Intro

In this homework we look at the similarities in structure between fourier series, dtft, and dft in order to see how they are in a lot of ways the same thing.

Q1

```
clearvars, clear, clc

dt = 0.01;
dummyt = -8:dt:8-dt;
t = -24:dt:24-dt;
padded = -5*(dummyt<=8 & dummyt>5) + 3*(dummyt<=5 & dummyt>-5) + -5*(dummyt<=-5);
%padded is input (aperiodic) function
figure, plot(dummyt, padded)
title("input function (aperiodic)")
ax = gca;
ax.Position = [0.13, 0.1, 0.775, 0.55];
fprintf('you can get the fourier series coefficients \nlike below with T = 8 and
w_o = 2pi/8')
```

```
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like below with T = 8 and w_o = 2pi/8
```

```
fprintf('if i was to solve it by hand i would probably\ndifferentiate once to
impulse train')
```

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```
fprintf('then as a sum of impulse trains, c_n = 1/period,\n then *1/jw once and
manually get the dc offset')
```

```
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```
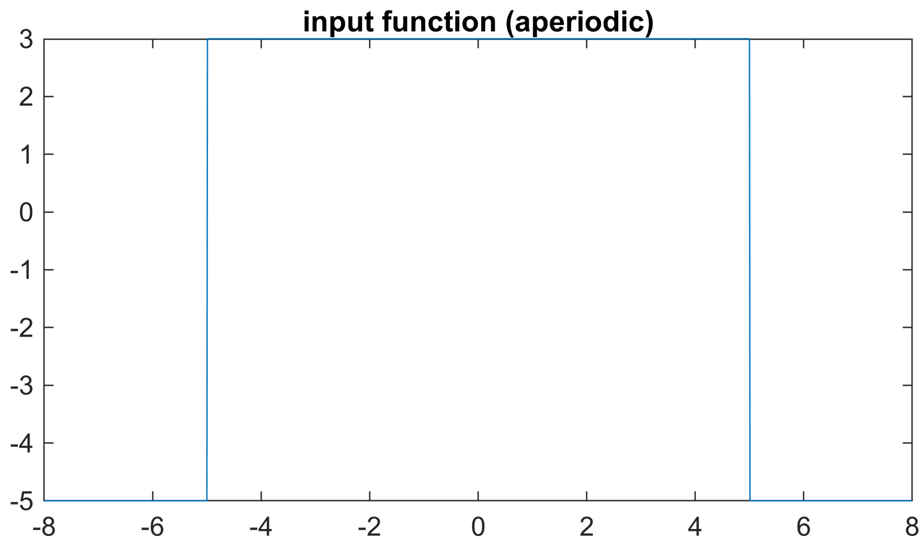
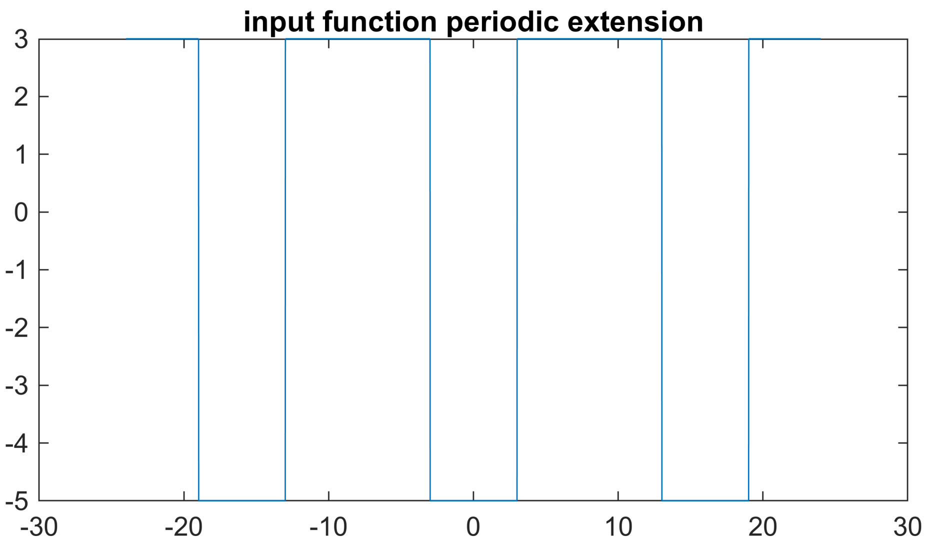

```
fourier_series = '$c_n = \frac{1}{T} \int_{T} f(t) e^{-j n \omega_0 t} \, dt$';
annotation('textbox', [0.1, 0.85, 0.8, 0.1], 'String', fourier_series, ...
           'Interpreter', 'latex', 'FontSize', 14, 'EdgeColor', 'none',
'HorizontalAlignment', 'center');
```

