Solutions for Tutorial #1
Example -1

Natural frequency of a SHM is $1/\pi\ s^{-1}$
At t=0, displacement from equilibrium position is 0.3m & velocity 0.7 ms^{-1}
We have to find phase (ϕ) & Amplitude (A)

$$x = Ae^{i(\omega_0 t+\phi)} - (1)$$

We can write eqn (1) as $$x = A[cos(\omega_0 t + \phi) + isin(\omega_0 t + \phi)]$$

At t=0, x is 0.3 m $$x = A[cos\phi + isin\phi] - (2)$$

We can take imaginary part as zero $$0.3 = Acos\phi - (3)$$

Velocity $$v = \frac{dx}{dt} = A\omega_0[icos\phi - sin\phi] = 0.7(at\ t = 0)$$

Again imaginary part is zero $$0.7 = -A\omega_0 sin\phi - (4)$$

$$Now, \frac{(4)}{(3)},\ we\ get\ tan\phi = -\frac{0.7}{0.3}\frac{1}{\omega_0} - (5)$$

We know, $$\omega_0 = 2\pi f\ and\ f = \frac{1}{\pi}\ thus\ \omega_0 = 2$$

Therefore from (5) $$\phi = tan^{-1}[-\frac{0.7}{0.6}] = -49.4^\circ$$

Now we can put value of ϕ in (3) and get amplitude A = 0.46

Example -2

$$x(t) = Acos(\omega t + \phi)$$

(a)

$$x^2 = A^2cos^2(\omega t + \phi)$$

$$< x^2 >= \frac{1}{T}\int_0^T x^2dt = \frac{A^2}{T}\int_0^T cos^2(\omega t + \phi)dt = \frac{A^2}{T}\frac{T}{2} = \frac{A^2}{2}$$

$$[Using\ 2cos^2x = (1 + cos2x)]$$

Again,

$$x^4 = A^4cos^4(\omega t + \phi) = \frac{A^4}{4}[1 + cos2(\omega t + \phi)]^2 = \frac{A^4}{4}[1 + 2cos2(\omega t + \phi) + cos^2x(\omega t + \phi)]$$

$$< x^4 >= \frac{1}{T}\int_0^T x^4dt$$

$$= \frac{A^4}{4T}\int_0^T [1 + 2cos2(\omega t + \phi) + cos^22(\omega t + \phi)]dt$$

$$= \frac{A^4}{4T}[T + 0 + \frac{T}{2}]$$

$$= \frac{3A^4}{8}$$

Example -2 Continued

(b) $$V(x) = 2e^{a^2x^2} = 2[1 + a^2x^2 + \frac{a^4x^4}{2} + ..]$$

For small displacement x is small .. $V(x) \simeq 2[1 + a^2x^2]$

$$Force,\ F = -\frac{dv}{dx} = -4a^2x = -kx$$

$$F \propto -x \rightarrow S.H.O$$

Differential eqn of S.H.O -

$$m\frac{d^2x}{dt^2} + \omega^2 x = 0,\ where\ \omega = \sqrt(\frac{k}{m})$$

Time period, $T = \frac{2\pi}{m} = 2\pi\sqrt(\frac{m}{k})$

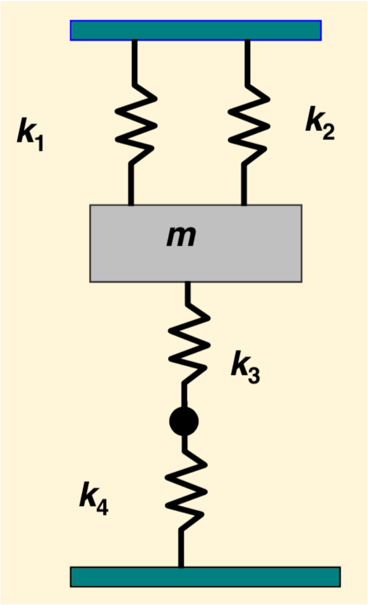
Here, m=0.3 kg and k = -4 for a=1

Example -3

(a) $m = 10Kg; k_1 = 1000N/m, K_2 = 3000N/m; K_3 = K_4 = 0$

In this case since K_3 and $K_4 = 0$, it becomes a **parallel combination** of springs , Unlike resistors in this case spring constants are **added in series**

$$K_{eq} = K_1 + K_2 = 4000N/m$$



Frequency of oscillation is .. $\omega = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{4000}{10}} = 20\ r.p.s.$

