<u>Homework #3</u>

Robot Dynamics, Spring 2018

Due Wednesday, March 14 by 11:59pm EST

Submit the assignment itself as a single scanned or typed pdf file, along with any code or supplemental material in a zip file through myWPI.

In HW #3 you will analyze a robot that combines a mobile robot base with a manipulator arm. The assignment is inspired by the Neobotix Mobile Manipulator MM-500 with a Schunck Lightweight Arm LWS 4D (http://www.neobotix-robots.com/mobile-manipulator-mm-500.html). However, <u>please use the specified configuration below where it differs from the real system</u>. You will solve for the position and velocity kinematics of the wheeled mobile manipulator (WMM) shown below. Although it is modeled after a physical system, some simplifications have been made and you are expected to <u>use the dimension and configurations listed in this assignment sheet</u> (see following page) which is some cases may be different than the physical robot.

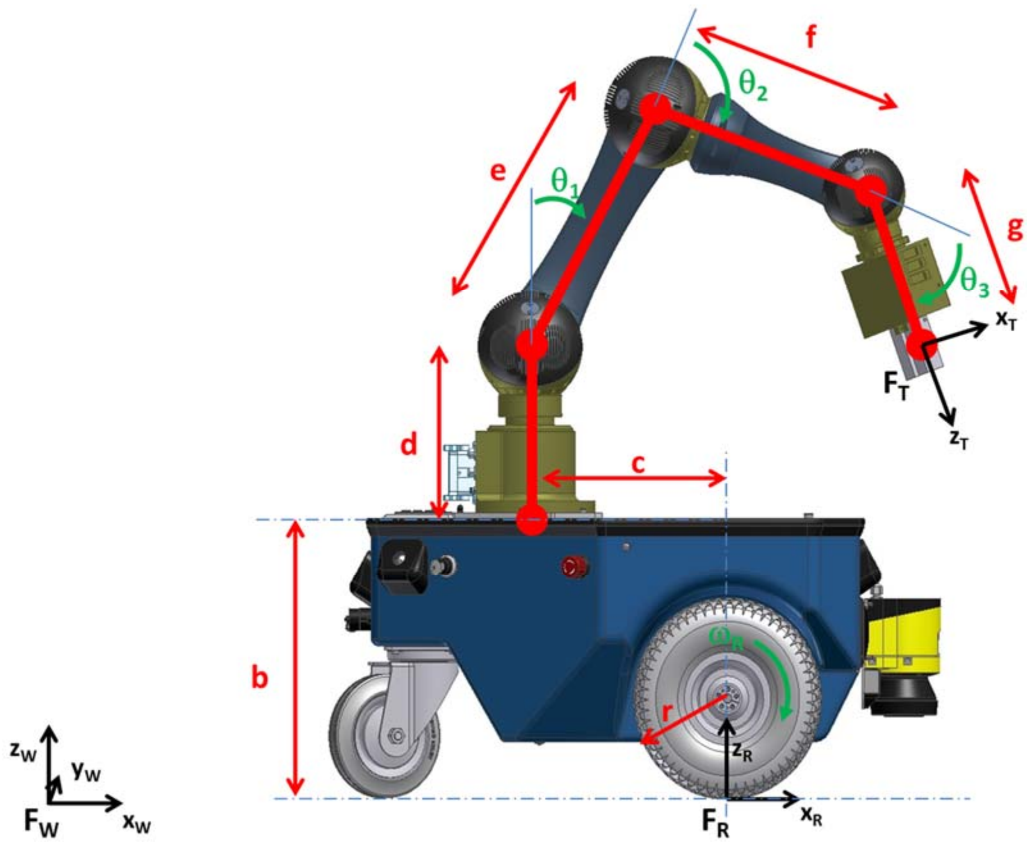
Neobotix Mobile Manipulator MM-500 with Schunck Lightweight Arm LWS

As shown in the following page, the robot's mobile base has two independently driven wheels with a wheelbase of ***a*** between the center of the wheels, and with angular velocities ω_R and ω_L for the right and left wheels, respectively. The direction of the angular velocities is such that if ω_R and ω_L are both the same, such that positive values the robot will drive in the forward x_R direction. The castor wheel (as shown) should instead be treated as single omniwheel centered below where the arm is attached to the base. All wheels are of radius ***r***.

