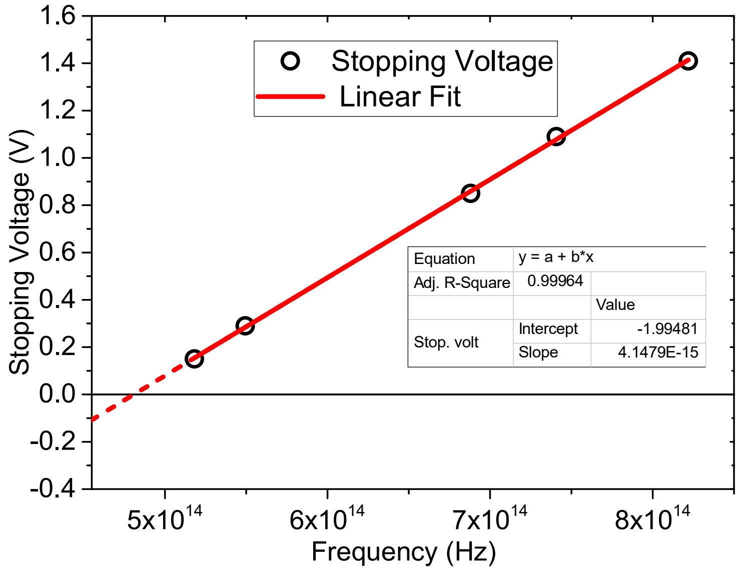
TUTORIAL 10

1. When a metal surface is irradiated with light of different wavelengths (λ in nanometer) from a mercury lamp, the stopping voltages (Vs in Volt) of the photoelectrons are measured as shown in the following table.

λ (nm)	365	405	436	546	579
V_s (V)	1.41	1.09	0.85	0.29	0.15

Plot the stopping voltage versus the frequency of the light and use the graph to determine the threshold frequency, the threshold wavelength, the work function of the metal, and the quotient h/e.

Solution:



The threshold frequency (ν_0) is the frequency for which stopping voltage becomes zero.

From,

$$KE_{\max} = h\nu - \Phi = h\nu - h\nu_o = eV_s$$

$$V_s = \frac{h}{e}\nu - \frac{h}{e}\nu_o$$

$$Slope = \frac{h}{e} \approx 4.15\times10^{-15}\ \text{V.s}$$

$$Intercept = -\frac{h}{e}\nu_o \approx -1.995\ \text{V}$$

$$So, \nu_o = -\frac{Intercept}{Slope} = \frac{1.995}{4.15\times10^{-15}}\ \text{Hz}$$

$$=4.8\times10^{14}\ Hz$$

And the threshold wavelength is

$$\lambda_0 = \frac{c}{\nu_0} = \frac{3\times10^8}{4.8\times10^{14}} m = 625 \text{ nm}$$

The electron work function of the metal,

$$\Phi = h\nu_0 = 6.626\times10^{-34}\times4.8\times10^{14} \text{ J}$$
$$=3.18\times10^{-19} \text{ J} \approx 1.99 \text{ eV}$$

The quotient $\frac{h}{e} = 4.15\times10^{-15}\ V.s$ is directly obtained from the slope.

2. Blue light of wavelength λ = 456 nm and power P = 1 mW is incident on a photosensitive surface of cesium. The electron work function of cesium is Φ = 1.95 eV.

(a) Determine the maximum velocity of the emitted electrons and the stopping voltage.

(b) If the quantum efficiency of the surface is η = 0.5 %, determine the magnitude of the photocurrent. The quantum efficiency is defined as the ratio of the number of photoelectrons to that of incident photons.

Solution:

a) Threshold wavelength for photoelectric effect of Cesium,

$$\lambda_0 = \frac{hc}{\Phi} = \frac{6.626\times10^{-34}\times3\times10^8}{1.95\times1.6\times10^{-19}} m = 636 \text{ nm}$$

As $\lambda_0 > \lambda_{incident}$, so electrons are emitted.

