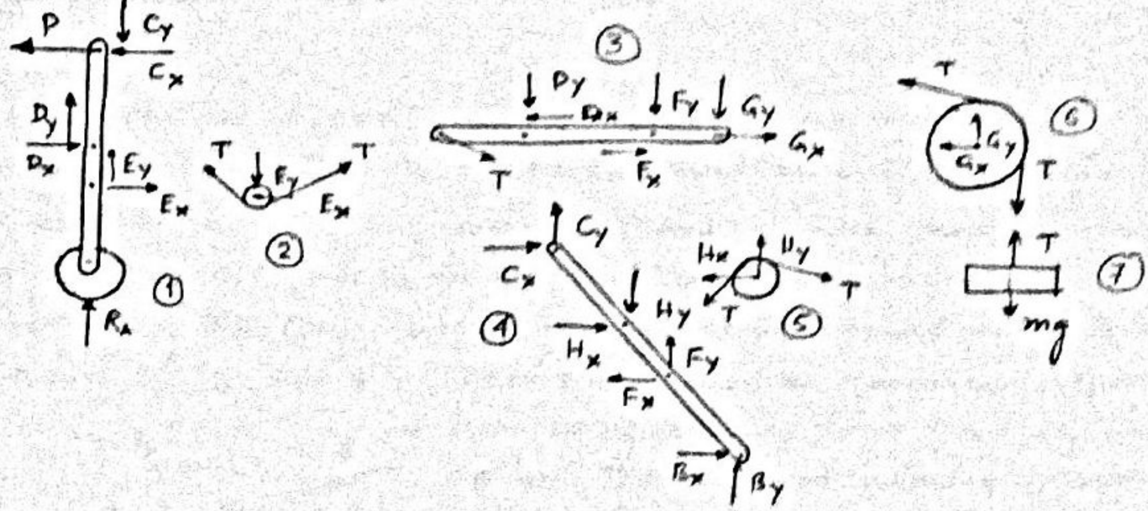
TUTORIAL PROBLEMS

SOLUTION

TUTORIAL - 2

1. The Free Body Diagram (FBD)'s of different members are shown below



Explaination:

Body-① : In the FBD the external force P is considered to be on this member. This is not essential since the same force could have been on member-④ also. The joints are also frictionless hinge joints. As they prevent movement along any direction (see attached figure) there are two independent constraint forces along two orthogonal direction (These are the components of a single force whose magnitude as well as direction are not known). The roller is taken as a part of body ①. Since there is no friction between the ground and the roller only normal reaction is present. However if the roller is taken separately then two reaction forces appear at the joint. The FBD of the roller will look like the one shown in figure 1.2.

Joint C   Joint-D

Fig 1.1

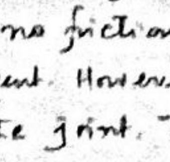
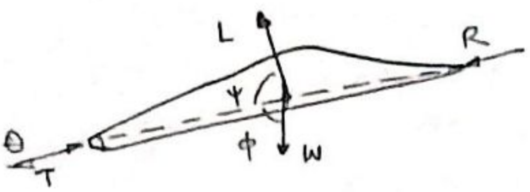


Figure 1.2

But since rollers are usually of negligible mass (in comparison with that of the structure) equations of statics can be used which yields $A_x = 0$ and $A_y = R_A$. Thus even if the roller is removed from the member the free body diagram still shows a single reaction force at the joint. The small pulley at E is drawn separately as a free body. If the pulley is considered as an integral part of the member then only string tension T needs to be shown. However, it must be remembered that the lines of action of these tension forces do not pass through point E. But since the tensions are the same their lines of action can be shifted to point E. The additional couples that act because of the shifting of the lines of action cancel each other. Hence it makes no harm if the tensions are shown at E directly. Be careful that this happens only because the string tensions are same due to absense of any friction at the hinge joint.

FBD's of other members do not need any further explaination. The above comments apply to them as well.

2. Since the aircraft is in equilibrium (as it is moving with constant speed) all the equilibrium conditions must be satisfied. This implies that in addition to the given forces R, T and W there must be another force, say L (called lift force for aircraft similar to buoyancy force for ships) to balance the component of force W along normal to the common line of action of T and R. These forces must be concurrent because two of them being collinear have three different lines of action. The moment balance equation will not be satisfied if this is not the case. It is easy to see that taking moment about the intersection of any two lines of action one can not make the net moment zero.