$$c_n = \frac{1}{T} \int_T f(t) e^{-jn\omega_0 t} \, dt$$



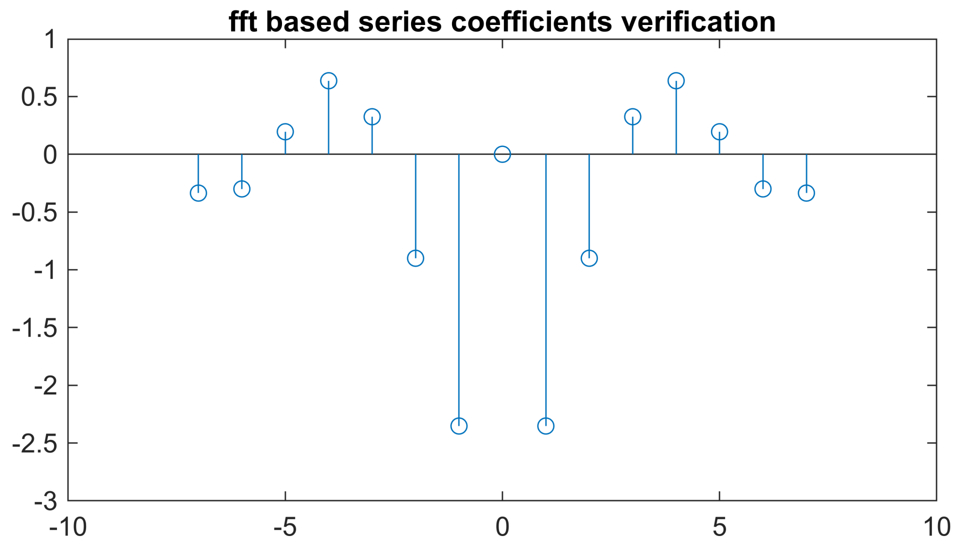

```
N = length(padded);
padded_mod = padded((mod(floor(t/dt), N)) + 1);
%padded mod is periodic extension

plot(t, padded_mod)
title("input function periodic extension")
```



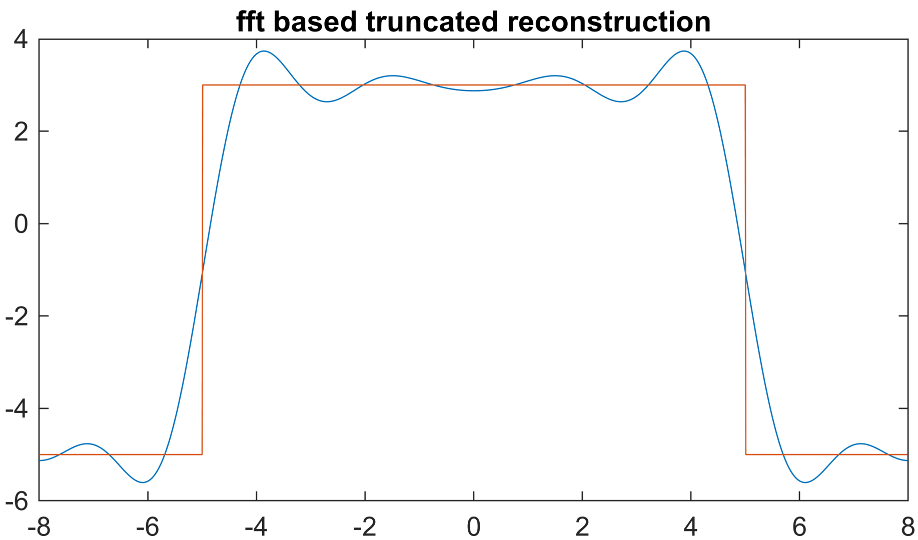

```
K = 7;
%fft verification (just to check work to make sure im on right track)
verif = fftshift((fft(padded)));

n = (-K:K);
fprintf("fft based verification just to make sure")
```

```
fft based verification just to make sure
```

```
stem(n, real((verif(N/2 -K + 1: N/2 + K + 1))*dt/16))
title("fft based series coefficients verification")
```

