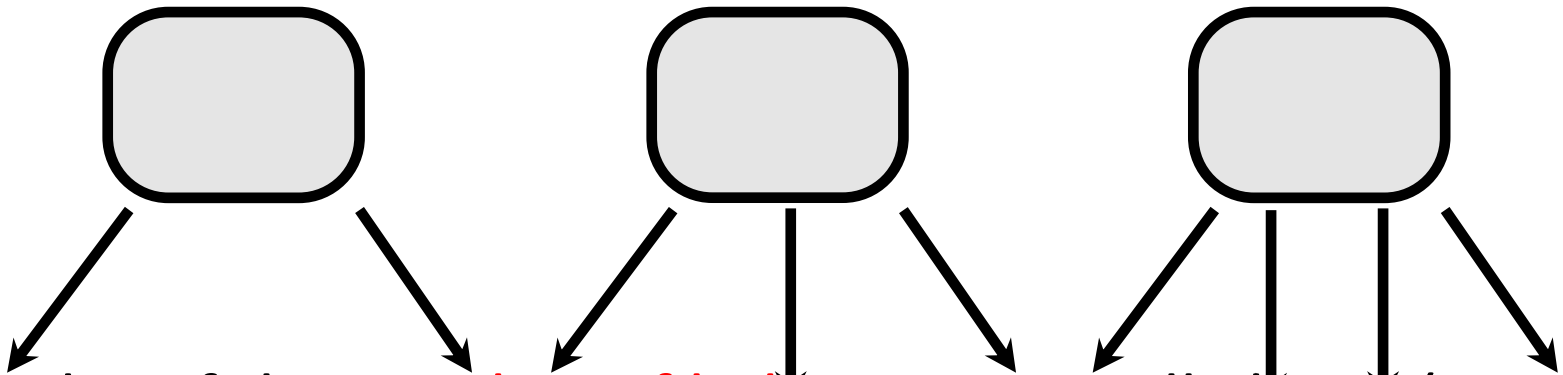


Binary **tree** nodes:



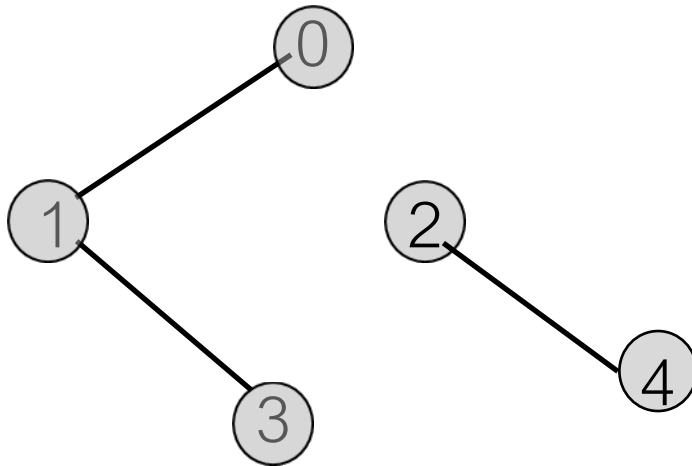
# Vertices

2-3-4 tree nodes



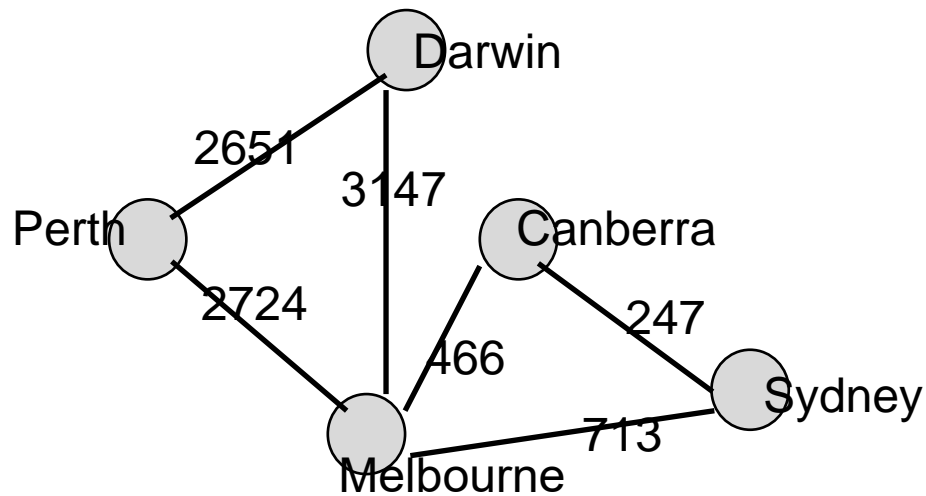
But what if the **number of links** is potentially large (**up to  $V-1$** )?

