Tutorial-7

Tutorial 7: Concept of Stress and Strain - II

Problem-1

An 80-m-long wire of 5-mm diameter is made of a steel with E = 200 GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (a) the largest allowable tension in the wire, (b) the corresponding elongation of the wire.

Solution

$$\sigma_U = 400\times10^6\,\text{Pa}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(5)^2 = 19.635\text{ mm}^2 = 19.635\times10^{-6}\text{ m}^2$$

$$P_U = \sigma_U A = (400\times10^6)(19.635\times10^{-6}) = 7854\text{ N} \qquad \Rightarrow P_{\text{all}} = \frac{P_U}{F.S} = \frac{7854}{3.2} = 2454\text{ N} = 2.45\text{ kN}$$

Ans.

$$\delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635\times10^{-6})(200\times10^9)} = 50.0\times10^{-3}\text{ m}$$ Ans.

Problem-2

Two gauge marks are placed exactly 250 mm apart on a 12 mm diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm; determine the modulus of elasticity of the aluminum used in the rod.

Solution

$$\delta = \Delta L = L - L_0 = 250.18 - 250.00 = 0.18 \text{ mm}$$

$$\delta = \frac{PL}{AE} = \frac{(6000)(250)}{\frac{\pi}{4}(12)^2 E} = \frac{13262.912}{E} = 0.18 \text{ mm}$$

$$\Rightarrow \text{Modulus of elasticity, } E = \frac{13262.912}{0.18} \text{ N/mm}^2 = 73.682 \text{ N/mm}^2 = 73.68 \text{ GPa}$$

Ans.

Problem-4

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that *E* = 105 GPa and that the allowable tensile strength is 180 MPa, determine (a) the maximum allowable length of the bar, (b) the required dimensions of the cross section if the tensile load is 40 kN.

<u>Solution</u>

We know $\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$ $\Rightarrow L_{allow} = \frac{\delta E}{\sigma_{allow}} = \frac{(2.5)(105\times10^3)}{180} = 1458$ mm = 1.458 m

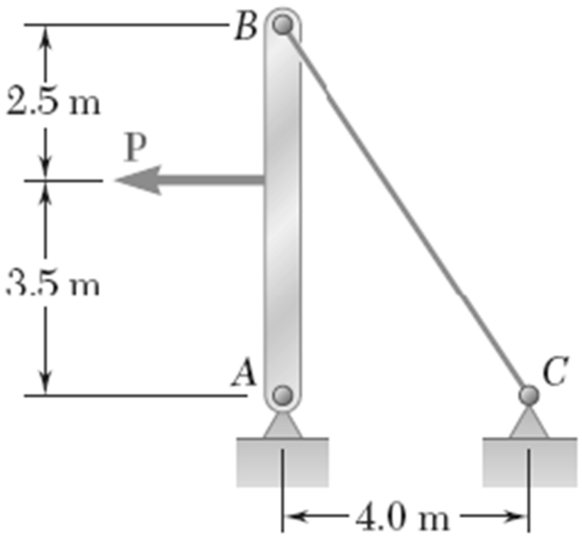
Ans.

Required cross section area = $\frac{Tensile\ load}{Allowable\ tensile\ strength} = \frac{40000}{180}$ mm^2 = 222.22 mm^2.

$\therefore$ Side of the square section = $\sqrt{222.22}$ mm = 14.91 mm Ans.

Problem-5

The 4-mm-diameter cable BC is made of a steel with E = 200 GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load P that can be applied as shown.



Solution

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

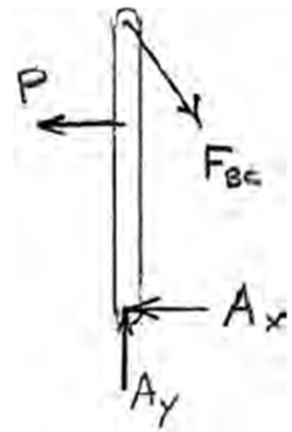
At equilibrium of the bar AB: $\Sigma M_A = 0$: $\quad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$

$\Rightarrow \quad P = 0.9509 F_{BC}$

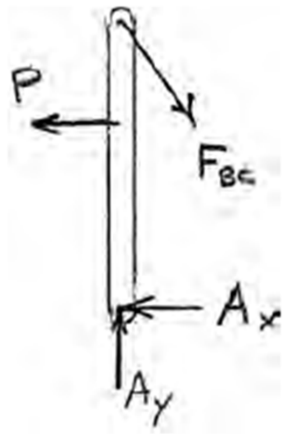
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{m}^2$$

Since, $\sigma = \frac{F_{BC}}{A}$ $\quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{N}$

