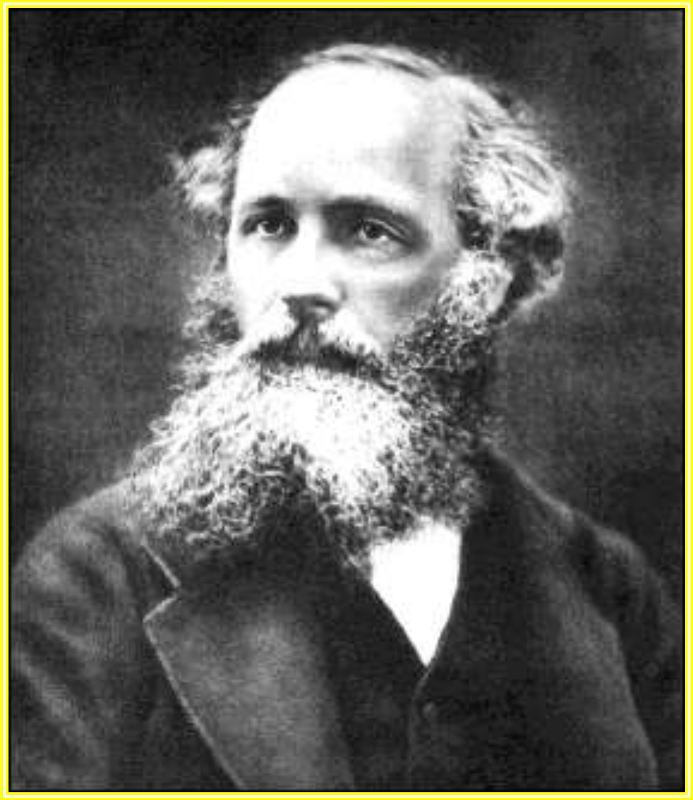
Electromagnetic Waves

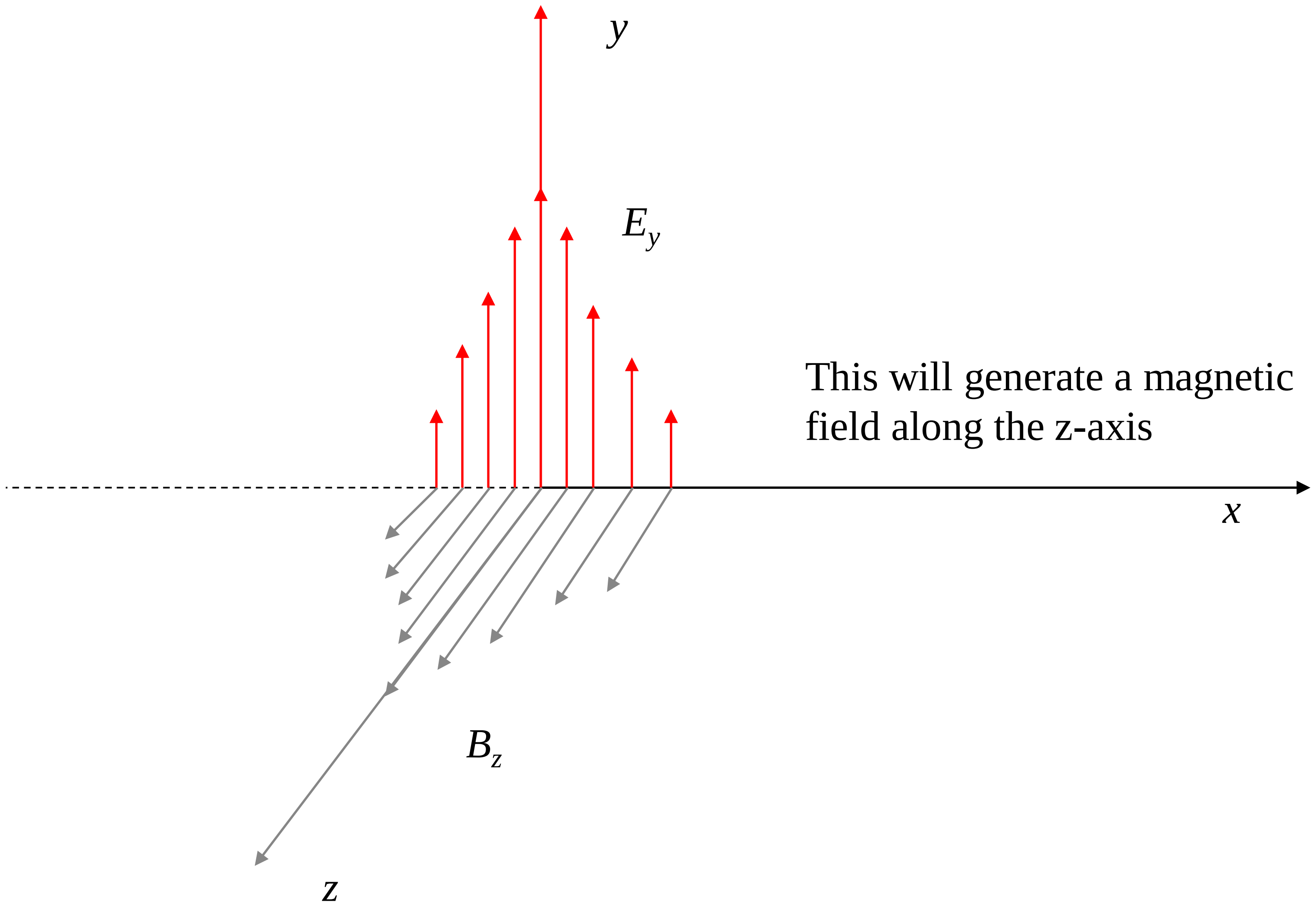


“Let there be electricity and magnetism and there is light”

J.C. Maxwell

EM Waves

Consider an oscillating electric field E_y



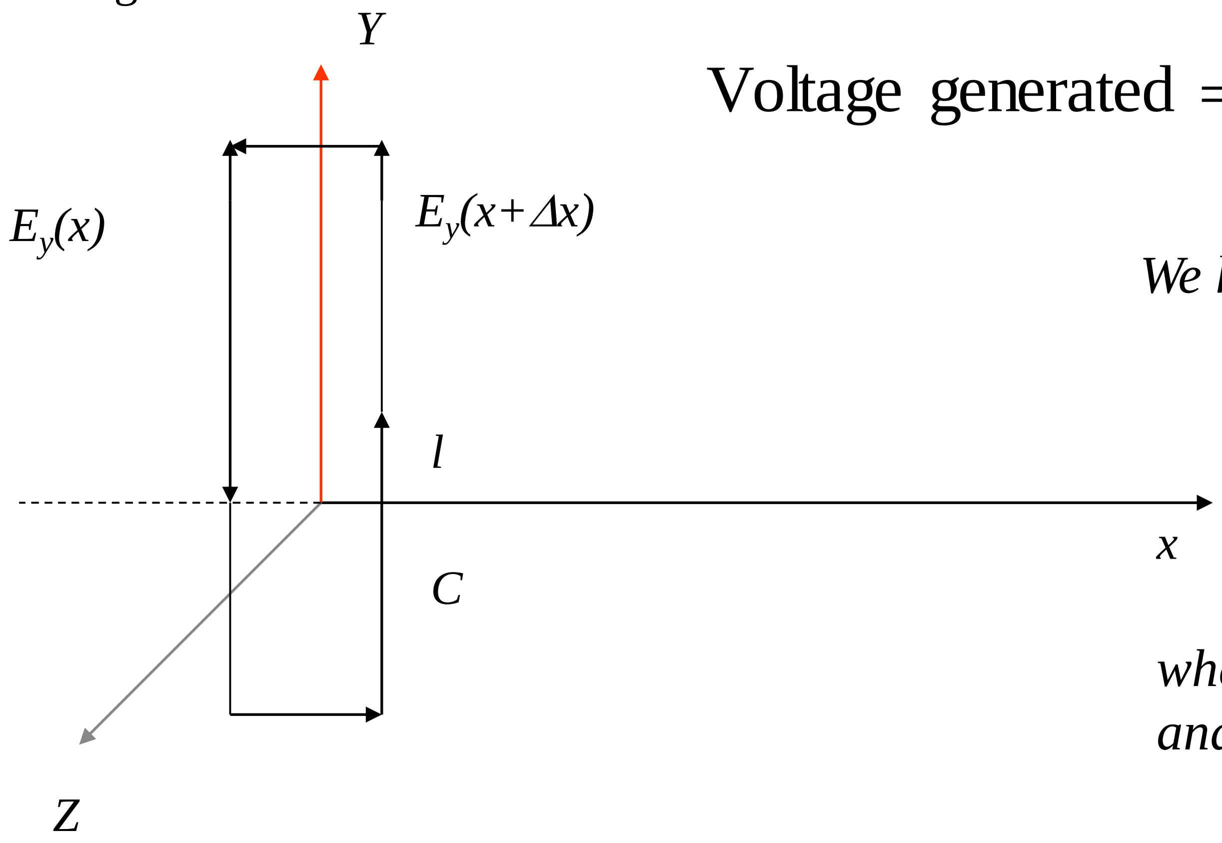
This will generate a magnetic field along the z-axis

Faraday's Law

The induced electromotive force in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit.

$$\text{Voltage generated} = -N\frac{\Delta BA}{\Delta t}$$

N: Number of turns
B: External magnetic field
A: Area of coil



We know that Faraday´s law in the integral form is given as:

$$\oint_C \vec{E}\,.\,\vec{dl} = -\frac{\partial}{\partial t}\iint_S \vec{B}\,.\,\vec{ds}$$

where C is the rectangle in the XY plane of length l, width Δx, and S is the open surface spanning the contour C

Using the Faraday´s law on the contour C, we get:

$$\left[E_y(x+\Delta x) - E_y(x)\right]l = -\frac{\partial B_z}{\partial t}l\Delta x$$

this implies...

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

Keep this in mind...

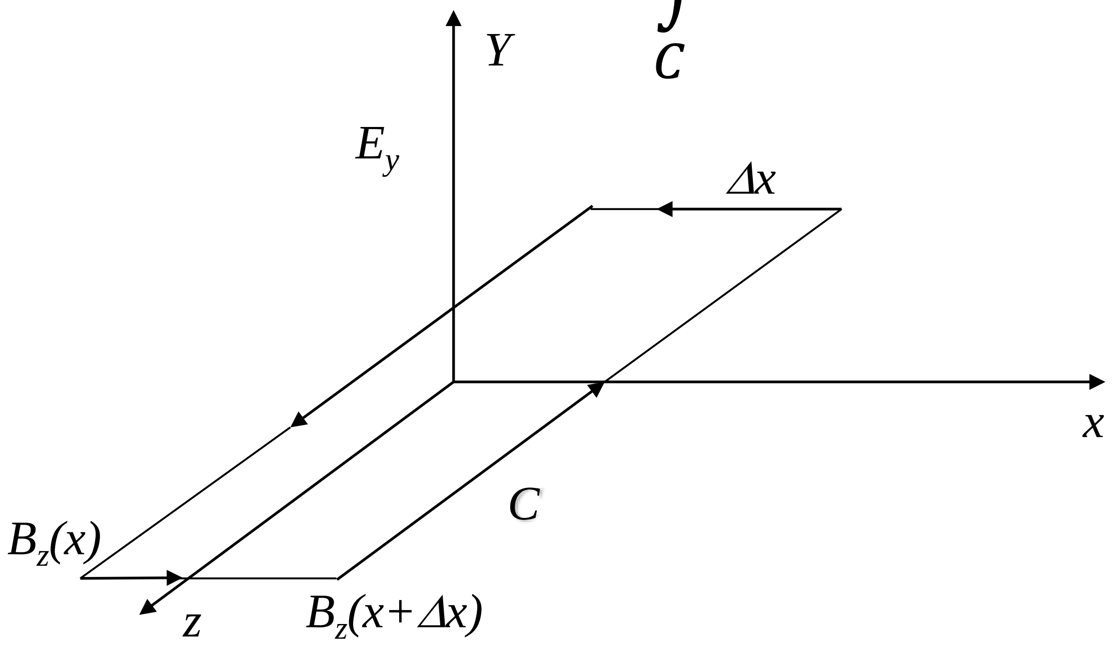
Ampere's Law

Ampère's circuital law relates the integrated magnetic field around a closed loop to the electric current passing through the loop.

The Ampere´s law with displacement current term can be written as:

$$\oint_C \vec{B}\,.\,\vec{dl} = \varepsilon_o \mu_o \frac{\partial}{\partial t} \iint_s \vec{E}\,.\,\vec{ds}$$

In free space, the displacement current is related to the time rate of change of electric field.



Using Ampere´s law, for the contour C, we get:

$$[-B_z(x+\Delta x) + B_z(x)]l = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t} l \Delta x$$

this implies...

$$-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$$

Outcome of Faraday's and Ampere's laws

Using the eqns. obtained earlier:

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

$$\boxed{-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}}$$

$$\boxed{\begin{gathered}\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial^2 E_y}{\partial x^2} \\ where \\ c^2 = \frac{1}{\varepsilon_0 \mu_0}\end{gathered}}$$

Form of wave equation

Note: Similar Equation can be derived for B_z

Solution of EM Wave equation

EM Wave Equation

Electromagnetic waves

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

for E field

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

for B field

In general, electromagnetic waves

$$\boxed{\nabla^2 \psi = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}}$$

Where ψ represents E or B or their components

Solution of 3D wave equation

A plane wave satisfies wave equation in Cartesian coordinates

A spherical wave satisfies wave equation in spherical polar coordinates

A cylindrical wave satisfies wave equation in cylindrical coordinates

In Cartesian coordinates

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

Separation of variables $\psi(x, y, z, t) = X(x)Y(y)Z(z)T(t)$

Substituting for ψ we obtain $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \frac{1}{c^2}\left(\frac{1}{T}\frac{\partial^2 T}{\partial t^2}\right)$

Variables are separated out. Each variable-term independent, and must be a constant

Solution of 3D wave equation

So we may write $$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2; \quad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2;$$

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -k_z^2; \quad \left(\frac{1}{T}\frac{\partial^2 T}{\partial t^2}\right) = -\omega^2$$

where we use

$$\omega^2/c^2 = k_x^2 + k_y^2 + k_z^2 = k^2$$

Solutions are then $$X(x) = e^{\pm i k_x x}; \quad Y(y) = e^{\pm i k_y y};$$

$$Z(z) = e^{\pm i k_z z}; \quad T(t) = e^{\pm i \omega t}$$

Total Solution is $$\psi(x, y, z, t) = X(x)Y(y)Z(z)T(t)$$

$$= Ae^{i[\omega t \mp (k_x x + k_y y + k_z z)]}$$

$$= Ae^{i[\omega t \mp \vec{k}.\vec{r}]}$$ **Plane wave**

3D Plane waves

$$\psi(\mathbf{r}) = A\sin(\mathbf{k}\cdot\mathbf{r})$$

$$\psi(\mathbf{r}) = B\cos(\mathbf{k}\cdot\mathbf{r})$$

$$\psi(\mathbf{r}) = C\exp(i\mathbf{k}\cdot\mathbf{r})$$

The surface of constant phase is:

$$\mathbf{k}\cdot\mathbf{r} = \phi_C$$

$$k_x x + k_y y + k_z z = \phi_C$$

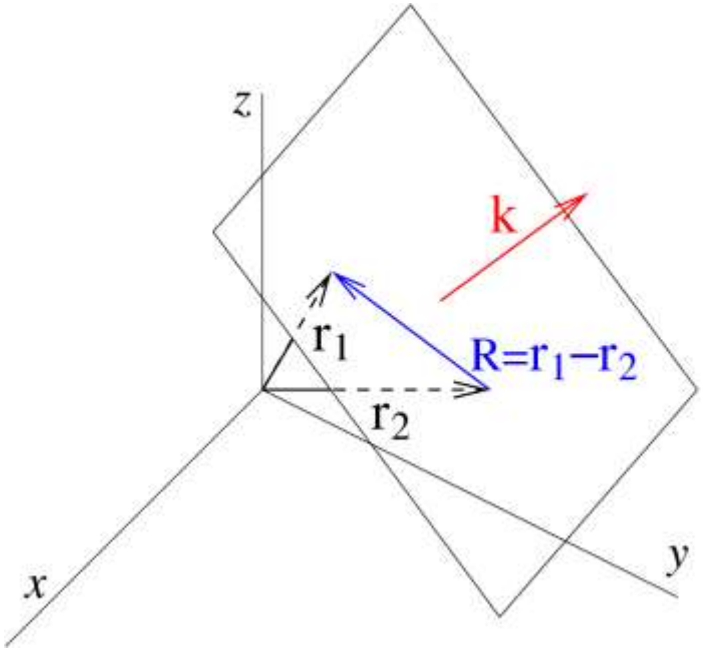
$$\mathbf{r_1}, \quad \mathbf{r_2}$$

$$\mathbf{k}\cdot\mathbf{r_1} = \mathbf{k}\cdot\mathbf{r_2} = \phi_C$$

$$\mathbf{k}\cdot(\mathbf{r_1} - \mathbf{r_2}) = 0$$

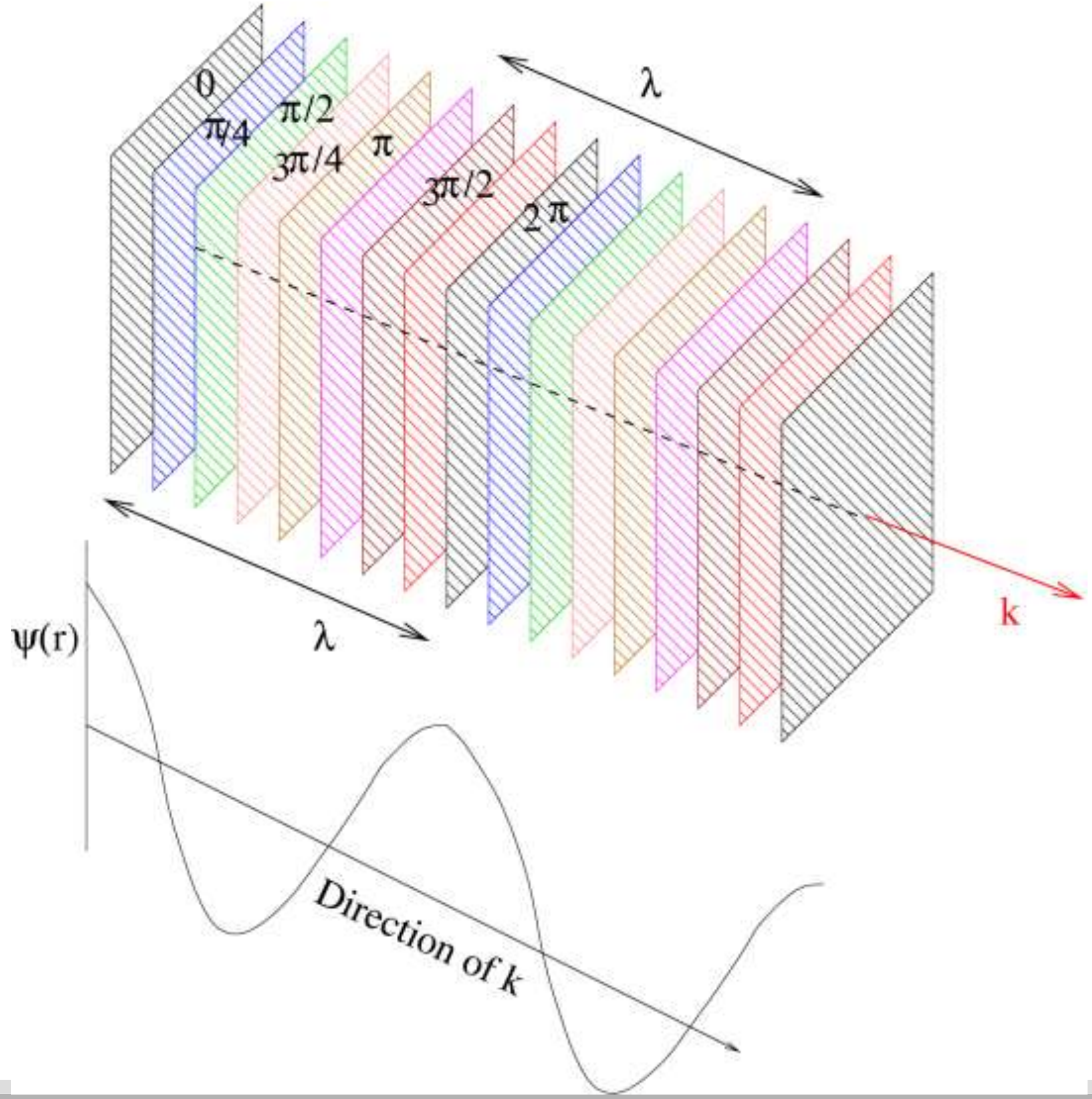
$$\mathbf{r_1} - \mathbf{r_2} = \mathbf{R}$$

$$\mathbf{k}\cdot\mathbf{R} = 0$$



Vectors **k** and **R** are orthogonal to each other. So, the surface swapped by a constant phase is a two dimensional plane and the vector **k** is normal to that plane.

3D Plane waves



Vector Analysis

(Refresh)

Before we go ahead to understand the properties of EM waves, let us learn to write the Laws of Electromagnetism in ***different forms (Differential and Integral forms)*****.**

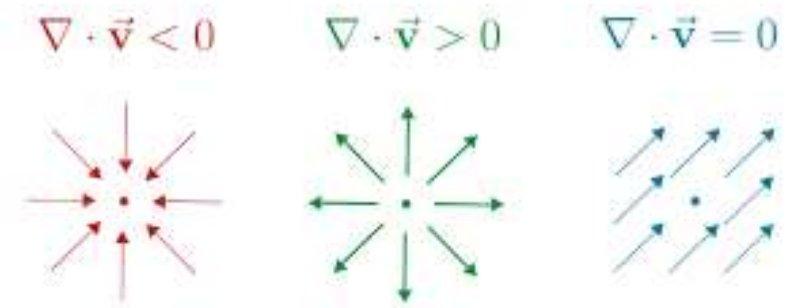
DIVERGENCE OF A VECTOR FIELD

For a vector field ***T*** the divergence of ***T*** is given by:

$$\nabla \cdot \vec{T} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \cdot \left(T_x\hat{x} + T_y\hat{y} + T_z\hat{z} \right)$$

$$= \left(\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z} \right)$$

It is a measure of how much the vector ***T*** diverges / spreads out from the point in question.



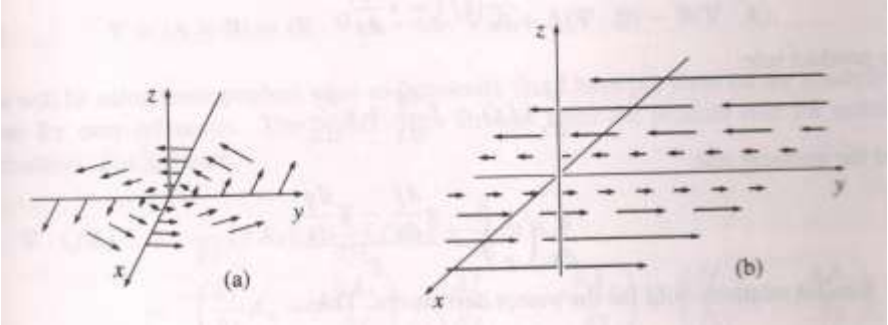
CURL

For a vector field ***T,*** the Curl of ***T*** is given by:

$$\nabla \times \vec{T} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \times \left(T_x \hat{x} + T_y \hat{y} + T_z \hat{z} \right)$$

$$= \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ T_x & T_y & T_z \end{pmatrix}$$

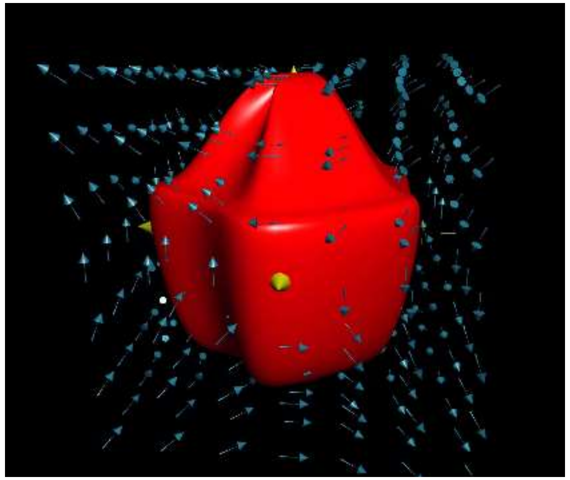
It is a measure of how much the vector ***T*** curls around the point in question.



DIVERGENCE THEOREM

/ Green's Theorem / Gauss's Theorem

- $\mathbf{F}(x, y, z)$ is a three-dimensional vector field.
  - V is some three-dimensional volume (think of a blob floating in space).
  - S is the surface of V.



- The divergence theorem relates the divergence of $\mathbf{F}$ within the volume V to the outward flux of $\mathbf{F}$ through the surface S:

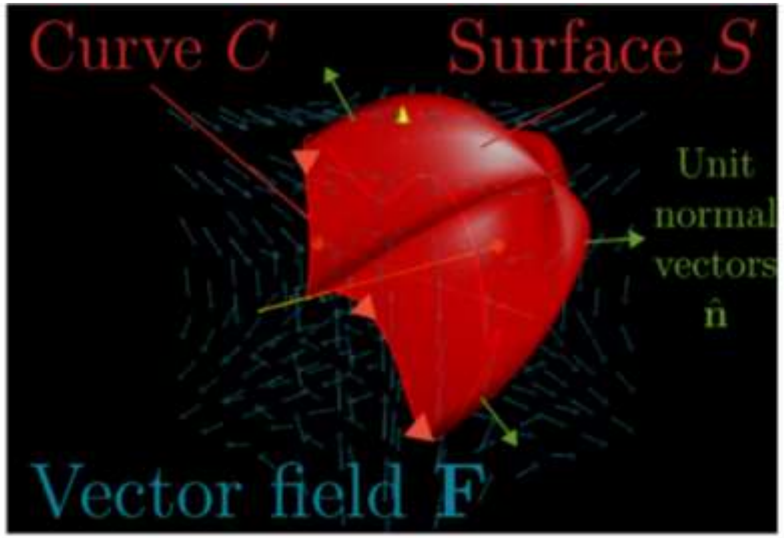
$$\underbrace{\iiint_V \operatorname{div} \mathbf{F}\, dV}_{\substack{\text{Add up little bits} \\ \text{of outward flow in } V}} = \underbrace{\overbrace{\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, d\Sigma}^{\text{Flux integral}}}_{\substack{\text{Measures total outward} \\ \text{flow through } V\text{'s boundary}}}$$

3D divergence theorem (article) | Khan Academy

$$\int_V (\nabla \cdot \vec{E}) d\tau = \oint_S \vec{E} \cdot d\vec{a}$$

Integral of a derivative (in this case the divergence) of a vector over a volume is equal to the outward flux of the vector through the surface that bounds the volume.

STOKES' THEOREM



Stokes' theorem (article) | Khan Academy

- It relates the surface integral of the curl of a vector field with the line integral of that same vector field around the boundary of the surface:

$$\overbrace{\iint_{\underbrace{S}_{S \text{ is a surface in 3D}}} (\text{curl}\,\mathbf{F}\cdot\hat{\mathbf{n}})\,d\Sigma}^{\text{Surface integral of a curl vector field}} = \underbrace{\int_C \mathbf{F}\cdot d\mathbf{r}}_{\text{Line integral around boundary of surface}}$$

[Breakdown of terms]

$$\int_S (\nabla\times\vec{E})\cdot d\vec{a} = \oint_P \vec{E}\cdot d\vec{l}$$

Laws of Electromagnetism

Laws of Electromagnetism in **Integral and Differential** forms:

Formulation in SI units

Name	Integral equations	Differential equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot\mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_\Omega \rho\,\mathrm{d}V$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$	The electric field leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot\mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{B}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell} = \mu_0\iint_\Sigma \mathbf{J}\cdot\mathrm{d}\mathbf{S} + \mu_0\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{E}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

https://i.stack.imgur.com/2QoMD.png

Laws of Electromagnetism

Put together, these are "Maxwell's equations' in vacuum

Gauss's laws

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere's law (modified)

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Plane EM waves in vacuum

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}\exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t))$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}\exp(i(k_x x + k_y y + k_z z - \omega t))$$

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}\exp(i(k_x x + k_y y + k_z z - \omega t))$$

Plane EM waves in vacuum

$$\mathbf{k}\cdot\mathbf{k} = k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2/c^2$$

$$\nabla\cdot\mathbf{E}(\mathbf{r},t) = i\mathbf{k}\cdot\mathbf{E} = 0$$

Wave vector **k** is perpendicular to **E**

$$\nabla\cdot\mathbf{B}(\mathbf{r},t) = i\mathbf{k}\cdot\mathbf{B} = 0$$

Wave vector **k** is perpendicular to **B**

Plane EM waves in vacuum

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}$$

$$\nabla \times \mathbf{B}(\mathbf{r},t) = \mu_0\epsilon_0 \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t}$$

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2}\mathbf{E}$$

$$\hat{\mathbf{k}} \times \mathbf{E} = \frac{\omega}{k}\mathbf{B} = c\mathbf{B}$$

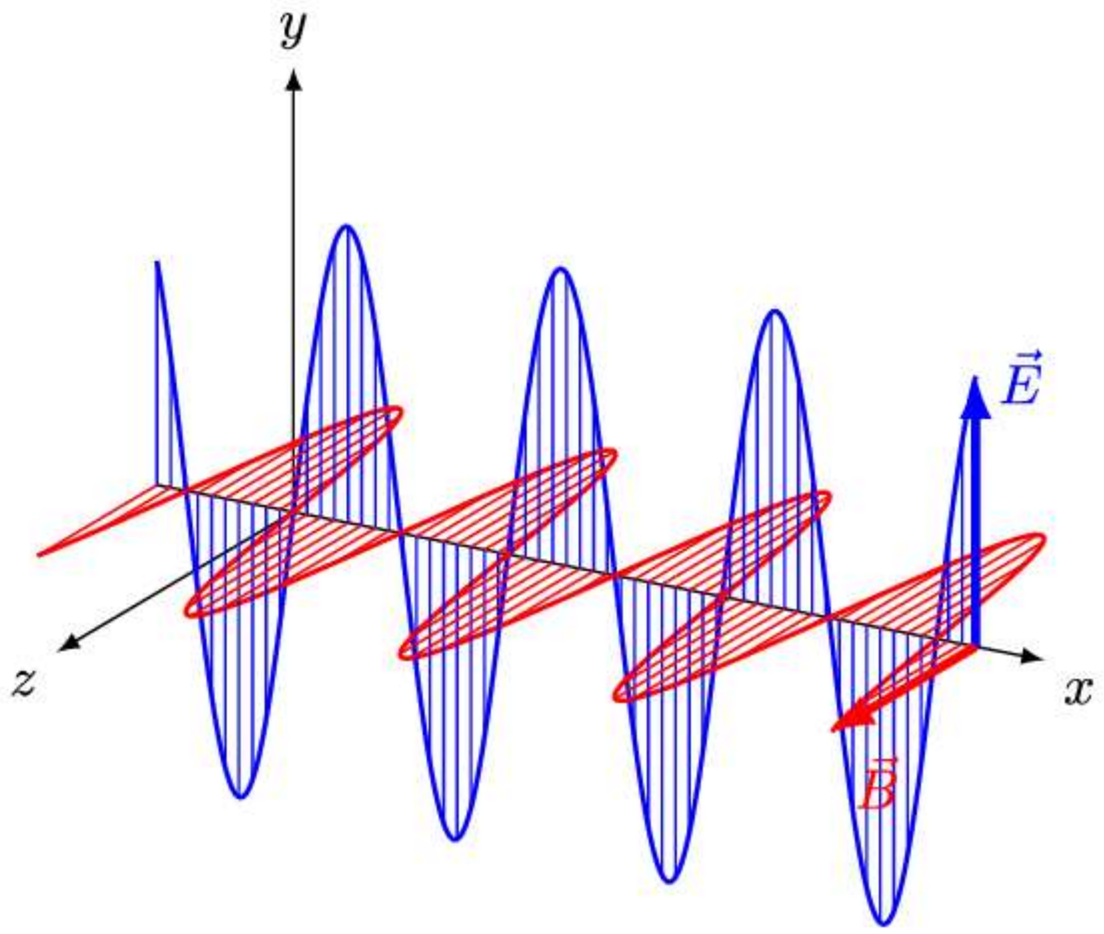
$$\mathbf{B} \times \hat{\mathbf{k}} = \frac{\omega}{kc^2}\mathbf{E} = \frac{1}{c}\mathbf{E}$$

B **is perpendicular to** **E**

$$c\mathbf{B} \times \hat{\mathbf{k}} = \mathbf{E}$$

B, k and E make a right handed Cartesian co-ordinate system

Plane EM waves in vacuum



em waves animated gif - Google Search

Energy and Momentum (EM waves)

EM Waves Transport Energy and Momentum

The energy density of the E field:

$$u_E = \frac{1}{2}\varepsilon_o E^2$$

Similarly, the energy density of the B field:

$$u_B = \frac{1}{2\mu_o} B^2$$

Using: ***E* = *cB*** and $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$ $u_E = u_B$

The energy streaming through space in the form of EM wave is shared equally between constituent electric and magnetic fields.

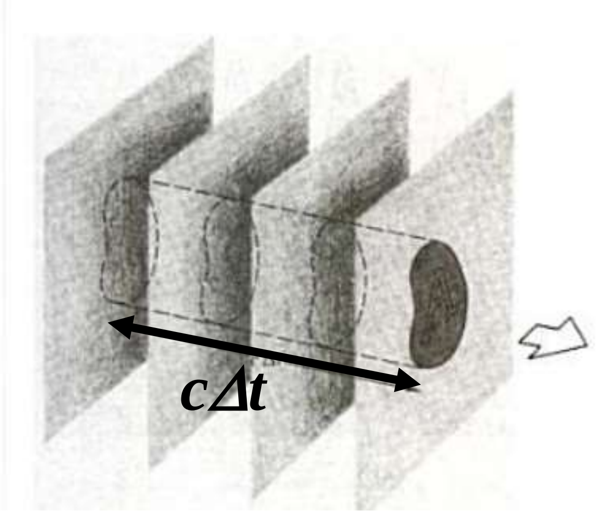
EM Waves Transport Energy and Momentum

Total energy density of the EM field: $u = u_E + u_B = \varepsilon_o E^2 = \dfrac{1}{\mu_o} B^2$

Let a vector **S** represent the flow of electromagnetic energy associated with a traveling wave. Specifically, let **S** represent the transport of energy per unit time per unit area: **Poynting Vector**

$$S = \frac{uc\Delta tA}{\Delta tA} = uc$$

$$S = \frac{1}{\mu_o} EB$$



In isotropic media, the energy flows in the direction of the propagation of wave

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B} \qquad \vec{S} = c^2 \varepsilon_o \vec{E} \times \vec{B}$$

The magnitude of **S** is the power per unit area crossing a surface whose normal is parallel to **S**.

EM Waves Transport Energy and Momentum

Given:

$$\vec{E} = \vec{E}_o \cos(\vec{k} \cdot \vec{r} - \omega t)$$

$$\vec{B} = \vec{B}_o \cos(\vec{k} \cdot \vec{r} - \omega t)$$

Instantaneous flow of energy per unit area per unit time

$$\vec{S} = c^2 \varepsilon_o \vec{E}_o \times \vec{B}_o \cos^2(\vec{k} \cdot \vec{r} - \omega t)$$

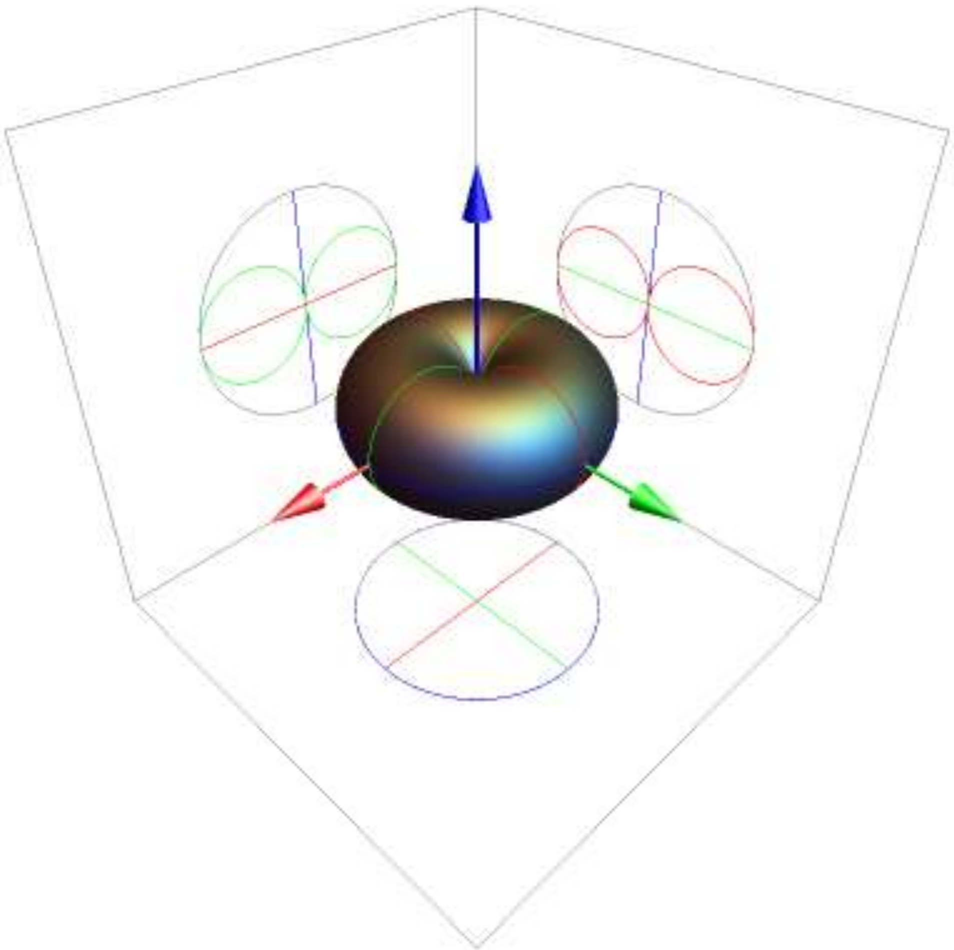
Time averaged value of the magnitude of the Poynting vector

$$\langle S \rangle = \frac{c^2 \varepsilon_o}{2} \left| \vec{E}_o \times \vec{B}_o \right|$$

The irradiance is proportional to the square of the amplitude of the electric field:

$$I \equiv \langle S \rangle = \frac{c\varepsilon_o}{2} E_o^2 \qquad I = c\varepsilon_o \langle E^2 \rangle$$

DIPOLE RADIATION PATTERN (POYNTING VECTOR)



Electric Dipole Propagation Pattern [Dipole Antenna] 📡 - YouTube

EM Waves Transport Energy and Momentum

EM waves transport energy:

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$

EM wave transport momentum:

$$p = U / c$$

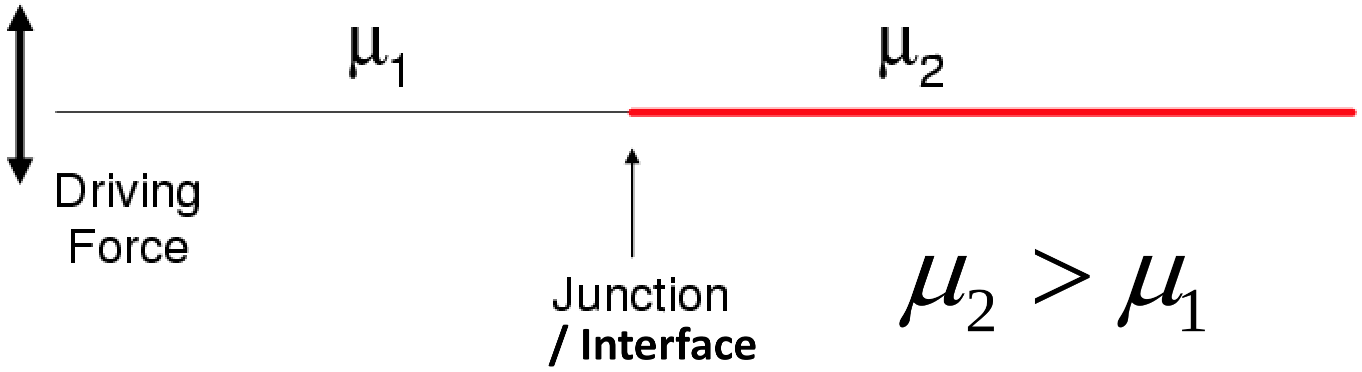
U: Energy of the EM wave
c: Speed of the EM wave

They exert a pressure:

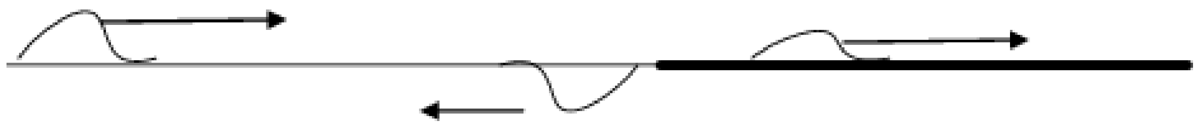
$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{cA}\frac{dU}{dt} = \frac{S}{c}$$

Reflection and Transmission

Reflection and Transmission



$$\mu_2 > \mu_1$$



$$y_{inc} = A\cos(k_1\, x - \omega\, t)$$

$$y_{ref} = B\cos(k_1\, x + \omega\, t)$$

$$y_{trans} = C\cos(k_2\, x - \omega\, t)$$

Reflection and Transmission

On the left side of the junction we have

$$y_l = y_{inc} + y_{ref}$$
$$= A\cos(k_1\,x - \omega\,t) + B\cos(k_1\,x + \omega\,t)$$

and on the right side of the junction we have

$$y_r = y_{trans} = C\cos(k_2\,x - \omega\,t).$$

At the boundary $x = 0$ the wave must be continuous, (as there are no kinks in it). Thus we must have

$$y_l(0,t) = y_r(0,t)$$
$$\left.\frac{\partial y_l(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial y_r(x,t)}{\partial x}\right|_{x=0}$$

So from the first equation

$$A\cos(\omega\,t) + B\cos(\omega\,t) = C\cos(\omega\,t)$$

$$\boxed{A + B = C}$$

$$\left.\frac{\partial y_l(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial y_r(x,t)}{\partial x}\right|_{x=0}$$
$$-A\,k_1\sin(-\omega\,t) - k_1\,B\sin(\omega\,t) = -k_2\,C\sin(-\omega\,t)$$
$$(A - B)\,k_1\sin\omega\,t = C\,k_2\sin\omega\,t$$

$$\boxed{A - B = \frac{k_2}{k_1}C}$$

Transmission Coefficient

$$A + B = C$$

$$A - B = \frac{k_2}{k_1}\, C$$

$$2\, A = \left(1 + \frac{k_2}{k_1}\right) C$$

We can define the transmission coefficient: (C/A)

$$t_r \equiv C / A = \frac{2\, k_1}{k_1 + k_2}$$

$$y_{inc} = A \cos(k_1\, x - \omega\, t)$$

$$y_{ref} = B \cos(k_1\, x + \omega\, t)$$

$$y_{trans} = C \cos(k_2\, x - \omega\, t)$$

Reflection Coefficient

$$y_{inc} = A\cos(k_1 x - \omega t)$$
$$y_{ref} = B\cos(k_1 x + \omega t)$$
$$y_{trans} = C\cos(k_2 x - \omega t)$$

We can define the Reflection coefficient: (B/A)

$$A + B = C$$

$$r \equiv B/A = \frac{C}{A} - 1 = \frac{k_1 - k_2}{k_1 + k_2}$$

$$t_r \equiv C/A = \frac{2\,k_1}{k_1 + k_2}$$

Rigid End: $\mu_2 \to \infty\ (\mu_2 >> \mu_1)$

$k_2 \to \infty$

$$r = \frac{k_1 - k_2}{k_1 + k_2} = \frac{\frac{k_1}{k_2} - 1}{\frac{k_1}{k_2} + 1}$$

$$r \to -1$$

When $\mu_2 > \mu_1$,

$r < 0$

Change in sign of the reflected pulse

External Reflection

Reflection Coefficient

Free End: $\mu_2 \rightarrow 0$ $(\mu_2 << \mu_1)$

$k_2 \rightarrow 0$

$$r = \frac{k_1 - k_2}{k_1 + k_2}$$

$$= \frac{k_1}{k_1}$$

$$r \rightarrow +1$$

When $\mu_2 < \mu_1$,

$r > 0$

No Change in sign of the reflected pulse

Internal Reflection

Transmission Coefficient

$$t_r \equiv C/A = \frac{2\,k_1}{k_1 + k_2}$$

In either case: $\boldsymbol{t_r \geq 0}$

No Change in phase of the transmitted pulse

Stoke's relations

It can be shown that $r_{12} = -r_{21}$

$$r \equiv B/A = \frac{C}{A} - 1 = \frac{k_1 - k_2}{k_1 + k_2}$$

$$1 - r_{12}^2 = t_{12} t_{21}$$ **Stoke's relations**

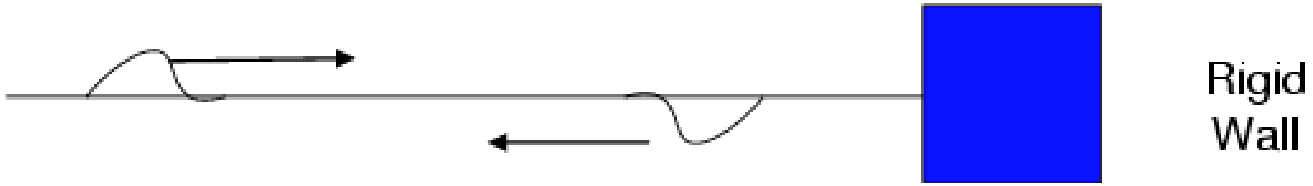
The reflectance and transmittance of Intensity is proportional to square of Amplitude

$$R_{12} = r_{12}^2$$

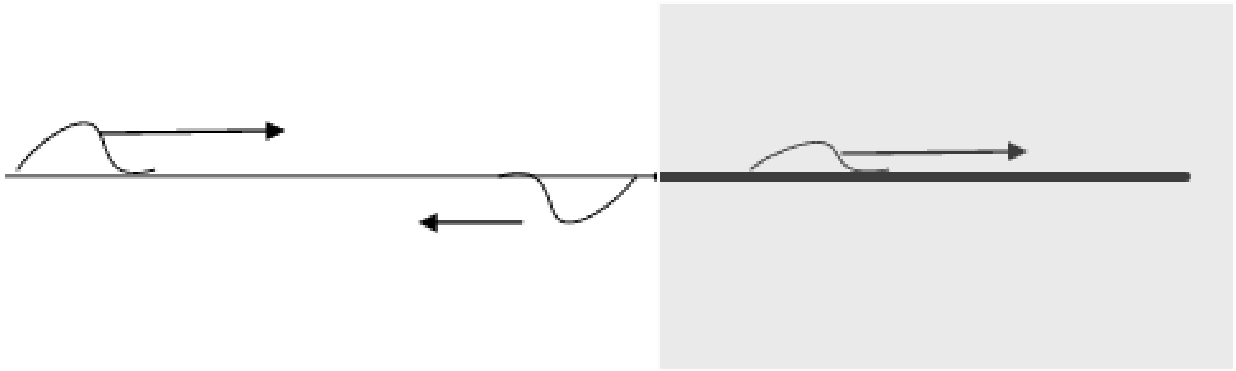
$$T_{12} = 1 - R_{12}$$

Inhomogeneous Medium (Reflection and Transmission): A Summary

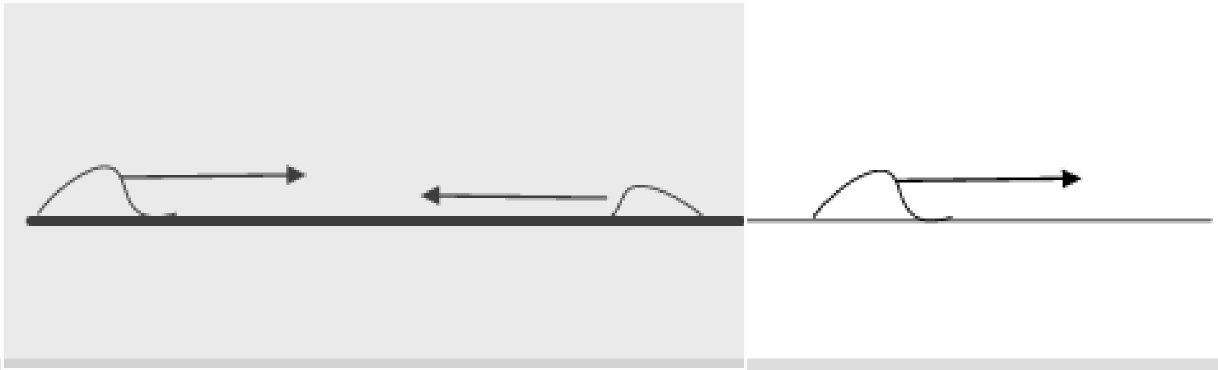
Various possibilities for Wave incident at interface of materials with different density / refractive index



Wave incident at interface of
Low to High density / refractive index:
π **phase change**
External Reflection



Wave incident at interface of
High to Low density / refractive index:
No phase change
Internal Reflection



Huygens's and Fermat's Principles

Fermat’s Principle

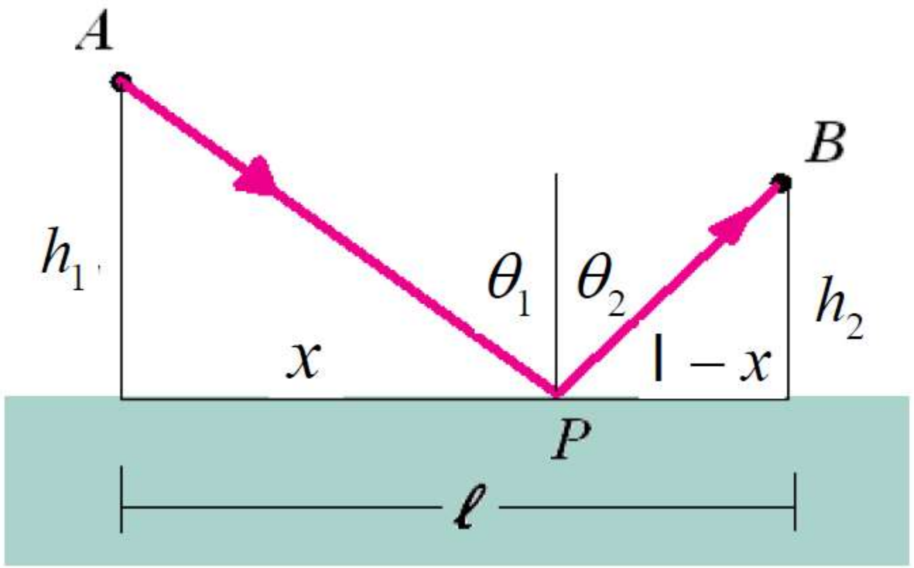
In optics, **Fermat's principle** or the **principle of least time** is the principle that the path taken between two points by a ray of light is the path that can be traversed in the least time. This principle is sometimes taken as the definition of a ray of light.

