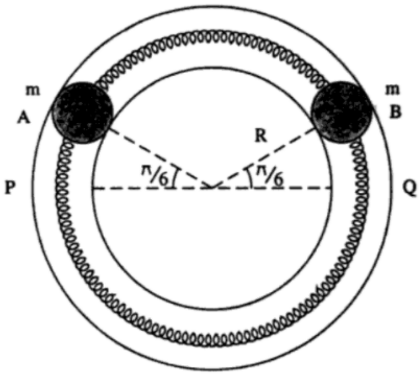
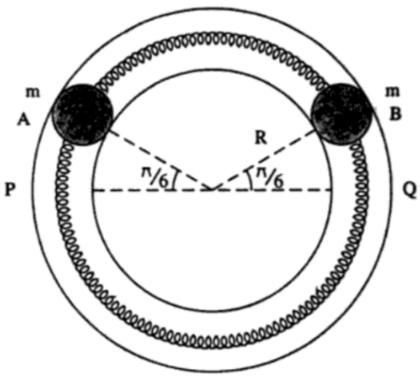
<table>
<tr>
<td>1A</td>
<td>Two identical balls A and B each of mass 100 gm are attached to two identical springs as shown in Fig. The spring-mass system is constrained to move in the horizontal plane in a rigid smooth pipe bent in the form of a circle. The centre of the balls move in a circle of radius 6 cm. Each spring has a natural length of 6π cm and spring constant 0.1 N/m. Initially both balls are displaced by an angle θ = π/6 radians with respect to the diameter PQ of the circle and released from rest. Calculate the frequency of oscillation in Hz of the ball B.


Ans. = 0.32 (Range: 0.3 to 0.34)
Solutions (1A) :
At an angular displacement θ of the balls, the compression or extension of the respective springs = $2R\theta$. Thus the force on B = $4kR\theta$.
The angular acceleration: $\frac{d^2\theta}{dt^2}$ and the linear acceleration is : $R\frac{d^2\theta}{dt^2}$
The equation of motion of the mass is $mR\frac{d^2\theta}{dt^2} = -4kR\theta$ or $\frac{d^2\theta}{dt^2} = -\frac{4k}{m}\theta$.
Therefore $\omega = \sqrt{\frac{4k}{m}}$ and $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi} = 0.318$</td>
<td>2 marks</td>
<td>Part B</td>
</tr>
<tr>
<td>1B</td>
<td>Two identical balls A and B each of mass 100 gm are attached to two identical springs as shown in Fig. The spring-mass system is constrained to move in the horizontal plane in a rigid smooth pipe bent in the form of a circle. The centre of the balls move in a circle of radius 6 cm. Each spring has a natural length of 6π cm and spring constant 0.1 N/m. Initially both balls are displaced by an angle θ = π/6 radians with respect to the diameter PQ of the circle and released from rest. Calculate the speed in cm/s of the ball A when A and B are furthest apart i.e., at PQ.


Ans. = 6.28 (Range: 5.9 to 6.6)
Solutions (1B) :
At an angular displacement θ of the balls, the compression or extension of the respective springs = $2R\theta$. Thus the force on B = $4kR\theta$.
The angular acceleration: $\frac{d^2\theta}{dt^2}$ and the linear acceleration is : $R\frac{d^2\theta}{dt^2}$
The equation of motion of the mass is $mR\frac{d^2\theta}{dt^2} = -4kR\theta$ or $\frac{d^2\theta}{dt^2} = -\frac{4k}{m}\theta$.
Therefore $\omega = \sqrt{\frac{4k}{m}}$
Now at $\theta = 0, KE = PE_{\theta=\frac{\pi}{6}}$ or $2\times\frac{1}{2}mv^2 = 2\times\frac{1}{2}k(2R\theta)^2$ or $v = 2R\theta\sqrt{\frac{k}{m}} = 2\times 0.06\times\frac{\pi}{6}$
$=0.02\pi$ m/s</td>
<td>2 marks</td>
<td>Part B</td>
</tr>
</table>

