Physics of Waves

PH11003

Tutorial 8 Solutions

Topic : Interference

12 January 2023

[8.1] Light with wavelength $\lambda = 0.55\ \mu$m from a distant point source falls normally on the surface of a glass wedge. A fringe pattern whose neighbouring maxima on the surface of the wedge are separated by a distance Δx = 0.21 mm is observed in reflected light. Find: (a) the angle between the wedge faces; (b) the degree of light monochromatism $(\Delta\lambda/\lambda)$ if the fringes disappear at a distance $l \approx$1.5 cm from the wedge's edge.

Solution

(a) For normal incidence, we have, using the above formula,

$$\Delta x = \frac{\lambda}{2n\alpha}$$

so,

$$\alpha = \frac{\lambda}{2n\Delta x} = 3 \text{ (on substituting values)}$$

(b) In a distance l on the wedge there are $N = l/\Delta x$ fringes. If the fringes disappear there, it must be due to the fact that the maxima due to the component of wavelength λ coincides with the minima due to the component of wavelength $\lambda + \Delta\lambda$. Thus,

$$N\lambda = \left(N - \frac{1}{2}\right)(\lambda + \Delta\lambda) \quad \text{or} \quad \Delta\lambda = \frac{\lambda}{2N}$$

so,

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2N} = \frac{\Delta x}{2l} = \frac{0.21}{30} = 0.007$$

[8.2] The convex surface of a plano-convex glass lens with curvature radius R = 40 cm comes into contact with a glass plate. A certain ring observed in reflected light has a radius r = 2.5 mm. Watching the given ring, the lens was gradually removed from the plate by a distance Δh = 5.0 μm. What has the radius of that ring become equal to?

Solution

The path traversed in air film of the wave constituting the k^{th} ring is

$$\frac{r^2}{R} = \frac{1}{2}k\lambda$$

When the lens is moved a distance Δh the ring radius changes to r' and the path length becomes

$$\frac{r'^2}{R} + 2\Delta h = \frac{1}{2}k\lambda$$

Thus,
$$r' = \sqrt{r^2 - 2R\Delta h} = 1.5 \text{ mm}$$

[8.3] Two thin symmetric glass lenses, one biconvex and the other biconcave, are brought into contact to make a system with optical power $\Phi = 0.50$ D. Newton's rings are observed in reflected light with wavelength $\lambda = 0.61\ \mu$m. Determine: (a) the radius of the tenth dark ring; (b) how the radius of that ring will change when the space between the lenses is filled up with water.

Solution

(a) Here
$$\Phi = (n-1)\left(\frac{2}{R_1} - \frac{2}{R_2}\right)$$

so,
$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{\Phi}{2(n-1)}$$

As in the previous problem, for the dark rings, we have

$$r_k^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\Phi}{2(n-1)}r_k^2 = k\lambda$$

As $k = 0$ is a dark spot; excluding it, we take $k = 10$ here.

Then,
$$r = \sqrt{\frac{20\lambda(n-1)}{\Phi}} = 3.49 \text{ mm}$$

(b) Path difference in water film will be

$$n_0\bar{r}^2 - \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(where $\bar{r}$ = new radius of the ring.) Thus,

$$n_0\bar{r}^2 = r^2$$

or
$$\bar{r} = \frac{r}{\sqrt{n_0}} = 3.03 \text{ mm}$$

(Here n_0 = R.I. of water = 1.33.)

[8.4] In Michelson's interferometer the yellow sodium line composed of two wavelengths $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm was used. In the process of translational displacement of one of the mirrors the interference pattern vanished periodically (why?). Find the displacement of the mirror between two successive appearances of the sharpest pattern.

Solution

Interference pattern vanishes when the maxima due to one wavelength mingle with the minima due to the other. Thus,

$$2\Delta h = k\lambda_2 = (k+1)\lambda_1$$

(where Δh = displacement of the mirror between the sharpest patterns of rings).

Thus, $$k(\lambda_2 - \lambda_1) = \lambda_1$$

or $$k = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

So, $$\Delta h = \frac{\lambda_1\lambda_2}{2(\lambda_2 - \lambda_1)} \cong \frac{\lambda^2}{2\Delta\lambda} \cong 0.29 \text{ mm}$$

[8.5] In a two-beam interferometer the orange mercury line composed of two wavelengths λ_1 = 576.97 nm and λ_2 = 579.03 nm is employed. What is the least order of interference at which the sharpness of the fringe pattern is the worst?

Solution

Sharpness of the fringe pattern is the worst when the maxima and minima intermingle

So, $$n_1\lambda_1 = \left(n_1 - \frac{1}{2}\right)\lambda_2$$

Using $\lambda_1 = \lambda$, $\lambda_2 = \lambda + \Delta\lambda$, we get

$$n_1\Delta\lambda = \frac{\lambda}{2}$$

or $$n_1 = \frac{\lambda}{2\Delta\lambda} = \frac{\lambda_1}{2(\lambda_2 - \lambda_1)} = 140$$