In the problem it is further stated that the force T is function of only R and W. This demands that L must act orthogonal to the line of action of T. Otherwise from the force balance equation along the direction of T we get

$$T = R - W \cos\phi - L \cos\psi,$$

ϕ and ψ being the angles between W and T and L and T respectively (see figure above).

3. The Free Body Diagram of the roller is shown in figure 3.1. The following points may be noted

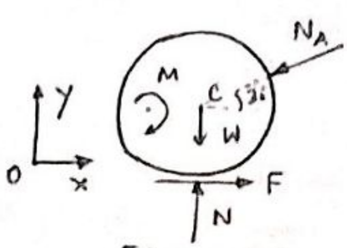


Figure 3.1

(i) In addition to the normal reaction, N, between the roller and the ground there is a friction force F. The direction of the latter force is not known since there is no slip at the point.

(ii) The is only normal reaction at the contact point between the wheel and the roller. The only condition needed is that there is no friction at the pin where the roller is connected to the ground. Even if the contact between the two rolling surfaces be rough no tangential friction force acts. If a sliding friction is included it will soon be found out to be zero simply by writing down the equilibrium equations of the roller from its FBD (shown in figure 3.2)

Figure 3.2

Equilibrium equations from figure 3.1

$$\Sigma F_x = 0 \quad : \quad F - N_A \cos 30^\circ = 0 \qquad \text{--- } (1)$$

$$\Sigma F_y = 0 \quad : \quad N - W - N_A \sin 30^\circ = 0 \qquad \text{--- } (2)$$

$$\Sigma M_c = 0 \quad : \quad -M + F \cdot r = 0 \qquad \text{--- } (3)$$

Solving equations (1) and (3) one gets

$$N_A = \frac{F}{\cos 30^\circ} = \frac{M}{r \cos 30^\circ} = \frac{60 \times 2}{0.3 \times \sqrt{3}} = 230.94 \text{ (N)}$$

The above result could have been obtained in one step by taking moment about the point of contact between the wheel and the roller and equating it to zero. The equation looks like

$$M = N_A \cdot d \qquad \text{--- } (4)$$

where d is the shortest distance between the point and the line of action of N_A which is found using simple geometrical calculation as $r \sin 60^\circ$.

4. In this problem the condition of the cylinder is one of impending motion. The state of a body can be either rest or movement with acceleration (We do not differentiate between rest and uniform motion because they are equivalent). When the body begins its movement from the rest its acceleration takes a finite value from zero. At the instant when the value of acceleration (and/or angular acceleration) just crosses zero the body is said to be in impending motion. Since all the accelerations are taken to be zero at that instant the equations of equilibrium are still applicable. However the force relationships between the body and its surrounding become different from the same at an instant when the body remains comfortably at rest. In the given problem the reaction force between the wheel and the horizontal ground acts when the wheel remains at rest, ie, shows no sign of movement. However, at an instant when it begins to roll up with the corner point as pivot the contact disappears. The free body diagram of the wheel at that instant can be shown as in figure 4.1

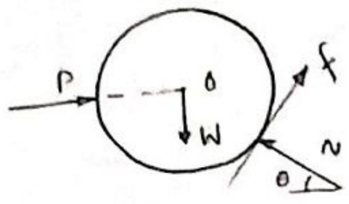


Fig 4.1

Note that the direction of the normal force N is perpendicular to both the contacting surfaces. In this case the normal can be defined only for one surface while the same can not be defined for the other surface. We take the direction of N to be along the normal to any of the plane that is well-defined.

The friction force, f, whose magnitude and direction are not known, should be shown in FBD since in the most general situation friction force is likely to act.

Applying the conditions of equilibrium to the FBD we get

$$\sum F_x = 0 \Rightarrow P + f \sin\theta - N\cos\theta = 0 \quad \text{①}$$

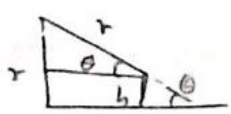
$$\sum F_y = 0 \Rightarrow N\sin\theta + f\cos\theta - W = 0 \quad \text{②}$$

$$\sum M_o = 0 \Rightarrow f r = 0 \quad \text{③}$$

Solving equations ① - ③ the following results are obtained

$$f = 0, \quad N = \frac{W}{\sin\theta} \quad \text{and} \quad P = \frac{W}{\tan\theta} \quad \text{④}$$

Using geometry it is easy to see (see figure 4.2) that



$$\sin\theta = \frac{r-h}{r}, \quad \cos\theta = \sqrt{1-\sin^2\theta} = \frac{\sqrt{2hr-h^2}}{r}$$

and consequently $P = \dfrac{W\sqrt{2hr-h^2}}{r-h} = mg\dfrac{\sqrt{2hr-h^2}}{r-h}$.

