

Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 12
Autumn 2022

1. Evaluate the integrals over the region D .

- (i) $\iint_D xy dA$, where D is region bounded by x -axis, ordinate $x = 2a$ and curve $x^2 = 4ay$,
- (ii) $\iint_D e^{\frac{x}{y}} dA$, where $D = \{(x, y) | 1 \leq y \leq 2, y \leq x \leq y^3\}$.
- (iii) $\iint_D (4xy - y^3) dA$, where D is region bounded by $y = \sqrt{x}$ and $y = x^3$,
- (iv) $\iint_D (6x^2 - 40y) dA$, where D is the triangle with vertices $(0, 3)$, $(1, 1)$ and $(5, 3)$,
- (v) $\iint_D (x^2 + 2xy^2 + 2) dA$, where D is region bounded by $y = x - x^2$, $y = 0$, $x = 0$ and $x = 2$.

2. Evaluate the following integrals by changing the order of integration

- (i) $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$,
- (ii) $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx$,
- (iii) $\int_0^3 \int_{x^2}^9 x^3 e^{y^3} dy dx$,
- (iv) $\int_0^8 \int_{3\sqrt{y}}^2 \sqrt{x^4 + 1} dx dy$.

3. Evaluate $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dy dx$, using the transformation $x + y = u$ and $y = uv$.

4. Consider the transform T from the xy -plane to the uv -plane given by

$$T : x = \frac{1}{2}(u + v), y = \frac{1}{2}(u - v).$$

(i) Calculate the Jacobian of the transform T .

(ii) Evaluate $\iint_D (x - y) \cos^2(x + y) dA$ using transformation T , where D is the square in xy -plane with vertices $(0, 0)$, (π, π) , $(0, 2\pi)$ and $(-\pi, \pi)$.

5. Evaluate the integral by making an appropriate change of variables

- (i) $\iint_D x^2 dx dy$, D is elliptic region $\{(x, y) : \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$.
- (ii) $\iint_D y^2 dx dy$, D is region bounded by curves $xy = 1$, $xy = 2$, $xy^2 = 1$ and $xy^2 = 2$.
- (iii) $\iint_D (x + y)^2 dx dy$, D is parallelogram bounded by the lines $x + y = 0$, $x + y =$

1, $2x - y = 0$ and $2x - y = 3$.

6. Find the area lying between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.
7. Find the volume of the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$, using double integral.
8. Find the area of the paraboloid $2z = \frac{x^2}{a} + \frac{y^2}{b}$ inside the cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
9. Calculate the area of the region bounded by the upper half of the circle $x^2 + y^2 = 25$, the x-axis and the ordinates $x = -3$ and $x = 4$.
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