TUTORIAL PROBLEMS

SOLUTION

TUTORIAL-1

1. The force can be expressed as a vector

$$\vec{F} = F\hat{n} \qquad -(1)$$

where F is its magnitude and $\hat{n}$ is the direction. Now, in the given problem two points P and M are given whose co-ordinates can be obtained from the given figure. Therefore, the unit vector $\hat{n}$ can be expressed most easily as

$$\hat{n} = \frac{\bar{r}_P - \bar{r}_M}{|\bar{r}_P - \bar{r}_M|} \qquad -(2)$$

The vector $\bar{r}_M$ can be most easily found out when one considers the plane x-y. Since the line OM lies on the plane, the x- and y- coordinates are found out (see figure 1.1) as $x_M = OM\cos\theta$, $y_M = OM\sin\theta$.

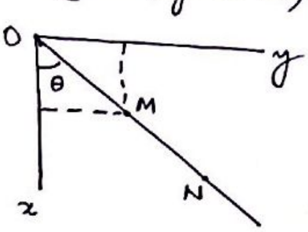


Fig 1.1

According to the given data $OM = \frac{ON}{2} = \frac{R}{2}$.

To find the position vector of P, we first note that the line OP lies in the meridianal plane plassing through points P and N. The location of P is shown in figure 1.2. The vector can be resolved into two components, one of magnitude $R\cos\phi$ along z-axis and another of magnitude $R\sin\phi$ along the line ON whose direction can be represented as $\hat{n}_N$. Thus

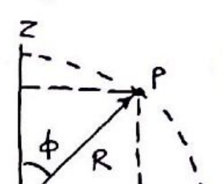


Fig 1.2

$$\bar{r}_P = \vec{OP} = R\cos\phi\,\hat{k} + R\sin\phi\,\hat{n}_N . \qquad --(3)$$

Now refer again to figure 1.1. The unit vector that points from O to N can be written as $\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$, which is nothing but $\hat{n}_N$. Thus, one gets

$$\bar{r}_P = R\cos\phi\,\hat{k} + R\sin\phi\cos\theta\,\hat{i} + R\sin\phi\sin\theta\,\hat{j} \qquad --(4)$$

The unit vector $\hat{n}$ can be written using equations (2) and (4) as

$$\hat{n} = \frac{\left(R\sin\phi\cos\theta - \frac{R}{2}\cos\theta\right)\hat{i} + \left(R\sin\phi\sin\theta - \frac{R}{2}\sin\theta\right)\hat{j} + R\cos\phi\,\hat{k}}{\left[\left(R\sin\phi\cos\theta - \frac{R}{2}\cos\theta\right)^2 + \left(R\sin\phi\sin\theta - \frac{R}{2}\sin\theta\right)^2 + R^2\cos^2\phi\right]^{1/2}} . \qquad (5)$$

which can be simplified to get

$$\hat{n} = \frac{(2\sin\phi - 1)(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + 2\cos\phi\,\hat{k}}{\sqrt{5 - 4\sin\phi}} . \qquad --(7)$$

With this expression for $\hat{n}$, the force can be written using equation (1).

2. Since the forces along a-a and b-b direction produce the same effect as the original force, the task is to resolve the force 600N into two components along the two given directions. The resolution of the force can be seen from the parallelogram law, shown for a general case in figure 2.1.

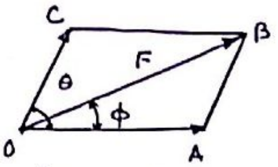
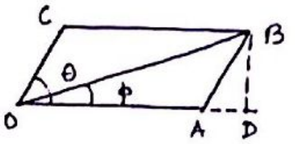


Figure-2.1.

The magnitude of the component forces can be obtained as the lengths of the sides of the parallelogram whose diagonal (OB in figure 2.1) represents the original force.

