Week 8: Interference (continued)

Division of amplitude interferometry:

In Young's experiment we achieved interference effects by **dividing the waverfont**. Alternatively, one may consider **dividing the amplitude**. A simple way of dividing the amplitude is by considering the superposition of different reflected waves. For example, to begin, let us consider a parallel plate or a film of uniform thickness d (transparent plate/film) and refractive index n_f. The refractive index outside the plate is n_1. Light is incident obliquely (at an angle, say θ_i) on this film. It is reflected and refracted at the point A. The refracted ray travels inside the plate/film and gets reflected again. It is once again refracted into the medium n_1. We would like to see what happens if we superpose these two waves–the one directly reflected and the other refracted-reflected-refracted.

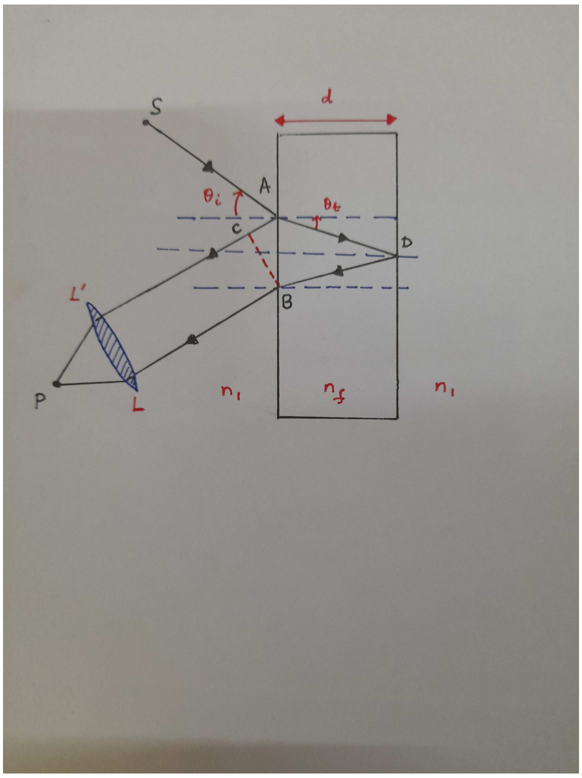

FIG. 1. Division of amplitude, parallel film.

It is easy to consider the effect of superposition by looking at the net **path/phase difference**. Look at Figure 9. What is the optical path difference between the paths SADBP and

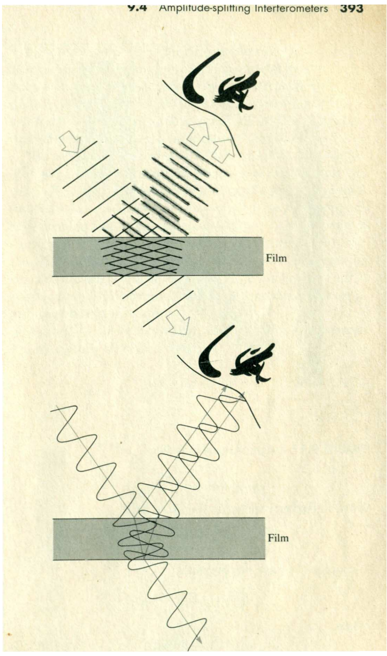

FIG. 2. Division of amplitude, parallel film, figure from Hecht, Optics.

SAP? We have

$$\Lambda = n_f\,(AD + DB) - n_1\,(AC) \tag{1}$$

We know that

$$AD = \frac{d}{\cos\theta_t} = AB \tag{2}$$

Therefore

$$\Lambda = \frac{2n_f d}{\cos\theta_t} - n_1\ AC \tag{3}$$

We also have $AC = AB\sin\theta_i$ and $AB = 2d\tan\theta_t$. Therefore,

$$\Lambda = \frac{2n_f d}{\cos\theta_t} - 2n_1 d\sin\theta_i\tan\theta_t \tag{4}$$

From Snell's law of refraction $n_1\sin\theta_i = n_f\sin\theta_t$. Using this, we get

$$\Lambda = 2n_f d\cos\theta_t \tag{5}$$

If the angle of incidence is such that it is less than θ_p (recall the polarisation, Brewster angle) and $n_1 < n_f$ (say, air and glass or air and water) then for both components of the incident and reflected fields (the in-plane of incidence and $\perp$ to plane of incidence), there will be a net phase difference of π between the two reflected waves meeting at P. Therefore, we have the net phase difference as:

$$\delta = \frac{4\pi d \cos\theta_t}{\lambda_f} - \pi \tag{6}$$

Exercise: Work out the path difference if $n_1 > n_f$ and $\theta_i < \theta_p$, $n_1 < n_f$ and $\theta_i > \theta_p$.

Since, this is a superposition of two waves, we will have the maxima and minima governed by $\delta = 2m\pi$ and $\delta = (2m+1)\pi$, respectively. Hence, we have

$$d\cos\theta_t = (2m+1)\,\frac{\lambda_f}{4} \quad (Maxima) \qquad d\cos\theta_t = (2m)\,\frac{\lambda_f}{4} \quad (Minima) \tag{7}$$

The interference fringes will be seen over a small region if the source is point source and the lens used to focus the rays has a small aperture. For an extended source, light will reach the lens from all directions and the maxima-minima will be spread out over a larger region of the plate/film. Figure 10 shows how the waves interfere to form the fringes.

What controls δ in this case? It is the value of θ_t or θ_i. Thus, the fringes appearing at P are for a specific θ_i. If we change θ_i the maximum or minimum appears at another point. This leads to the nomenclature –**fringes of equal inclination**. For an extended source, each point on it is a source which is incoherent w.r.t. any other point. Therefore, when we see the image of the extended source in reflected light, we find dark and bright bands. Each band is on the arc of a circle centred on the intersection of a perpendicular dropped from the eye to the film.

Note that we have considered the interference of only two waves whose amplitudes are obviously different due to reflection and transmission. In reality, of course the process of successive reflections and transmissions continue. We have made an assumption that the subsequent reflected and transmitted waves do not contribute because their amplitudes have become very small. This assumption does seem to work.

When we increase d, the quantity AB increases and one of the rays may not reach the eye. In such an event, we will not see any interference fringes. A larger lens has to be used. We can also reduce AB by reducing θ_i (nearly normal incidence). The fringes of equal inclination seen for nearly normal incidence and with thick plates are known as **Haidinger fringes**. Since θ_i is close to zero, we have circular symmetry and you see concentric circular fringes in the field of view.

Figures 11, 12 show the various cases for an extended source. Figure 13 illustrates Haidinger fringes.

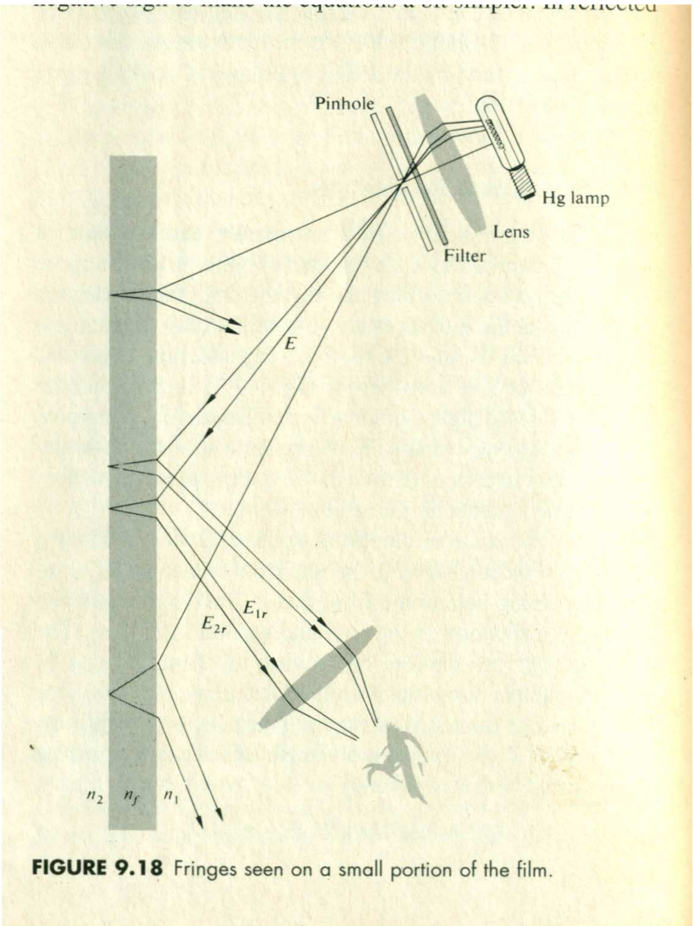

FIGURE 9.18 Fringes seen on a small portion of the film.

