Lehrstuhl für Informatik 1
Friedrich-Alexander-Universität
Erlangen-Nürnberg

MASTER THESIS

The validity of current contact force models for the collision of viscoelastic spheres

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Erlangen, June 27, 2016

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Abstract

An important model for granular particles are elastic and viscoelastic spheres. The macroscopic interaction forces for such objects are commonly obtained from the continuum mechanical equations of motion for elastic and viscoelastic material in quasi static approximation. The same holds true for the coefficients of restitution of colliding spheres which are, in turn, obtained from the macroscopic interaction forces. The quasi static assumption implies that the characteristic deformation rate is much smaller than the speed of sound in the material and that the relaxation time of the particle's material is negligible compared to the duration of the contact. In this work the validity of these assumptions is probed for realistic impact scenarious by comparing to a direct numerical solution of the underlying continuum mechanical equations of motion by means of finite elements.

List of Tables

List of Figures

CONTENTS

1 INTRODUCTION

Some intro

1.1 Granulates

Some stuff about granulates

1.2 Particle Simulations of Granulates

1.3 Particle Models

1.4 Aims

1.5 Acknowledgments

A big thank you for the support to Dr.Patric Mueller

2 Background

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Figure 2.1: Some sample caption

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A more complex figure is shown in Figure 2.3. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

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As an example of a complex enumeration, here is the kernel tree of Linux:

- **`arch/x86`**: x86_32 and x86_64 specific source code

$$
\begin{aligned}
\hat{B}_0 &:= IP(P) \\
\hat{B}_{i+1} &:= R_i(\hat{B}_i) \\
C &:= FP(\hat{B}_{32})
\end{aligned}
$$

where

$$
\begin{aligned}
R_i(X) &= L(\hat{S}_i(X \oplus \hat{K}_i)) & i &= 0, \ldots, 30 \\
R_i(X) &= \hat{S}_i(X \oplus \hat{K}_i) \oplus \hat{K}_{32} & i &= 31
\end{aligned}
$$

Figure 2.2: Left part of a complex figure

$$
\begin{aligned}
X_0, X_1, X_2, X_3 &:= \hat{S}_i(\hat{B}_i \oplus \hat{K}_i) \\
X_0 &:= X_0 <<< 13 \\
X_2 &:= X_2 <<< 3 \\
X_1 &:= X_1 \oplus X_0 \oplus X_2 \\
X_3 &:= X_3 \oplus X_2 \oplus (X_0 << 3) \\
X_1 &:= X_1 <<< 1 \\
X_3 &:= X_3 <<< 7 \\
X_0 &:= X_0 \oplus X_1 \oplus X_3 \\
X_2 &:= X_2 \oplus X_3 \oplus (X_1 << 7) \\
X_0 &:= X_0 <<< 5 \\
X_2 &:= X_2 <<< 22 \\
\hat{B}_{i+1} &:= X_0, X_1, X_2, X_3
\end{aligned}
$$

Figure 2.3: Right part of the figure

```
unsigned char s0[16] = {
         3,  8, 15,  1,
        10,  6,  5, 11,
        14, 13,  4,  2,
         7,  0,  9, 12
};
```

Figure 2.4: Serpent S-box S_0 written as array

```
#define S0(x0, x1, x2, x3, x4) ({ \
                        x4  = x3; \
  x3 |= x0;  x0 ^= x4;  x4 ^= x2; \
  x4 = ~x4;  x3 ^= x1;  x1 &= x0; \
  x1 ^= x4;  x2 ^= x0;  x0 ^= x3; \
  x4 |= x0;  x0 ^= x2;  x2 &= x1; \
  x3 ^= x2;  x1 = ~x1;  x2 ^= x4; \
  x1 ^= x2;                       \
})
```

Figure 2.5: S_0 written as logical sequence

  - **`crypto`**: x86 specific implementation of ciphers
  - **`include/asm`**: x86 specific kernel headers
- `block`: Block I/O layer
- **`crypto`**: Crypto API
- `drivers`: Device drivers
- `firmware`: Device firmware
- `fs`: Filesystem implementations
- **`include`**: Kernel headers
  - **`crypto`**: Crypto API headers
- `init`: Kernel boot and initialization code
- `ipc`: Interprocess communication
- `kernel`: Core subsystems (e.g. scheduling)
- `lib`: Helper routines
- `mm`: Memory Management subsystem
- `net`: Networking subsystem (Ethernet, IPv4, IPv6, ...)
- `security`: Linux Security Module
- `sound`: Sound subsystem
- `virt`: Virtualization infrastructure

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3 SIMULATION METHOD

3.1 FEM

3.2 Simulation Setup

3.2.1 Sphere vs Rigid Plane

In this simulation the collision of two viscoelastic spheres is studied. Both the spheres have the same magnitude of velocity but opposite directions. Therefore to simplify the model and computation, instead of simulation two spheres colliding, a single sphere colliding against a rigid plane can be simulated. This setup would be equivalent to the original problem as both the spheres are the same in all aspects except for having different directions of velocities.

3.2.2 Symmetry

To further simplify the model, instead of considering the complete sphere, only a 2D semi-circular cross-section is considered. As the spheres are symmetric about the central rotational axis and the angle of contact is 90 degrees, there would not be any velocity in the Y direction.

3.3 Measurement Quantities

3.4 Verification

4 Evaluation

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5 CONCLUSION AND FUTURE WORK

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