Since $\delta = \frac{F_{BC} L_{BC}}{AE}$ $\quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{N}$



FBD of bar AB



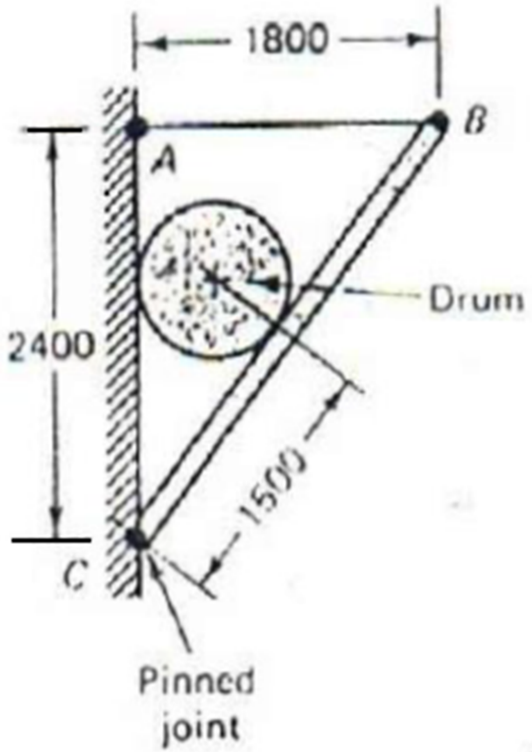
FBD of bar AB

Therefore, maximum tensile load which the cable BC can sustain is : $F_{BC} = 2.091 \times 10^3$ N

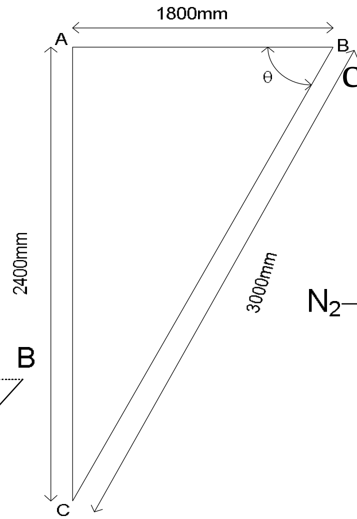
$\therefore \quad P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3\,\text{N}$ Ans.

Problem-6

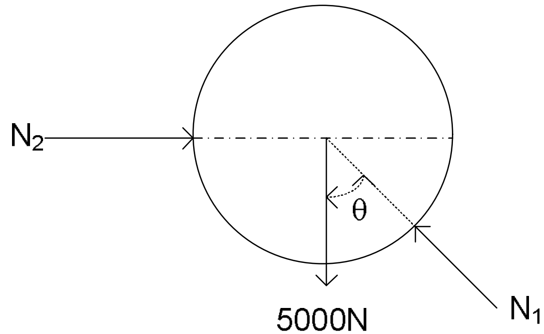
All the joints of the wall bracket may be considered as pin connected. Steel rod AB (E = 200 GPa) has a cross sectional area of 5 mm^2 and the member BC is a rigid beam. If a 1000 mm diameter frictionless drum of weight 5000 N is placed in the position shown, what will be the elongation of rod AB?



Solution



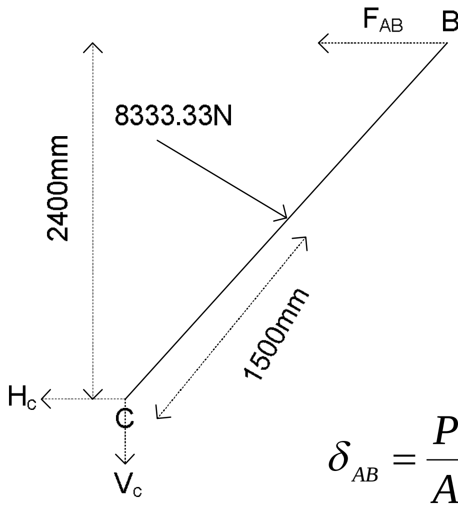
$$\cos\theta = \frac{1800}{3000} = 0.6$$



