ECE 148 Homework 5

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Background

The periodic signal $f(t)$ has a period $T = 1$ and is defined as:

$$f(t) = \sum_{m=1}^{7} a_m \sin(m\omega_0 t)$$

where $\omega_0 = \frac{2\pi}{T}$ is the fundamental frequency, and the amplitudes of the harmonics are given by:

$$a_m = [\ 9\ 5\ 8\ 7\ 1\ 8\ 9\ ].$$

The function $f(t)$ is sampled at 32 uniform intervals over one period.

$$f(n) = f\left(\frac{n}{32}\right)$$

for $n = 0, 1, \ldots, 31$.

```matlab
T = 1; % Period of the signal
w_0 = 2 * pi / T; % Fundamental frequency
a_m = [9 5 8 7 1 8 9]; % Amplitude of harmonics

f = @(t) a_m(1) * sin(1 * w_0 * t) + a_m(2) * sin(2 * w_0 * t) + ...
         a_m(3) * sin(3 * w_0 * t) + a_m(4) * sin(4 * w_0 * t) + ...
         a_m(5) * sin(5 * w_0 * t) + a_m(6) * sin(6 * w_0 * t) + ...
         a_m(7) * sin(7 * w_0 * t);

% We will plot the function f(t) over one period.
t = linspace(0, T, 1000); % Time vector
f_t = f(t); % Function values

% We then take 32 uniform samples of the function f(t) over one period:
delta_T = T / 32; % Sampling interval
t_samples = 0:delta_T:(T-delta_T); % Sampled time vector
f_n = f(t_samples); % Sampled function values {f(n = 0, 1, ..., 31)}
```

Listing 1: MATLAB Script to Initialize the Function

Problem 1: Time Function and Sampling

The function $f(t)$ is plotted over one period ($0 \leq t < T$), along with its sampled values $f(n)$ for $n = 0, 1, \ldots, 31$.

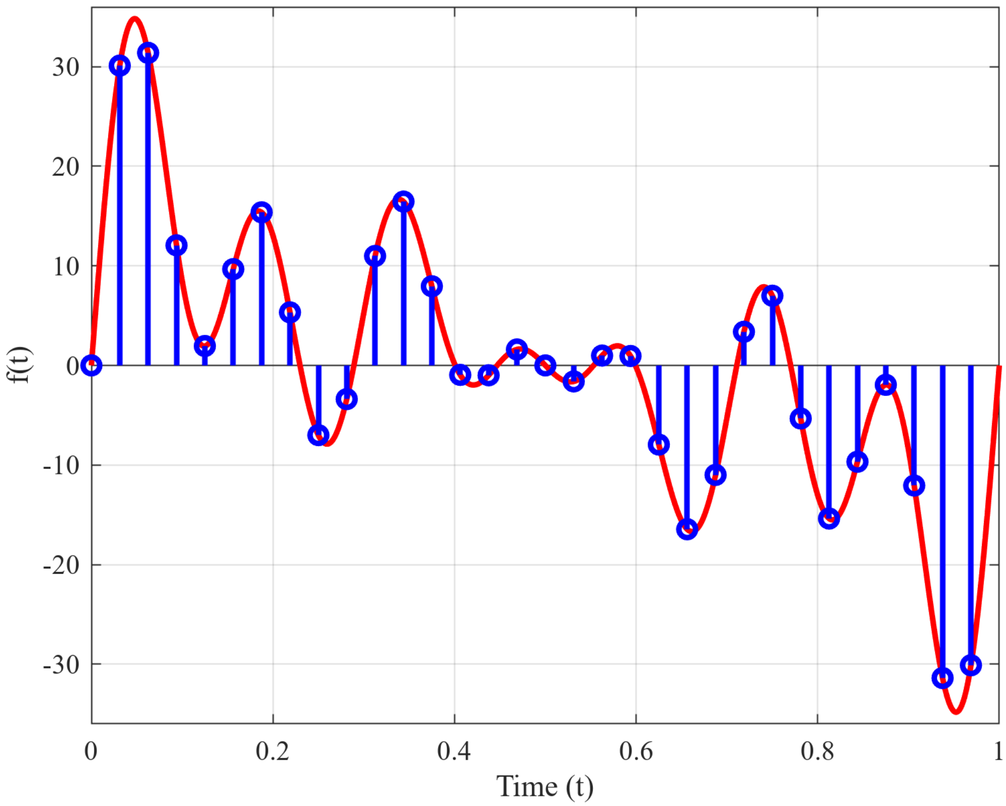

Figure 1: Periodic Signal $f(t)$ and Sampled Values

Problem 2: Fourier Series Expansion

The periodic signal $f(t)$ is expressed in the form of a complex Fourier series expansion:

$$F_m = \frac{1}{T}\int_0^T f(t)e^{-jm\omega_0 t}dt$$

where $m = -7, \ldots, 7$ are the harmonic indices. The Fourier coefficients F_m are computed numerically, and their magnitudes are plotted below:

