Solution 1:

In a medium characterized by electric permittivity ε and magnetic permeability μ, the speed of light is given by $v = \sqrt{\frac{1}{\varepsilon\mu}}$.

The values of ε and μ for a medium is related to the corresponding values for free space ε_0 and μ_0 by $\varepsilon = K\varepsilon_0$ and $\mu = K_m\mu_0$ where K and K_m are constants.

For diamond, K= 5.84 and K_m = 1,

So, the speed of light in diamond is

$$v = \sqrt{\frac{1}{\varepsilon\mu}} = \sqrt{\frac{1}{K\varepsilon_0 K_m \mu_0}} = c\sqrt{\frac{1}{KK_m}}$$

(where $c = \sqrt{\frac{1}{\varepsilon_0\mu_0}}$ is the speed of light in free space)

$$= 3\times 10^8 \times \sqrt{\frac{1}{5.84}} \text{ m/s}$$

$$=1.24\times 10^8 \text{ m/s}$$

Solution 2:

Maxwell's wave equation in three-dimension for E field and B field is given by

$\nabla^2 E(\vec{r},t) = \frac{1}{c^2}\frac{\partial^2 E(\vec{r},t)}{\partial t^2}$ and $\nabla^2 B(\vec{r},t) = \frac{1}{c^2}\frac{\partial^2 B(\vec{r},t)}{\partial t^2}$ respectively.

Taking the wave equation as $\nabla^2 \psi(\vec{r},t) = \frac{1}{c^2}\frac{\partial^2 \psi(\vec{r},t)}{\partial t^2}$ where ψ can represent either E or B,

we solve the equation in spherical polar coordinate (r,θ,ϕ) where the operator ∇^2 take the following form.

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

For simplification, considering radial symmetry i.e. assuming the solution $\psi(\vec{r},t)$ is not a function of θ and ϕ and depends only on r,

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$$

So the wave equation becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi(r,t)}{\partial r}\right)=\frac{1}{c^2}\frac{\partial^2\psi(r,t)}{\partial t^2} \qquad \ldots(1)$$

We assume a solution in separable form of function of r and t i.e. $\psi(r,t)=\psi_r(r)\psi_t(t)$ and putting in (1) we get,

$$\psi_t(t)\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_r(r)}{\partial r}\right)=\psi_r(r)\frac{1}{c^2}\frac{\partial^2\psi_t(t)}{\partial t^2}$$

$$=> \frac{1}{\psi_r(r)}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_r(r)}{\partial r}\right)=\frac{1}{\psi_t(t)}\frac{1}{c^2}\frac{\partial^2\psi_t(t)}{\partial t^2}$$

Here as LHS is a function of only r and RHS is a function of only t, so both must be a constant independent of both r and t.

$\frac{1}{\psi_r(r)}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_r(r)}{\partial r}\right)=\frac{1}{\psi_t(t)}\frac{1}{c^2}\frac{\partial^2\psi_t(t)}{\partial t^2}=-K^2$ (as positive constant gives solution that dies out very fast, called evanescent field)

For $\psi_r(r)$,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_r(r)}{\partial r}\right)=-K^2\psi_r(r)$$

$$=> \frac{1}{r^2}\left(2r\frac{\partial\psi_r(r)}{\partial r}+r^2\frac{\partial^2\psi_r(r)}{\partial r^2}\right)=-K^2\psi_r(r)$$

$$=> \frac{1}{r}\left(2\frac{\partial\psi_r(r)}{\partial r}+r\frac{\partial^2\psi_r(r)}{\partial r^2}\right)=-K^2\psi_r(r)$$

$$=> \frac{1}{r}\frac{\partial^2}{\partial r^2}\left(r\psi_r(r)\right)=-K^2\psi_r(r)$$

$$=> \frac{\partial^2}{\partial r^2}\left(r\psi_r(r)\right)=-K^2\left(r\psi_r(r)\right)$$

Which gives solution

$$r\psi_r(r)=\psi_{r0}\exp(\pm iKr)$$

$$\psi_r(r)=\frac{\psi_{r0}}{r}\exp(\pm iKr)$$

(where ψ_{r0} is a constant.)

For $\psi_t(t)$,

$$\frac{1}{\psi_t(t)}\frac{1}{c^2}\frac{\partial^2\psi_t(t)}{\partial t^2} = -K^2$$

$$\frac{\partial^2\psi_t(t)}{\partial t^2} = -K^2c^2\psi_t(t) = -\omega^2\psi_t(t)$$

Which gives $\psi_t(t) = \psi_{t0}\exp(\pm i\omega t)$; (where ψ_{t0} is a constant.)

So, total solution:

$$\psi(r,t) = \psi_r(\mathrm{r})\ \psi_t(t) = \frac{\psi_{r0}\ \psi_{t0}}{r}\exp\left(i(\pm Kr \pm \omega t)\right)$$

$$\psi(r,t) = \frac{\psi_0}{r}\exp\left(i(\pm Kr \pm \omega t)\right)$$

Where $\psi_0 = \psi_{r0}\ \psi_{t0}$.