2	The function representing the displacement x(t) of an oscillator is given: $x(t) = e^{-0.02t}(3 \cos t + 4 \sin t)$. Find (a) the λ (log. decrement), (b) the Q (quality factor). Ans: 25 (Range: 24 to 26) Solution: Here β=.02, $\omega^2 = 1 = \omega_0{}^2 - \beta^2$. Therefore T=2π and $\lambda = \beta T = 0.04\pi$. Therefore, $Q = \frac{\pi}{\lambda} =$ 25	1 Marks	Part A
3	Two identical spring mass systems (mass m, spring constant k) attached to rigid walls at two ends are coupled by a spring of spring constant k'. The springs with constant k are replaced with new ones with constant $\frac{k}{2}$ and that with constant k' with another one with constant $2k'$. If the new normal frequencies ($\omega_0{}'$ and $\omega_1{}'$) in terms of the old ones (ω_0, ω_1) are given as $\omega_0'^2 = A\omega_0^2$ and $\omega_1'^2 = B\omega_1^2 - C\omega_0^2$, then the values of B=_______ (Ans: 2; Range: NA) C=_______ (Ans: 1.5; Range: NA) Solution: Since $k \to \frac{k}{2}$ and $k' \to 2k'$, the new normal frequencies are $\frac{k}{2m}$ and $\frac{k+8k'}{2m}$. The old o $\omega_0^2 = \frac{k}{m}$ and $\omega_1^2 = \frac{k+2k'}{m}$ Therefore, $$\omega'^2_0 = \frac{\omega_0^2}{2} \quad ; \quad \omega'^2_1 = 2\omega_1^2 - \frac{3}{2}\omega_0^2$$	2 Marks	Part B
4	A travelling wave is represented by $\psi = \psi_0 \cos(7x+6y+3z-10t+\varphi_0)$. The wave at time t = 0 is crossing the origin (0,0,0) and the phase at this point is $\varphi = \frac{\pi}{3}$. The phase of the point (1, 0, 0) at time t = 1 s is ________ rad Ans: -1.95 (Range: -1.9 to -2) Solution (a) The phase is $\frac{\pi}{3} - 3 = -1.9528$	1 Marks	Part A
5	Consider an undamped mass-spring system (mass m = 1 kg and spring constant k = 4 N/m) on which a piston acts to provide a harmonic driving force F (t) = 2 cos 2t. The time-averaged power $\langle \frac{dE}{dt} \rangle$ (in SI units) is Ans: 1.57 (Range:1.47 to 1.67)	2 Marks	Part B

(d) The instantaneous power is given by $\frac{dE}{dt} = \dot{x}F(t)$.Here we get

$$\frac{dE}{dt} = 2\cos 2t\ \dot{x} = \cos 2t \sin 2t + 2\cos^2 2t \tag{6}$$

(e) The time-averaged power is found by noting that the time average of $\cos 2t \sin 2t$ is zero, so only the second term contributes.

$$\langle \frac{dE}{dt} \rangle = \frac{2}{T} \int_0^T t \cos^2 2t\ dt \tag{7}$$

The value of the integral, with $T = \pi$ for this case,

$$\int_0^\pi t \cos^2 2t\ dt = \int_0^\pi \frac{t}{2} (1 + \cos 4t)\ dt = \frac{t^2}{4}|_0^\pi + \int_0^\pi t \cos 4t\ dt \tag{8}$$

Using integration by parts one can show that the integral $\int_0^\pi t \cos 4t\ dt = 0$. Hence, finally the value of the integral is $\frac{\pi^2}{4}$. Therefore,

$$\langle \frac{dE}{dt} \rangle = \frac{\pi}{2} \tag{9}$$

6A. A string, tied horizontally at its both ends is vibrating in its fundamental mode. The travelling waves have amplitude 5 cm and frequency 4 Hz. The magnitudes of maximum transverse velocity and maximum transverse acceleration of the point located at $\frac{\lambda}{4}$ from the left-hand end of the string (origin).