From Fermat’s principle, one can derive

(a) the law of reflection [the angle of incidence is equal to the angle of reflection] and

(b) the law of refraction [Snell’s law]

Law of Reflection



The time required for the light to traverse the path

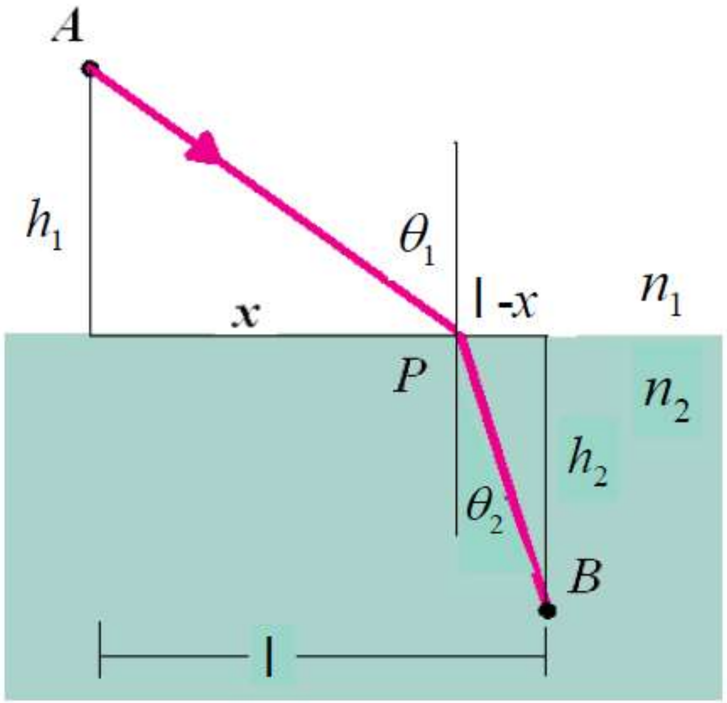
$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c}$$

To minimize the time set the derivative to zero

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2 + h_1^2}} + \frac{-(l - x)}{c\sqrt{(l - x)^2 + h_2^2}} \quad \rightarrow$$

$$\frac{x}{\sqrt{x^2 + h_1^2}} = \frac{(l - x)}{\sqrt{(l - x)^2 + h_2^2}} \quad \rightarrow \quad \sin\theta_1 = \sin\theta_2 \quad \rightarrow \quad \boxed{\theta_1 = \theta_2}$$

Law of Refraction



The time required for the light to traverse the path

$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c/n_2}$$

To minimize the time set the derivative to zero

$$0 = \frac{dt}{dx} = \frac{n_1 x}{c\sqrt{x^2 + h_1^2}} + \frac{-n_2 (l - x)}{c\sqrt{(l - x)^2 + h_2^2}} \rightarrow \frac{n_1 x}{\sqrt{x^2 + h_1^2}} = \frac{n_2 (l - x)}{\sqrt{(l - x)^2 + h_2^2}}$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

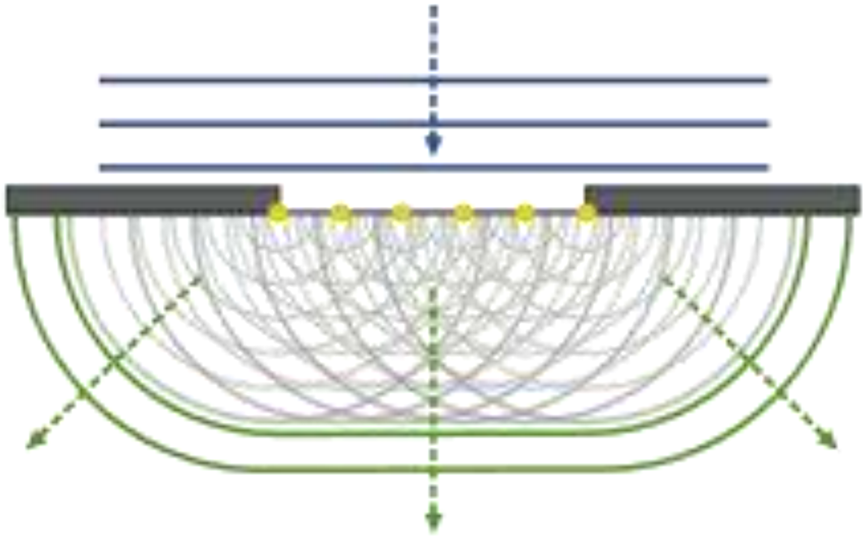
Huygens's Principle

Every point on a propagating wavefront serves as a source of spherical secondary wavelets, such that the wavefront at some later time is the envelope of these wavelets.

- Every unobstructed point on a wavefront will act as a source of secondary spherical waves.
- The new wavefront is the surface tangent to all the secondary spherical waves.

Huygens's Principle

When a part of the wave front is cut off by an obstacle, and the rest admitted through apertures, the wave on the other side is just the result of superposition of the Huygens wavelets emanating from each point of the aperture, ignoring the portions obscured by the opaque regions.



Huygens–Fresnel principle - Wikipedia

Principle of Superposition

Principle of Superposition

Superposition of several sinusoidal waves with same frequency and polarization (oscillations in the same plane) but different amplitudes and phases is again a sinusoidal wave with the same frequency. Resultant amplitude and phase are obtained by adding the individual phasors vectorially.

Consider: $|A_n| \cos(\omega t + \phi_n) = Re\, \boldsymbol{A_n} \exp(i\omega t)$

where, $\boldsymbol{A_n} = |A_n| \exp(i\phi_n)$

Superposition:

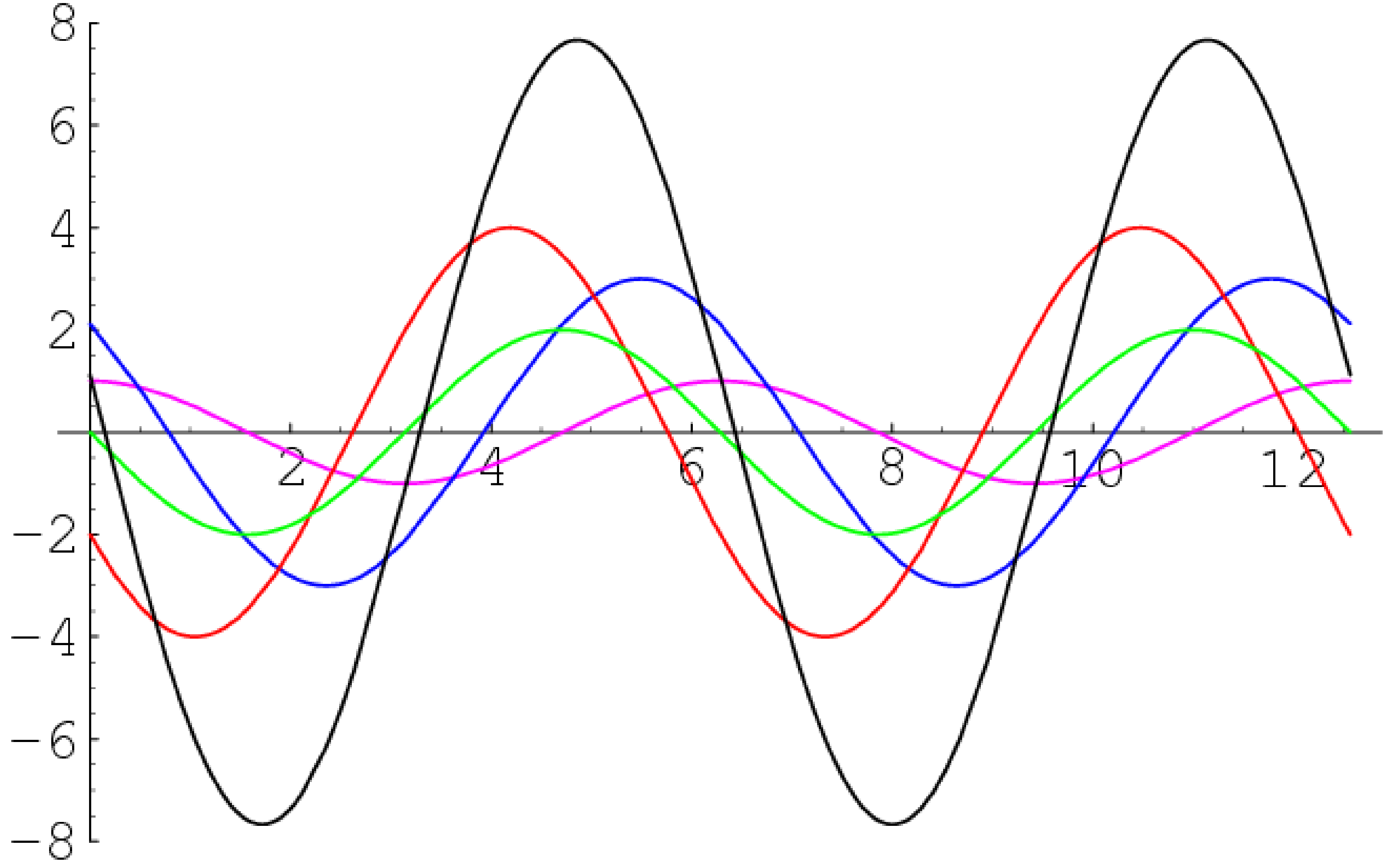
$$\mathbf{A}_1 \exp(i\omega t) + \mathbf{A}_2 \exp(i\omega t) + \mathbf{A}_3 \exp(i\omega t) + \cdots = \sum_n \mathbf{A}_n \exp(i\omega t)$$
$$= \mathbf{A} \exp(i\omega t)$$

Resultant phasor: $\mathbf{A} = |A| \exp(i\phi) = \sum_n \mathbf{A}_n = \sum_n |A_n| \exp(i\phi_n)$

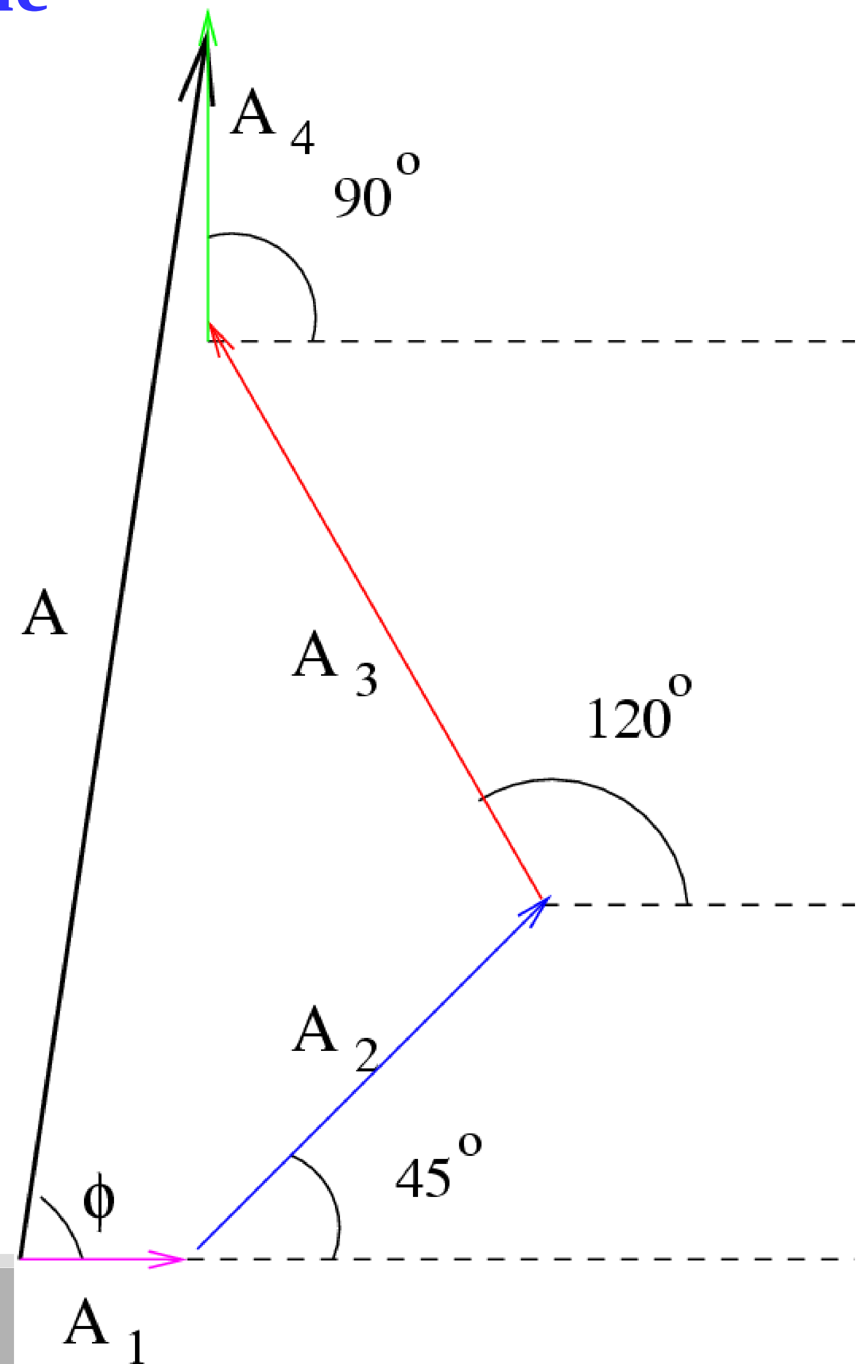
Intensity (irradiance): $I = \mathbf{A}\mathbf{A}^* = |A|^2$

Example $\underline{\cos t} + \underline{3\cos(\pi/4 + t)} + \underline{4\cos(2\pi/3 + t)}$

$$+\underline{2\cos(\pi/2 + t)}$$



Example



$|A_1| = 1$

$|A_2| = 3$

$|A_3| = 4$

$|A_4| = 2$

$|A| \cong 7.5$

$\phi \cong 81^{\circ}$

$$\cos t + 3\cos(\pi/4 + t) + 4\cos(2\pi/3 + t)$$
$$+2\cos(\pi/2 + t)$$

Problem

Superposition of a large number of phasors of equal amplitude *a* and equal successive phase difference θ. Find the resultant phasor.

$$A = |A| \exp(i\phi) = a + a\exp(i\theta) + a\exp(i2\theta)$$
$$+a\exp(i3\theta) + \cdots a\exp(i(n-1)\theta)$$

Remember: The sum of the first *n* terms of a geometric series is:

$$a + ar + ar^2 + ar^3 + \ldots + ar^{(n-1)} = \sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$$

Addition of Phasors

$$A = |A|\exp(i\phi) = a + a\exp(i\theta) + a\exp(i2\theta)$$
$$+a\exp(i3\theta) + \cdots a\exp(i(n-1)\theta)$$

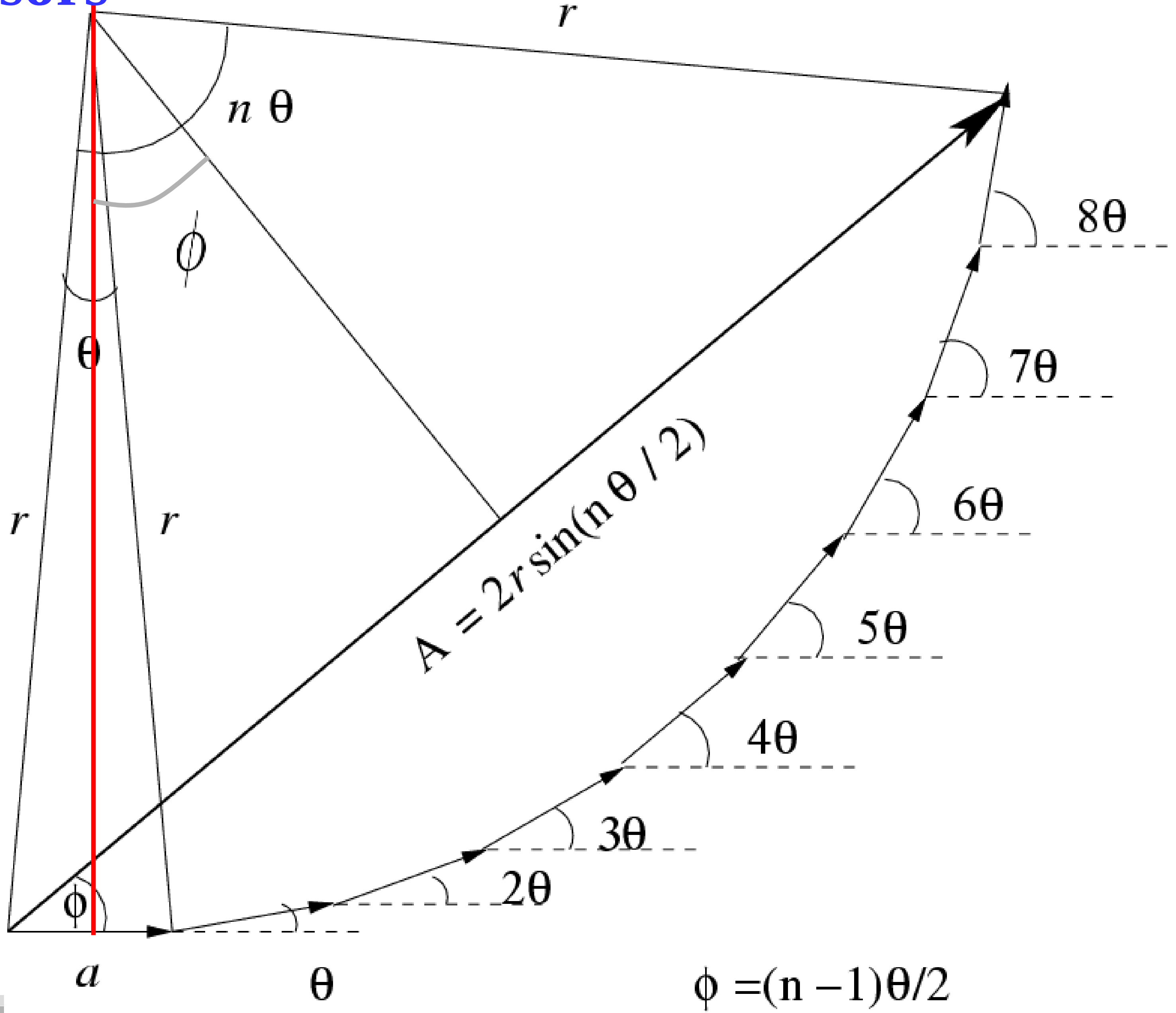
$$A = \frac{a[1-e^{in\theta}]}{[1-e^{i\theta}]} = \frac{ae^{in\theta/2}[e^{in\theta/2}-e^{-in\theta/2}]}{e^{i\theta/2}[e^{i\theta/2}-e^{-i\theta/2}]}$$

Geometric series

$$= a\frac{\sin(n\theta/2)}{\sin(\theta/2)}e^{i(n-1)\theta/2}$$

$$|A| = a\frac{\sin(n\theta/2)}{\sin(\theta/2)} \qquad \phi = (n-1)\theta/2$$

Addition of Phasors



$a = 2r \sin(\theta/2)$

Coherence:

Spatial, Temporal

Coherence

The concept of coherence is related to stability or predictability of phase

Spatial coherence describes the correlation between signals at different points in space.

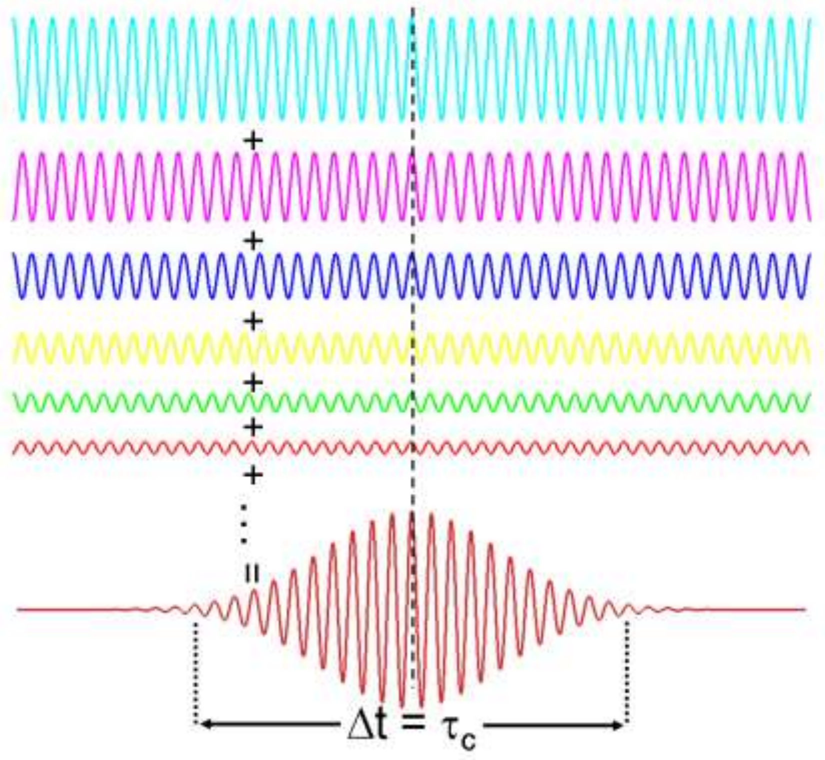
Temporal coherence describes the correlation between signals at different moments of time.

Quantifying Coherence

Physically, monochromatic sources are *fictitious*.

Band of frequencies = $\Delta\nu = 1/(\Delta t)_c$

Wave train / Wave packet: Formed by the superposition of many sinusoidal waves of nearby frequencies



Quantifying Coherence

Temporal coherence:

Coherence time:

The **coherence time** is the time over which a propagating wave may be considered coherent. In other words, it is the time interval within which its phase is, on average, predictable.

$$(\Delta t)_c = {}^{1}/_{\Delta\nu}$$

$\Delta\nu$: Spectral width of the source in units of frequency.

Coherence length:

$$l_c = c(\Delta t)_c$$

The **coherence length** is the coherence time times the vacuum velocity of light, and thus also characterizes the temporal (not spatial!) coherence via the propagation length (and thus propagation time) over which coherence is lost.

Quantifying Coherence

Red Cadmium $\lambda = 6438$ Å

$\Delta\nu = 10^9$ Hz, $l_c = 30$ cm

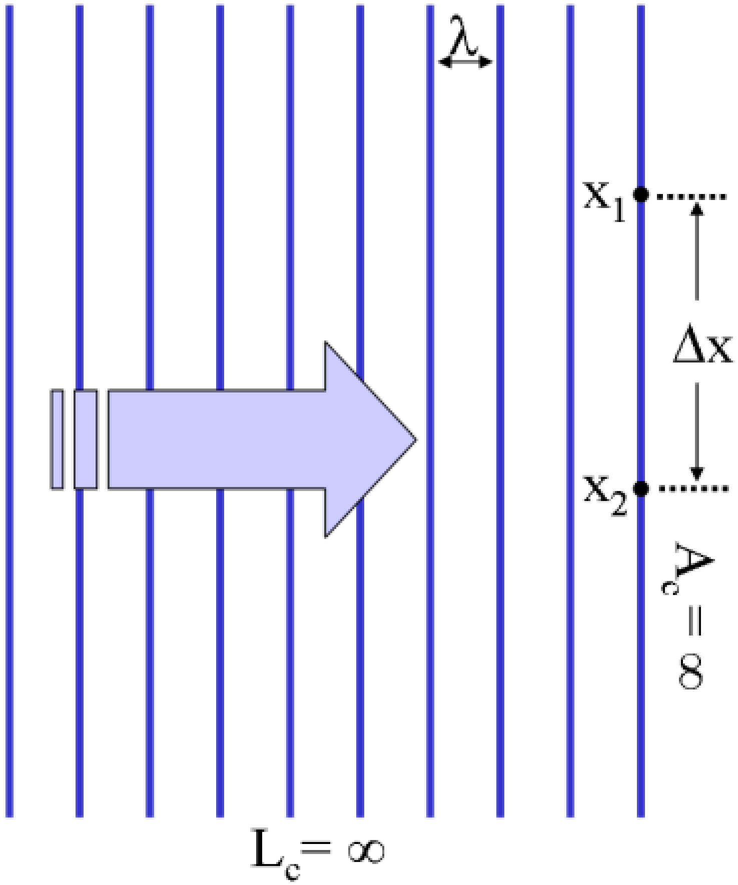
Yellow Sodium $\lambda = 5893$ Å

$\Delta\nu = 10^{10}$ Hz, $l_c = 3$ cm

He-Ne Laser $\lambda = 6328$ Å

$\Delta\nu = 10^6$ Hz, $l_c = 300$ m

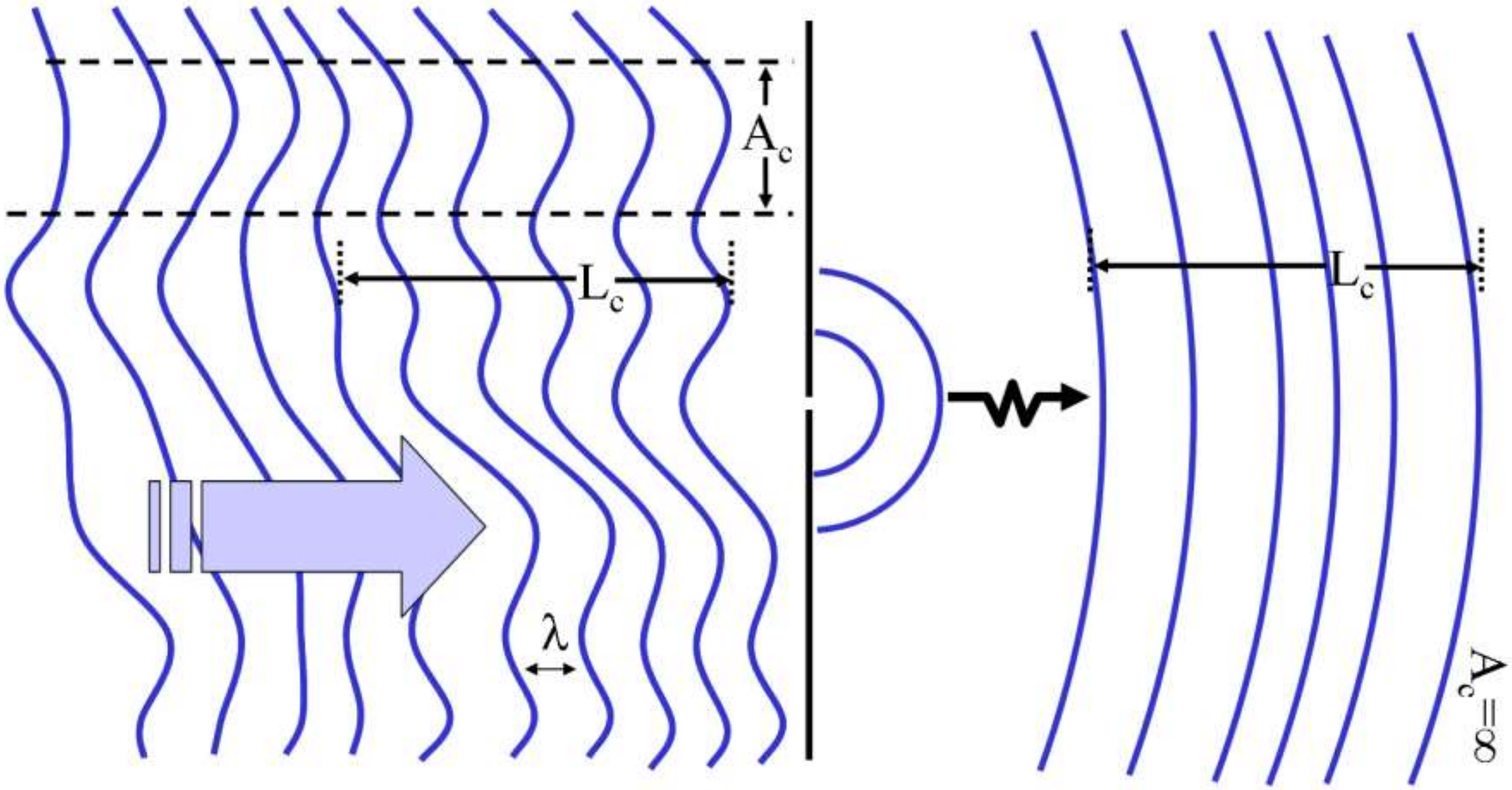
Quantifying Coherence



A plane wave with an infinite coherence length.

Since there are two transverse dimensions, we can define a coherence area (A_c).

Quantifying Coherence



The wave with finite coherence length is passed through a pinhole. The emerging wave has infinite coherence area. The coherence length (or coherence time) are unchanged by the pinhole.

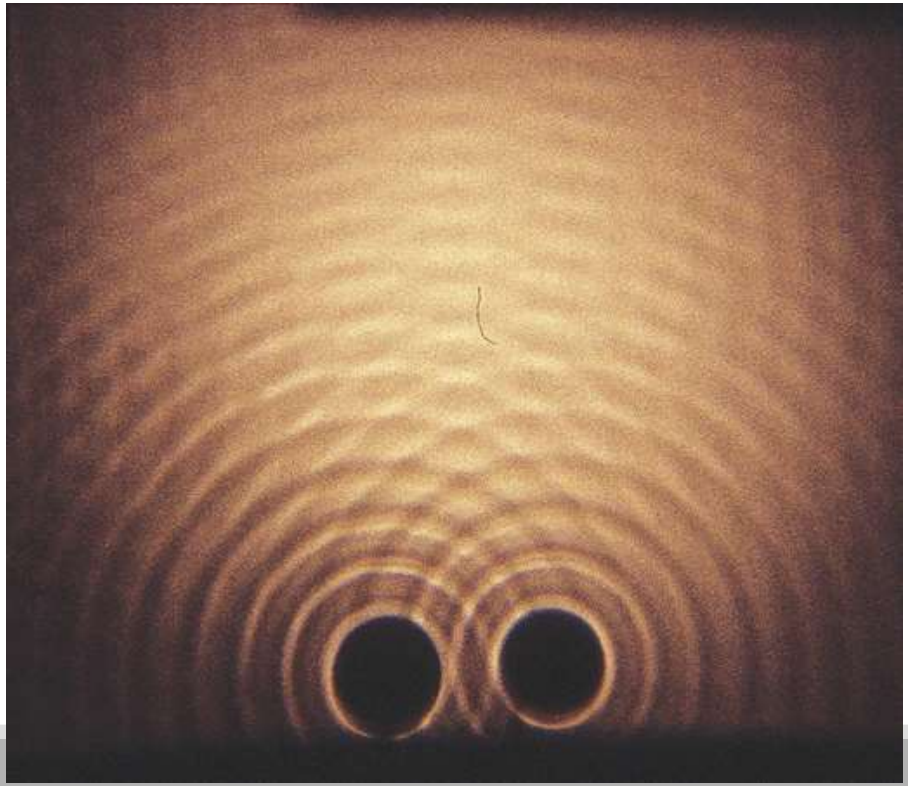
Interference

Interference

Interference is the effect produced by the superposition of waves from two coherent sources passing through the same region.

Coherent Sources

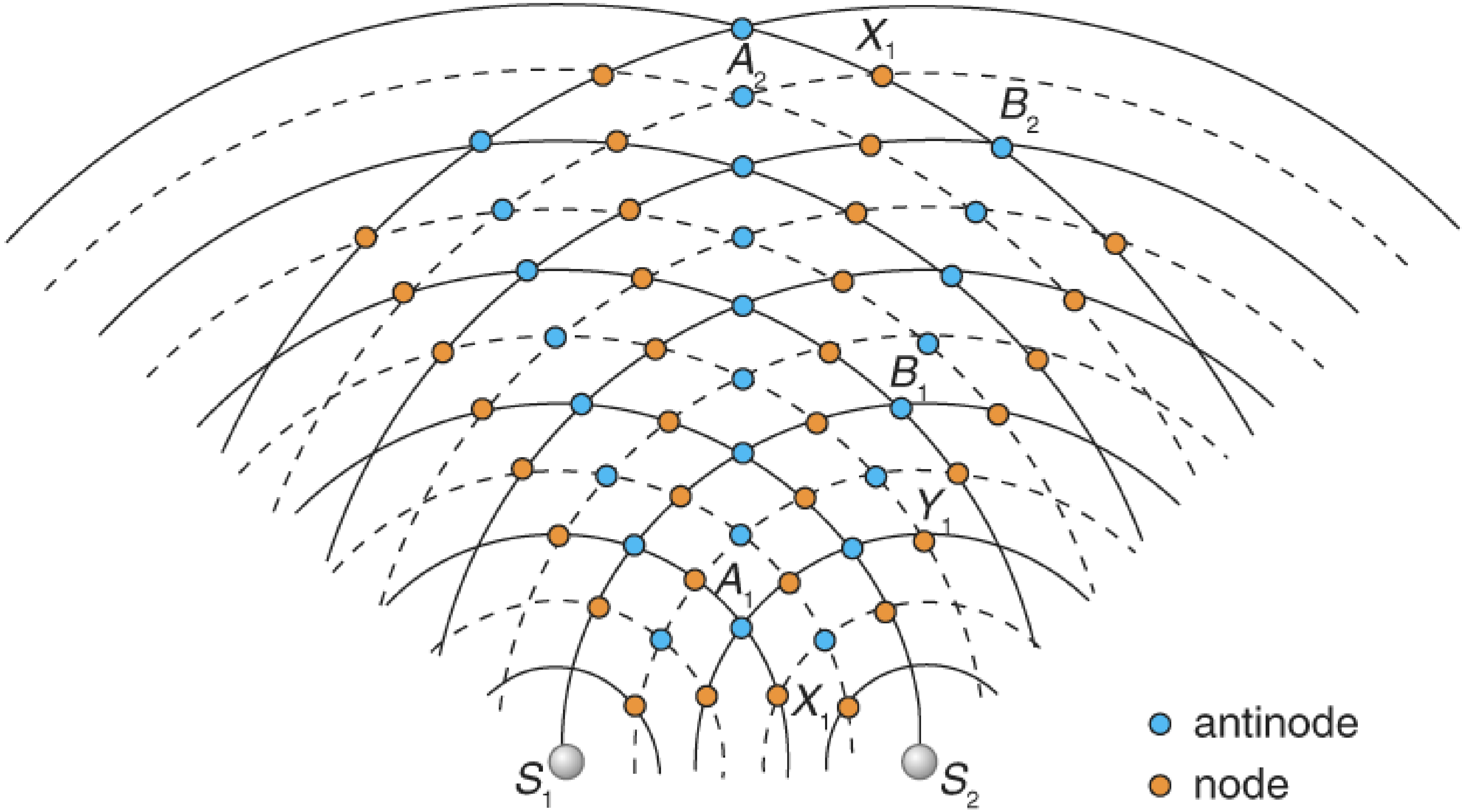
- They maintain a constant phase difference.
- Such sources may or may not be in step but are always marching together.
- They have same frequency.



Interference of water waves

The interference pattern produced in a ripple tank using two sources of circular waves which are in phase with each other.

Constructive & destructive interference



The two sources S_1 and S_2 are in phase and coherent. Therefore, the wavelengths of waves from S_1 and S_2 are the same, say λ.

Optical Interference

Optical interference corresponds to the interaction of two or more light waves yielding a resultant irradiance that deviates from the sum of component irradiance.

- Light waves interfere with each other much like mechanical waves do.
- All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.
- LINEAR SUPERPOSITION!