# Array representation



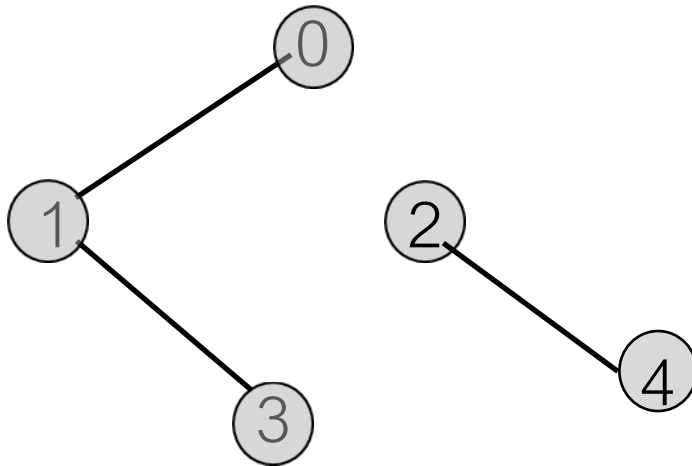
A	0	1	2	3	4
0	0	1	0	0	0
1	1	0	0	1	0
2	0	0	0	0	1
3	0	1	0	0	0
4	0	0	1	0	0

# Weighted undirected graph

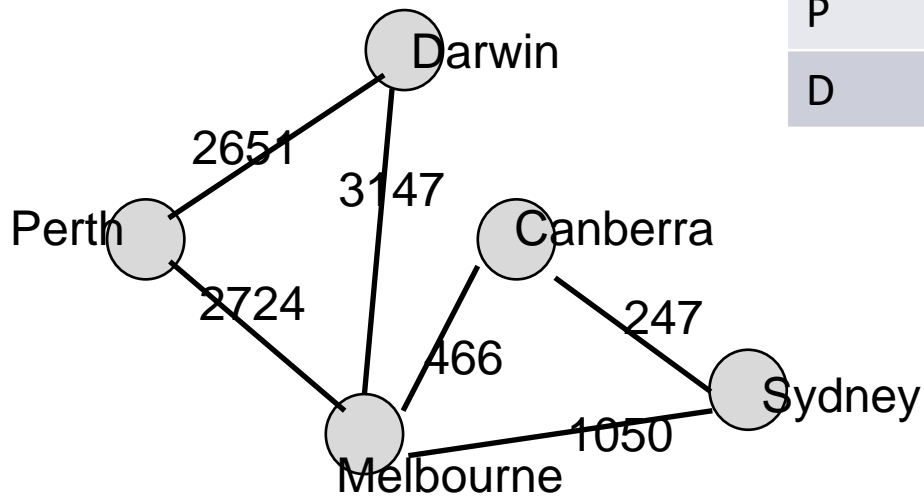


713

# Array representation of weighted undirected graph



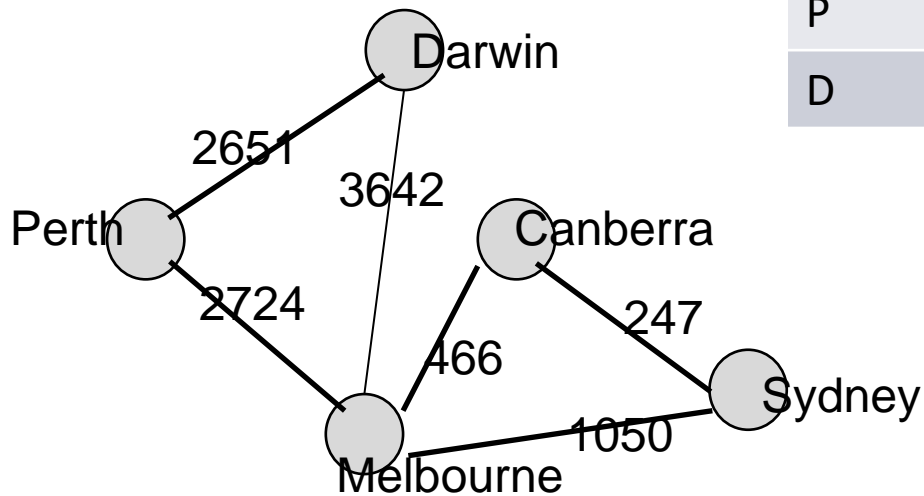
A	0	1	2	3	4
0		25			
1	25			40	
2					10
3		40			
4			10		



