Lehrstuhl für Informatik 1
Friedrich-Alexander-Universität
Erlangen-Nürnberg

MASTER THESIS

The validity of current contact force models for the collision of viscoelastic spheres

SANJAY CHAMOLI

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Examiner: Prof. Dr. Thorsten Pöschel
Advisor: Dr. Patric Müller

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SANJAY CHAMOLI

Abstract

An important model for granular particles are elastic and viscoelastic spheres. The macroscopic interaction forces for such objects are commonly obtained from the continuum mechanical equations of motion for elastic and viscoelastic material in quasi static approximation. The same holds true for the coefficients of restitution of colliding spheres which are, in turn, obtained from the macroscopic interaction forces. The quasi static assumption implies that the characteristic deformation rate is much smaller than the speed of sound in the material and that the relaxation time of the particle's material is negligible compared to the duration of the contact. In this work the validity of these assumptions is probed for realistic impact scenarious by comparing to a direct numerical solution of the underlying continuum mechanical equations of motion by means of finite elements.

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1 INTRODUCTION

Some intro

1.1 Granulates

A granular material is a conglomeration of discrete solid, macroscopic particles characterized by a loss of energy whenever the particles interact (the most common example would be friction when grains collide). The constituents that compose granular material must be large enough such that they are not subject to thermal motion fluctuations. Thus, the lower size limit for grains in granular material is about $1\mu m$. On the upper size limit, the physics of granular materials may be applied to ice floes where the individual grains are icebergs and to asteroid belts of the Solar System with individual grains being asteroids. [1]

1.2 Particle Simulations of Granulates

1.3 Particle Models

1.4 Aims

1.5 Acknowledgements

A big thank you for the support to Dr.Patric Mueller

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BACKGROUND

3 SIMULATION METHOD

3.1 FEM

The finite element method (FEM) is a numerical technique for solving problems which are described by partial differential equations or can be formulated as functional minimization. A domain of interest is is divided into an assembly of finite elements. Approximating functions in finite elements are determined in terms of nodal values of a physical field which is sought. A continuous physical problem is transformed into a discretized finite element problem with unknown nodal values. For a linear problem a system of linear algebraic equations should be solved. Values inside finite elements can be recovered using nodal values. FEM then uses variational methods from the calculus of variations to approximate a solution by minimizing an associated error function. Two features of the FEM are worth to be mentioned: 1) Piece-wise approximation of physical fields on finite elements provides good precision even with simple approximating functions (increasing the number of elements we can achieve any precision). 2) Locality of approximation leads to sparse equation systems for a discretized problem. This helps to solve problems with very large number of nodal unknowns.

3.1.1 Steps in FEM

Finite Element Method can be generaliyed into these steps.

- The physical problem has to be defined. All the desired results and the geometry of the problem have to be specified.
- The mathematical model has to be formulated The physical problem has to be formulated as an (initial) boundary value problem, e.g. by means of partial differential equations with boundary and, if required, initial conditions.
- The FE model has to be setup, which means that the mathematical model has to be discretised and the equations are solved. The structure is subdivided into finite elements, the solution is approxi-

mated element-wise and computed at the nodes of these elements. By this procedure, the differential equations are transferred into a system of algebraic equations

$$K.u = F \tag{3.1}$$

For a static stress analysis, the unknown nodal values subsumed in u are the nodal displacements, K is the stiffness matrix and f contains the nodal forces. This system of equations has to be solved for the unknown nodal values

$$u = K^{-1}.f \tag{3.2}$$

Subsequently, the dependent variables have to be computed in a so-called postprocessing step. For a stress analysis the strains, the stresses, equivalent stresses and further variables are derived from the nodal displacement values and are usually displayed graphically.

- The FE model yields a numerical solution which is an approximation of the exact solution of the mathematical problem. It has to be verified that the numerical solution is accurate enough and, if not, the FE model has to be improved. A more accurate solution can be obtained by choosing a larger number of nodes, which results in smaller finite elements. This, however, leads to higher computational effort such that an FE solution always represents a reasonable compromise between accuracy and computational effort.
- When a satisfying numerical solution is found, the results have to be interpreted and validated. This is usually done by comparing them to those obtained from experiments. If the numerical solution does not sufficiently capture the relevant phenomena, the physical problem or the mathematical model might have to be adapted. The comparison between numerical and experimental results can also be used to e.g. identify material parameters.

