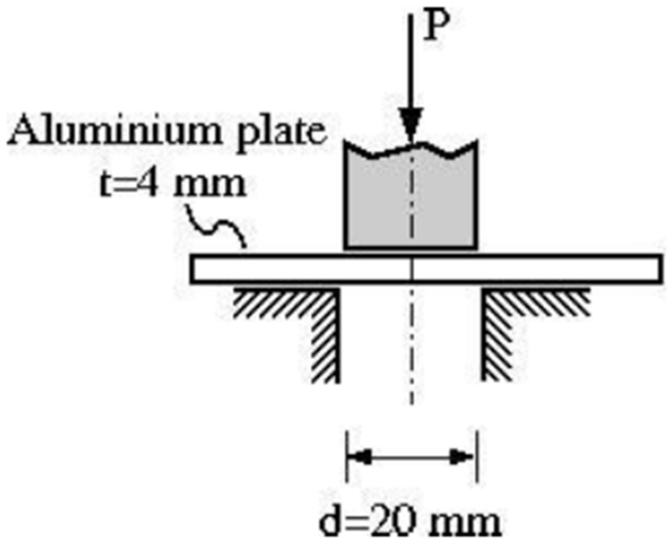
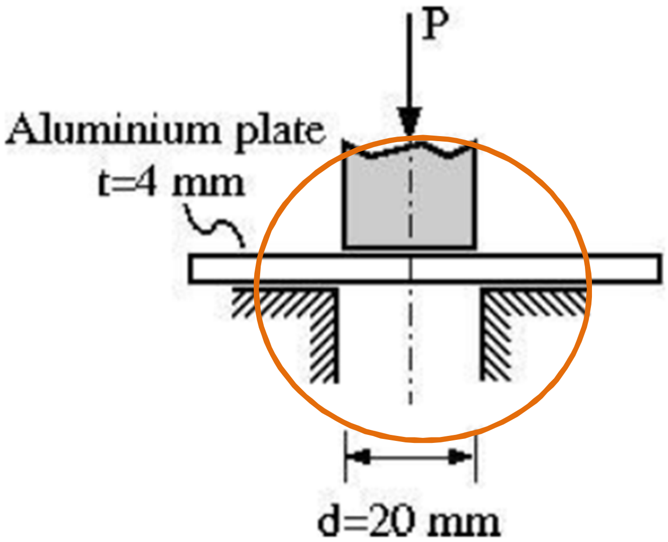
Tutorial -6

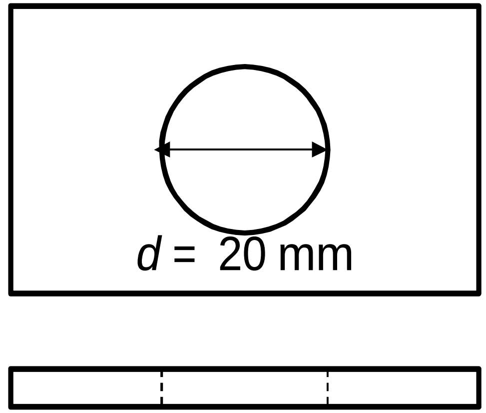
Problem -7

A punch of diameter d = 20 mm is used to make holes in an Aluminum plate of thickness t = 4 mm. If the ultimate shear stress for Aluminum is 275 MPa, estimate the force P required for punching through the plate.