FBD of drum

$$\sum F_y = 0$$

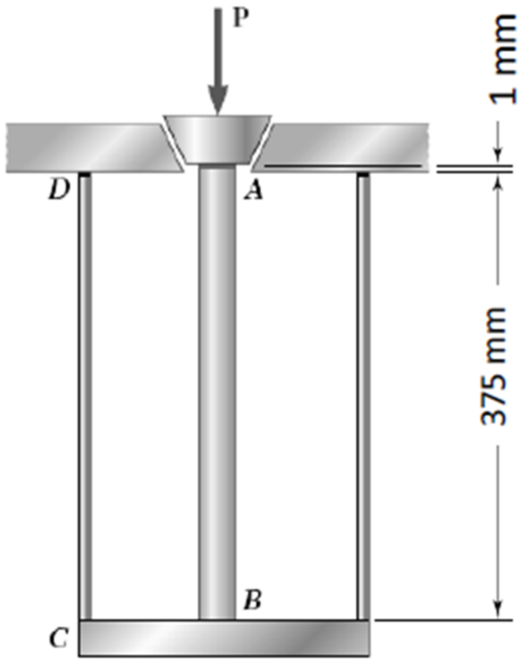
$$\Rightarrow N_1 = \frac{5000}{\cos\theta} = \frac{5000}{0.6} = 8333.33\text{N}$$



$$\sum M_C = 0 \quad \Rightarrow F_{AB} = \frac{8333.33 \times 1500}{2400} = 5208.33N$$

$$\delta_{AB} = \frac{PL}{AE} = \frac{5208.33 \times 1800}{5 \times 200 \times 10^3} = 9.375\,\text{mm}$$

Problem-7



The brass tube AB (E = 105 GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder (E = 72 GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force P that must be applied to the tube.

<u>Solution</u>

Shortening of brass tube AB

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2 \; E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Elongation of Aluminum cylinder CD

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

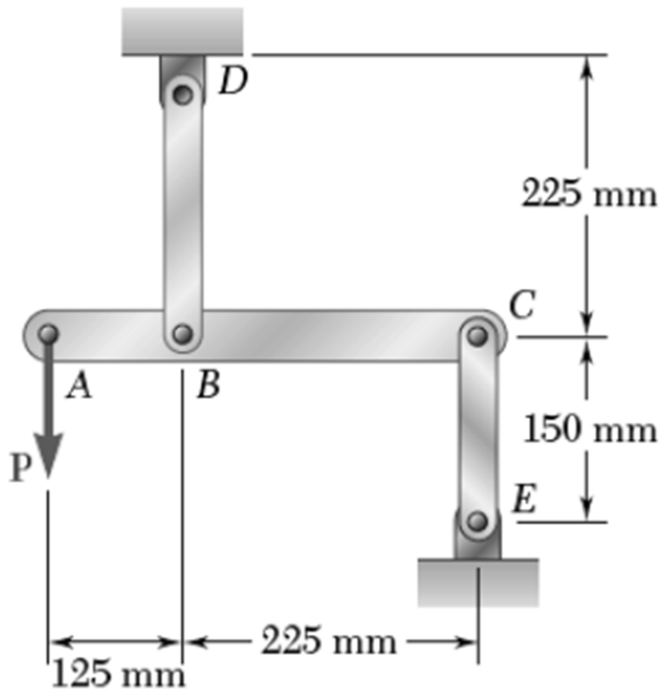
$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

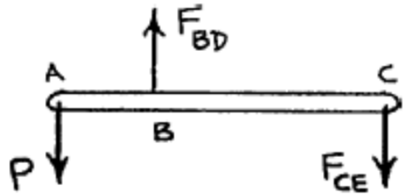
$\Rightarrow P = 21.547 \times 10^3$ N

Problem-10

Link BD is made of brass (E = 105 GPa) and has a cross-sectional area of 240 mm^2. Link CE is made of aluminum (E = 72 GPa) and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member ABC, determine the maximum force P that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.



Solution



FBD of member ABC

$\sum M_c = 0 \quad \Rightarrow \quad 0.350P - 0.225F_{BD} = 0$

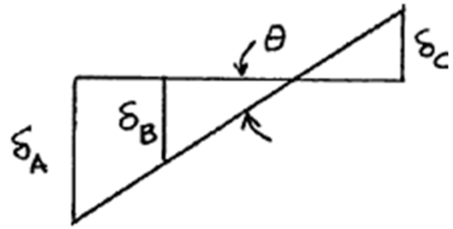
$\Rightarrow \quad F_{BD} = 1.55556P$

Again, $\sum M_B = 0 \quad \Rightarrow \quad 0.125P - 0.225F_{CE} = 0$

$\Rightarrow \quad F_{CE} = 0.55556P$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times10^9)(240\times10^{-6})} = 13.8889\times10^{-9}P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times10^9)(300\times10^{-6})} = 3.8581\times10^{-9}P$$