Kinematics [2]

The basic variables to describe the motion and deformation flexible bodies are Deformation, Velocity and Acceleration.

In continuum mechanics we can consider a body as set of particles, defined by the position vector X and material configuration β_0 of body at initial time t_0 (material configuration) and position vector x and configuration β_t at certain time t (spatial configuration).

[5]configurations and motion of a body (figure 2.1)

We define a nonlinear deformation map φ, which describes the motion of the body. The deformation map has to be unique, continuous and differentiable and maps the points X in the material configuration to the places x in the spatial configuration

$$x = \varphi(X, t)\, with\, \varphi\colon \beta_0 \to \beta_t \tag{3.3}$$

This description is common in solid mechanics and is denoted as the Lagrangian description of motion: the motion is characterized with respect to the material coordinates and one follows the movement of a particle in time. Inverse mapping is described using

$$X = \varphi(x, t), with, \varphi : \beta_t \to \beta_0 \tag{3.4}$$

Using Lagrangian description displacement field u can be determined as the difference of spatial and material coordinate system.

$$u(X, t) = x(X, t) - X \tag{3.5}$$

Kinetics [2]

The derivation of the kinetic formulation for FE structure is based upon above principle of structural displacement and D'Alembert principle in Lagrangian formulation.

According to D'Alembert principle the sum of the virtual work of all forces acting on the body must be zero. Hence, the sum of the virtual work of the inertial forces, the internal forces and the applied external body force over an arbitrary volume V_0, and the applied surface force over an infinitesimal area A_0 of V_0 equals zero, i.e.

$$-\int_{V_0} \delta u^T \rho_0 \ddot{u} dV_0 - \int_{V_0} \delta \epsilon^T H \varepsilon dV_0 + \int_{V_0} \delta u^T b_0 dV_0 + \int_{A_0} \delta u^T p_0 dA_0 = 0 \tag{3.6}$$

let ρ_0 represent the material density, b_0 the vector of the applied external body force over V_0, p_0 the vector of the applied external surface force over A_0 and the strain tensor. The term H is the, so called, material matrix defined for homogeneous, elastic and isotropic materials, with which the material law is described.

The substitution of Eq.(3.5) into the first summand of Eq.(3.6) leads to the following series of equations:

$$\int_{V_0} \delta u^T \rho_0 \ddot{u} dV_0 \; = \delta z_F^T \overset{[}{e} = 1] n_E \sum \left(T^{e^T} \int_{V_0^e} N^{eT}(x) \rho_0^e(x) N^e(x) dV T^e \right) \ddot{z}_F \tag{3.7}$$

$$= \delta z_F^{\mathrm{T}} \overset{[}{\mathrm{e}} = 1] \mathrm{n_E} \sum \mathrm{T^{eT} M^e T^e \ddot{z}_F} \tag{3.8}$$

$$= \delta \boldsymbol{z_F^T M}_{\mathrm{F}} \ddot{\boldsymbol{z}}_{\mathrm{F}} \tag{3.9}$$

where,

$$\mathbf{M^e} = \int_{\mathbf{V_{0e}}} \mathbf{N^{eT}}(\mathbf{x}) \rho^{\mathrm{e}}(\mathbf{x}) \mathbf{N^e}(\mathbf{x}) \mathrm{dV}$$

represents mass matrix for each element of FE structure.

$$\mathbf{M_F} = \sum_{\mathbf{e=1}}^{\mathbf{n_E}} \mathbf{T^{eT} M^e T^e}$$

represents mass matrix for entire FE structure.