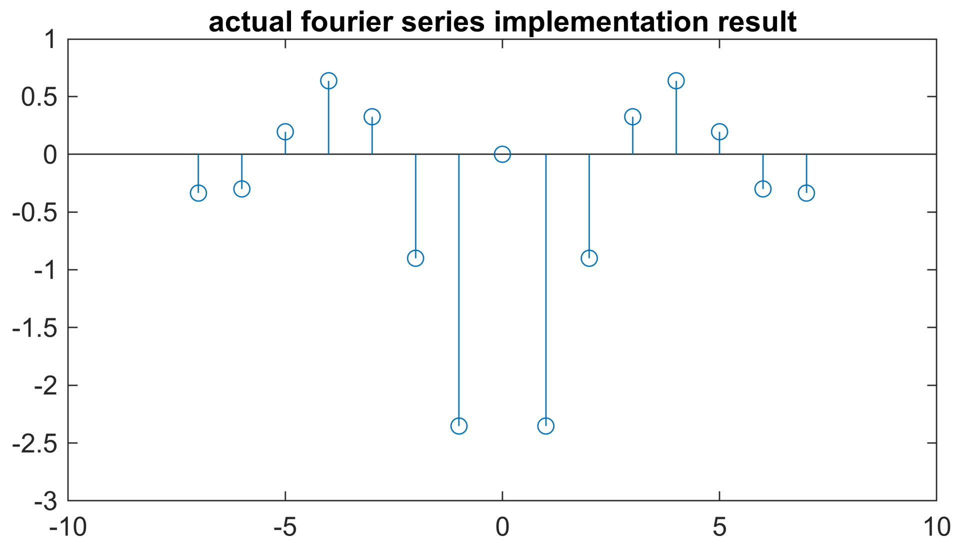



```
%fft reconstruction verification
%center harmonics
verif_padded = zeros([1, N]);
center = (N/2) + 1;
center_indexes = (center-K+1:center+K+1);
verif_padded(center_indexes) = verif(center_indexes);

verif_unshifted = ifftshift(verif_padded);
recon = real(ifft(verif_unshifted));

plot((-8:dt:8-dt), recon)
hold on
plot((-8:dt:8-dt), padded)
title("fft based truncated reconstruction")
hold off
```



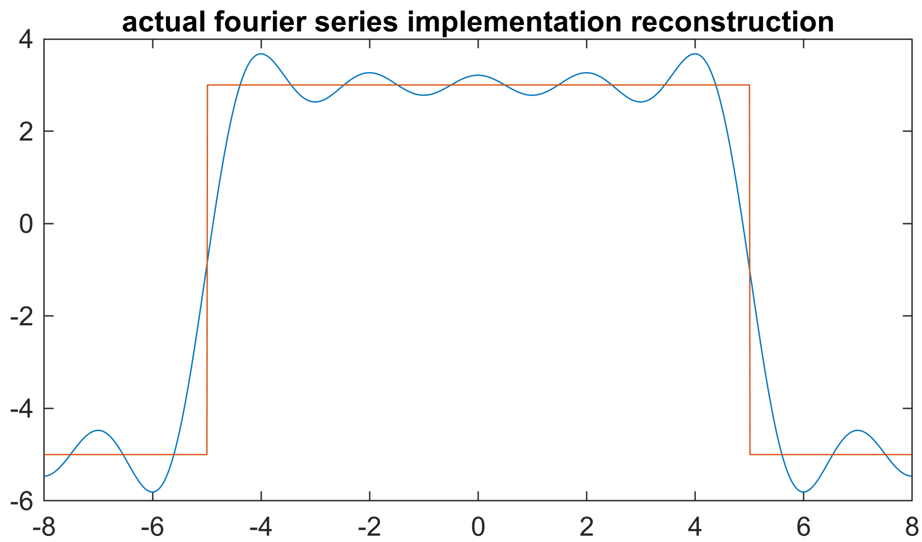
fourier series (implemented with inner products)

```
k = (-N/2: N/2-1);
basis = exp(-1j * 2 * pi / 16 * (dummyt+8)' * k);
%the plus eight is added because it was flipping every other coefficient
modulated = (1/N)*basis' * padded';
ck = modulated;
ck_trunc = (ck(N/2 -K + 1: N/2 + K + 1));
%ck = sum(modulated, 2);
stem((-7:7), real(ck_trunc))
title("actual fourier series implementation result")
```