FIG. 3. Division of amplitude, parallel film, figure from Hecht, Optics.

Another type of fringes can be observed in reflected light. This involves the case of a **wedge shaped film** (see Figure 14). The fringes seen here are known as **fringes of equal thickness** since the dominant parameter here is the **optical thickness** $n_f d$. The colours of thin films, oil slicks upon illumination by white light are largely understood as fringes of equal thickness.

If we assume normal incidence (**Fizeau fringes**), then $\theta_t = \theta_i = 0$. If the wedge angle is α, then we know that the thickness at the horizontal location x (see Figure 14) is given by

$$d = x\alpha \tag{8}$$

Using the path/phase difference calculated earlier, we find that the condition for a maximum at a specific d_m (or x_m) is,

$$\left(m + \frac{1}{2}\right)\lambda_0 = 2n_f d_m = 2n_f \alpha x_m \tag{9}$$

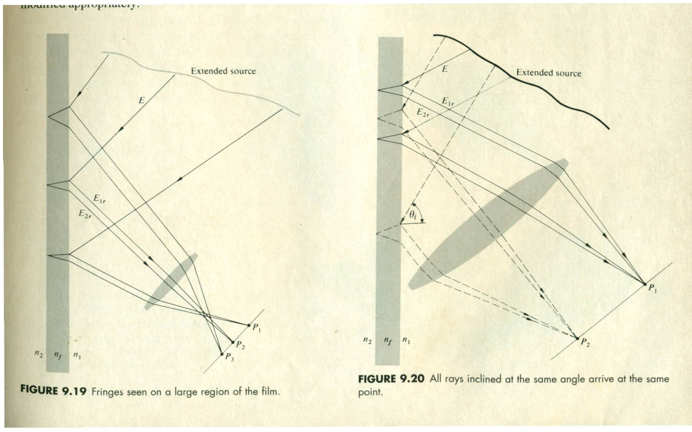

FIG. 4. Division of amplitude, parallel film, figure from Hecht, optics.

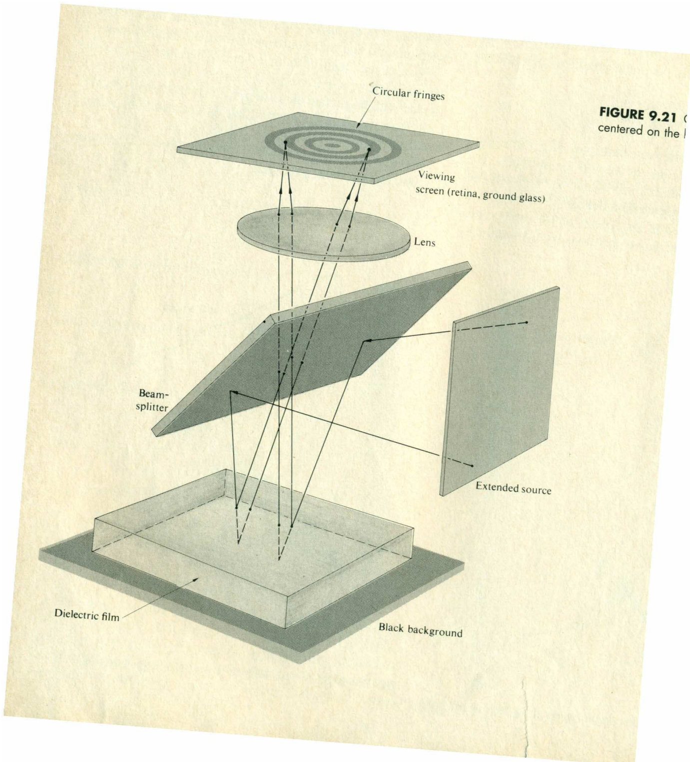

FIG. 5. Haidinger fringes, figure from Hecht, Optics

We can rewrite this as

$$x_m = \frac{m + \frac{1}{2}}{2\alpha} \lambda_f \tag{10}$$

Therefore, as we move along the wedge, after every $\frac{\lambda_f}{2\alpha}$ we will encounter a maximum. Hence

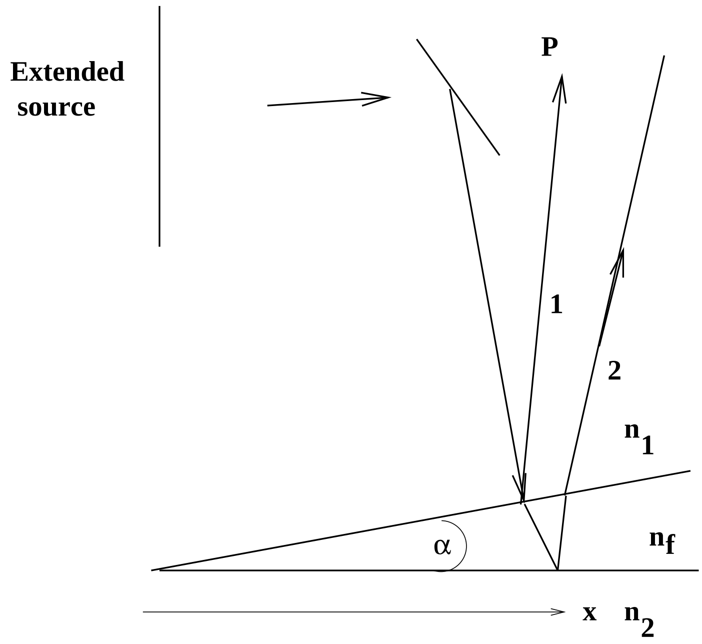

FIG. 6. Wedge shaped film

the separation between the maxima will be

$$\Delta x = \frac{\lambda_f}{2\alpha} \tag{11}$$

The thickness of the film at the maxima will be

$$d_m = \left(m + \frac{1}{2}\right)\frac{\lambda_f}{2} \tag{12}$$

which is an odd mutiple of the quarter of a wavelength (take $m = 0, 1, 2....$).
If you hold a soap film in a frame vertically and illuminate it with white light–you notice that there are various colours but the top portion turns out to look black. This is because the thickness of the top part has become less than $\frac{\lambda_f}{4}$ (due to drainage caused by gravity) and, therefore, there is no scope of having an intensity maximum there.
The most well known application of the fringes of equal thickness is the case of Newton's rings that you see in your laboratory class. Figure 15 shows the set-up. If R is the radius of curvature of the lens and d, x as shown in Figure 15, then,

$$x^2 = R^2 - (R-d)^2 \approx 2Rd \tag{13}$$

since R is much greater than d. If we, as before, look at only the first two reflected rays (see Figure 15), then the mth order interference maxima occur when the thickness follows