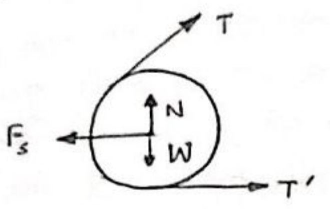
The same result could have been obtained by considering moment balance equation about the contact point between the wheel and the obstruction. In this case one gets

$$mg \cdot \sqrt{r^2-(r-h)^2} = P\,(r-h) \quad (5)$$

which immediately leads to the result.

Note that in this case friction force is zero, ie, the wheel shows no tendency for slip. This is accidental and arises because of the particular way of applying the force P. If the force were applied at any other point or if a ~~moment~~ torque were applied then friction force would have appeared. A free body diagram without the friction force would have become unmanageable!

5. The free body diagram of the disc is shown in figure 5.1



Note the following points

(i) At the centre there is only one constraint force between the disc and the frictionless guide.

(ii) The weight of the disc, W, the normal force N and the spring force F_s may not act at the same point but their lines of action have offsets by very small amounts depending on the physical construction of the collar as well as the spring attachment. The effect of these offsets are to produce a couple that may turn the disc about vertical axis. The effect of the couple is not considered here because of its smallness. Moreover, if the ~~attachm~~ loads are symmetric about the vertical surface passing through disc's centre then no couple exists as they cancell each other. The forces shown in the FBD are the sum of the forces, ie, equivalent force systems at the disc centre.

From the conditions of equilibrium we get

$M_0 = 0 \Rightarrow T r = F' r$ --- (1)

$\sum F_x = 0 \Rightarrow T \cos 45° + T' = F_s$ --- (2)

$\sum F_y = 0 \Rightarrow N + T \sin 45° = W$ --- (3)

From the above three equations one finally gets

$$T = \frac{F_s}{(1 + 1/\sqrt{2})}, \quad N = W - \frac{F_s/\sqrt{2}}{(1 + 1/\sqrt{2})}. \quad \text{---(4)}$$

The spring force is given by

$$F_s = K \cdot \Delta,$$

where Δ is the stretch of the spring. According to the data given

$\Delta = 10 + x = 160$ mm.

Thus, from equation (4) we get

$$T = \frac{0.16 \times 3.5}{(1 + 0.707)} = 0.328 \text{ KN} = 328 \text{ (N)}$$

and $N = 3 \times 9.81 - 232$ (N) $= -202.57$ (N)

The magnitude of the normal force is seen to be 203 N while its direction is downward. Now, in the problem there may be two possi- -bilities. Either the disk touches the guide at one point (if there is very small clearance between the disk collar and the guide) or at both points (if fitted without any clearance). The answer suggests that in the former case the disk touches the guide at the upper surface. For the latter case we calculate the net force. The individual forces acting on the two sides of the slide have the same line of action and can not be determined by the principle of statics alone. The result shows that the net force acts downward.

6. The free body diagram of the beam is shown in figure 6.1. The equilibrium equations are written in the following:

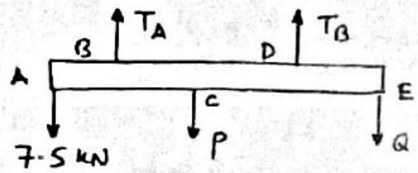


Fig 6.1

$\sum F_y = 0 \Rightarrow P + Q + 7.5 = T_A + T_B$

$\sum F_x = 0 \Rightarrow 0 = 0$

$M_A = 0 \Rightarrow T_A \cdot (AB) - P \cdot (AC) + T_B \cdot (AD) - Q \cdot (AE) = 0$

From the above equations the values of T_A and T_B can be solved. However this tedious process of solving the equations can be avoided if the following equations of equilibrium are used in stead of above.

$$M_B = 0 \Rightarrow 7.5 \times (AB) - P \cdot (BC) + T_B \cdot (BD) - Q \cdot (BE) = 0 \quad \cdots (1)$$

$$M_D = 0 \Rightarrow 7.5 \times (AD) - T_A\,(BD) + P \cdot (CD) - Q\,(DE) = 0 \quad \cdots (2)$$

