ECE 148 **Signal Processing Applications** **Spring 2025**

Assignment 5: Spectrum Estimation

Due: Tuesday, May 13

The objective of this exercise is to establish the frequency-mapping correspondences to perform spectral estimation of continuous-time signals with *DFT* and finite time-domain data samples.

Background:

Consider a real and periodic signal $f(t)$, of period T. The signal has 7 harmonics, for $m = 1, 2, ..7$.

$$f(t) = \sum_{m=1}^{7} a_m \sin(m\omega_0 t)$$

where $\omega_o = 2\pi/T$, and the 7 Fourier coefficients $\{a_m\}$ are formed with your personal *7-digit UCSB perm number*.

Then we take 32 uniform samples within one period with sample spacing $\Delta t = T/32$ to form a short 32-point sequence $\{f(n)\}$, where $n = 0, 1, 2, \ldots 31$.

Subsequently, to observe the spectrum of the function $f(t)$, we take a 32-point *DFT* of the sequence to obtain the 32-point spectral sequence $F(k)$, where $k = 0, 1, 2, \ldots 31$.

$$F(k) = DFT_{N=32}\{f(n)\}$$

Questions:

1. *Time function and sampling:* Plot one full period of the time function $f(t)$, from $t = 0$ to $t = T$, and list the 32-point sample sequence $\{f(n)\}$.

2. *Fourier series expansion:* Formulate the periodic signal $f(t)$ in the form of complex Fourier series expansion. List and sketch the Fourier coefficients $\{F_m\}$ as a sequence of m. And then identify the corresponding physical frequencies.

3. *Fourier transform of periodic functions:* Determine and sketch the Fourier transform $F(j\omega)$ of the periodic function $f(t)$. Identify the amplitudes and physical frequencies of the peaks.

4. *DTFT:* Plot the *DTFT* $F(e^{j\Omega})$ of the 32-point time-domain sequence $f(n)$ over the interval $(-\pi, +\pi)$. Identify the amplitudes and locations of the peaks.

5. *DFT:* Compute and plot the 32-point *DFT* spectral sequence $F(k)$. Identify the amplitudes and locations of the peaks.

6. *Spectral analysis:* Given the background information (a) T, the period of the original periodic function, (b) sampling spacing $T/32$, (c) and *DFT* length $N=32$, formulate the correspondences to map the integer index k in the *DFT* spectrum $F(k)$ back to the original physical frequency index ω in $F(j\omega)$ in order to identify the harmonics (including both amplitude and frequency) of the original periodic waveform $f(t)$.

   (Hints: Each component of the Fourier series expansion of the periodic function $f(t)$ consists of a complex amplitude and its corresponding frequency. Components of the series show in the form of spectral peaks over the Fourier spectrum $F(j\omega)$ over the interval $(-\infty, +\infty)$ in Problem 3. These spectral peaks also show up in the *DTFT* spectrum $F(e^{j\Omega})$ over the interval $(-\pi, +\pi)$ as indicated by your result from Problem 4. Again, in Problem 5, these spectral peaks show up in the *DFT* spectrum $F(k)$ over the interval $(0, N-1)$, where $N = 32$. The most important task is to formulate the correspondences of the frequency index systems of (a) Fourier transform $F(j\omega)$, (b) *DTFT* $F(e^{j\Omega})$, and (c) *DFT* $F(k)$.)

   Fourier transformation: $F(j\omega) = \int_{-\infty}^{\infty} f(t)\, exp(-j\omega t)\, dt$

   Fourier series expansion: $f(t) = \sum_{m=-\infty}^{\infty} F_m\ exp(jm\omega_0 t)$

   where $F_m = \frac{1}{T}\int_0^T f(t)\, exp(-jm\omega_0 t)\, dt$

   DTFT (Discrete-time Fourier transform): $F(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} f(n)\, exp(-jn\Omega)$

   DFT (Discrete Fourier transform): $F(k) = \sum_{n=0}^{N-1} f(n)\ exp(-j2\pi nk/N)$