(b) $m = 10Kg; K_1 = K_2 = 0; K_3 = 1000N/m, K_4 = 3000N/m$

In this case since K_1 and $K_2 = 0$, it becomes a **series combination** of springs , Unlike resistors in this case spring constants are **added in parallel**

$$K_{eq} = \frac{K_3 K_4}{K_3 + K_4} = 750N/m$$

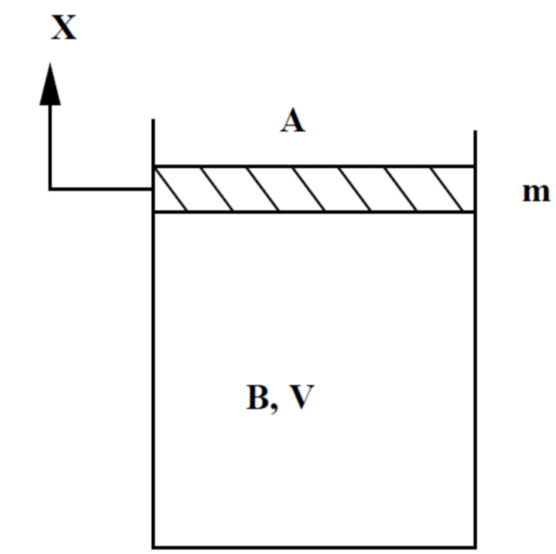
Frequency of oscillation is .. $\omega = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{750}{10}} = 8.66\ r.p.s.$

Example -4

Bulk modulus $$B = -\frac{dP}{\frac{dV}{V_0}} = -V_0\frac{dP}{dV}$$ V_0 is the initial volume

For a small displacement of piston by an amount of dx, $dV = Adx$

So, $$B = -\frac{V_0}{A}\frac{dP}{dx} \qquad \Rightarrow dP = -\frac{AB}{V_0}dx$$



$$\Rightarrow dF = -\frac{A^2B}{V_0}dx \qquad \Rightarrow F = -\frac{A^2B}{V_0}x$$

Equation of motion, $$F = m\frac{d^2x}{dt^2} \qquad \Rightarrow m\frac{d^2x}{dt^2} + \frac{A^2B}{V_0}x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$$

So frequency of vibration, $$\omega = \sqrt{(\frac{A^2B}{V_0})}$$

Example -4 continued

In isothermal process, $PV = Constant \quad \Rightarrow PdV + VdP = 0$

For small oscillation from equilibrium condition, $P \simeq P_0 = 1\ atm\ \& V \simeq V_0$

So from , $PdV + VdP = 0 \quad \Rightarrow -V_0 \dfrac{dP}{dV} = P_0 \quad \Rightarrow B = P_0$

So vibrational frequency (comparing previous case), $\omega = \sqrt{(\dfrac{A^2 P_0}{V_0})}$

In adiabetic process,

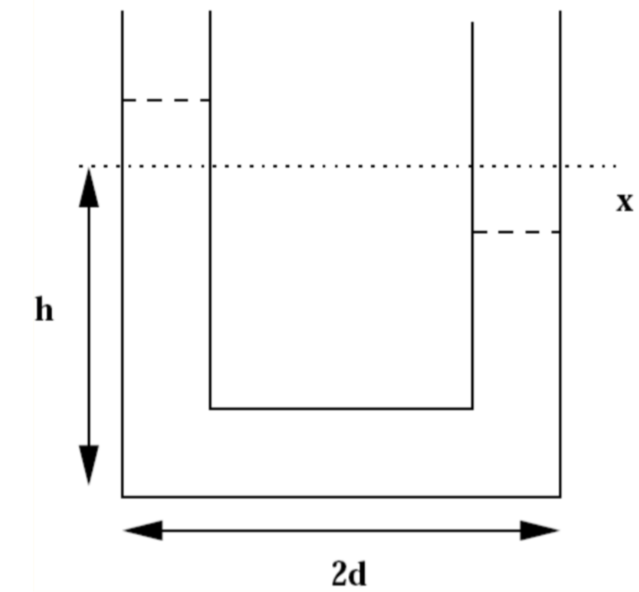
$$PV^\gamma = Constant \quad \Rightarrow P\gamma V^{\gamma-1} dV + V^\gamma dP = 0$$

For small oscillation from equilibrium condition, $P \simeq P_0 = 1\ atm\ \& V \simeq V_0$

$$\Rightarrow P_0 \gamma \frac{dV}{V_0} + dP = 0 \quad \Rightarrow -V_0 \frac{dP}{dV} = \gamma P_0 \quad \Rightarrow B = \gamma P_0$$

So vibrational frequency (comparing previous case), $\omega = \sqrt{(\dfrac{A^2 \gamma P_0}{V_0})}$

Example -5



Let's assume liquid in the U tube has density ρ and

A is cross-sectional area

Total change in length due to vertical displacement is (2x)

Restoring force (F_R) acting on the liquid due to this displacement will be -mg,
Where g is acceleration due to gravity and m is the mass of the liquid in column (2x)

$$F_R = -mg = -[volume \times density]g = -[a \times (2x) \times \rho]g$$

According to work energy principle, F_R must be equal to M [total mass of the entire liquid] times acceleration ..

$$M\frac{d^2x}{dt^2} = -\rho A(2x)g - (1)$$ Where, $$M = \rho A(2h + 2d) = 2\rho A(h + d)$$

From eqn (1) we get, $$\frac{d^2x}{dt^2} + \frac{g}{(h+d)}x = 0 - (2)$$

Eqn of motion in SHM written as, $$\frac{d^2x}{dt^2} + \omega^2 x = 0 - (3)$$

Comparing (2) & (3) we get , $$\omega^2 = \frac{g}{(h+d)}$$

$$Then,\ T\ (time\ period\ ) = \frac{2\pi}{\omega} \qquad And,\ f\ (frequency\ ) = \frac{1}{T}$$