	M	C	S	P	D
M	0	466	1050	2724	3642
C	466	0	247		
S	1050	247	0		
P	2724			0	2651
D	3642			2651	0

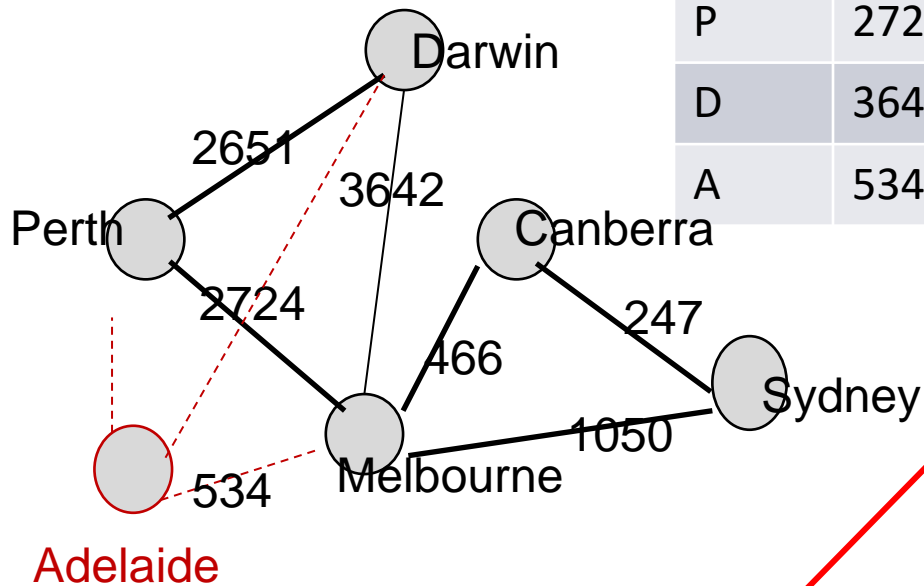
713

	M	C	S	P	D
M	0	466	1050	2724	3642
C	466	0	247		
S	1050	247	0		
P	2724			0	2651
D	3642			2651	0



Canberra to Darwin?  
 $466 + 3642 = 4108$

	M	C	S	P	D	A
M	0	466	1050	2724	3642	534
C	466	0	247			1160
S	1050	247	0			1163
P	2724			0	2651	2134
D	3642			2651	0	2620
A	534	1160	1163	2134	2620	0



Canberra to Darwin?

Still  $466 + 3642$ ?

Or  $466 + 534 + 2724$ ?



# Shortest route between Melbourne and Dubbo?



Considerations:

- Distance
- Kind of road
- Traffic points

# Shortest route between Melbourne and Dubbo?



**Dubbo**

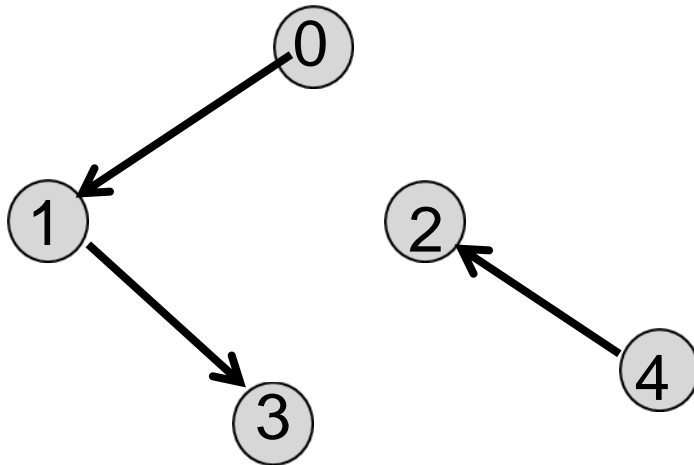
**Melbourne**



**Dubbo**

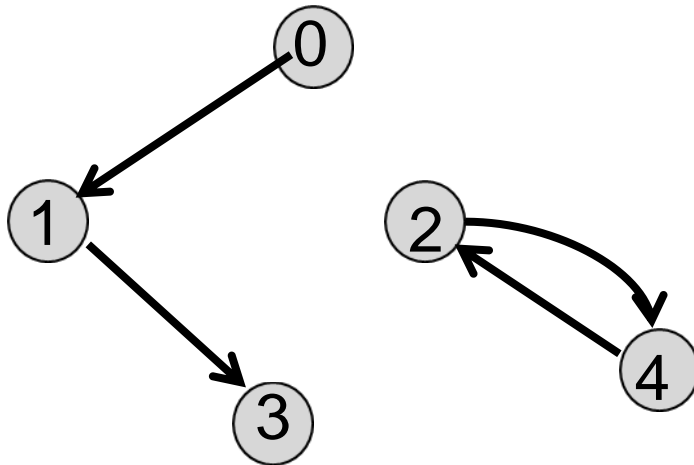
**Melbourne**

# Array representation: weighted directed graph



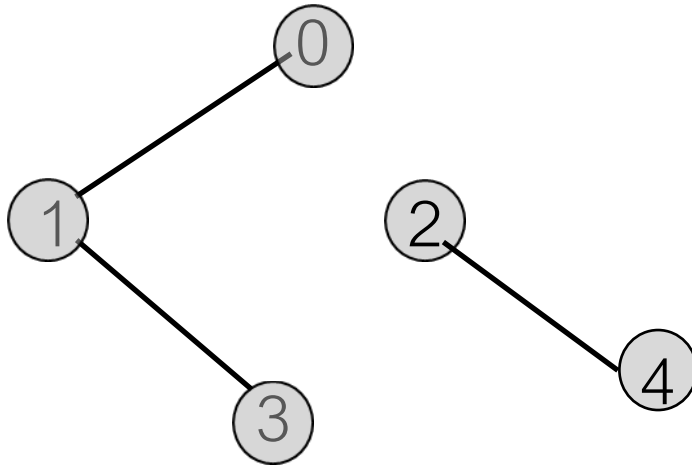
A	0	1	2	3	4
0		25			
1				40	
2					
3					
4			10		