Maximum K.E of electron

$$= h\nu - \Phi = \frac{hc}{\lambda} - \Phi$$
$$= \frac{6.626\times10^{-34}\times3\times10^8}{456\times10^{-9}} - 1.95\times1.6\times10^{-19} \text{ J}$$
$$=(4.36\text{-}3.12)\times10^{-19} \text{ J} = 1.24\times10^{-19} \text{ J}$$
$$=0.775 \text{ eV}$$

So, stopping voltage is 0.775 V.

Maximum velocity of electron

$$v_{\max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times1.24\times10^{-19}}{9.11\times10^{-31}}} \text{ m/s}$$
$$= 5.22\times10^5 \text{ m/s}$$

b) Quantum efficiency (η) is defined as the ratio of the number of photoelectrons (n_e) to that of incident photons (n_p). So,

$$n_e = \eta \times n_p$$

$$as,\ n_p = \frac{P}{hc/\lambda}$$

$$So,\ n_e = \eta \times n_p = \frac{P \times \lambda \times \eta}{hc}$$

Now Current,

$$I = n_e e = \frac{P \times \lambda \times \eta \times e}{hc}$$

$$= \frac{1\times10^{-3}\times456\times10^{-9}\times5\times10^{-3}\times1.6\times10^{-19}}{6.626\times10^{-34}\times3\times10^{8}}\ \text{A}$$

$$= 1.84\ \mu\text{A}$$

3. X-rays of wavelength 70.7 pm are scattered from a graphite block. (a) Determine the energy of a photon. (b) Determine the shift in the wavelength for radiation leaving the block at an angle of 90^0 from the direction of the incident beam. (c) Determine the direction of maximum shift in wavelength and the magnitude of this shift.

<u>Solution</u>:

a) $\lambda = 70.7\times10^{-12}$ m

So, energy of a photon

$$E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{70.7\times10^{-12}}\ \text{J}$$

$=2.81\times10^{-15}$ J

$=1.75\times10^{4}$ eV

b) Shift in the wavelength, $\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\theta)$

λ: incident photon wavelength

λ′: scattered photon wavelength

λ_c: Compton wavelength $= \frac{h}{mc} = 2.426\times10^{-12}$ m

when, $\theta=90^0$

$$\Delta\lambda = \lambda_c\left(1-\cos 90^0\right)$$
$$= \lambda_c = 2.426\times10^{-12}\ \text{m}$$
=2.426 pm

c) The direction of maximum shift is θ=π, i.e. backward scattered photon.

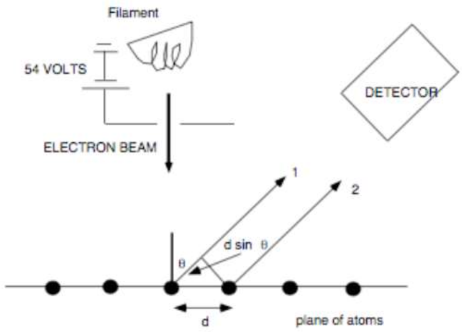
$$\Delta\lambda_{\max} = \lambda_c\left(1-\cos 180^0\right)$$
$$= 2\lambda_c = 2\times2.426\times10^{-12}\ \text{m}$$
=4.852 pm

4. In the Davisson-Germer experiment, 54 eV electrons were diffracted from a nickel crystal. Consider the case when the electron beam impinges normal to the nickel crystal surface (see figure). A plot of the intensity of the diffracted electrons as a function of the angle from the normal to the surface shows the first peak at $\theta = 50^0$.

(a) Calculate the spacing between the atoms on the nickel surface from the peak in the intensity distribution.

(b) The electron beam energy is now changed to 100 eV. Find the angle from the surface normal where the maximum intensity is expected. Is there a second angle at which the intensity is again a maximum? If there is, what is the second angle?