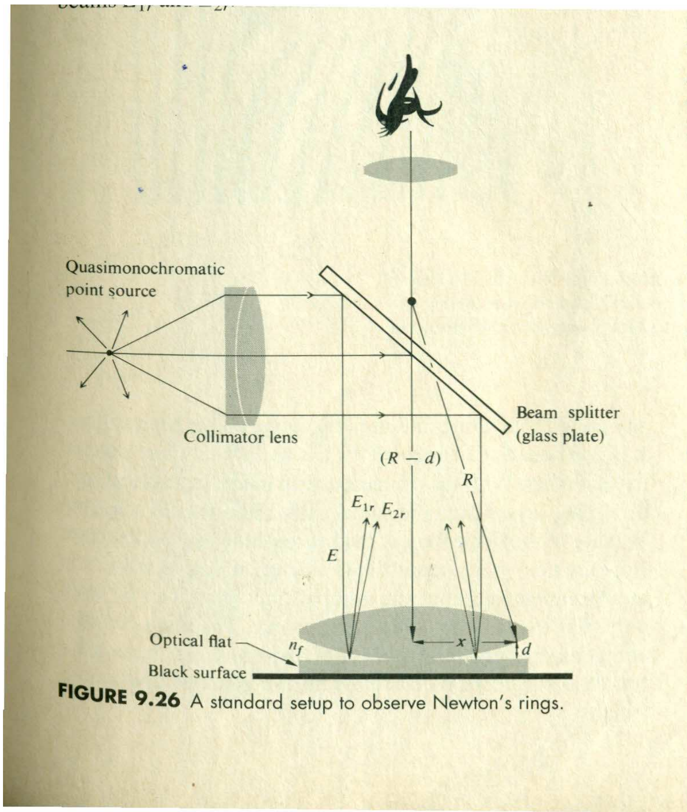

FIG. 7. Newton's rings, figure from Hecht, Optics

the relation

$$2n_f d_m = \left(m + \frac{1}{2}\right)\lambda_0 \tag{14}$$

The radius of the mth bright ring is therefore found as

$$x_m = \left[\left(m + \frac{1}{2}\right)\lambda_f R\right]^{\frac{1}{2}} \tag{15}$$

Similarly, for a dark ring, we have

$$x_m = (m\lambda_f R)^{\frac{1}{2}} \tag{16}$$

Hence, the central fringe $m = 0$ will clearly turn out to be dark. It will be a maximum (bright) if we observe in transmitted light. Newton's rings are another example of **Fizeau fringes**. An application of Newton's rings, which you do in your lab, involves experimentally determining the **radius of curvature** of the lens from the radii of the rings, provided the wavelength is given. In the same way, if the radius of curvature is known, one can find the wavelength. If the region between the lens and the lower plate is filled with a liquid of a certain refractive index, then one can find the refractive index by determining the radii of the bright and dark rings.

We will now discuss two very well-known devices–the Michelson interferometer and the Fabry-Perot interferometer in some detail. Both these devices have numerous applications in science and engineering.

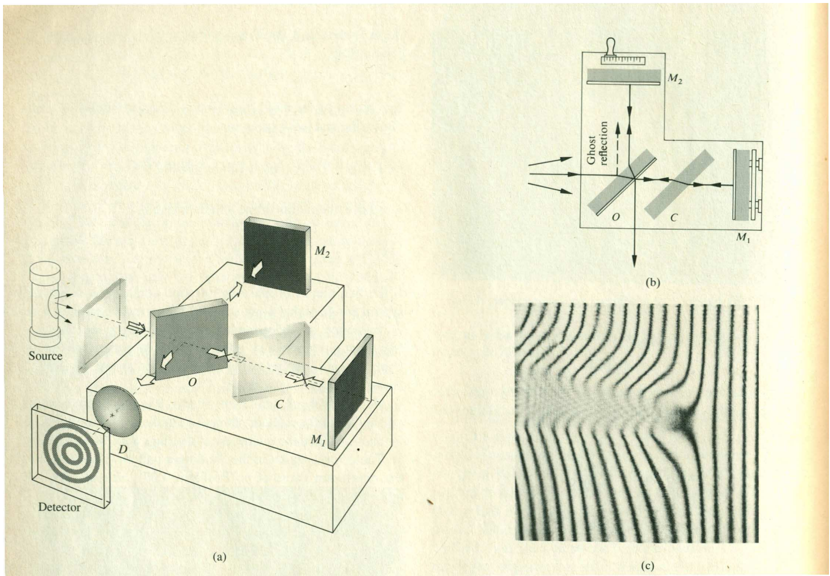

FIG. 8. Michelson interferometer, figure from Hecht, Optics

Michelson interferometer: Let us begin with the Michelson interferometer. Figure 1 (figure on left and right-top) shows the set-up. M_1 and M_2 are two mirrors, O is a beam splitter and C is the compensator plate. One of the beams (the one reflected by mirror M_1) travels three times through the beam splitter), whereas the other beam travels once. In order to **compensate** this difference the **compensator** is used in the path of beam 1 (as shown). The compensator is identical in optical properties to the beam splitter, thereby ensuring that both beams travel the same total optical paths as long as the arm lengths of the interferometer are equal. Effects of dispersion are also taken care off by the compensator since it is designed to work for a quasi-monochromatic source.

Let us assume that we use an extended source. To understand fringe formation, we construct an optically equivalent **linear** diagram where all optical elements (or their images) are brought along the line joining mirror M_2 and the detector (see Figure 2). In this diagram, M_1' is the image of M_1 in the beam splitter, Σ represents the image of the extended source

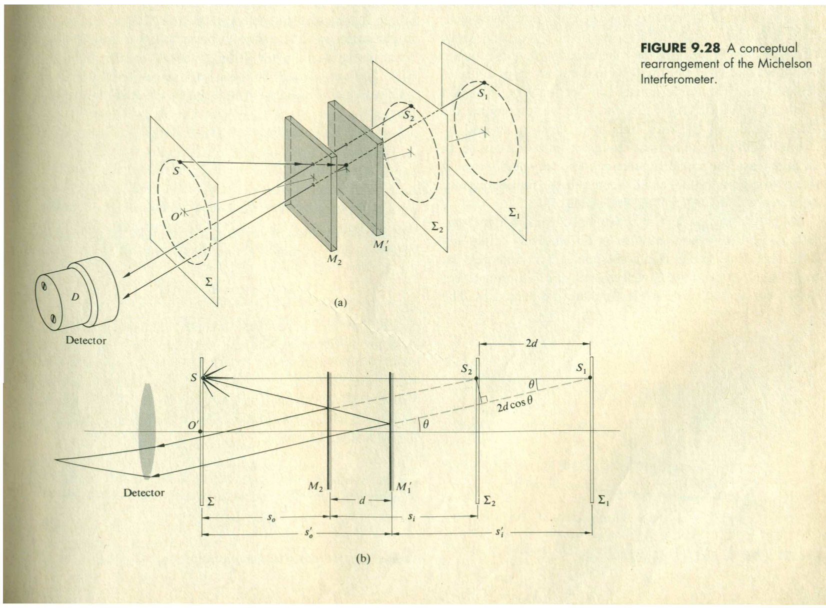

FIG. 9. Linear diagram, figure from Hecht, Optics

as seen on the beam splitter. The distance between M_2 and M_1' is assumed to be d–this being the difference between the arm-lengths which introduces a path difference between the traversals of the two beams. In the Figure 2, the images of the source point S on Σ, as seen in M_1' and M_2 are S_1 (on Σ_1) and S_2(on Σ_2), respectively. The parallel rays are collected via a wide lens in a detector (which could just be the human eye). The path difference between the two rays reflected off mirror M_1 and M_2 is simply $2d\cos\theta_m$ where θ_m is as shown in the Figure 2. Note that because the two reflections occur from rare to dense and dense to rare, there is an extra phase of π. Hence, the condition for a **minimum** is

$$2d\cos\theta_m = m\lambda \tag{17}$$

This gives, by symmetry, circular dark rings as fringes. The fringes are virtual and localised at infinity. One will need a lens to focus the parallel rays into the eye because because the aperture of the eye's lens is small. The fringes seen are the familiar Haidinger fringes mentioned earlier.