# Array representation: weighted directed graph



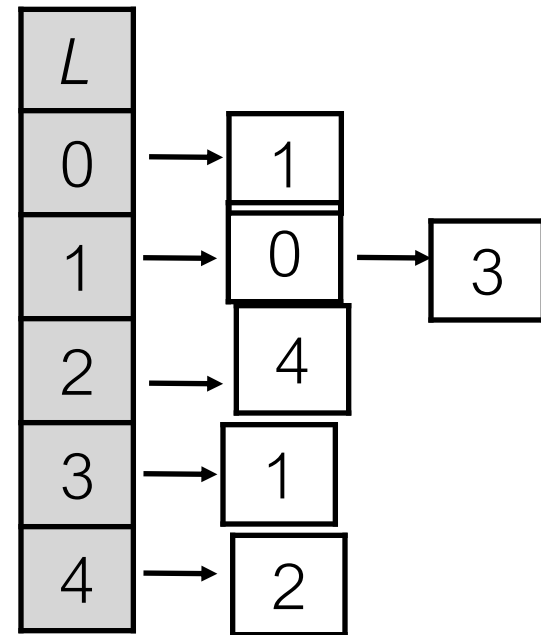
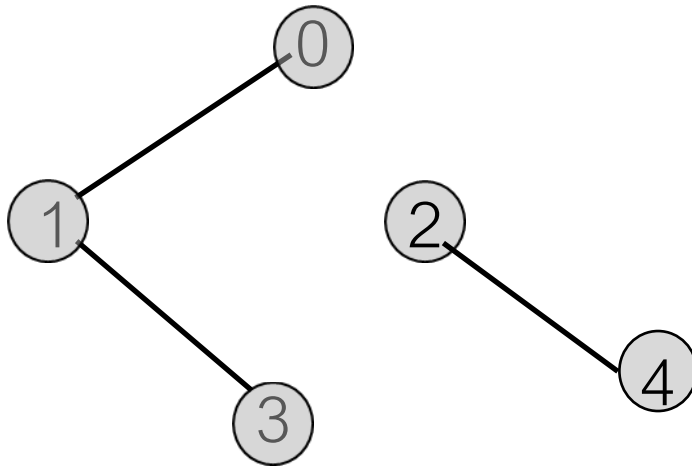
A	0	1	2	3	4
0		25			
1				40	
2					15
3					
4			10		