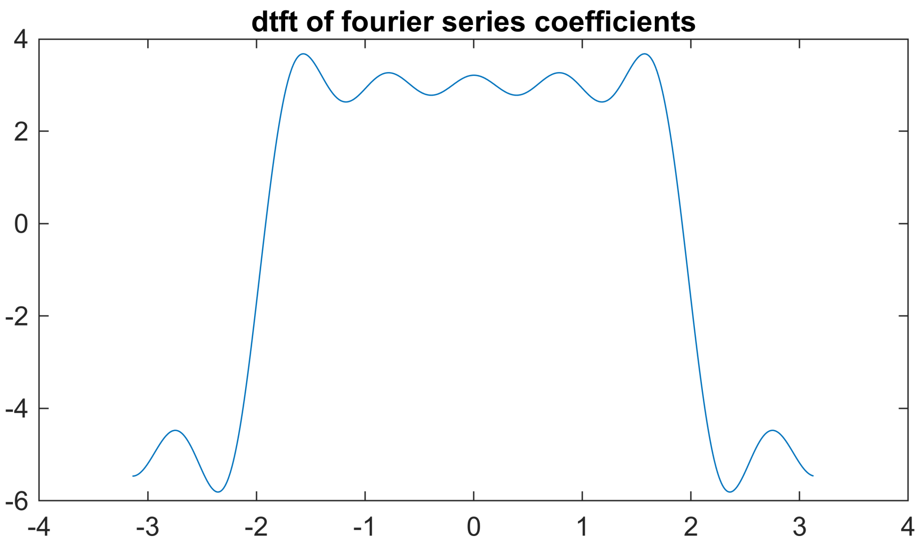
reconstruction (Q2)

```
k = -K:K;
basis = exp(1j * 2 * pi / 16 * (dummyt+8)' * k);
recon = basis * ck_trunc;
plot((dummyt), real(recon))
hold on
plot((dummyt), padded)
title("actual fourier series implementation reconstruction")
hold off
```



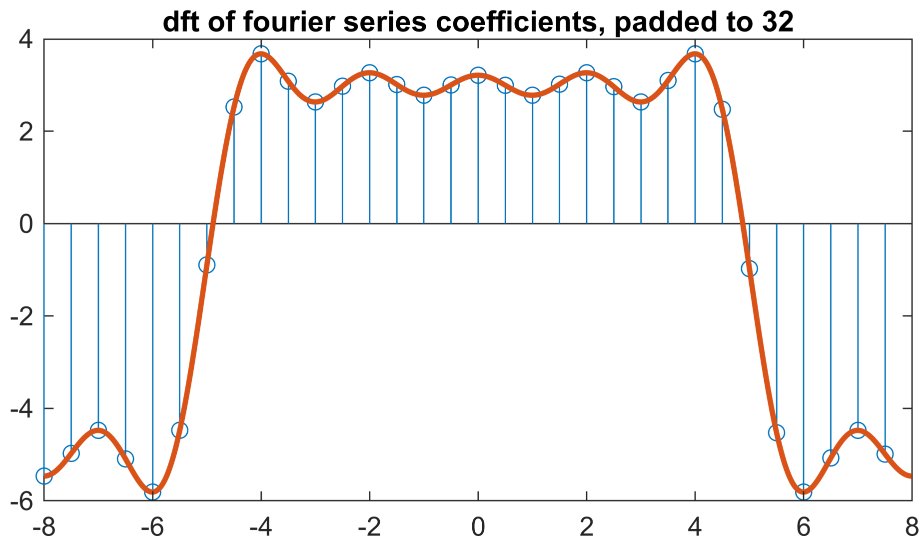
Question 3

```
df = 0.01;
omega = (-pi:df:pi-df);
dtft_basis = exp(-j * (omega+pi)' * k);
dtft = dtft_basis * ck_trunc ;
plot(omega, real(dtft))
title("dtft of fourier series coefficients")
```