(a) Magnitude of maximum transverse velocity = ______ m/s

Ans: 2.51 (Range: 2.41 to 2.61)

2 Marks

(b) Magnitude of maximum transverse acceleration = ______ m/s^2

Ans: 63.16 (Range: 62.16 to 64.16)

Part B

Solution:

Maximum transverse velocity $= 4\pi Af sin\left(\frac{2\pi x}{\lambda}\right) = 4\pi Af sin\left(\frac{2\pi\left(\frac{\lambda}{4}\right)}{\lambda}\right) = 4\pi Af$

Maximum transverse acceleration = $8\pi^2 Af^2 sin\left(\frac{2\pi x}{\lambda}\right) = 8\pi^2 Af^2 sin\left(\frac{2\pi\left(\frac{\lambda}{4}\right)}{\lambda}\right) = 8\pi^2 Af^2$

6B. A string, tied horizontally at its both ends is vibrating in its fundamental mode. The travelling waves have amplitude 4 cm and frequency 2 Hz. The magnitudes of maximum transverse velocity and maximum transverse acceleration of the point located at $\frac{\lambda}{8}$ from the left-hand end of the string (origin).

(a) Magnitude of maximum transverse velocity = ______ m/s

Ans: 0.71 (Range: 0.61 to 0.81)

2 Marks

(b) Magnitude of maximum transverse acceleration = ______ m/s^2

Ans: 8.93 (Range: 7.93 to 9.93)

Part B

Solution:

Maximum transverse velocity $=4\pi A f sin\left(\frac{2\pi x}{\lambda}\right) = 4\pi A f sin\left(\frac{2\pi\left(\frac{\lambda}{8}\right)}{\lambda}\right) = 2\sqrt{2}\pi A f$

Maximum transverse acceleration = $8\pi^2 A f^2 sin\left(\frac{2\pi x}{\lambda}\right) = 8\pi^2 A f^2 sin\left(\frac{2\pi\left(\frac{\lambda}{8}\right)}{\lambda}\right) = 4\sqrt{2}\pi^2 A f^2$

6B. A string, tied horizontally at its both ends is vibrating in its fundamental mode. The travelling waves have amplitude 4 cm and frequency 2 Hz. The magnitudes of maximum transverse velocity and maximum transverse acceleration of the point located at $\frac{\lambda}{6}$ from the left-hand end of the string (origin).

(a) Magnitude of maximum transverse velocity = ______ m/s

Ans: 0.87 (Range: 0.77 to 0.97)

2 Marks

(b) Magnitude of maximum transverse acceleration = ______ m/s^2

Ans: 10.94 (Range: 9.94 to 11.94)

Part B

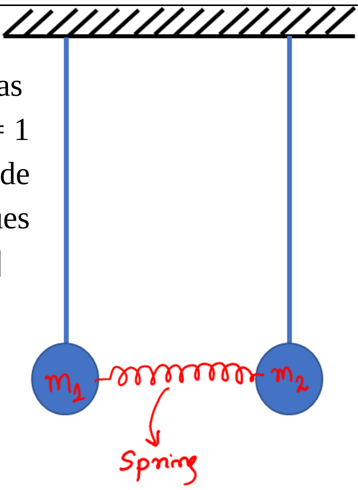
Solution:

Maximum transverse velocity $=4\pi A f sin\left(\frac{2\pi x}{\lambda}\right) = 4\pi A f sin\left(\frac{2\pi\left(\frac{\lambda}{6}\right)}{\lambda}\right) = 2\sqrt{3}\pi A f$

Maximum transverse acceleration = $8\pi^2 A f^2 sin\left(\frac{2\pi x}{\lambda}\right) = 8\pi^2 A f^2 sin\left(\frac{2\pi\left(\frac{\lambda}{6}\right)}{\lambda}\right) = 4\sqrt{3}\pi^2 A f^2$