# Array representation: initialization

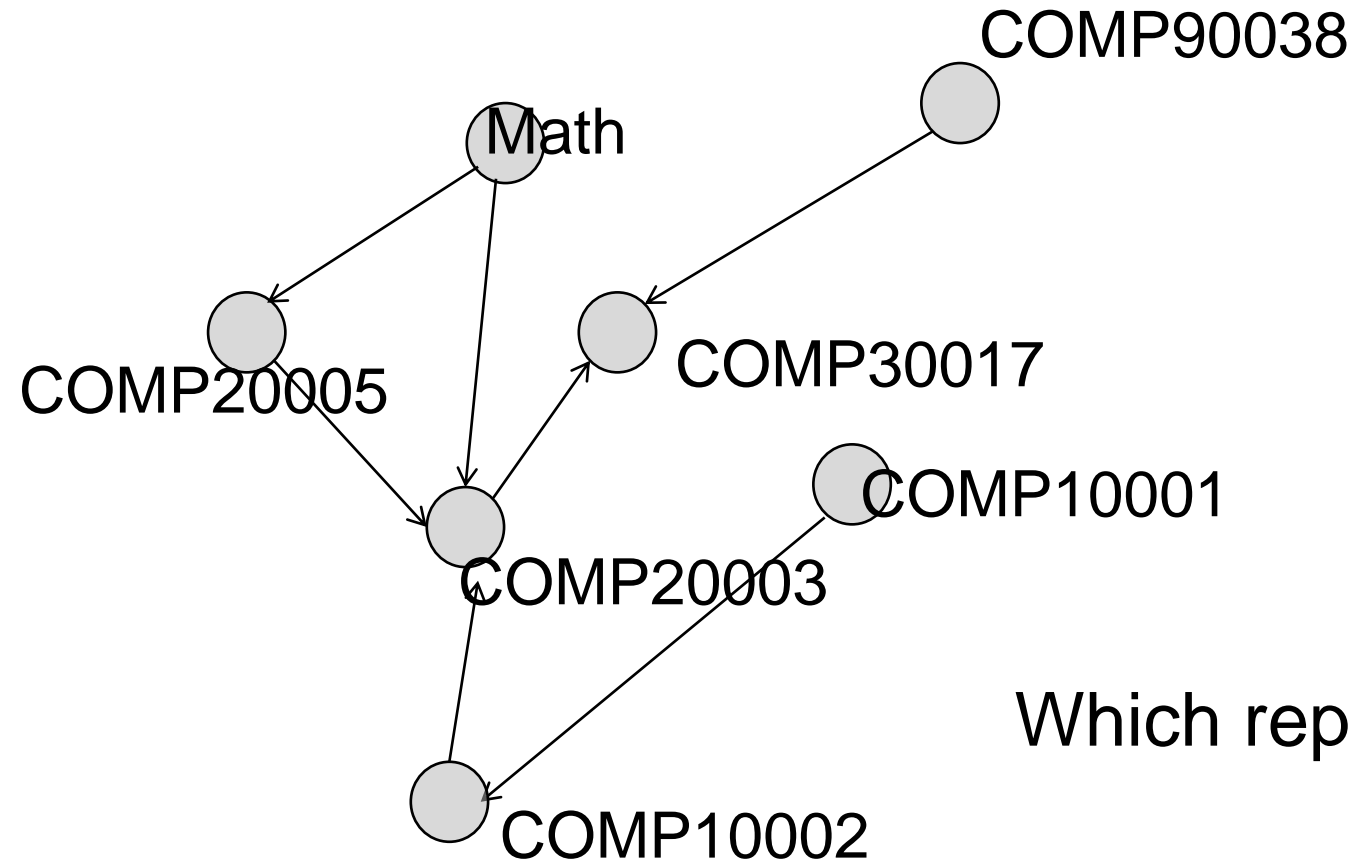


A	0	1	2	3	4
0	$\infty$	25	$\infty$	$\infty$	$\infty$
1	25	$\infty$	$\infty$	40	$\infty$
2	$\infty$	$\infty$	$\infty$	$\infty$	10
3	$\infty$	40	$\infty$	$\infty$	$\infty$
4	$\infty$	$\infty$	10	$\infty$	$\infty$

# Adjacency list representation



# Directed graph: Subject prerequisites



Which representation?



## Size of matrix and list

Size of representation in terms of

- $|V|$  number of vertices
- $|E|$  number of edges

Matrix

- $O(V^2)$

Adjacency list

- $O(V + E)$





## Size of matrix and list

Matrix

- $O(|V|^2)$

Adjacency list

- $O(|V| + |E|)$

Dense graph, lots of edges, use matrix representation

Sparse graph, use list



# Some interesting graph path problems

Reachability

Single shortest path

Single source shortest path

All pairs shortest paths

Travelling Salesman Problem



## Other interesting graph problems

Minimum Spanning Tree

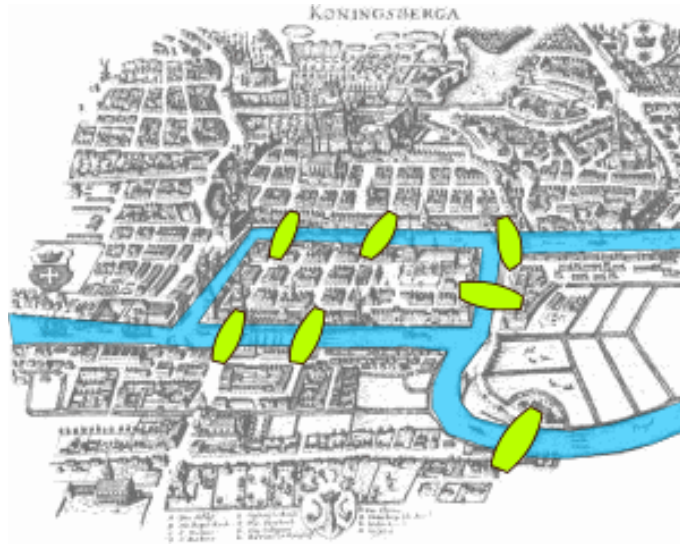
Topological sort

Map coloring

Matching

# History

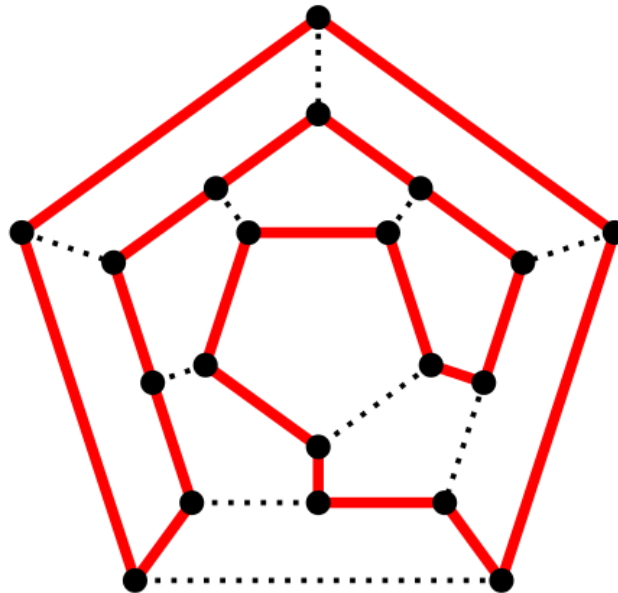
Graph theory started with Euler (1736) who was asked to find a nice path across the seven Königsberg bridges