Elementary trigonometry (see figure 2.2) gives the following two equations



$$OA = OD - AD = OB \cos\phi - AB \cos\theta = F\cos\phi - OC\cos\theta \quad \cdots(1)$$

$$BD = AB \sin\theta = OB \sin\phi = F\sin\phi \quad \cdots(2)$$

From equations (1) and (2) we get

$$OC = AB = F\frac{\sin\phi}{\sin\theta} \quad \cdots(3)$$

and $$OA = F\cos\phi - F\frac{\sin\phi}{\sin\theta}\cos\theta = F\frac{\sin(\theta-\phi)}{\sin\theta} \quad \cdots(4)$$

In fact, we see the following nice relationship

$$\frac{OB}{\sin\theta} = \frac{AB}{\sin\phi} = \frac{OA}{\sin(\theta-\phi)} \quad \cdots(5)$$

Which is a known trigonometric relationship for triangle OAB.

Considering above discussion and the data given in figure of the problem we get

$$F_a = 600\,\frac{\sin(180^\circ-60^\circ-30^\circ)}{\sin(180^\circ-60^\circ)}\,(N) = 600\times\frac{2}{\sqrt{3}}\ N = 692.82\ N$$

and $$F_b = 600\,\frac{\sin 30^\circ}{\sin(180^\circ-60^\circ)} = \frac{600}{\sqrt{3}}\ N. = 346.42\ N.$$

Although the problem is solved by geometric method, the following vector approach can also be used. Let F_a and F_b be the magnitude of the component forces along directions represented as $\hat{n}_a$ and $\hat{n}_b$, respectively.

Thus,

$$\vec{F} = F_a \hat{n}_a + F_b \hat{n}_b \qquad - (6)$$

From equation (6) we get the following equations

$$\vec{F} \cdot \hat{n}_a = F_a + F_b (\hat{n}_a \cdot \hat{n}_b) \qquad \cdot\cdot (7)$$

and $$\vec{F} \cdot \hat{n}_b = F_a (\hat{n}_a \cdot \hat{n}_b) + F_b . \qquad \cdot\cdot (8)$$

Solving the above two equations one gets

$$\frac{(\vec{F} \cdot \hat{n}_a)(\hat{n}_a \cdot \hat{n}_b) - (\vec{F} \cdot \hat{n}_b)}{(\hat{n}_a \cdot \hat{n}_b)^2 - 1} = F_b \qquad \cdot\cdot (9)$$

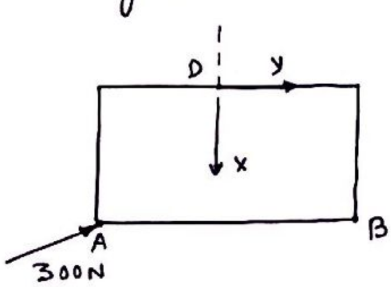
and $$\frac{(\vec{F} \cdot \hat{n}_b)(\hat{n}_a \cdot \hat{n}_b) - (\vec{F} \cdot \hat{n}_a)}{(\hat{n}_a \cdot \hat{n}_b)^2 - 1} = F_a \qquad \cdot\cdot\cdot (10)$$

Since $\hat{n}_a \cdot \hat{n}_b = \cos 120^\circ = -\frac{1}{2}$, $\vec{F} \cdot \hat{n}_b = F \cos (120^\circ - 30^\circ) = 0$, $\vec{F} \cdot \hat{n}_a = F \cos 30^\circ$

we get

$$F_a = -\frac{F \sqrt{3}/2}{\left(\frac{1}{2}\right)^2 - 1} = \frac{2F}{\sqrt{3}}, \qquad F_b = \frac{\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2 - 1} F = \frac{F}{\sqrt{3}} . \qquad \cdots (11)$$

P.3. (a) To calculate the moment about point D we place the coordinate system as shown in figure 3.1 with origin at D. The moment of the force about point D can be calculated in the straight forward method as



$$\bar{M}_D = \vec{r}_{DA} \times \vec{F} \qquad \cdot\cdot (1)$$

where $\vec{r}_{DA} = -0.1\hat{j} + 0.2\hat{i}$ (m) and

$\vec{F} = 300(\cos 25^\circ \hat{j} - \sin 25^\circ \hat{i})$ (N).

Carrying out the vector multiplication we get

$$\bar{M}_D = 300 (0.2 \cos 25^\circ - 0.1 \sin 25^\circ)\hat{k} \ (\text{N-m})$$
$$= 41.699\hat{k} (\text{N-m})$$

The moment is 41.7 N-m in the counter-clockwise direction.