For a wave travelling radially outward from a point source,

$$\psi(r,t) = \frac{\psi_0}{r}\exp\left(i(Kr - \omega t)\right)$$

Now,

as $E(r,t) = \frac{E_0}{r}\exp\left(i(Kr - \omega t)\right)$ and $\mathrm{B}(r,t) = \frac{B_0}{r}\exp\left(i(Kr - \omega t)\right)$

So, Poynting vector,

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$|S| = \frac{1}{\mu_0}|E|\ \ |B|$$

$$|S| \propto \frac{1}{r^2}$$

Poynting vector varies as inverse square of the distance from the source.

Solution 3:

Poynting vector (S) gives the energy per unit time per unit area.

From the previous solution $|S| = \frac{K}{r^2}\hat{r}$. (where K is a proportionality constant)

So the total energy crossing a spherical surface of radius r is

$$U = \oiint_s \vec{S}.d\vec{A} |= \oiint \frac{K}{r^2}\hat{r}.\, \mathrm{r}^2 \sin\theta d\theta d\phi \hat{r}$$

$$= \frac{K}{r^2}.\mathrm{r}^2 \int_0^{\pi} \sin\theta d\theta. \int_0^{2\pi} d\phi$$

$$= \frac{K}{r^2}.4\pi \mathrm{r}^2 = 4\pi K$$

Which is independent of r, the distance from the source.

But, for a static source of E and B field we have,

$|E| \propto \frac{1}{\mathrm{r}^2}$ (from Coulomb's law)

and $|B| \propto \frac{1}{\mathrm{r}^2}$ (from Biot-savart law)

If we similarly define a Poynting vector, then

$|S| = \frac{1}{\mu_0} |E| \;\; |B| \;\; = \frac{K_1}{r^4}$ (where K_1 is a proportionality constant)

So, now the total energy crossing a spherical surface would be

$$\frac{K_1}{r^4}.4\pi r^2 = \frac{4\pi K_1}{r^2}$$

Which falls as inverse square of the distance from the source.

In the former case, the energy detaches from the source once it gets emitted from the source. Whereas in the later static case, the energy always remains coupled to the source.

Solution4:

For a plane wave

$$\vec{E} = E_0 \cos(kz - \omega t)\hat{i}$$

$$\vec{B} = B_0 \cos(kz - \omega t)\,\hat{j}$$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})\ \hat{k}$$

$$|\vec{S}| \quad = \quad \frac{1}{\mu_0} |\vec{E}||\vec{B}|$$

$$|\vec{S}| = \frac{1}{\mu_0} E_0 B_0 \cos^2(kz - \omega t)$$

The time average Poynting vector

$$\langle S \rangle = \frac{1}{T}\int_0^T \left|\vec{S}\right| dt$$

$$= \frac{1}{2\mu_0} E_0 B_0 \qquad (\text{as } \left\langle \cos^2(kz - \omega t) \right\rangle = \frac{1}{2})$$

$$\langle S \rangle = \frac{1}{2\mu_0 c} E_0^2 \qquad (\text{as } B_0 = \frac{1}{c} E_0)$$

$$\langle S \rangle = \frac{1}{2\mu_0} E_0 B_0 = \frac{1}{2\mu_0 c} E_0^2$$

Solution5:

Pulse duration $\Delta t = 4ns = 4 \times 10^{-9}\, s$

Pulse average power $< p > = 2.0 \times 10^{12} W$

Diameter of the cell = 0.5 μm

Energy of the pulse is spread uniformly over the faces of 100 cells (N=100)

a) Energy given to a cell $= \dfrac{< \mathrm{P} > \times \Delta \mathrm{t}}{\mathrm{N}}$

$$= \frac{2 \times 10^{12} \times 4 \times 10^{-9}}{100}\ \mathrm{J}$$

$$= 80\ \mathrm{J}$$

b) Intensity delivered to the cell = $\dfrac{\text{Power per cell}}{\text{Area of the cell}}$

$$= \frac{2\times10^{12} \times 4}{100\times \pi \times (0.5\times10^{-6})^2}$$

$$\approx 10.19\times10^{22} W / m^2$$

c) The average intensity is given by

$$I_{avg} = \frac{c\varepsilon_0 E_{\max}^2}{2}$$

$$E_{\max} = \sqrt{\frac{2I_{avg}}{c\varepsilon_0}}$$

$$E_{\max} = \sqrt{\frac{2\times1\times10^{21}}{3\times10^8 \times 8.85\times10^{-12}}}$$

$$\approx 8.7\times10^{11} v / m$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{8.7\times10^{11}}{3\times10^8}$$

$$\approx 2.9\times10^3 T$$