The (Eulerian) path should cross over each of the seven bridges (edge) exactly once

## History

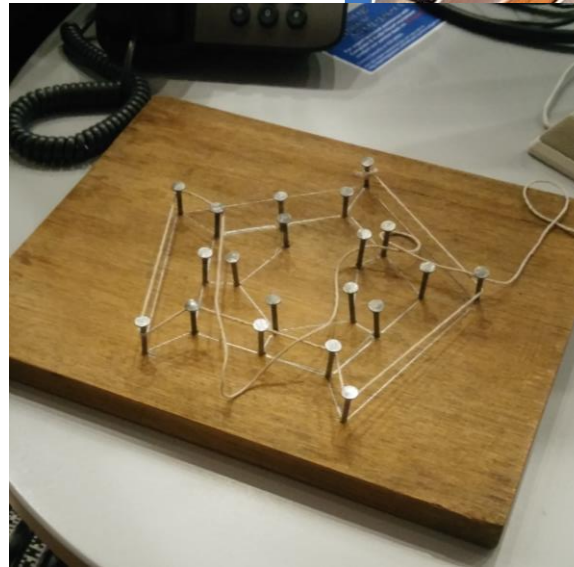
Another early precursor was Sir William Rowan Hamilton (1805-1865)



In 1859 he developed a **toy** based on **finding a path visiting all cities (vertexes) in a graph exactly once** and sold it to a toy maker in Dublin. It never was a big success

# History

Build your own game



Or play it here: <http://neamar.fr/Res/Icosien/> (Flash)

- Euler Game:

- Traverse an **Edge** once: <https://youtu.be/ztGElutUIJQ>

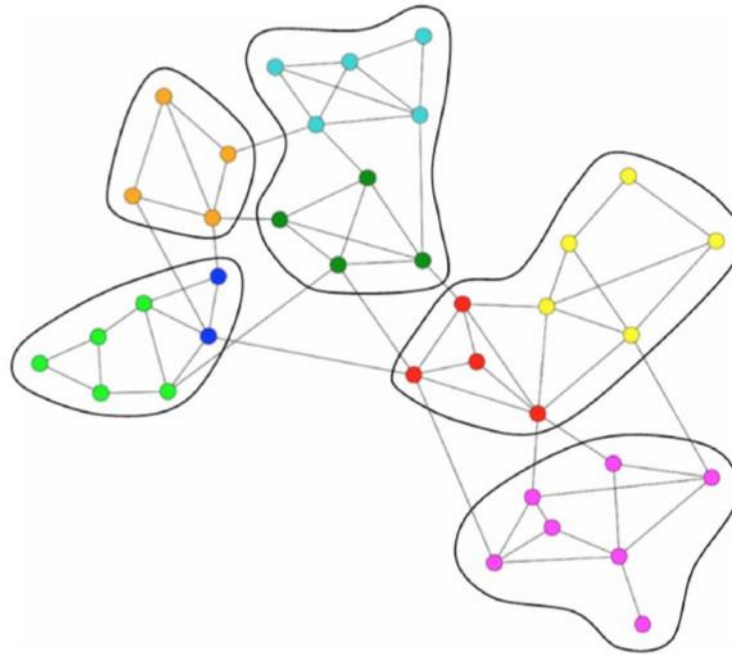
- Hamiltonian Cycle:

- Traverse an **Vertex** once: [https://youtu.be/PU7ZIA0\\_Z5w](https://youtu.be/PU7ZIA0_Z5w)

- Analysis: <https://www.naturelovesmath.com/en/games/icosian-a-graph-theory-game/>