(b) The magnitude of moment about a point can be written as

$$M = F\, r \sin\theta$$

where θ is the angle between the vectors $\vec{F}$ and $\vec{r}$ and with $\vec{r}$ as the vector joining the point and any point on the line of action of the force. Thus, for given values of M, ~~r and θ~~ and r, F attains minimum value when $\theta = \pi/2$. In the given problem the line of action of F to be applied at point B is known but since Point B lies on the line of action $r = DB$. Hence, the smallest force that is required to be applied at B to get the same moment as that due to 300 N, i.e, 41.7 N-m, is given by

$$F = \frac{M}{BD} = \frac{41.7}{0.2\sqrt{2}}\,(N) = 147.428\,(N).$$

The direction of F is perpendicular to the line BD, ie the line of action makes an angle $\tan^{-1}\left(\frac{0.2}{0.2}\right) = 45°$ with the horizontal. Further, since the moment is in the counter-clockwise direction the required force can be written in vector form as

$$\vec{F} = 147.428\left(-0.707\,\hat{i} + 0.707\,\hat{j}\right)\ (N).$$

The same problem can be solved purely analytically by assuming the force as $F_x\hat{i} + F_y\hat{j}$. The moment about point D is then

$$\bar{M} = 0.2(\hat{i}+\hat{j}) \times (F_x\hat{i} + F_y\hat{j}) = 0.2\,(F_y - F_x)\,\hat{k}.$$

Since $0.2\,(F_y - F_x) = 41.7 = m$ (say), we get $F_y = F_x + 5m$. We are needed to find the minimum force, ie

$$F^2 = F_x^2 + F_y^2 = F_x^2 + (F_x + 5m)^2$$

must be minimized. It is easy to see that the value of F^2 attains minimum value when $F_x = -\frac{5}{2}m = -104.248\ (N)$ and hence $F_y = \frac{5}{2}m = 104.248\ (N)$. The required force is therefore

$$\vec{F} = +104.248\left(-\hat{i} + \hat{j}\right)\ (N).$$

4. The moment of $\vec{T}$ about point O can be written as

$$\bar{M}_o = \vec{r}_{OA} \times \vec{T},$$

or $\quad \bar{M}_o = \vec{r}_{OB} \times \vec{T}.$

Note that, for the given data the mathematical manipulation required is almost the same whether the first formula is used or the second. In both the cases the radius vector has two non-vanishing components along the axes.

If point A is assumed then

$$\bar{M}_o = \left(18\hat{j} + 30\hat{k}\right) \times T\left(\frac{6\hat{i} - 5\hat{j} - 30\hat{k}}{\sqrt{6^2+5^2+30^2}}\right) \text{ (kN-m)}$$

where $T = 24$.

By carrying out the vector multiplication of the above vectors we get

$$\bar{M}_o = \left(-301.9355\hat{i} + 139.3548\hat{j} - 83.6129\hat{k}\right)\text{(kN-m)}.$$

If point B is assumed, then

$$\bar{M}_o = \left(6\hat{i} + 13\hat{j}\right) \times T\left(\frac{6\hat{i} - 5\hat{j} + 30\hat{k}}{\sqrt{6^2+5^2+30^2}}\right) \text{ kN-m.}$$

It can be verified after vector multiplication that the moment becomes the same as obtained previously, as it should be. ■

5. It is known that the weight of a body passes through a point, which is called its centre of gravity. However, finding the CG of a body of complex shape, such as the one shown in figure, requires a bit of calculation. When the body is made of a number of simple bodies it becomes sometimes useful to work with the weights of the individual constituents members rather than the whole body. The given structure is seen to ~~ma~~ be made up of three straight rods whose centres of gravity are known to be at their mid-points. Thus the weight of the structure is made up of three weights, namely

(i) $2 \times 7 = 14$ kg passing through the origin of the coordinate system
(ii) $1.1 \times 7 = 7.7$ kg passing through $(0, 1.1/2, 0)$ m and
(iii) $2 \times 7 = 14$ kg passing through point $(-0.2, 1.1, 0)$ m.