Deformation Diagram

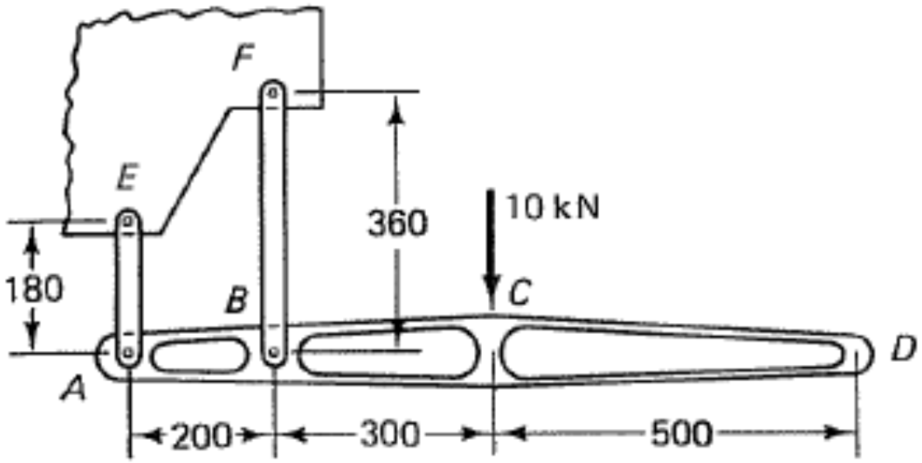
Slope, $\theta = \dfrac{\delta_B + \delta_C}{L_{BC}} = \dfrac{17.7470\times10^{-9}P}{0.225} = 78.876\times10^{-9}P$

$$\delta_A = \delta_B + L_{AB}\theta$$
$$= 13.8889\times10^{-9}P + (0.125)(78.876\times10^{-9}P)$$
$$= 23.748\times10^{-9}P$$

Since, $\delta_A = 0.35$ mm $= 0.35\times10^{-3}$ m $\quad \delta_A = 0.35\times10^{-3}\,\text{m} = 23.748\times10^{-9}P \quad \Rightarrow P = 14.7381\times10^3\,\text{N}$

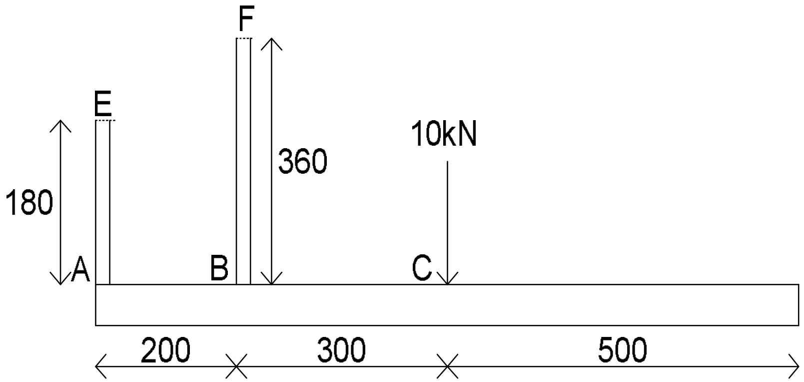
Ans.

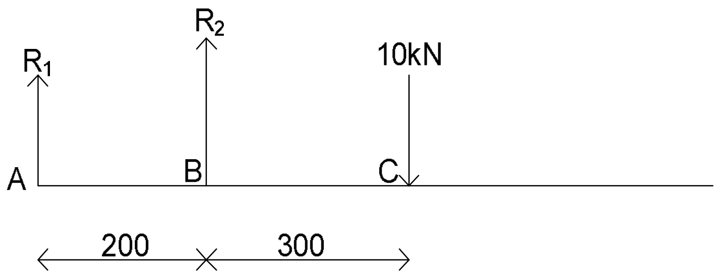
Problem-11



A rigid machine part AD is suspended by double hangers AE of cross sectional area of 50 mm^2 each and BF of cross sectional area of 100 mm^2 each respectively. The elastic modulus of hanger material is 180 GPa and yield stress is 600 MPa. Determine the deflection that would occur at D by applying a downward force of 10 kN at C. Check hanger stress to assure that an elastic solution is applicable.

Solution





All dimensions are in mm

$$\sum F_y = 0 \quad R_1 + R_2 = 10\times10^3\,\text{N}$$

$$\sum M_A = 0 \qquad R_2 \times 200 = 10\times10^3 \times 500 \qquad \Rightarrow \; R_2 = \frac{10\times10^3\times500}{200} = 25\times10^3\,\text{N}$$

$$R_1 = (10-25)\times10^3 = -15\times10^3\,\text{N}$$

$$\delta_1 = \frac{PL}{AE} = \frac{-15\times10^3\times180}{(50/2)\times180\times10^3} = -0.15\,\text{mm} \quad \text{Therefore} \quad \varepsilon_1 = \frac{\delta_1}{L} = \frac{0.15}{180} = 8.33\times10^{-4}$$

