Physics of Waves

PH11003

Tutorial 7

Topic : Interference

6 January 2023

[7.1] A plane monochromatic light wave falls normally on a diaphragm with two narrow slits separated by a distance d = 2.5 mm. A fringe pattern is formed on a screen placed at a distance l = 100 cm behind the diaphragm. By what distance and in which direction will these fringes be displaced when one of the slits is covered by a glass plate of thickness h = 10 um?

Solution

Extra phase difference introduced by the glass plate is

$$\frac{2\pi}{\lambda} = (n-1)b$$

This will cause a shift equal to $(n-1)b/\lambda$ fringe width,

i.e., by $$(n-1)\frac{b}{\lambda} \times \frac{l\lambda}{d} = \frac{(n-1)bl}{d} = 2 \text{ mm}$$

The fringes move down if the lower slit is covered by the plate to compensate for the extra phase shift introduced by the plate.

[7.2] Find the minimum thickness of a film with refractive index 1.33 at which light with wavelength 0.64 um experiences maximum reflection while light with wavelength 0.40 um is not reflected at all. The incidence angle of light is equal to 30Â°..

Solution

Given that

$$2d\sqrt{n^2 - \left(\frac{1}{4}\right)} = \left(k + \frac{1}{2}\right) \times 0.64\ \mu\text{m} \quad \text{(bright fringe)}$$

$$2d\sqrt{n^2 - \left(\frac{1}{4}\right)} = k' \times 0.40\ \mu\text{m} \quad \text{(dark fringe)}$$

(where k, k' are integers.)

Then, $$64\left(k + \frac{1}{2}\right) = 40k' \quad \text{or} \quad 4(2k+1) = 5k'$$

This means, for the smallest integer solutions, $k = 2$ and $k' = 4$

Hence, $$d = \frac{4 \times 0.40}{2\sqrt{n^2 - 1/4}} = 0.65\ \mu\text{m}$$

[7.3] The ratio of the amplitudes of two beams forming an interference fringe pattern is 2/1. What is the visibility? What ratio of amplitudes produces a visibility of 0.5?

Solution

$$\text{visibility} \equiv V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2E_{01}E_{02}}{E_{01}^2 + E_{02}^2}$$

Then,

$$V = \frac{2E_{01}E_{02}\ 1/E_{02}^2}{E_{01}^2 + E_{02}^2\ 1/E_{02}^2} = \frac{2R}{R^2+1}, \text{with } R = E_{01}/E_{02}$$

For $R = 2$,

$$V = 0.8$$

If $V = 0.5$,

$$V(1+R^2) = 2R \Rightarrow 0.5 + 0.5R^2 = 2R$$

$$R^2 - 4R + 1 = 0 \Rightarrow R = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3} = 3.732, 1/3.732$$

[7.4] Two slits are illuminated by light that consists of two wavelengths. One wavelength is known to be 436 nm. On a screen, the fourth minimum of the 436-nm light coincides with the third maximum of the other light. What is the wavelength of the other light?

Solution

Let $\lambda_1 = 436$ nm and the unknown wavelength be λ_2. At the position of overlap,

$$(3+1/2)\lambda_1 = a\sin\theta = 3\lambda_2 \Rightarrow \lambda_2 = (3.5/3)\lambda_1 = (7/6)(436\ \text{nm}) = 508.7\ \text{nm}$$

[7.5] Suppose a monochromatic coherent source of light passes through three parallel slits, each separated by a distance d from its neighbor. The waves have the same amplitude E_0 and angular frequency ω, but a constant phase difference

$\phi = 2\pi d sin\theta/\lambda$. (a) Show that the intensity is

$$I = \frac{I_0}{9}[1 + 2cos(\frac{2\pi d sin\theta}{\lambda})]^2$$

where I_0 is the maximum intensity associated with the primary maxima. (b) What is the ratio of the intensities of the primary and secondary maxima?

Solution

(a) Let the three waves emerging from the slits be

$$E_1 = E_0 \sin \omega t, \quad E_2 = E_0 \sin(\omega t + \phi), \quad E_3 = E_0 \sin(\omega t + 2\phi)$$

Using the trigonometric identity

$$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)$$

the sum of E_1 and E_3 is

$$E_1 + E_3 = E_0\left[\sin \omega t + \sin(\omega t + 2\phi)\right] = 2E_0 \cos\phi \sin(\omega t + \phi)$$

The total electric field at the point P on the screen is

$$\begin{aligned} E = E_1 + E_2 + E_3 &= 2E_0 \cos\phi \sin(\omega t + \phi) + E_0 \sin(\omega t + \phi) \\ &= E_0(1 + 2\cos\phi)\sin(\omega t + \phi) \end{aligned}$$

where $\phi = 2\pi d \sin\theta / \lambda$. The intensity is proportional to $\langle E^2 \rangle$:

$$I \propto E_0^2 (1 + 2\cos\phi)^2 \langle \sin^2(\omega t + \phi) \rangle = \frac{E_0^2}{2}(1 + 2\cos\phi)^2$$

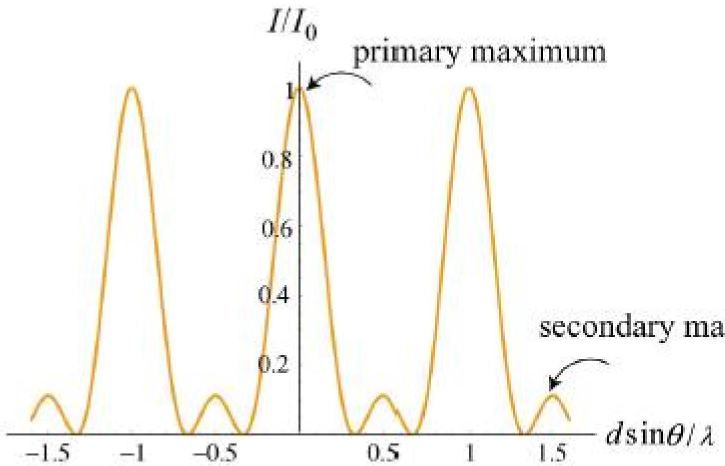
where we have used $\langle \sin^2(\omega t + \phi) \rangle = 1/2$. The maximum intensity I_0 is attained when $\cos\phi = 1$. Thus,

$$\frac{I}{I_0} = \frac{(1+2\cos\phi)^2}{9}$$

which implies

$$I = \frac{I_0}{9}(1+2\cos\phi)^2 = \frac{I_0}{9}\left[1+2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2$$

(b) The interference pattern is shown in Figure



From the figure, we see that the minimum intensity is zero, and occurs when $\cos\phi = -1/2$. The condition for primary maxima is $\cos\phi = +1$, which gives $I/I_0 = 1$. In addition, there are also secondary maxima which are located at $\cos\phi = -1$. The condition implies $\phi = (2m+1)\pi$, or $d\sin\theta/\lambda = (m+1/2)$, $m = 0, \pm 1, \pm 2, \ldots$ The intensity ratio is $I/I_0 = 1/9$.

Answer Keys