The moment of all the forces can be calculated using vectors ($g \approx 9.81$ m/s²)

$$\bar{M}_o = \bar{0} \times (-14\hat{k}) + 0.55\hat{j} \times (-7.7\hat{k}) + (-0.2\hat{i} + 1.1\hat{j}) \times (-14\hat{k})\, g \text{ N-m}$$

$$= \left(-19.635\hat{i} - 2.8\hat{j}\right) \times g \text{ N-m} \approx -192.62\hat{i} - 27.47\hat{j} \text{ (N-m)}$$

Note that the weights of the members act at points different from the ones considered here. Since for calculation of moment we need only to know any point on the line of action of the force, we take those points which amount to minimum calculation. These points are chosen to be the ones where the lines of action of the weights intersect x-y plane.

6. The moment of a force $\vec{F}$ about an axis whose direction is represented by unit vector $\hat{n}$ is given by

$$M = \hat{n} \cdot (\vec{r} \times \vec{F})$$

where $\vec{r}$ is the vector from <u>any point</u> on the specified axis to <u>any point</u> on the line of action of the force.

For the given problem we can choose any two points in different ways. For example (O and A) or (O and B) can be chosen for calculation. However if we take point B and (0, 0, 0.4) as the points then the calculation becomes simple because in this case $\vec{r} = 0.5\hat{j}$ (m). The points are shown in figure 6.1 below

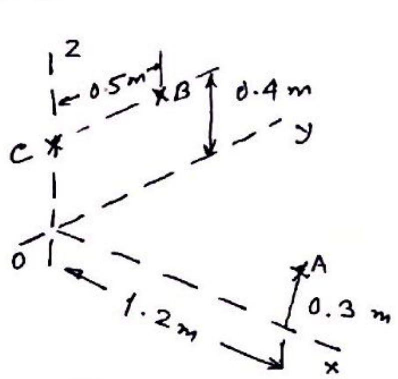


Figure 6.1.

The force vector can be written as

$$\vec{F} = 2\,\hat{n}_{AB}\ (\text{kN}) = 2\,\frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|}\ (\text{kN})$$

$$= 2\left(\frac{-1.2\,\hat{i} + 0.5\,\hat{j} + 0.1\,\hat{k}}{\sqrt{(1.2)^2 + (0.5)^2 + (0.1)^2}}\right) \text{kN}$$

Thus $$\vec{r} \times \vec{F} = \frac{1}{\sqrt{(1.2)^2 + (0.5)^2 + (0.1)^2}}\left(1.2\,\hat{k} + 0.1\,\hat{i}\right) \text{kN-m}$$

Since $\hat{n} = \hat{k}$, $$M = \hat{k} \cdot (\vec{r} \times \vec{F}) = \frac{1.2}{\sqrt{(1.2)^2 + (0.5)^2 + (0.1)^2}} \text{kN-m}$$

$$= 0.92 \text{ kN-m}$$

If you try with different sets of points you must get the same result, though the amount of calculation may vary.

It is instructive to visualize the situation in more details. The force $\vec{F}$ can be resolved into three components as $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ where $F_x = -1.8407$ kN, $F_y = 0.767$ kN, and $F_z = 0.1534$ kN. The forces pass through point A. If you now imagine the pipe to be able to rotate about z-axis, then it is easy to see that only F_y is able to cause rotation. The component force F_x passes through z-axis and F_z acts along the z-axis. Both these components can not cause the rotation. Therefore, the moment about the z-axis, which is the force's effectiveness in causing rotation about the same axis is given by the product of the force component F_y and the moment arm which in this case is 1.2 m. The required moment is, therefore,

$M_o = 1.2 \times 0.767$ kN-m $= 0.92$ kN-m.

7. In the given problem the platform is pulled by the same cable at points C and D. The tension is same throughout the cable since there is no friction at the ~~hinge~~ hook E. (For a rusty hook where considerable friction is expected to work the tensions in the sections CE and DE are not equal) Although the magnitudes of the force are the same they must be represented by two independent forces because their lines of actions are different. The forces can be represented in the following vector forms:

$$\vec{F}_1 = \text{Force in } CE = T\frac{\bar{r}_E - \bar{r}_C}{|\bar{r}_E - \bar{r}_C|} = 1349\left(\frac{0.9\hat{i} + 1.5\hat{j} - 2.25\hat{k}}{\sqrt{(0.9)^2 + (1.5)^2 + (2.25)^2}}\right) \text{ N}.$$

and

$$\vec{F}_2 = \text{Force in } DE = T\frac{\bar{r}_E - \bar{r}_D}{|\bar{r}_E - \bar{r}_D|} = 1349\left(\frac{-2.3\hat{i} + 1.5\hat{j} - 2.25\hat{k}}{\sqrt{(2.3)^2 + (1.5)^2 + (2.25)^2}}\right) \text{ N}.$$

Note that the string is capable of pulling only. Thus the direction of the force is from C to E or D to E only.