These equations yield the following

$$T_A = 7.5 \times \frac{AD}{BD} + P \cdot \frac{CD}{BD} - Q \cdot \frac{DE}{BD} \quad (kN)$$

$$= 7.5 \times \frac{2.75}{2.25} + 5 \times \frac{1.5}{2.25} - Q \cdot \frac{0.75}{2.25} \quad (kN) \quad \cdots (3)$$

and

$$T_B = -7.5 \times \frac{AB}{BD} + P \cdot \frac{BC}{BD} + Q\,\frac{BE}{BD} \quad (kN)$$

$$= -7.5 \times \frac{0.5}{2.25} + 5 \times \frac{0.75}{2.25} + Q \cdot \cancel{0.75}\,\frac{3}{2.25} \quad (kN) \quad - (4)$$

The requirement that $0 \leq T_A \leq 12$ (kN) and $0 \leq T_B \leq 12$ (kN) leads to the following results

(i) $Q \leq 37.5$ $\quad (T_A \geq 0)$

(ii) $Q \geq 1.5$ kN $\quad (T_A \leq 12$ kN$)$

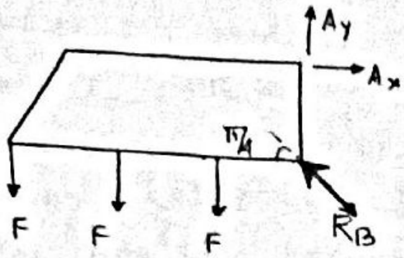
(iii) $Q \geq 0$ $\quad (T_B \geq 0)$

(iv) $Q \leq 9$ kN $\quad (T_B \leq 12$ kN$)$

These above conditions are all satisfied if $1.5\text{ kN} \leq Q \leq 9$ kN.

The inequality $T_A \geq 0$ and $T_B \geq 0$, although not stated in the problem, are important. The tensions in the cable can only take positive values. A cable becomes slack when its tension becomes negative. In the free body diagram the cable force does not appear as its value becomes less than zero. Thus, the FBD which has been drawn for the given problem implicitly assumes the cable tensions to be non-negative. It bears no special significance that the non-negativity condition does not affect the final result.

7. The free body diagram of the entire truss is shown in figure 7.1.



Note that at B only a single constraint force R_B acts perpendicular to the plane on which the rollers can move freely.

To determine R_B it is most profitable if the moment equation about point A is considered. The equilibrium equation yields

$$F(3\times2+2\times2+1\times2) - R_B \cos 45^\circ \times 2 = 0$$

$$R_B = \frac{6F}{1/\sqrt{2}} = 6\sqrt{2}\,F \quad \text{---(1)}$$

For $R_B \leq 3$ kN one gets $F \leq \frac{1000}{2\sqrt{2}}$ N $= 353.55$ (N).

8. It can be seen that when C weighs less than its maximum value all the three pipes remain in equilibrium. The FBD's of the three pipes can be drawn as shown in figure 8.1. The symmetry of the problem immediately suggests the following

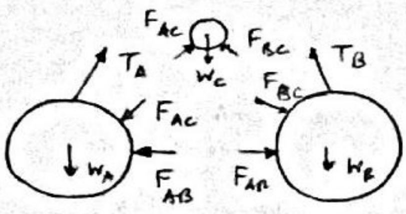


Fig. 8.1

$$T_A = T_B, \quad F_{AC} = F_{BC}$$

For a given value of W_C the unknown forces can be obtained by considering the FBD of any of the bigger tubes and that of the smaller tube. The four equations thus obtained (note that the forces are concurrent and we get two equations of equilibrium per a rigid body) are sufficient to evaluate the values of unknowns T_B, F_{BC} and F_{AB}. Note that one of the four equations are trivially satisfied if symmetry condition $F_{AC} = F_{BC}$ is imposed. In fact, this is the force balance equation in horizontal direction for the smaller tube.

The equilibrium equations are written in the following from

$$F_{AB} = T_B \sin\phi_1 - F_{BC} \sin\phi_2 \qquad (\Sigma F_x = 0 \text{ in } B) \quad \cdots (1)$$

$$W_B = T_B \cos\phi_1 - F_{BC} \cos\phi_2 \qquad (\Sigma F_y = 0 \text{ in } B) \quad \cdots (2)$$

$$W_C = 2 F_{BC} \cos\phi_2 \quad \cdots (3)$$

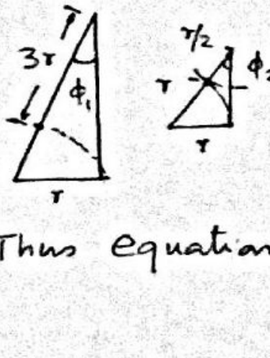
Eliminating T_B and F_{BC} from the above equations one gets

