Tutorial Sheet - 13 B

Autumn 2023

Advanced Calculus (MA11003)

1. Find the gradient and the unit normal vector to the following surfaces

   (a) $x^2 + y - z = 4$ at the point $(2, 0, 0)$.

   (b) $x^2 + 2y^2 + 3z^2 = 0$ at the point $(\sqrt{10}, 0, 0)$.

   (c) $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

2. Find the directional derivatives of the following scalar valued functions

   (a) $f(x, y) = e^x \cos y$ at the point $(0, \frac{\pi}{4})$ in the direction of $(\hat{\imath} + 3\hat{\jmath})/\sqrt{10}$.

   (b) $f(x, y, z) = e^x + yz$ at the point $(1, 1, 1)$ in the direction of $\hat{\imath} - \hat{\jmath} + \hat{k}$.

   (c) $f(x, y, z) = \frac{1}{x^2+y^2+z^2}$ at the point $(2, 3, 1)$ in the direction of $\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

3. Find the directional derivative of the scalar valued function $f(x, y) = \dfrac{y}{x^2 + y^2}$ at the point $(0, 1)$ in the direction of a vector which makes an angle of $30°$ with the positive x-axis.

4. (a) In what direction from the point $(1, 3, 2)$ the directional derivatives of $\phi = 2xz - y^2$ is maximum? What is the magnitude of this maximum?
   (b) Find the values of the constant a, b and c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at the point $(1, 2, -1)$ has maximum of magnitude 64 in the direction of the z-axis.

5. If $r = |\vec{r}|$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then prove that

   (a) $\nabla(\dfrac{1}{r}) = -\dfrac{\vec{r}}{r^3}$.

   (b) $\nabla(\log(|\vec{r}|)) = \dfrac{\vec{r}}{r^2}$.

   (c) $\nabla(r^n) = nr^{n-2}\vec{r}$.

6. Let $\vec{F} = 2xz^2\hat{\imath} + \hat{\jmath} + xy^3z\hat{k}$ and $f = x^2y$. Then compute the following

   (a) $curl(\vec{F})$

   (b) $\vec{F} \times \nabla f$

   (c) $\vec{f} \cdot (\nabla f)$

7. For any two vector fields $\vec{F}$ and $\vec{G}$ show that

   (a) $\nabla \cdot (\nabla \times \vec{F}) = 0$

   (b) $div(\vec{F} \times \vec{G}) = curl(\vec{F}) \cdot \vec{G} - curl(\vec{G}) \cdot \vec{F}$

   (c) $\nabla \times (\nabla \vec{F}) = \vec{0}$