Now, in the problem one needs to find out the moment of $\vec{F}_1$ about each axis. Since finding the moment about any axis requires any point on the same axis, it is obviously advantageous to take the same point for all the three axes, if there exists any. It is seen that the origin O is that point. Thus we take the moment about point O, which is given by

$$\bar{M}_0 = 2.25\,\hat{k} \times \vec{F}_1$$
$$= -1597.5\,\hat{i} + 958.5\,\hat{j} \;(\text{N-m})$$

The moment about x-axis is $M_x = \bar{M}_0 \cdot \hat{i} = -1597.5$ N-m while the moment about y-axis is 958.5 N-m. Moment about z-axis is zero. ■

8. It is known that the moments of a force about three coordinate axes are nothing but the components of the moment of the same force about origin along the respective axes. Thus

$$\bar{M}_0 = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

where M_x, M_y and M_z are the moments of the force about x-, y- and z-axis, respectively.

~~For the given problem $\vec{F} = P(-\sin\phi\,\hat{j} + \cos\phi\,\hat{k})$ (N)~~

and $\bar{r}_c$ (c is the point of application of the force) $= \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$.

From the given data it can be easily checked that

$\overline{OA} = 0.15\,\hat{i}$ (m), $\overline{BC} = 0.1\,\hat{i}$ (m) and $\vec{r}_{AB} = 0.2(\sin\theta\,\hat{j} + \cos\theta\,\hat{k})$ (m)

Thus, $\overline{M}_o = \vec{r}_c \times \vec{F}$

$$= (0.25\,\hat{i} + 0.2\sin\theta\,\hat{j} + 0.2\cos\theta\,\hat{k}) \times P(-\sin\phi\,\hat{j} + \cos\phi\,\hat{k}) \text{ (N-m)}$$

By carrying out vector product we get

$$\overline{M}_o = (0.2(\sin\theta\cos\phi + \cos\theta\sin\phi)\,\hat{i} - 0.2\cos\theta\cos\phi\,\hat{j} - 0.25\sin\phi\,\hat{k})P \text{ N-m.}$$

According to the given data

$$\left.\begin{aligned} M_x &= +0.2\,P\sin(\theta+\phi) = 20 \quad \cdots (a) \\ M_y &= -0.25\,\cancel{\cos\theta}\,\cos\phi\,P = -8.75 \quad \cdots (b) \\ M_z &= -0.25\sin\phi\,P = -30 \quad \cdots (c) \end{aligned}\right\}$$

From (b) and (c)

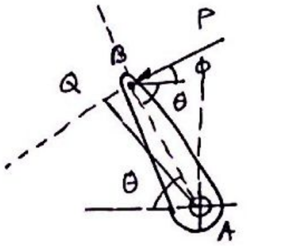
$$\tan\phi = \frac{30}{8.75} \Rightarrow \phi = 73.74° \approx 74°$$

and $$P = \sqrt{30^2 + (8.75)^2}\,/\,0.25 = 125 \text{ (N)}$$

From (a) $$\theta = \sin^{-1}\left(\frac{100}{125}\right) - \phi = 53.13° - 74° \text{ or } (180° - 53.13°) - 74°$$
$$= -21° \text{ or } 53°.$$

Discarding the negative value of θ we get $\theta = 53°$.

In stead of using vectors the above three equations (a - c) can be obtained considering only physical significance of the moment of a force about any axis. The rotation about y-axis is caused by only component $P\cos\phi$ while that about z-axis is due to $P\sin\phi$. Equations (b) and (c) are resulted when one considers the moment arm for both the cases as 0.25 m. The rotation about x-axis is resulted by the force P. The moment arm in this case is seen from figure 8.1 to be $0.2\sin(\theta+\phi)$ (m). Equation (a) is thus obtained straight forward.