The substitution of Eq.(3.5) and linearized strain-displacement relation in the second summand of Eq.(3.6)leads to the following:

$$\int_{\mathrm{V_0}} \delta \varepsilon^{\mathbf{T}} \mathbf{H} \varepsilon \mathrm{dV_0} = \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \sum_{\mathrm{e=1}}^{\mathrm{n_E}} (\mathbf{T}^{\mathrm{eT}} \int_{\mathrm{V_{0^e}}} \mathbf{B}^{\mathrm{eT}}(\mathbf{x}) \mathbf{H}^{\mathrm{e}} \mathbf{B}^{\mathrm{e}}(\mathbf{x}) \mathrm{dV} \mathbf{T}^{\mathrm{e}}) \mathbf{z}_{\mathrm{F}} \tag{3.10}$$

$$= \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \sum_{\mathrm{e=1}}^{\mathrm{n_E}} \mathbf{T}^{\mathrm{eT}} \mathbf{K}^{\mathrm{e}} \mathbf{T}^{\mathrm{e}} \mathbf{z}_{\mathrm{F}}$$

$$= \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \mathbf{K}_{\mathrm{F}} \mathbf{z}_{\mathrm{F}} \tag{3.11}$$

where,

$$\mathbf{K}^{\mathrm{e}} = \int_{\mathrm{V_0}} \mathbf{B}^{\mathrm{eT}}(\mathbf{x}) \mathbf{H}^{\mathrm{e}} \mathbf{B}^{\mathrm{e}}(\mathbf{x}) \mathrm{dV}$$

represents linear stiffness matrix for each element of FE structure.

$$\mathbf{K}_{\mathrm{F}} = \sum_{\mathrm{e}=1}^{\mathrm{n_E}} \mathbf{T}^{\mathrm{e}} \mathbf{K}^{\mathrm{e}} \mathbf{T}^{\mathrm{e}}$$

represent linear stiffness matrix of whole FE structure.

For the third summand of Eq.(3.6), the definition of the body force for each element of the FE structure is used, using b_0=Element material density (ρ) * gravity (g)

$$\int_{\mathbf{A_0}} \delta \mathbf{u^T b_0} \mathrm{dV}_0 = \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \sum_{\mathrm{e}=1}^{\mathrm{n}_\mathcal{E}} (\mathrm{T}^{\mathrm{eT}} \int_{\mathrm{V}_0^{\mathrm{e}}} \mathbf{N^{eT}}(\mathbf{x}) \rho_0^{\mathrm{e}}(\mathbf{x}) \mathrm{dV} \mathbf{\Gamma^e}) \mathrm{g} \tag{3.12}$$

$$= \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \sum_{\mathrm{e}=1}^{\mathrm{n_E}} \mathbf{T^{eT}} \mathbf{f}_{\mathrm{g}}^{\mathrm{e}} = \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \mathbf{F_g} \tag{3.13}$$

where,

$$f_g^e = \int_{V_0^e} N^{eT}(x) \rho_0^e(x) dV \Gamma^e g$$

represent body force vector for each element.

$$\mathbf{F}_{\mathrm{g}} = \sum_{\mathrm{e}=1}^{\mathrm{n_E}} \mathbf{T^{eT}} \mathbf{f}_{\mathrm{g}}^{\mathrm{e}}$$

represent body force vector for whole FE structure.

Finally, for the last summand of Eq. (3.6) the utilization of Eq. (3.5) gives

$$\int_{\mathbf{V_0}} \delta \mathbf{u^T p_0} \mathrm{dA}_0 = \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \sum_{\mathrm{e}=1}^{\mathrm{n}_\mathcal{E}} \mathrm{T}^{\mathrm{eT}} \int_{\mathrm{V}_0^{\mathrm{e}}} \mathbf{N^{eT}}(\mathbf{x}) \rho_0^{\mathrm{e}}(\mathbf{x}) \mathrm{dA} \mathbf{\Gamma^e} \tag{3.14}$$

$$= \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \sum_{\mathrm{e}=1}^{\mathrm{n_E}} \mathbf{T^{eT}} \mathbf{f}_{\mathrm{p}}^{\mathrm{e}} = \delta \mathbf{z}_{\mathrm{F}}^{\mathrm{T}} \mathbf{F}_{\mathrm{p}} \tag{3.15}$$

where,

$$\mathbf{f}_{\mathrm{p}}^{\mathrm{e}} = \int_{\mathrm{V}_0^{\mathrm{e}}} \mathbf{N^{eT}}(\mathbf{x}) \rho_0^{\mathrm{e}}(\mathbf{x}) \mathrm{dA} \mathbf{\Gamma^e}$$

represent external applied on surface of each element.

$$\mathbf{F}_{\mathrm{p}} = \sum_{\mathrm{e}=1}^{\mathrm{n_E}} \mathbf{T^{eT}} \mathbf{f}_{\mathrm{p}}^{\mathrm{e}}$$

represent external force applied on surface of whole FE structure.

