Physics of Waves

PH11003

Tutorial 9

Topic : Diffraction

20 January 2023

[9.1] Light with wavelength $\lambda = 0.50\ \mu m$ falls on a slit of width $b = 10\ \mu m$ at an angle $\theta_0 = 30^\circ$ to its normal. Find the angular position of the first minima located on both sides of the central Fraunhofer maximum.

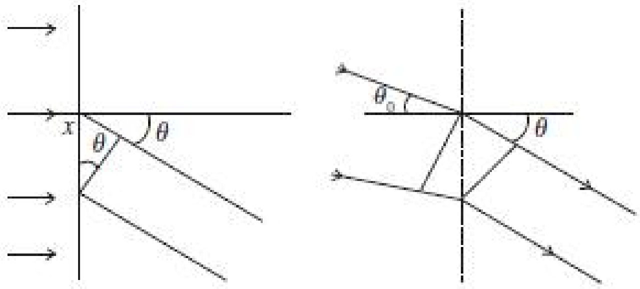
Solution

The relation $b \sin\theta = k\lambda$ for minima (when light is incident normally on the slit) has a simple interpretation: $b \sin\theta$ is the path difference between extreme rays emitted at angle θ. When light is incident at an angle θ_0, the path difference is

$$b(\sin\theta - \sin\theta_0)$$

and the condition of minima is

$$b(\sin\theta - \sin\theta_0) = k\lambda$$



For the first minima

$$b(\sin\theta - \sin\theta_0) = \pm\lambda \quad \text{or} \quad \sin\theta = \sin\theta_0 \pm \frac{\lambda}{b}$$

Putting in values $\theta_0 = 30^\circ$, $\lambda = 0.50\ \mu\text{m}$, $b = 10\ \mu\text{m}$, we get

$$\sin\theta = \frac{1}{2} \pm \frac{1}{20} = 0.55 \quad \text{or} \quad 0.45$$

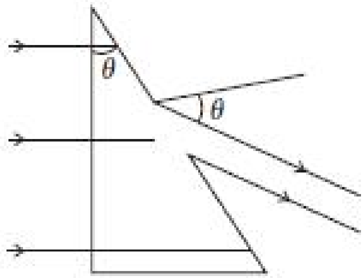
and $$\theta_{+1} = 33^\circ 20' \text{ and } \theta_{-1} = 26^\circ 44'$$

[9.2] A plane light wave with wavelength $\lambda = 0.60\ \mu m$ falls normally on the face of a glass wedge with refracting angle $\theta = 15^\circ$. The opposite face of the wedge is opaque and has a slit of width $b = 10\ \mu m$ parallel to the edge. Find: (a) the angle $\Delta\theta$ between the direction to the Fraunhofer maximum of zeroth order and that of incident light; (b) the angular width of the Fraunhofer maximum of the zeroth order.

Solution

(a) This case is analogous to the previous one except that the incident wave moves in glass of R.I. n. Thus, the expression for the path difference for light diffracted at angle θ from the normal to the hypotenuse of the wedge is

$$b(\sin\theta - n\sin\Theta)$$



We write $\theta = \Theta + \Delta\theta$

Then, for the direction of Fraunhofer maximum

$$b(\sin(\Theta + \Delta\theta) - n\sin\Theta) = 0$$

or $\Delta\theta = \sin^{-1}(n\sin\Theta) - \Theta$

Using $\Theta = 15°, n = 1.5,$ we get

$$\Delta\theta = 7.84°$$

(b) The width of the central maximum is obtained from

$$b(\sin\theta_1 - n\sin\Theta) = \pm\lambda \ \ (\text{where } \lambda = 0.60\ \mu\text{m}, b = 10\ \mu\text{m})$$

Thus, $\theta_{+1} = \sin^{-1}\left(n\sin\Theta + \dfrac{\lambda}{b}\right) = 26.63°$