Solution



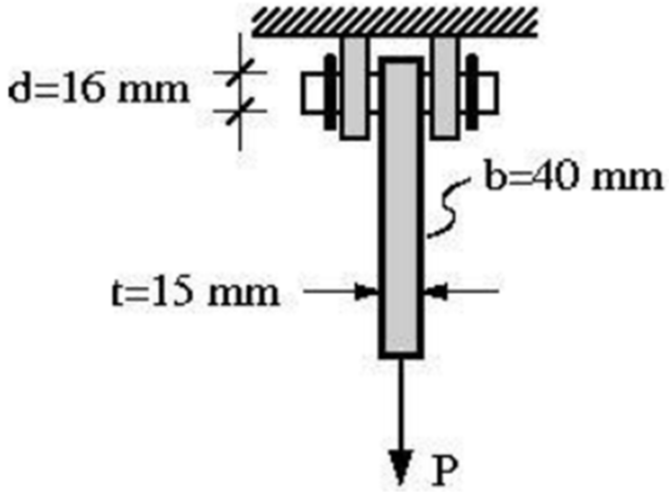


τ_{max} = 275 MPa = 275 N/mm²

Shearing area= $\pi dt = \pi \times 20 \times 4$ mm²

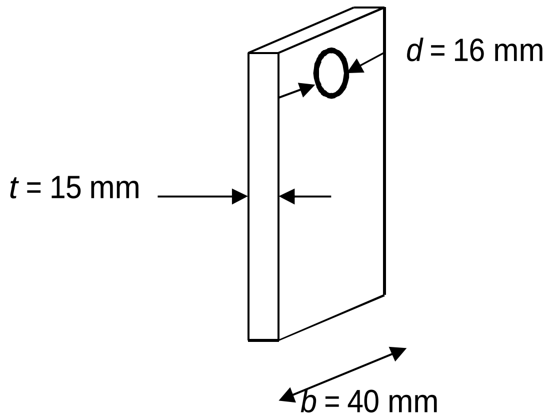
So, Total Load = 275 x π x 20 x 4 = 69.12 kN

Problem-8



An Aluminum bar is attached to its support by a 16 mm diameter pin, as shown in the figure. The thickness *t* of the bar is 15 mm, and its width *b* is 40 mm. If the allowable tensile normal stress in the bar is 85 MPa, find the allowable load *P*.

Solution



Effective area in tension = (40-16)x15 = 360 mm²

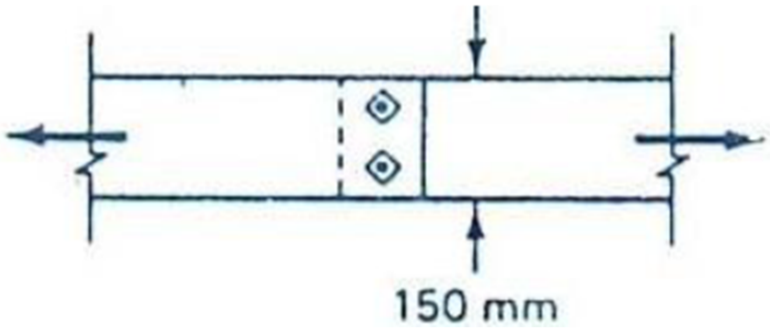
Maximum tensile stress in bar = $\sigma_t = \frac{P}{360}$ N/mm²

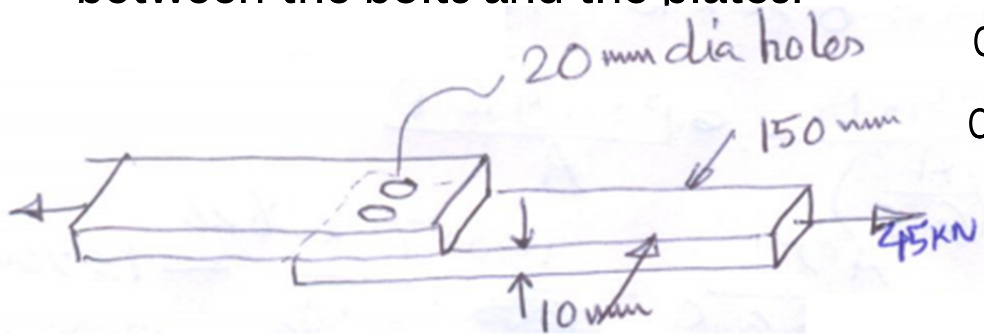
Allowable tensile normal stress = 85 MPa

$$\frac{P}{360} = 85 \qquad \Rightarrow P = 85 \times 360 \text{ N} = 30.6 \text{ kN}$$

Problem-9

Two 10 mm thick steel plates are fastened together by means of two 20 mm bolts that fit tightly into the holes. If the joint transmits a tensile force of 45 kN, determine (a) average normal stress in the plates at the section where no holes occur; (b) the average normal stress at the critical section; (c) the average shear stress in the bolts and (d) the average bearing stress between the bolts and the plates.