1 | 2 marks

Two simple pendulum, each of length (l) having masses (m_1) and (m_2) are coupled by a spring of spring constant (k) as shown in the figure . Take l = 5 m, m_1 = 1 kg, m_2 = 3 kg, k = 1 Nm^{-1} and g = 10 ms^{-2} . If ω_1 and ω_2 are the normal mode frequencies of the system in rad/sec and $\omega_1 < \omega_2$ then the values are: [Please provide your answer upto two places of decimal]



ω_1 = -- : ω_2 = --

Ans: ω_1 = *1.41 rad/sec [Range: 1.30 to 1.50]*

ω_2 = *1.83 rad/sec [Range: 1.75 to 1.95]*

Soln:

$$m_1\ddot{x}_1 = -m_1 \frac{g}{l}x_1 - k(x_1 - x_2)$$

$$m_2\ddot{x}_2 = -m_2 \frac{g}{l}x_2 - k(x_2 - x_1)$$

Assumed soln :- $x_1 = Ae^{i\omega t}$

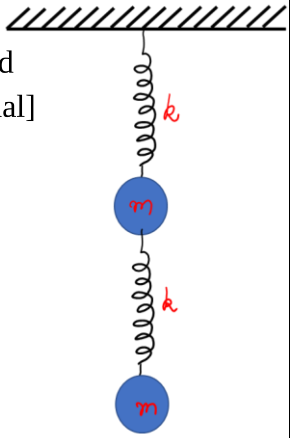
$x_2 = Be^{i\omega t}$

Thus

$$\begin{vmatrix} -m_1\omega^2 + m_1\frac{g}{l} + k & -k \\ -k & -m_2\omega^2 + m_2\frac{g}{l} + k \end{vmatrix} = 0.$$

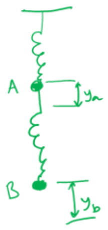
2 | 2 marks

Two equal masses m are connected to two identical massless springs of spring constant k as shown in the figure. If the masses are slightly disturbed they undergo small vertical oscillations about their equilibrium positions. Find the angular frequency corresponding to the faster normal mode. [m = 1 kg and k = 1 Nm^{-1}] [Please provide your answer upto two places of decimal]



Ans: 1.61 rad/sec [Range : 1.50 to 1.70]

Soln:



Restoring force on mass A ⇒

$-ky_a + k(y_b - y_a)$

Restoring force on mass B ⇒

$-k(y_b - y_a)$

$\therefore\ m\ddot{y}_a = -ky_a + k(y_b - y_a)$

$m\ddot{y}_b = -k(y_b - y_a)$

$$\therefore \begin{vmatrix} -m\omega^2 + 2k & -k \\ -k & -m\omega^2 + k \end{vmatrix} = 0. \rightsquigarrow \omega^4 - \frac{3k}{m}\omega^2 + \frac{k^2}{m^2} = 0$$

Since $k = 1$ & $m = 1$.