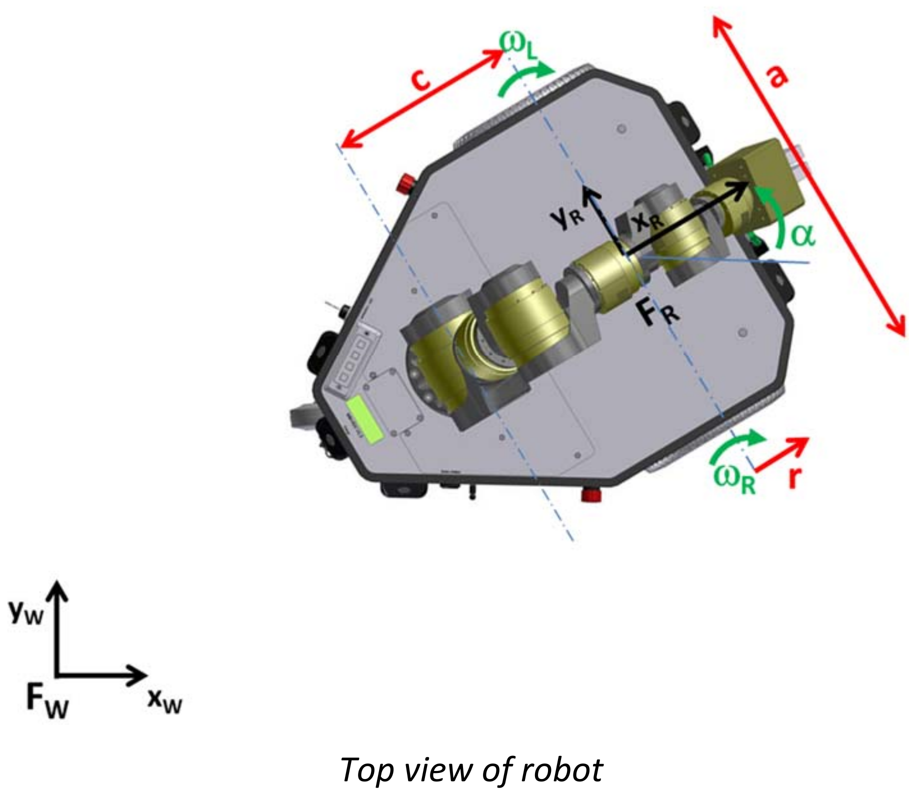
A 3-DOF planar arm (similar to a simplified version of the Schunk LWS arm with the three out-of-plane motions disabled) is mounted to the mobile robot base with translational offsets ***b***, ***c***, and ***d*** with respect to the robot-fixed base frame, F_R , as shown. The link lengths are labeled ***e***, ***f***, and ***g***, and the joint angles for the three joints are θ_1, θ_2, and θ_3 as shown. *Please note that there is no manipulator base rotation, this is a 3-link planar arm with respect to the robot platform.*

This assignment incorporates most of what we have covered so far in class, and it is highly recommended that you follow the steps in order, as they will help you successively build up to the solution. Please follow through the following steps, clearly showing all your work and be sure to include units where applicable. Be sure to clearly answer all questions asked – some questions request multiple responses. If parameters are not defined, clearly state any assumptions made.

You should use Matlab to complete the assignment, being sure to clearly publish the final output. Any sketches, figures, or other non-Matlab material should be included in the published Matlab output. Your submission should contain the published pdf output as well as the .m file and source in a .zip file.



Lateral view of robot



Top view of robot

Part A: Robot Arm Kinematics

1. FRAMES & D-H PARAMETERS:

   Label all of the relevant frames, transformations, and parameters for determining the 6-DOF pose of the robot arm's tip (***F_T***) with respect to the robot's base (***F_R***) on a figure of the robot. *Be sure to note where **F_R** is located in the figures above.*

   Identify the D-H parameters required to determine the robot's forward position kinematics and present in a table. Note that this is for a 3-DOF planar arm as shown. *You may use intermediate/dummy links that do not incorporate variable joint parameters if necessary. Be sure to make your naming convention clear and consistent with the remainder of the assignment.*

2. POSITION KINEMATICS:

   Using the function for converting D-H table parameters to transformations from HW #2, determine the symbolic forward position kinematics of the arm that describes ***F_T*** with respect to the frame fixed, ***F_R*** (located on the robot directly between the wheels at floor level as shown). That is, symbolically solve for the transformation between ***F_R*** and ***F_T*** . *Be sure to show how you calculate all intermediate transformations and identify all steps in the process in order to receive full credit.*

3. VELOCITY KINEMATICS:

   Determine the corresponding forward velocity kinematics. That is, write out the full symbolic 6-DOF relationship (including all 3 linear and 3 angular velocities) between joint velocities of the 3-DOF arm and the tip velocities of the robot with respect to the robot base, ***F_R*** .

   Then write out the reduced 3-DOF form of the Jacobian relationship (in matrix-vector form) for a planar manipulator (which we will treat this as). *Note that all robot motion is only in the x-z plane with respect to the robot base, **F_R***. Again, *be sure to show and identify all steps in the process in order to receive credit.*

4. FORCE PROPAGATION:

   Determine the relationship between the task space Wrench acting on the robot tip (i.e. the 6-DOF Cartesian forces and moments). That is, write out the full symbolic 6-DOF relationship (including all 3 forces and 3 moments) between task space Wrench as measured with respect to the robot base, $\boldsymbol{F_R}$, and the joint torques.