Cross-section of plate= 150x10 =1500 mm²

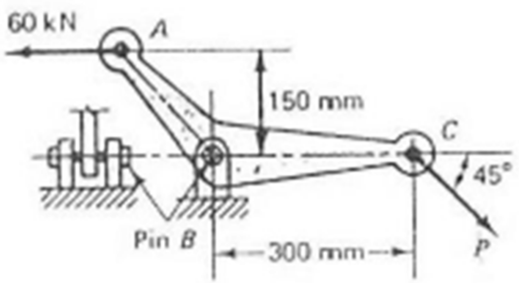
Cross-section of bolt =2x $\frac{\pi}{4}$x 20^2 =628.32 mm²

Avg. normal stress= $\frac{(45)(10^3)}{1500}$ =30 MPa

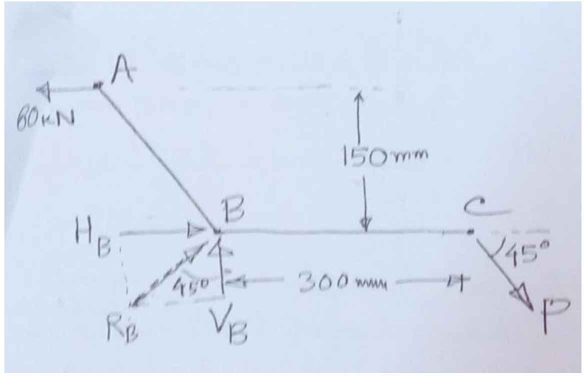
Average shear stress = $\frac{(45)(10^3)}{628.32}$ = 71.6 MPa

Average Bearing stress = $\frac{(45)\,(10^3)}{(2)(10)(20)}$ = 112.5 MPa

Problem-10



What is the required diameter of pin B for the bell crank mechanism, if an applied force of 60 kN is resisted by a force P at C? The allowable shear stress is 100 MPa.



$$\sum M_B = 0 \quad \Rightarrow P = \frac{(60)(150)}{\text{Sin}45\ (300)} = 42.43 \text{ kN}$$

$\sum F_y = 0$ gives, $\qquad V_B = P\ \text{Sin}45 = 30 \text{ kN}$

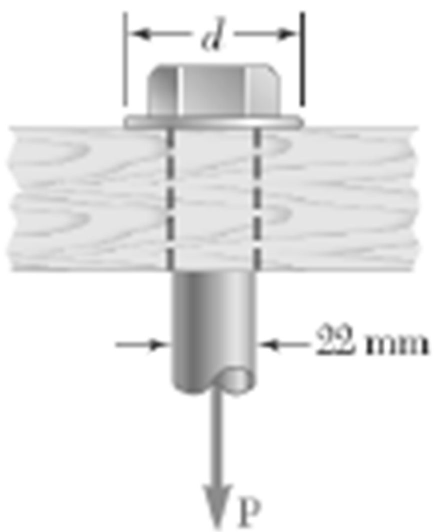
$\sum F_x = 0$ gives, $\qquad H_B = 60 \text{ kN} \qquad \therefore R_B = \sqrt{30^2 + 30^2} = 30\sqrt{2}$ kN

Since, area of bolt in double shear $\qquad \therefore \dfrac{(30\sqrt{2})(10^3)}{(\frac{\pi}{4}\text{d}^2)(2)} < 100$

$\Rightarrow d \quad > 16.43$ mm

Problem-11

The load P applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

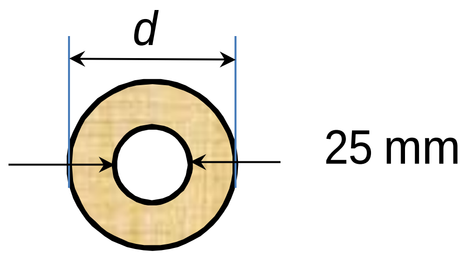


Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13\times10^{-6}\text{m}^2$ $\qquad \sigma = 35\times10^6\text{Pa}$