Moment arm = AQ

$= AB\sin\angle ABQ$

$= AB\sin(180° - \theta - \phi)$

Figure 8.1

9. According to the information given in the problem the directions of two component forces as well as that of the resultant are ~~gt~~ known. In addition the magnitude of one of the component forces is given. This information are sufficient to construct the parallelogram of the forces. The parallelogram is shown in figure 9.1. The lengths of the sides of the parallelogram are proportional to the magnitudes of the forces. Thus $PS = QR = 8$

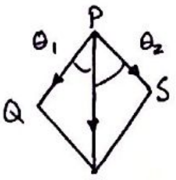


Figure 9.1

Now, according to sine law for triangle PRS we get

$$\frac{PR}{\sin(\theta_1+\theta_2)} = \frac{RS}{\sin\theta_2} = \frac{PS}{\sin\theta_1}.$$

The angles θ_1 and θ_2 are easily calculated from the geometry given in the problem (ie, dimensions of the physical system). They are

$$\theta_2 = \tan^{-1}\left(\frac{40}{60}\right), \quad \theta_1 = \tan^{-1}\left(\frac{50}{40}\right).$$

Thus, we get

$$RS = 8 \times \frac{\sin\theta_2}{\sin\theta_1} = 5.6829. \text{ and}$$

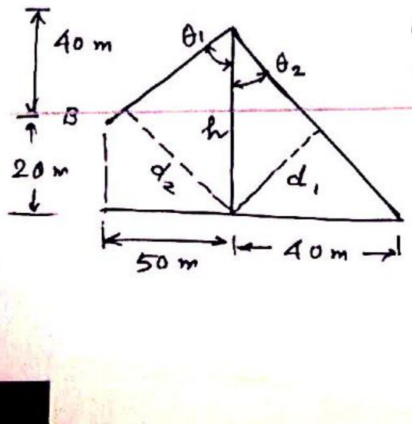
$$PR = 8 \times \frac{\sin(\theta_1+\theta_2)}{\sin\theta_1} = 10.2065.$$

The required tension in the cable AB is $5.68(N)$ and the magnitude of the resultant force is $10.21\,(N)$.

Alternatively, Varignon's theorem can be used for solving the problem. According to the same theorem the moment about a point of a number of concurrent forces is equal to the moment of the resultant force about the point. Since the resultant force, in the given problem, acts downward from point A, the moment of the two forces, namely 8 kN and T, about the point lying on the ground directly below A is zero. The moments are given by $8 \times d_1$ and $T d_2$ in opposite sense, where d_1 and d_2 are the smallest distances from the base point to lines AC and AB, respectively. According to the condition given

$$T = 8\,\frac{d_1}{d_2}.$$

The values of d_1 and d_2 are obtained from geometry (see figure 9.2)



$$d_1 = h \sin\theta_2, \quad d_2 = h \sin\theta_1.$$

Thus $T = 8\,\dfrac{\sin\theta_2}{\sin\theta_1} = 5.68\,(N).$

The value of the resultant force is

$$R = 8\cos\theta_2 + T\cos\theta_1 = 8\left[\cos\theta_2 + \frac{\cos\theta_1 \sin\theta_2}{\sin\theta_1}\right]$$

$$= 8\,\frac{\sin(\theta_1+\theta_2)}{\sin\theta} = 10.21\,(N).$$

10. It is known that the resultant of a number of co-planar forces and a number of couples in the same plane can be represented most simply by a single force, known as the simplest resultant, passing through some fixed point. It is, therefore, obvious that if the moment of all the forces and couples are taken about the same point, then the moment will be zero, since the resultant force passes through the point.

In the given problem the moment of the forces and the couple about O is zero. The moments of all the forces can be calculated by the vector method or by simply using the fact that the magnitude of the moment is the product of the force and the shortest distance between the point and the line of action of the free. The direction of the moment is determined by the tendency to rotate a right-handed screw located at the point.

Thus, the moment of 320 N force is given as

$$M_1 = 320 \times 0.3 \text{ N-m (CW)} \quad \left[\begin{array}{l}\text{The force is perpendi-} \\ \text{-cular to the lever arm}\end{array}\right]$$

The moment of 400 N is

$$M_2 = 400 \times 0.15 \cos 30^\circ \text{ (CW)}$$

Thus, the required couple has the moment

$M = M_1 + M_2$ in Counter-clockwise direction.