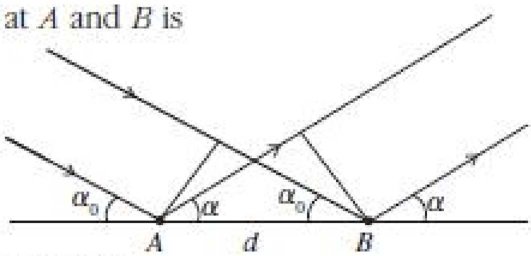
and $\theta_{-1} = \sin^{-1}\left(n\sin\Theta - \dfrac{\lambda}{b}\right) = 19.16°$

Therefore, $\delta\theta = \theta_{+1} - \theta_{-1} = 7.47°$

[9.3] A monochromatic beam falls on a reflection grating with period $d = 10$ *mm* at a glancing angle $\alpha_0 = 1.0°$. When it is diffracted at a glancing angle $\alpha = 3.0°$ a Fraunhofer maximum of second order occurs. Find the wavelength of light.
Solution

The path difference between waves reflected at A and B is

$$d(\cos\alpha_0 - \cos\alpha)$$



and for maxima

$$d(\cos\alpha_0 - \cos\alpha) = k\lambda \ (k = 0, \pm 1, \pm 2, \ldots)$$

In our case, $k = 2$ and α_0, α are small in radians. Then,

$$2\lambda = d\left(\frac{\alpha^2 - \alpha_0^2}{2}\right)$$

or
$$\lambda = \frac{(\alpha^2 - \alpha_0^2)d}{4}$$

$$= 0.61\ \mu\text{m}$$

(for $\alpha = 3\pi/180$, $\alpha_0 = \pi/180$ and $d = 10^{-3}$ m).

[9.4] Find the wavelength of monochromatic light falling normally on a diffraction grating with period $d = 2.2\ \mu m$ if the angle between the directions to the Fraun-

hofer maxima of the first and the second order is equal to $\Delta\theta = 15°$.
Solution

Given that

$$d \sin\theta_1 = \lambda$$

and

$$d \sin\theta_2 = d\sin(\theta_1 + \Delta\theta) = 2\lambda$$

Thus,

$$\sin\theta_1 \cos\Delta\theta + \cos\theta_1 \sin\Delta\theta = 2\sin\theta_1$$

or

$$\sin\theta_1 (2 - \cos\Delta\theta) = \cos\theta_1 \sin\Delta\theta$$

or

$$\tan\theta_1 = \frac{\sin\Delta\theta}{2 - \cos\Delta\theta}$$

or

$$\sin\theta_1 = \frac{\sin\Delta\theta}{\sqrt{\sin^2\Delta\theta + (2 - \cos\Delta\theta)^2}}$$

$$= \frac{\sin\Delta\theta}{\sqrt{5 - 4\cos\Delta\theta}}$$

Thus,

$$\lambda = \frac{d\sin\Delta\theta}{\sqrt{5 - 4\cos\Delta\theta}}$$

Substitution gives

$$\lambda \approx 0.534\ \mu\text{m}$$

[9.5] A transmission grating is expected to provide an ultimate first-order resolution of at least 1Å anywhere in the visible spectrum (400 to 700 nm). The ruled width of the grating is to be 2 cm. (a) Determine the minimum number of grooves required. (b) If the diffraction pattern is focused by a 50-cm lens, what is the linear separation of a 1-Å interval in the vicinity of 500 nm? Solution

(a) The resolution is given by $\Re = \lambda/\Delta\lambda_{\min}$. Taking the worst case, or $\lambda = 700\,\text{nm}$,

$$\Re = \frac{7000\ \text{Å}}{1\ \text{Å}} = 7000 = mN = (1)N \Rightarrow N = 7000$$

Then, $a = 2\,\text{cm}/7000\,\text{grooves}$.

(b) The grating equation gives $m\lambda = a\sin\theta \approx a\theta \approx ay/f$. Thus, $m\Delta\lambda = a\Delta y/f$. ;

$$\Delta y = \frac{mf\Delta\lambda}{a} = \frac{(1)\,(50\,\text{cm})\,(1\times10^{-8}\,\text{cm})}{(2/7000)\,\text{cm}} = 0.00175\text{cm}$$

Answers:

[9.1] 33°and 27°
[9.2] (a) 7.9°, (b) 7.3°
[9.3] 0.6 μm
[9.4] 0.54 μm
[9.5] (a) 7000 , (b) 0.00175 cm