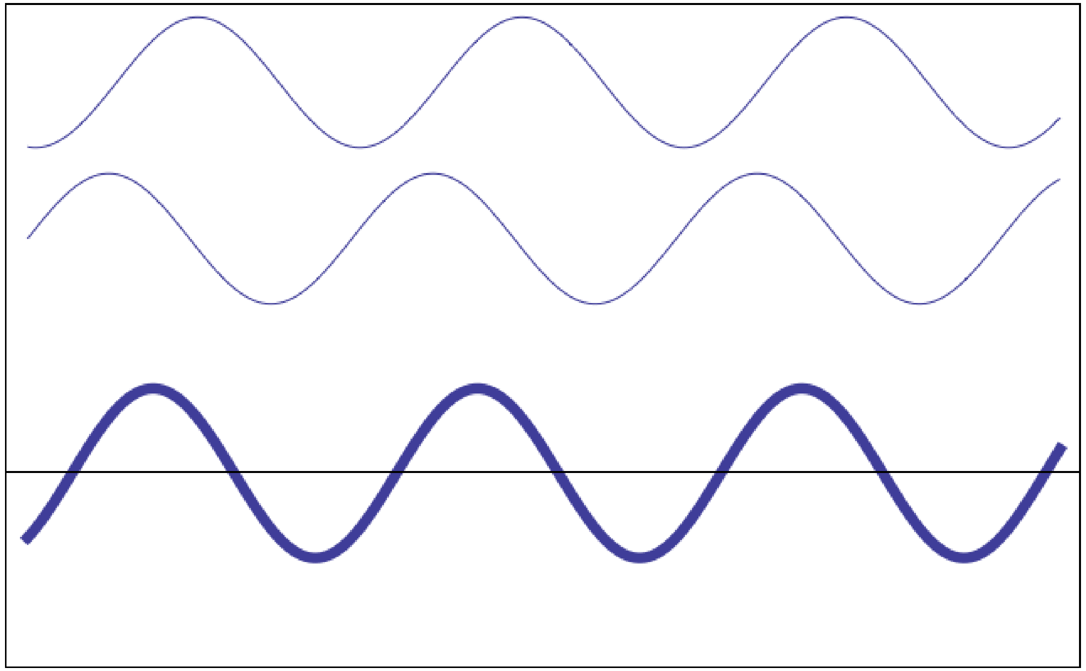
- Wavefront splitting
- Amplitude splitting

Wavefront Division: Involves taking one wavefront and dividing it up into more than one wave.
Ex: Young's double slit interference; Diffraction grating

Amplitude Division: Involves splitting a light beam into two beams at a surface of two media of different refractive index.
Ex: Michelson interferometer

Optical Interference

Resultant: $$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4 + \ldots..$$



http://www.acs.psu.edu/drussell/demos/superposition/superposition.html

Optical Interference

$$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{k}_1.\vec{r} - \omega t + \varepsilon_1)}$$

$$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{k}_2.\vec{r} - \omega t + \varepsilon_2)}$$

Irradiance $I \propto \langle E^2 \rangle$

Strictly speaking irradiance is power/area
And intensity is power/solid angle.

$$E^2 = (\vec{E}_1 + \vec{E}_2).(\vec{E}_1 + \vec{E}_2)$$

$$= E_1^2 + E_2^2 + 2(\vec{E}_1.\vec{E}_2)$$

Taking time average on both sides

$$I = I_1 + I_2 + I_{12}$$

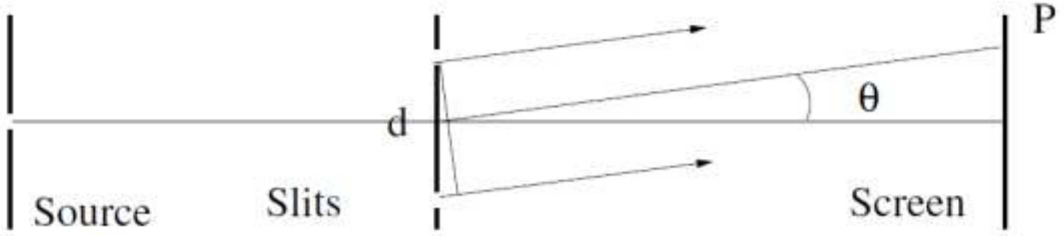
$$I_1 = \langle E_1^2 \rangle$$

$$I_2 = \langle E_2^2 \rangle$$

$$I_{12} = 2\langle \vec{E}_1.\vec{E}_2 \rangle$$

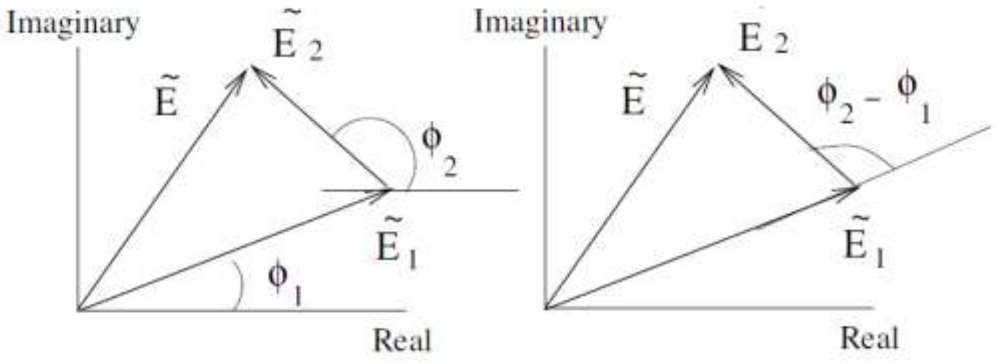
Interference term

Optical Interference



$$\vec{E}_1 = E_{01} e^{i(\vec{k}_1.\vec{r} - \omega t + \varepsilon_1)}\ \hat{\theta}$$

$$\vec{E}_2 = E_{02} e^{i(\vec{k}_2.\vec{r} - \omega t + \varepsilon_2)}\ \hat{\theta}$$



$$\vec{E}_1 . \vec{E}_2$$
$$= E_{01} E_{02} \cos\left(\vec{k}_2 . \vec{r} + \varepsilon_2 - \vec{k}_1 . \vec{r} - \varepsilon_1\right) e^{-2i\omega t}$$

Time average gives: $\langle \vec{E}_1 . \vec{E}_2 \rangle = \dfrac{1}{2} E_{01} E_{02} \cos\delta$

The interference term $I_{12} = 2\langle \vec{E}_1 . \vec{E}_2 \rangle = E_{01} E_{02} \cos\delta = 2\sqrt{I_1 I_2} \cos\delta$

$$\delta = \left(\vec{k}_2 . \vec{r} + \varepsilon_2 - \vec{k}_1 . \vec{r} - \varepsilon_1\right)$$

Optical Interference

$$\vec{E}^2 = \vec{E}_1^2 + \vec{E}_2^2 + 2(\vec{E}_1 \cdot \vec{E}_2)$$

$$I = I_1 + I_2 + I_{12}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$I_1 = \left\langle \bar{E}_1^2 \right\rangle = \frac{E_{01}^2}{2}$$

$$I_2 = \left\langle \bar{E}_2^2 \right\rangle = \frac{E_{02}^2}{2}$$

$$I_{12} = 2\left\langle \bar{E}_1 . \bar{E}_2 \right\rangle = 2\sqrt{I_1 I_2}\cos\delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}Cos\delta$$

Optical Interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,Cos\delta$$

For maximum irradiance $\cos\delta = 1$

$$I_{\max} = I_2 + I_2 + 2\sqrt{I_1 I_2}$$

Total constructive interference $\delta = 0, \pm 2\pi, \pm 4\pi, \ldots\ldots\ldots$

For minimum irradiance $\cos\delta = -1$

$$I_{\max} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Total destructive interference $\delta = \pi, \pm 3\pi, \pm 5\pi, \ldots\ldots\ldots$

Twin Source Interference Pattern

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,Cos\delta$$

For $I_1 = I_2$

$$I = 2I_0(1+\cos\delta)$$

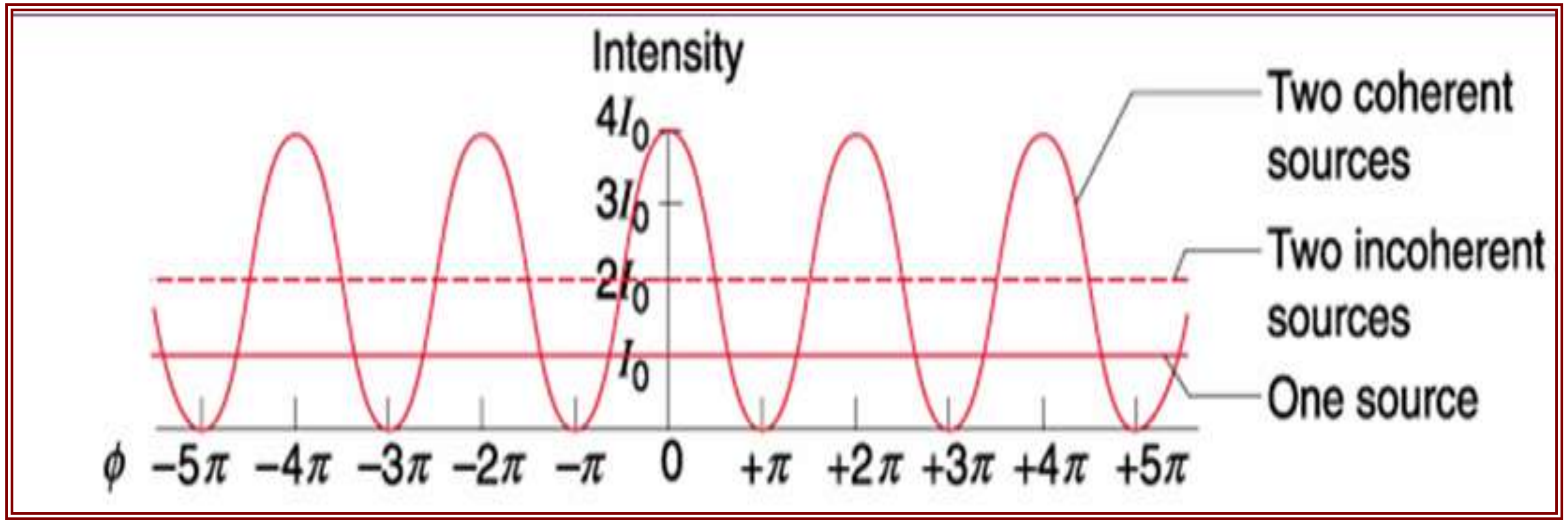
$$= 4I_0\cos^2\frac{\delta}{2}$$

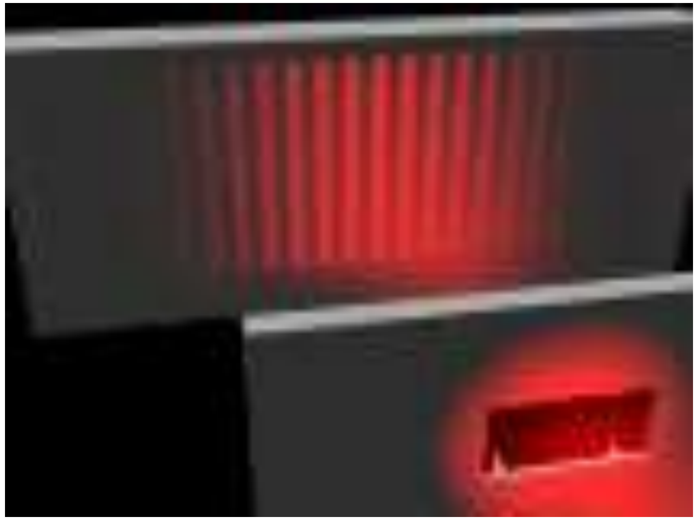
For the spherical wave emitted by two sources, in-phase at the emitter

$$\delta = \bar{k}_1.\bar{r} + \varepsilon_1 - \bar{k}_2.\bar{r} - \varepsilon_2$$

$$\delta = k(\bar{r}_1 - \bar{r}_2)$$

Optical Interference



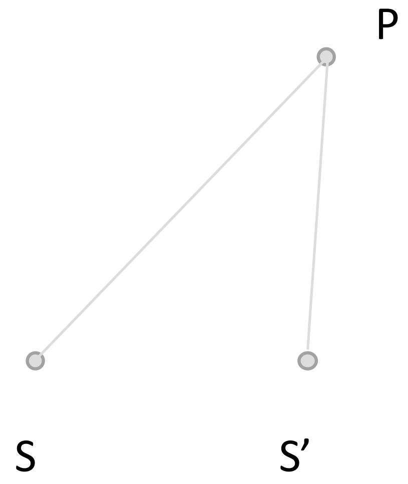


The angular spacing of the fringes, θ_f, is given by:

$$\theta_f \approx \frac{\lambda}{d}$$

where d is the separation between slits

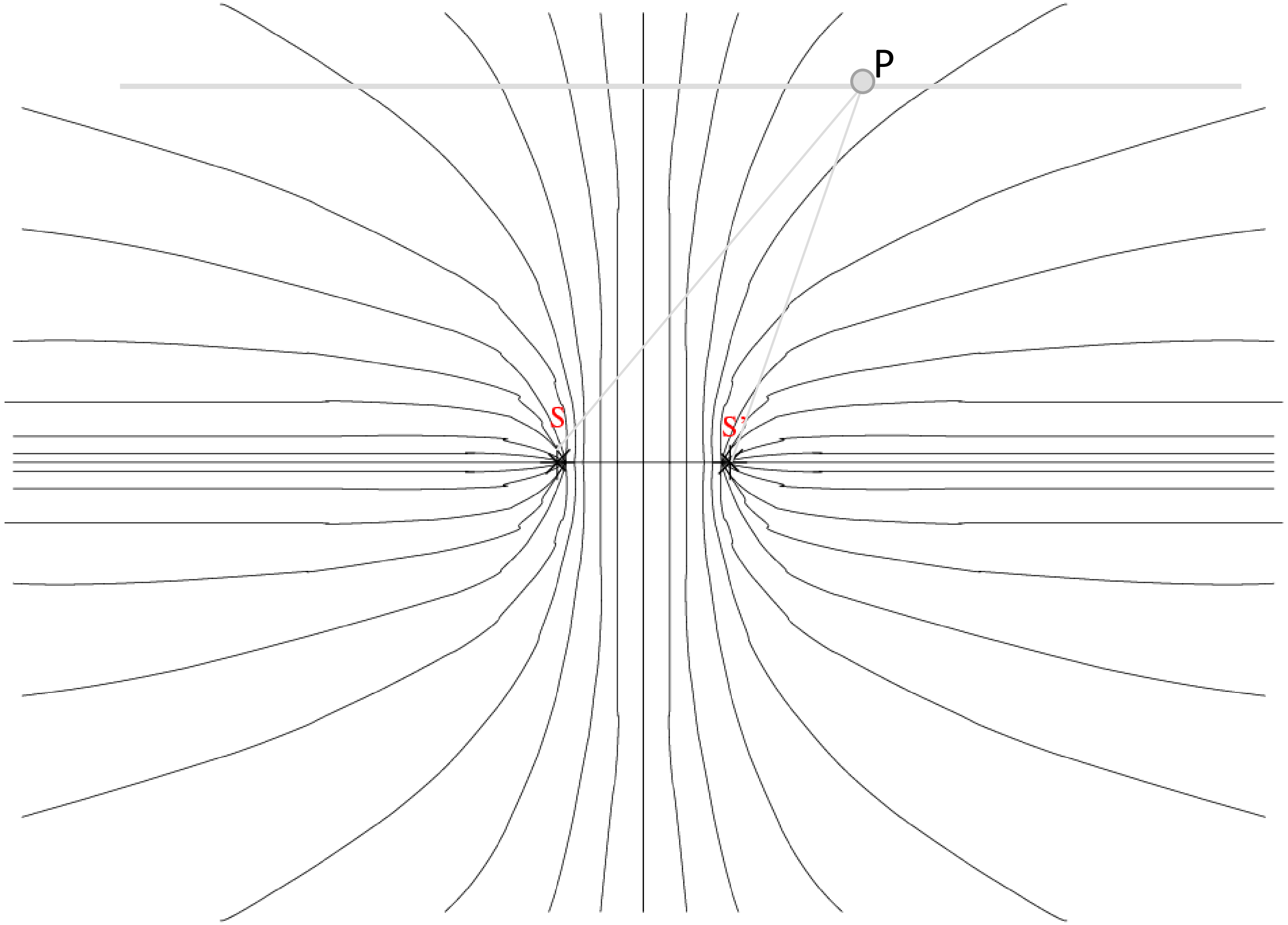
Twin Source Interference:



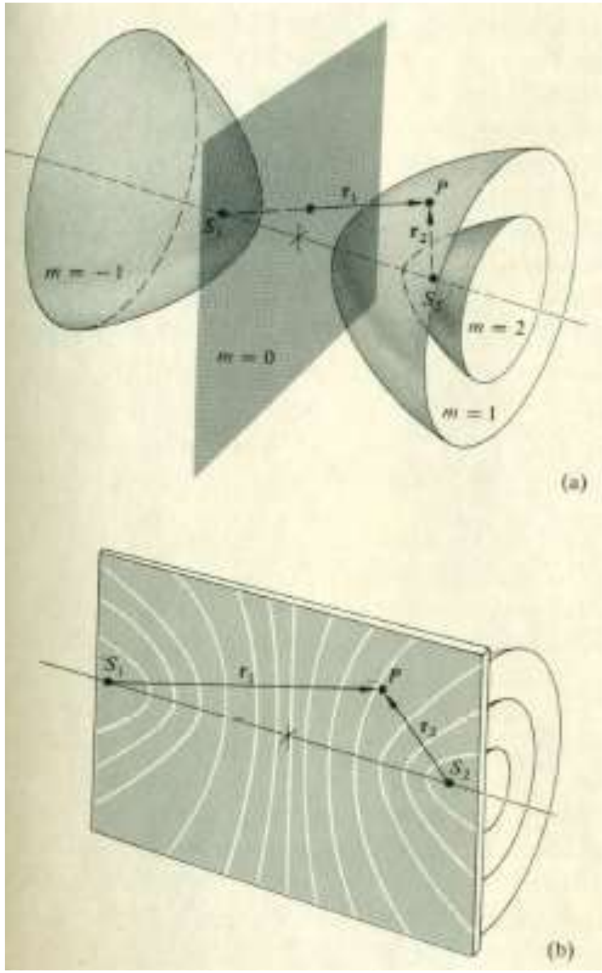
P can be any point around the sources

Path Difference: SP-S’P

Twin Source Interference



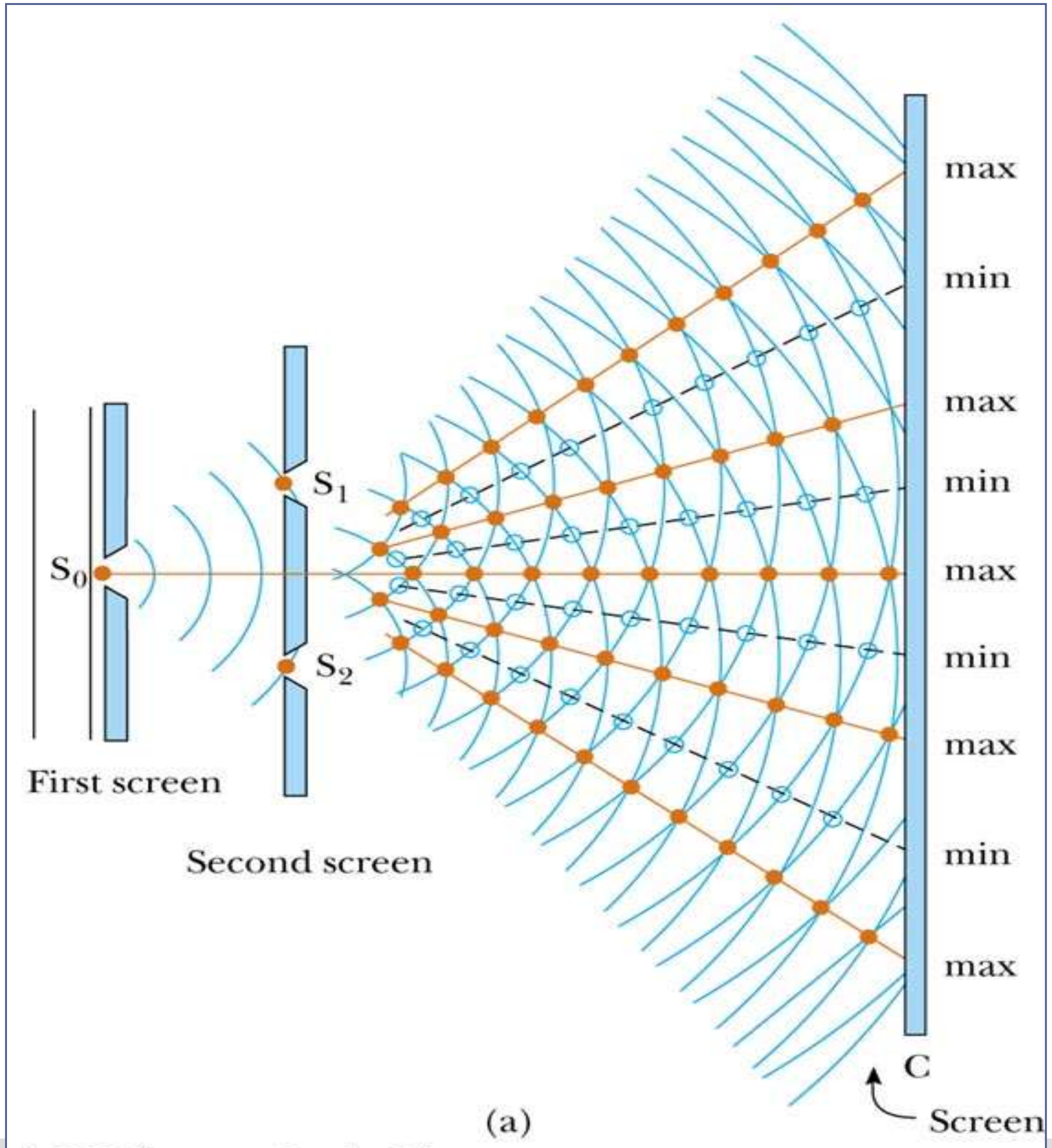
SP-S'P=mλ (condition for intensity maximum)
(Confocal Hyperbolae with S and S' as the common foci)


S_1
$\mathbf{r}_1$
P
$\mathbf{r}_2$
S_2
$m = -1$
$m = 0$
$m = 1$
$m = 2$
(a)
S_1
$\mathbf{r}_1$
P
$\mathbf{r}_2$
S_2
(b)

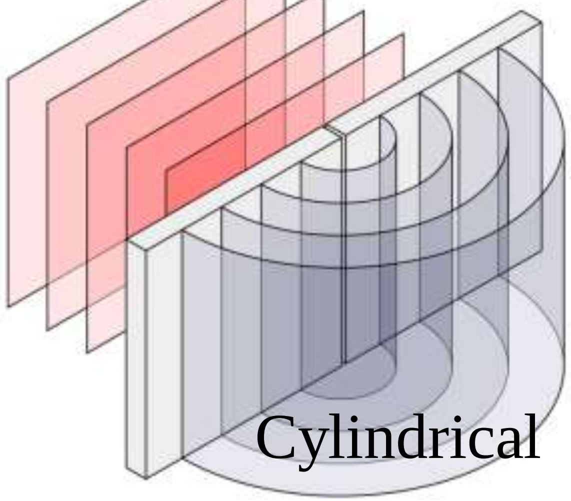
Young's Double Slit Experiment

(Division of Wavefront)

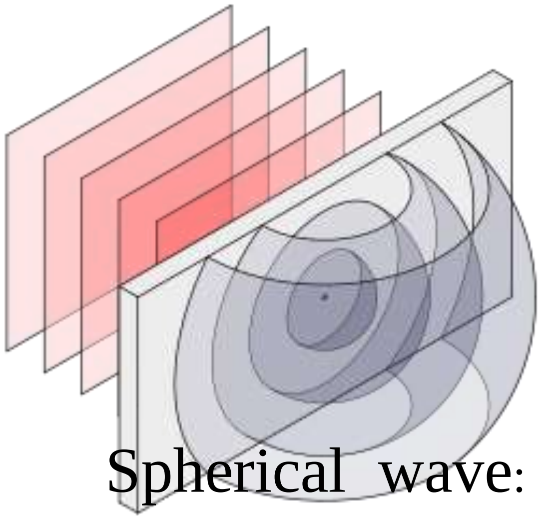
Young's Double Slit Experiment



Plane wave:



Cylindrical wave:



Spherical wave:

Plane wave:

$$\psi_P(\vec{r},t) = Ae^{i(\omega t - \vec{k}.\vec{r})}$$

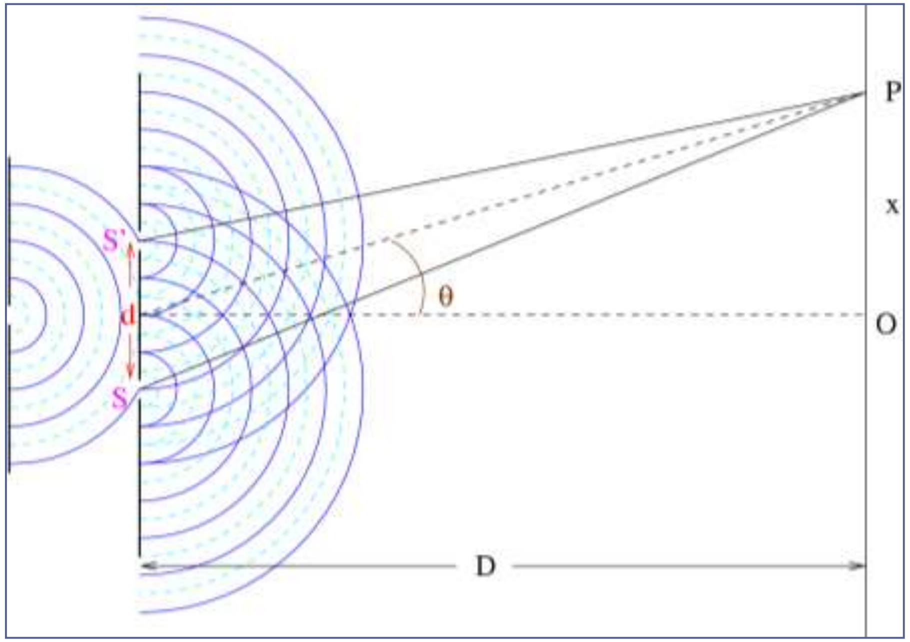
Cylindrical wave:

$$\psi_C(\vec{\rho},t) = \frac{A}{\sqrt{\rho}}e^{i(\omega t - \vec{k}.\vec{\rho})}$$

Spherical wave:

$$\psi_S(\vec{r},t) = \frac{A}{r}e^{i(\omega t - \vec{k}.\vec{r})}$$

Young's double slit



Path difference:

$$SP - S'P = \sqrt{D^2 + (x + d/2)^2} - \sqrt{D^2 + (x - d/2)^2}$$

$$= D\left[1 + \frac{(x + d/2)^2}{D^2}\right]^{1/2} - D\left[1 + \frac{(x - d/2)^2}{D^2}\right]^{1/2}$$

$$D >> x, d$$

For $1 >> x$

$$(1 + x)^n \approx 1 + nx$$

$$= \frac{(x + d/2)^2 - (x - d/2)^2}{2D}$$

$$SP - S'P = \left[(2x)d\right]/2D = xd/D$$

Total irradiance $$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\delta = \frac{2\pi}{\lambda}(SP - S'P)$$

For two beams of equal irradiance (I_0)

$$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

$$(\delta = \textit{phase difference})$$

Path difference $= d\sin\theta$

Phase difference $\delta = d\sin\theta \times \frac{2\pi}{\lambda}$

$$\sin\theta = \frac{x}{D}$$

$$\Rightarrow \quad I = 4I_0\cos^2\frac{\pi x d}{\lambda D}$$

Path difference: $SP - S'P = \dfrac{xd}{D}$

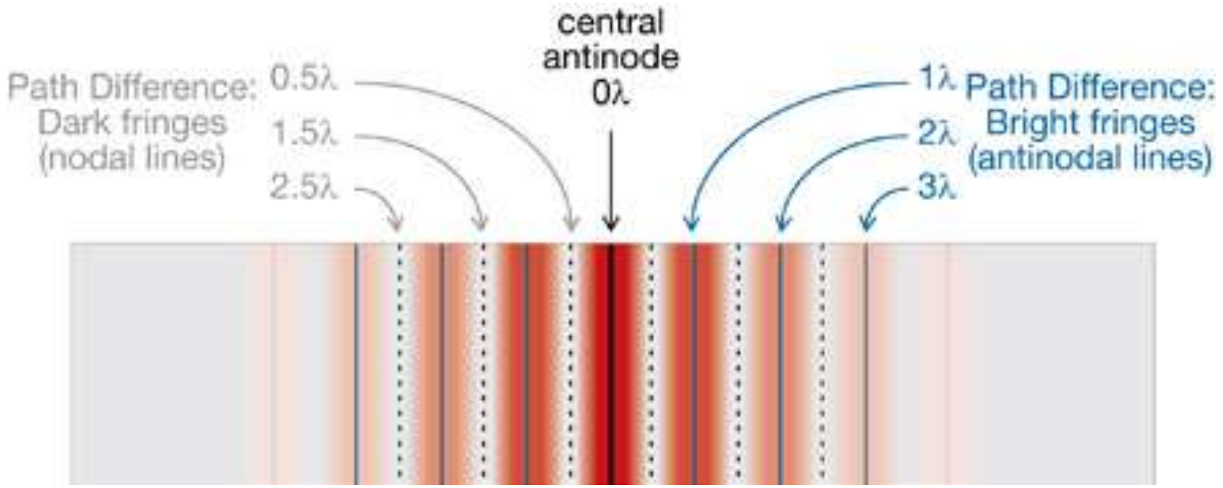
Interference pattern: $I = 2I_0(1+\cos\delta) = 4I_0\cos^2\dfrac{\delta}{2}$ $\quad \delta = \dfrac{2\pi}{\lambda}(SP - S'P)$

For a bright fringe, $SP - S'P = m\lambda$

m: any integer

For a dark fringe, $SP - S'P = (2m+1)\lambda/2$

Fringe width = $\Delta x = \dfrac{D\lambda}{d}$

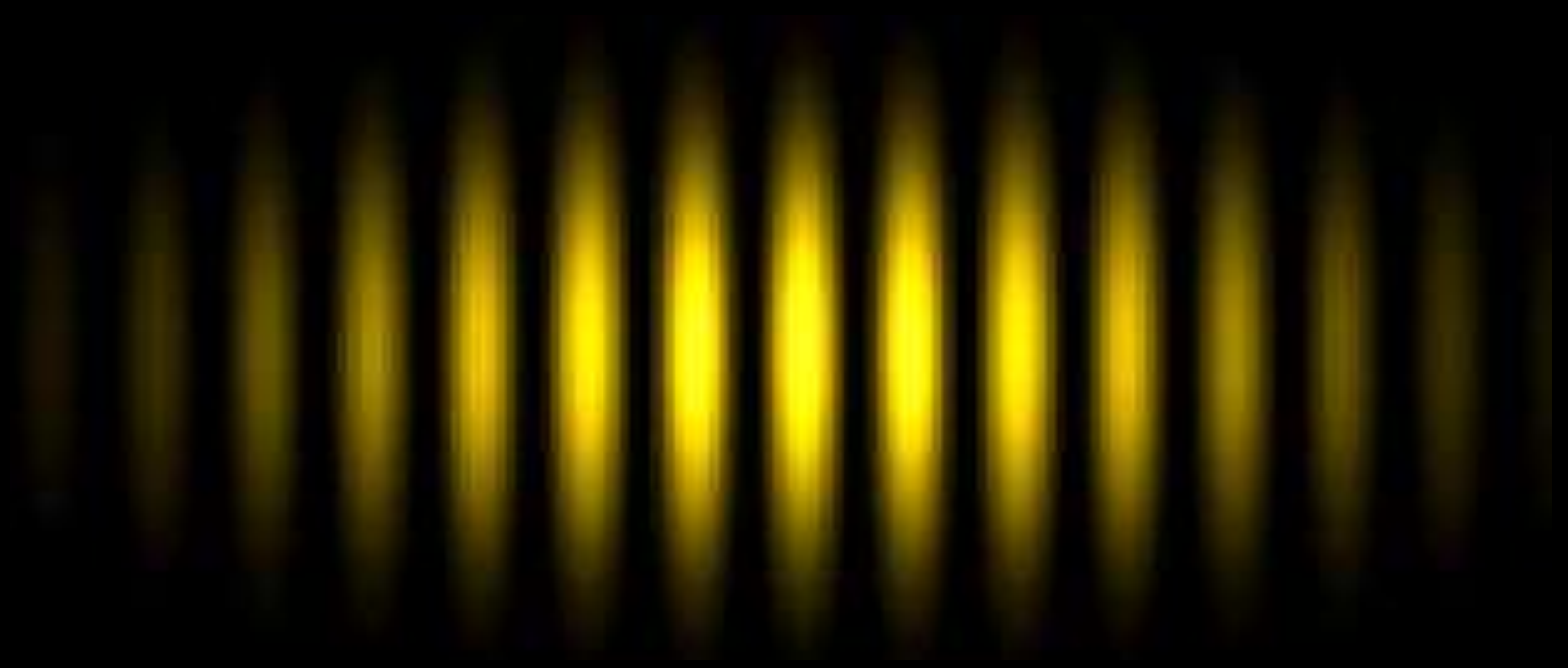


Visibility of the fringes (*V*)

$$V \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Maximum and adjacent minimum of the fringe system

Photograph of real fringe pattern for Young's double slit

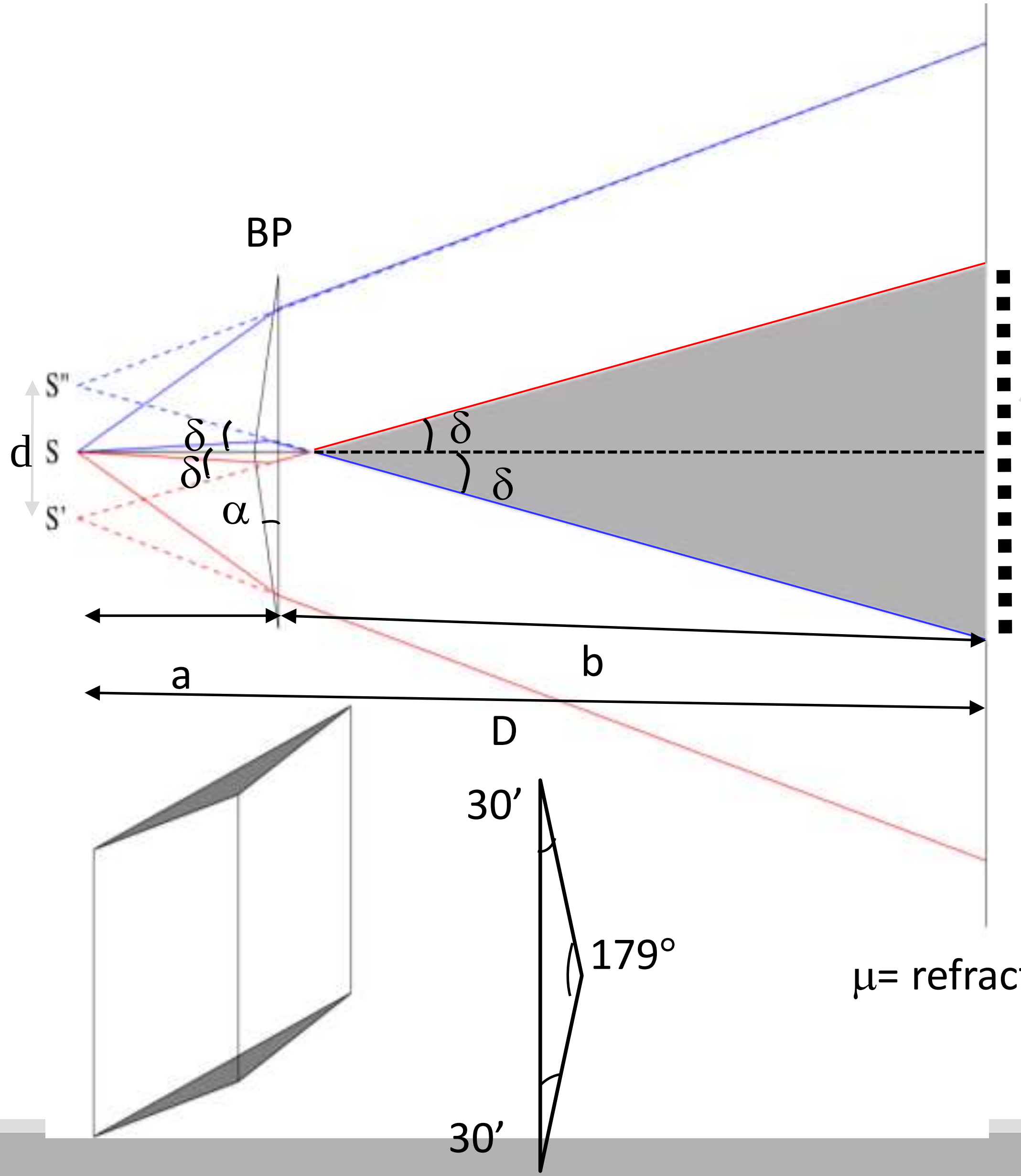


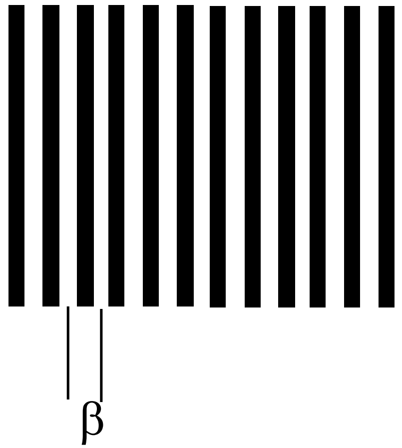
Uses for Young’s Double Slit Experiment

- Young’s Double Slit Experiment provides a method for measuring wavelength of the light

- This experiment gave the wave model of light a great deal of credibility.

Fresnel's biprism





Fringe width $\beta = \frac{\lambda D}{d}$

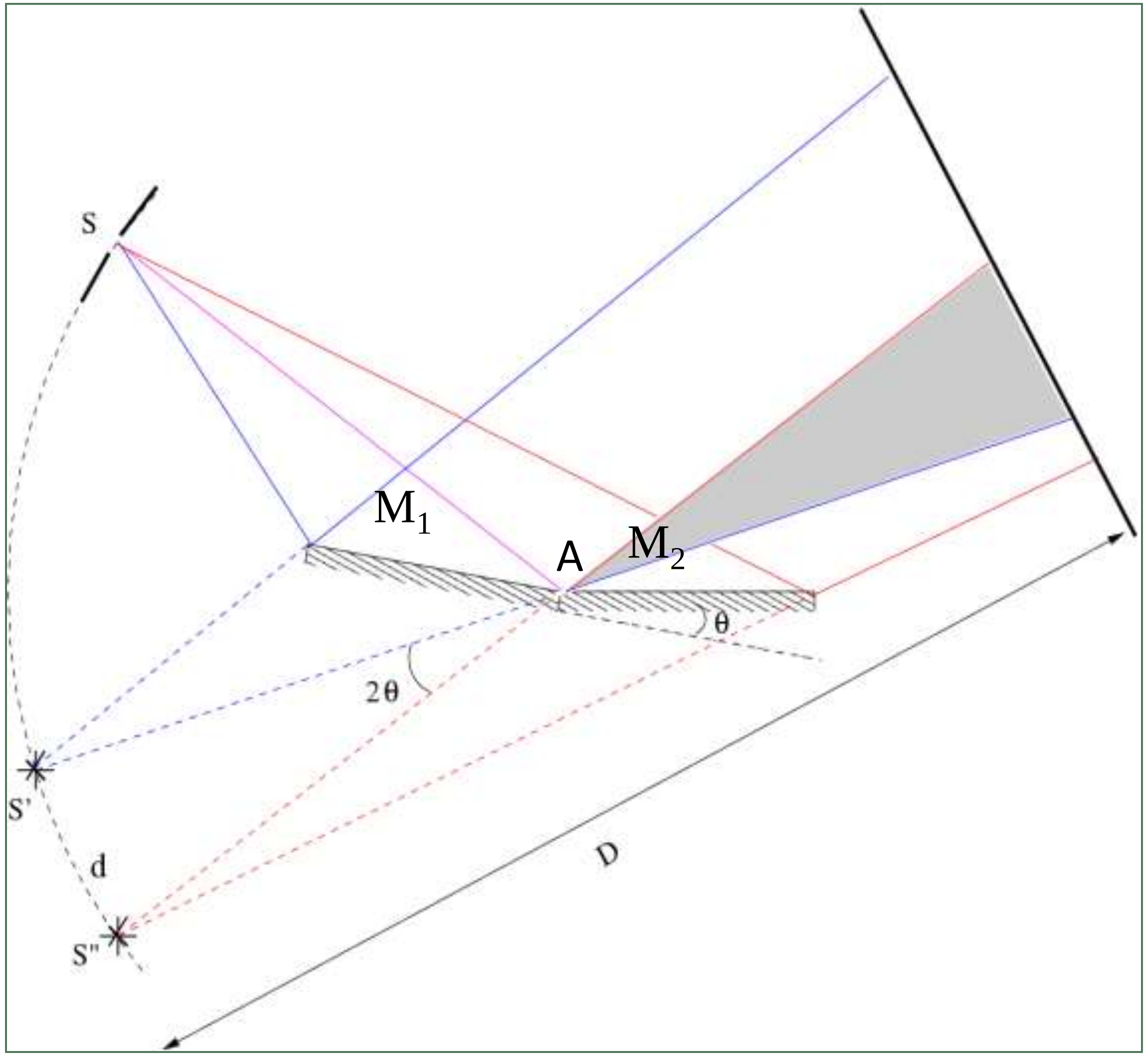
Deviation angle $\delta = (\mu-1)\alpha$

The distance between the two virtual sources is : $d= 2a\delta$

$$d= 2a(\mu-1)\alpha$$

μ= refractive index of the material of the prism

Fresnel double mirror



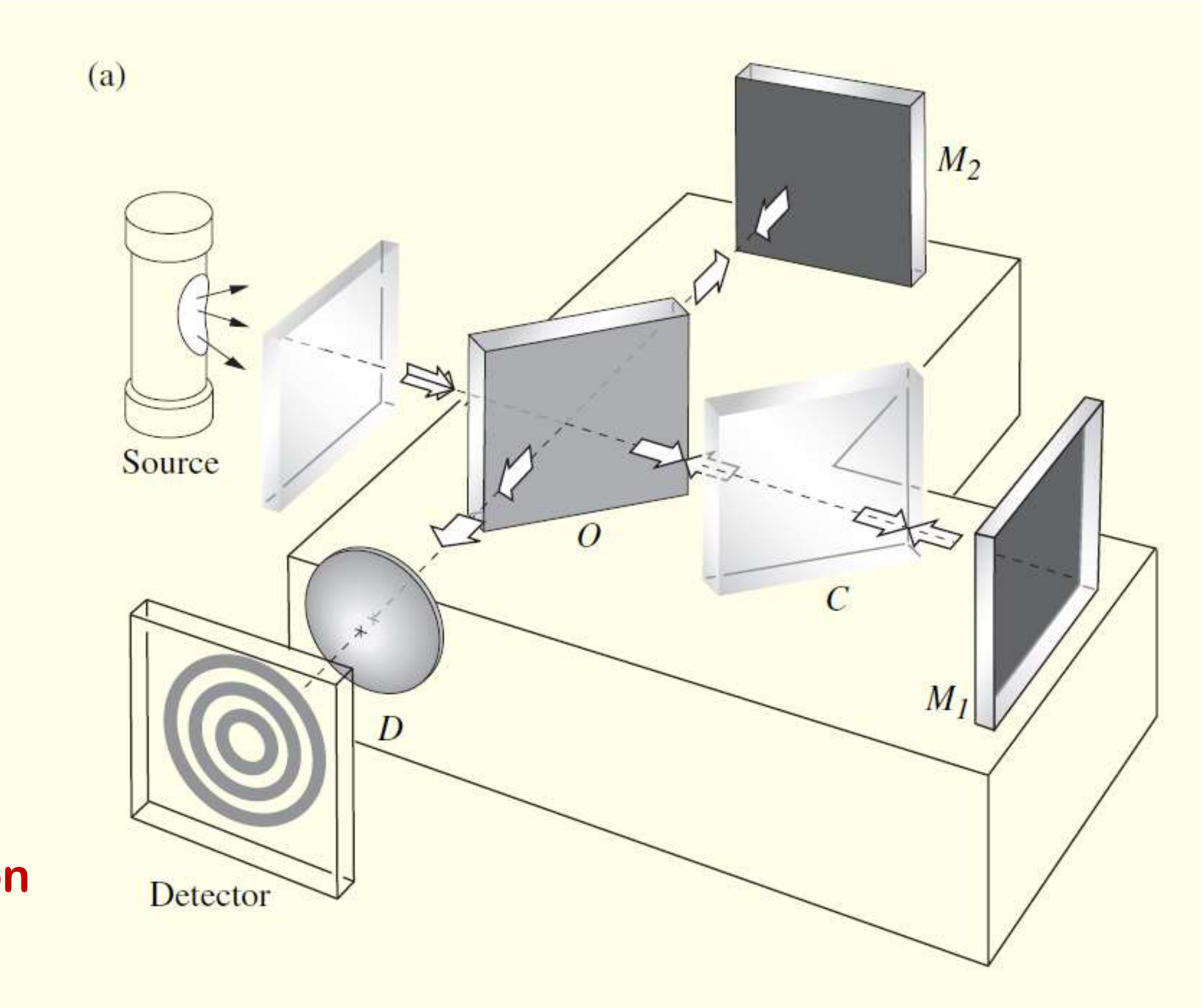
- If SA= b then S’A=S”A=b
- d= 2b sin θ
- The Conditions for bright and dark fringes are same as YDSE

Interference
by
Division of Amplitude

Michelson Interferometer

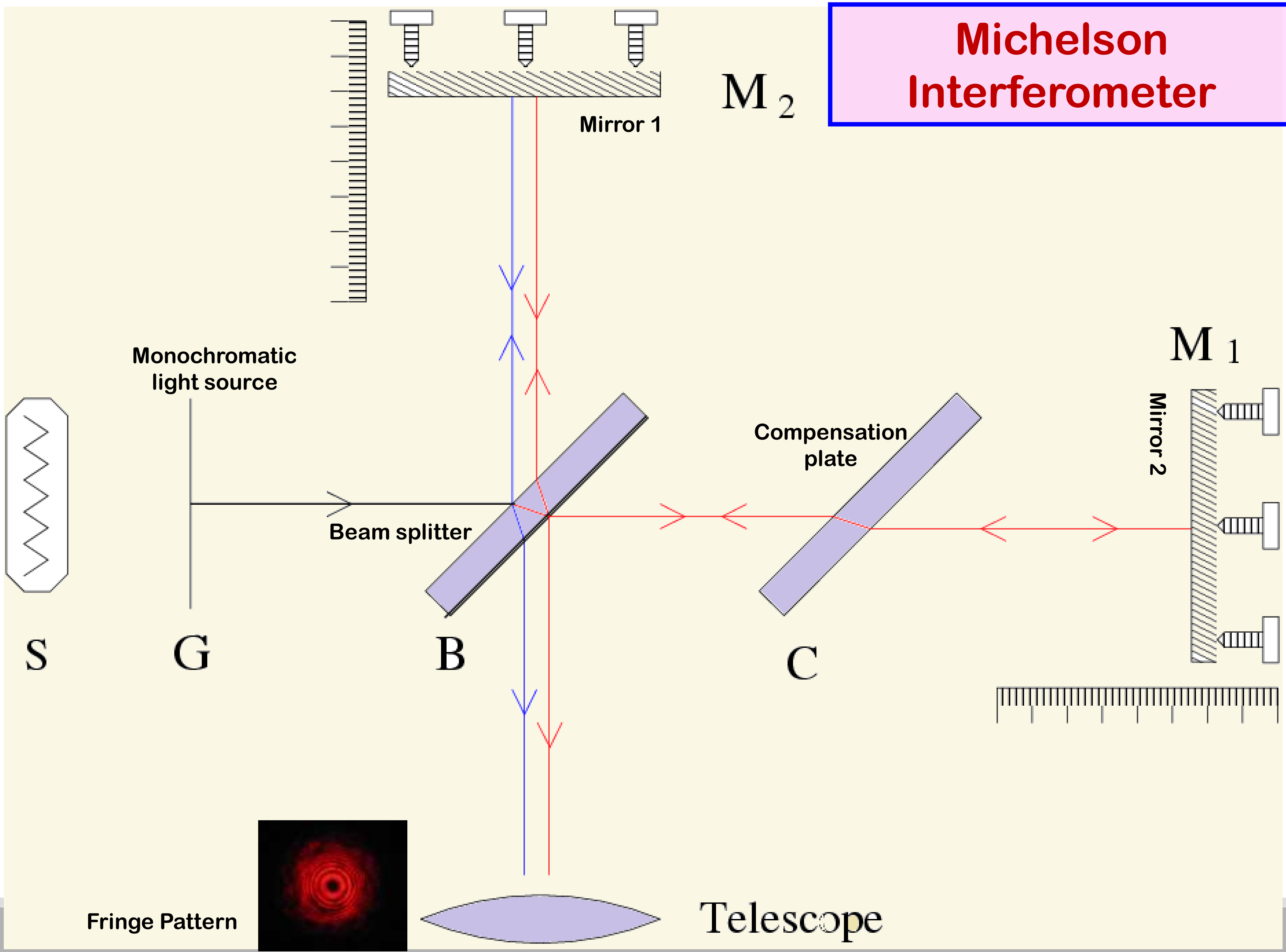
Experimental set up

Albert Abraham Michelson
(1852-1931)