Gathering Eq. (3.9), (3.11), (3.13), and (3.15) and substituting them in Eq. (3.6) leads to

$$\delta \mathbf{z}_F^T (\mathbf{M}_F \ddot{\boldsymbol{z}}_F + \mathbf{K}_F \mathbf{z}_F + \mathbf{F_g} + \mathbf{F}_p) = 0 \tag{3.16}$$

which is general second order LTI equation of motion for undamped FE structure.

If damping effects are to be considered, the definition of the associated parameter is essential. The simplest, and most commonly used, damping model assumes the damping to be linearly proportional to the structure's velocity. This leads to the following equation for linearly damped FE structures in matrix form

$$\mathbf{M}_{\mathrm{F}} \ddot{\boldsymbol{z}}_{\mathrm{F}} + \mathbf{D}_{\mathrm{F}} \dot{\mathbf{z}}_{\mathrm{F}} + \mathbf{K}_{\mathrm{F}} \mathbf{z}_{\mathrm{F}} = \mathbf{F} \tag{3.17}$$

with $\mathbf{D_F}$ being the structural damping matrix and $\mathbf{F = F_g + F_p}$ used for total force acting on structure. The elemental damping can be formulated in similar pattern as for other matrix described above.

From now on we will drop the suffix F used to represent system matrix and vectors of FE structure, so equation can be written as

$$\mathbf{M\ddot{z} + D\dot{z} + Kz = F} \tag{3.18}$$

3.1.2 FEM Tools

3.2 Simulation Setup

3.2.1 Sphere vs Rigid Plane

In this simulation the collision of two viscoelastic spheres is studied. Both the spheres have the same magnitude of velocity but opposite directions. Therefore to simplify the model and computation, instead of simulation two spheres colliding, a single sphere colliding against a rigid plane can be simulated. This setup would be equivalent to the original problem as both the lspheres are the same in all aspects except for having different directions of velocities.

3.2.2 Symmetry

To further simplify the model, instead of considering the complete sphere, only a 2D semi-circular cross-section is considered. As the spheres are symmetric about the central rotational axis and the angle of contact is 90 degrees, there would not be any velocity in the Y direction.

3.2.3 Mesh

The 3.1 shows the mesh used in the simulation. We can see that the mesh is finer at the bottom of the sphere as it is the point of contact and would undergo massive deformation during the impact.

The 3.2 shows an enlarged image of the bottom half of the mesh. We can see the degree of fineness of the mesh compared to the other half of the sphere.

3.2.4 Rigid Plane

The figure shows the rigid plane against which the sphere would be colliding.

3.3 Measurement Quantities

3.3.1 Displacement

To measure the displacement of the body, the displacement of the center of mass was measured. The center of mass of the of a sphere is located at the sphere. The figure shows the center of the 2D semi-circle which was considered as the sphere of the model. To measure the displacement center of mass of the sphere, the displacement of center of the semicircle was considered.