$$F_{AB} \cos\phi_1 - W_B \sin\phi_1 = \frac{W_C}{2\cos\phi_2}\left(\cos\phi_2 \sin\phi_1 - \cos\phi_1 \sin\phi_2\right) \quad \cdots (4)$$

where ϕ_1 and ϕ_2 are the angles made by the lines of action of T_B and F_{BC} with the vertical, respectively. It may be seen that the numerator of the right hand side of equation (4), being equal to $W_c \sin(\phi_1 - \phi_2)$ is always negative since $\phi_1 < \phi_2$. Thus, as the value of W_c is increased the force F_{AB} becomes less until it vanishes. If W_c is increased more than this critical value F_{AB} becomes negative according to equation (4). Since F_{AB} is the contact force that must always act inward on each ~~sph~~ cylinder its value can never become negative. Physically as F_{AB} becomes zero the equilibrium gets upset and the tubes separate allowing the smaller tube to fall. One can say that the system arrives a state of impending motion as $F_{AB} = 0$. The value of W_c required to attain this condition is obtained by putting $F_{AB} = 0$ in equation (4). This results

$$W_C = \frac{2\ W_B}{\left(\frac{\tan\phi_2}{\tan\phi_1} - 1\right)} \quad \cdots (5)$$

From the geometry it is seen (see figure 8·2) that

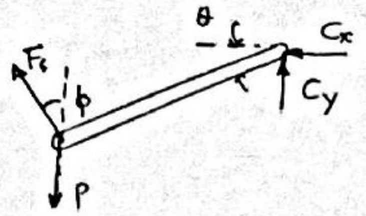


$$\tan\phi_1 = \frac{r}{(\sqrt{4^2-1})r} = \frac{1}{\sqrt{15}}$$

and $$\tan\phi_2 = \frac{r}{\left(\sqrt{\left(\frac{3}{2}\right)^2 - 1}\right)r} = \frac{2}{\sqrt{5}}.$$

Thus equation (5) yields $$W_c = \frac{2\ W_B}{2\sqrt{3} - 1} = 0.812\ W.$$

9. The free body diagram of the rod is shown in figure 9.1



Considering balance of moment about point C we get

$$M_c = 0 \Rightarrow \quad P l \cos\theta - (F_s \cos\phi)\, l \cos\theta - (F_s \sin\phi)\, l \sin\theta = 0 \quad \text{--- (1)}$$

where ϕ is the angle between the spring and the vertical line.

From equation (1) one gets

$$P = F_s \frac{\cos(\theta - \phi)}{\cos\theta} \quad \text{--- (2)}$$

This equation could have been obtained by considering the fact that the rod is a two force member. The resultant of F_s and P must be then along the rod. The components of F_s and P along the line perpendicular to the rod must cancel each other. One can verify that this process leads directly to equation (2).

To obtain F_s consider $F_s = k\Delta$, where Δ is the spring stretch which in this case is the length AB. It is easy to see from eq the triangle ΔABC that

$$AB = 2 l \sin\theta/2 \quad \text{--- (3)}$$

and

$$\phi = \left(\frac{\pi - \theta}{2}\right) - \left(\pi/2 - \theta\right) = \theta/2 \quad \text{--- (4)}$$

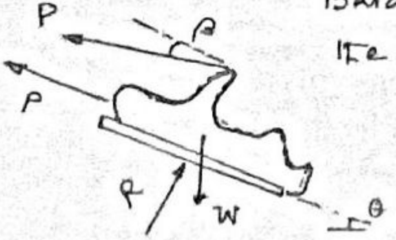
Thus from equation (2) we get

$$P/kl = \frac{2 \sin\theta/2 \cos\theta/2}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta,$$

ie $\quad \theta = \tan^{-1}(P/kl)$.

If $P = 2kl$ then $\theta = \tan^{-1}(2) = 63.44^\circ$.

10. The free body diagram of the cart along with the athletic is shown in figure 10.1



Balancing the total force along the incline gives the following result

$$P(1 + \cos\beta) = W \sin\theta$$

ie $\quad P = W \dfrac{\sin\theta}{(1 + \cos\beta)} = mg \dfrac{\sin\theta}{1 + \cos\beta}$

Considering $m = 70$, $g = 9.81\ m/s^2$, $\theta = 15^\circ$ and $\beta = 18^\circ$

We get $\quad P = 92.09\,(N)$.