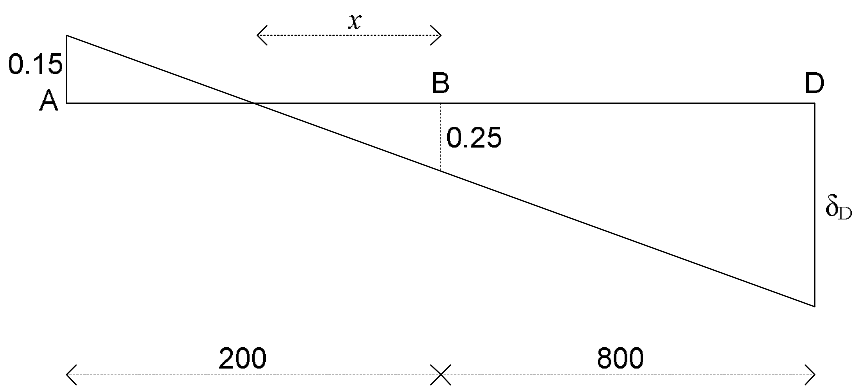
Hence, $\sigma_1 = \varepsilon_1 E = 8.33\times10^{-4}\times180\times10^3 = 150\ \text{MPa} < 600\,\text{MPa}$

$$\delta_2 = \frac{PL}{AE} = \frac{25\times10^3\times360}{(100/2)\times180\times10^3} = 0.25\ \text{mm} \quad \varepsilon_2 = \frac{\delta_2}{L} = \frac{0.25}{360} = 6.94\times10^{-4}$$

Hence, $\sigma_2 = \varepsilon_2 E = 6.94\times10^{-4}\times180\times10^3 = 125\ \text{MPa} < 600\ \text{MPa}$

$\Rightarrow$ Elastic condition is assured

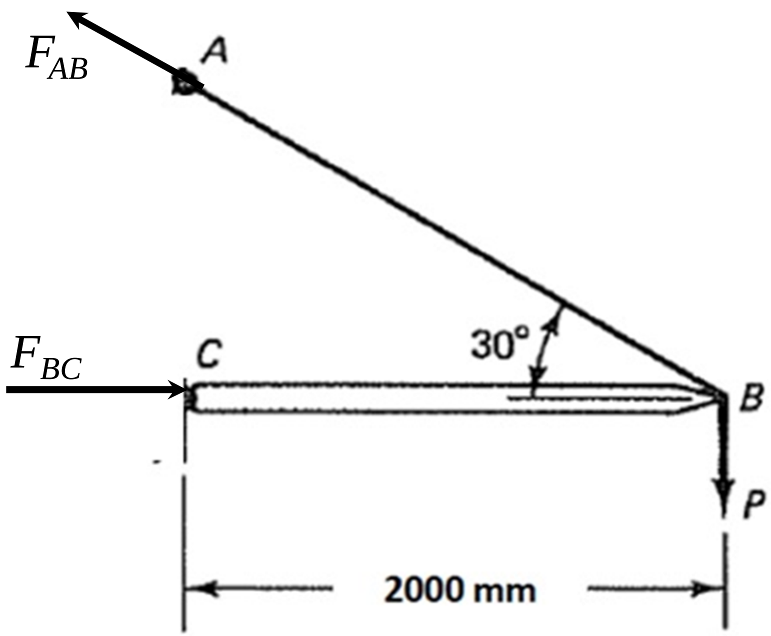
Solution Cont…



All dimensions are in mm

$$\frac{x}{0.25} = \frac{200 - x}{0.15} \qquad \Rightarrow x = \frac{50}{0.4} = 125 \text{ mm}$$

$$\frac{125}{0.25} = \frac{800 + 125}{\delta_D} \qquad \Rightarrow \delta_D = \frac{231.25}{125} = 1.85 \text{ mm} \quad \text{Ans.}$$