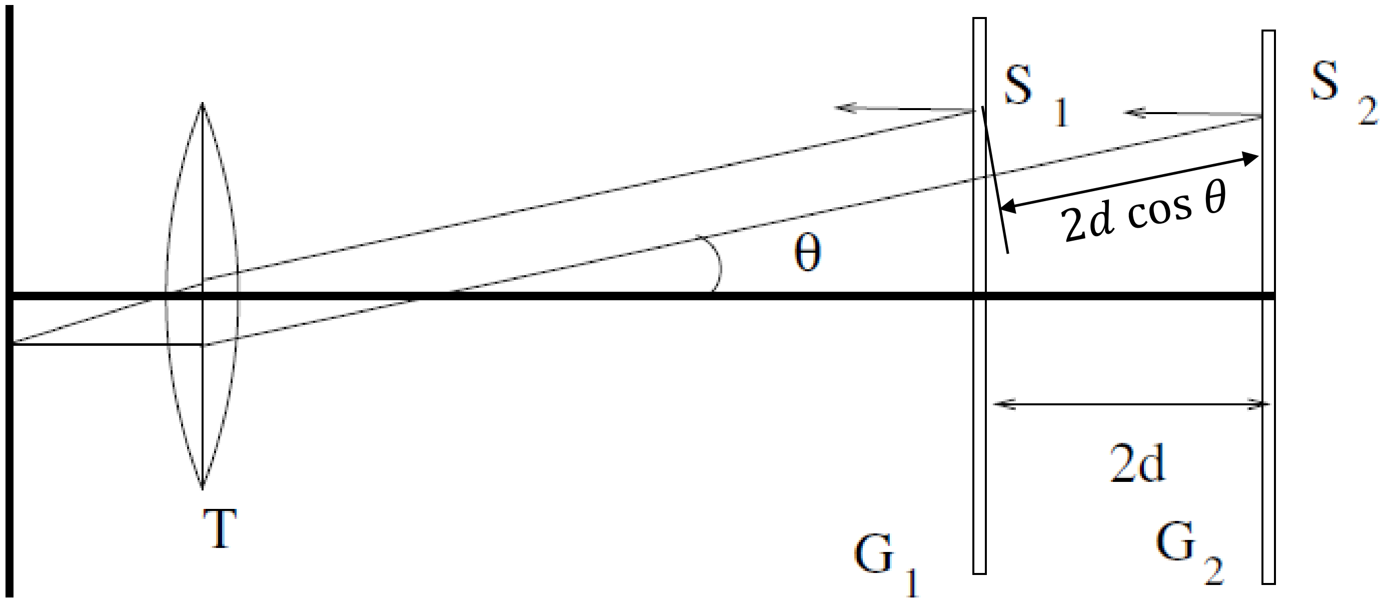
Michelson Interferometer

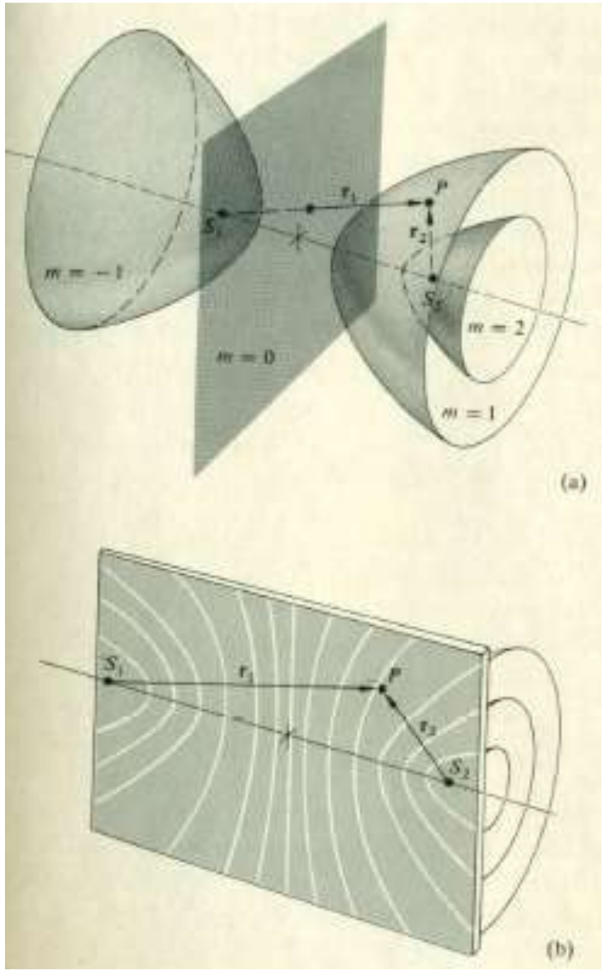
Michelson Interferometer



Effective arrangement of the interferometer

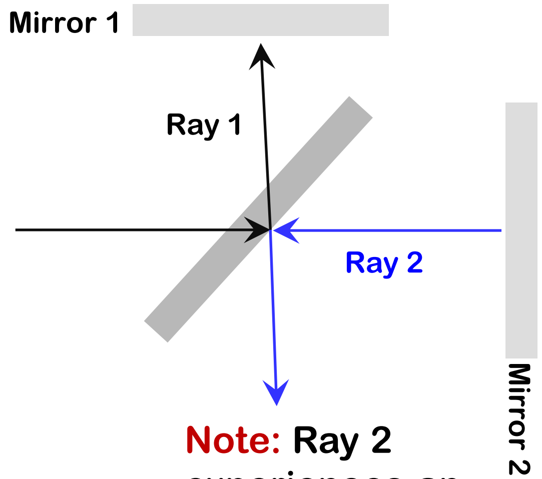
Circular fringes





In Michelson interferometer

(when the phase change of ray 2 is considered)



$$2d\cos\theta_m = m\lambda \; (\mathrm{m} = 0,1,2,\ldots) : \mathrm{Minima}$$

$$2d\cos\theta_m = \left(m + \frac{1}{2}\right)\lambda \; (\mathrm{m} = 0,1,2,\ldots) : \mathrm{Maxima}$$

Note: Ray 2 experiences an additional π phase change due to external reflection and as a result the conditions of maxima and minima are exchanged

Order of the fringe:

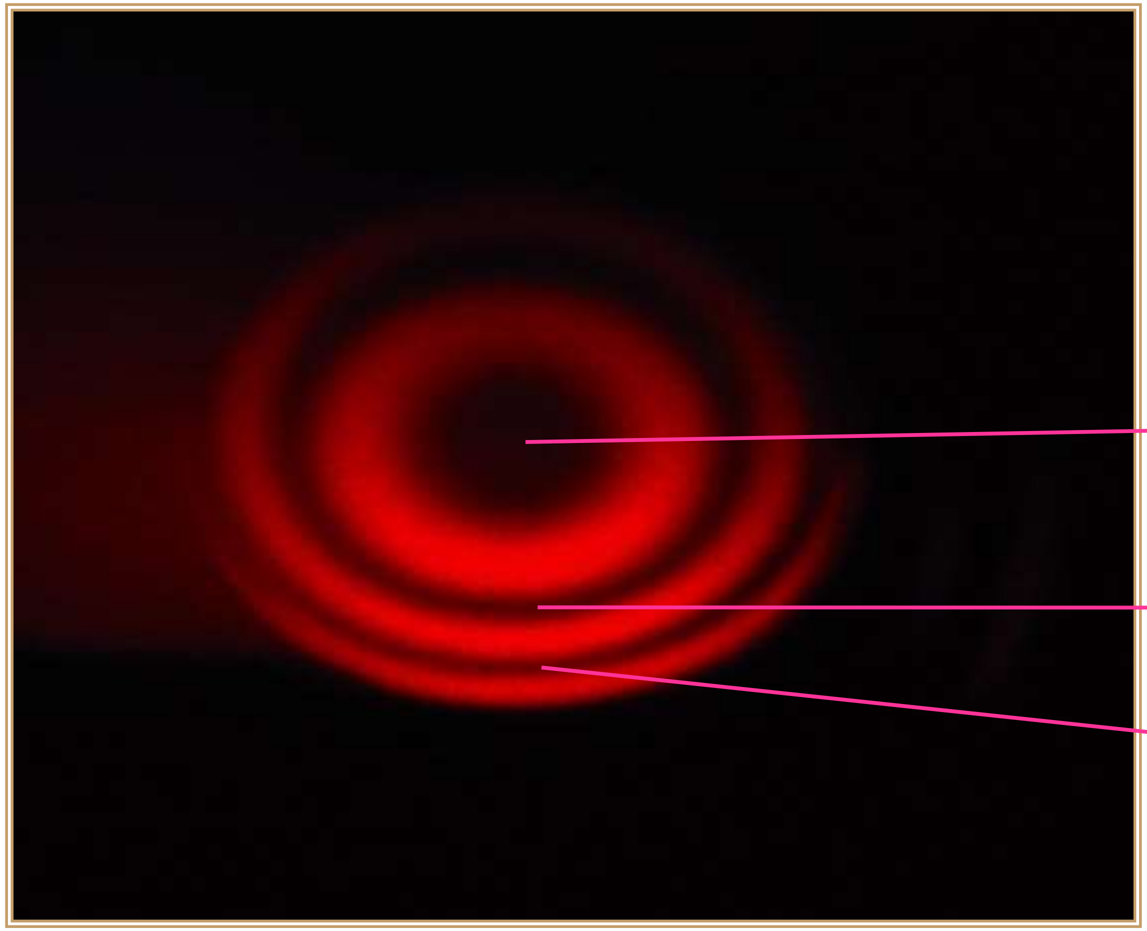
When the central fringe is dark the order of the fringe is

$$m = \frac{2d}{\lambda}$$

As *d* is increased new fringes appear at the centre and the existing fringes move outwards, and finally move out of the field of view.

For any value of *d*, the central fringe has the largest value of *m*.

Fringe shape



Central dark fringe

$$2d = m_o\lambda$$

1st dark ring

$$2d\cos\theta_1 = (m_0 - 1)\lambda$$

2nd dark ring

$$2d\cos\theta_2 = (m_0 - 2)\lambda$$

And so on............

In Michelson interferometer

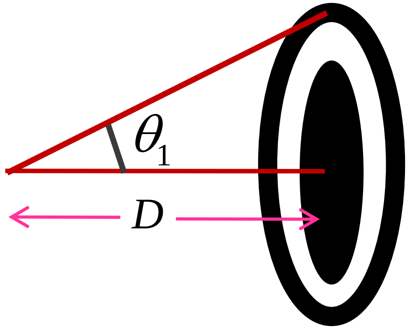
$$2d\cos\theta_m = m\lambda$$

For central dark fringe: $2d = m_o\lambda$

The **first dark ring** satisfies: $2d\cos\theta_1 = (m_0 - 1)\lambda$

For small θ $\cos\theta_1 \approx 1 - \frac{\theta_1^2}{2}$

$$2d\left(1 - \frac{\theta_1^2}{2}\right) = (m_0 - 1)\lambda$$



$$d\theta_1^2 = \lambda$$

$$\boxed{r_1^2 \approx D^2\theta_1^2 = \frac{D^2\lambda}{d}}$$

Radius of first dark ring

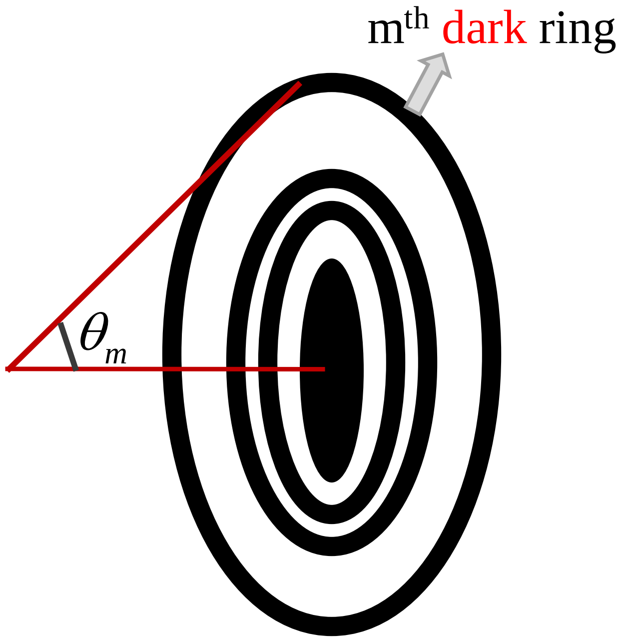
The **m^{th} dark ring** satisfies: $2d\cos\theta_m = (m_0 - m)\lambda$

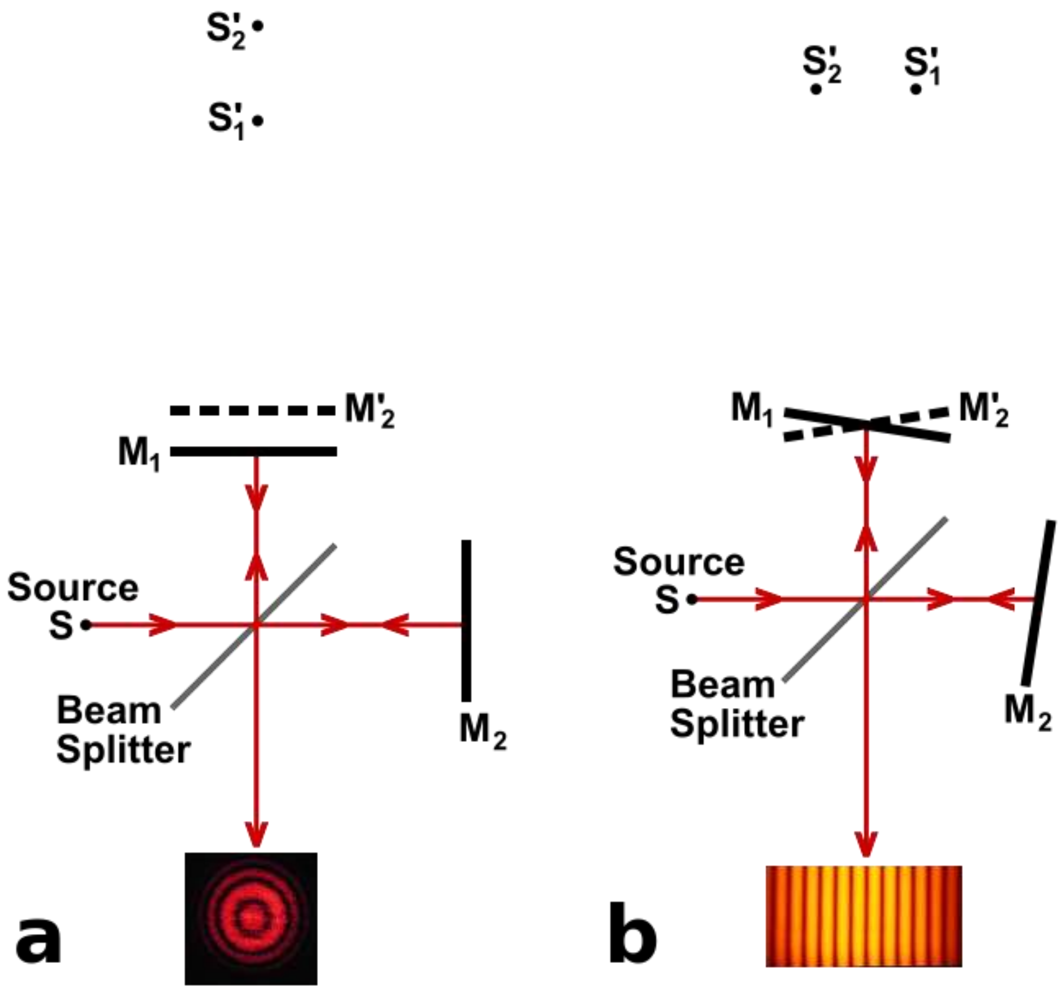
$$2d\left(1-\frac{\theta_m^2}{2}\right)=(m_0-m)\lambda \Longrightarrow d\theta_m^2 = m\lambda \qquad (2d = m_o\lambda)$$

Radius of m^{th} dark ring:

$$d\theta_m^2 \approx m\lambda$$

$$r_m^2 \approx D^2\theta_m^2 = \frac{D^2 m\lambda}{d}$$





This instrument can produce both types of interference fringes i.e., *circular fringes* and *Straight fringes*

1. Measurement of wavelength of light

$$2d\cos\theta_m = m\lambda$$

$$2d = m_0\lambda \qquad (\theta = 0)$$

Move one of the mirrors to a new position *d'* so that the order of the fringe at the centre is changed from m_o to m.

$$2d' = m\lambda$$

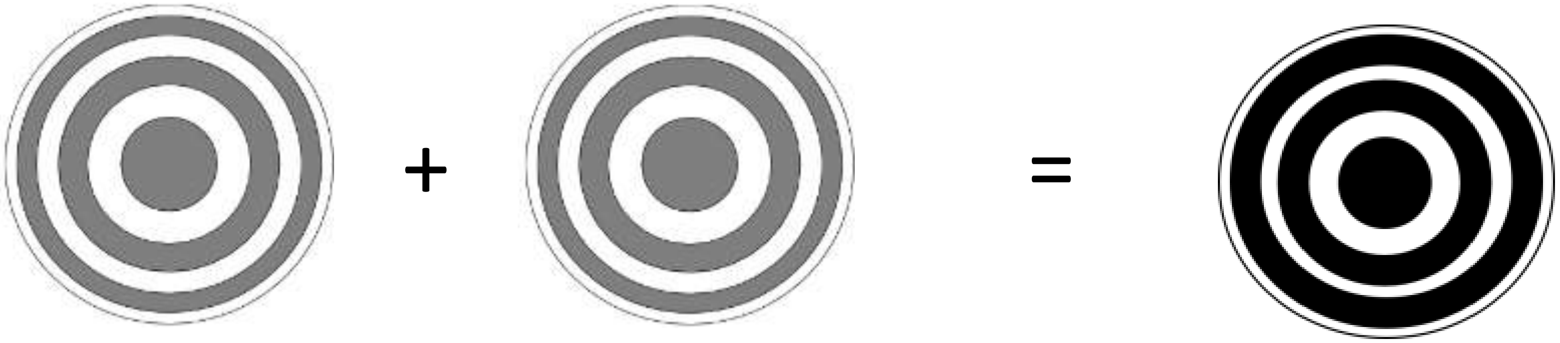
$$2|d' - d| = |m - m_0|\lambda = \Delta m\lambda$$

$$\boxed{\lambda = 2\frac{\Delta d}{\Delta m}}$$

2. Measurement of wavelength separation of a doublet

- For a given value of d and λ, the order of the central spot is fixed
- In general, if one component of the doublet gives an integer value for m_0, the other component doesn't.
- If we move one of the mirrors away from the beam splitter, at a particular position, the two systems coincide exactly and the visibility of the fringes is highest (Concordance)

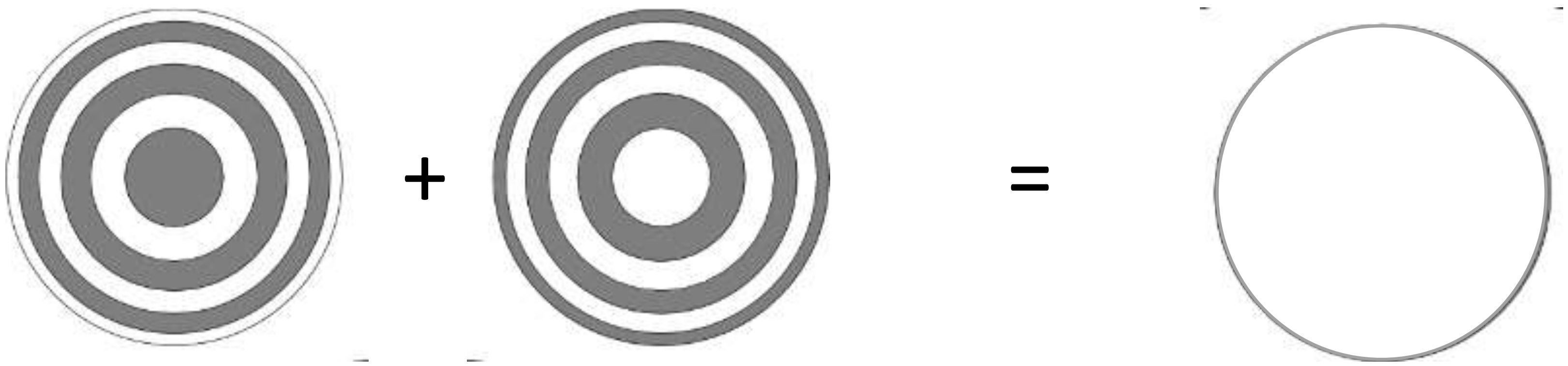
$$2d_1 = p\lambda_1 = q(\lambda_1 + \Delta\lambda)$$



$2d_1 = p\lambda_1$ $\quad$ $2d_1 = q(\lambda_1 + \Delta\lambda)$ $\quad$ Concordance

- If the mirror is moved further away from the beam splitter, the bright rings of one component coincides with the dark rings of other and the fringes of the system are washed away (discordance)

$$2d' = p\lambda_1; \;\; 2d' = \left(q + \frac{1}{2}\right)(\lambda_1 + \Delta\lambda)$$



$2d_1 = p\lambda_1$ $\quad$ $2d_1 = (q+1/2)(\lambda_1 + \Delta\lambda)$ $\quad$ Discordance

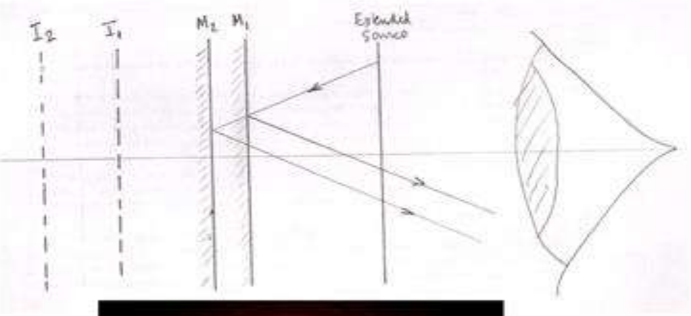
- If the mirror is moved further a new concordance will occur and this happens when the number of new rings of the two components that emerge differ by unity

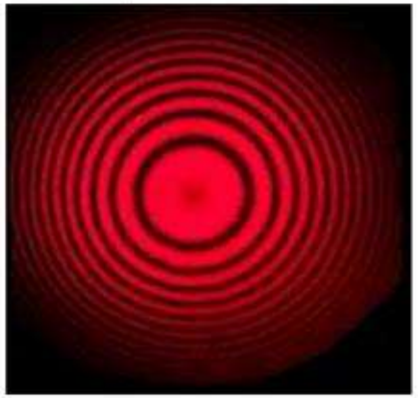
$$2d_2 = (p+n)\lambda_1 = (q+n-1)(\lambda_1 + \Delta\lambda)$$

$$2(d_2 - d_1) = n\lambda_1 = (n-1)(\lambda_1 + \Delta\lambda)$$

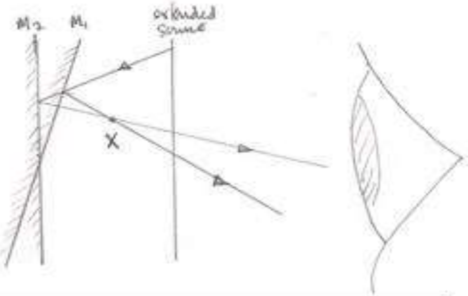
$$\Delta\lambda \approx \frac{\lambda_1^2}{2(d_2 - d_1)}$$

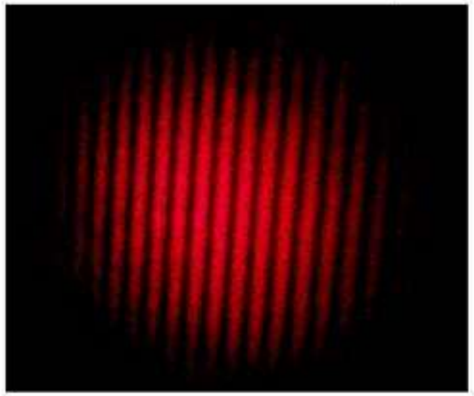
Fringes of equal inclination



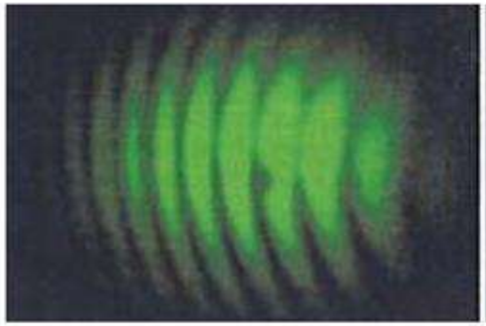


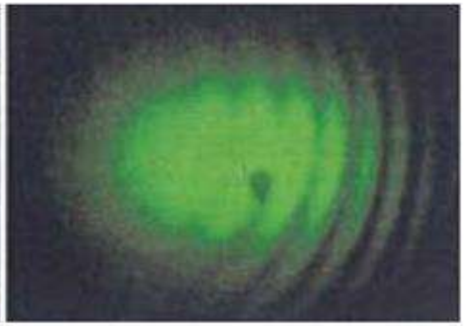
Fringes of equal thickness



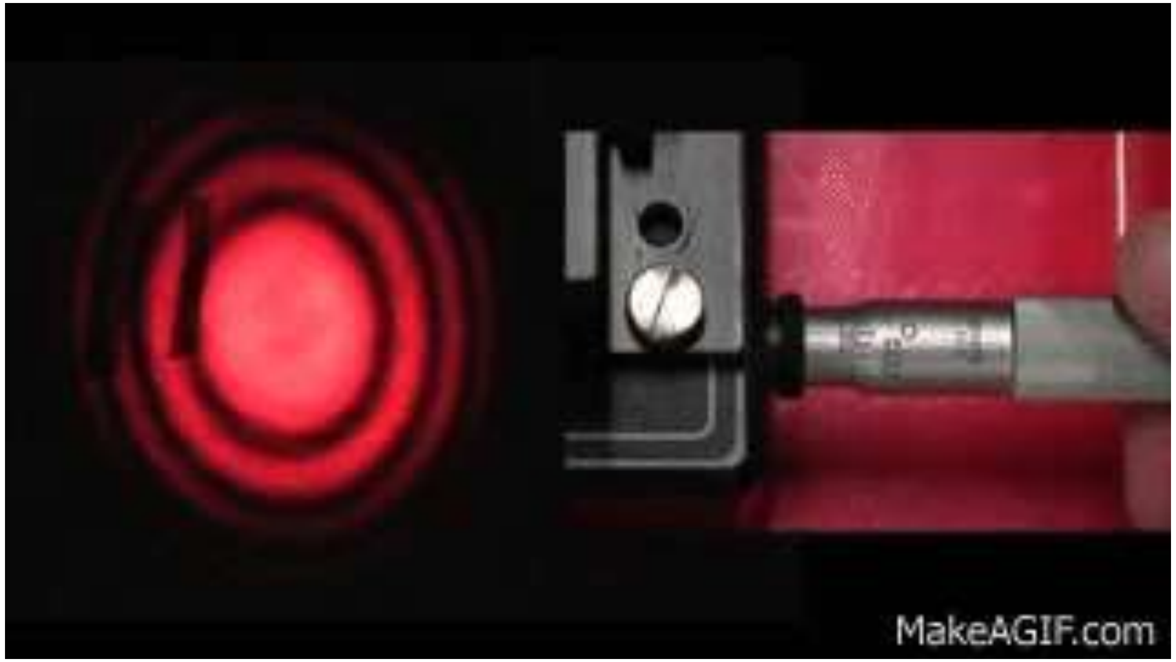


A bit of each





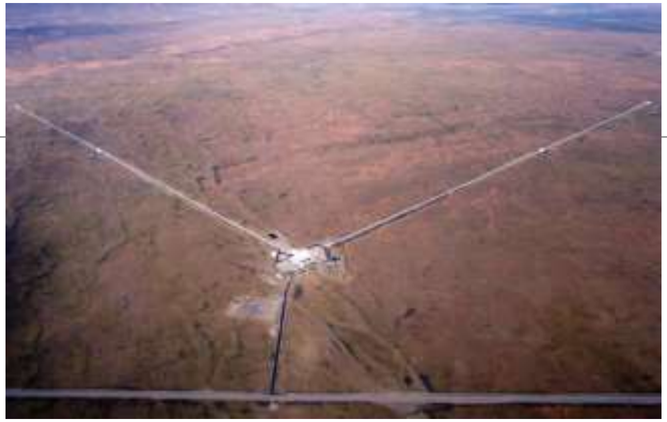
https://physics.stackexchange.com/questions/317051/michelson-interferometer-circular-fringes



https://makeagif.com/gif/michelson-interferometer-p8_x_O

Laser Interferometer Gravitational-wave Observatory (LIGO)

- General relativity predicts the existence of gravitational waves.



- In Einstein's theory, gravity is equivalent to a distortion of space. These distortions can then propagate through space.

- The LIGO apparatus is designed to detect the distortion produced by a disturbance that passes near the Earth.

- The interferometer uses laser beams with an effective path length of several kilometers.

- At the end of an arm of the interferometer, a mirror is mounted on a massive pendulum.

- When a gravitational wave passes, the pendulum moves, and the interference pattern due to the laser beams from the two arms changes.

Interferometric Detectors

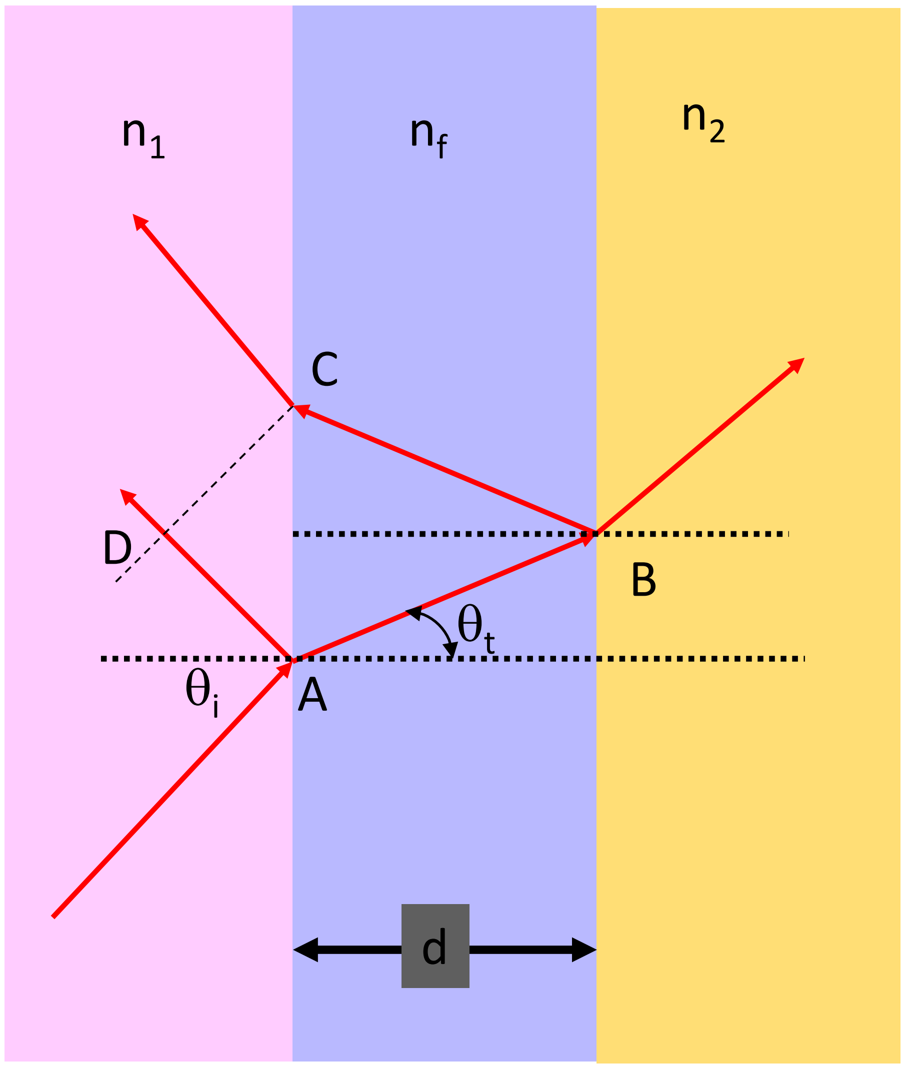
Interferometric Detectors

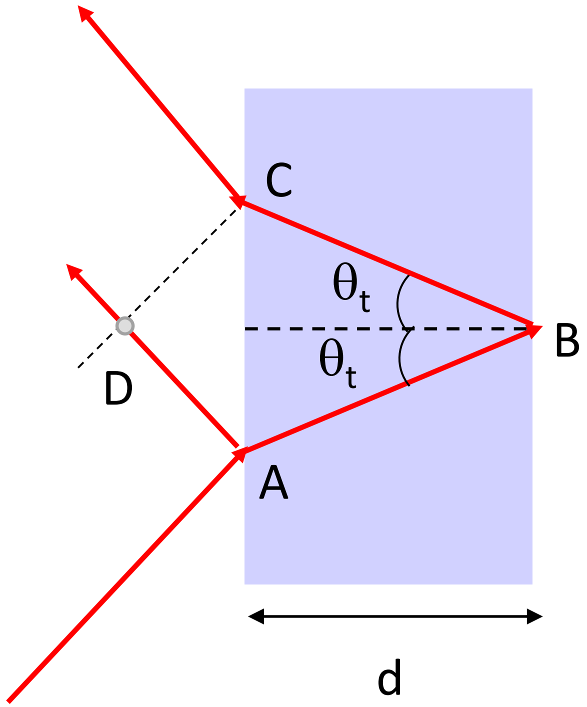
LIGO Louisiana 4km, USA

Interferometric Detectors

LIGO Washington 2km& 4km, USA

Thin Film Interference





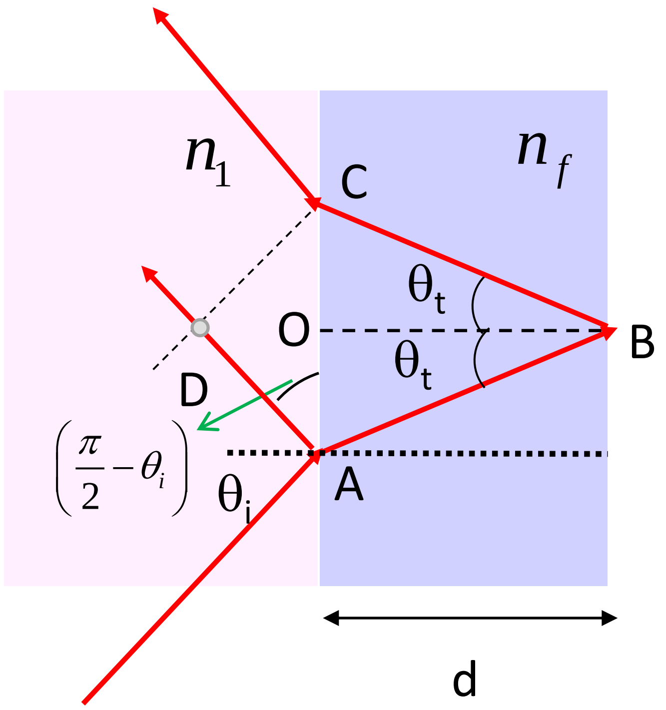
Optical path difference for the first two reflected beams

$$\Lambda = n_f[AB + BC] - n_1(AD)$$

$$AB = BC = d/\cos\theta_t$$

$$AD = AC\sin\theta_i$$

$$AD = 2d\ \tan\theta_t \times \frac{n_f}{n_1}\ \sin\theta_t$$



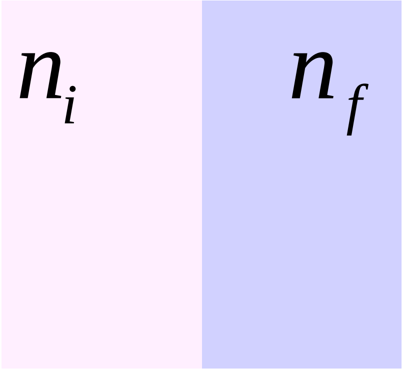
$$\boxed{\begin{aligned} &\cos(\pi/2-\theta_i) = \sin(\theta_i) = \frac{AD}{AC} \\ &AC = AO + OC = 2d\tan\theta_f \\ &n_1\sin\theta_i = n_f\sin\theta_t \end{aligned}}$$

$$\Lambda = \frac{2dn_f}{\cos\theta_t}\left[1-\sin^2\theta_t\right]$$

$$= 2dn_f\cos\theta_t$$

Path Difference

$$\Lambda = 2dn_f \cos\theta_t$$



$$n_i < n_f \Rightarrow \pi \ \ phase\ shift$$
$$n_i > n_f \Rightarrow 0 \ \ phase\ shift$$

Phase shift (in the case of external reflection)

$$\delta = k_0\Lambda \pm \pi$$

$$\delta = \frac{4\pi n_f}{\lambda_o} d\cos\theta_t \pm \pi$$

For $n_1 > n_f > n_2$, or $n_1 < n_f < n_2$, the $\pm\pi$ phase shift will not be present

Phase shift $\Rightarrow$ $$\boxed{\delta = \frac{4\pi n_f}{\lambda_o} d\cos\theta_t \pm \pi}$$

$$\left(\lambda_f = \frac{\lambda_0}{n_f}\right)$$

Condition for maxima ($\delta = 2m\pi$)

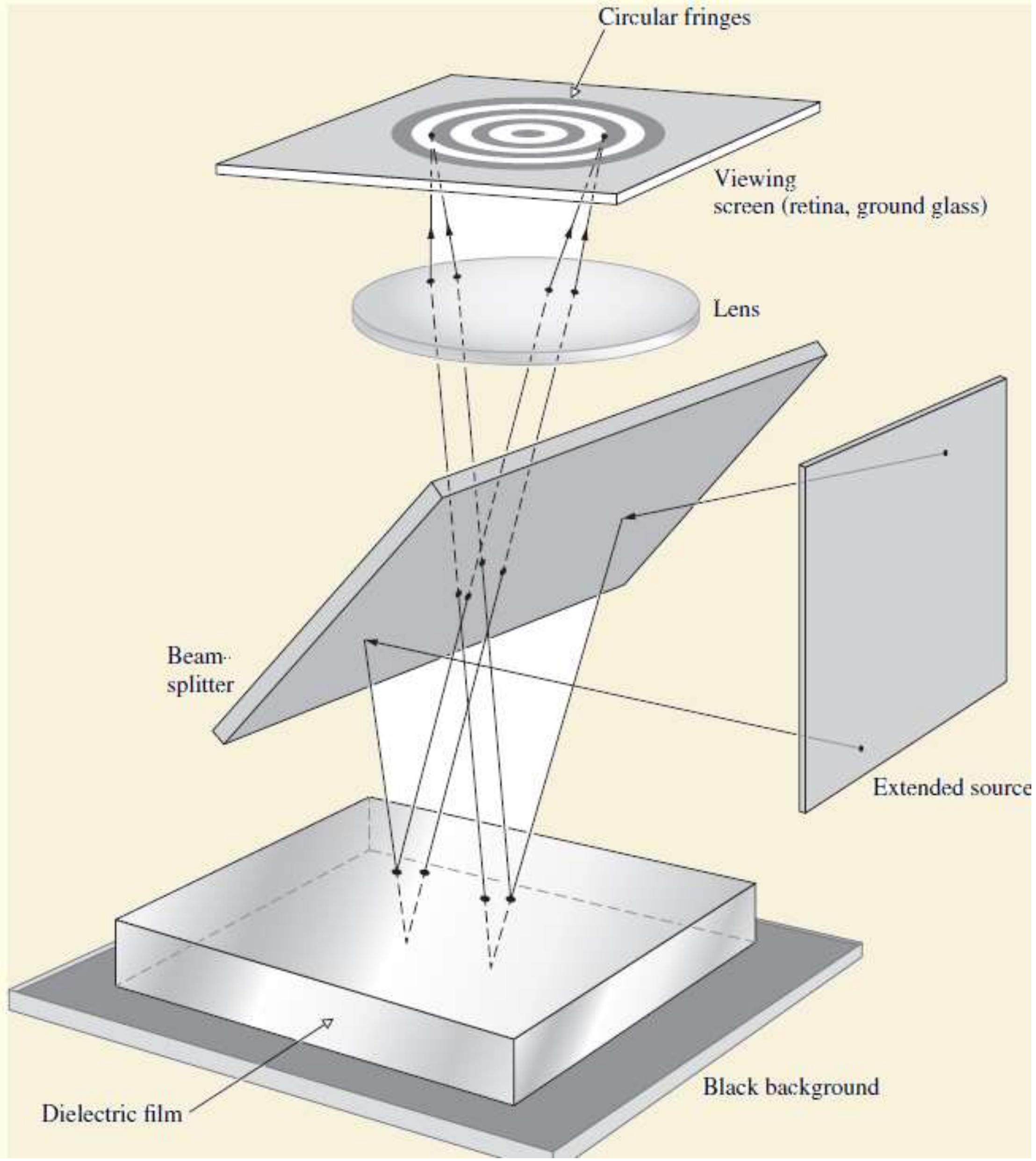
$$d\cos\theta_t = (2m+1)\frac{\lambda_f}{4} \qquad m = 0,\ 1,\ 2, \ldots$$

Condition for minima ($\delta = (2m+1)\pi$)

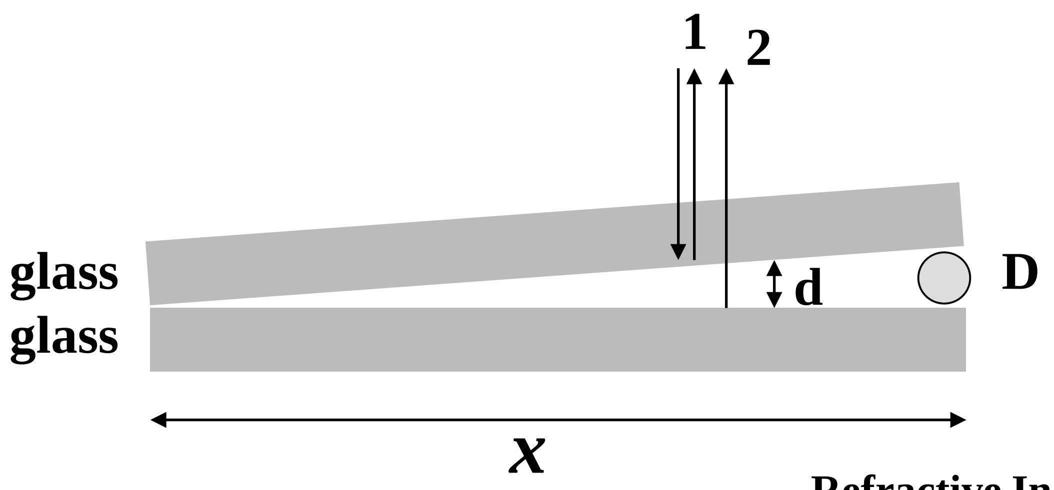
$$d\cos\theta_t = 2m\frac{\lambda_f}{4} \qquad m = 0,\ 1,\ 2, \ldots$$

Note: Odd and even multiple of ($\lambda_f/4$)

