ECE 148 **Signal Processing Applications** **Spring 2025**

Assignment 1: Signal Sampling and Aliasing (Part A)

Due: Tuesday, April 15

1. (Orthogonal decomposition)

   The relationship between the lowpass time-domain signal $f(t)$ and the samples $\{f(n\Delta t)\}$ can be described as a transformation pair. These two versions represent the same information content. The basis functions of the transformation are

$$\psi_n(t) = sinc\left(\frac{\omega_0(t-n\Delta t)}{2}\right) \qquad (1)$$

   where Δt is the sample spacing. The forward conversion from the time function $f(t)$ to the sample sequence $\{f(n\Delta t)\}$ is conducted in the form of an inner product

$$f(n\Delta t) = < f(t),\ \psi_n(t) > = \int_{-\infty}^{\infty} f(t)\, \psi_n{}^*(t)\, dt \qquad (2)$$

   which is equivalent to the sampling process

$$f(t) \cdot s(t) = \sum_{n=-\infty}^{\infty} f(n\Delta t)\, \delta(t - n\Delta t) \qquad (3)$$

   And the inverse transformation (reconstruction process) from the sequence $\{f(n\Delta t)\}$ back to $f(t)$ is in the form of another inner product

$$f(t) = < f(n\Delta t),\ \psi_n(t) > = \sum_{n=-\infty}^{\infty} f(n\Delta t)\, \psi_n{}^*(t) \qquad (4)$$

   (a) Show $\{\psi_n(t)\}$ are orthogonal basis functions of the transformation.
   (b) Show *Eqs. (2)* and *(4)* are valid transformations of the orthogonal conversion procedure.

2. (Application of aliasing, time-domain partition)

   The time-domain function $f(t)$ is a lowpass signal. It can be partitioned into N sub-components.

$$f(t) = \sum_{n=1}^{N} f_n(t)$$

   The frequency spectra of $f(t)$ and all sub-components $f_n(t)$ span from $-\omega_o/2$ to $+\omega_o/2$ for a total bandwidth of

$$B = \omega_0$$

   Suppose we modulate each of the N sub-components with an extra single-frequency term $exp(jk_n\omega_0 t)$ to form another function

$$\hat{f}(t) = \sum_{n=1}^{N} f_n(t)\, exp(jk_n\omega_0 t)$$

   where k_n are arbitrary integers. Determine the result, if you sample $\hat{f}(t)$ with sample spacing Δt and then lowpass the sampled function through a lowpass filter with cutoff frequencies at $\pm\,\omega_0/2$.

3. (Application of aliasing, frequency-domain partition)

   The function $g(t)$ a lowpass periodic function. It can be represented in the form of Fourier series expansion, consisting of $2N+1$ terms, where ω_x is the fundamental frequency of the expansion and G_n are the Fourier coefficients.

$$g(t) = \sum_{n=-N}^{N} G_n\, exp(jn\omega_x t)$$

   The frequency spectrum spans from $-N\omega_x$ to $+N\omega_x$ for a total bandwidth of

$$B = 2N\omega_x < \omega_0$$

   Suppose we modulate each of the $2N+1$ components with an extra single-frequency term $exp(jk_n\omega_0 t)$ to form another function

$$\hat{g}(t) = \sum_{n=-N}^{N} G_n\, exp(jn\omega_x t)\, exp(jk_n\omega_0 t) = \sum_{n=-N}^{N} G_n\, exp(j(n\omega_x + k_n\omega_0)t)$$

   where k_n are arbitrary integers.

   (a) Sketch the Fourier spectrum of $g(t)$.
   (b) Describe the Fourier spectrum of $\hat{g}(t)$.
   (c) Determine the result, if you sample $\hat{g}(t)$ with sample spacing Δt and then lowpass the sampled function through a lowpass filter with cutoff frequencies at $\pm\,\omega_0/2$.

4. (Summar)

Provide a brief summary outlining the basic concept and techniques based on your analysis of *Problems 2* and *3*.