Exercise: Find the condition for a bright spot at the centre.

When $\theta_m = 0$ we have

$$2d = m_0\lambda \tag{18}$$

which can be re-written as,

$$\frac{d}{\frac{\lambda}{2}} = m_0 \tag{19}$$

Thus, the highest order m_0 is for the central minimum ($\theta_m = 0$) and this will occur as long as d is an integral multiple of $\frac{\lambda}{2}$. For subsequent orders, with the same d, we have

$$2d\cos\theta_1 = (m_0 - 1)\,\lambda \tag{20}$$

$$2d\cos\theta_2 = (m_0 - 2)\,\lambda \tag{21}$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$2d\cos\theta_p = (m_0 - p)\,\lambda \tag{22}$$

Thus, we obtain

$$2d\,(1 - \cos\theta_p) = p\lambda \tag{23}$$

For small θ_p one can expand the cosine upto it quadratic term and therefore one gets

$$\theta_p = \sqrt{\frac{p\lambda}{d}} \tag{24}$$

θ_p is the angular position of the pth dark ring.

Exercise: Find the condition for a bright spot at the centre and the general condition for brught rings.

When we have inclined mirrors we see straight fringes (as in a wedge) which are nothing but the well-known Fizeau fringes. These fringes are real and localised.

Figures 3-6 (taken from a course at Univ. of Colorado) show the details of fringe formation in a Michelson interferometer. The figures are self-explanatory.

We now turn to the study of temporal coherence using the Michelson interferometer. The path difference between the two beams in the interferometer can be interpreted as a **time delay** between the respective electric fields. It is this time delay which produces the fringes. If the source is ideally monochromatic, then we could write the electric field before it reaches the beam splitter, as

$$E(t) = Ae^{i\omega t} \tag{25}$$

Here we are ignoring the spatial dependence of the electric field for the time being. The total electric field after the beam has been split and has travelled along the respective paths, with one of them having a time-delay w.r.t. the other, is

$$E(t) = E_1(t) + E_2(t+\tau) = \frac{A}{2}e^{i\omega t} + \frac{A}{2}e^{i\omega(t+\tau)} \tag{26}$$

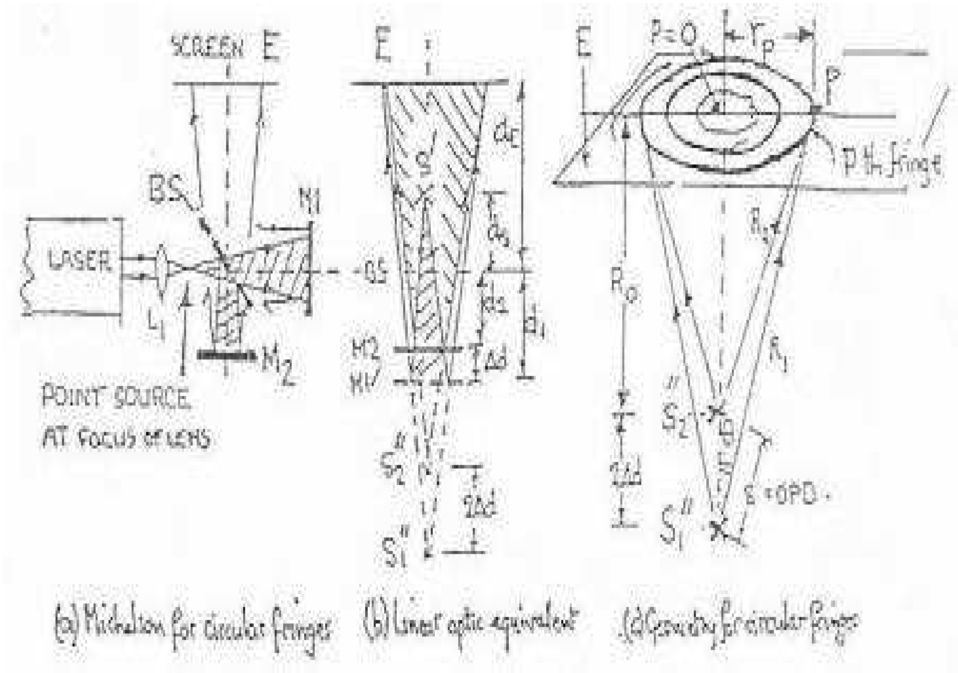

FIG. 10. Circular fringes

where the time delay is denoted as τ and the minus sign is introduced to take care of the phase difference due to reflection. On calculating the total intensity one recovers the standard result

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2}\cos\omega\tau = 2I_0\left(1 - \cos\omega\tau\right) \tag{27}$$

The question of temporal coherence is related to a finite band-width of the source. Let us first see what may happen if we have two frequencies $\omega_{1,2} = \omega \pm \frac{\Delta\omega}{2}$. The total electric field (before the beam splitter) would then become

$$E(t) = A(t)e^{i\omega t} \tag{28}$$

where now $A(t)$ is a function of t (not a constant) and is reminiscent of **amplitude modulation**. If we have a spread of frequencies, then we will have the same form but $A(t)$ will be functionally different. This total electric field, on entering the beam splitter, will traverse the two paths, have a time delay and finally enter the detector. Thus, we will now have, for the intensity,

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2}\; C_{12}(\tau)\; cos\omega\tau \tag{29}$$

The new quantity introduced here is similar to the one we had when we studied spatial

