Vectors - 20 Questions: Step-by-step Solutions

1. Define a vector and give two real-life examples.

Solution: Vector ek quantity hoti hai jisme magnitude (size) aur direction dono hote hain. Examples: 5 km North (magnitude=5 km, direction=North), 10 m/s East.

2. Find the magnitude of vector a = (6, 8).

Solution: $|a| = \text{sqrt}(6^2 + 8^2) = \text{sqrt}(36 + 64) = \text{sqrt}(100) = 10$.

3. Find the direction of vector v = (3, 3).

Solution: Direction angle $\theta = \arctan(y/x) = \arctan(3/3) = \arctan(1) = 45°$.

4. Add vectors a = (2, 5) and b = (-1, 4).

Solution: a + b = (2 + (-1), 5 + 4) = (1, 9).

5. Subtract vector b = (7, 2) from a = (10, 5).

Solution: a - b = (10 - 7, 5 - 2) = (3, 3).

6. Find the unit vector of a = (4, 2).

Solution: $|a| = \text{sqrt}(4^2 + 2^2) = \text{sqrt}(16 + 4) = \text{sqrt}(20) = 2*\text{sqrt}(5)$. Unit vector = a/|a| = $(4,2)/(2\sqrt{5}) = (2/\sqrt{5}, 1/\sqrt{5})$.

7. Check if vectors (2, 4) and (1, 2) are parallel.

Solution: If one is scalar multiple of other. (2,4) = 2*(1,2). Therefore they are parallel.

8. If vector a = (3, 4), find 5a.

Solution: 5a = 5*(3,4) = (15, 20).

9. Find dot product of a = (1, 2, 3) and b = (4, 5, 6).

Solution: $a \cdot b = 1*4 + 2*5 + 3*6 = 4 + 10 + 18 = 32$.

10. Find angle between a = (1, 0) and b = (0, 1).

Solution: $a \cdot b = 1*0 + 0*1 = 0$. |a|=1, |b|=1. $\cos\theta = 0/(1*1) = 0 \rightarrow \theta = 90°$. They are perpendicular.

11. For vectors a = (2, 3) and b = (-8, -12), check if they point in opposite directions.

Solution: Check if b = k*a with k negative. b = -4*(2,3) = (-8,-12). k = -4 (negative) → they point in opposite directions.

12. Find projection of a = (3, 4) on b = (1, 0).

Solution: Projection formula: $\text{proj}_b(a) = (a \cdot b / |b|^2) * b$. $a \cdot b = 3*1 + 4*0 = 3$. $|b|^2 = 1$. So proj = 3*(1,0) = (3,0).

13. If |a| = 7 and |b| = 5 and angle is 60°, find a·b.

Solution: $a \cdot b = |a||b|\cos\theta = 7*5*\cos 60° = 35*(1/2) = 17.5$.

14. Find vector AB if A(4, 2) and B(10, 9).

Solution: AB = B - A = (10 - 4, 9 - 2) = (6, 7).

15. Determine if vectors (1, 3, 4) and (-3, -9, -12) are collinear.

Solution: Check scalar multiple: (-3, -9, -12) = -3*(1,3,4) → yes collinear (k = -3).

16. Find cross product of vectors a = (1, 2, 3) and b = (4, 5, 6).

Solution: a×b = | i j k ; 1 2 3 ; 4 5 6 | = i*(2*6 - 3*5) - j*(1*6 - 3*4) + k*(1*5 - 2*4) = i*(12 -15) - j*(6 -12) + k*(5 -8) = i*(-3) - j*(-6) + k*(-3) = (-3, 6, -3).

17. Find area of parallelogram formed by a = (2, 1, -1) and b = (1, -1, 2).

Solution: Area = |a×b|. First compute cross product: a×b = | i j k ; 2 1 -1 ; 1 -1 2 | = i*(1*2 - (-1)*(-1)) - j*(2*2 - (-1)*1) + k*(2*(-1) - 1*1) = i*(2 -1) - j*(4 -(-1)) + k*(-2 -1) = i*1 - j*5 + k*(-3) = (1, -5, -3). Magnitude = sqrt(1^2 + (-5)^2 + (-3)^2) = sqrt(1 +25 +9) = sqrt(35). So area = √35.

18. Compute scalar triple product of a = (1, 0, 1), b = (2, 1, 0), c = (3, -1, 2).

Solution: Scalar triple = a·(b×c). First b×c = | i j k ; 2 1 0 ; 3 -1 2 | = i*(1*2 - 0*(-1)) - j*(2*2 - 0*3) + k*(2*(-1) - 1*3) = i*(2 - 0) - j*(4 - 0) + k*(-2 -3) = (2, -4, -5). Now a·(b×c) = (1,0,1)·(2,-4,-5) = 1*2 + 0*(-4) + 1*(-5) = 2 -5 = -3.

19. If a = (1, 2), b = (2, k) are perpendicular, find k.

Solution: Perpendicular ⇒ a·b = 0. 1*2 + 2*k = 0 ⇒ 2 + 2k = 0 ⇒ k = -1.

20. Find unit vector perpendicular to vectors a = (1, 1, 0) and b = (0, 1, 1).

Solution: Vector perpendicular = a×b. a×b = | i j k ; 1 1 0 ; 0 1 1 | = i*(1*1 - 0*1) - j*(1*1 - 0*0) + k*(1*1 - 1*0) = i*(1) - j*(1) + k*(1) = (1, -1, 1). Magnitude = sqrt(1 +1 +1) = sqrt(3). Unit vector = (1/√3, -1/√3, 1/√3).

Note: Yeh solutions step-by-step diye gaye hain. Agar aap chaho to har question ka separate detailed PDF ya Python code ke sath bhi bana dunga.