



Lehrstuhl für Informatik 1  
Friedrich-Alexander-Universität  
Erlangen-Nürnberg



## MASTER THESIS

# **The validity of current contact force models for the collision of viscoelastic spheres**

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Erlangen, August 5, 2016

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## **Abstract**

An important model for granular particles are elastic and viscoelastic spheres. The macroscopic interaction forces for such objects are commonly obtained from the continuum mechanical equations of motion for elastic and viscoelastic material in quasi static approximation. The same holds true for the coefficients of restitution of colliding spheres which are, in turn, obtained from the macroscopic interaction forces. The quasi static assumption implies that the characteristic deformation rate is much smaller than the speed of sound in the material and that the relaxation time of the particle's material is negligible compared to the duration of the contact. In this work the validity of these assumptions is probed for realistic impact scenarios by comparing to a direct numerical solution of the underlying continuum mechanical equations of motion by means of finite elements.



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# INTRODUCTION

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Some intro

## 1.1 Granulates

A granular material is a collection of distinct macroscopic particles, such as sand in an hourglass or peanuts in a container. The evolution of the particles follows Newton's equations, with repulsive forces between particles that are non-zero only when there is a contact between particles. Although granular materials are very simple to describe they exhibit a tremendous amount of complex behavior, much of which has not yet been satisfactorily explained. They behave differently than solids, liquids, and gases which has led many to characterize granular materials as a new form of matter.

## 1.2 Particle Simulations of Granulates

## 1.3 Particle Models

## 1.4 Aims

## 1.5 Acknowledgments

A big thank you for the support to Dr.Patric Mueller



## BACKGROUND

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## SIMULATION METHOD

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### 3.1 FEM

### 3.2 Simulation Setup

#### 3.2.1 Sphere vs Rigid Plane

In this simulation the collision of two viscoelastic spheres is studied. Both the spheres have the same magnitude of velocity but opposite directions. Therefore to simplify the model and computation, instead of simulation two spheres colliding, a single sphere colliding against a rigid plane can be simulated. This setup would be equivalent to the original problem as both the spheres are the same in all aspects except for having different directions of velocities.

#### 3.2.2 Symmetry

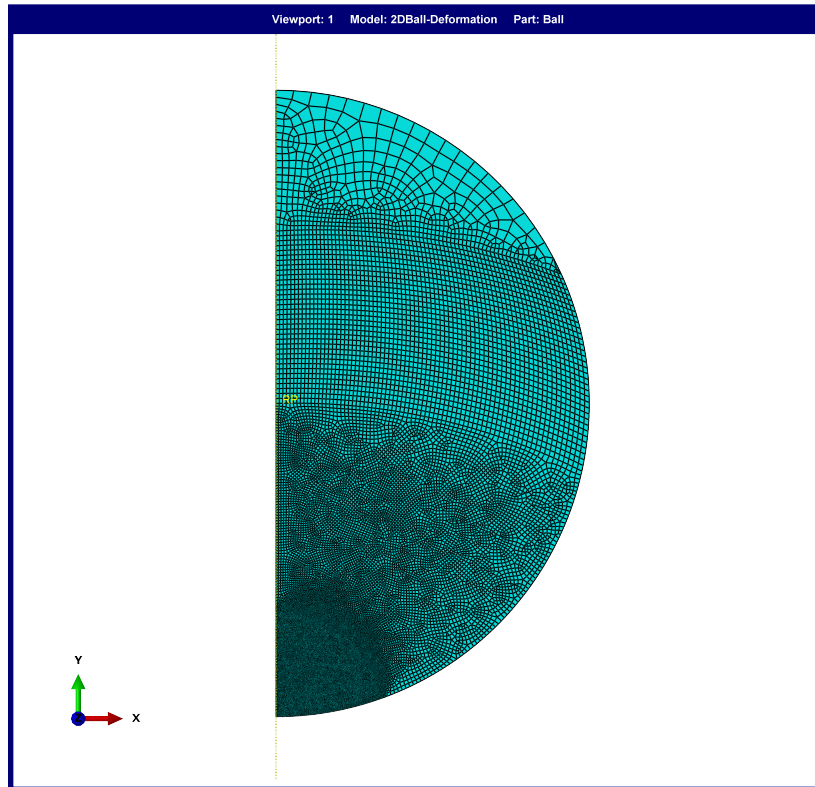
To further simplify the model, instead of considering the complete sphere, only a 2D semi-circular cross-section is considered. As the spheres are symmetric about the central rotational axis and the angle of contact is 90 degrees, there would not be any velocity in the Y direction.

#### 3.2.3 Mesh

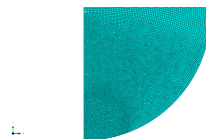
The 3.1 shows the mesh used in the simulation. We can see that the mesh is finer at the bottom of the sphere as it is the point of contact and would undergo massive deformation during the impact.

The 3.2 shows an enlarged image of the bottom half of the mesh. We can see the degree of fineness of the mesh compared to the other half of the sphere.





**Figure 3.1:** Mesh



**Figure 3.2:** Bottom Half of Mesh

### 3.2.4 Rigid Plane

The figure shows the rigid plane against which the sphere would be colliding.

## 3.3 Measurement Quantities

### 3.3.1 Displacement

To measure the displacement of the body, the displacement of the center of mass was measured. The center of mass of the of a sphere is located at the sphere. The figure shows the center of the 2D semi-circle which was considered as the sphere of the model. To measure the displacement center of mass of the sphere, the displacement of center of the semicircle was considered.

### 3.3.2 Kinetic Energy

The kinetic energy of an object is the energy that it possesses due to its motion. As the aim of the thesis is to measure the co-efficient of restitution, which is directly related to the restitution kinetic energy in the body after the collision is completed.

$$KineticEnergy(KE) = \frac{mv^2}{2} \quad (3.1)$$

where  $m$  is the mass of the object and  $v$  is the velocity of the object.

### 3.3.3 Strain Energy

The strain energy is the energy stored by a system undergoing deformation. During a collision, a part of the kinetic energy is converted into strain energy. This strain energy can also be observed as vibration on the body of the object. When the load is removed,

$$StrainEnergy(U) = \frac{V\sigma\epsilon}{2} \quad (3.2)$$

where  $V$  is volume,  $\sigma$  is stress and  $\epsilon$  is strain.

### 3.3.4 Co-efficient of Restitution

The co-efficient of restitution describes the energy transfer

## 3.4 Verification



## RESULTS

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## CONCLUSION AND FUTURE WORK

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## BIBLIOGRAPHY

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