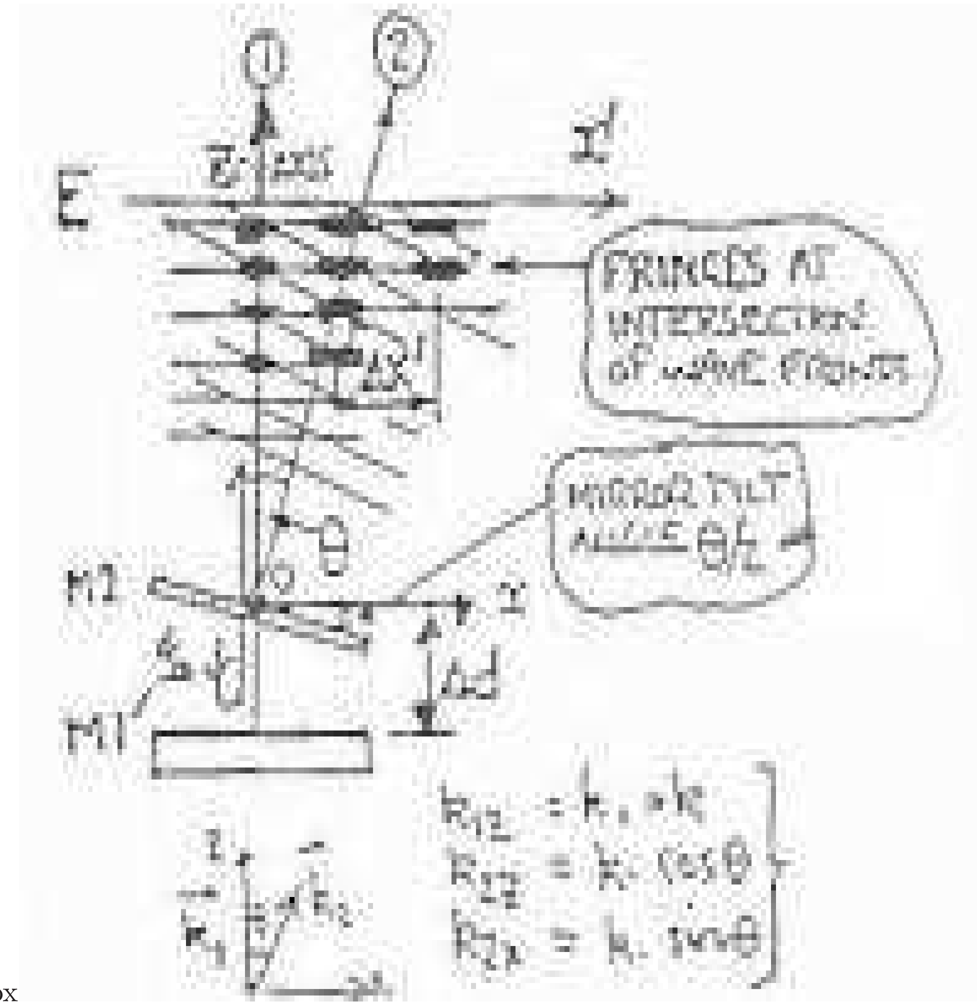

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FIG. 11. Straight fringes, tilted mirrors

coherence. As before C_{12} will be related to the visibility and thus, measurements of visibility will be able to tell us about the nature of coherence.

If $C_{12} = 0$, then there is no interference, i.e. there is no coherence. If $0 \leq C_{12} \leq 1$ then we have partial coherence and finally when $C_{12} = 1$ we have perfect temporal coherence. The related time-scale $\tau_c = \frac{2\pi}{\Delta\omega}$ or, equivalently, the length scale $l_c = c\tau_c$, are know as the coherence time and the coherence length (temporal), respectively.

The Michelson interferometer can be used to measure various quantities such as the wavelength of light, very small wavelength differences, thickness and refractive indices of films of various types etc. One of its most famous uses is in the well-known Michelson–Morley experiment which was done in the later part of the 19th century to show that the speed of light is not direction dependent and there is no such thing as an **all-pervading medium**

named ether. More recently, the Michelson interferometer is being used in a project called LIGO (Laser Interferometric Graviational-wave Observatory) with the purpose of detecting a prediction of Einstein's theory of gravity (General Relativity), namely the gravitational wave. Such a wave when it falls on one arm of the interferometer creates a disturbance and a fringe-shift. The shift is very very small and therefore the experiments have to take care of noise in an extremely precise way. This is an ongoing experiment which is yet to deliver results.

Exercise: If N fringes collapse at the centre when d changes by an additional Δd, then show that the wavelength of the source can be found from the formula $\lambda = \frac{2\Delta d}{N}$.

Exercise: If λ_1 and $\lambda_2 = \lambda_1 + \Delta\lambda$ are two wavelengths with a small difference $\Delta\lambda$ then explain how $\Delta\lambda$ $(= \frac{\lambda_1^2}{2\Delta d})$ can be found using the Michelson interfereometer.

Not included in syllabus for this semester

Multiple beam interference: Till now we have been exclusively concerned with two beams and their interference. There are situations where the effects of more than two reflected beams (or transmitted beams) can be significant. We shall take up this study now. We shall confine ourselves to **scalar amplitudes** for the electric fields of the incident, reflected and refracted waves. The figure shows a very simple arangement, the familiar transparent dielectric slab of thickness d. Light is incident at a certain angle and then reflected and refracted. The process generates multiple reflected and transmitted rays which are then collected through a lens. What are the transmitted and reflected intensities in terms of the incident intensity and the phase difference?

Let us assume that the reflection and transmission coefficients are as follows. To keep things simple, assume air and glass. For reflection off a air-glass interface let r denote the reflection coefficient. For glass-air let is be r'. Similarly for air-glass transmission the coefficient is t and for glass-air it is t'. Between these quantities, there are certain relations which are known as the **Stokes' relations**. To prove them look at the figure. We have an incident ray of amplitude a. The reflected and transmitted rays have amplitudes ar and at. We now reverse the direction of the reflected and refracted rays. This gives a reflected ray of amplitude ar^2 and a transmitted ray of amplitude art. Reversing the transmitted ray gives a reflected ray of amplitude atr' and a refracted ray of amplitude att'. Adding the amplitudes

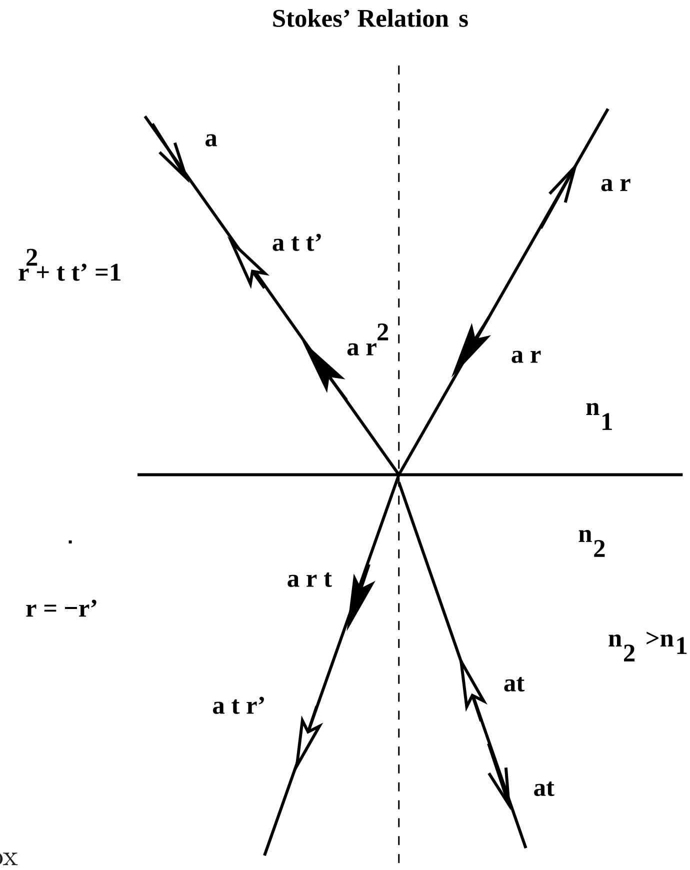

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FIG. 12. The Stokes’ relations

(with appropriate signs indicating directions) along the refracted and reflected rays give:

$$r^2 + tt' = 1 \quad ; \quad r = -r' \tag{30}$$

These are the Stokes’ relations. Note the minus sign which speaks of the **phase change of** π discussed earlier.

We now go back to the amplitudes of the reflected rays. It is easy to check that

$$E_{1r} = E_0 r e^{i\omega t} \tag{31}$$

$$E_{2r} = E_0 t r' t' e^{i(\omega t - \delta)} \tag{32}$$

$$E_{3r} = E_0 t r'^3 t' e^{i(\omega t - 2\delta)} \tag{33}$$

....