$$\therefore \omega_{fast}^2 = \frac{3+\sqrt{5}}{2}$$

$$\therefore \omega_{fast} = 1.61 \text{ rad/s}$$

3	Suppose two harmonic vibrations : $y_1 = cos\ (t)$ and $y_2 = cos\ (3t)$ [where t is in seconds] are superimposed so that the phenomenon of beating is observed. If T_A represents the time period of the beating amplitude and T_I represents the time period of the beating intensity then find T_A and T_I. [Give your answer upto two places of decimal] Ans: T_A = 6.28 sec [Range : 6.20 to 6.36] T_I = 3.14 sec [Range: 3.10 to 3.18] Soln: $y_1 = \cos(t)$ $\quad y_2 = \cos(3t)$ $\omega_1 = 1$ $\quad \omega_2 = 3.$ $\therefore$ Beat freq. of the amplitude $\Rightarrow \frac{\omega_2 - \omega_1}{2}$ $= 1.$ $\therefore T_A = \frac{2\pi}{\omega} = 2\pi$. sec. Beat freq. of intensity $\Rightarrow 2\left(\frac{\omega_2 - \omega_1}{2}\right)$ $= 2.$ $\therefore T_I = \frac{2\pi}{2} = \pi$ sec	1 mark	
4	~~Nickel has a Young's modulus of 21.4 x 10^{10} Nm^{-2} and a density of 8.9 x 10^3 kgm^{-3}. The speed of long~~itudinal sound waves in nickel is: Ans : 4903 ms^{-1} [Range: 4895 t0 4910] Soln: Speed of sound $= v = \sqrt{\frac{Y}{\rho}}$	1 mark	
5	Periodic mechanical disturbances $[\zeta(x,t)]$ are created at $x = 0$ of a long tube filled with a fluid in which the speed of propagation of sound wave is 4000 ms^{-1}. If $\zeta(0,t) = \sin^2(\omega t)$ where $\omega = 2\ rad/sec$ then the value of $\zeta(8000,1)$ is: Ans: 0.83 [Range : 0.82 to 0.84] Soln: $v = 4000\ ms^{-1}$ $\Psi(0,t) = \sin^2 2t.$ $\therefore \Psi(x,t) = \sin^2 2\left(\frac{x}{v} - t\right)$ $= \sin^2 2\left(\frac{8000}{4000} - 1\right)$ $= \sin^2 2$	1 mark	
6	A steel wire is stretched between two clamps 100 cm apart with a tension which produces an extension of 0.608 cm in it. Calculate the fundamental frequency (in Hz) of transverse vibration. Density of steel = 7600 kgm^{-3} and the Young's modulus = 2 x $10^{11}$$Nm^{-2}$.	1 mark	

Ans: 200 Hz [Range: 198 Hz to 202 Hz]

Soln:

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/\alpha}{\Delta L/L} = \frac{TL}{\alpha \Delta L}$$ where $\alpha \rightarrow$ cross sectional area

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$ where $\mu \rightarrow \alpha \rho$, ρ ← density

$$= \frac{1}{2L}\sqrt{\frac{\alpha \Delta L Y}{L \alpha \rho}} = \frac{1}{2L}\sqrt{\frac{Y}{\rho}\frac{\Delta L}{L}}.$$

Q1A

An ideal damped oscillator follow the given equation in SI unit, $0.25\,\ddot{x} + 0.07\dot{x} + 85\,x = 0$. The ratio of the amplitude of the damped oscillations to the initial amplitude at the end of 20 cycles is

Ans: 0.385 Range: 0.385±0.02

The given equation is,

$$0.25\,\ddot{x} + 0.07\,\dot{x} + 85\,x = 0;$$

2 Marks

Part A

The amplitude is given by,

$$A = A_0 e^{-\beta t};$$

$$\omega_0^2 = \frac{k}{m} = \frac{85}{0.25} = 340.$$

$$\beta^2 = \left(\frac{b}{2m}\right)^2 = \left(\frac{0.07}{0.5}\right)^2 = 0.0196.$$

$$\omega_0^2 >> \beta^2 \Rightarrow \omega \approx \omega_0.$$

$$\frac{A_{20}}{A_0} = e^{-\beta(20T)} = e^{-\frac{0.07}{2\times 0.25}\times 20\times\frac{2\times 3.14}{\sqrt{85/0.25}}};$$

$$= e^{-\frac{17.58}{18.44}} = e^{-0.953} = 0.385$$

Q1B

An ideal damped oscillator follow the given equation in SI unit, $0.25\,\ddot{x} + 0.07\dot{x} + 85\,x = 0$. The ratio of the amplitude of the damped oscillations to the initial amplitude at the end of 10 cycles is