(c) If a beam of He atoms is used instead of electrons, what is the energy of the He atoms required to yield a maximum at the same angle as for the electrons?



Davisson-Germer experiment

(d) If a beam of photons is used, calculate the energy of the photons to obtain the same angle of diffraction for the maximum intensity as in the figure above.

Solution:

The crystal acts like a diffraction grating for the incoming electron beam with energy (E) 54eV.

The condition for maxima, $d\sin\theta = n\lambda;$ n=1,2,3...

a)

$$\lambda_{electron} = \frac{\text{h}}{p} = \frac{\text{h}}{\sqrt{2mE}}$$
$$= \frac{6.626\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times54\times1.6\times10^{-19}}}$$
$$= 0.167\times10^{-9}\ \text{m}$$

Therefore, for n=1,

$$d = \frac{\lambda}{\sin\theta} = \frac{0.167\times10^{-9}}{\sin 50^0} = 0.218 \text{ nm}$$

b)

If electron beam energy changed to E′=100 eV, then de Broglie wavelength changed to λ′, angle corresponding to maxima changed to θ′.

$$\frac{\lambda'}{\lambda} = \frac{\sin\theta'}{\sin\theta} = \sqrt{\frac{E}{E'}} = \sqrt{\frac{54}{100}}$$
$$So,\ \sin\theta' = \sin 50^0 \times 0.735 = 0.563$$
$$\theta' = 34.26^o$$

For n=2,

$$\sin\theta_2' = 2\times\sin 50^0\times 0.735 = 1.12$$

As n=2 gives $\sin\theta_2' > 1$, so no second maxima.

c)

Since, diffraction angle (θ) and spacing (d) remains same, then

$$\lambda_{He} = \lambda_e$$
$$\frac{h}{\sqrt{2m_{He}E_{He}}} = \frac{h}{\sqrt{2m_e E_e}}$$
$$So, E_{He} = E_e\frac{m_e}{m_{He}}$$

Now, $m_{He} = 2m_p + 2m_n \approx 4m_p$; ($m_p$=mass of proton, m_n=mass of neutron)

So,

$$E_{He} = E_e\frac{m_e}{m_{He}} = E_e\frac{m_e}{4m_p}$$
$$= 54\times\frac{9.11\times10^{-31}}{4\times1.67\times10^{-27}}eV$$
$$= 7.36\ meV$$

d)

$$\lambda_{photon} = d\sin\theta$$
$$= 0.218 \times \sin 50^o$$
$$= 0.167 \text{ nm}$$

So, $E = \dfrac{hc}{\lambda} = 1.19 \times 10^{-15}$ J = 7.44 KeV

5. The thermal kinetic energy of a hydrogen atom is roughly equal to kT, where $k=1.38\text{x}10^{-23}$ J/K and T is the absolute temperature. The radius of the atom is roughly equal to the radius of the n = 1 Bohr orbit, $r_1 = 0.53 \text{ x } 10^{-10}$ m. For what temperature will the de Broglie wavelength of the hydrogen atom be equal to its diameter? Take the mass of the atom to be that of the proton, $1.66 \text{ x } 10^{-27}$ kg.

Solution:

The de Broglie wavelength of the atom,

$$\lambda = \frac{\text{h}}{p} = \frac{\text{h}}{\sqrt{2mE}} = \frac{\text{h}}{\sqrt{2mkT}}$$
$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times T}}$$
$$= \frac{3.1 \times 10^{-9}}{\sqrt{T}} \text{ m}$$

For the temperature for which $\lambda=2r_1$, we have

$$\frac{3.1 \times 10^{-9}}{\sqrt{T}} = 2 \times 0.53 \times 10^{-10}$$
$$T = \left(\frac{3.1 \times 10^{-9}}{2 \times 0.53 \times 10^{-10}} \right)^2 \text{ K}$$
$$= 852.6 \;\; K$$
$$= 852.6 - 273 \; C$$
$$= 579.6^o \;\; C$$