Department of Electrical Engineering, IIT (BHU)

EE-312 Physical Significance of Mathematical Methods

End Term Examination (Even Semester) 2021-22
Date: 27/04/2022

Note:

- ***Attempt all questions. All necessary information is already provided. All symbols have their usual meaning.***
- ***One of the physical interpretation should match with the interpretation discussed during the lecture, if necessary you can write more than one interpretation.***
- ***If physical interpretation does not match with the lectures then no marks are given.***

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Q-1 A group of IIT (BHU) students is trying to understand the correlation between stability, asymptotic stability, and exponential stability about the equilibrium point of the dynamical system

$$\dot{x} = f(x); \quad x \in \mathbb{R}^n.$$

They found that the concept of A drives the idea of stability. Further, they found that when the autonomous system is stable about the equilibrium point, then the property B satisfies. However, B and C should hold for asymptotic stability properties. Furthermore, they found that B, C, and D ensure exponential stability. Answer the following questions:

a Identify A, B, C and D.

b Explain stability, asymptotic stability, and exponential stability using the concept of balls.

Q-2 Two identical masses m are connected by a massless rigid bar of length l, and they are constrained to move in two frictionless slides, one vertical and the other horizontal as shown in the figure (1). Assume that the conservative gravitational force acts along the negative y axis and is incorporated into the scalar potential U. The generalized coordinate can be chosen to be the angle α corresponding to a single degree of freedom. The relative cartesian coordinates of the blocks are given by

$$x = l\cos\alpha$$
$$y = l\sin\alpha$$

Thus

$$\dot{x} = -l(\sin\alpha)\dot{\alpha}$$
$$\dot{y} = l(\cos\alpha)\dot{\alpha}$$

This constraint, that is absorbed into the generalized cohorizontal channels. ordinate, is holonomic, scleronomic, and conservative.

a. Derive the equation of motion using Euler-Lagrange equation.

$$ml^2\ddot{\alpha} + mgl\cos\alpha = 0$$

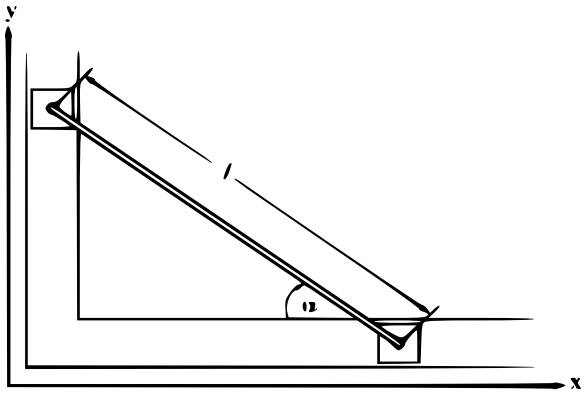

Figure 1:

b. Linearized the above equation about the equilibrium point.

Q-3 Find the stability of the following system at equilibrium point using epsilon-delta concept.

$$\dot{x}(t) = -x(t)^3$$

Q-4 For what numbers b is the following matrix positive semi-definite?

$$\begin{pmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{pmatrix}$$