Formation of circular fringes



Wedge between two plates



Refractive Index of wedge film: n_f

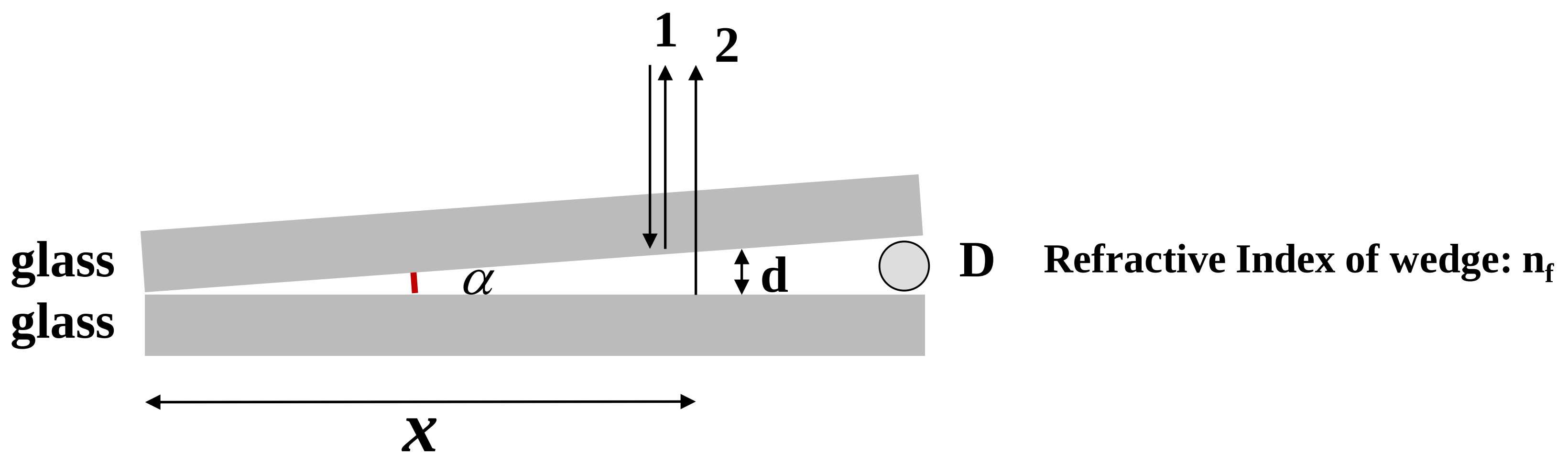
Path difference $= 2d$

Phase difference $\delta = 2kd - \pi$ **(phase change for 2, but not for 1)**

Maxima $2d_m = (m + ½)\,\lambda_o/n_f$

Minima $2d_m = m\lambda_o/n_f$

Conditions for maximum (For small values of θ_i)



$$(m+\frac{1}{2})\lambda_0 = 2n_f d_m$$

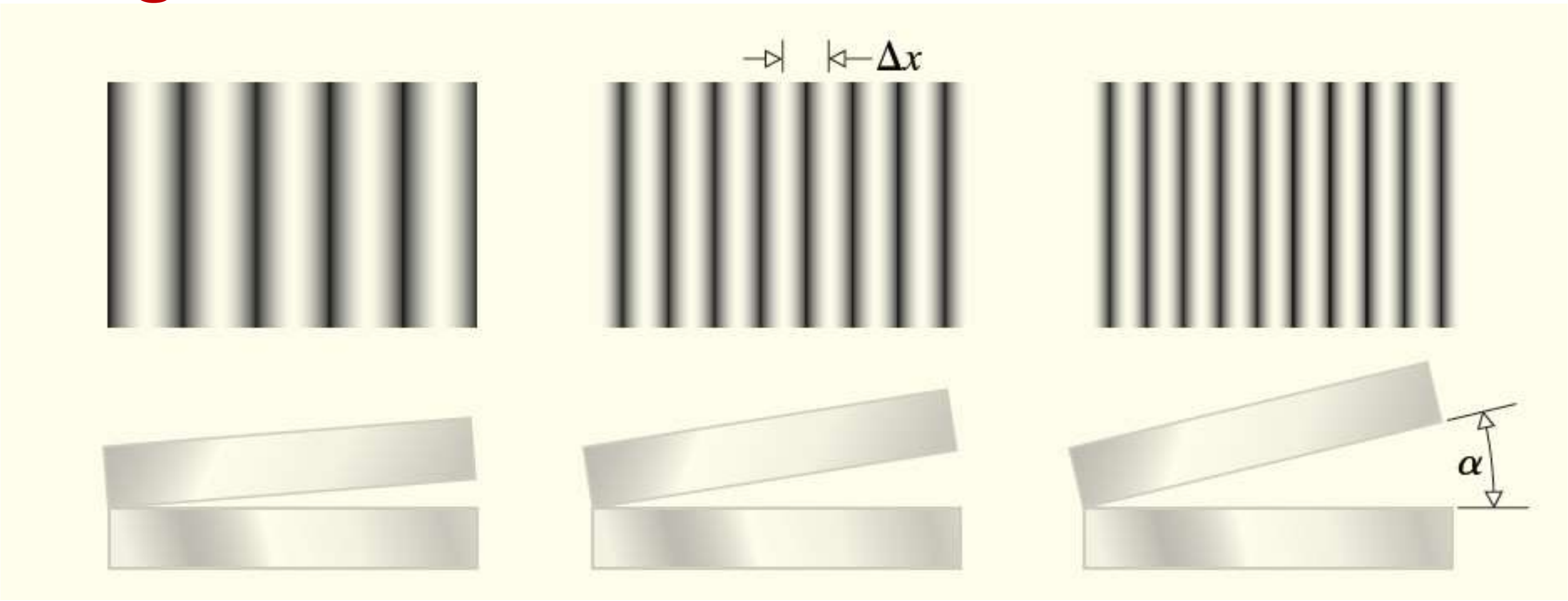
d is the thickness at a particular point

$$d = x\alpha$$

$$x_m = \left(\frac{m+1/2}{2\alpha}\right)\lambda_f$$

- x_m : Position of the bright fringe

Fringe width



Fringe width decreases with increasing wedge angle

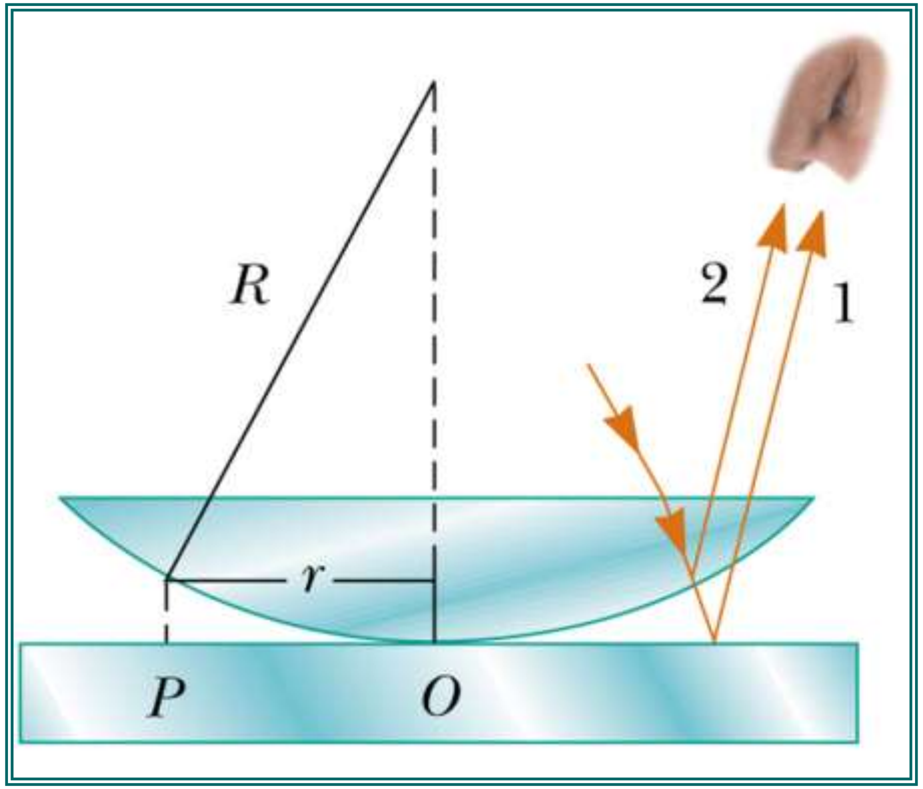
$$x_m = \left(\frac{m+1/2}{2\alpha}\right)\lambda_f$$

$$\Delta x = x_{m+1} - x_m$$

$$\Delta x = \frac{\lambda_f}{2\alpha}$$

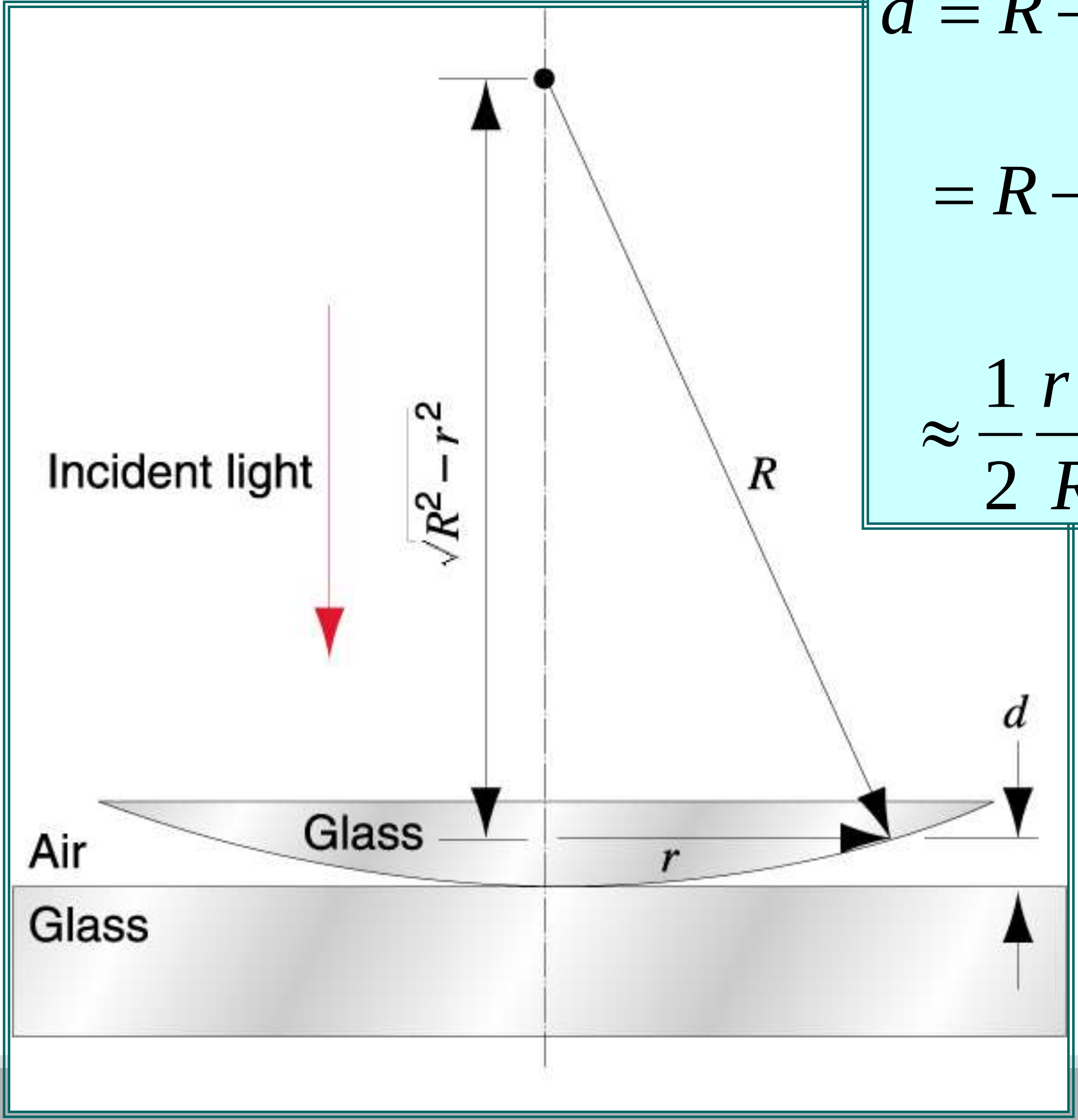
Newton's Ring

Ray 1 undergoes a phase change of 180° on reflection, whereas ray 2 undergoes no phase change



R = radius of curvature of lens

r = radius of Newton's ring

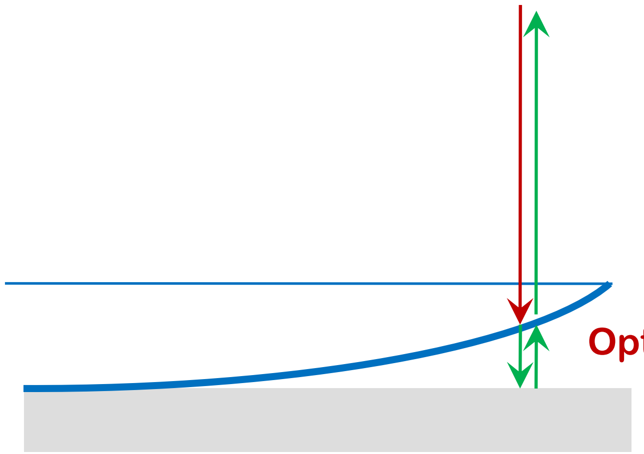


$$d = R - \sqrt{R^2 - r^2}$$

$$= R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + \ldots\right]$$

$$\approx \frac{1}{2}\frac{r^2}{R}$$

For bright rings



$$2dn$$

- n is refractive index of the film

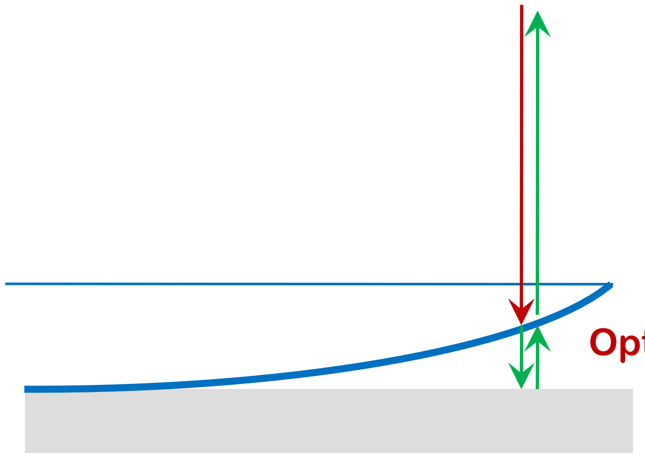
$$2dn = (2m+1)\frac{\lambda}{2}$$

$$2\times\frac{1}{2}\frac{r^2}{R}n = (2m+1)\frac{\lambda}{2}$$

$$r_{bright} = \sqrt{(2m+1)R\frac{\lambda}{2n}} = \sqrt{(2m+1)R\frac{\lambda_n}{2n}}, \quad m = 0,1,2\ldots$$

For dark rings

$$\boxed{d = \frac{1}{2}\frac{r^2}{R}}$$

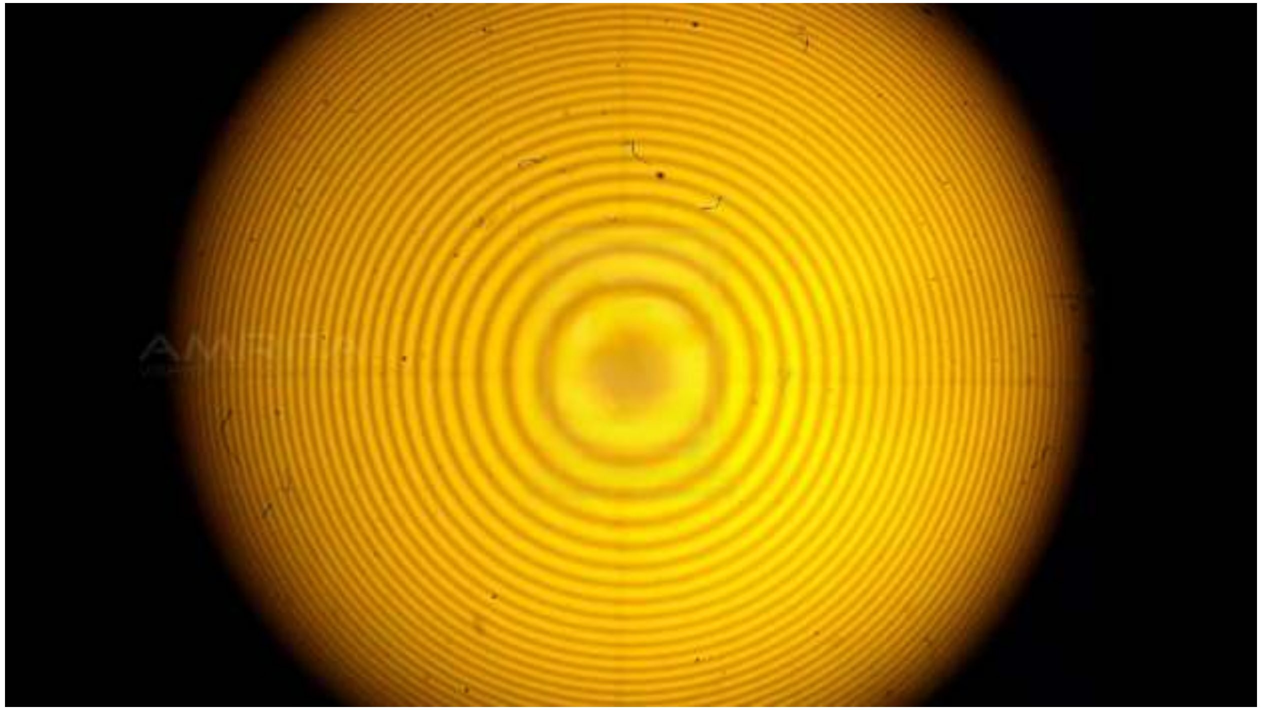


$$2dn$$

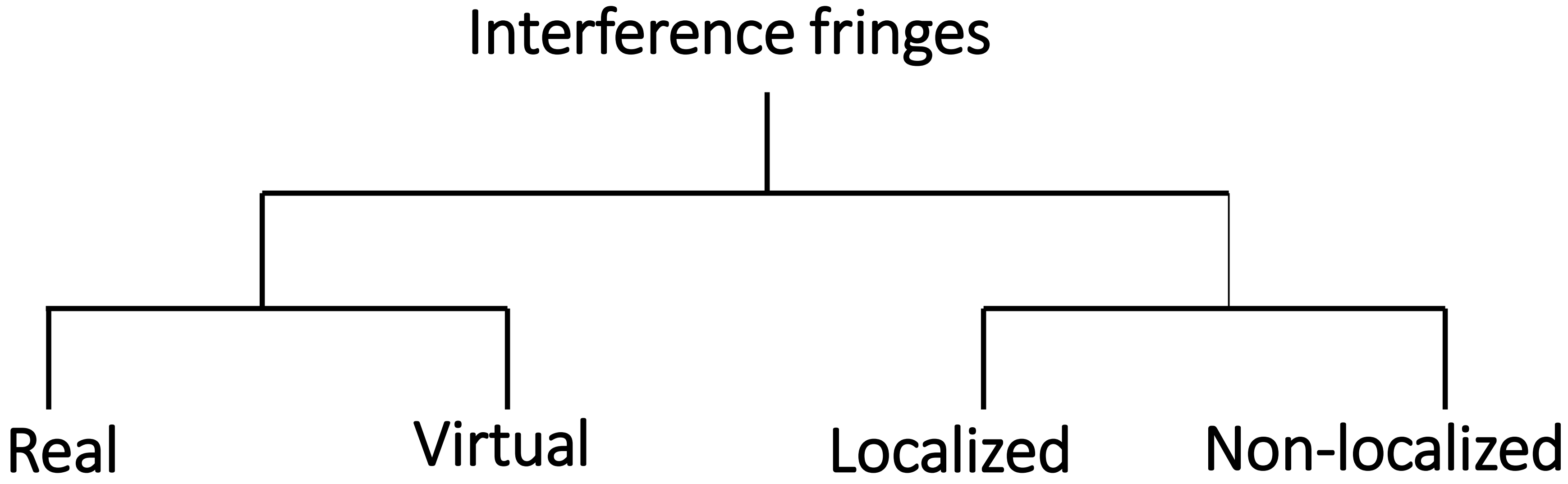
$$2dn = 2m\frac{\lambda}{2}$$

$$r_{dark} = \sqrt{2mR\frac{\lambda_n}{2}},\ m = 0,1,2\ldots$$

Newton’s Ring



Types of fringes


Interference fringes
Real
Virtual
Localized
Non-localized

Real fringes/images

- Can be intercepted on a screen (placed anywhere in the vicinity of the interferometer without a condensing lens system).

Virtual fringes/images

- Cannot be projected onto a screen.

Non-localized fringes

- Exists everywhere
- Result of point/line source

Localized fringes

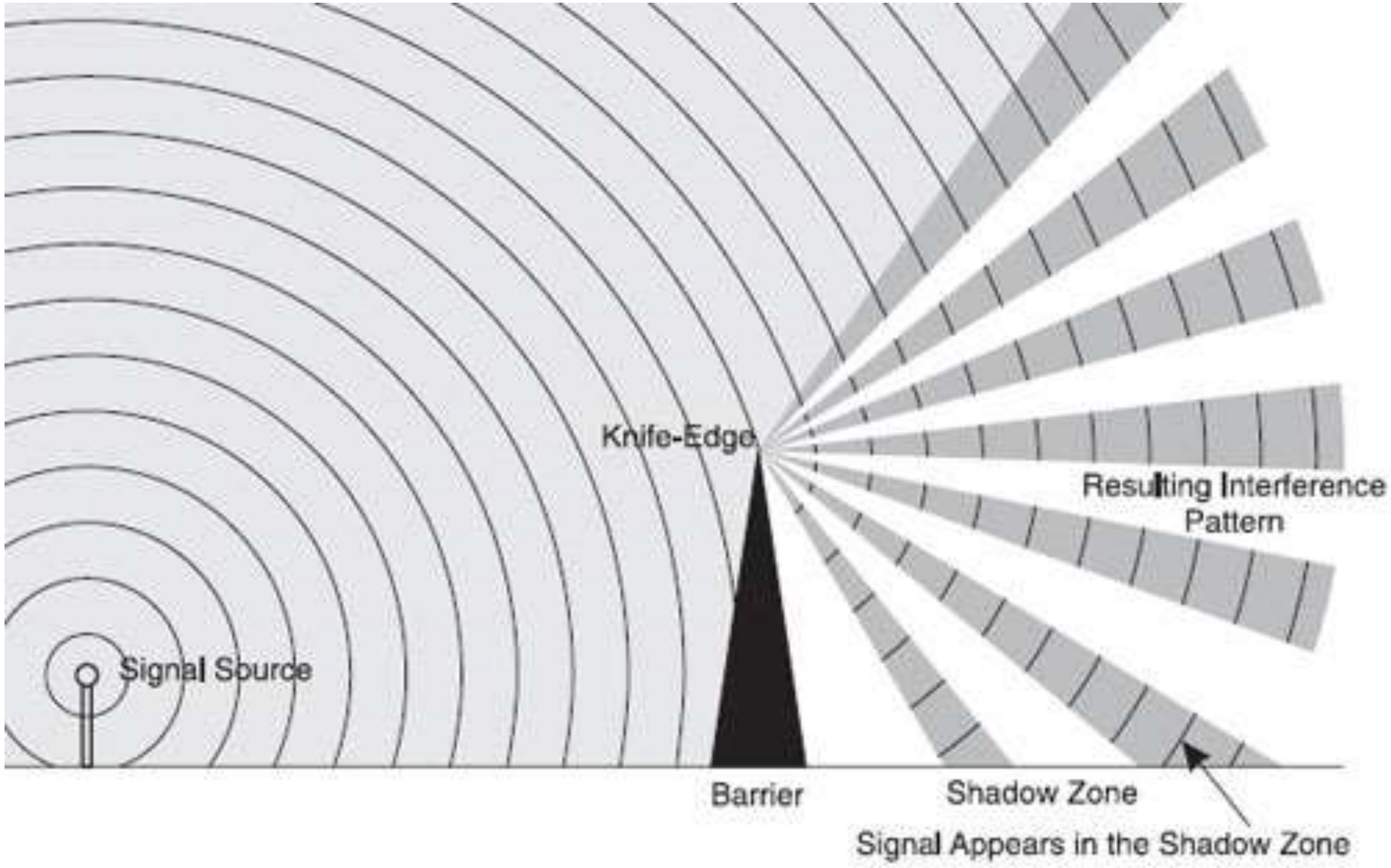
- Observed over particular surface

- Result of extended source

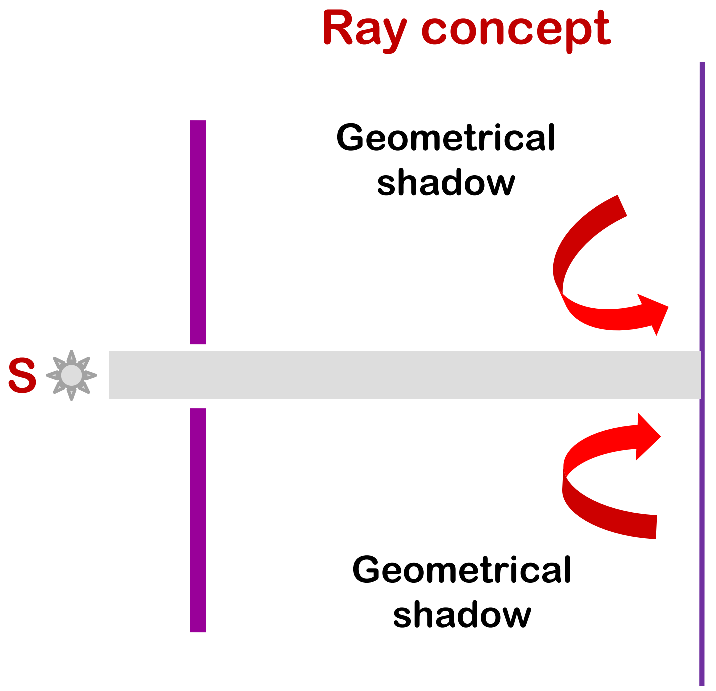
Diffraction

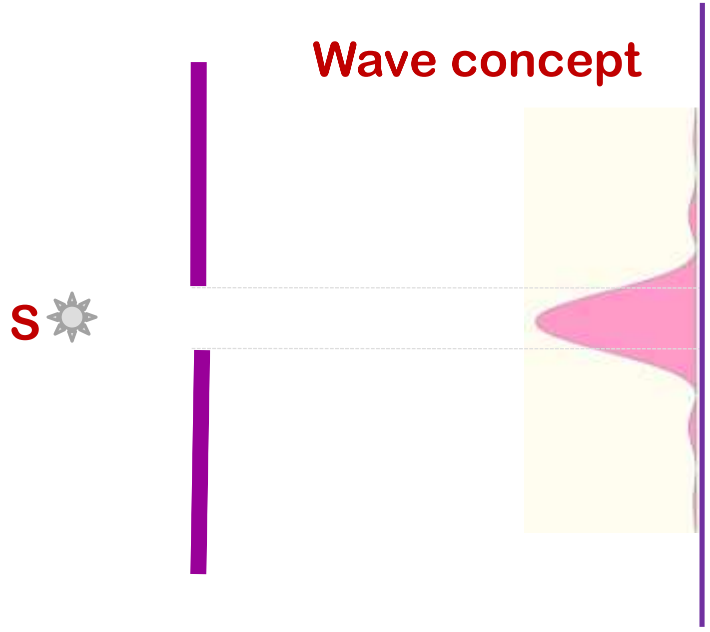
Diffraction

"Any deviation of light rays from rectilinear path which is neither reflection nor refraction is known as diffraction." (Sommerfeld)



Diffraction

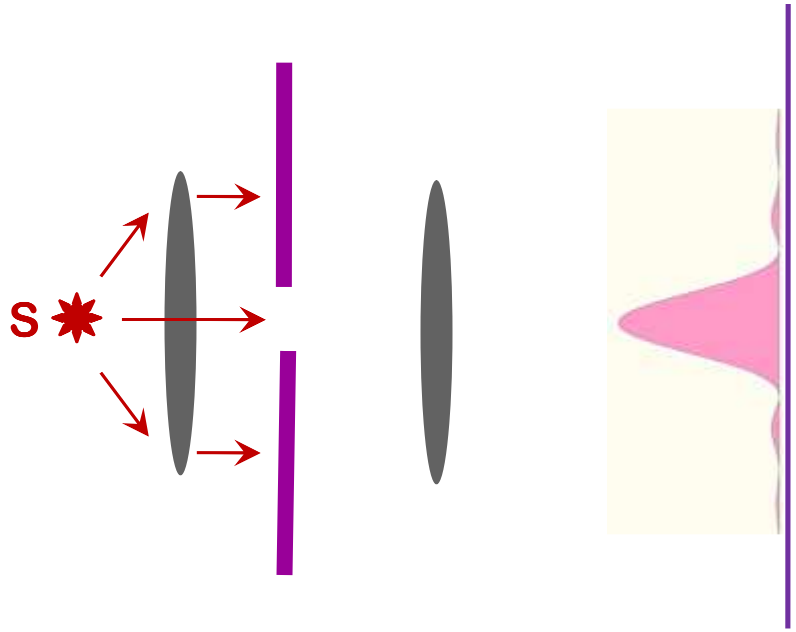




Types or kinds of diffraction:

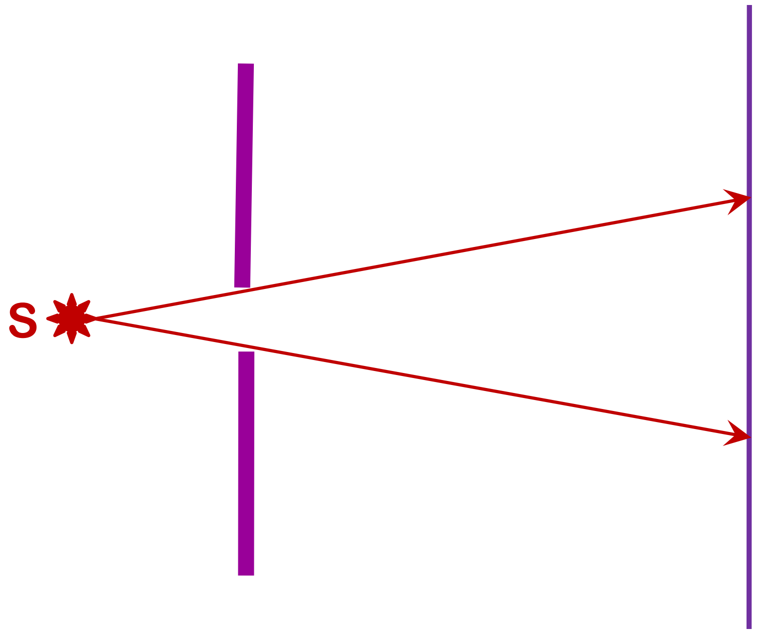
1. Fraunhofer (1787-1826)
2. Fresnel (1788-1827)

Fraunhofer Diffraction



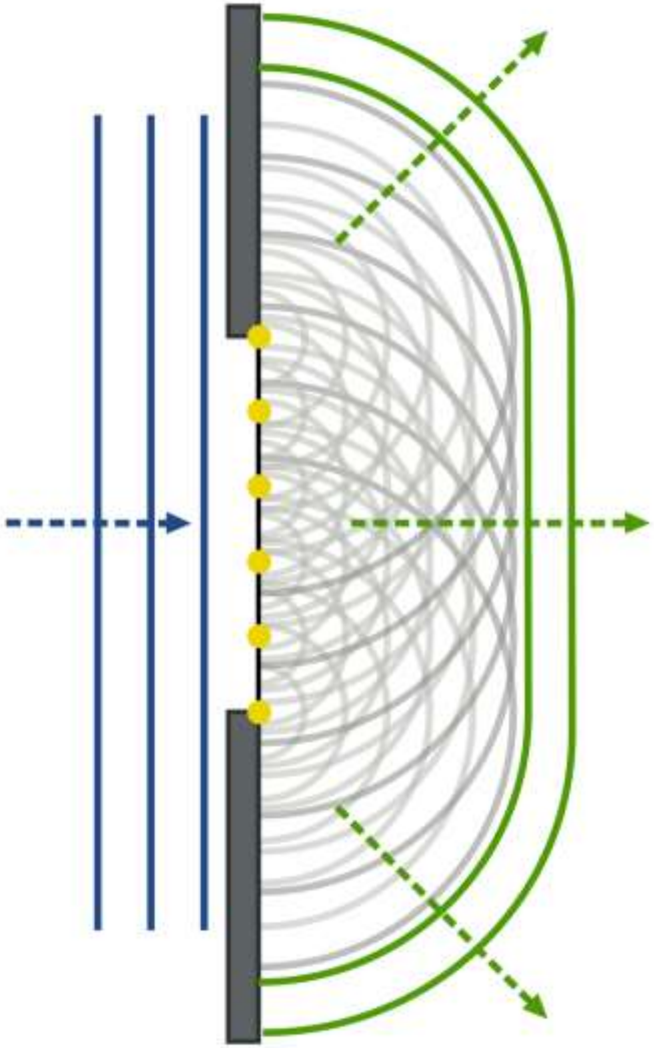
Both source and screen are in infinity- Fraunhofer class

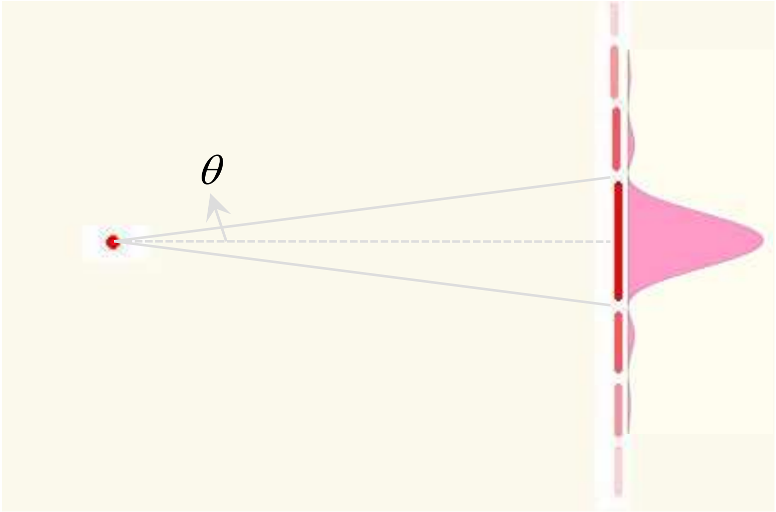
Fresnel Diffraction



Either the source or the screen (or both) are at finite distance - Fresnel class

(Fraunhofer) Diffraction: Huygens' explanation





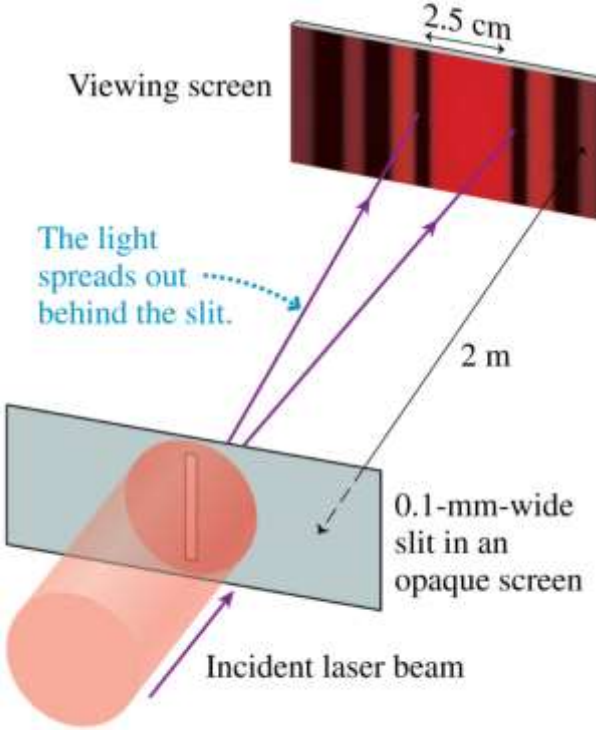
Interference Vs Diffraction

- WAVES: Frank S. Crawford Jr.

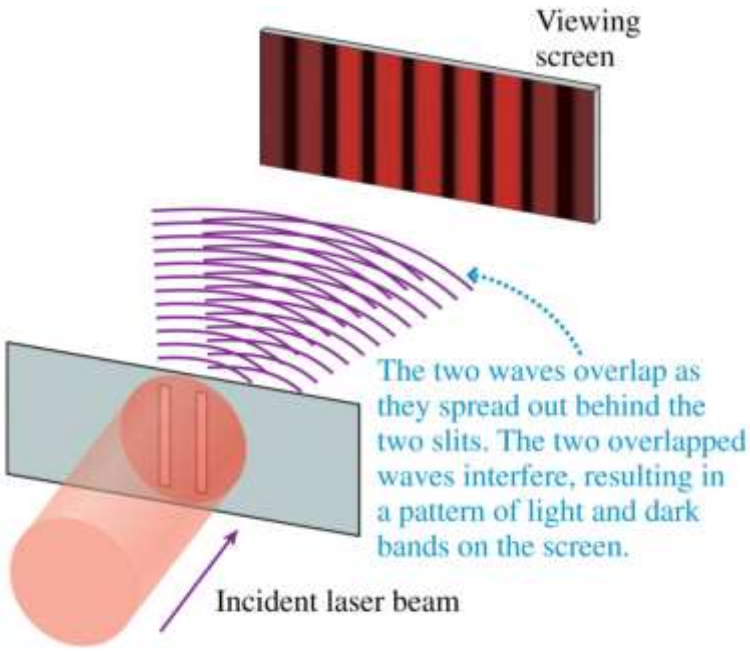
- What is the difference between an interference pattern and a diffraction pattern?
  - None, really.
- For historical reasons,
  - the amplitude or intensity pattern produced by superposing contributions from a finite number of discrete coherent sources is usually called an interference pattern;
  - the amplitude or intensity pattern produced by superposing contributions from a continuous distribution of coherent sources is usually called a diffraction pattern.
- Thus one speaks of the interference pattern from two narrow slits, or the diffraction pattern from one wide slit, or the combined interference and diffraction pattern from two wide slits.