$$E_{Nr} = E_0 t r'^{2N-3} e^{i(\omega t - (N-1)\delta)} \tag{34}$$

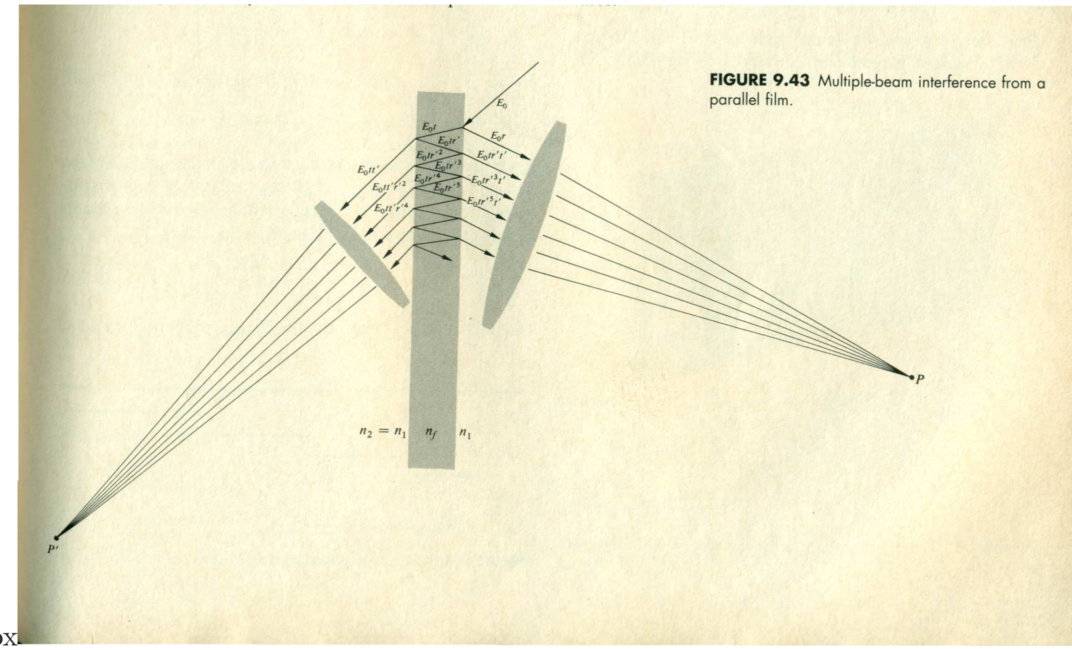
mbox

FIG. 13. Multiple beam interference, figure from Hecht, Optics

If we add the amplitudes and make $N \to \infty$ we get

$$E_r = E_o e^{i\omega t}\left[r - \frac{rtt'}{1 - r'^2 e^{-i\delta}}\right] \tag{35}$$

At this point, we make use of the Stokes' relations. We find that

$$E_r = E_0 e^{i\omega t}\frac{r\left(1 - e^{-i\delta}\right)}{1 - r^2 e^{-i\delta}} \tag{36}$$

The total intensity with be $\frac{1}{2}E_r E_r^*$. This turns out to be:

$$I_r = I_i \frac{2r^2(1 - \cos\delta)}{1 + r^4 - 2r^2\cos\delta} \tag{37}$$

This can be further simplified by using the fact that $1 - \cos\delta = 2\sin^2\frac{\delta}{2}$ and defining

$$F = \left(\frac{2r}{1 - r^2}\right)^2 \tag{38}$$

which gives

$$I_r = I_i \frac{F\sin^2\frac{\delta}{2}}{1 + F\sin^2\frac{\delta}{2}} \tag{39}$$

In a similar fashion, one can find out what will be the transmitted intensity. We have

$$E_{1t} = E_0 tt' e^{i\omega t} \tag{40}$$

$$E_{2t} = E_0 t r'^2 t' e^{i(\omega t - \delta)} \tag{41}$$

$$E_{3t} = E_0 t r'^4 t' e^{i(\omega t - 2\delta)} \tag{42}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$E_{Nt} = E_0 t r'^{2(N-1)} e^{i(\omega t - (N-1)\delta)} \tag{43}$$

As before we add the electric fielda and then take $N \to \infty$. This gives, for the total electric field as

$$E_t = E_0 t t' e^{i\omega t} \frac{1}{1 - r'^2 e^{-i\delta}} \tag{44}$$

After some sraightforward algebra, the transmitted intensity turns out to be

$$I_t = I_i \frac{1}{1 + F \sin^2 \frac{\delta}{2}} \tag{45}$$

The quantity on the R. H. S. is known as the **Airy function**. It is easy to note that $I_r + I_t = I_i$ The maxima of the reflected intensity occur at $\delta = \frac{4\pi n_f d \cos\theta_t}{\lambda} = \delta = (2m+1)\pi$. The maximum value reaches 1 only in the limit $r \to 1$. On the other hand, the maximum value for the transmitted intensity is 1 and this occurs at $\delta = 2m\pi$.

The sharpness of the transmitted intensity depends on how large F is. F is known as the coefficient of finesse. For large F the fringes (circular) in transmitted light would look like bright rings in a dark background. In reflected light, it would look complimentary, i.e. dark rings in a otherwise bright background. The Fabry-Perot interferometer was developed on the basis of the above-stated theory of multiple beam interference.

Let us now evaluate the FWHM for the transmitted intensity. This is obtained from the equation

$$\frac{1}{2} = \frac{1}{1 + F \sin^2 \frac{\delta}{2}} \tag{46}$$

If the intensity drops to half its value for $\delta = \delta_{max} \pm \delta_{\frac{1}{2}}$ then from the above equation we get

$$\delta_{\frac{1}{2}} = 2 \sin^{-1} \frac{1}{\sqrt{F}} \tag{47}$$

For large values of F we can write this as

$$\delta_{\frac{1}{2}} = \frac{2}{\sqrt{F}} \tag{48}$$

The FWHM is therefore given as

$$\gamma = 2\delta_{\frac{1}{2}} = \frac{4}{\sqrt{F}} \tag{49}$$

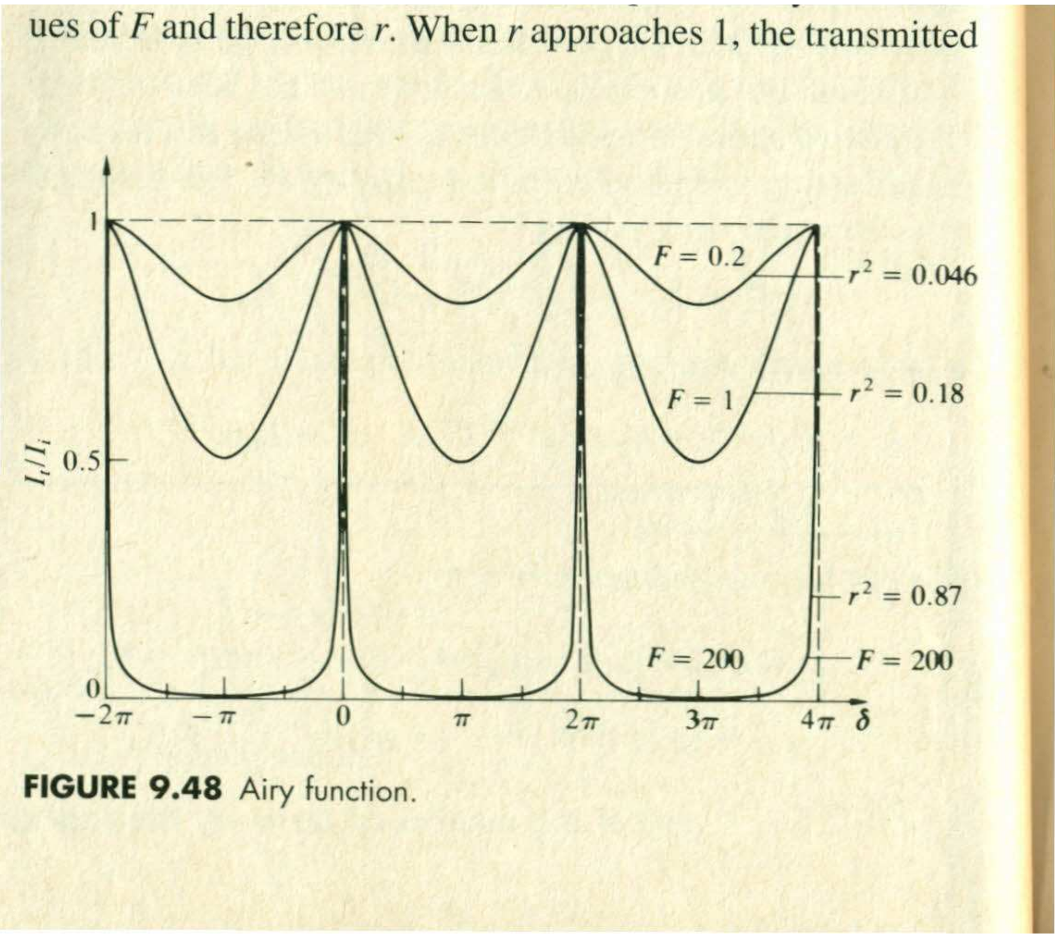
mbox

FIG. 14. Transmitted intensity, figure from Hecht, Optics

Since $F = \left(\frac{2r}{1-r^2}\right)^2$, the closer r is to 1, the larger the F is and hence the γ becomes smaller. A smaller γ implies a sharper intensity distribution and obviously sharp fringes. The quantity