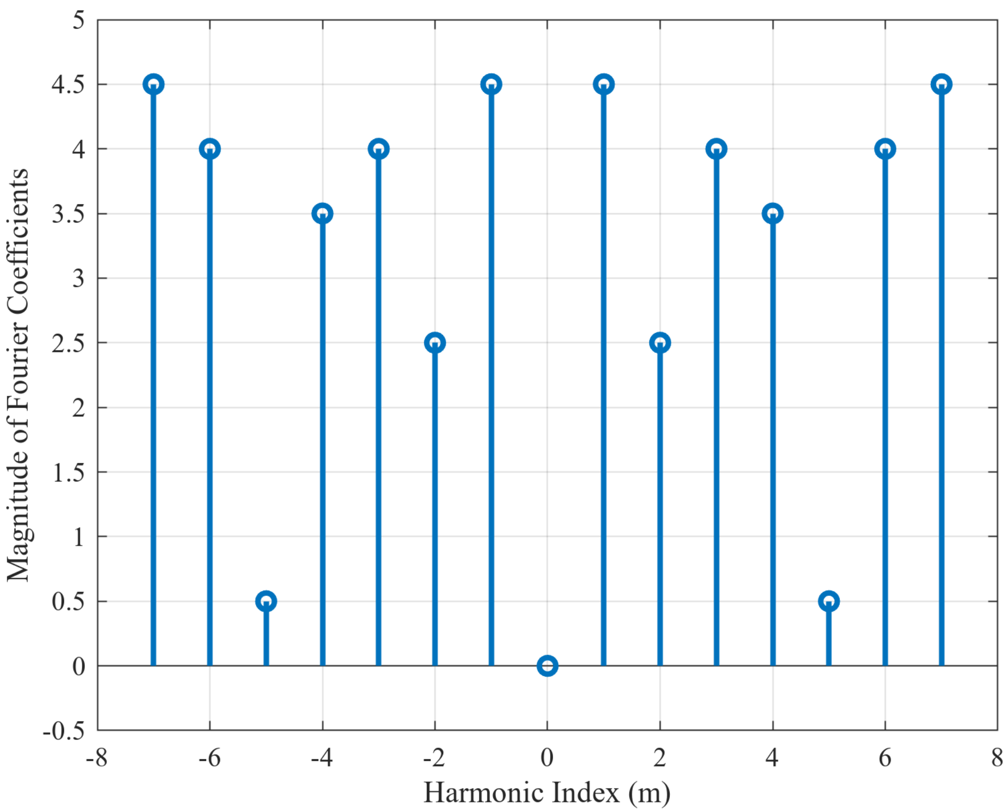

Figure 2: Fourier Coefficients F_m

The Fourier coefficients are computed using the following MATLAB code:

```
N = 7;  % Number of harmonics to compute (from -N to N)
m = -N:N;  % Harmonic indices corresponding to F_m
F_m = zeros(size(m));  % Complex Fourier coefficients (vector from -N to N)
dt = t(2) - t(1);   % Calculate dt for numerical integration

% Calculate each coefficient using rectangular integration
for i = 1:length(m)
    integrand = f_t .* exp(-1j * m(i) * w_0 * t);
    F_m(i) = (1/T) * sum(integrand) * dt;
end
clear i integrand;
```

Listing 2: MATLAB Script to Compute Fourier Coefficients

The complex Fourier coeffiecients are:

$$F_m = \begin{bmatrix} 0+4.5j \\ 0+4j \\ 0+0.5j \\ 0+3.5j \\ 0+4j \\ 0+2.5j \\ 0+4.5j \\ 0+0j \\ 0-4.5j \\ 0-2.5j \\ 0-4j \\ 0-3.5j \\ 0-0.5j \\ 0-4j \\ 0-4.5j \end{bmatrix}$$

The physical frequencies corresponding to the Fourier coefficients are given by $f_m = m\omega_0/(2\pi)$. The physical frequencies are:

$$F_m = \begin{bmatrix} -7 \\ -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} \text{Hz}$$

Problem 3: Fourier Transform of Periodic Functions

The Fourier transform $F(j\omega)$ of the periodic function $f(t)$ is determined and plotted. The amplitudes and physical frequencies of the peaks are identified.