Ans: 0.62 Range: 0.62±0.03

The given equation is,

$$0.25\,\ddot{x} + 0.07\,\dot{x} + 85\,x = 0;$$

2 Marks

Part A

The amplitude is given by,

$$A = A_0 e^{-\beta t};$$

$$\omega_0^2 = \frac{k}{m} = \frac{85}{0.25} = 340.$$

$$\beta^2 = \left(\frac{b}{2m}\right)^2 = \left(\frac{0.07}{0.5}\right)^2 = 0.0196.$$

$$\omega_0^2 >> \beta^2 \Rightarrow \omega \approx \omega_0.$$

$$\frac{A_{20}}{A_0} = e^{-\overset{10}{\beta(20T)}} = e^{-\frac{0.07}{2\times 0.25}\times 10\times \frac{2\times 3.14}{\sqrt{85/0.25}}};$$

$$= e^{\frac{4\times 0.7\times 3.14}{18.44}} e^{-\frac{8.8}{18.44}} = e^{-0.48} = 0.62$$

Q1C

An ideal damped oscillator follow the given equation in SI unit, $0.25\,\ddot{x} + 0.07\dot{x} + 85\,x = 0$. The ratio of the amplitude of the damped oscillations to the initial amplitude at the end of 5 cycles is

Ans: 0.79Range: 0.79±0.04

The given equation is,

$$0.25\,\ddot{x} + 0.07\,\dot{x} + 85\,x = 0;$$

2 Marks

Part A

The amplitude is given by,

$$A = A_0 e^{-\beta t};$$

$$\omega_0^2 = \frac{k}{m} = \frac{85}{0.25} = 340.$$

$$\beta^2 = \left(\frac{b}{2m}\right)^2 = \left(\frac{0.07}{0.5}\right)^2 = 0.0196.$$

$$\omega_0^2 >> \beta^2 \Rightarrow \omega \approx \omega_0.$$

$$\frac{A_{20\;5}}{A_0} = e^{-\beta(20\overset{5}{T})} = e^{-\frac{0.07}{2\times0.25}\times5\times\frac{2\times3.14}{\sqrt{85/0.25}}};$$

$$= e^{\frac{4\times0.35\times3.14}{18.44}} e^{-\frac{4.4}{18.44}} = e^{-0.24} = 0.79$$

Q2A

The amplitude of a lightly damped oscillator decreases by 10% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Ans: 19, Range: NA

The amplitude is given by,

$$A = A_0\, e^{-\beta t}; E = E_0\, e^{-2\beta t}.$$

$$\frac{A_1 - A_2}{A_1} \times 100 = 10, \Rightarrow \frac{A_2}{A_1} = 0.9 = e^{-\beta T};$$

1 Mark

$$\frac{E_1 - E_2}{E_1} = \frac{E_0 e^{-2\beta T} - E_0 e^{-2\beta(2T)}}{E_0 e^{-2\beta T}}$$

$$= 1 - e^{-2\beta T} = 1 - (0.9)^2 = 0.19;$$

Part A

$$\frac{E_1 - E_2}{E_1} \times 100 = 19\%.$$

Q2B

The amplitude of a damped oscillator decreases by 15% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Ans: 27.75, Range: 27.75±0.5

The amplitude is given by,

$$A = A_0\, e^{-\beta t}; E = E_0\, e^{-2\beta t}.$$

$$\frac{A_1 - A_2}{A_1} \times 100 = 15, \Rightarrow \frac{A_2}{A_1} = 0.85 = e^{-\beta T};$$

$$\frac{E_1 - E_2}{E_1} = \frac{E_0 e^{-2\beta T} - E_0 e^{-2\beta(2T)}}{E_0 e^{-2\beta T}}$$

1 Marks

$$= 1 - e^{-2\beta T} = 1 - (0.85)^2 = 0.2775;$$

Part A

$$\frac{E_1 - E_2}{E_1} \times 100 = 27.75\%.$$

Q2C

The amplitude of a damped oscillator decreases by 20% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Ans: 36, Range: 36±1