5. NUMERIC SOLUTION:

   For Problem A.5, we model the robot off of the dimensions as defined in the arm and mobile base datasheets. As such, you are expected to use the fixed parameters: b=424mm, c=300mm, d=380mm, e=328mm, f=323mm, and g=82.4mm, and that the robot base is fixed. At the configuration where $\theta_1=20^o$, $\theta_2=90^o$, and $\theta_3=30^o$ (similar to that shown in the figure), determine:

   a. The numeric forward position kinematics as a 4x4 transformation matrix representing the 6-DOF tip position and orientation, $\boldsymbol{F_T}$, with respect to the robot base, $\boldsymbol{F_R}$.

   b. The numeric instantaneous tip velocity (6-DOF, translation and rotation) if the instantaneous joint velocities of the three joints in the arm are all 30 deg/sec in their respective positive directions. Assume that the mobile base is fixed.

   c. The numeric arm joint torques required to apply a force straight ahead of 30N (x_R direction wrt robot base). Assume that the mobile base is fixed.

6. INVERSE VELOCITY KINEMATICS:

   For the configuration of Problem A.5, solve for the inverse velocity kinematics of the robot so as to numerically determine a set of instantaneous joint velocities necessary to move the robot tip downward ($-\mathbf{z}_r$ direction) with a velocity of 10cm/sec.

Part B: Mobile Platform Kinematics

7. CONSTRAINT EQUATIONS:

   Determine the constraint equations with respect to the robot-fixed frame, $\boldsymbol{F_R}$, for each of the three wheels on the mobile base (6 equations total).

8. CONSTRAINT MATRIX:

   Generate the constraint matrices J & C. Then compile the constraints into a single matrix constraint equation. Reduce the equation to eliminate any redundant constraints.

9. MOBILE KINEMATICS:

   Determine the velocity kinematics of the mobile robot base (defined by the robot-fixed frame, $\boldsymbol{F_R}$, on the robot between the wheels) with respect to the world coordinate $\boldsymbol{F_W}$ in terms of wheel angular velocities as defined in the figure. *Be sure to define any assumptions or unlabeled information.*

10. NUMERIC SOLUTION:

   For Problem B.10, based on the mobile base datasheet, you will use the fixed parameters: a=507mm and r=143mm. The robot's base frame $\boldsymbol{F_R}$ starts at an (x,y) location with respect to the world frame $\boldsymbol{F_W}$ of (2.5m, 1.5m) and oriented such that it is rotated about z_w by 30^o (similar to that shown in the figure).

   For this configuration, numerically solve for the instantaneous linear and angular velocity of the robot base frame $\boldsymbol{F_R}$ with respect to the world/home frame $\boldsymbol{F_W}$ if the instantaneous wheel velocities are ω_L=30rpm and ω_R=60rpm *(That is, solve for the 3x1 vector* $\dot{\xi}_0$*).*

Part C: Hybrid System Kinematics

11. COMBINED POSITION KINEMATICS:

For the complete combined system (arm + mobile base), incorporating your solutions from Part A and Part B, symbolically solve for the position forward kinematics that symbolically represent the tip frame $\boldsymbol{F_T}$ with respect to the world coordinate $\boldsymbol{F_W}$. *Be sure to define any assumptions or unlabeled information.*

12. NUMERIC SOLUTION:

Using the arm configuration form Problem A.5 and the mobile base configuration from Problem B.10, solve for the numeric forward position kinematics as a 4x4 transformation matrix representing the 6-DOF tip position and orientation, $\boldsymbol{F_T}$, with respect to the world/home frame, $\boldsymbol{F_W}$. *Be sure to keep track of your units and reference frames.*

13. COMBINED VELOCITY KINEMATICS:

Solve symbolically for the full forward velocity kinematics that represent the 6-DOF velocity (linear and angular velocities) of the robot tip frame $\boldsymbol{F_T}$ with respect to the world coordinate $\boldsymbol{F_W}$ as a function of all of the drive wheel and joint angular velocities (5 joint variables).

Write out the corresponding symbolic Jacobian and matrix-vector equation that gives the linear and angular velocities of the tip as a function of the two wheel angular velocities and the three joint angular velocities. *You will combine the mobile base and manipulator equations into one.*

14. FORCE PROPAGATION:

For the arm configuration from Problem A.5 and the mobile base configuration from Problem B.10, numerically determine the required torque at each wheel to keep the position of the mobile base stable as it is pushing against the wall with 30N (assuming no slip)?