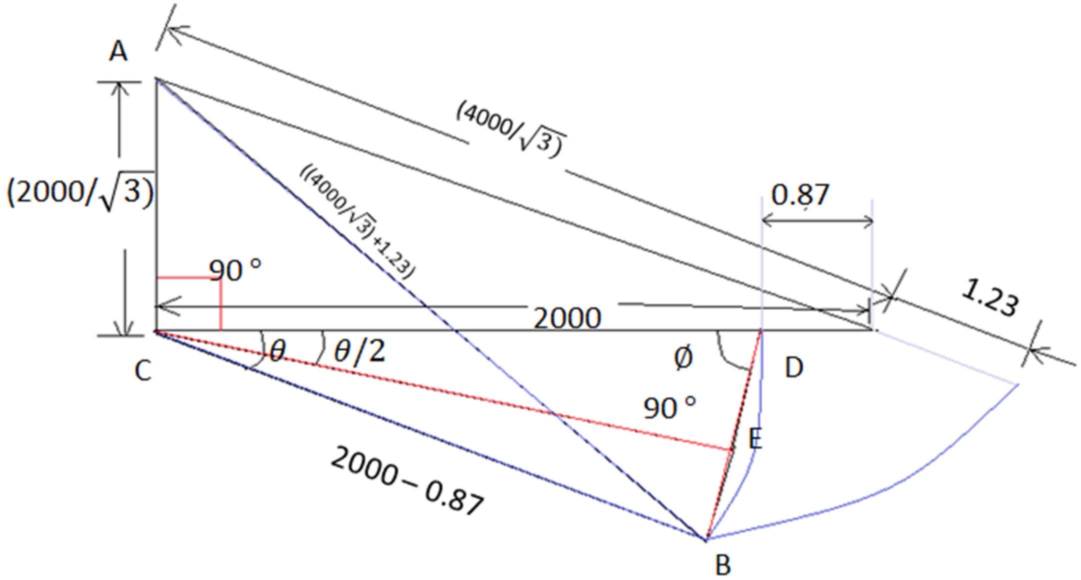
FBD of member ABC

Solution

F_{AB} = 32 kN F_{BC} = 32 kN

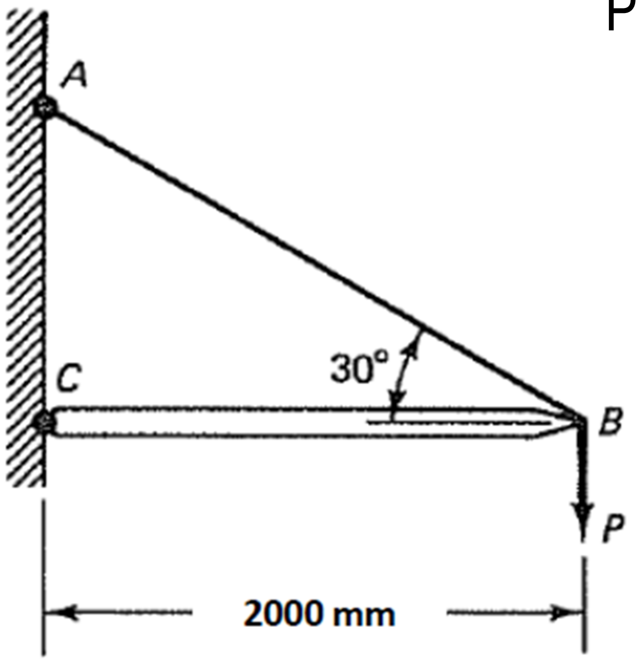
Δ_{AB} = 1.23 mm

Δ_{BC} = -0.87 mm



Deformed shape of member ABC

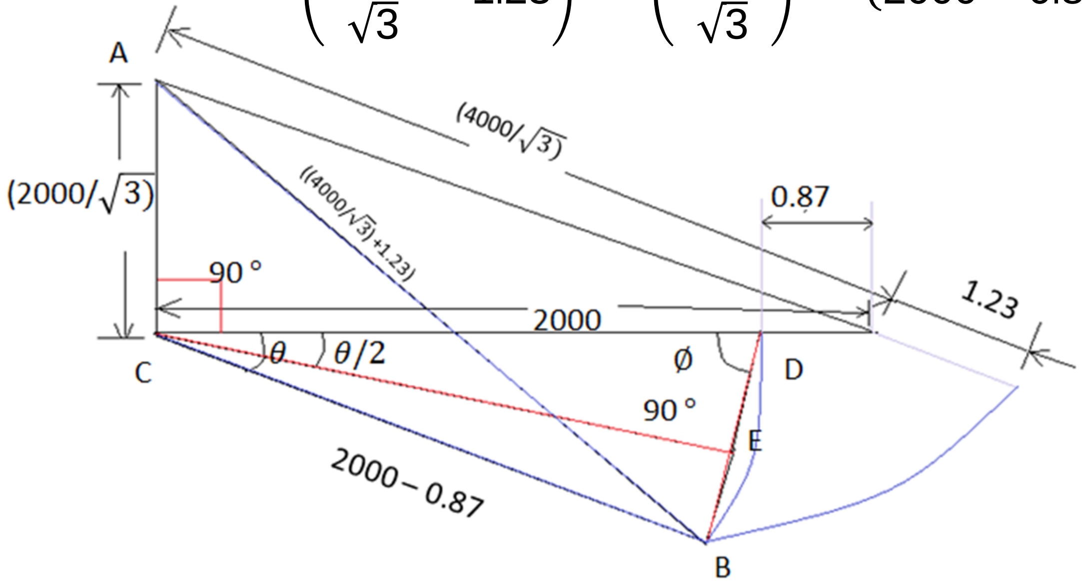
Problem-12



The jib crane shown in the figure has the cable AB of cross-sectional area of 300 mm^2 and the bar BC of cross-sectional area of 320 mm^2. (a) Determine the deflection vector at B caused by the application of a force P = 16 kN. (b) Hence, estimate the vertical stiffness of the crane at

point B. Take E = 200 GPa.