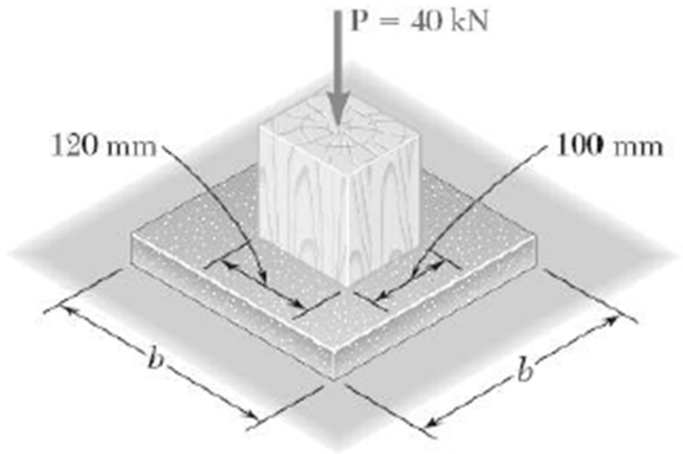
$$P = \sigma A = (35\times10^6)(380.13\times10^{-6}) = 13.305\times10^3\text{N}$$

Required bearing area $A_b = \frac{P}{\sigma_b} = \frac{13.305\times10^3}{5\times10^6} = 2.6609\times10^{-3}\text{m}^2$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$ $\qquad \Rightarrow d = 63.3$ mm



Problem-12



A 40 kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (a) the maximum bearing stress on the concrete footing, (b) the size of the footing for which the average bearing stress in the soil is 145 kPa.

Bearing stress on concrete footing $\sigma = \dfrac{P}{A} = \dfrac{40 \times 10^3}{12 \times 10^{-3}} = 3.333 \times 10^6 \text{Pa}$

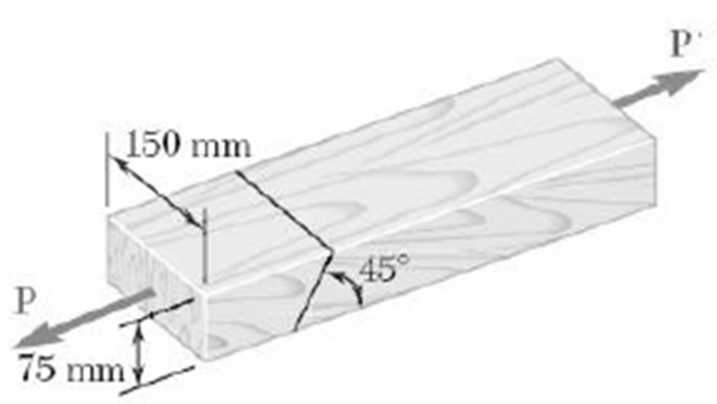
Footing area Avg. bearing stress on soil $\sigma = 145 \text{ kPa} = 45 \times 10^3 \text{Pa}$

$$\therefore \ \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

$$\Rightarrow b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m}$$

Problem-13

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (a) the largest load *P* that can be safely applied, (b) corresponding tensile stress in splice

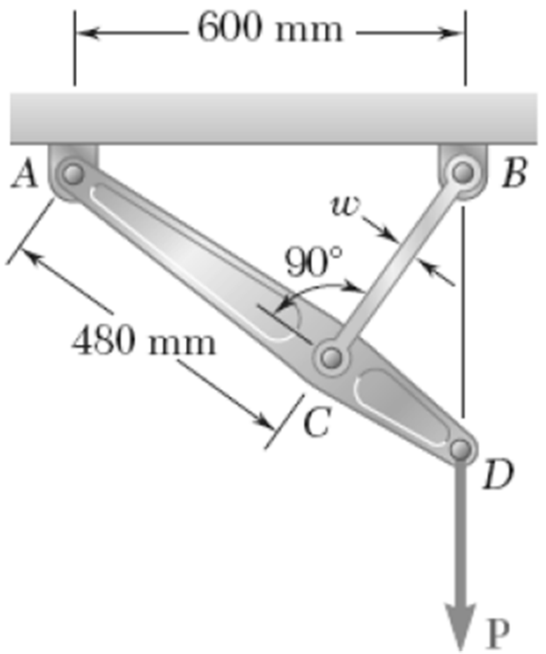


$$\theta = 90° - 45° = 45° \qquad A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