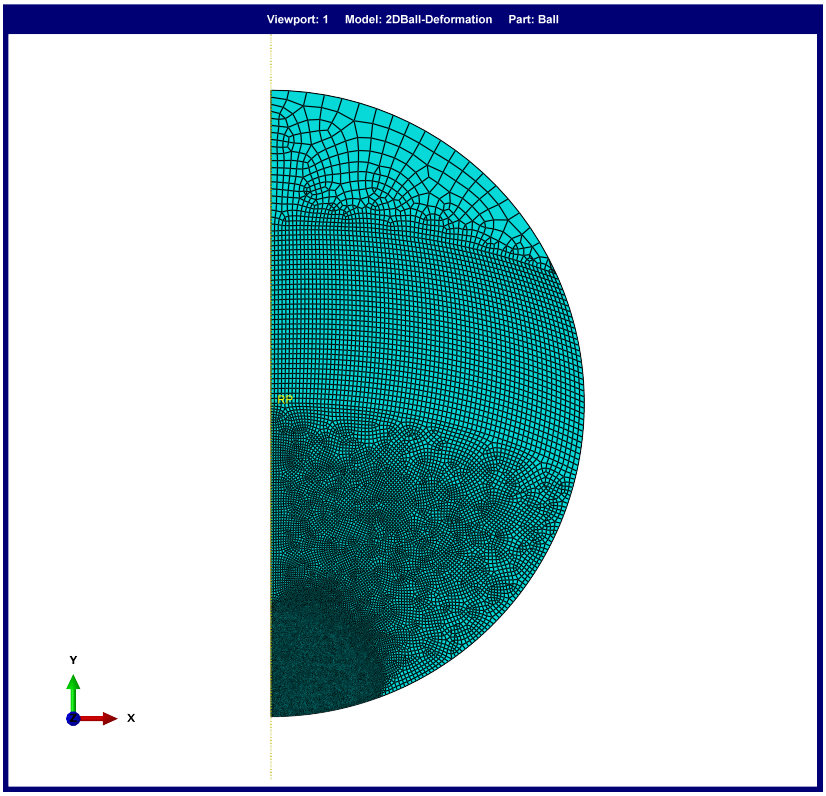


Figure 3.1: Mesh

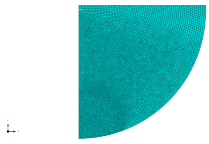


Figure 3.2: Bottom Half of Mesh

3.3.2 Kinetic Energy

The kinetic energy of an object is the energy that it possesses due to its motion. As the aim of the thesis is to measure the co-efficient of restitution, which is directly related to the restitution kinetic energy in the body after the collision is completed.

$$KineticEnergy(KE) = \frac{mv^2}{2} \tag{3.19}$$

where m is the mass of the object and v is the velocity of the object.

3.3.3 Strain Energy

The strain energy is the energy stored by a system undergoing deformation. During a collision, a part of the kinetic energy is converted into strain energy. This strain energy can also be observed as vibration on the body of the object. When the load is removed,

$$StrainEnergy(U) = \frac{V\sigma\epsilon}{2} \tag{3.20}$$

where V is volume, σ is stress and ϵ is strain.

3.3.4 Co-efficient of Restitution

The co-efficient of restitution describes the energy transfer

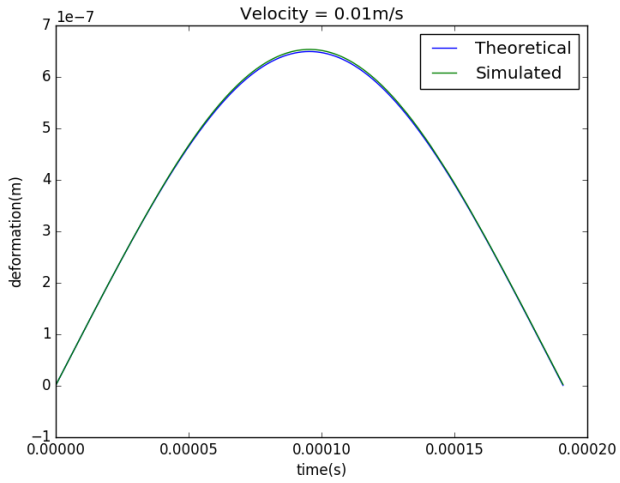
3.4 Verification

To verify the correctness of the simulation setup, various meshes were tried. As Abaqus CAE has an option to automatically choose a suitable timestep, the automatic option was chosen.