$$\left(\frac{4000}{\sqrt{3}}+1.23\right)^2=\left(\frac{2000}{\sqrt{3}}\right)^2+(2000-0.87)^2-2\left(\frac{2000}{\sqrt{3}}\right)(2000-0.87)\cos(90+\theta)$$



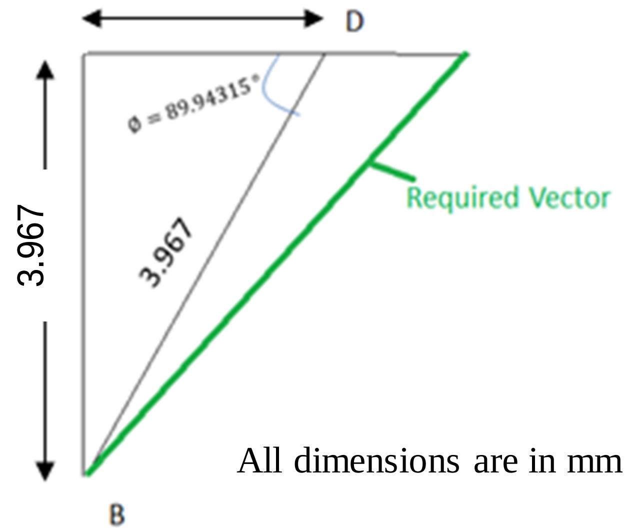
$$\Rightarrow \quad \theta = 0.1137\,° \text{ (magnitude)}$$

$$\Rightarrow \quad \emptyset = 90-\frac{\theta}{2} = 89.94315\,°$$

$\therefore$ DE = $(2000-0.87)\ \cos\emptyset$ = 1.9836 mm (E is the middle point of DB)

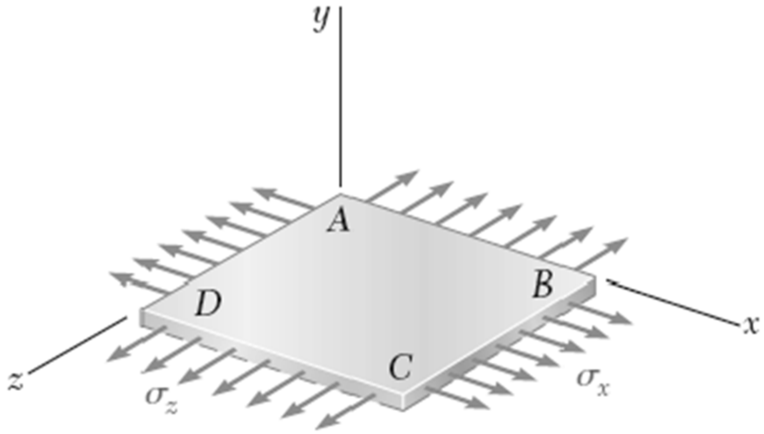
DB = (2)(1.9836) = 3.967 mm

Hence, displacement vector = $(-\ 0.8739i - 3.967\,j\ )$ mm



All dimensions are in mm

Problem-15



The homogeneous plate ABCD is subjected to a biaxial loading as shown. It is known that $\sigma_z = k$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. If the modulus of elasticity is E and Poisson's ratio is ν, determine (a) the required magnitude of σ_x, (b) the ratio k/ε_z.

<u>Solution</u>

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

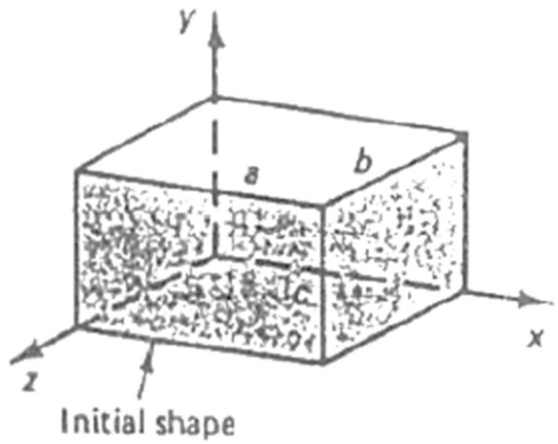
since $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0) = 0 \qquad \Rightarrow \sigma_x = \nu\sigma_0$

since $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \qquad \therefore \ \varepsilon_z = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$$\Rightarrow \frac{k}{\varepsilon_z} = \frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$$

