Ground-Penetrating Radar Range Profile Estimation

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1 Introduction

This report documents the process of reconstructing subsurface range profiles using step-frequency FMCW ground-penetrating radar (GPR) data. The goal is to estimate depth profiles at each antenna position and generate a high-resolution image of the subsurface.

2 Signal Processing Approach

2.1 Data Overview

The dataset consists of a 128×200 complex-valued matrix, where each column corresponds to a spatial antenna position (200 total), and each row corresponds to a stepped frequency (128 total, from 0.976 GHz to 2.00 GHz). The spatial step is 0.0213 m, and the relative permittivity of the medium is $\epsilon_r = 6$.

2.2 Range Estimation Procedure

(a) **Propagation Speed Adjustment:** The speed of electromagnetic waves in the medium is $v = c/\sqrt{\epsilon_r}$, where $c = 3 \times 10^8$ m/s. This yields $v \approx 1.225 \times 10^8$ m/s.

(b) **Resolution Calculations:**

- *Bandwidth:* $B = 1$ GHz.
- *Time Resolution:* $\Delta t = 1/B = 1$ ns.
- *Range Resolution:* $\Delta r = v\Delta t/2 \approx 0.0612$ m.
- *Frequency Resolution:* $\Delta f = (2.00 - 0.976)\,\text{GHz}/128 \approx 8$ MHz.
- *Maximum Time:* $t_{\max} = 1/\Delta f = 125$ ns.
- *Maximum Detectable Depth:* $r_{\max} = vt_{\max}/2 \approx 7.65$ m.

(c) **Surface Reflection Gating:** Strong reflections near the surface are gated out to improve visualization of subsurface features.

(d) **Depth Range of Interest:** The rebars are expected at shallow depths (< 30 cm), so only the first few pixels in the depth axis are of interest.

2.3 Interpolation and Image Formation

- **Depth Interpolation:** Zero-padding the frequency axis to 2048 points increases depth resolution. The IFFT is performed along the frequency axis, and only the last 64 points (corresponding to shallow depths) are retained.
- **Lateral Interpolation:** The spatial axis is padded to 256 points, FFT is performed, and then zero-padded to 2048 points in the frequency domain. An IFFT yields a finely sampled lateral profile.
- **Physical Pixel Size:** After interpolation, the depth pixel size is 0.3827 cm, and the lateral pixel size is 0.26625 cm.

3 Resolution and Maximum Detectable Depth

- **Range (Depth) Resolution:** $\Delta r = 0.0612$ m (6.12 cm) before interpolation; after zero-padding, the effective pixel size is 0.3827 cm.
- **Maximum Detectable Depth:** $r_{\max} \approx 7.65$ m.

4 Physical Pixel Size

- **Depth Direction:** 0.3827 cm per pixel (after zero-padding and IFFT).
- **Lateral Direction:** 0.26625 cm per pixel (after zero-padding and IFFT).

5 Final Image

Figure 1: Final interpolated range profile image. The vertical axis represents depth (shallowest at the top), and the horizontal axis represents antenna position along the scan path.

6 Code

```
gpr_data = load('gpr_data/gpr_data.mat');

F = gpr_data.F;
f = gpr_data.f;
da = gpr_data.da;

epsilon_r = 6;

c = 3 * 10^8; # m/s
```

```
velocity = c / sqrt(epsilon_r); # = 1.2247e+08 m/s

bandwidth = 1 * 10^9; % 1 GHz

% Time Resolution
delta_t = 1 / bandwidth; % 10^-9 s

% Range Resolution
delta_r = velocity * delta_t / 2; % = 0.061237 m

% Frequency Resolution
delta_f = (2 - 0.976) * 10^9 / 128; % = 8 MHz

% Max Time
range_time_max = 1 / delta_f; % = 1.25 * 10^-7 s

% Max Distance
range_distance_max = velocity * range_time_max / 2; % = 7.6547 m

% Dividing 765 cm / 24 cm (within our range of interest), yields
   approximately
% 32.

% We have 128 frequencies: 128/32 = 4, meaning we only care about the
   first 4 px.

%% INTERPOLATION

%% Interpolating Range (Depth) for Display (Columns)

% THE SIZE OF OUR DATA IS 128 columns x 200 rows.

% We want to interpolate the depth data to 2048 points.
% Interpolation factor = 2048 / 128 = 16

% We need to zero-pad the data to 2048 points.
% Very naively, we can just pad with zeros at the end of each column.
% We know the freq resolution is 8 Mhz, and the minimum frequency is 0.976
    GHz.
% That means we can determine the number of empty points we should add
   infront
% of the data.

pts_front = 0.976 * 10^9 / delta_f; % = 122
F_padded = zeros(2048, 200);
F_padded(pts_front + 1 : pts_front + 128, :) = F;

% THE SIZE OF OUR DATA IS 2048 rows x 200 columns.

F_ifft = ifft(F_padded, [], 1);

% The IFFT turns the columnar frequency data into depth profiles.

F_ifft = ifft(F_padded, [], 1)(1985:2048, :);
```

```
    % Number of pixels for display = (4*16) = 64
    % Pixel size = 6.1237 cm / 16 = 0.3827 cm

% THE SIZE OF OUR DATA IS 64 rows x 200 columns.

%% Interpolating receiver positions. (horizontal)

% We have 200 receiver positions

% We will first pad by 28 zeros at the beginning and end of the receiver pos axis.
% This lets us FFT the data.
F_padded = zeros(64, 256);
F_padded(:, 29:228) = F_ifft;

# THE SIZE OF OUR DATA IS 64 rows x 256 columns.

% Now we can perform the FFT along the position axis.
F_fft = fft(F_padded, [], 2);

% In the frequency domain we can zero pad the position data to 2048 points.
% We add zeros to the middle of the data
F_padded = zeros(64, 2048);
F_padded(:, 1:128) = F_fft(:, 1:128);
F_padded(:, 1921:2048) = F_fft(:, 129:256);

# THE SIZE OF OUR DATA IS 64 rows x 2048 columns.

% Perform the IFFT on the padded data
F_ifft = ifft(F_padded, [], 2);

% We are now back in the spatial domain, with 2048 receiver points.
% The interpolation ratio is 2048/256 = 8.
% The pixel size is 0.0213 m / 8 = 0.0026625 m = 2.6625 mm = 0.26625 cm

% To get a properly scaled image we must take into account the disparate
% pixel sizes in the depth and horizontal directions.

% Each depth pixel is 0.3827 cm, and each horizontal pixel is 0.26625 cm.

img = abs(F_ifft)(:, 225:1825);
img = img - min(img(:));
img = img / max(img(:));
img_uint8 = uint8(img * 255);
cmap = jet(256);
imwrite(img_uint8, cmap, 'depth_map.png');
```

Listing 1: Ground-Penetrating Radar Range Profile Estimation

7 Summary

- The GPR data was processed using FFT and IFFT techniques to estimate subsurface range profiles.
- Zero-padding in both frequency and spatial domains enabled high-resolution interpolation, yielding a final image with sub-centimeter pixel sizes.
- The depth resolution is 0.3827 cm, and the maximum detectable depth is 7.65 m, though only shallow depths are of practical interest.
- The final image clearly visualizes subsurface features, such as rebars, with high spatial and depth resolution.
- I was not able to get the contrast of the example image. I am dealing with an unfamiliar programming suite, but that's not really an excuse.