Balancing the force along perpendicular direction we get

$$R = W\cos\theta + P\sin\beta$$
$$= mg\left[\cos\theta + \frac{\sin\theta\,\sin\beta}{1+\cos\beta}\right]$$
$$= 691.45\,(N)$$

It should be noted that we have solved the problem using only the force balance equations. The moment balance equation has been dis-regarded. With the given information this equation can not be used. Firstly, the line of action of R is taken to be arbitrary. In fact, the force exerted by the cart on the incline plane is distributed with unknown distribution. What we have written as R is the simplest resultant. Since the distribution is not known the line of action of R can not be found out. The line of action of the incline force P is also not known.

The moment balance equation is important when the rotational motion is considered. Since the cart and athletic do not show any sign of rotation the moment balance equation can be disregarded with-out having any great consequence.

11. The free body diagrams of the shaft and the hook wrench are shown in figure 11.1

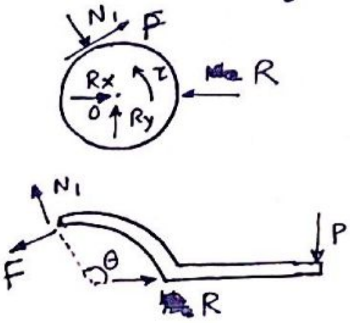


Fig 11.1

Note that in the free body diagram a torque τ is shown whose origin is not given in the problem. However, this is the resistance torque that must be overcome by the wrench with the help of force F.

The moment balance equation about point O leads to the following result

$$F = \tau / r \quad \cdots (1)$$

To find R in the easiest manner we may take the moment of all forces about pin B in the FBD of the wrench, although it involves a little bit of geometric manipulations. We thus get

$$P\left(l + r\cos(\pi-\theta)\right) = R\,r\sin(\pi-\theta) \quad \cdots (2)$$

Further taking moment about O in the same FBD we get

$$F r = P L \quad \cdots (3)$$

Combining equations ① - ③ we get

$$R = \frac{\tau}{r}\left(1 + \frac{r}{L}\cos(\pi-\theta)\right) / \sin\theta .$$

With the given data $\tau = 80$ Nm, $r = 0.1$ m, $L = 0.375$ m, $\theta = 120°$

one gets

$$R = 1046.9 \text{ (N)}$$

12. The free body diagram of the jig along with the pipe is shown in figure 12.1. The FBD of the gear at B is shown in figure 12.2

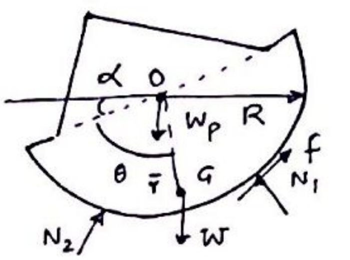


Figure 12.1

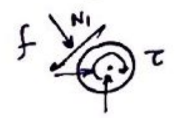


Figure 12.2.

From the moment balance equation applied to the gear one gets ...①

$$f = \tau / r_B .$$

The moment balance equation of the jig about point O leads to the following equation:

$$fR = W\bar{r}\cos(\pi - (\theta+\alpha)) \quad \text{...②}$$

From equation ① and ② we get

$$\tau\frac{R}{r_B} = W\bar{r}\cos(\pi - (\theta+\alpha)) . \quad \text{...③}$$

The following data are given

(i) $\alpha = 0$, $-\tau = 2460$ N-m,

(ii) $\alpha = 30°$, $\tau = +4680$ N-m, }

From equation ③ we get

$$-2460\left(\frac{5}{0.240}\right) = 80 \times 10^3 \times \bar{r}\cos(\pi-\theta) \times g \quad \text{...④}$$

and

$$4680\left(\frac{5}{0.240}\right) = 80 \times 10^3 \times r\cos(\pi - \pi/6 - \theta) \times g \quad \text{...⑤}$$

From equations ④ and ⑤ one gets

$$\frac{\cos\theta}{\cos\left(\frac{5\pi}{6}-\theta\right)} = \frac{2460}{4680} \Rightarrow \tan\theta = ~~79.76~~\ 1.392 \text{ or}$$

$$\theta = 79.76°.$$

and

$$\bar{r} = \frac{2460 \times 5/0.240}{80\times10^3 \times \cos 79.76°} \times \frac{1}{9.81} = 0.3674 \text{ m}$$

$$= 367.4 \text{ mm}$$

— x —