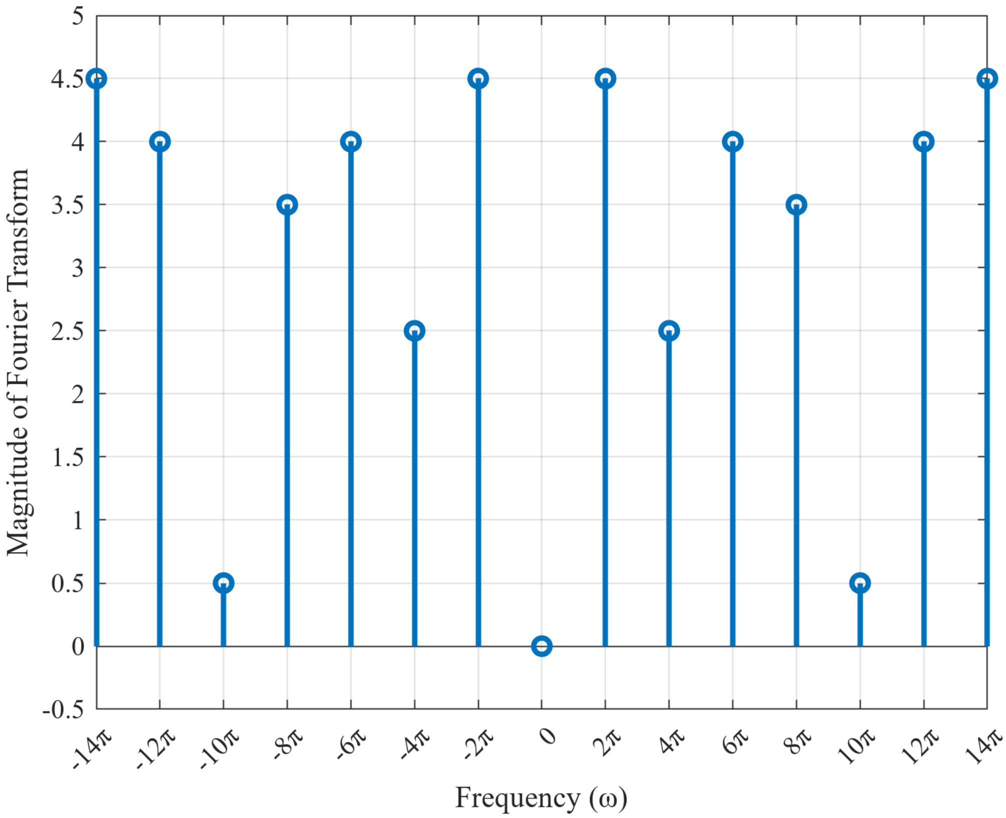

Figure 3: Fourier Transform $F(j\omega)$

The Fourier transform is formulated as following:

$$F(j\omega) = \sum_{m=-7}^{7} F_m \delta(\omega - m\omega_0)$$

where $\delta(\cdot)$ is the Dirac delta function.

Problem 4: DTFT

The Discrete-Time Fourier Transform (DTFT) $F(e^{j\Omega})$ of the 32-point time-domain sequence $f(n)$ is computed and plotted over the interval $(-\pi, \pi)$.