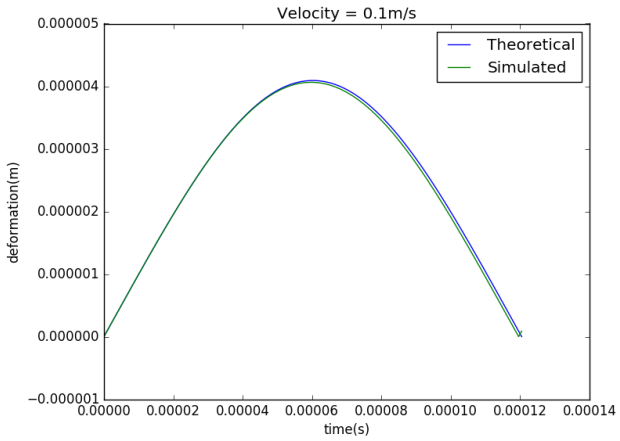
4

RESULTS

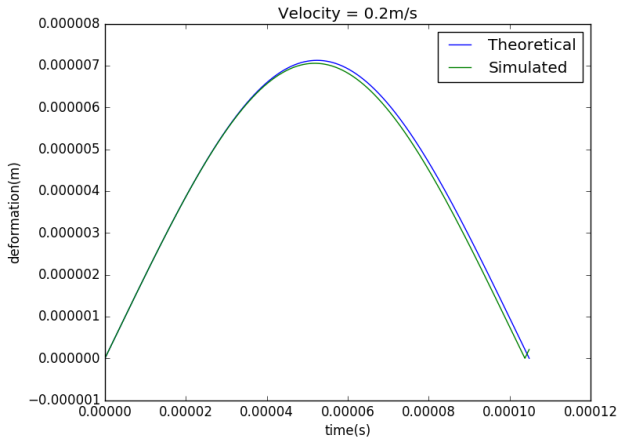
4.1 Results



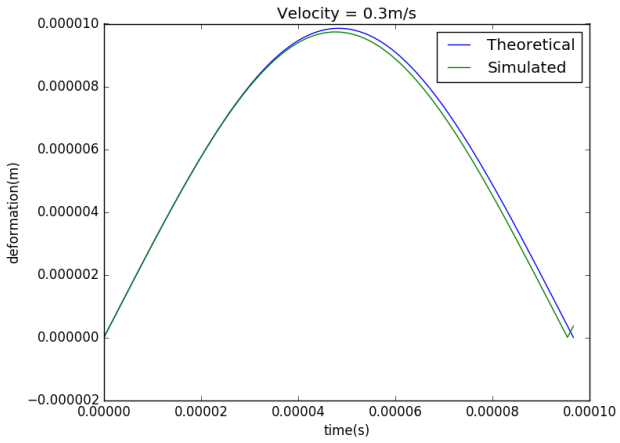
(a) Velocity 0.01m/s



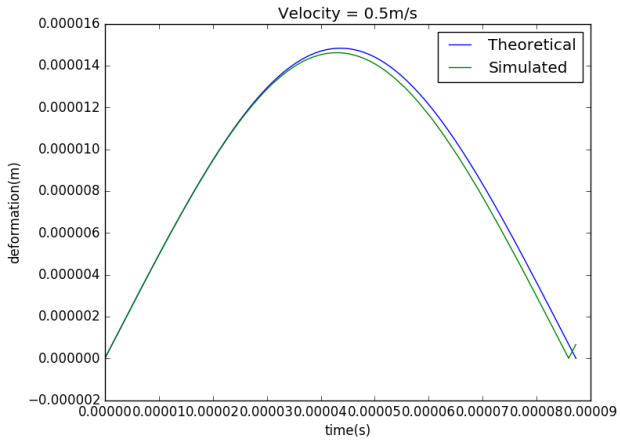
(b) Velocity 0.10m/s



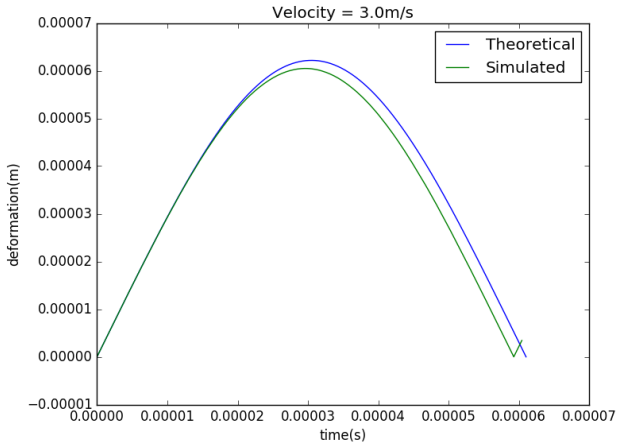
(c) Velocity 0.20m/s



(d) Velocity 0.30m/s



(e) Velocity 0.50m/s



(f) Velocity 3.0m/s

Figure 4.1: Time evolution of the temperature at x=y=0.4

4.2 COR

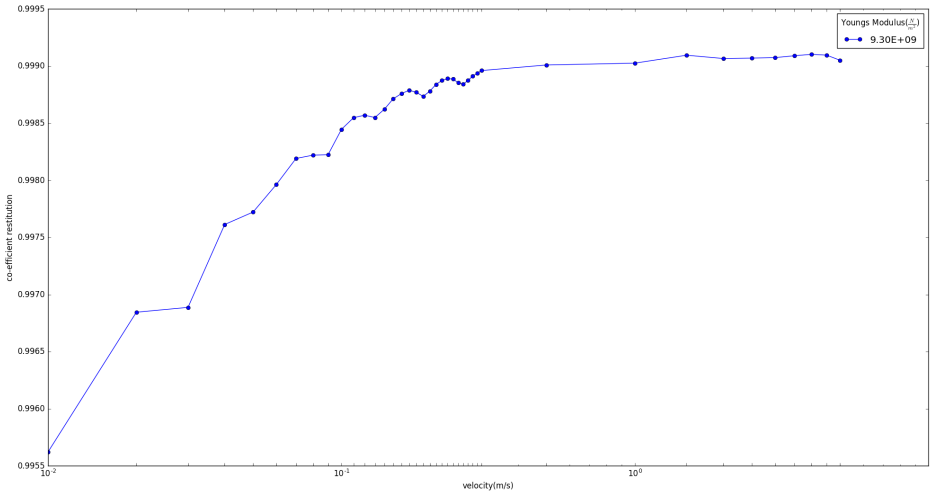
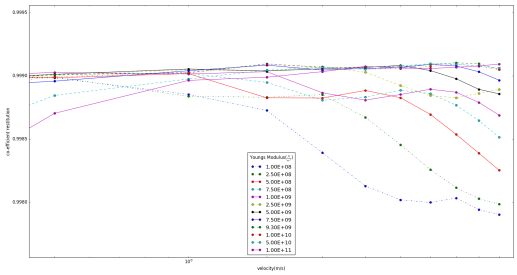