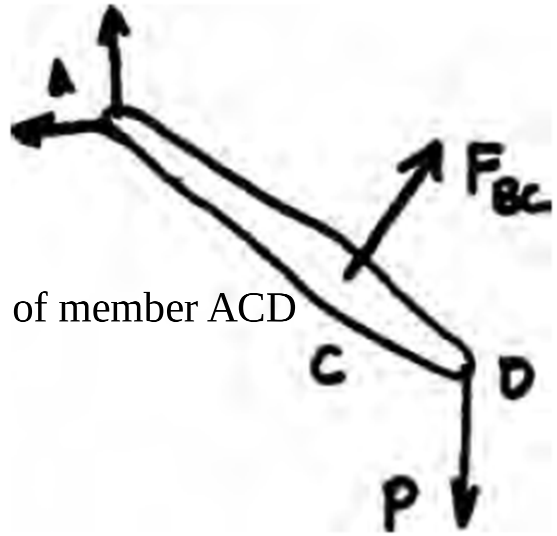
Since, $\tau = \dfrac{P \sin 2\theta}{2A_0}$ $\qquad \therefore \quad P = \dfrac{2A_0\tau}{\sin 2\theta} = \dfrac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°} = 13.95 \times 10^3 \text{N}$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}} = 620 \times 10^3 \text{Pa}$$

Problem-14



Link BC is 6 mm thick and is made of steel with a 450 MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20 kN load *P* with a factor of safety of 3?



FBD of member ACD

$$\Sigma M_A = 0$$

$$\Rightarrow \quad F_{BC} = \frac{600P}{480} = \frac{(600)(20\times10^3)}{480} = 25\times10^3\,\text{N}$$

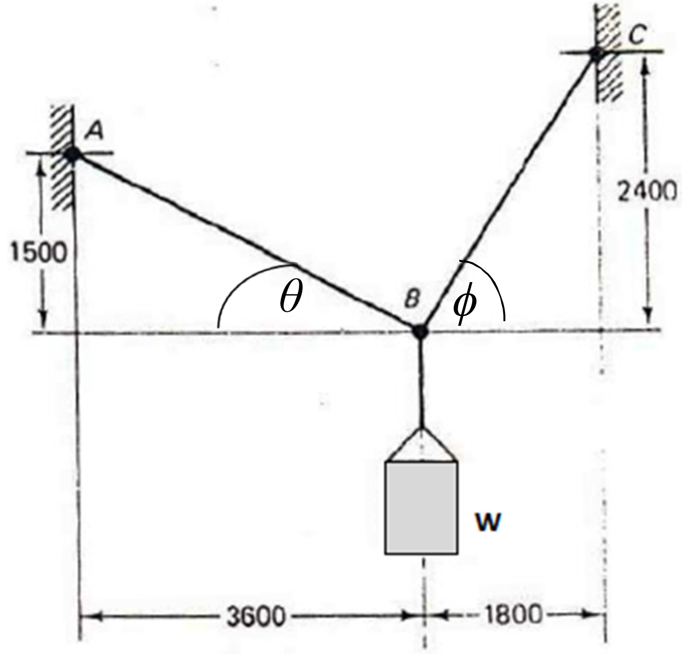
ultimate load of member BC is $F_U = (\text{F.S.})(F_{BC}) = (3)(25\times10^3) = 75\times10^3\,\text{N}$

$$\therefore A = \frac{F_U}{\sigma_U} = \frac{75\times10^3}{450\times10^6} = 166.67\times10^{-6}\,\text{m}^2$$

For a rectangular section $\quad A = wt$ or $w = \dfrac{A}{t}$ $\qquad \Rightarrow w = 27.8$ mm

Problem-15

Two high strength steel rods of different diameters are attached at A and C and support a weight *W*. The ultimate strength of the rods is 800 MPa. Rods AB and BC have cross-sectional areas of 200 mm^2 and 400 mm^2 respectively. If the factor of safety is 2, what weight *W* can be supported by the wires?



Given: $\sigma_{ult} = 800$ MPa $= 800$ N/mm^2 $A_{AB} = 200$ mm^2 and $A_{BC} = 400$ mm^2