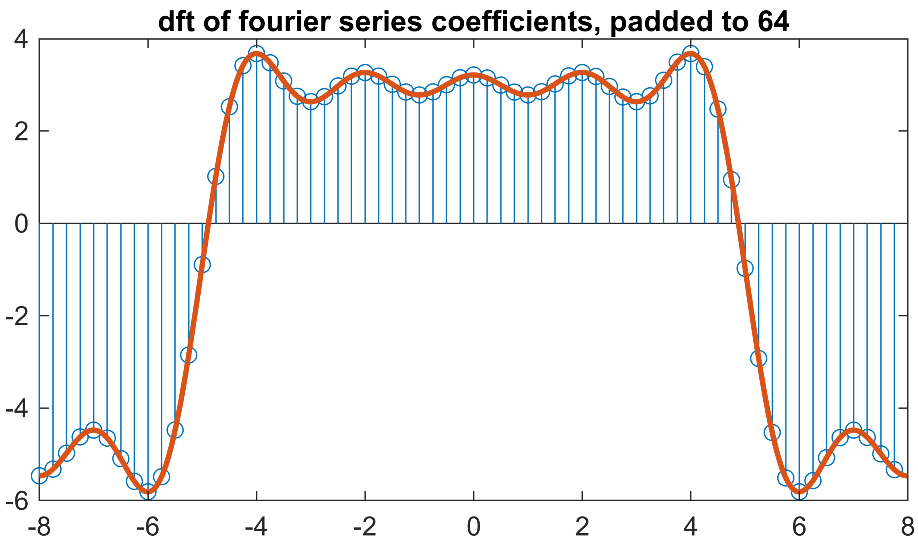
Question 4

```
thirtytwo = zeros([1, 32]);
n = (-16:15);
k = (-16:15);
center_index = 17;
center = center_index-K:center_index+K;
thirtytwo(center)=ck_trunc;
dft_basis = exp(-j * 2*pi/32 * (n)' * k);
output = dft_basis * thirtytwo';
stem((n/2.0), fftshift(real(output)))  %fftshift is used not fft
hold on
plot(dummyt, real(recon), 'LineWidth',2)
hold off
%n/2 because we zero padded signal by 2 and want it to match up
title("dft of fourier series coefficients, padded to 32")
```



Question 5

```
sixtyfour = zeros([1, 64]);
n = (-32:31);
k = (-32:31);
center_index = 33;
center = center_index-K:center_index+K;
sixtyfour(center)=ck_trunc;
dft_basis = exp(-j * 2*pi/64 * (n)' * k);
output = dft_basis * sixtyfour';
stem((n/4.0), fftshift(real(output)))
hold on
plot(dummyt, real(recon), 'LineWidth',2)
hold off
%stem(real(fft(fftshift(sixtyfour))))
title("dft of fourier series coefficients, padded to 64")
```



Question 6

Problem 2, when reconstructing the signal it gave a similar looking traincar back but it was wigglier on account of truncating some harmonics.

Plotting the dtft of the harmonics ended up giving the same result as reconstructing the function from the fourier series coefficients.

DFT also seemed to give a sampled version of that function back. Not the original function but a sampled version of the wiggly function. And the zero padding of the DFT determined how resolved the wiggly function was.

It seems like the DTFT is basically the same as reconstructing a fourier series from its coefficients. (like if fourier series is cts time->discrete freq, dtft is (in this case) the reverse, discrete freq -> cts time) And the DFT is effectively sampling the DTFT. It does get stretched out though on account of having more points and each point being separated by 1 unit.

Conclusion:

In this homework assignment we found how master salesman Joseph Fourier is a master salesman who packaged one analysis technique into four (3 in this assignment) different analysis techniques.