Interference Vs Diffraction

Diffraction

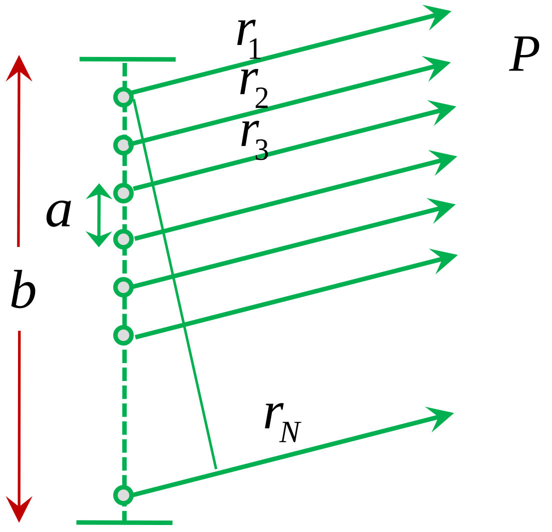


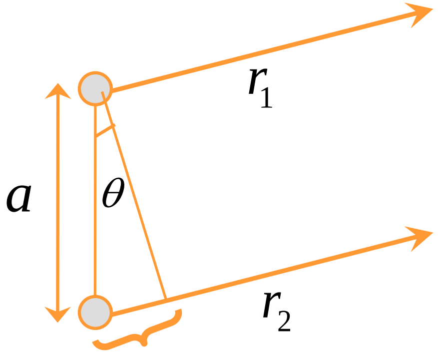
Interference



Single slit diffraction pattern (Fraunhofer)

N coherent oscillator model





$$\Lambda = a\sin\theta$$

$b = (N-1)a =$ **Slit width**

The sum of the interfering wavelet at P is

$$E = E_0 e^{i(kr_1-\omega t)} + E_0 e^{i(kr_2-\omega t)} + E_0 e^{i(kr_3-\omega t)} + \ldots + E_0 e^{i(kr_N-\omega t)}$$

$$= E_0 e^{i(kr_1-\omega t)}\left[1 + e^{ik(r_2-r_1)} + e^{ik(r_3-r_1)} \ldots + e^{ik(r_N-r_1)}\right]$$

Phase difference between the adjacent source

$$\boxed{\phi = k\Lambda = ka\sin\theta}$$

$$E = E_0 e^{i(kr_1-\omega t)}\left[1 + e^{i\phi} + e^{2i\phi} \ldots + e^{i(N-1)\phi}\right]$$

$$= E_0 e^{i(kr_1-\omega t)}\left(\frac{e^{iN\phi}-1}{e^{i\phi}-1}\right)$$

$$= E_0 e^{-i\omega t} e^{i[kr_1 + (N-1)\frac{\phi}{2}]} \frac{\sin\left(\frac{N}{2}\phi\right)}{\sin\left(\frac{\phi}{2}\right)}$$

$$\boxed{I = A\frac{\sin^2\left(\frac{N}{2}\phi\right)}{\sin^2\left(\frac{\phi}{2}\right)}}$$

We have........

$$I = A\frac{\sin^2\left(\frac{N}{2}\phi\right)}{\sin^2\left(\frac{\phi}{2}\right)}$$

$\phi = k\Lambda = ka\sin\theta$

$b = (N-1)a =$ **Slit width**

In the limit...... $N \to \infty \quad a \to 0$

$$I = A\frac{\sin^2\left(\frac{N\,k\,a\,\sin\theta}{2}\right)}{\sin^2\left(\frac{k\,a\,\sin\theta}{2}\right)} = A\frac{\sin^2\left(\frac{\pi\,b\,\sin\theta}{\lambda}\right)}{\left(\frac{\pi\,a\,\sin\theta}{\lambda}\right)^2} = A\frac{\sin^2\left(\frac{\pi\,b\,\sin\theta}{\lambda}\right)}{\left(\frac{\pi\,b\,\sin\theta}{N\lambda}\right)^2} = N^2 A\frac{\sin^2\left(\frac{\pi\,b\,\sin\theta}{\lambda}\right)}{\left(\frac{\pi\,b\,\sin\theta}{\lambda}\right)^2}$$

Now if $I_0 = N^2 A$ **and** $\beta = \frac{\pi b}{\lambda}\sin\theta$

$$\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}}$$

Condition of maxima and minima

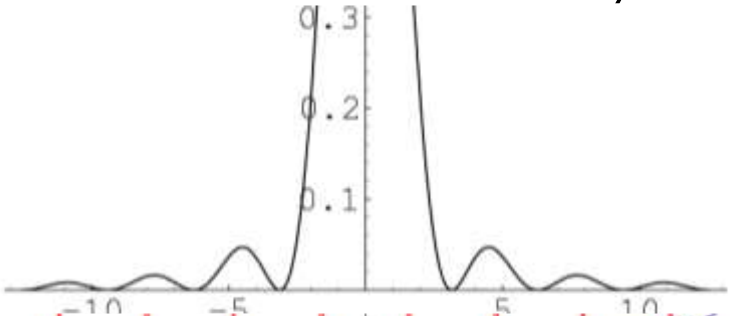
$$\boxed{I = I_0 \frac{\sin^2 \beta}{\beta^2}}$$

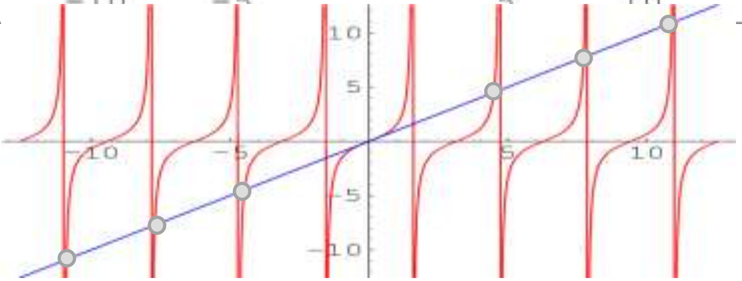
$$\beta = m\pi, \quad (\mathrm{m} \neq 0)$$ ⇨ **Condition of minima**

$$\beta = \frac{\pi b}{\lambda} \sin\theta$$

For Principal maxima $\theta = 0; \quad \frac{\sin\beta}{\beta} = 1, \;\; I(\theta) = I(0)$

For secondary maxima $\frac{dI}{d\beta} = I(0) \frac{2\sin\beta(\beta\cos\beta - \sin\beta)}{\beta^3} = 0$ ⇨ $\boxed{\tan\beta = \beta}$



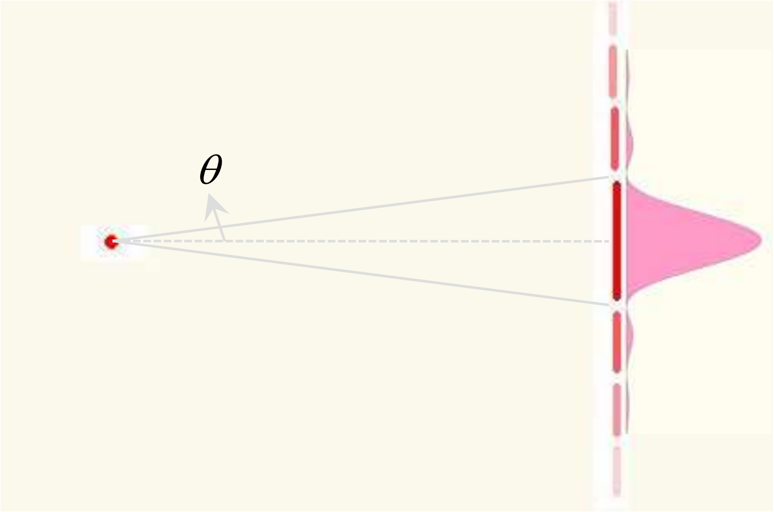


For secondary maxima

⇩

$$\beta = \pm 1.4303\pi, \pm 2.4590\pi, \pm 3.470\pi, ...$$

Angular width of central maximum



$$\beta = m\pi, \quad (\mathrm{m} \neq 0)$$

$$\beta = \frac{\pi b}{\lambda}\sin\theta$$

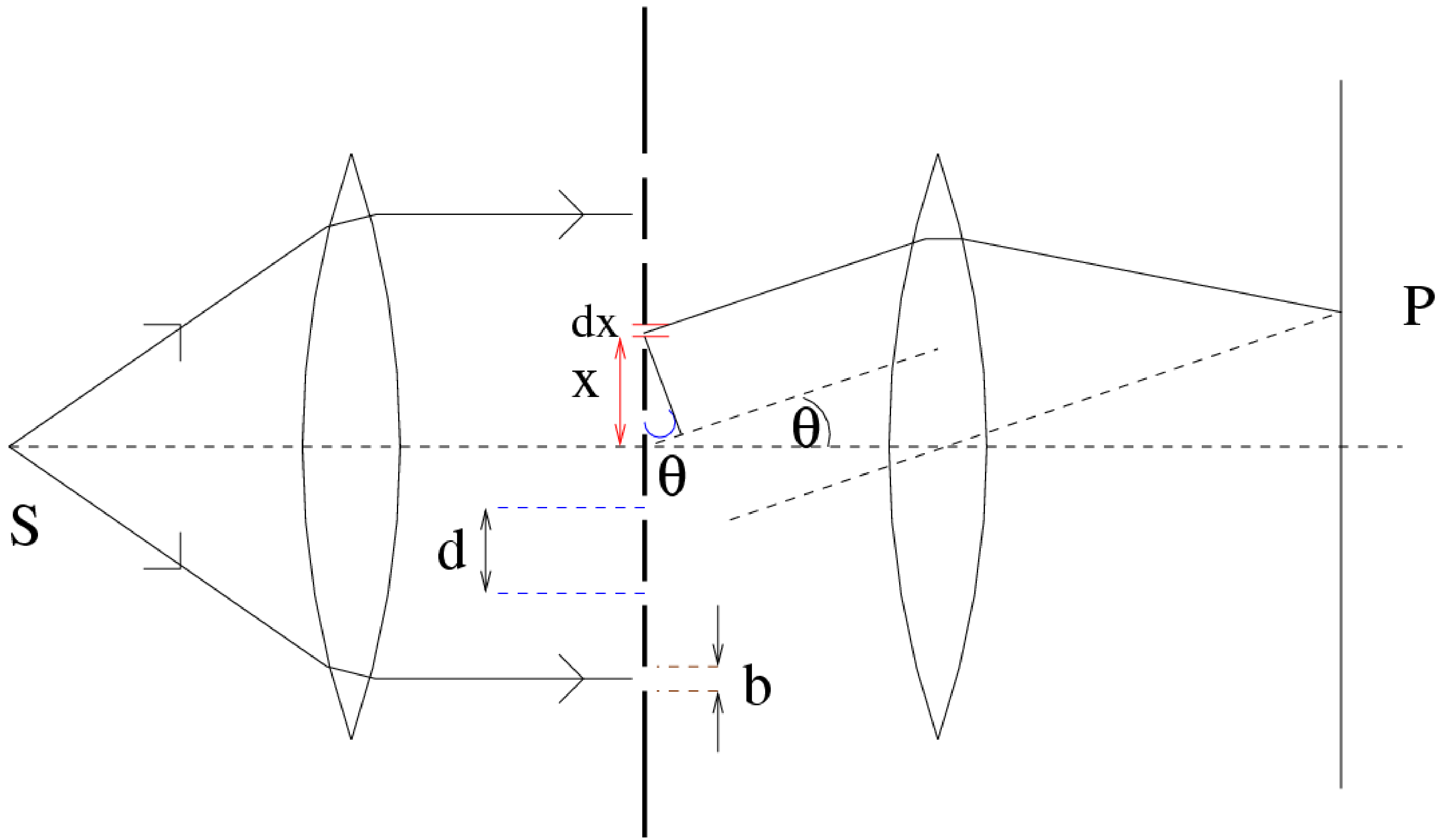
For first minima $m = 1 \Rightarrow \beta = \pi$

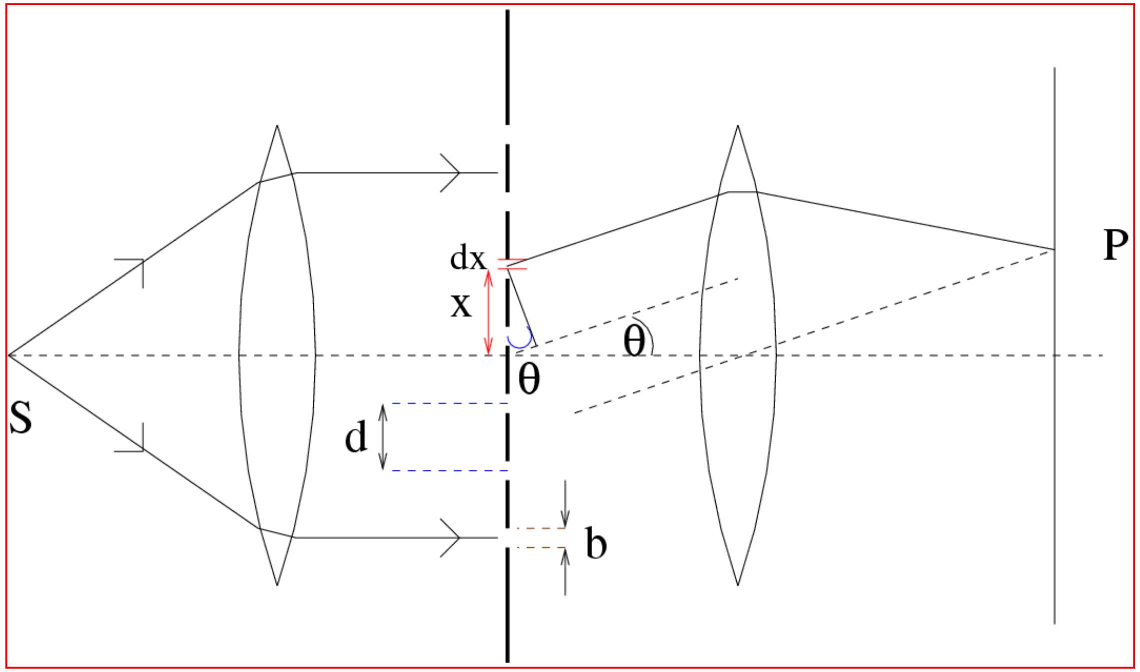
$$\beta = \frac{\pi b}{\lambda}\sin\theta = \pi \Rightarrow \sin\theta = \frac{\lambda}{b}$$

θ **is small** $\Rightarrow \sin\theta \approx \theta = \dfrac{\lambda}{b}$

Angular width $2\theta = \dfrac{2\lambda}{b}$

N slit diffraction/Diffraction grating





$$A \propto a + ae^{i\delta} + ae^{i2\delta} + ae^{i3\delta} + \cdots . + ae^{i(N-1)\delta}$$

$$a = \frac{\sin\beta}{\beta}$$

$$\beta = \frac{\pi}{\lambda} b \sin\theta$$

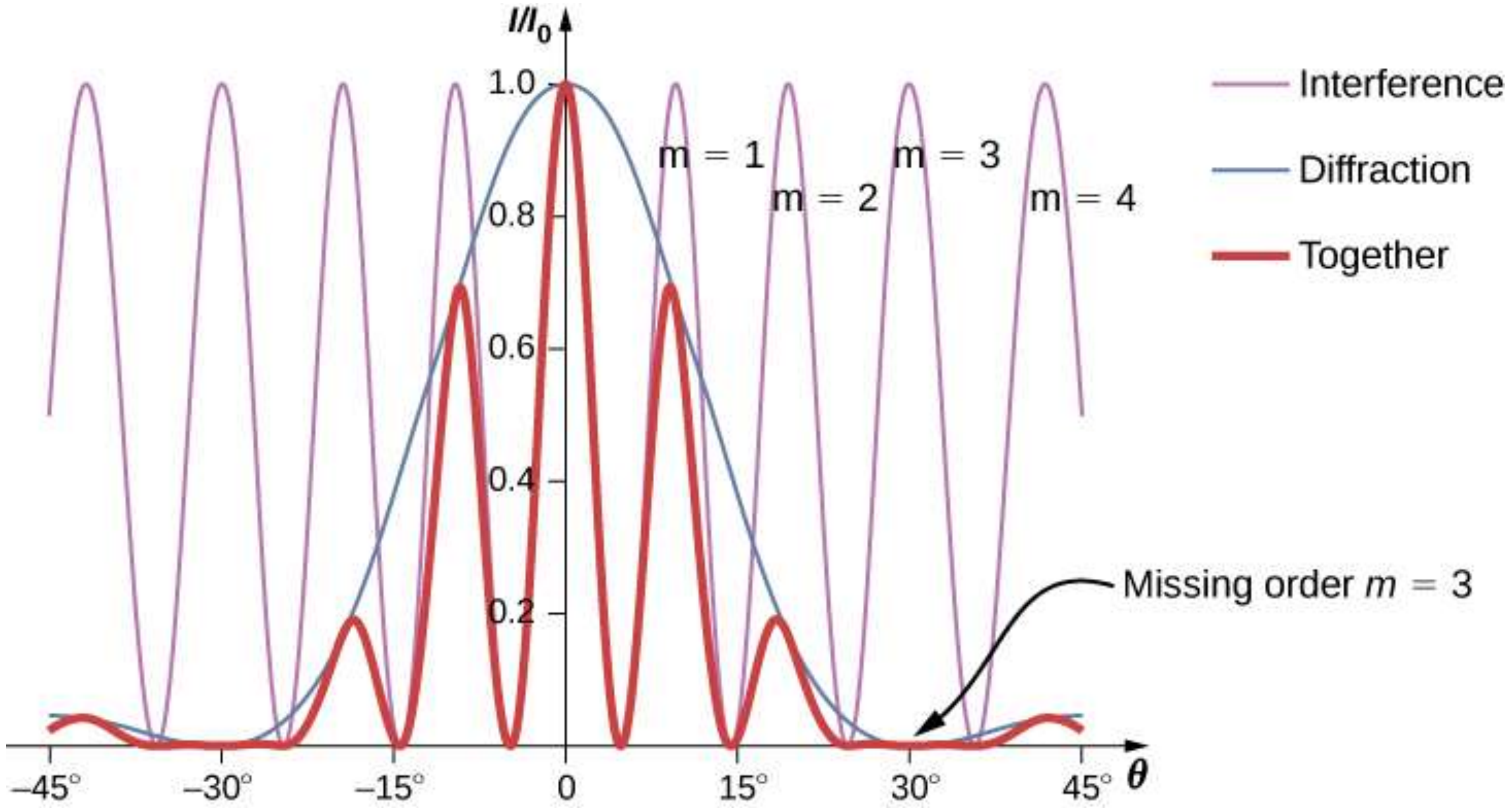
$$\delta = \frac{2\pi}{\lambda} d \sin\theta = 2\gamma$$

$$\Rightarrow A \propto a\frac{1-e^{iN\delta}}{1-e^{i\delta}} = a\frac{\sin\left(\frac{N\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} e^{\frac{i(N-1)\delta}{2}} = a\frac{\sin(N\gamma)}{\sin(\gamma)} e^{i(N-1)\gamma} = \frac{\sin\beta}{\beta}\frac{\sin(N\gamma)}{\sin(\gamma)} e^{i(N-1)\gamma}$$

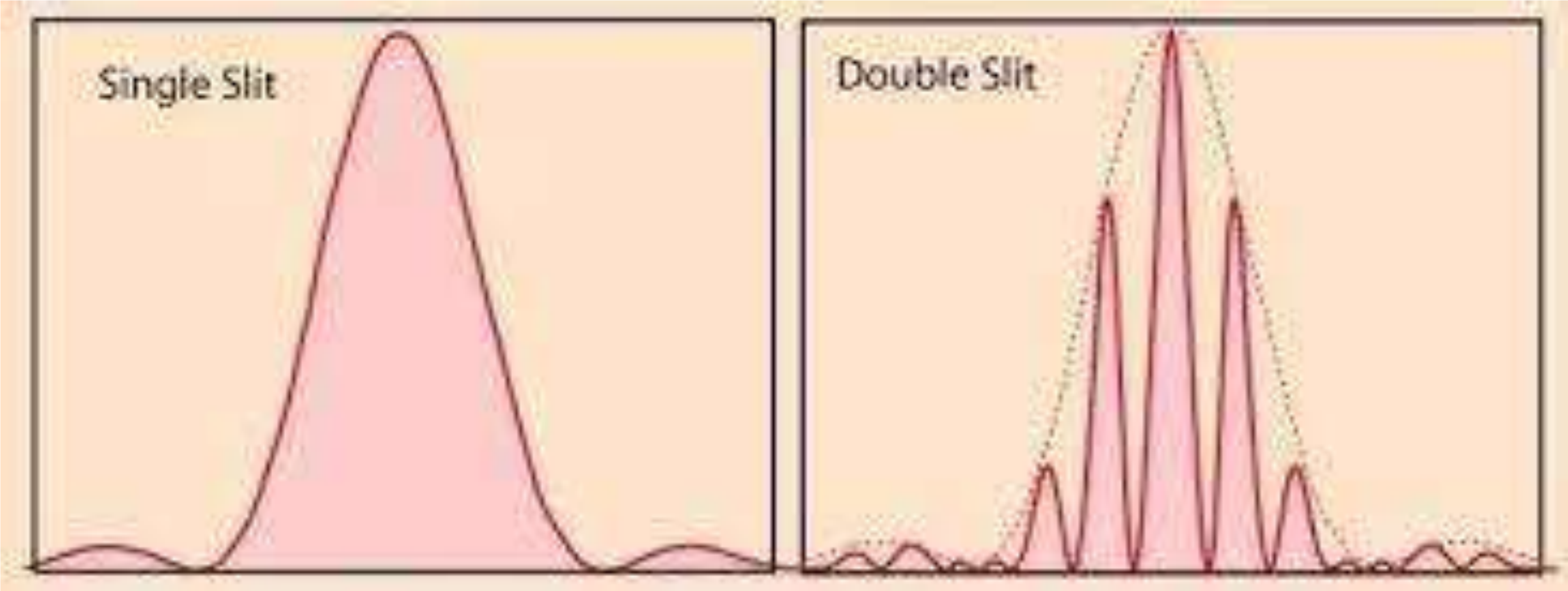
$$\Rightarrow \mathrm{I}(\theta) = I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{Single slit diffraction term}} \times \underbrace{\frac{\sin^2(N\gamma)}{\sin^2\gamma}}_{\text{N-slit interference term}}$$

$$\boxed{\mathrm{I}(\theta) = I_0 \frac{\sin^2(N\gamma)}{\sin^2\gamma}, \text{ for small } b}$$

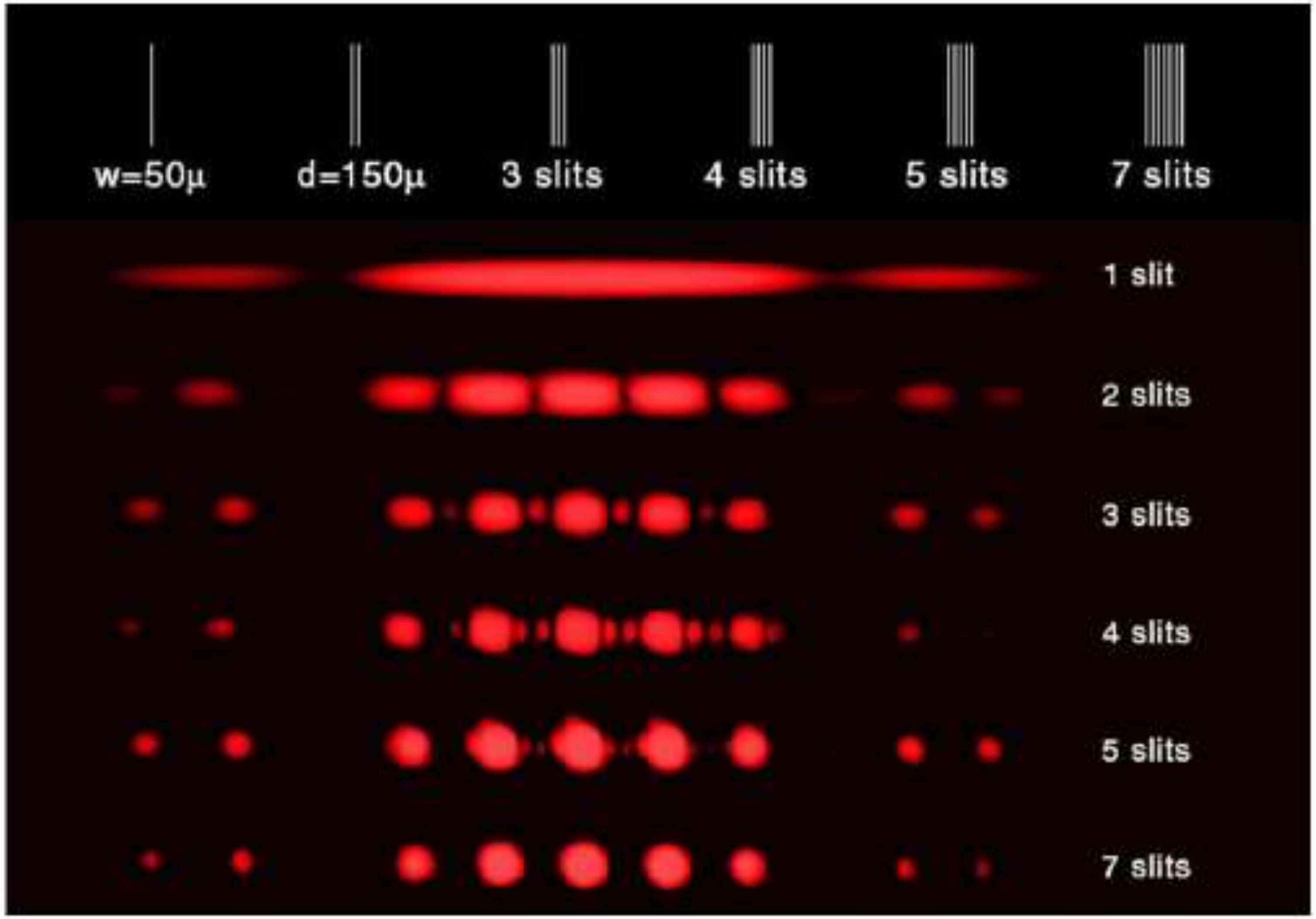
Single slit diffraction term × N-slit interference term



I/I0
1.0
0.8
0.6
0.4
0.2
m = 1
m = 2
m = 3
m = 4
Interference
Diffraction
Together
Missing order m = 3
–45°
–30°
–15°
0
15°
30°
45°
θ


Single Slit
Double Slit

Multi slit diffraction



$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2 ; I_0 = \frac{b^2 C^2}{2}.$$

Analysis

$$\Rightarrow \mathrm{I}(\theta) = I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}} \underbrace{\frac{\sin^2(N\gamma)}{\sin^2\gamma}} \qquad \gamma = \frac{\pi}{\lambda} d \sin\theta$$

Single slit diffraction term × N-slit interference term

The interference term $I_{int} = \frac{\sin^2(N\gamma)}{\sin^2\gamma}$:

for large N and small d, $\quad I_{int} = \frac{\sin^2(N\gamma)}{\sin^2\gamma} \approx \frac{\sin^2(N\gamma)}{\gamma^2}$

then, for $\gamma = 0$, $\quad I_{int} = \lim_{\gamma\to 0} \frac{\sin^2 N\gamma}{\gamma^2} = N^2 \lim_{\gamma\to 0} \frac{\sin^2 N\gamma}{\gamma^2} = N^2$ (a maxima $\equiv$ principal maxima)

In fact, for any $\gamma = m\pi$ $(m = 0, \pm1, \pm2, \dots)$, $I_{int} = N^2$ (**principal maxima**)

$$\boxed{\gamma_m = m\pi \;\Rightarrow d \sin\theta_m = m\lambda \; (m = 0, \pm1, \pm2, \dots) \equiv \text{principal maxima}}$$

For these θ values, the intensity has large value unless $\frac{\sin^2\beta}{\beta^2}$ itself is very small.

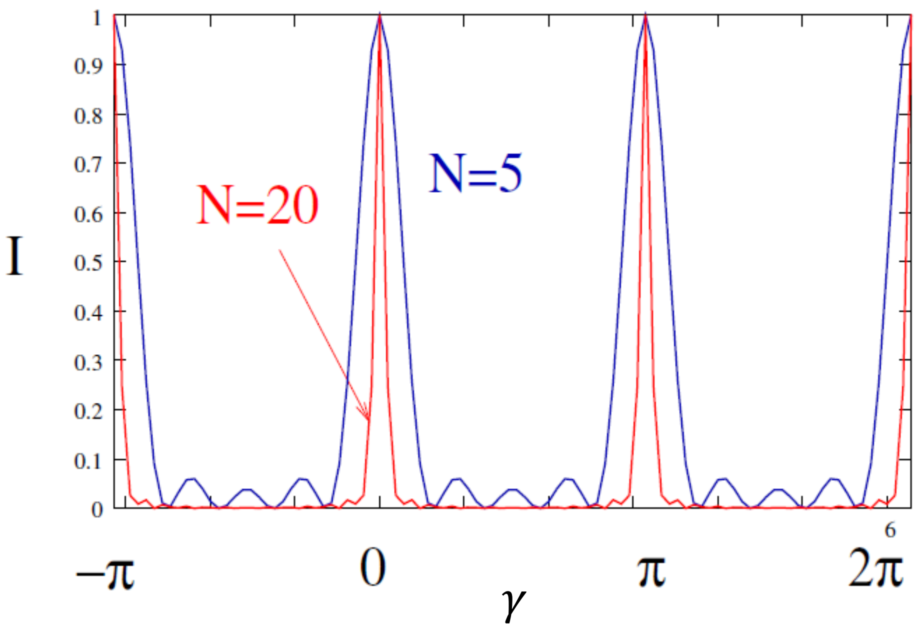
Since $|\sin\theta_m| \le 1$, there will only be a finite number of principal maxima

The interference term $I_{int} = \frac{\sin^2(N\gamma)}{\sin^2\gamma}$:

for large N and small d, $I_{int} = \frac{\sin^2(N\gamma)}{\sin^2\gamma} \approx \frac{\sin^2(N\gamma)}{\gamma^2}$

I_{int} will be zero (**minima**) whenever the numerator is zero, i.e.,

For $N\gamma = p\pi$ ($p = \pm1, \pm2, \ldots$, but $\neq \pm N, \pm2N, \ldots$ where it will be maxima),



Double slit diffraction (N=2)

$$I(\theta) = I_0 \frac{\sin^2\beta}{\beta^2}\frac{\sin^2(2\gamma)}{\sin^2\gamma} = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

where, $\beta = \frac{\pi}{\lambda} b \sin\theta$ and $\gamma = \frac{\pi}{\lambda} d \sin\theta$

The intensity is **zero** whenever

either $\beta = m\pi \; (m = \pm1, \pm2, \dots.)$

$\Rightarrow b\sin\theta = m\lambda\,; (m = \pm1, \pm2, \dots.)$

or $\gamma = (n + 1/2)\pi;\; (n = 0, \pm1, \pm2, \dots.)$

$\Rightarrow d\sin\theta = (n + 1/2)\lambda\,; (n = 0, \pm1, \pm2, \dots.)$

The intensity **maxima** whenever

$\gamma = n\pi;\; (n = 0, \pm1, \pm2, \dots.)$

$\Rightarrow d\sin\theta = n\lambda\,; (n = 0, \pm1, \pm2, \dots.)$

MISSING ORDER

$b \sin\theta = m\lambda\,; (m = \pm 1, \pm 2, \ldots.)$ - single slit diffraction minima

$d \sin\theta = n\lambda\,; (n = 0, \pm 1, \pm 2, \ldots.)$ - double slit interference maxima

If both equation satisfy for a given value of θ

$$\frac{d}{b} = \frac{n}{m}$$

Thus, the n^th order interference maximum will have a zero intensity and will be missing

- For, $d/b = 5$, the first missing fringe is fifth one and then multiple of 5 will be missing.
- For, $d/b = 1$, all the interference maxima should be missing except the central one. (Two slit coalesce into one bigger slit.)

$$I(\theta) = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma = 4I_0\left(\frac{\sin 2\beta}{2\beta}\right)^2, \text{since } \gamma = \beta$$

- For, $d/b = \infty$, no interference fringe should be missing (double slit interference condition)

Width of the principal maxima

The m^{th} order principal maxima occurs at $b \sin\theta_m = m\lambda$; where $m = 0; 1; 2\ldots$

If $\theta_m + \Delta\theta_{1m}$ & $\theta_m - \Delta\theta_{2m}$ represent the angles of diffraction corresponding to the first minimum on either side of the principal maximum, then $\frac{1}{2}(\boldsymbol{\Delta\theta_{1m}}$

For a large value of N, $\boldsymbol{\Delta\theta_{1m} \approx \Delta\theta_{2m} = \Delta\theta_m}$

$$\sin(\theta_m \pm \Delta\theta_m) = \sin\theta_m \cos\Delta\theta_m \pm \cos\theta_m \sin\Delta\theta_m$$

$$= \sin\theta_m \pm \Delta\theta_m \cos\theta_m .$$

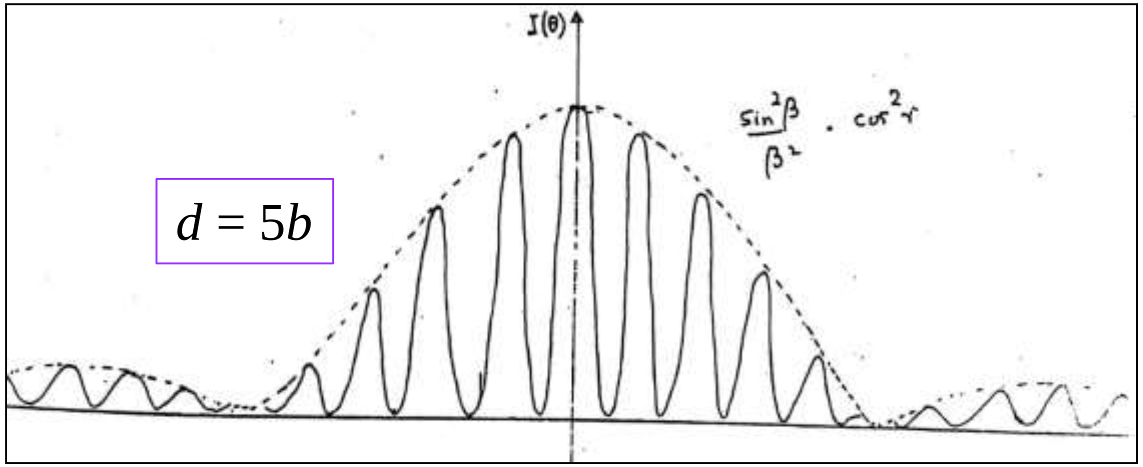
Since, $$b\sin(\theta_m \pm \Delta\theta_m) = m\lambda \pm \frac{\lambda}{N};$$

Hence,

$$\Delta\theta_m \approx \frac{\lambda}{N\, b\cos\theta_m} .$$

which shows the principal maxima becomes sharper with larger N

Double slit diffraction (N=2)

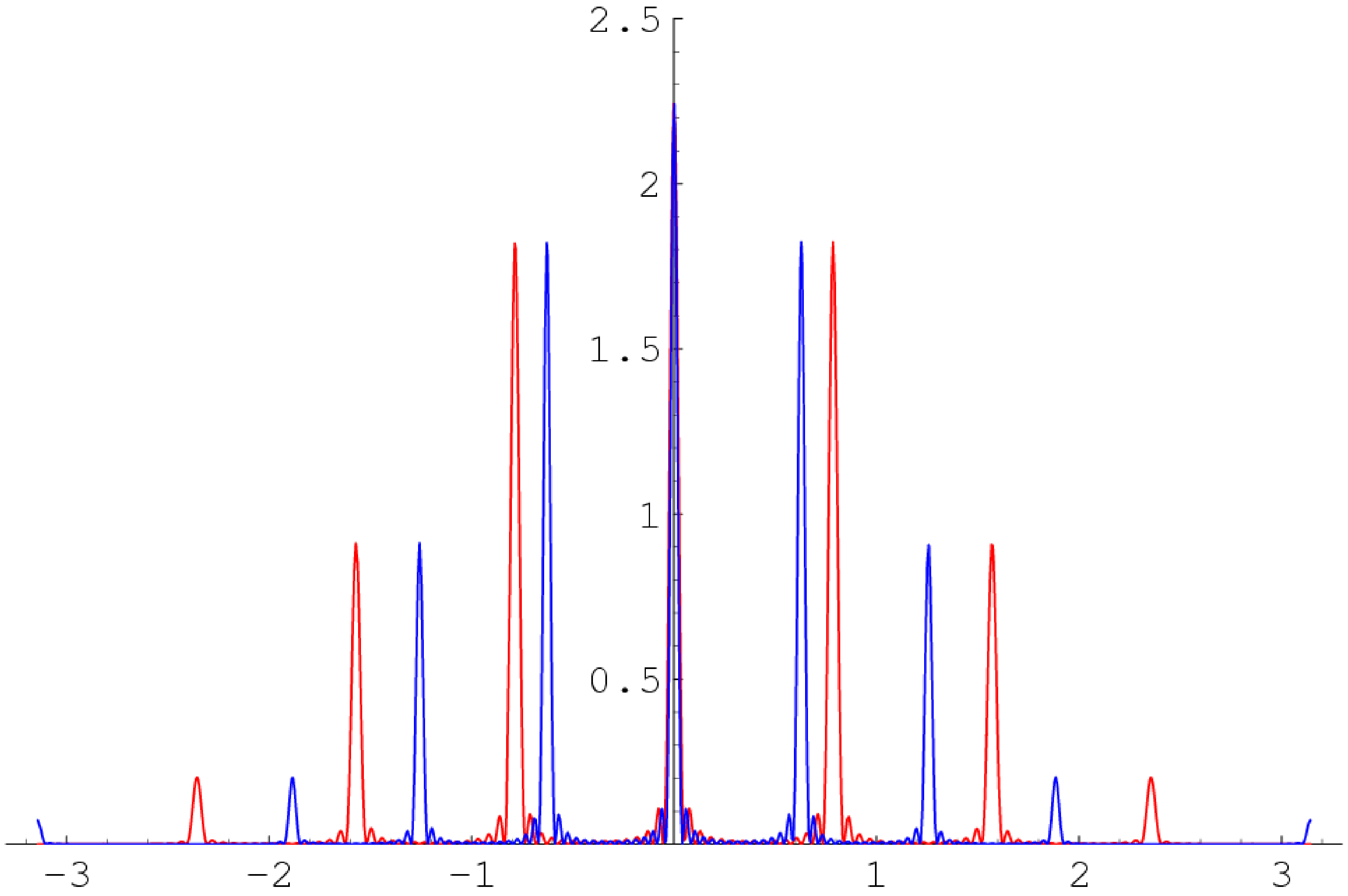


Number of interference maxima in principal diffraction maximum = 2(*d*/*b*) - 1

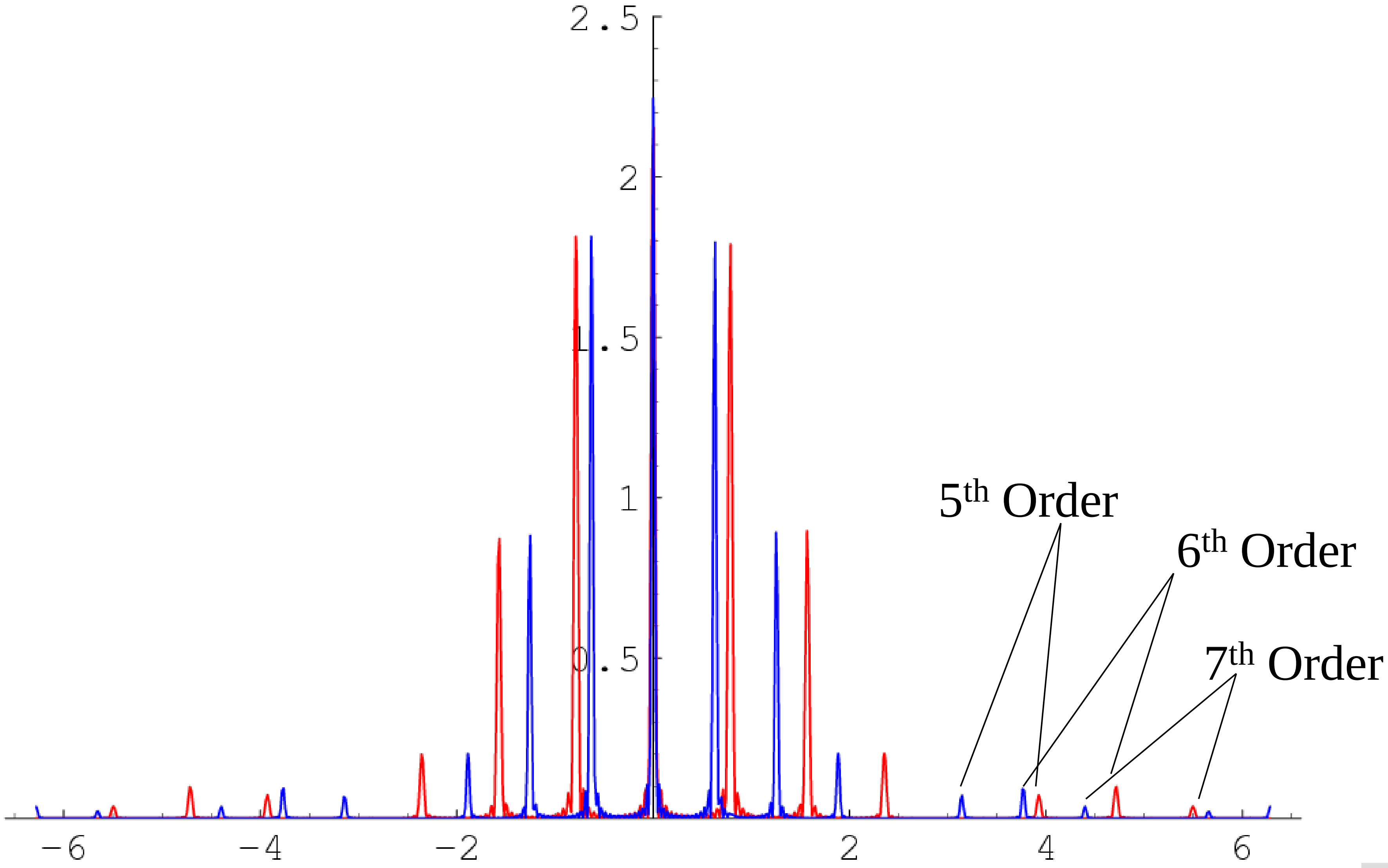
width of principal maxima, $\Delta\theta_m \approx \dfrac{\lambda}{N\,b\cos\theta_m} = \dfrac{\lambda}{2\,b\cos\theta_m}$,

Full width of principal maxima, $2\Delta\theta_m$

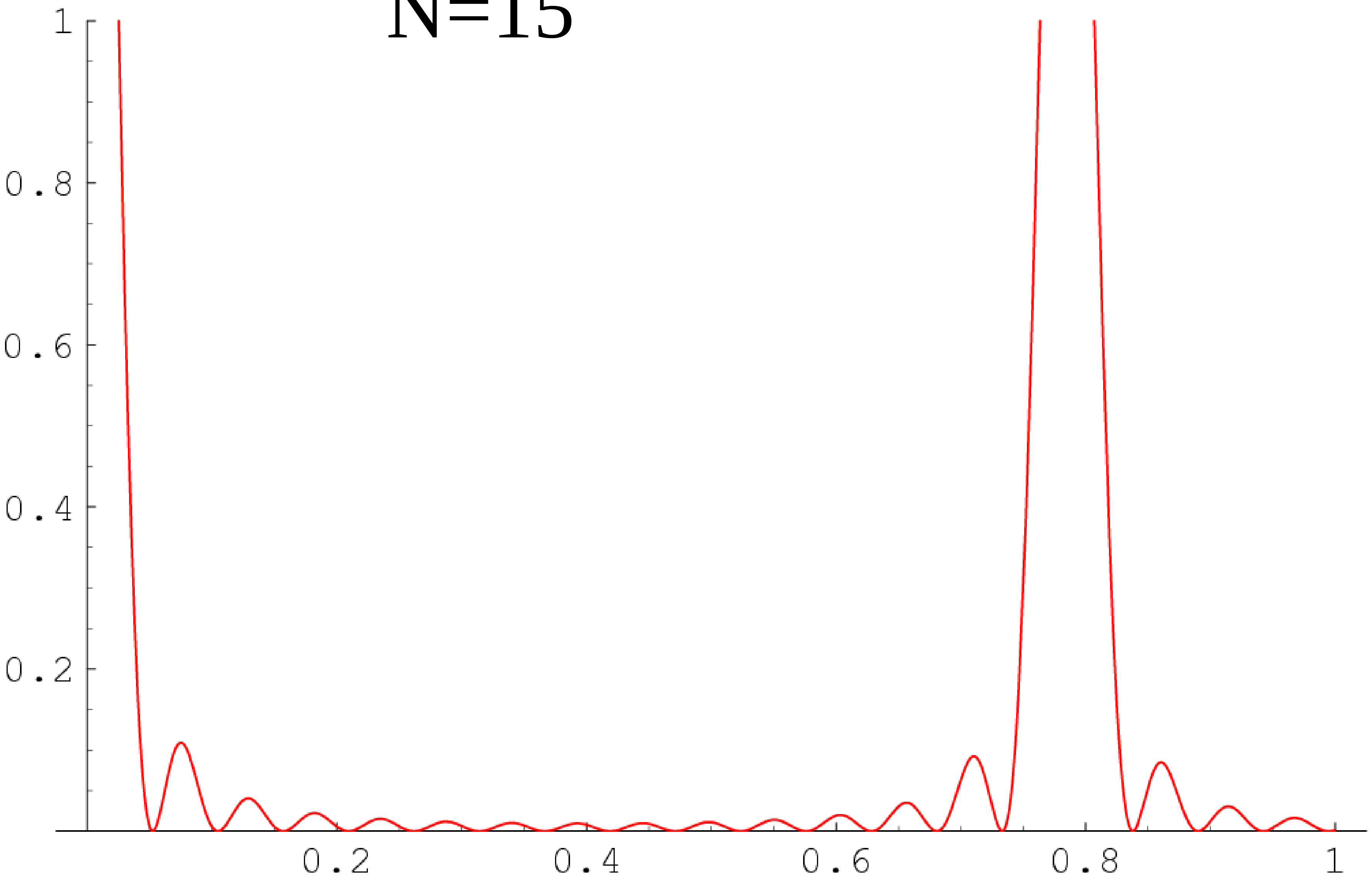
For two wavelengths



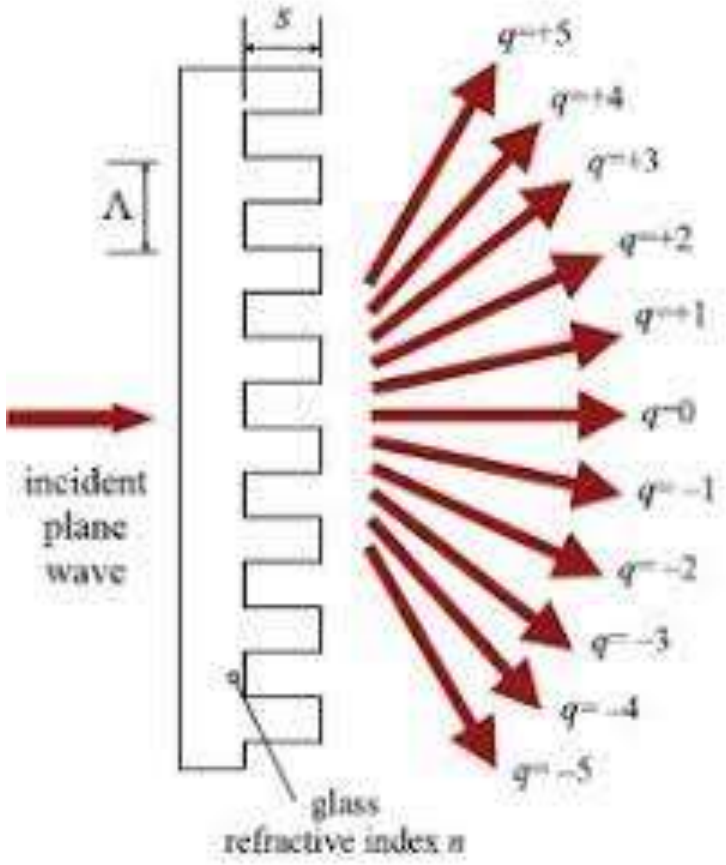
$a/b = 4$

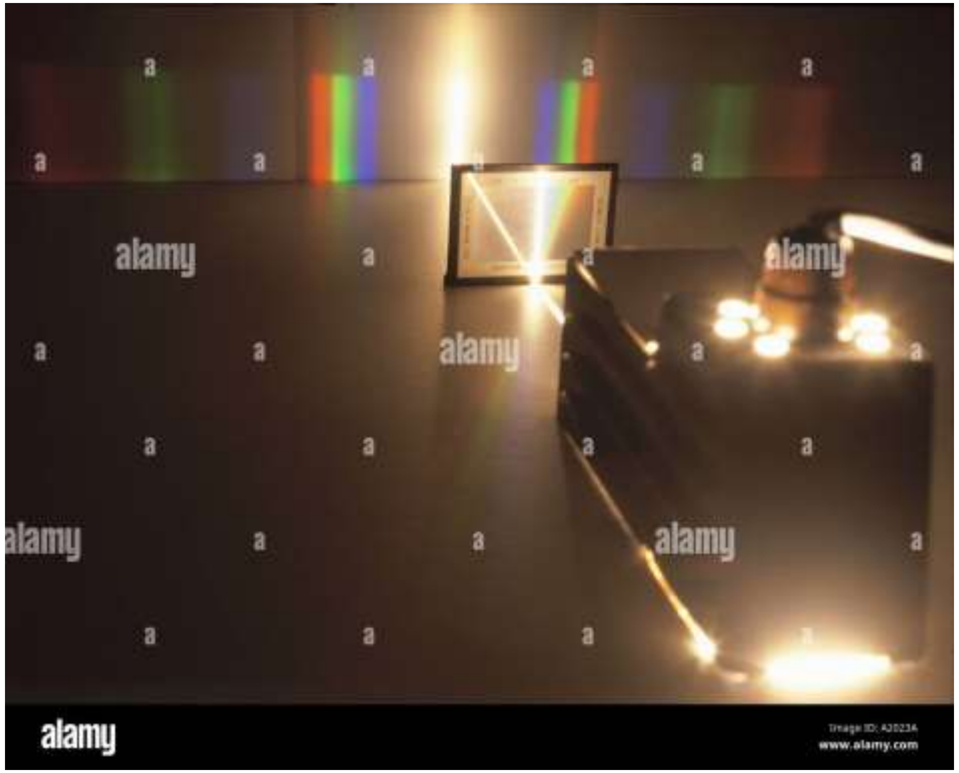


N=15



Diffraction Grating

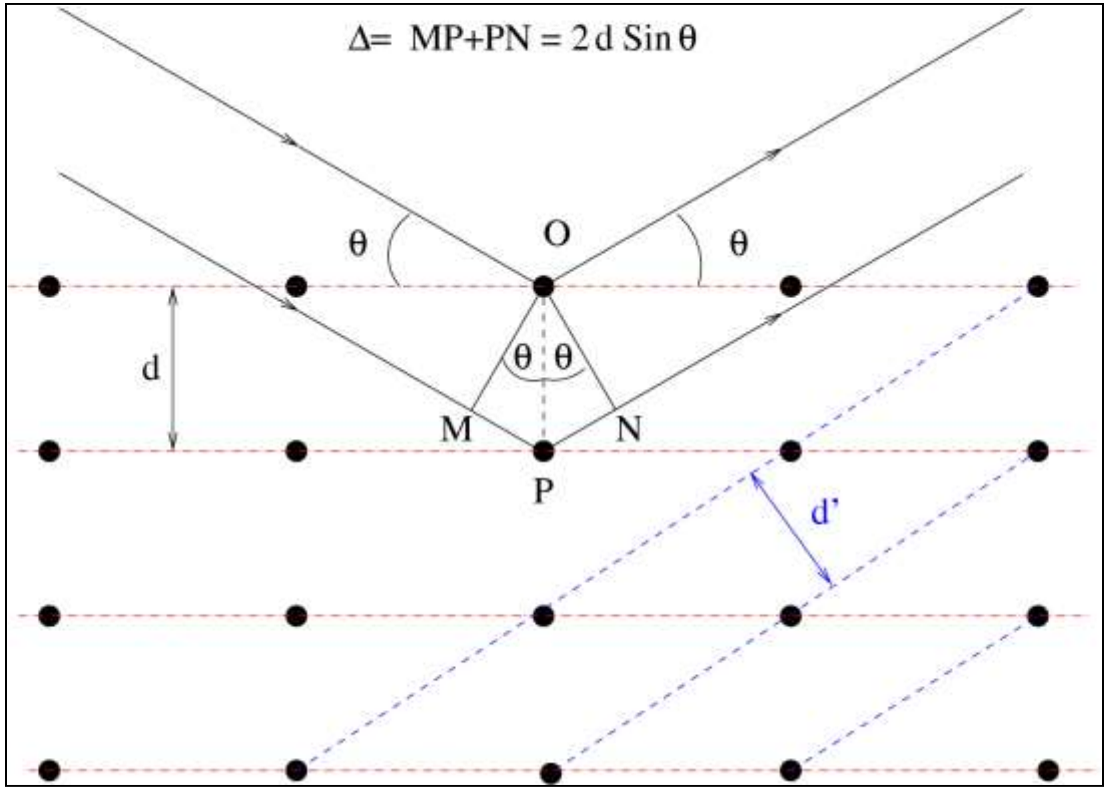




X-ray diffraction from crystals: Bragg’s law

$$2d \sin\theta = n\lambda$$

$$\Delta = MP + PN = 2d \sin\theta$$



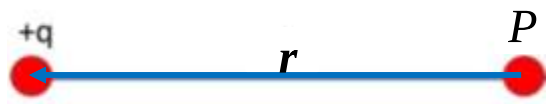
Polarization of light

RECALL

Dipole Oscillation, Radiation. Polarization

Dipole Radiation

RECALL



What is the electric field produced at a point P by a charge q located at a distance r?

$$\vec{E} = -\frac{q}{4\pi\varepsilon_0}\frac{\hat{e}_r}{r^2}$$

Where, $\hat{e}_r$ is the unit vector from P to the position of the charge.