Hence $M = 147.96$ N-m (CCW) $\simeq 148$ N-m (CCW).

11. In the problem two forces are given as follows

$\vec{F}_1 = T\hat{i}$ whose line of action is expressed as $y = 3$ (m).

and $\vec{F}_2 = T(\cos 15^\circ \hat{i} + \sin 15^\circ \hat{j})$ with point of application $-(10\hat{i}+3\hat{j})$

The equivalent force-couple system at point O is given by

(i) A force $\vec{F} = \vec{F}_1 + \vec{F}_2 = T\left((1+\cos 15^\circ)\hat{i} + \sin 15^\circ \hat{j}\right)$ N $= 1.9659\,\hat{i} + 0.2588\,\hat{j}$ (N)

(ii) A couple of moment $\vec{C} = 3\hat{j} \times (T\hat{i}) - (10\hat{i} + 3\hat{j}) \times T(\cos 15^\circ \hat{i} + \sin 15^\circ \hat{j}) = T(-3 - 10\sin 15^\circ + 3\cos 15^\circ)\,\hat{k}$ (N-m)

$= -2.6904\,\hat{k}$ (N-m)

Suppose the simplest resultant of the force-couple system passes through $x\hat{i}$. Evidently, the moment of the force couple system must vanish about the point. This implies

$$M = -2.6904 + (-x\hat{i}) \times \vec{F} = 0 \Rightarrow x = -\frac{2.6904}{0.2588} = -10.3957 \text{ (m)}.$$

12. At the given instant the aircraft is subjected to the following three forces.

(i) $\vec{F}_1 = 90\,\hat{i}$ (kN) with line of action $y = 21$ (m), $z = 2$ (m).

(ii) $\vec{F}_2 = 90\,\hat{i}$ (kN) with line of action $y = \overset{(-)}{12}$ (m), $z = 3$ (m)

(iii) $\vec{F}_3 = 90\,\hat{i}$ (kN) with line of action $y = -21$ (m), $z = 2$ (m)

In order to get the simplest resultant it may be useful, first, to find the equivalent force - couple system at a known point, say, the origin, O. The equivalent system consists of

(i) A force $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 270\,\hat{i}$ (kN). and

(ii) A couple of moment $\vec{C} = \left((21\hat{j} + 2\hat{k}) \times 90\,\hat{i}\right) + \left((-12\hat{j} + 3\hat{k}) \times 90\hat{i}\right)$

$$+ \left((-21\hat{j} + 2\hat{k}) \times 90\,\hat{i}\right) \text{ kN-m}$$

$$= (-12\hat{j} + 7\hat{k}) \times 90\,\hat{i} = 90(7\hat{j} + 12\hat{k}) \text{ (kN-m)}.$$

Let the simplest resultant of the force-couple system passes through the point $(y\hat{j} + z\hat{k})$. Hence the moment of the above force-couple system about the same point must vanish. This gives

$$\vec{M} = 90(7\hat{j} + 12\hat{k}) + (-y\hat{j} - z\hat{k}) \times 270\,\hat{i} = 0$$

This yields $\quad y = -\dfrac{90 \times 12}{270} = -4$ (m)

and $\quad z = \dfrac{90 \times 7}{270} = 2.33$ (m)

Thus, the resultant of the three forces ~~are~~ is a force $270\,\hat{i}$ (N) passing through point $(-4\hat{j} + 2.33\,\hat{k})$ (m).

Note that the above procedure of introducing an intermediate point (in this case point O) is not essential. If $(y\hat{j} + z\hat{k})$ is the point through which the resultant passes then, we get according to the above discussion,

$$\vec{M} = \left((21-y)\hat{j} + (2-z)\hat{k}\right) \times (90\,\hat{i}) + \left((-12-y)\hat{j} + (3-z)\hat{k}\right) \times (90\,\hat{i})$$

$$+ \left((-21-y)\hat{j} + (2-z)\hat{k}\right) \times 90\,\hat{i} = 0$$

~~It is easy to see that the above equation yields~~

$$(21-y) + (-12-y) + (-21-y) = 0 \quad \Rightarrow y = -\frac{12}{3} = -4$$

and $$(2-z) + (3-z) + (2-z) = 0 \quad \Rightarrow z = 7/3 = 2.33.$$

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