# Applications

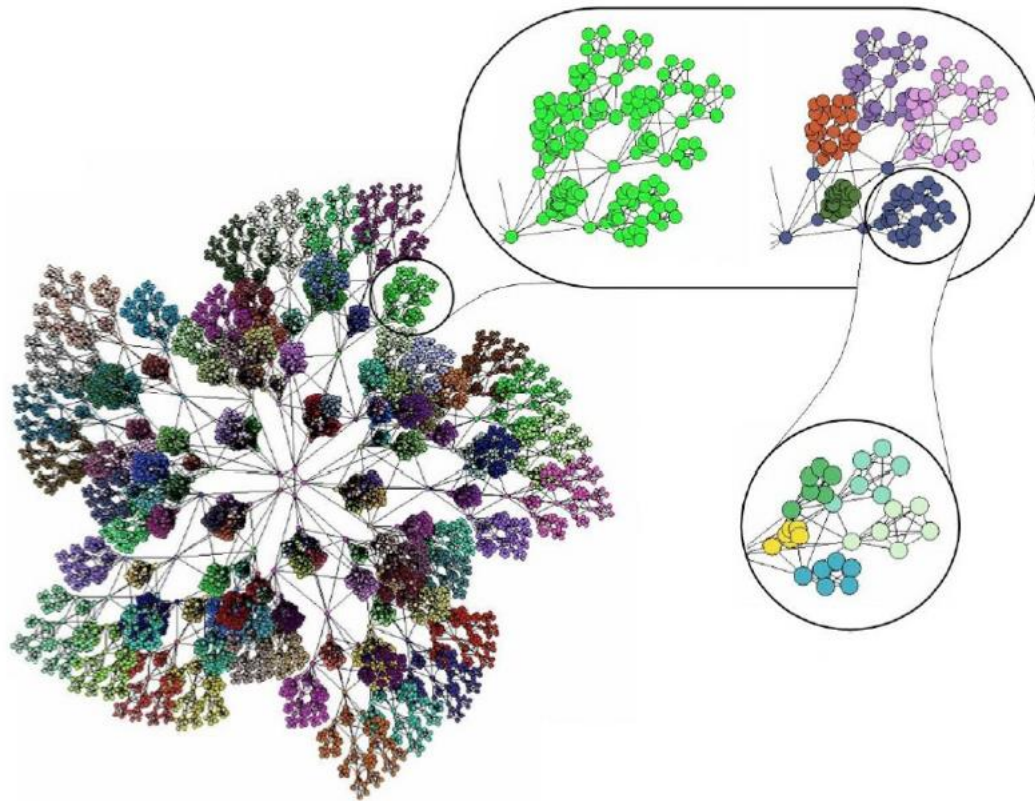
But **now** graph theory is used for **finding communities** in **networks**



where we want to **c**

# Applications

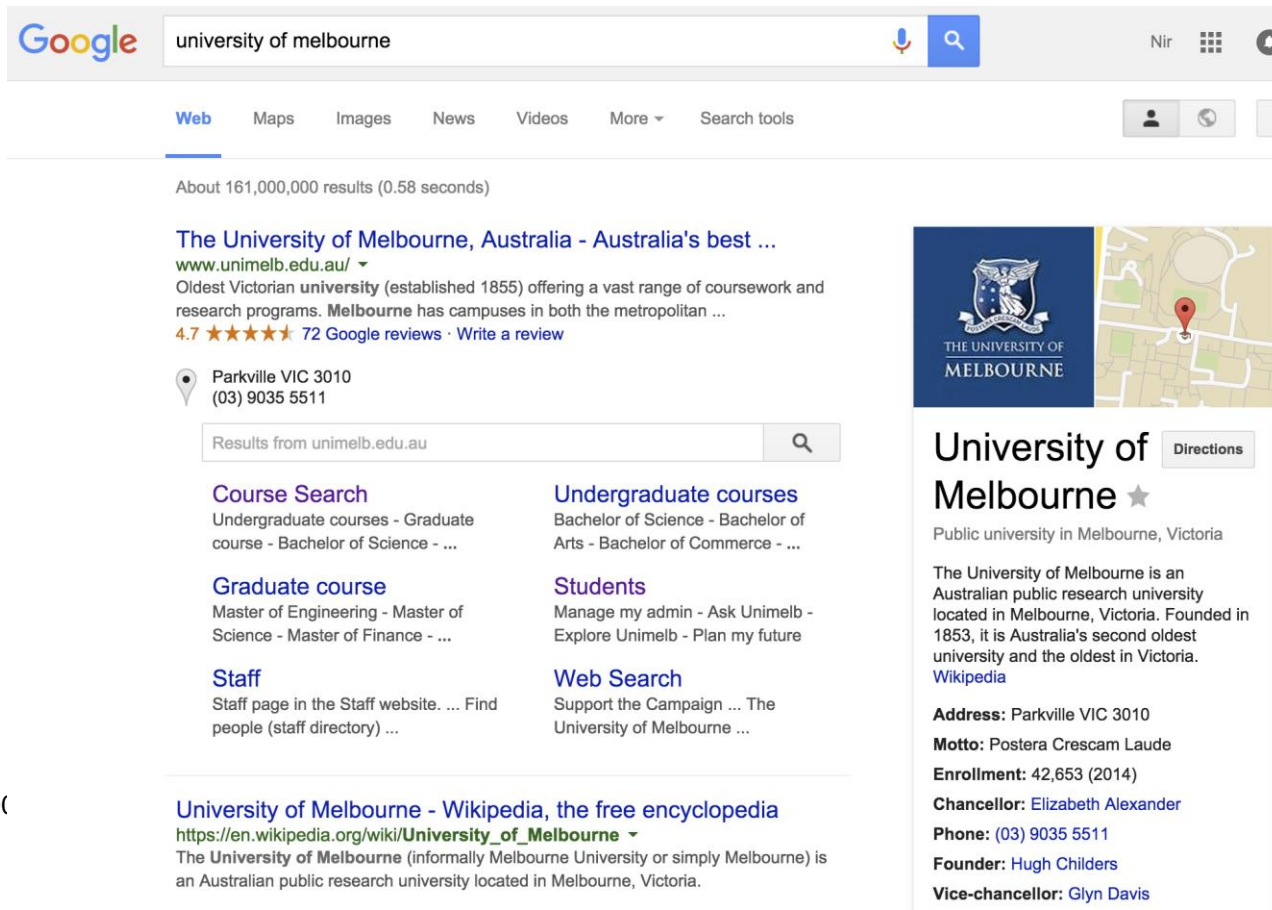
and their **sizes** can become **quite big** ...