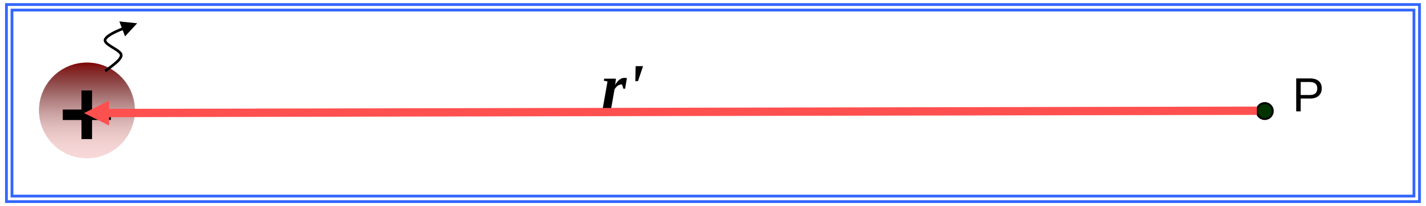
Assumption:	The field reaches the point instantaneously.
Reality:	The field propagates with the speed of light.
Required:	CORRECTION

The CORRECTION

RECALL

The correct formula for the electric field

$$\mathbf{E} = \frac{-q}{4\pi\epsilon_0}\left[\frac{\mathbf{e}_{\mathbf{r}'}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\mathbf{e}_{\mathbf{r}'}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$



$e_{r'}$:unit vector directed from *P* to the point charge at an earlier time

Important features

1. No information can propagate instantaneously

2. The electric field at the time *t* is determined by the position of the charge at an earlier time, when the charge was at *r'*, the retarded position.

3. First two terms falls off as $1/r'^2$ and hence are of no interest at large distances

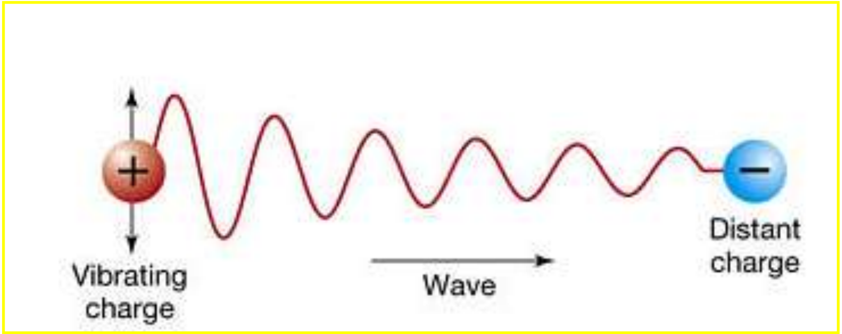
Correct Expression
(at large distances)

RECALL

$$\mathbf{E} = \frac{-q}{4\pi\epsilon_0 c^2}\left[\frac{d^2\mathbf{e}_{\mathbf{r}'}}{dt^2}\right]$$

This is electro-magnetic radiation or simply radiation.

It is also to be noted that only accelerating charges produce radiation.

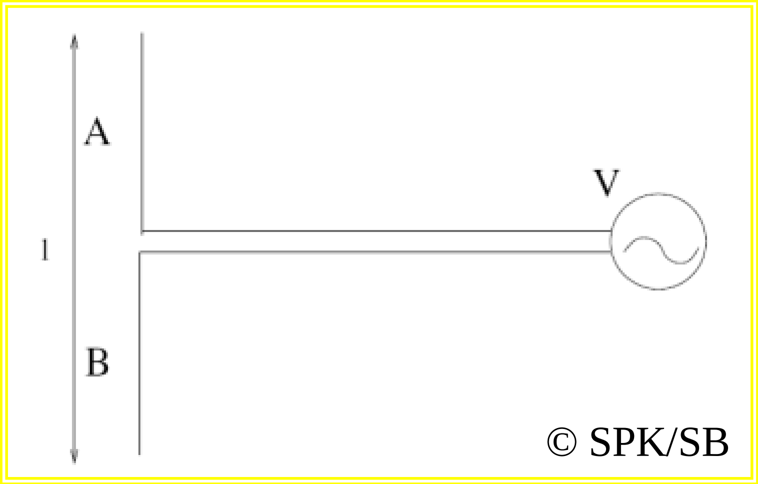


If a charge moves non-uniformly, it radiates.

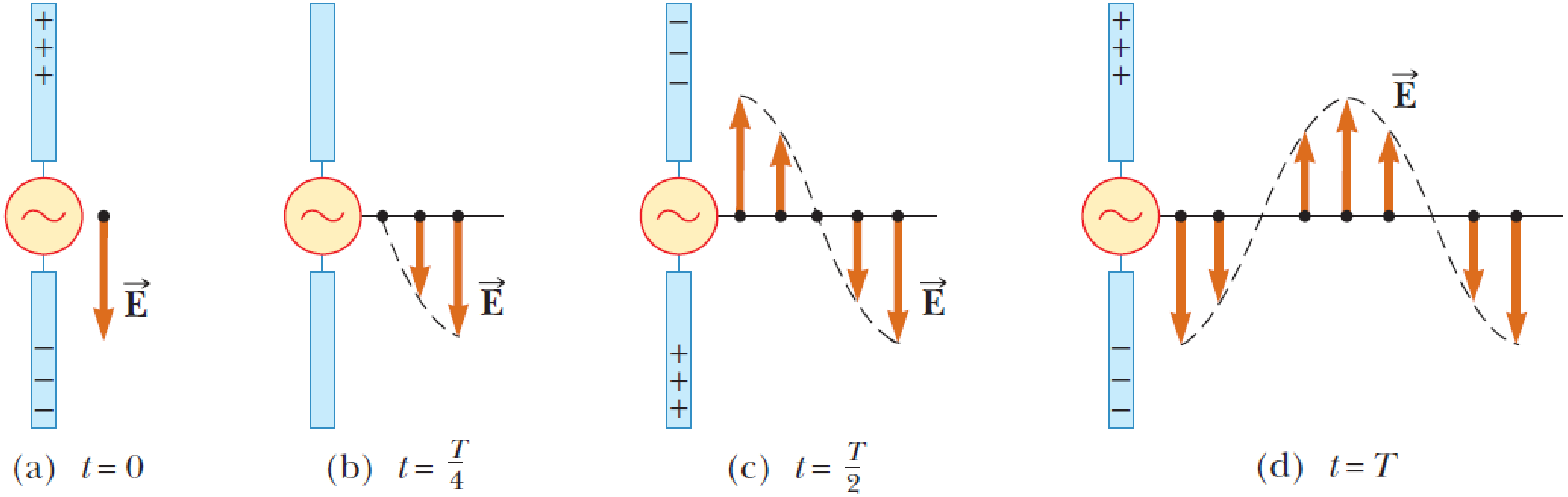
The magnetic field produced by the charge is $\mathbf{B} = -\mathbf{e}_{\mathbf{r}'} \times \frac{\mathbf{E}}{\mathbf{c}}$

Electric Dipole Oscillator

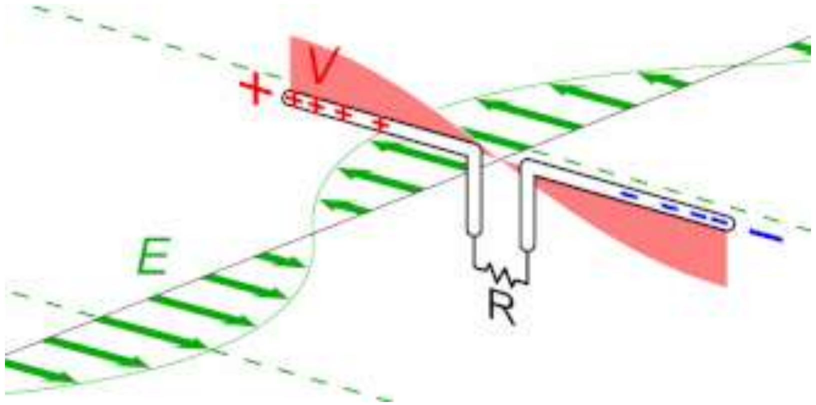
RECALL



Radio-wave transmission



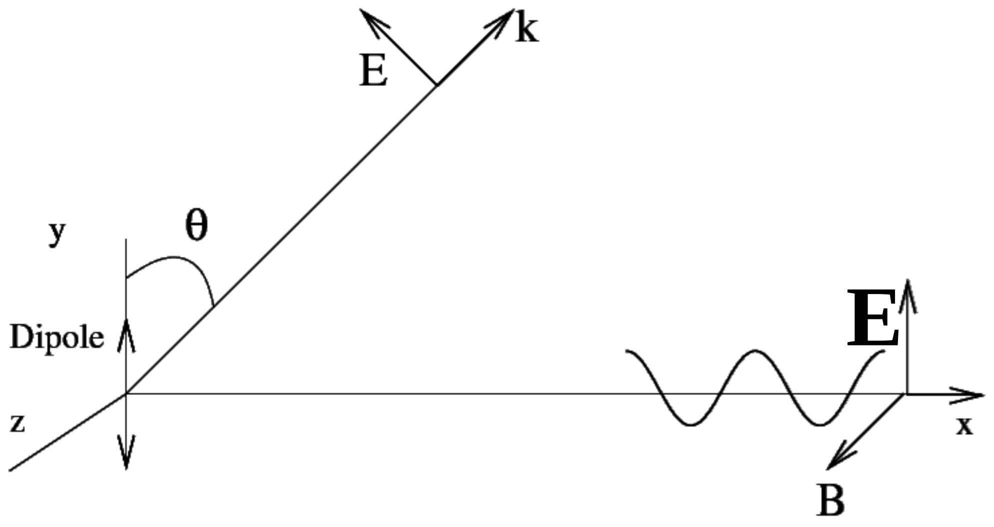
RECALL



Dipole antenna - Wikipedia

Sinusoidally Oscillating Dipole

RECALL



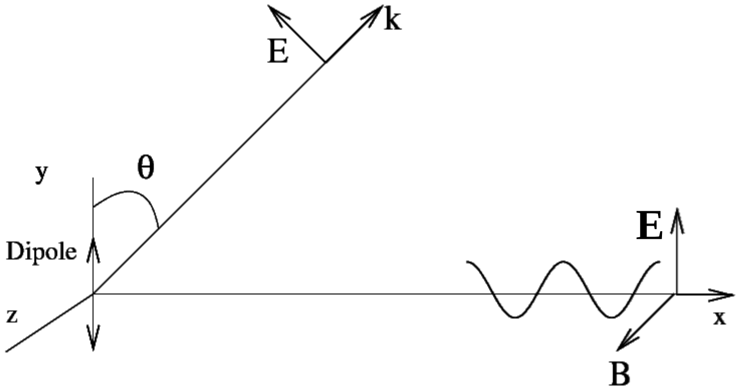
The voltage causes the charges to move up and down $y(t) = y_0 \cos(\omega t)$

producing an electric field

$$E(t) = \frac{q y_0 \omega^2}{4\pi\epsilon_0 c^2 r} \cos[\omega(t - r/c)] \sin\theta$$

Sinusoidally Oscillating Dipole

RECALL



$$E(t) = \frac{q y_0 \omega^2}{4\pi\epsilon_0 c^2 r} \cos[\omega(t - r/c)] \sin\theta$$

At large distances the electric field of the radiation can be well described by

$$E_y(x,t) = E\cos[\omega t - kx]$$ Plane wave $k = \omega/c$

$$\vec{B}(x,t) = \hat{i} \times \vec{E}_y(x,t)/c = \frac{E}{c}\cos(\omega t - kx)\,\hat{k}$$

Crossed Dipoles

RECALL

$$\vec{E}(x,t) = E_y(x,t)\hat{j} + E_z(x,t)\hat{k}$$

Suppose:

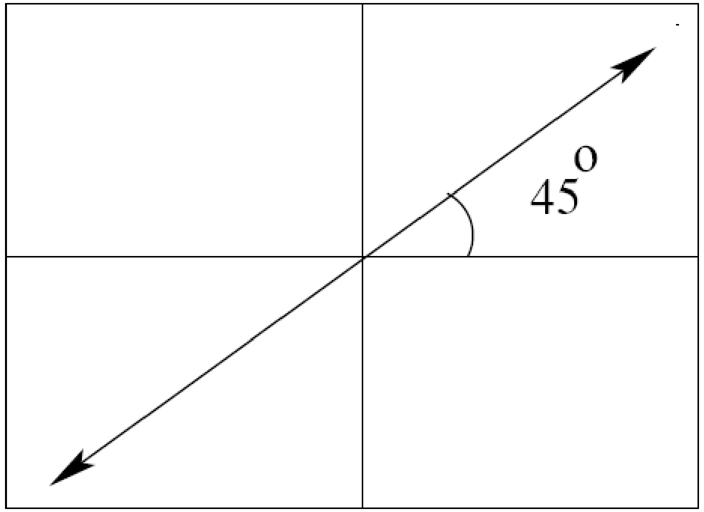
$$E_y(x,t) = E_{yo}\, cos(\omega t - kx)$$

$$E_z(x,t) = E_{zo} cos(\omega t - kx)$$

and, $E_{yo} = E_{zo} = E$

Then: $E_y(x,t) = E_z(x,t)$

$$\vec{E}(x,t) = E(\hat{j} + \hat{k})\cos(\omega t - kx)$$



Linear Polarization

RECALL

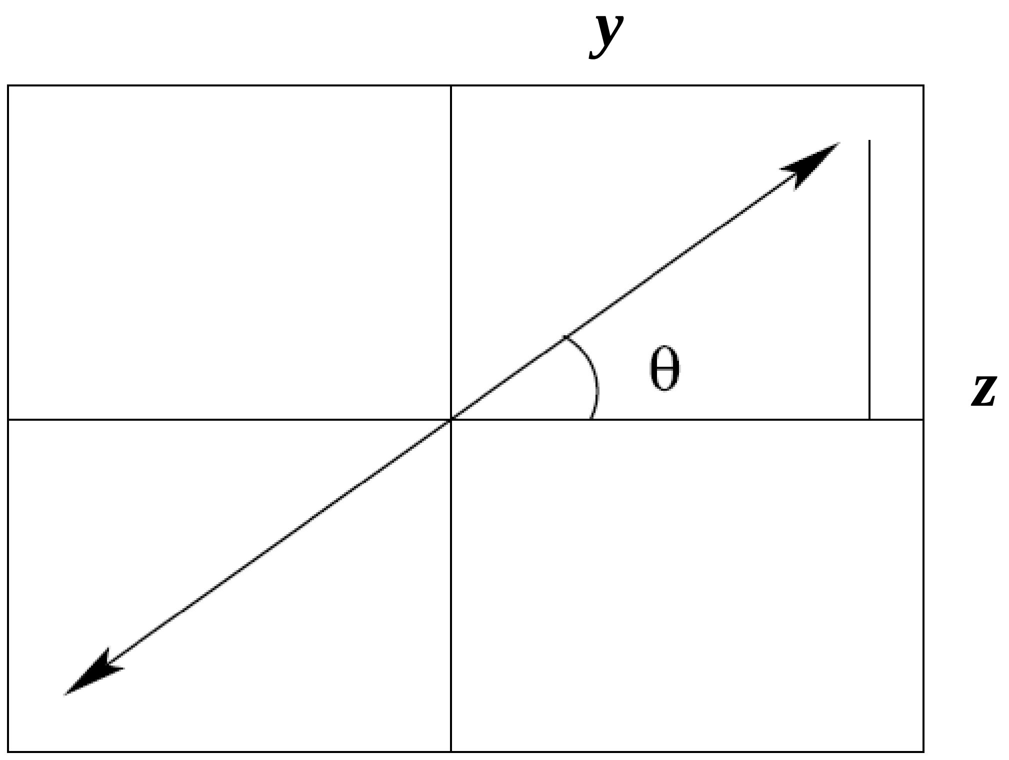
$$\vec{E}(x,t) = E_y(x,t)\hat{j} + E_z(x,t)\hat{k}$$

Suppose:

$$E_y(x,t) = E_{yo}\, cos(\omega t - kx)$$

$$E_z(x,t) = E_{zo} cos(\omega t - kx)$$

and, $E_{yo} \neq E_{zo}$



Then:

$$\vec{E}(x,t) = (E_{yo}\hat{j} + E_{zo}\hat{k}) cos(\omega t - kx)$$

$$E = \sqrt{E_{y0}^2 + E_{z0}^2}$$

$$\theta = tan^{-1}\left(\frac{E_{zo}}{E_{yo}}\right)$$

Circular Polarization

RECALL

Consider: Two orthogonally polarized waves of equal amplitude and with the field in the z-direction having an extra phase of π/2

$$\vec{E}(x,t) = E\left[\cos(\omega t - kx)\hat{j} + \cos(\omega t - kx + \pi/2)\hat{k}\right.$$

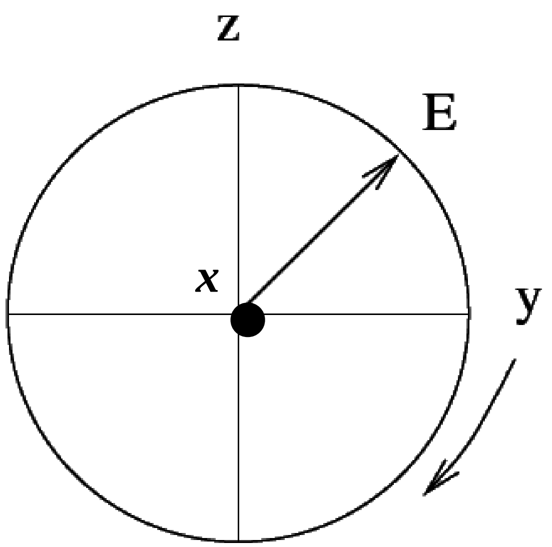
$$= E\left[\cos(\omega t - kx)\hat{j} - \sin(\omega t - kx)\hat{k}\right]$$

Circular Polarization

RECALL

$$\vec{E}(x,t) = E[cos(\omega t - kx)\hat{\jmath} - sin(\omega t - kx)\hat{k}]$$

At x = 0, the electric field vector traces a **Clockwise** circular path with increasing time, when the observer looks towards the source: **Right circularly polarized light**.

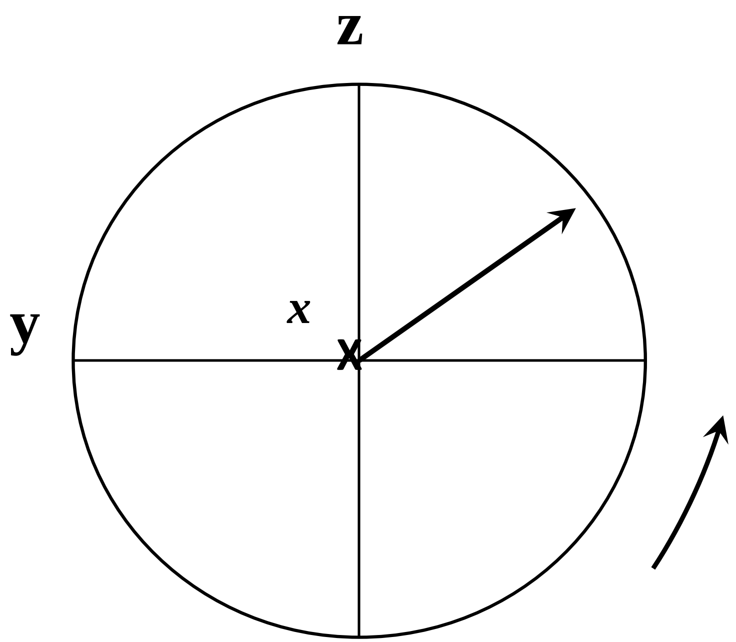


Right Circularly Polarized Light

Circular Polarization

RECALL

Consider: Two orthogonally polarized waves of equal amplitude and with the field in the z-direction having a phase lag of $\pi/2$

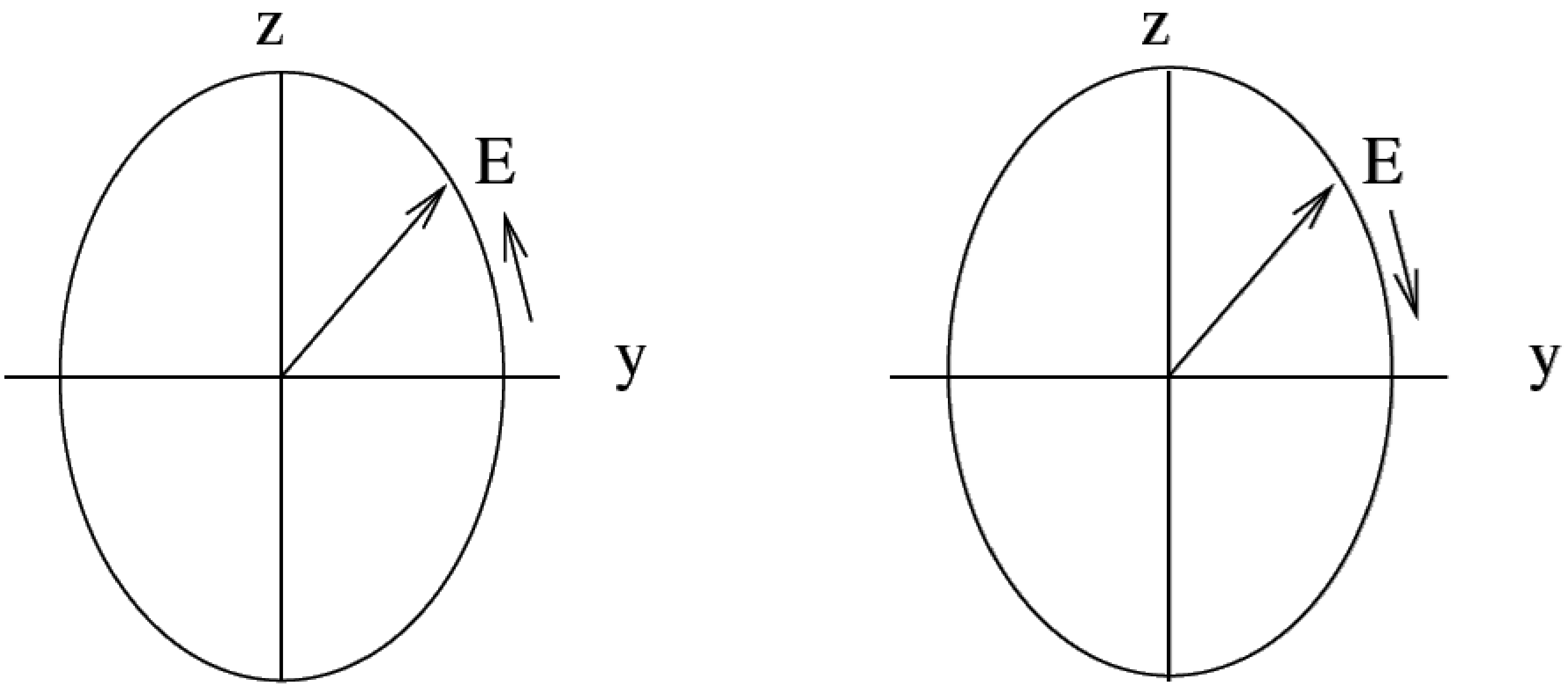


Left Circularly Polarized Light

Elliptical Polarization

RECALL

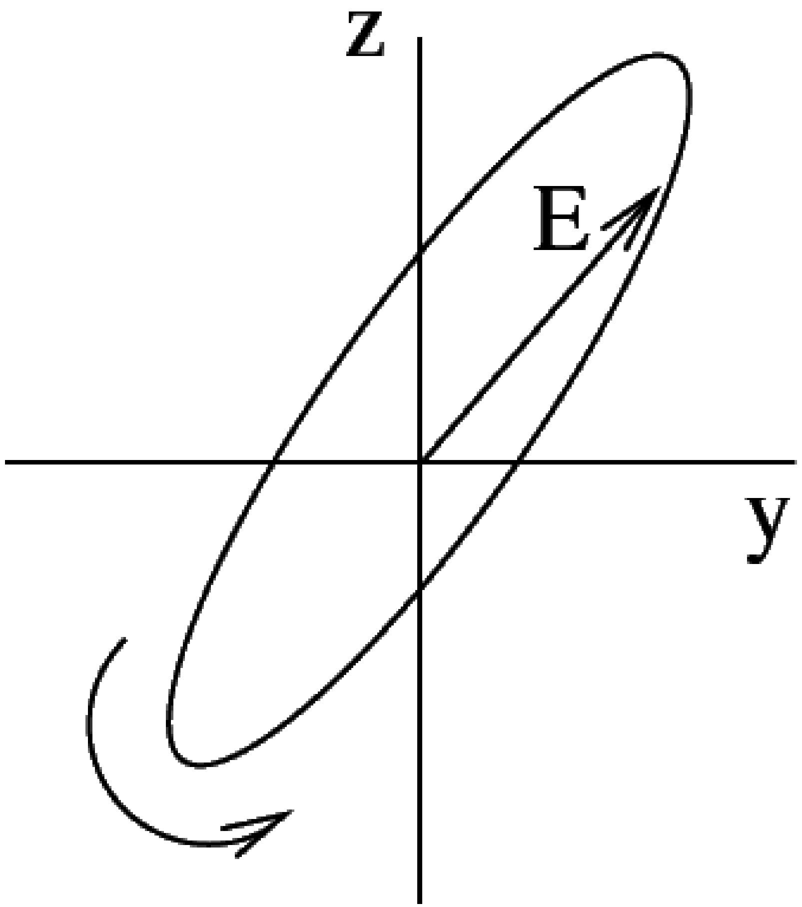
Different amplitudes and Phase difference = π/2



Elliptical Polarization

RECALL

Different amplitudes and Phase difference not $\pi/2$



Superposition of two plane polarized waves

$$\vec{E} = \hat{i}E_x + \hat{j}E_y$$

$$E_x = E_{x0}\cos(\mathrm{kz}-\omega \mathrm{t})$$

$$E_y = E_{y0}\cos(\mathrm{kz}-\omega \mathrm{t}+\delta)$$

$$\Downarrow$$

$$\frac{E_y}{E_{y0}} = \cos(\mathrm{kz}-\omega \mathrm{t})\cos\delta - \sin(\mathrm{kz}-\omega \mathrm{t})\sin\delta$$

$$\frac{E_y}{E_{y0}} = \frac{E_x}{E_{x0}}\cos\delta - \sqrt{1-\frac{E_x{}^2}{E_{x0}{}^2}}\sin\delta$$

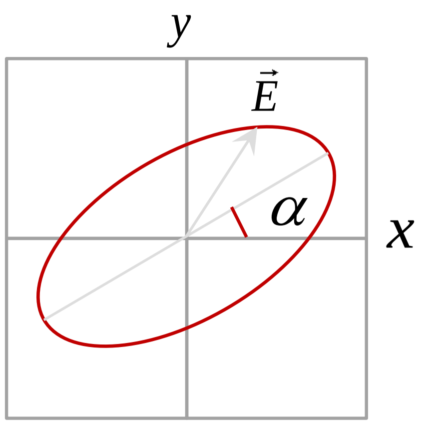
$$\left(\frac{E_y}{E_{y0}} - \frac{E_x}{E_{x0}}\cos\delta\right)^2 = \sin^2\delta - \frac{E_x{}^2}{E_{x0}{}^2}\sin^2\delta$$

$$\boxed{\frac{E^2{}_y}{E^2{}_{y0}} - 2\frac{E_x}{E_{x0}}\frac{E_y}{E_{y0}}\cos\delta + \frac{E_x{}^2}{E_{x0}{}^2} = \sin^2\delta}$$

$$\frac{E^2{}_y}{E^2{}_{y0}} - 2\frac{E_x}{E_{x0}}\frac{E_y}{E_{y0}}\cos\delta + \frac{E_x{}^2}{E_{x0}{}^2} = \sin^2\delta$$

This is an equation of ellipse whose major axis is making an angle say α

$$\tan 2\alpha = \frac{2E_{x0}E_{y0}\cos\delta}{E_{x0}{}^2 - E^2{}_{y0}}$$

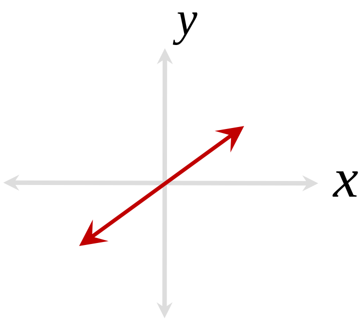


Linearly polarized

$$E_x = E_{x0}\cos(\mathrm{kz} - \omega \mathrm{t})$$

$$E_y = E_{y0}\cos(\mathrm{kz} - \omega \mathrm{t} + \delta)$$

$$\delta = 2m\pi \qquad m = 0,1,2,3....$$



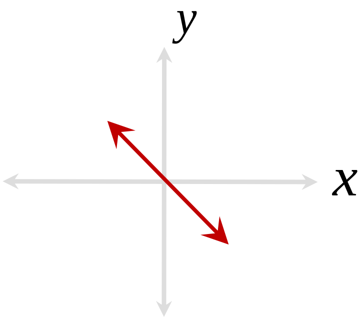
$$\frac{E_y}{E_x} = \frac{E_{y0}}{E_{x0}}$$

$$\theta = \tan^{-1}\left(\frac{E_{y0}}{E_{x0}}\right)$$

$$E_y = E_{y0}\cos(\mathrm{kz} - \omega \mathrm{t} + 2m\pi)$$

$$E_y = E_{y0}\cos(\mathrm{kz} - \omega \mathrm{t})$$

$$\delta = (2m+1)\pi \qquad m = 0,1,2,3....$$



$$\frac{E_y}{E_x} = -\frac{E_{y0}}{E_{x0}}$$

$$\theta = -\tan^{-1}\left(\frac{E_{y0}}{E_{x0}}\right)$$

$$E_y = E_{y0}\cos(\mathrm{kz} - \omega \mathrm{t} + 2m\pi + \pi)$$

$$E_y = -E_{y0}\cos(\mathrm{kz} - \omega \mathrm{t})$$

Elliptically polarized

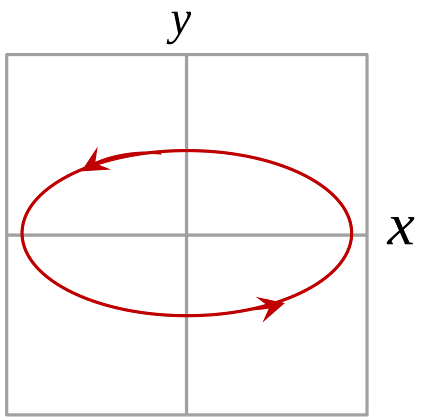
At $z = 0$

$$\delta = (2m+1)\frac{\pi}{2} \qquad m = 0,1,2,3.... \Rightarrow \frac{E_y^2}{E_{y0}^2} + \frac{E_x^2}{E_{x0}^2} = 1$$

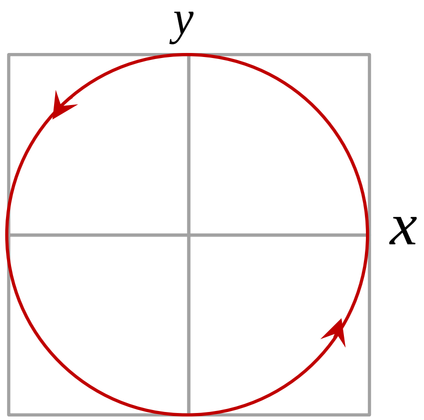
$$E_x = E_{x0}\cos(\omega t)$$
$$E_y = E_{y0}\cos(\omega t - \delta)$$

For $\delta = \frac{\pi}{2};\ 5\frac{\pi}{2};9\frac{\pi}{2}........$

$$E_x = E_{x0}\cos(\omega t)$$
$$E_y = E_{y0}\sin(\omega t)$$



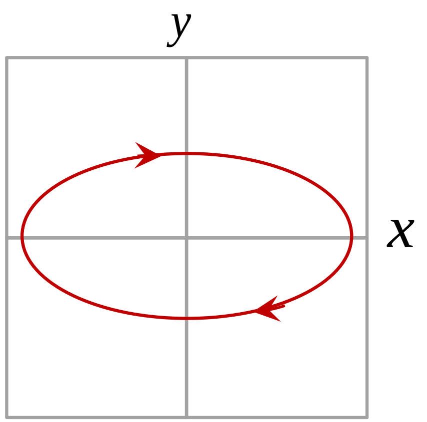
Counter-clock wise rotation with time



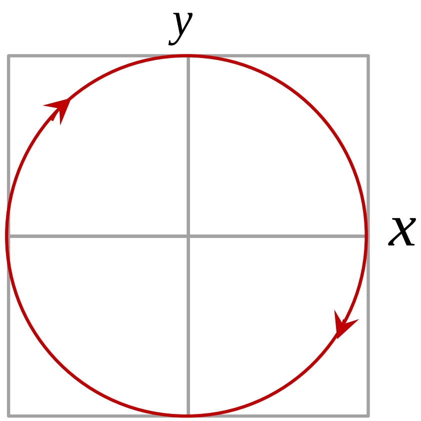
$E_{x0} = E_{y0}$

For $\delta = 3\frac{\pi}{2};\ 7\frac{\pi}{2};11\frac{\pi}{2}........$

$$E_x = E_{x0}\cos(\omega t)$$
$$E_y = -E_{y0}\sin(\omega t)$$



Clock wise rotation with time



$E_{x0} = E_{y0}$

Light

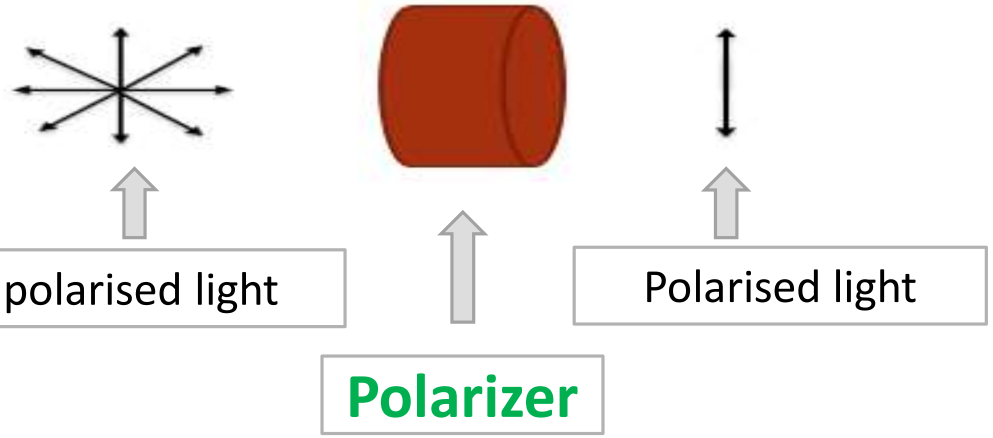
a transverse electromagnetic wave

Unpolarised Light

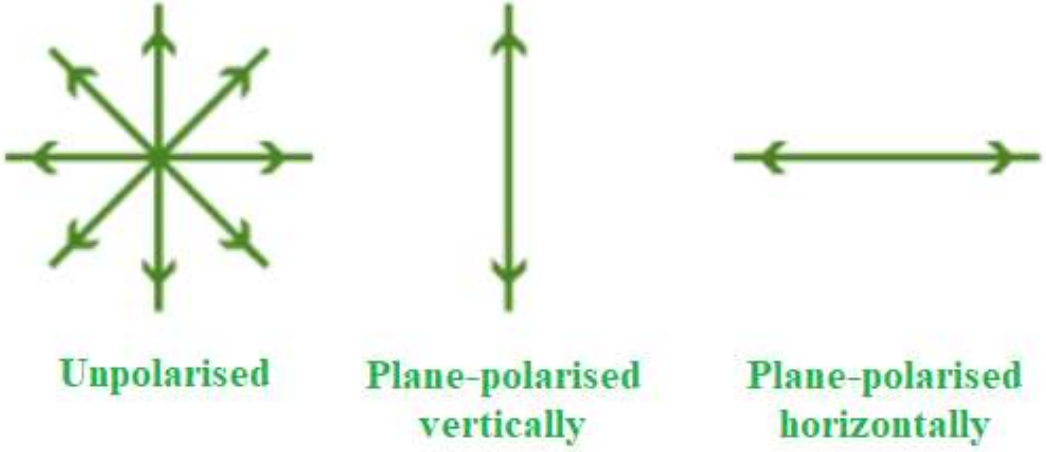
oscillations of electric fields are occurring in all possible directions