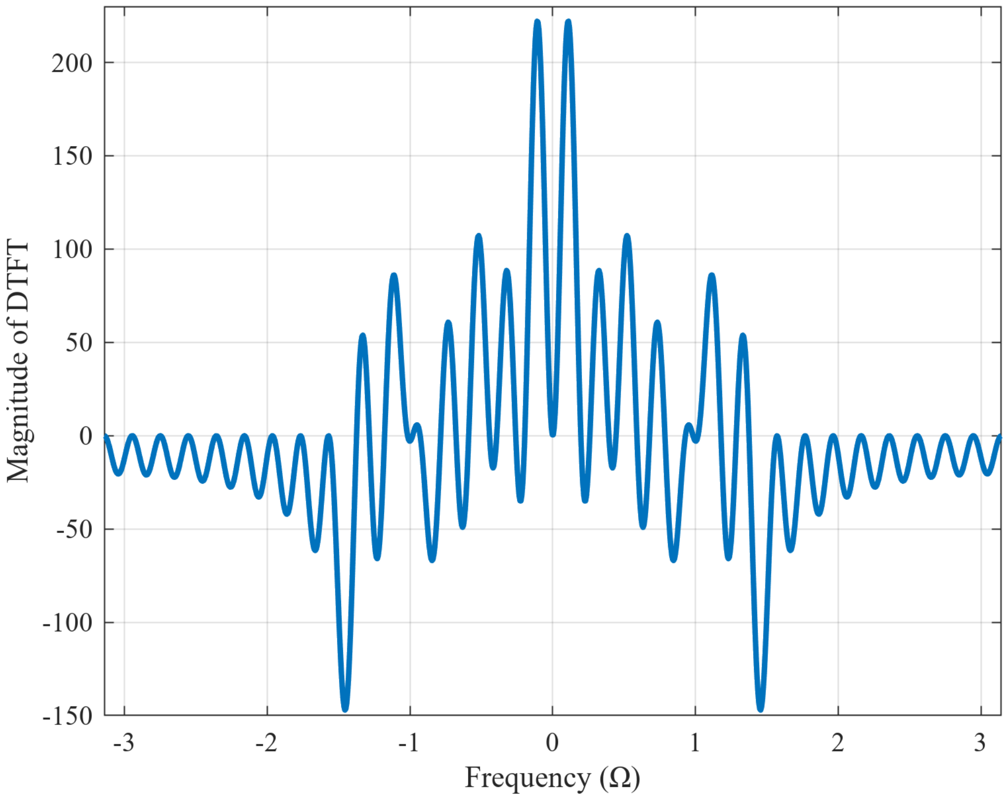

Figure 4: DTFT $F(e^{j\Omega})$

```
Omega = linspace(-pi, pi, 1000); % Frequency range

F_Omega = zeros(size(Omega));
% Compute the DTFT using the Fourier coefficients
for n = 1:length(Omega)
    F_Omega(n) = sum(f_n .* exp(-1j * Omega(n) * (0:31)));
end
```

Listing 3: MATLAB Script to Compute DTFT

Problem 5: DFT

The 32-point DFT of the sampled sequence $f(n)$ is computed and plotted. The amplitudes and locations of the peaks are identified.

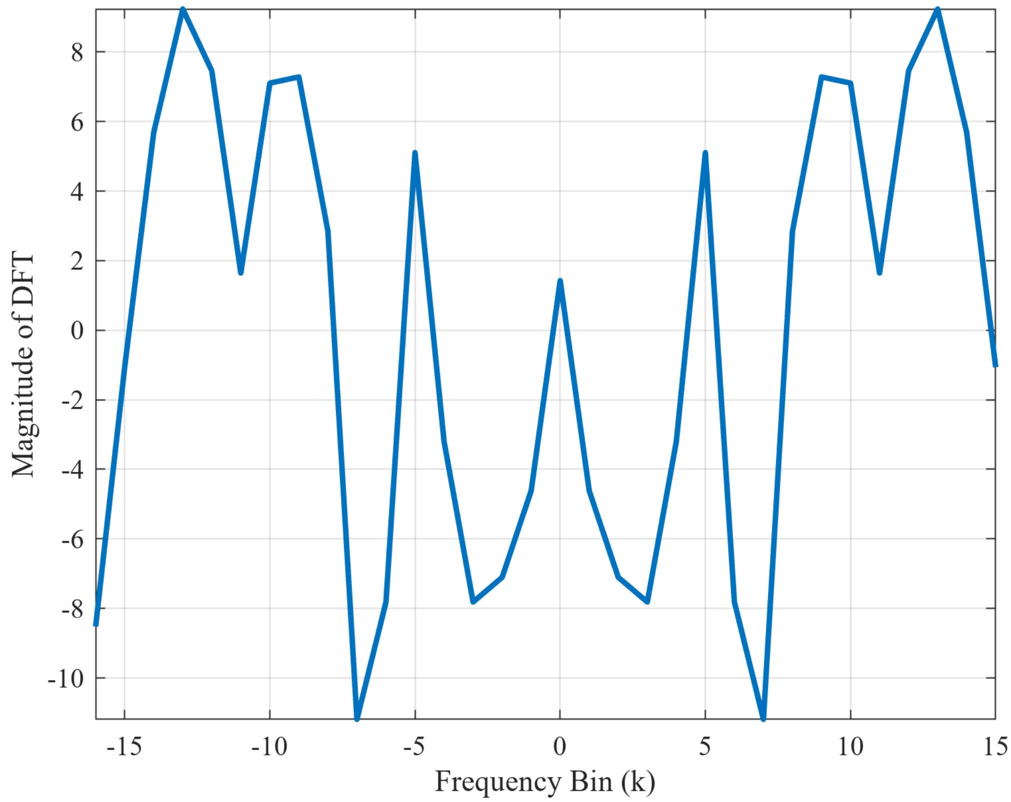

Figure 5: 32-point DFT of $f(n)$

Problem 6: Spectral Analysis

Summary

- The Fourier series expansion provides a representation of the periodic signal $f(t)$ in terms of its harmonic components.
- The Fourier transform $F(j\omega)$ reveals the spectral content of $f(t)$, with peaks corresponding to the harmonic frequencies.
- The DTFT $F(e^{j\Omega})$ and the DFT provide discrete representations of the spectrum, with the DFT being limited to 32 frequency bins.