Problem-16

A rectangular steel block has the following dimensions: *a* = 50 mm, *b* = 75 mm and *c* = 100 mm. The faces of this block are subjected to uniformly distributed forces of 180 kN (tension) in the *x*-direction, 200 kN (tension) in the *y*-direction and 240 kN (compression) in the *z*-direction. Determine the magnitude of a single system of forces acting only in the *y*-direction that would cause the same deformation in the *y*-direction as the initial forces. Consider ν =0.25.



Solution

$$F_X = +180 \text{ kN (tension)}; \text{ Area } (A_x) = b \times c = 75 \times 100 \text{ mm}$$

$$F_Y = +200 \text{ kN (tension)}; \text{ Area } (A_y) = a \times b = 50 \times 75 \text{ mm}$$

$$F_Z = -240 \text{ kN (compression)}; \text{ Area } (A_z) = a \times c = 50 \times 100 \text{ mm}$$

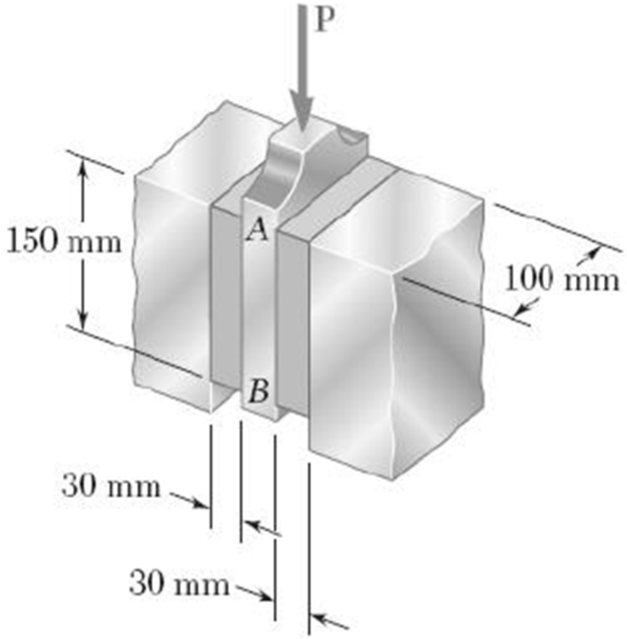
$$\sigma_x = \frac{F_X}{A_x}; \sigma_y = \frac{F_Y}{A_y}; \sigma_z = \frac{F_Z}{A_z}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \upsilon(\sigma_x + \sigma_z)\right] = \frac{1}{E}\left[\frac{200\times10^3}{50\times75} - 0.25\left(\frac{180\times10^3}{75\times100} - \frac{240\times10^3}{50\times100}\right)\right] = \frac{1}{E}[53.33+6] = \frac{59.33}{E}$$

For Equivalent force system, $\varepsilon_y^{'} = \frac{1}{E}\frac{F_y^{'}}{A_y} = \varepsilon_y^{'} = \frac{1}{E}\frac{F_y^{'}}{50\times75} = \frac{59.33}{E}$ Therefore, $\frac{1}{E}\frac{F_y^{'}}{50\times75} = \frac{59.33}{E}$

$$\Rightarrow \quad F_y^{'} = 222.49\times10^3 \text{ N} = 222.49 \text{ kN}$$

Problem-17



A vibration isolation unit consists of two blocks of rubber bonded to a rigid metal plate AB and to rigid supports as shown. Knowing that a force of magnitude *P* = 25 kN causes a deflection *d* = 1.5 mm of plate AB in the downward direction, determine the modulus of rigidity of the rubber used.

<u>Solution</u>

Given: $P = 25\text{ kN} = 25\times10^3\text{ N} \qquad \delta = 1.5\text{ mm}$

$$\text{Area} = (150)(100)\text{ mm}^2$$

$$\text{Shear strain} = \gamma = \frac{\delta}{h} \qquad \text{Shear stress} = \tau = G\gamma = G\frac{\delta}{h}$$

$$\text{Total force} = \frac{P}{2} = \text{Area}\times\tau = \text{Area}\times G\gamma = \text{Area}\times G\frac{\delta}{h}$$

$$\text{Or,}\quad \frac{(25)(10^3)}{2} = (150)(100)G\frac{1.5}{30}$$

$$\text{Therefore,}\quad G = \frac{25\times10^3\times30}{2\times150\times100\times1.5}\text{ N/mm}^2 = 16.67\text{ MPa}$$