$$\mathcal{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{F}}{2} \tag{50}$$

is called the **finesse**. It is particularly important to characterise a Fabry-Perot interferometer, for which, usually, the finesse is around 30.

The figure shows the Fabry-Perot interferometer with its various components. Conventionally, when the mirrors are not held fixed we call it an interferometer. When the mirrors are held fixed and can only be adjusted for parallelism, we call it a **Fabry Perot etalon**. The fringes can be seen by looking into the etalon and focusing at infinity. One can get real, non-localised fringes using a point source. For a broad source one gets the usual circular fringes.

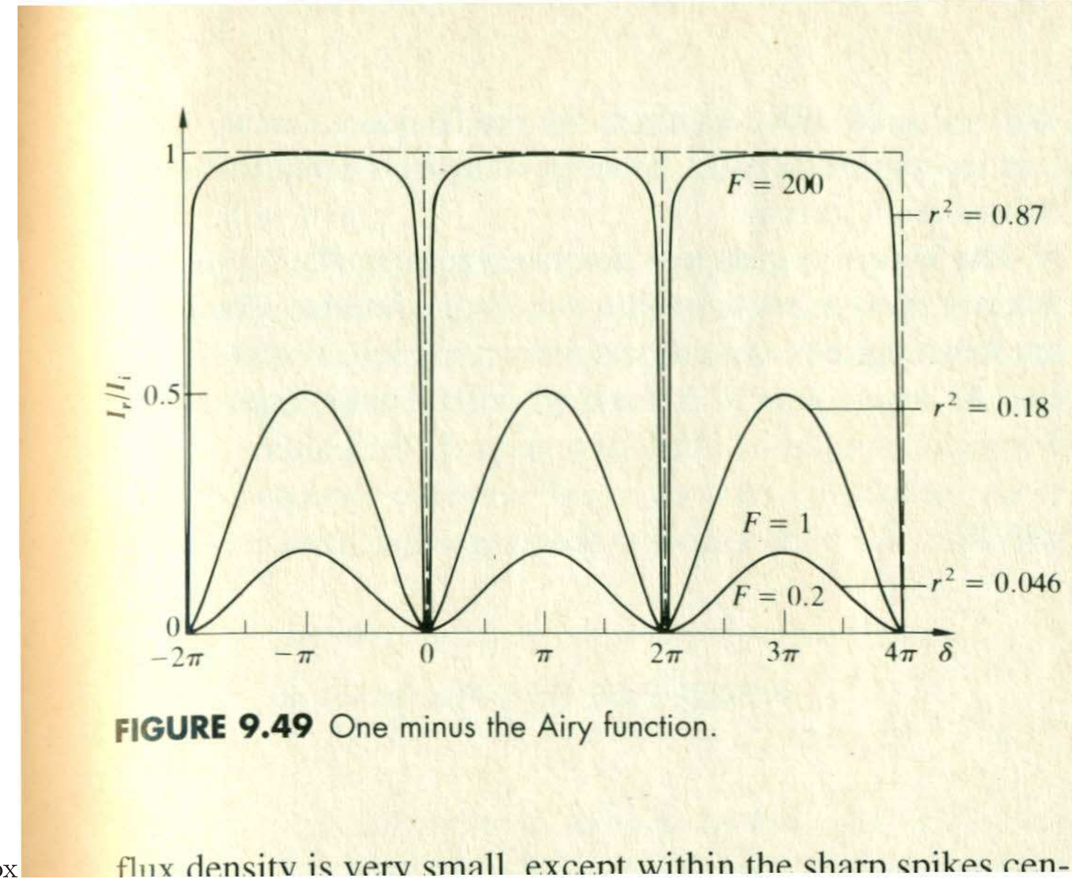
mbox

FIG. 15. Reflected intensity, figure from Hecht, Optics

How does a Fabry-Perot interferometer resolve small wavelength differences? If we have two diferent wavelengths, each will produce its own set of fringes. Obviously, these will overlap. How does one decide whether the resultant fringes are capable of resolving the two wavelengths? Let the individual intensities (transmitted) of the two wavelengths be given as

$$I_1(\delta_1) = \frac{1}{1 + F\sin^2\frac{\delta_1}{2}} \quad ; \quad I_2(\delta_2) = \frac{1}{1 + F\sin^2\frac{\delta_2}{2}} \tag{51}$$

where $\delta_1 = \frac{4\pi n_f d\cos\theta_t}{\lambda_1}$ and $\delta_2 = \frac{4\pi n_f d\cos\theta_t}{\lambda_2}$ with $\lambda_{1,2} = \lambda \pm \frac{1}{2}\Delta\lambda$. Therefore $\delta_2 > \delta_1$ and we may assume $\delta_2 - \delta_1 = \epsilon$. We could also write $\delta_1 = \delta - \frac{\epsilon}{2}$ and $\delta_2 = \delta + \frac{\epsilon}{2}$. The total intensity due to these two wavelengths is

$$I_t^{tot} = I_1 + I_2 = \frac{I_i}{1 + F\sin^2\frac{\delta_1}{2}} + \frac{I_i}{1 + F\sin^2\frac{\delta_2}{2}} \tag{52}$$

We will now plot I_t^{tot} as a function of δ. At a maximum of the total intensity ($\delta_1 = 2m\pi$