Figure 4.2: COR

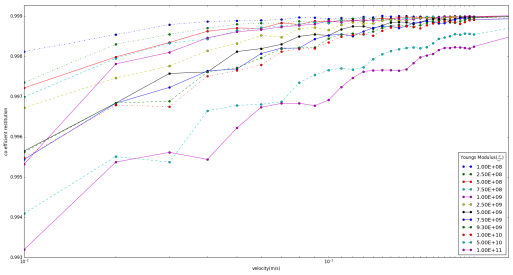
4.3 Parametric Study

4.3.1 Different Youngs Modulus

(a) COR



(b) COR High Velocity



(c) COR Low Velocity

4.3.2 Different Diameters

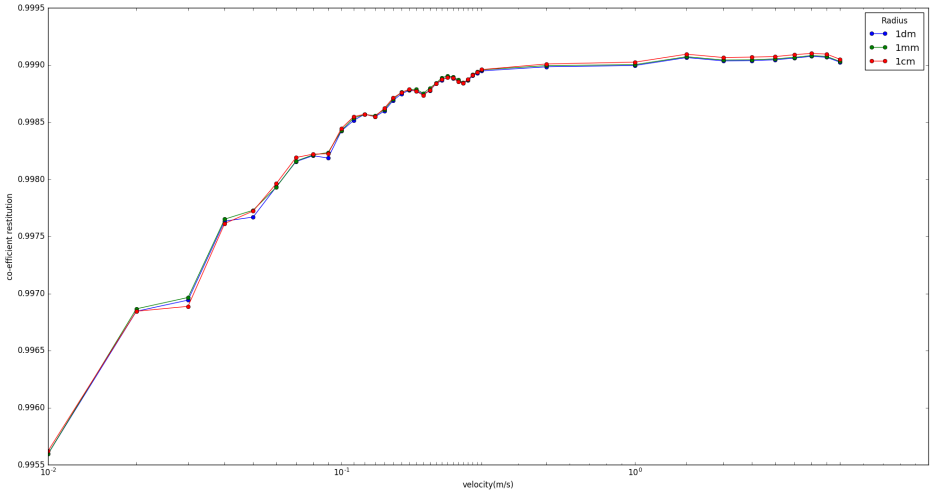


Figure 4.3: COR

5 Conclusion and Future Work

In this chapter we want to draw conclusions about the work, which has been done during this thesis. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

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