# Applications

It is also used for **ranking** (ordering) **hyperlinks**



Google university of melbourne Nir

Web Maps Images News Videos More Search tools

About 161,000,000 results (0.58 seconds)

**The University of Melbourne, Australia - Australia's best ...**  
[www.unimelb.edu.au/](http://www.unimelb.edu.au/)  
Oldest Victorian university (established 1855) offering a vast range of coursework and research programs. Melbourne has campuses in both the metropolitan ...  
4.7 ★★★★★ 72 Google reviews · Write a review

Parkville VIC 3010  
(03) 9035 5511

Results from unimelb.edu.au

<b>Course Search</b> Undergraduate courses - Graduate course - Bachelor of Science - ...	<b>Undergraduate courses</b> Bachelor of Science - Bachelor of Arts - Bachelor of Commerce - ...
<b>Graduate course</b> Master of Engineering - Master of Science - Master of Finance - ...	<b>Students</b> Manage my admin - Ask Unimelb - Explore Unimelb - Plan my future
<b>Staff</b> Staff page in the Staff website. ... Find people (staff directory) ...	<b>Web Search</b> Support the Campaign ... The University of Melbourne ...

**University of Melbourne** ★  
Public university in Melbourne, Victoria

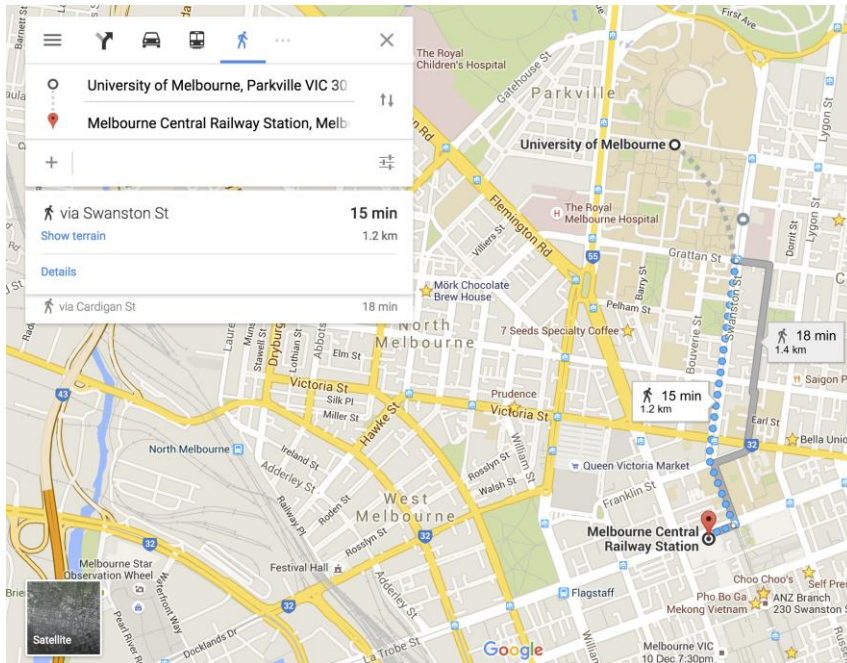
The University of Melbourne is an Australian public research university located in Melbourne, Victoria. Founded in 1853, it is Australia's second oldest university and the oldest in Victoria.  
[Wikipedia](#)

**Address:** Parkville VIC 3010  
**Motto:** Postera Crescam Laude  
**Enrollment:** 42,653 (2014)  
**Chancellor:** Elizabeth Alexander  
**Phone:** (03) 9035 5511  
**Founder:** Hugh Childers  
**Vice-chancellor:** Glyn Davis

**University of Melbourne - Wikipedia, the free encyclopedia**  
[https://en.wikipedia.org/wiki/University\\_of\\_Melbourne](https://en.wikipedia.org/wiki/University_of_Melbourne)  
The University of Melbourne (informally Melbourne University or simply Melbourne) is an Australian public research university located in Melbourne, Victoria.

# Applications

or by your **GPS** to find the **shortest path** home ...



... and the list should go on for hours