Physics of Waves

PH11003

Tutorial 6

ElectroMagneticWaves

31 December 2022

[6.1] A plane harmonic em wave with plane polarization propagates in vacuum. The electric component of the wave has a strength amplitude $E_m = 50\ mV/m$, the frequency is $\nu = 100\ MHz$, Find (a) the rms value of the displacement current density, (b) the mean energy flow averaged over an oscillation period.
solution:

We have $$E = E_m \cos(2\pi\nu t - kx)$$

(a) $$j_{\text{dis}} = \frac{\partial D}{\partial t} = -2\pi\varepsilon_0 \nu E_m \sin(\omega t - kx)$$

Thus, $$(j_{\text{dis}})_{\text{rms}} = < j_{\text{dis}}^2 >^{1/2}$$
$$= \sqrt{2}\pi\varepsilon_0 \nu E_m = 0.20\ \text{mA/m}^2$$

(b) As in Problem 4.196, we can write
$$< S_x > = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_m^2$$

Thus, $$< S_x > = 3.3\ \mu\text{W/m}^2$$

[6.2] Find the mean radiation power of an electron performing harmonic oscillations with amplitude $a = 0.10\ nm$ and frequency $\omega = 6.5 \times 10^{14}\ s^{-1}$.
solution:

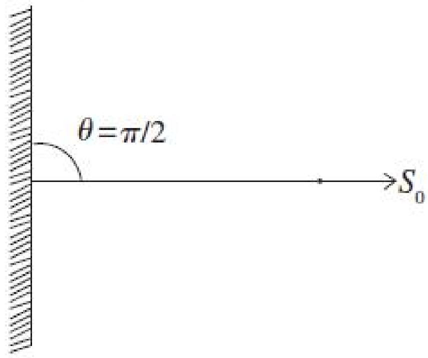
We have
$$P = \frac{1}{4\pi\varepsilon_0}\frac{2(\ddot{\mathbf{p}})^2}{3c^3}$$
$$|\ddot{\mathbf{p}}|^2 = (e\omega^2 a)^2 \cos^2 \omega t$$
Thus,
$$< P > = \frac{1}{4\pi\varepsilon_0}\frac{2}{3c^3}(e\omega^2 a)^2 \times \frac{1}{2}$$
$$= \frac{e^2\omega^4 a^2}{12\pi\varepsilon_0 c^3} = 5.1\times 10^{-1.5}\ \text{W}$$

[6.3] An em wave emitted by an elementary dipole propagates in vacuum so that in the far field zone the mean value of the energy flow density is equal is equal to S_0 at the point removed from the dipole by a distance r along the perpendicular drawn to the dipole's axis. Find the mean radiation power of the dipole.
solution:

We know that $S_0(r) \propto 1/r^2$. At other angles $S(r, \theta) \propto \sin^2\theta$, thus

$S(r, \theta) = S_0(r)\sin^2\theta = S_0 \sin^2\theta$

Average power radiated
$$P = S_0 \times 4\pi r^2 \times \frac{2}{3} = \frac{8\pi}{3} S_0 r^2$$
(Average of $\sin^2\theta$ over whole sphere is 2/3.)



[6.4] The mean power radiated by an elementary dipole is equal to P_0. Find the mean space density of energy of the em field in vacuum in the far field zone at the point removed from the dipole by a distance r along the perpendicular drawn to the dipole's axis.
solution:

From the previous problem
$$P_0 = \frac{8\pi S_0 r^2}{3}$$
or
$$S_0 = \frac{3P_0}{8\pi r^2}$$
Thus,
$$< w > = \frac{S_0}{c} = \frac{3P_0}{8\pi c r^2}$$
(Poynting flux vector is the energy contained in a box of unit cross-section and length c.)

[6.5] A system consists of two coherent point sources 1 and 2 located in a certain plane so that their dipole moments are oriented at right angles to that plane. The sources are separated by a distance d, the radiation wavelength is λ. Taking into account that the oscillations of source 2 lag in phase behind the oscillations of source 1 by ϕ ($\phi < \pi$), Find (a) the angles θ at which the radiation intensity is maximum. (b) the conditions under which the radiation in the direction $\theta = \pi$ is maximum and in the opposite direction, minimum.

solution:

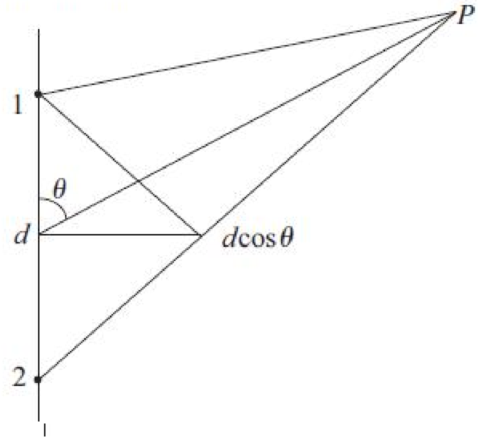
(a) With dipole moment perpendicular to plane, there is no variation with θ of individual radiation amplitude. Then the intensity variation is due to interference only. In the direction given by angle θ, the phase difference is

$$\frac{2\pi}{\lambda}(d\cos\theta) + \varphi = 2k\pi \quad \text{(for maxima)}$$

Thus, $$d\cos\theta = \left(k - \frac{\varphi}{2\pi}\right)\lambda$$

Here $k = 0, \pm 1, \pm 2, \ldots$

We have added φ to $2\pi/\lambda \; d\cos\theta$ because the extra path that the wave from 2 has to travel in going to P (as compared to 1) makes it lag more than it already has (due to φ).



(b) Maximum for $\theta = \pi$ gives

$$-d = \left(k - \frac{\varphi}{2\pi}\right)\lambda$$

Minimum for $\theta = 0$ gives

$$d = \left(k' - \frac{\varphi}{2\pi} + \frac{1}{2}\right)\lambda$$

Adding we get $$\left(k + k' - \frac{\varphi}{\pi} + \frac{1}{2}\right)\lambda = 0$$

This can be true only if $k' = -k$, $\varphi = \pi/2$, since $0 < \varphi < \pi$.

Then, $$-d = \left(k - \frac{1}{4}\right)\lambda$$

Here $k = 0, -1, -2, -3, \ldots$

(Otherwise R.H.S. will become $+ve$.)

Putting $k = -\bar{k}$, $\bar{k} = 0, +1, +2, +3, \ldots$, we get

$$d = \left(\bar{k} + \frac{1}{4}\right)\lambda$$