$L_{AB} = 3900$ mm $\qquad L_{BC} = 3000$ mm

$$\sin\theta = \frac{1500}{3900} = \frac{5}{13} \qquad \cos\theta = \frac{3600}{3900} = \frac{12}{13} \qquad \cos\phi = \frac{1800}{3000} = \frac{3}{5} \qquad \sin\phi = \frac{2400}{3000} = \frac{4}{5}$$

$$P_{AB}\,.\sin\theta + P_{BC}\,.\sin\phi = W \qquad \text{and} \qquad P_{AB}\,.\cos\theta = P_{BC}\cos\phi$$

$$\Rightarrow P_{Bc} = \frac{20}{21}W \qquad \Rightarrow P_{AB} = \frac{13}{21}W$$

$$\text{Stress in AB} = \frac{13}{(21)(200)} W \text{ N/mm}^2 < \frac{800}{2} \text{ N/mm}^2$$

$$\Rightarrow \; W < 129.23 \text{ kN}$$

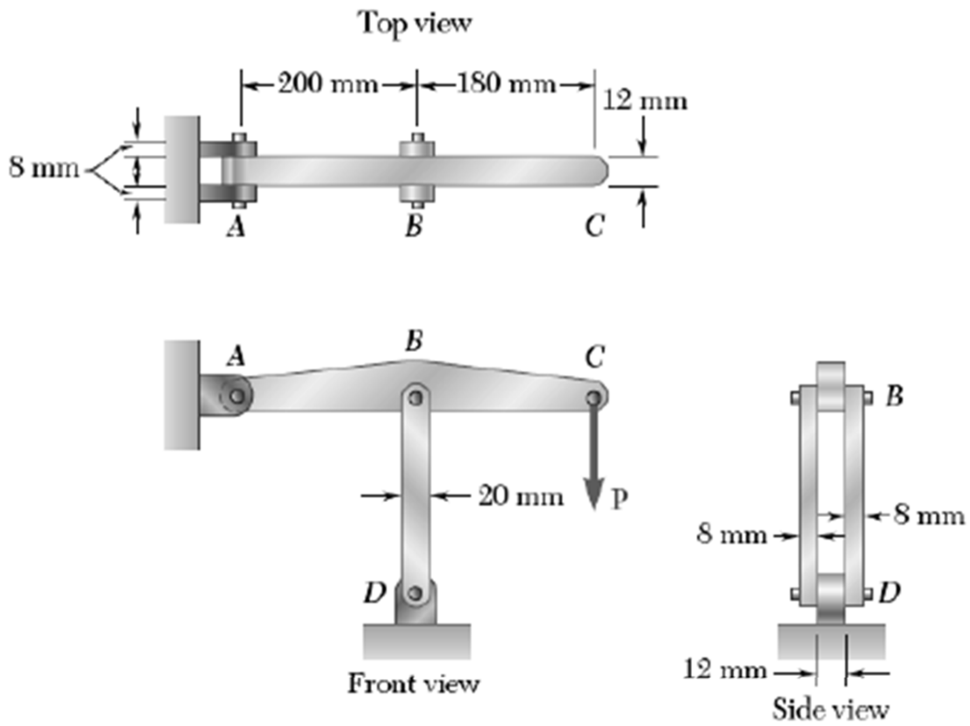
$$\text{Stress in BC} = \frac{20}{(21)(400)} W \text{ N/mm}^2 < \frac{800}{2} \text{ N/mm}^2$$

$$\Rightarrow \; W < 168 \text{ kN}$$

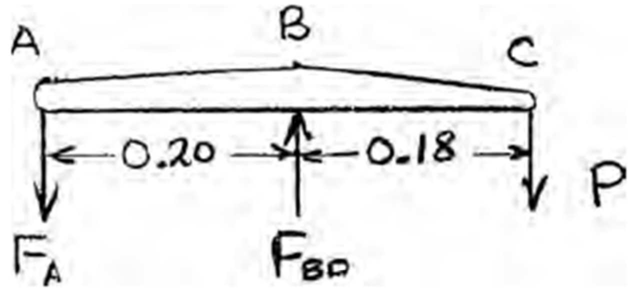
Since FOS = 2

Taking smaller of the two: W = 129.23 kN (AB will fail first)

Problem-16



In the structure shown, an 8 mm diameter pin is used at A, and 12 mm diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all pins and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired.



$$\Sigma M_B = 0: \quad 0.20\,F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$\Sigma M_A = 0: \quad 0.20F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin A: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.008)^2 = 50.266\times10^{-6}\text{m}^2$

$$F_A = \frac{2\tau_U A}{\text{F.S.}} = \frac{(2)(100\times10^6)(50.266\times10^{-6})}{3.0} = 3.351\times10^3\text{N} \qquad \Rightarrow \qquad P = \frac{10}{9}F_A = 3.72\times10^3\text{N}$$