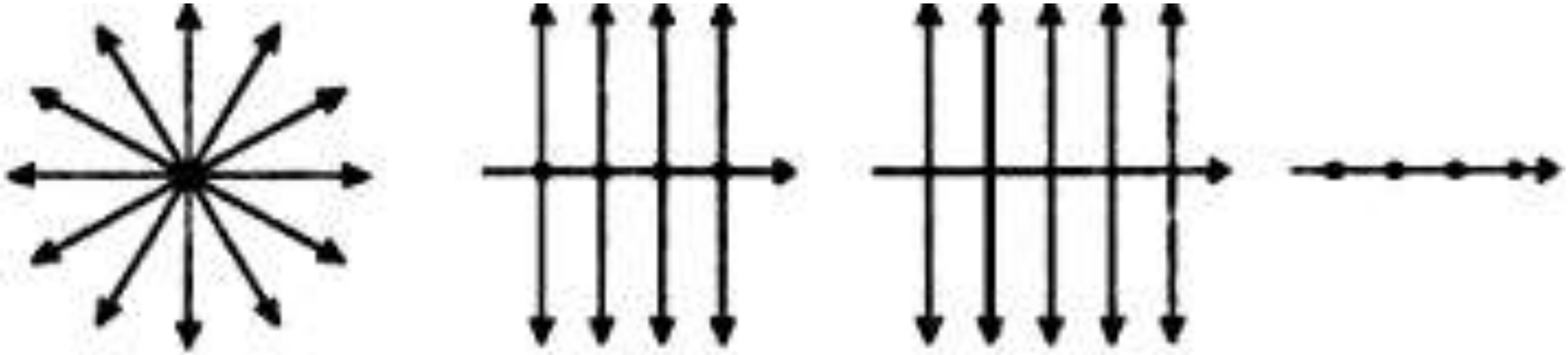
Polarised light

oscillations of electric fields are occurring in a particular direction

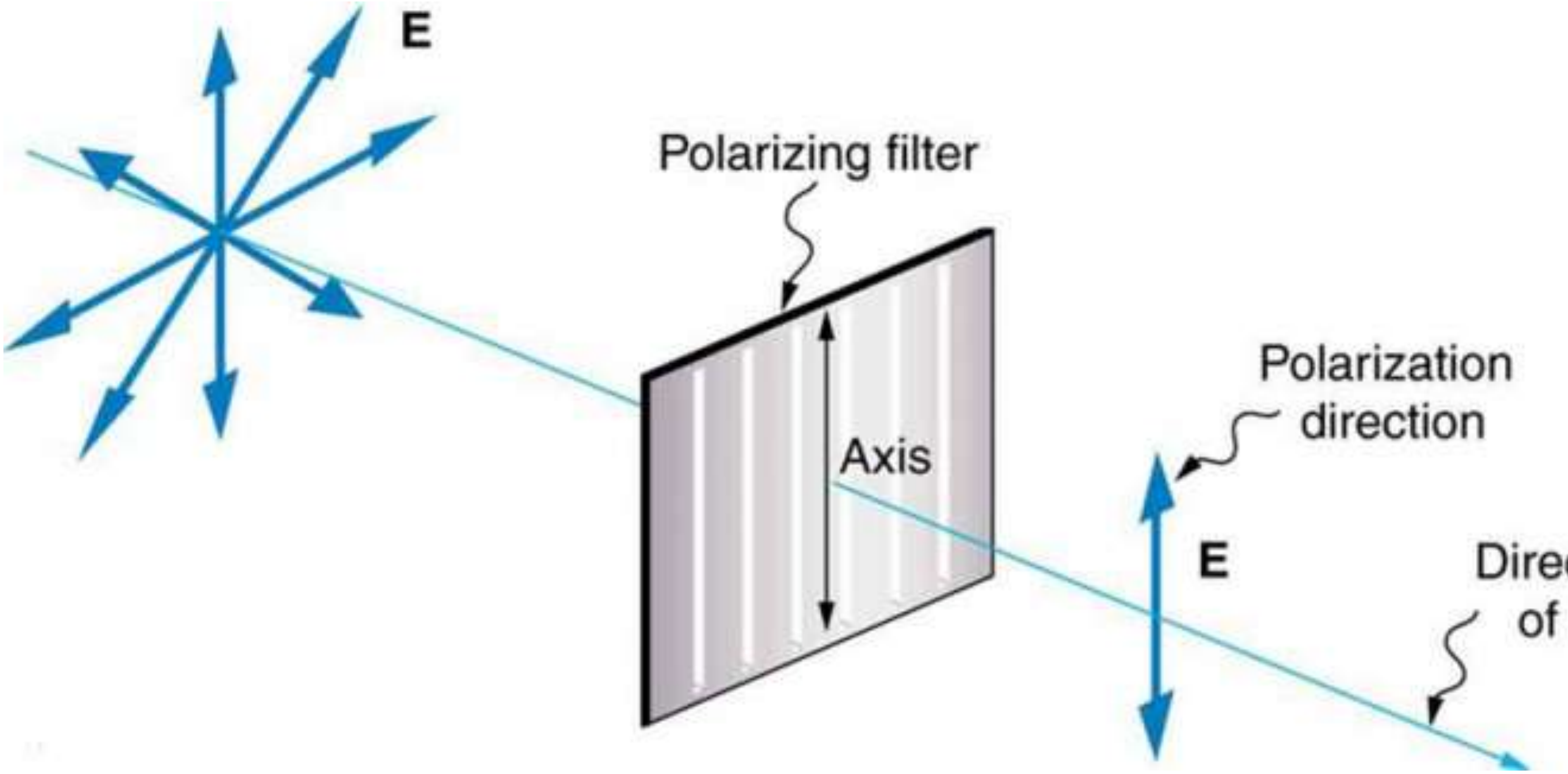


Representing unpolarized light

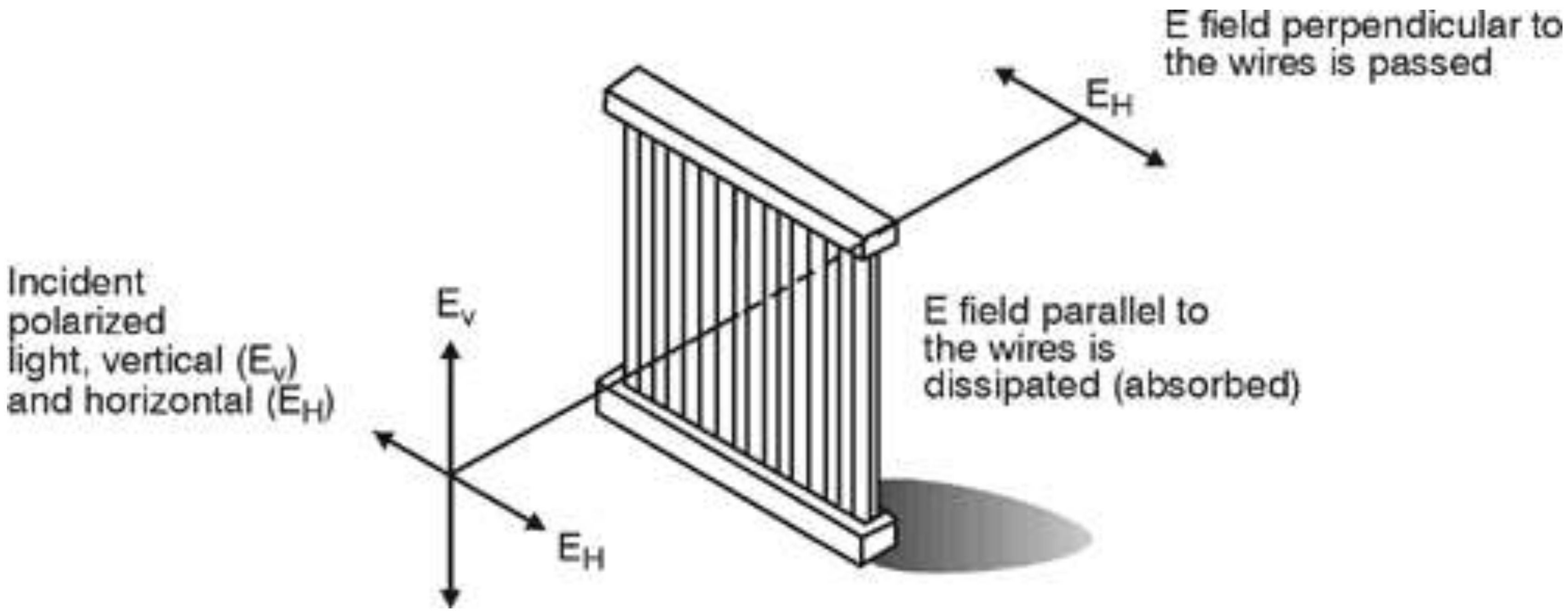




Polarizers

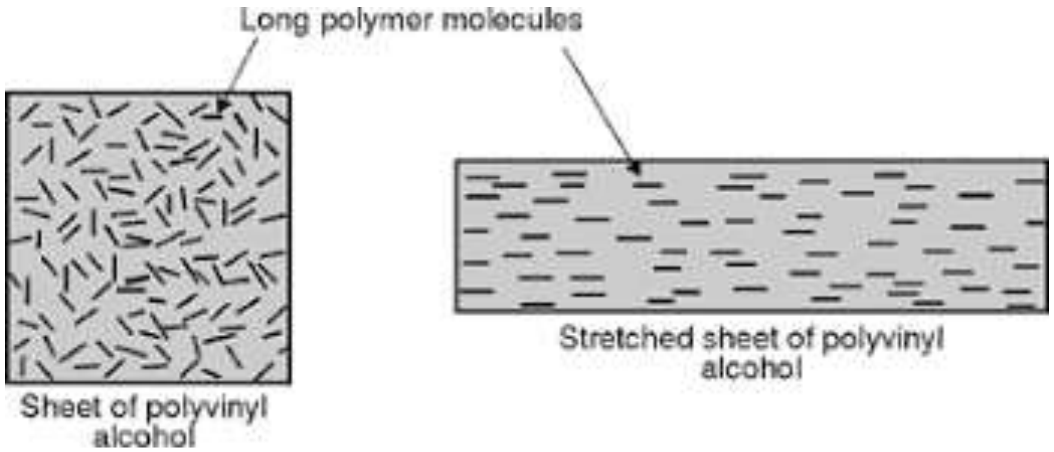

E
Polarizing filter
Axis
Polarization direction
E
Direction of ray

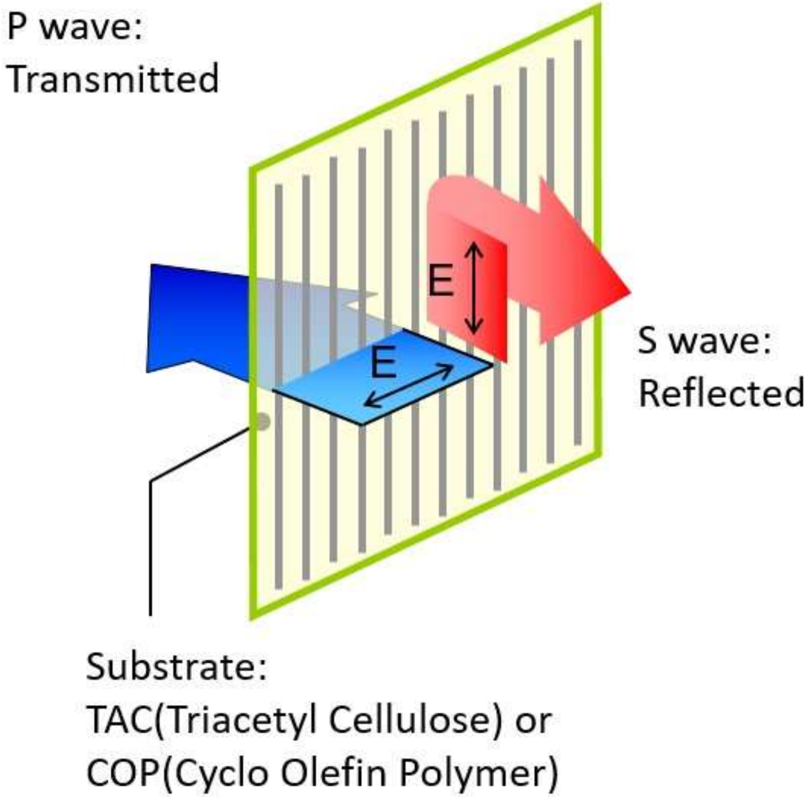
Wire Mesh Polarizers



Good for: Radio waves, microwaves

Visible light: Polymer Mesh Polarizers (Polaroid H-sheet)

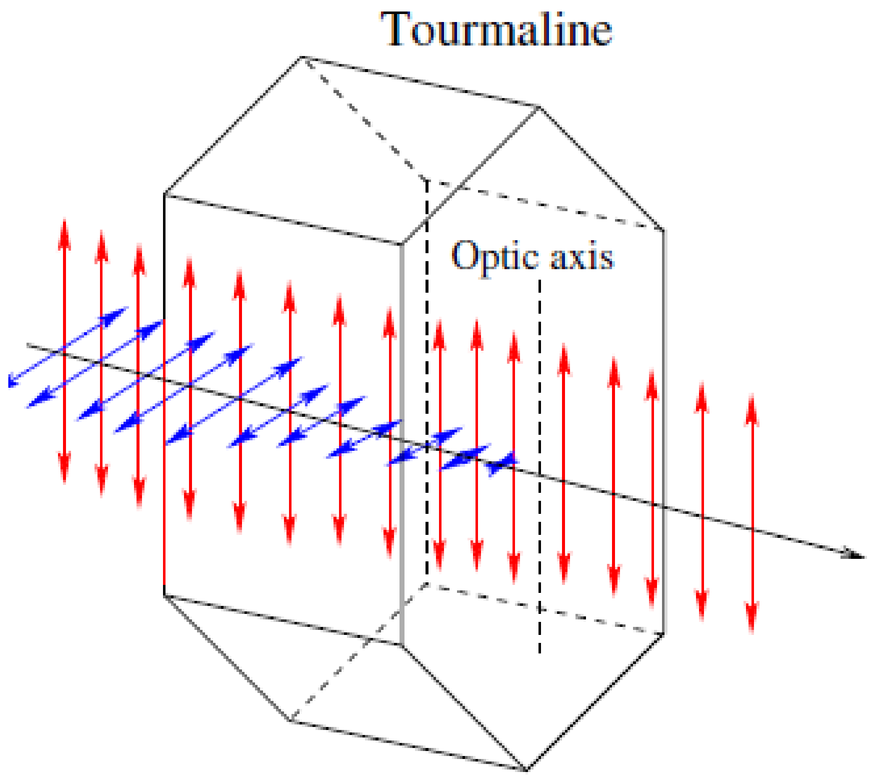




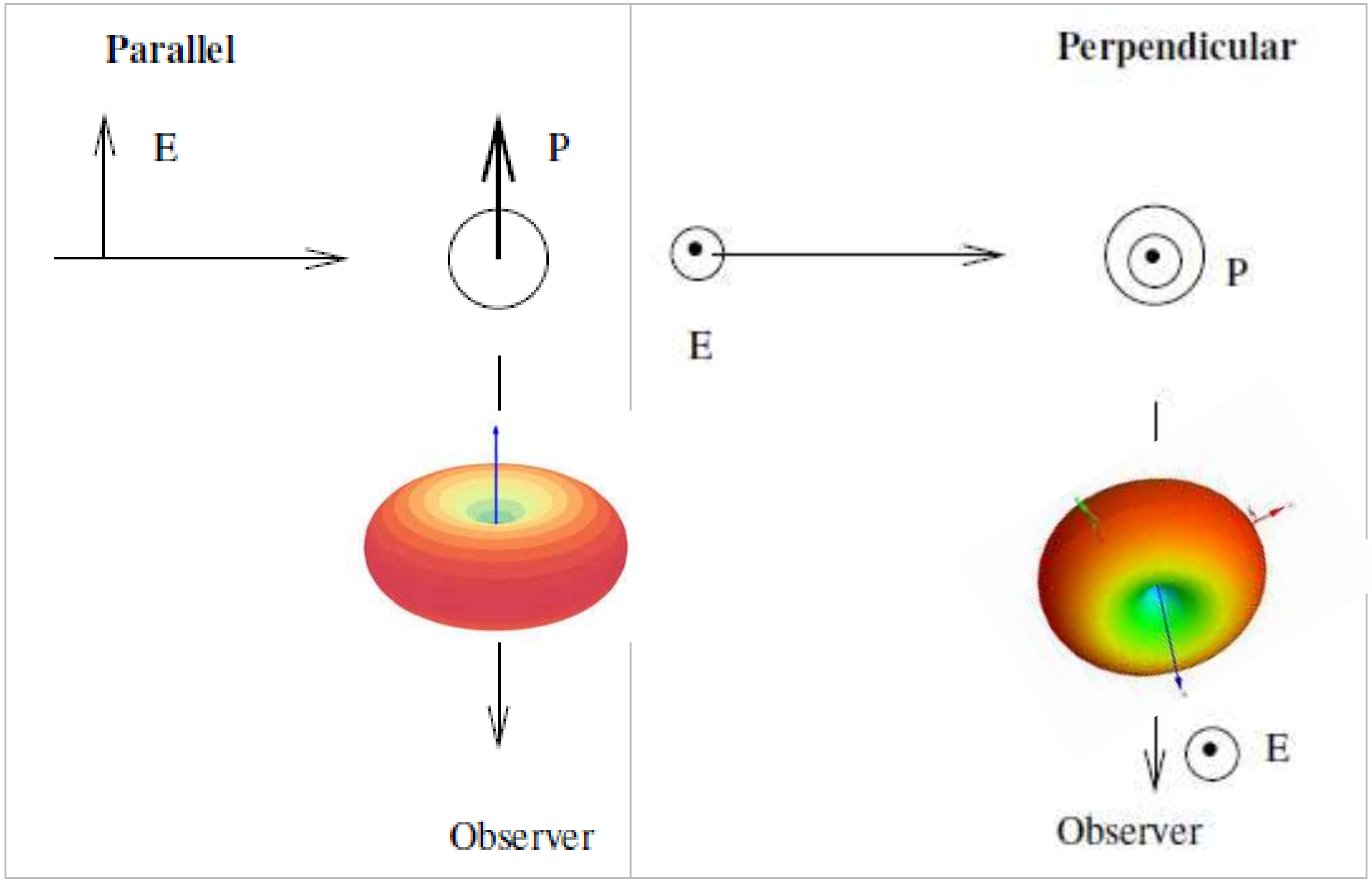
The stretched sheet is dipped into ink that is rich in iodine. The sheet absorbs iodine which forms chains along the polymer chains. These iodine chains act like conducting wires and the whole sheet acts like a wire mesh.

[Unbelievable handmade candy masters' candy making skills - A huge handmade candy factory - YouTube]

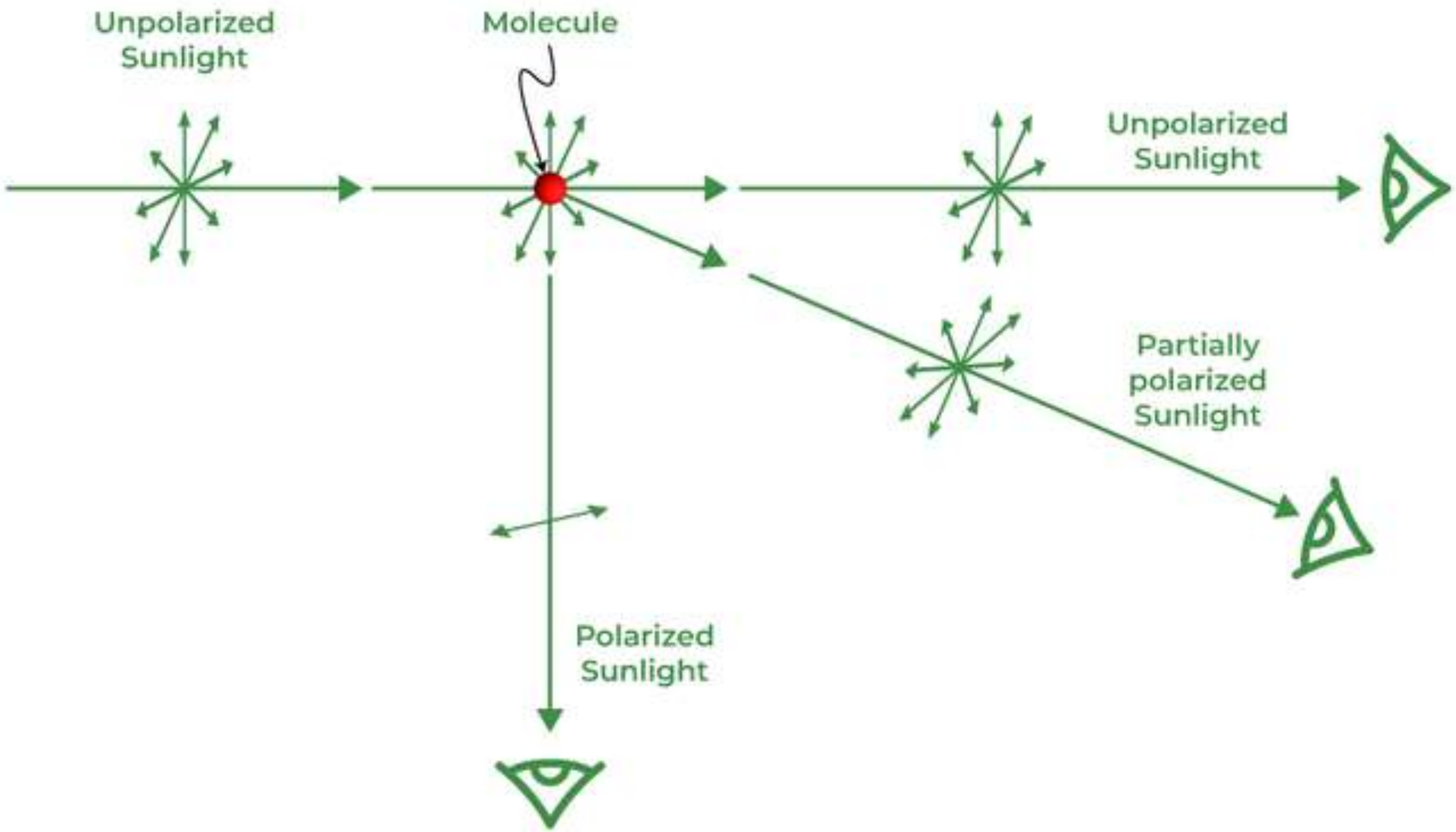
Visible light: Tourmaline (dichroic) crystal



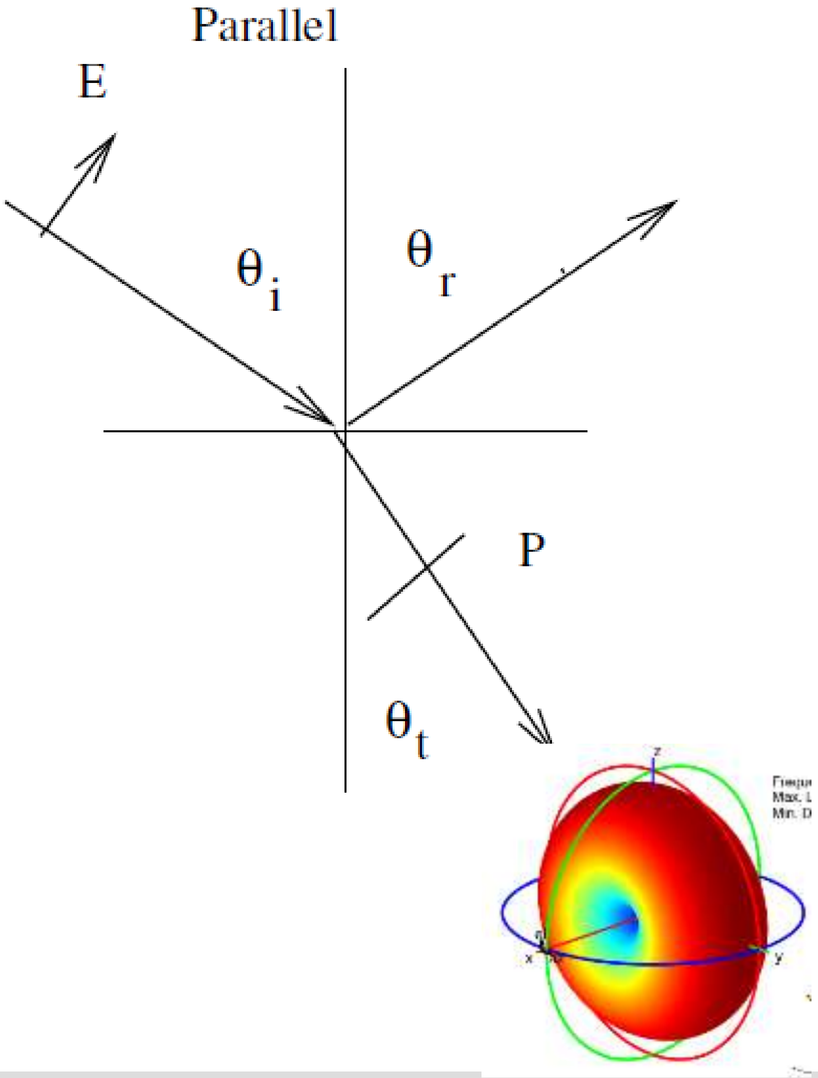
Polarization by scattering

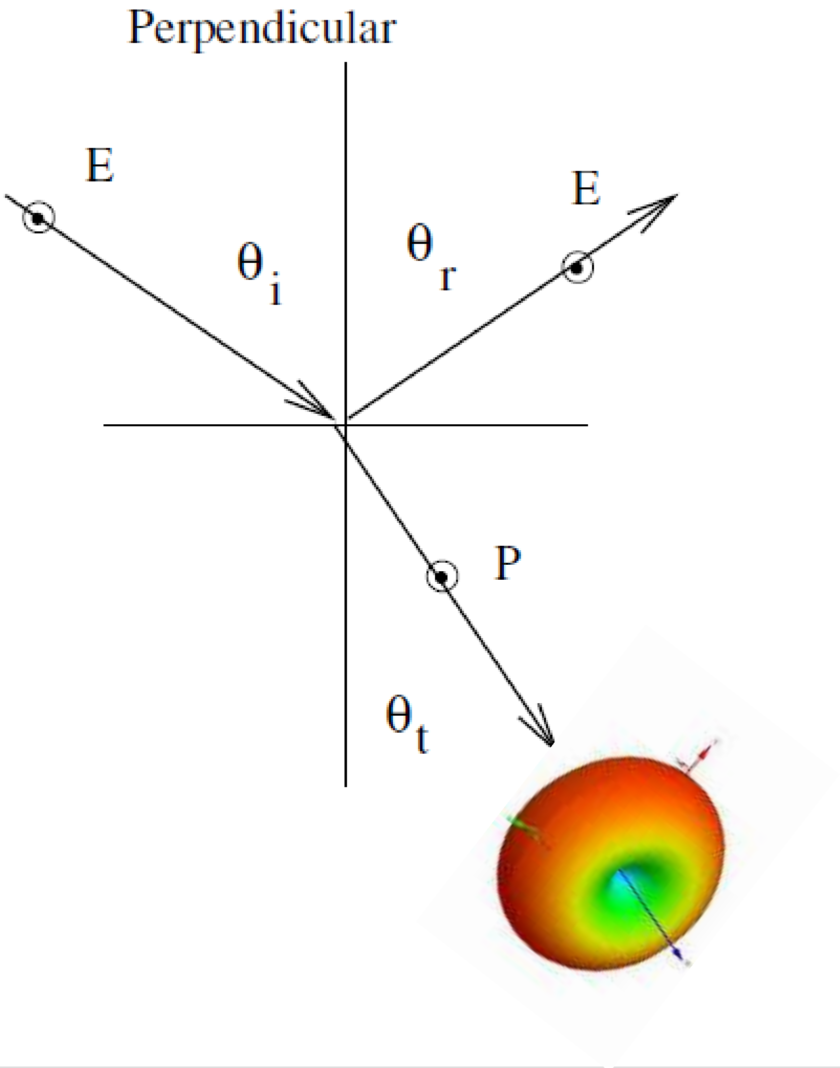


Polarization by scattering

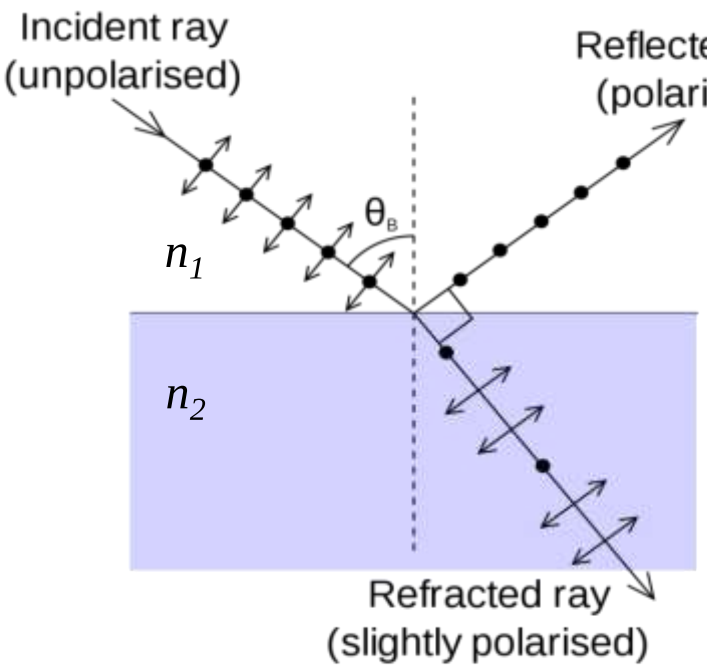


Polarization by reflection





Polarization by reflection: Brewster's law



$$\theta_r + \theta_B = \frac{\pi}{2}$$

Brewseter's angle

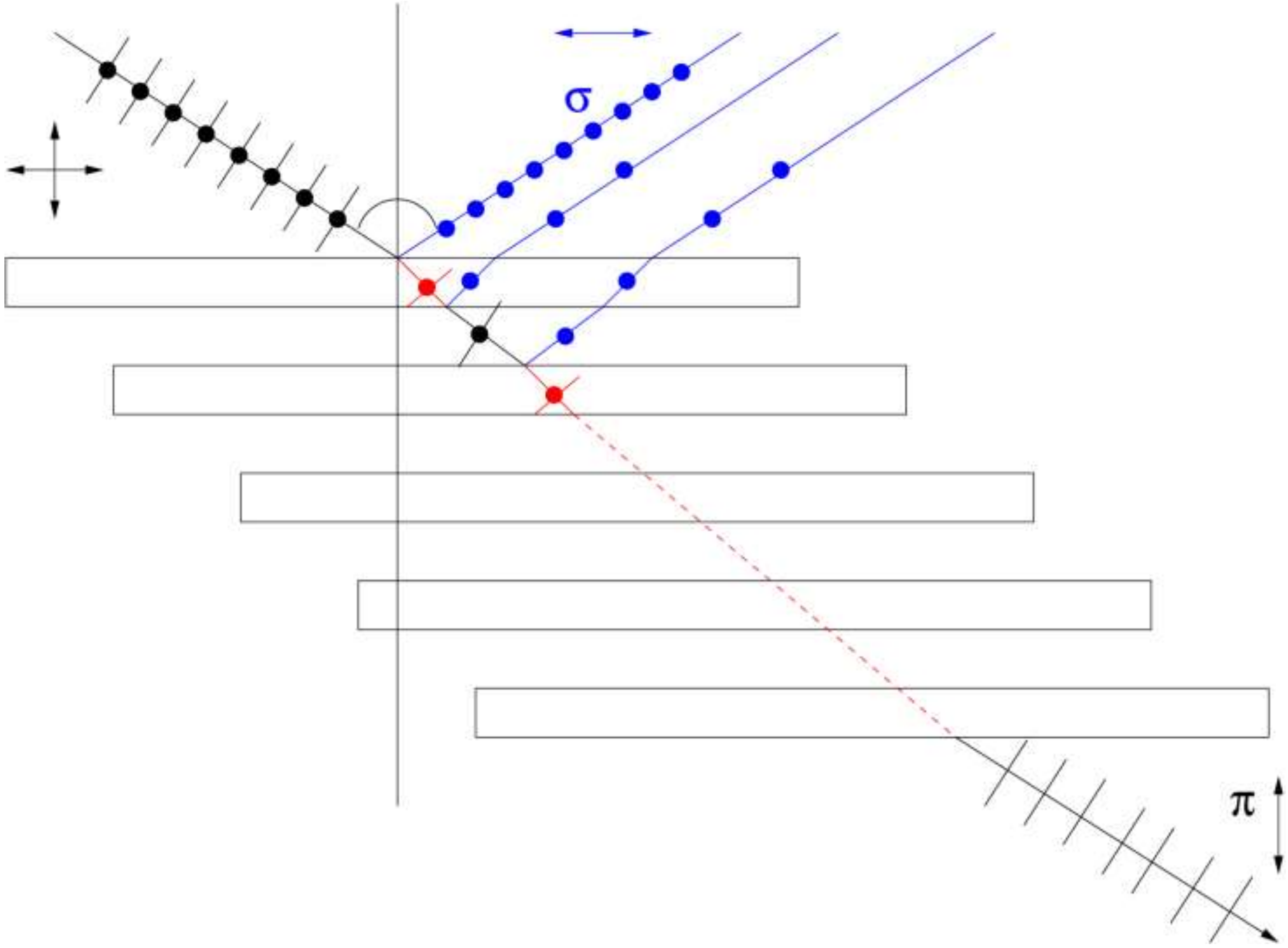
$$\theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$$

$$n_1 \sin\theta_B = n_2 \sin\theta_r$$

$$n_1 \sin\theta_B = n_2 \sin\left(\frac{\pi}{2} - \theta_B\right) = n_2 \cos\theta_B$$

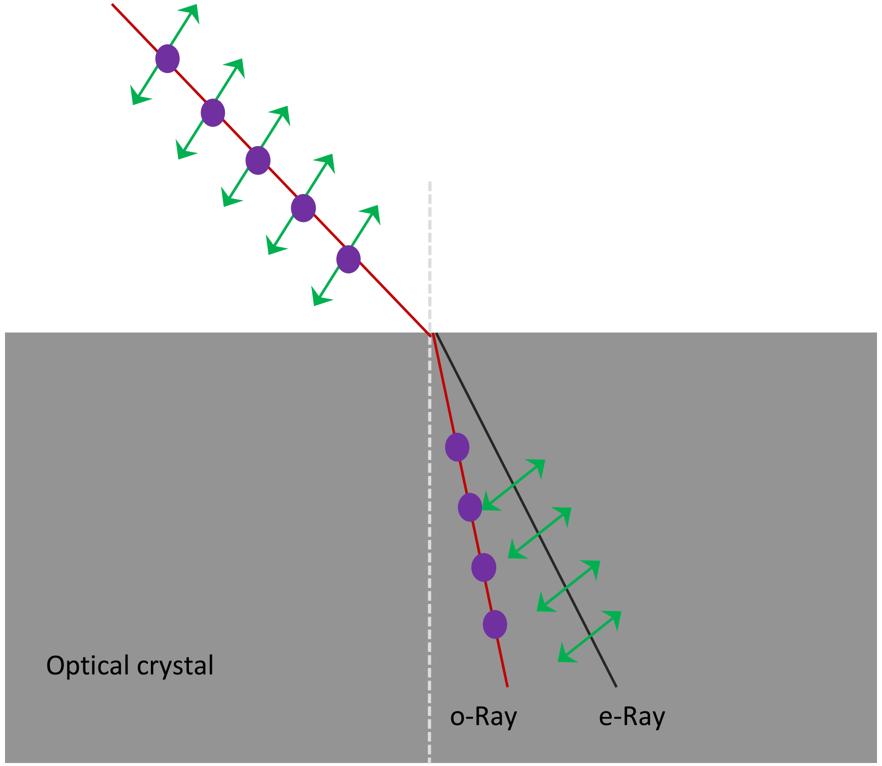
$$\tan\theta_B = \frac{n_1}{n_2}$$

Polarization by Transmission



Polarization by double refraction

- Two refracted beams emerge instead of one
- Two images instead of one



Birefringence/Double Refraction

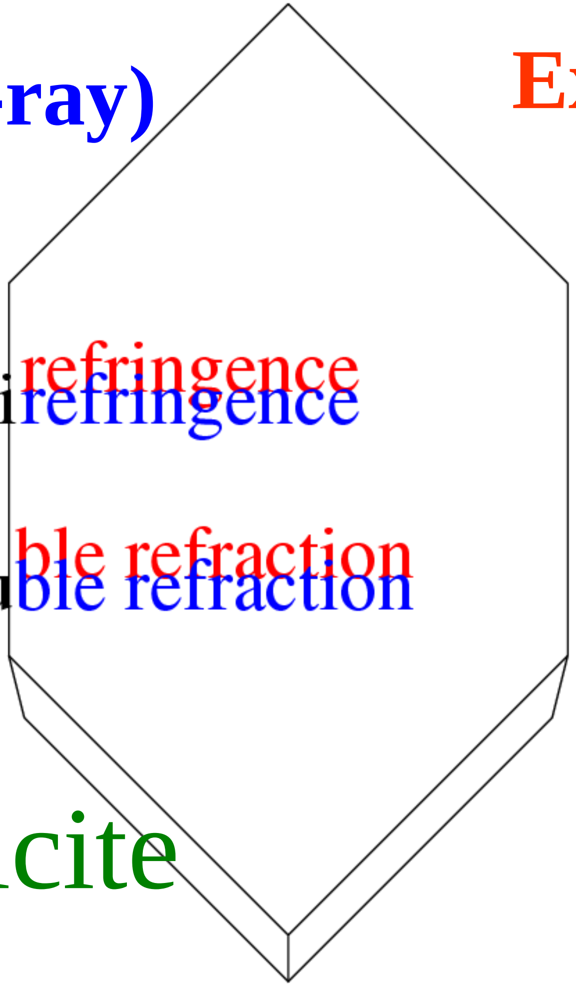
Calcite

Quartz

Optic Axis:
There is a unique direction in a uniaxial crystal called the optic axis.
Values of physical parameters along optic axis are different from the values perpendicular to it.

Ordinary ray (o-ray)

Extraordinary ray (e-ray)



Birefringence

Double refraction

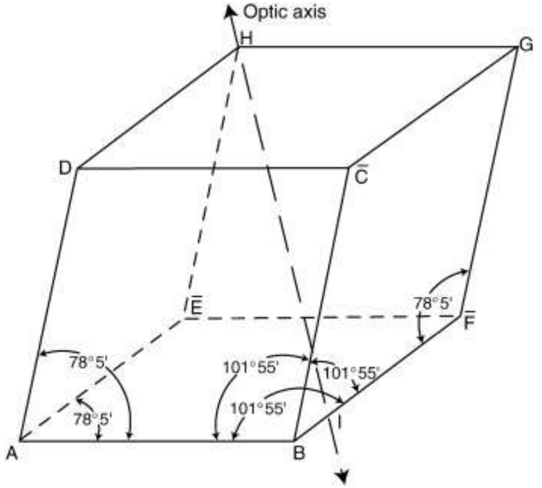
Principal Plane:

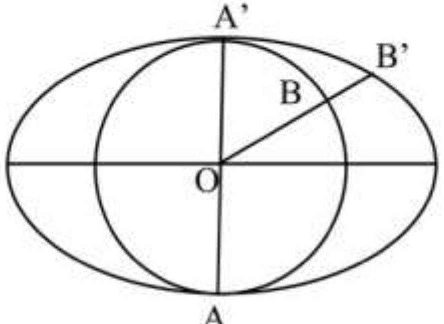
Plane contains optic axis and the direction of propagation

Calcite

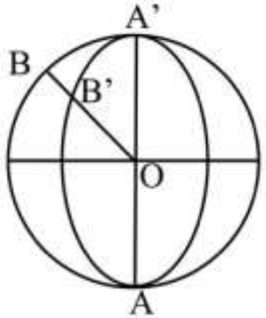
Optic axis

The image which is not displaced (refractive index n_0) is having a polarization perpendicular to the optic axis whereas the displaced image (with refractive index n_e) has a polarization along the optic axis of the crystal.

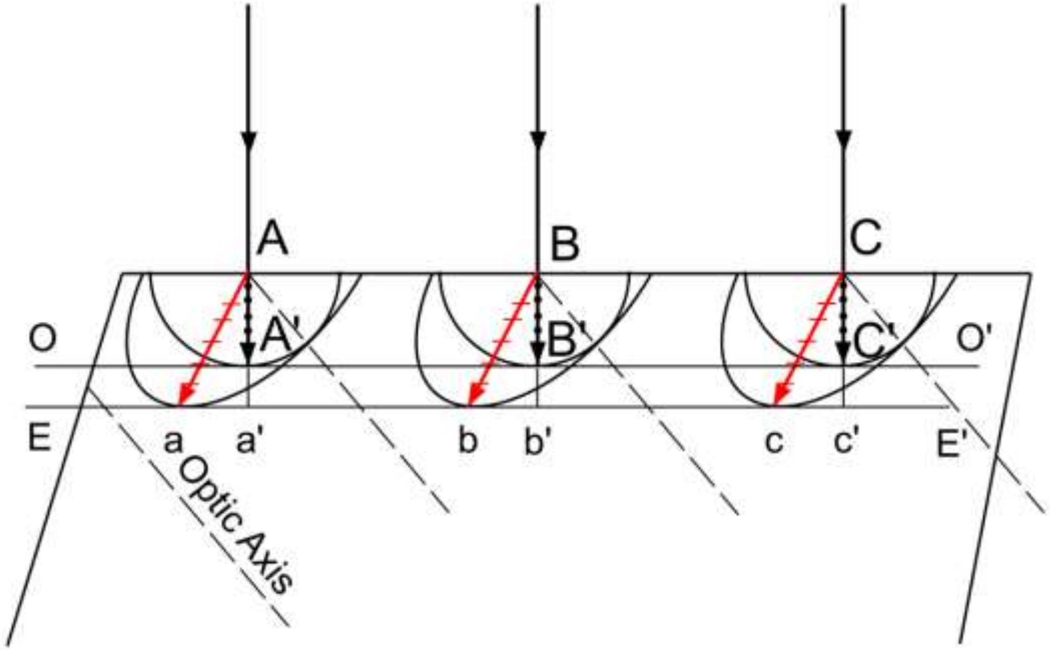


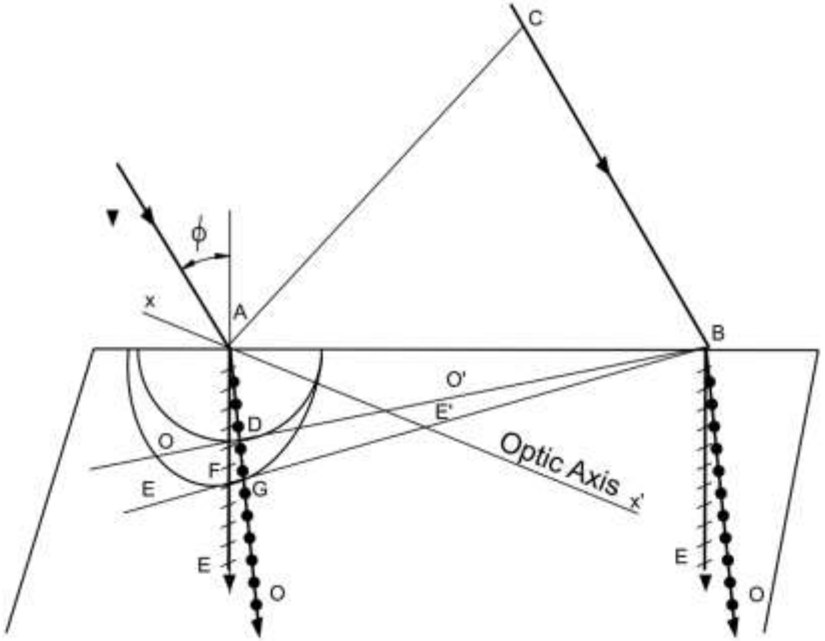


Wavefronts in a Negative Uniaxial Crystal

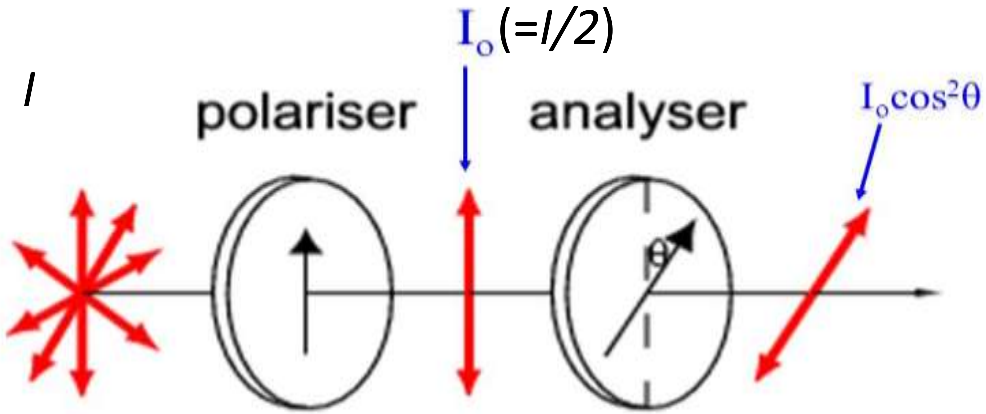


Wavefronts in a Positive Uniaxial Crystal





Malus' Law



1) The 1st polarizer is used to polarize unpolarised light in a plane
2) The 2nd polarizer (analyzer) is rotated w.r.t. the 1st polarizer by an angle θ

Unloparised light through polariser

The intensity of an unpolarized light across a plane polarizer also reduces following the relation $I_0 = I cos^2\theta$, I is the intensity before polarizer

When averaged over all possible angles, the total intensity reduces by half

$$I_0 = I < cos^2\,\theta > = \frac{I}{2}$$

Degree of polarization

If the incident light is a mixture of unpolarised light of intensity $\boldsymbol{I_u}$ and polarised light of intensity $\boldsymbol{I_p}$, then the transmitted light is given by:

$$I = \frac{I_u}{2} + I_p \cos^2\theta$$

$$I_{\max} = \frac{I_u}{2} + I_p; \qquad I_{\min} = \frac{I_u}{2}$$

$$P \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Optical Activity

A substance is optically active if it rotates the plane of polarized light.

Dextrorotatory Substance

- Rotation is clockwise
- Example: Glyceraldehyde

Levorotatory Substance

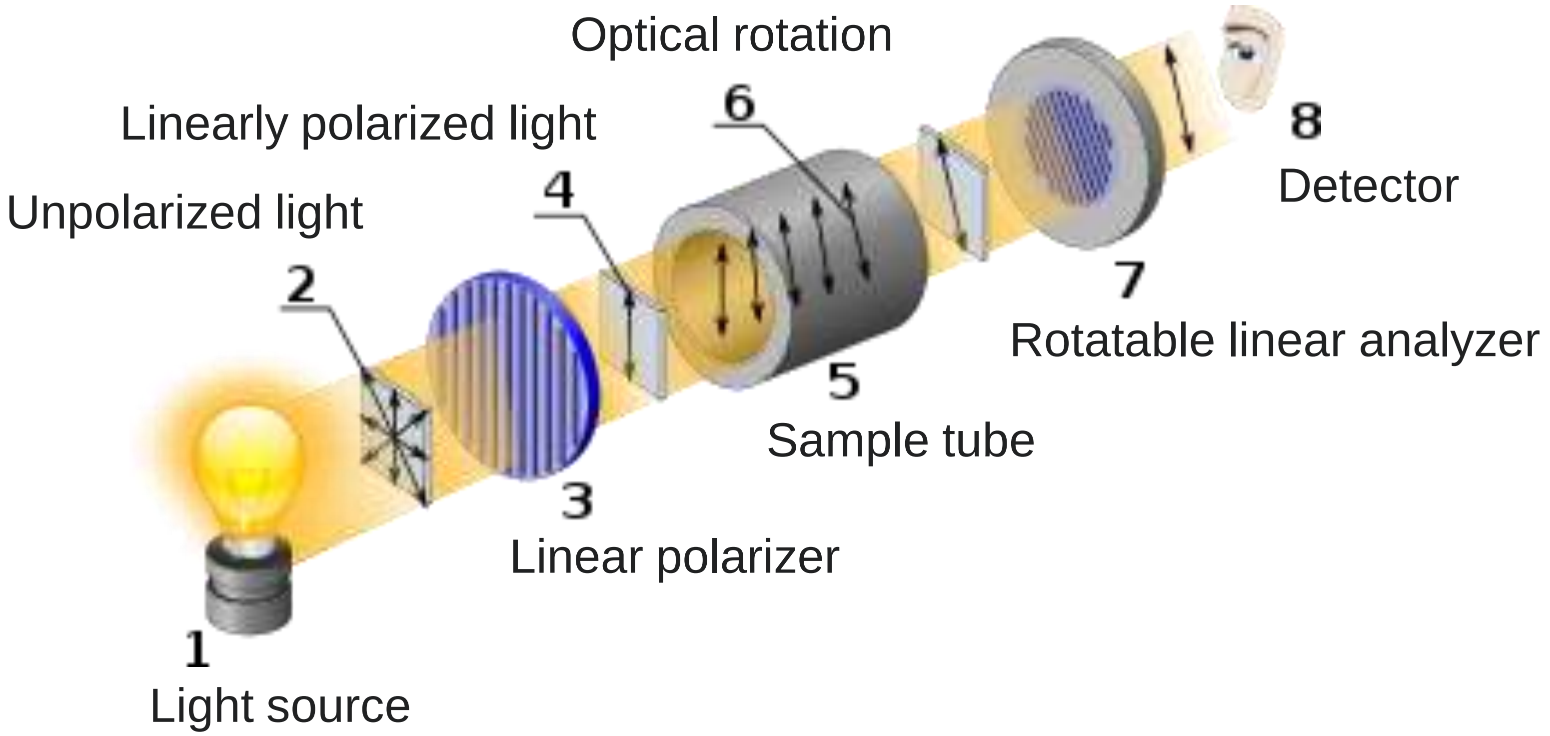
- Rotation is counterclockwise
- Example: D-fructose

Specific rotation [S] of a chiral substance

the rotation produced by a column of solution of length L decimeter and containing 1 gm of the active substance per cm^3 of the solution at a particular **temperature, wavelength, and concentration (c)**

- Length (L) measured in cm.
- θ is the angle of rotation.

$$S = \frac{10\theta}{L\,c}$$


Optical rotation
6
Linearly polarized light
4
Unpolarized light
2
8
Detector
7
Rotatable linear analyzer
5
Sample tube
3
Linear polarizer
1
Light source