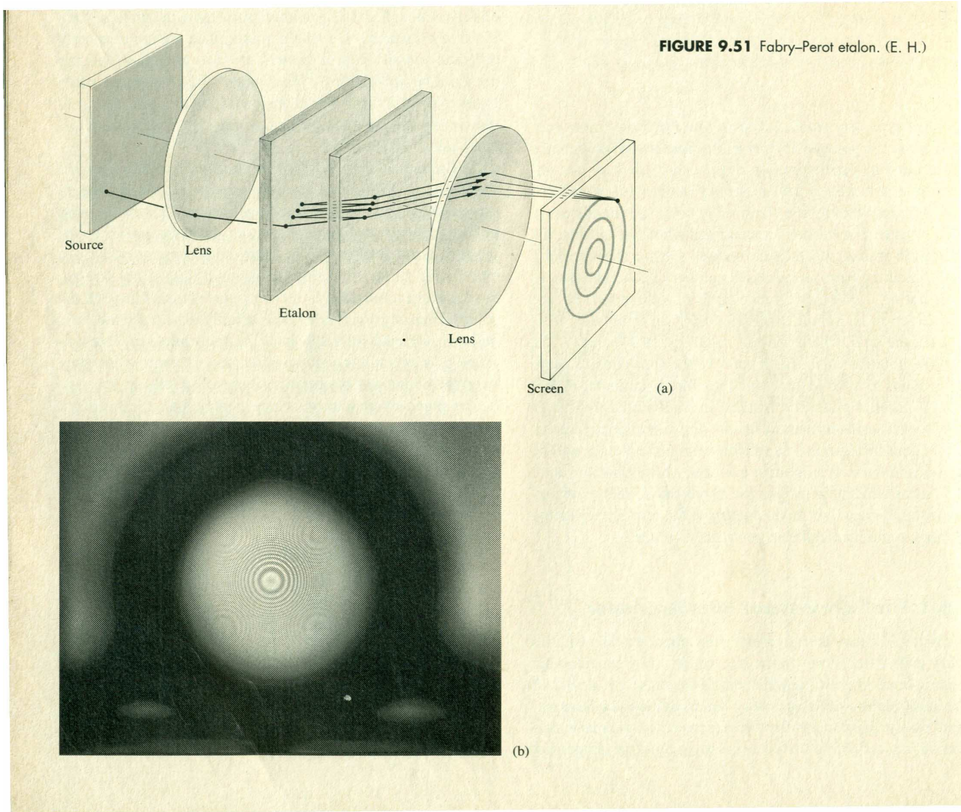


FIG. 16. Fabry-Perot interferometer, figure from Hecht, Optics

and $\delta_2 = 2m\pi + \epsilon$ and $\delta = 2m\pi + \frac{\epsilon}{2}$), the value of the intensity will be

$$I_t^{tot,max} = I_i + \frac{I_i}{1 + F\sin^2\frac{\epsilon}{2}} \tag{53}$$

The value of the intensity at the mid-point (saddle) between the two maxima of the total intensity is obtained using $\delta_1 = 2m\pi - \frac{\epsilon}{2}$, $\delta_2 = 2m\pi + \frac{\epsilon}{2}$ (and $\delta = 2m\pi$)

$$I_t^{saddle} = \frac{2I_i}{1 + F\sin^2\frac{\epsilon}{4}} \tag{54}$$

At the saddle point, what should be intensity in order to resolve the two peaks? Among various criteria, the oldest and most popular is the **Rayleigh criterion** which states that for resolution, the intensity at the saddle point must be $\left(\frac{8}{\pi^2}\right)(I_t)_{max}$. Using the Rayleigh criterion, we get the following relation ($\frac{8}{\pi^2} = 0.81$),

$$\frac{2I_i}{1 + F\sin^2\frac{\epsilon}{4}} = 0.81\left(I_i + \frac{I_i}{1 + F\sin^2\frac{\epsilon}{2}}\right) \tag{55}$$

which eventually gives, assuming ϵ small and $\sin\epsilon = \epsilon$

$$F^2\epsilon^4 - 15.5F\epsilon^2 - 30 = 0 \tag{56}$$

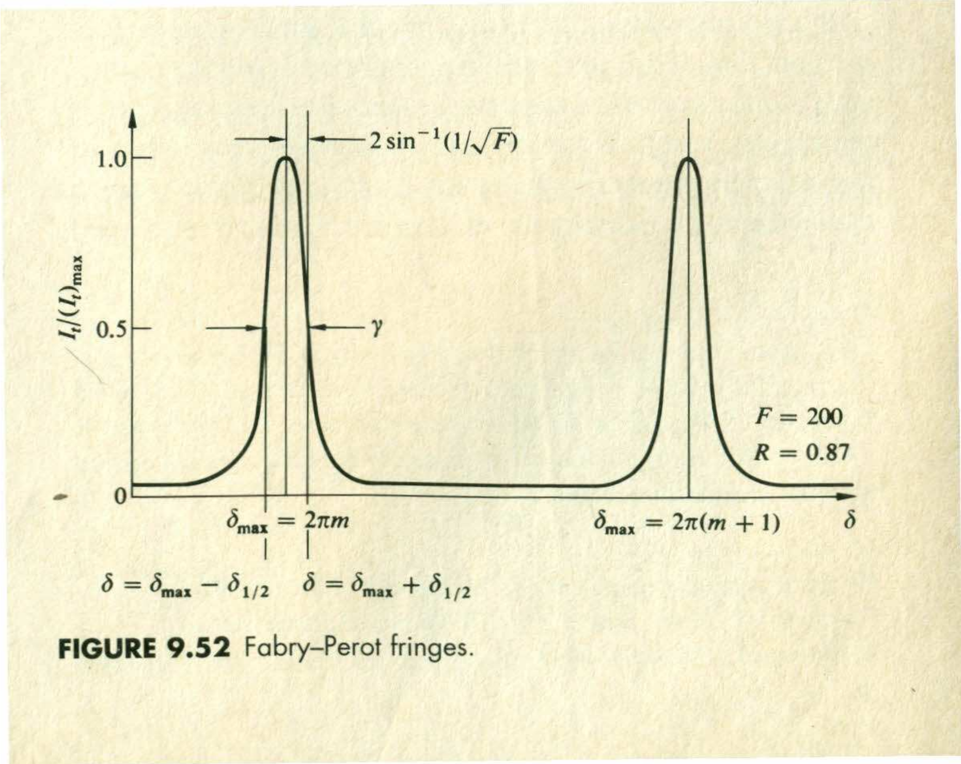


FIG. 17. FWHM, figure from Hecht, Optics

Exercise: Show that the above equation follows under the approximation of small ϵ.

The positive real solution of this equation is

$$\epsilon = \frac{4.15}{\sqrt{F}} \tag{57}$$

Now, since

$$\delta = \frac{4\pi n_f d \cos\theta_t}{\lambda} \tag{58}$$

we have

$$|\Delta\delta| = \frac{4\pi n_f d \cos\theta_t}{\lambda^2}\Delta\lambda = 2\pi m \frac{\Delta\lambda}{\lambda} \tag{59}$$

Hence, we have

$$\frac{\Delta\lambda}{\lambda} = \frac{|\Delta\delta|}{2\pi m} \tag{60}$$

Thus, the **chromatic resolving power** of the Fabry Perot interferometer is given as

$$\mathcal{R} = \frac{2\pi m}{|\Delta\delta|} \tag{61}$$

Using $|\Delta\delta| = \epsilon$ gives

$$\mathcal{R} = \frac{2\pi m}{\epsilon} = 0.97m\mathcal{F} \tag{62}$$

Assuming near-normal incidence, one can write the resolving power as

$$\mathcal{R} = \frac{2n_f d\mathcal{F}}{\lambda} \tag{63}$$

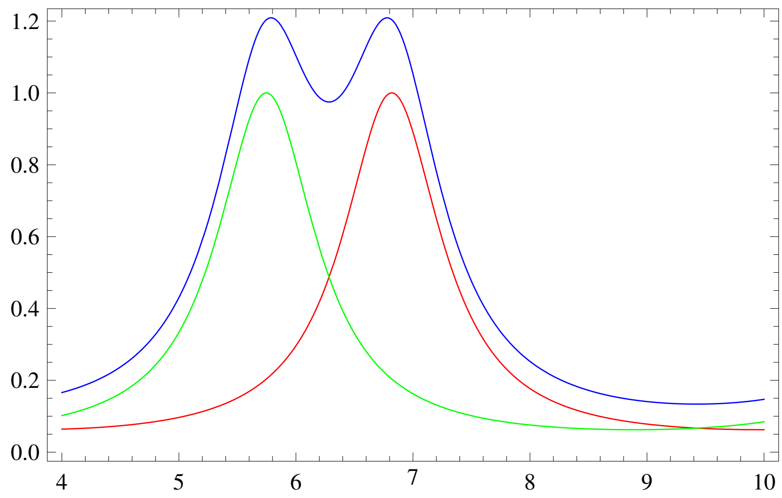


FIG. 18. Resolving two wavelengths: $F = 15$, $\frac{\epsilon}{2} = 0.536$. The red and green curves are the I_t for two different wavelengths, the blue curve is the sum at the limit of resolution.