Based on double shear in pins at B and D : $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\mathrm{m}^2$

$$F_{BD} = \frac{2\tau_U A}{\mathrm{F.S.}} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\mathrm{N} \qquad \Rightarrow \quad P = \frac{10}{19}F_{BD} = 3.97 \times 10^3\mathrm{N}$$

Based on compression in links BD, For one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\mathrm{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{\mathrm{F.S.}} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\mathrm{N}$$

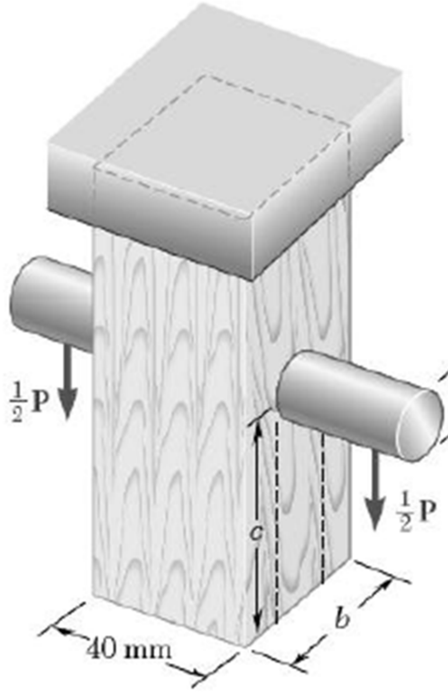
$$P = \frac{10}{19}F_{BD} = 14.04 \times 10^3\mathrm{N}$$

Allowable value of P is smallest,

$$\therefore P = 3.72 \times 10^3\mathrm{N}$$

Problem-17

A load *P* is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that b = 40 mm, c = 55 mm, and d = 12 mm, determine the load P if the overall factor of safety is 3.2.



Based on double shear in pin $P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U = \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\text{N}$

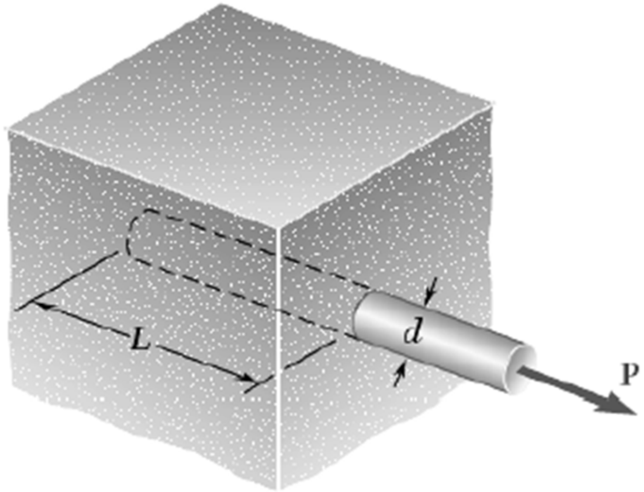
Based on tension in wood: $P_U = A\sigma_U = w(b-d)\sigma_U = (0.040)(0.040-0.012)(60\times10^6) = 67.2\times10^3\text{N}$

Based on double shear in the wood: $P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6) = 33.0\times10^3\text{N}$

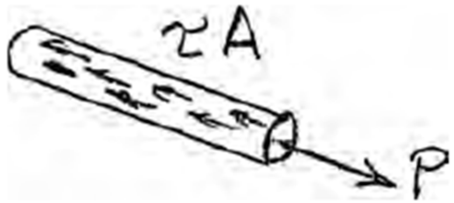
$\therefore P_U = 32.80\times10^3\ \text{N}$

$\Rightarrow$ Allowable: $P = \frac{P_U}{\text{F.S.}} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3\text{N}$

Problem-18



A force P is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{allow} in the steel, and the average allowable bond stress τ_{allow} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar).



For shear, $P = \tau_{\text{all}} A = \tau_{\text{all}} \pi d L$

For tension, $P = \sigma_{\text{all}} A = \sigma_{\text{all}} \left(\frac{\pi}{4} d^2 \right)$

$$\therefore \ \tau_{\text{all}} \pi d L = \sigma_{\text{all}} \frac{\pi}{4} d^2$$