The amplitude is given by,

$$A = A_0\, e^{-\beta t}; E = E_0\, e^{-2\beta t}.$$

$$\frac{A_1 - A_2}{A_1} \times 100 = 20, \Rightarrow \frac{A_2}{A_1} = 0.8 = e^{-\beta T};$$

$$\frac{E_1 - E_2}{E_1} = \frac{E_0 e^{-2\beta T} - E_0 e^{-2\beta(2T)}}{E_0 e^{-2\beta T}}$$

$$= 1 - e^{-2\beta T} = 1 - (0.8)^2 = 0.36;$$

1 Marks

Part A

$$\frac{E_1 - E_2}{E_1} \times 100 = 36\%.$$

Q3A

The damped spring-mass system follows the equation, $1.5\,\ddot{x} + 12\,\dot{x} + 24\,x = 0$ (in SI unit). The mass is pulled to one side and released from rest. The time (>0) at which the damping force exactly balances the spring force

Ans: 0.25 s; Range: NA

The given equation is,

$$1.5\ddot{x} + 12\dot{x} + 24x = 0;$$

$$x(t) = e^{-\beta t} a_0 (1 + \beta t);$$

The given system is in critical damped.

1 Marks

Part A

$$\omega_0^2 = 24/1.5 = 16; \beta^2 = \left(\frac{12}{2\times 1.5}\right)^2 = 16.$$

The damping force and spring force balances at equilibrium point.

$$2\beta = \frac{12}{1.5};\ \beta = 4;$$

$$t = 1/\beta = 0.25\ \text{s}.$$

Q3B

The damped spring-mass system follow the equation, $\ddot{x} + 10\,\dot{x} + 25\,x = 0$ (in SI unit). The mass is pulled to one side and released from rest. The time (>0) at which the damping force exactly balances the spring force

Ans: 0.2 s; Range: NA

1 Marks

Part A

The given equation is,

$$\ddot{x} + 10\,\dot{x} + 25\,x = 0;$$

$$x(t) = e^{-\beta t} a_0\,(1 + \beta\,t);$$

The given system is in critical damped.

$$\omega_0^2 = 25\,;\ \&\ \ \beta^2 = \left(\frac{10}{2}\right)^2 = 25.$$

$$\omega_0^2 = \beta^2.$$

The damping force and spring force balances at equilibrium point.

$$\beta = \pm 5;$$

$$t = 1/5 = 0.2\ \ \text{s}.$$

Q3C

The damped spring-mass system follows the equation, $\ddot{x} + 2\,\dot{x} + \ x = 0$ (in SI unit). The mass is pulled to one side and released from rest. The time (>0) at which the damping force exactly balances the spring force

Ans: 1 s; Range: NA

1 Marks

Part A

The given equation is,

$\ddot{x} + 2\,\dot{x} + 1\,x = 0;$

$x(t) = e^{-\beta t} a_0\,(1 + \beta\, t);$

The given system is in critical damped.

$\omega_0^2 = 1\,;\ \&\ \ \beta^2 = (2/2)^2 = 1.$

$\omega_0^2 = \beta^2.$

The damping force and spring force balances at equilibrium point.

$\beta = \pm 1;$

$t = 1/1 = 1\ \text{ s}.$

Q4A

An ideal LCR parallel circuit follows the equation, (in SI unit). $\ddot{V} + 2500\dot{V} + 10^{10}V = 0$. The quality factor of the circuit is

Ans: 40; Range 40±1

1 Mark

Part A

The standard equation for a circuit in which LCR in parallel is,

$$C\ddot{V} + \frac{1}{R}\dot{V} + \frac{1}{L}V = 0;$$

$$\frac{1}{CR} = 2500, \ \& \ \frac{1}{CL} = 10^{10}$$

$$Q = R\sqrt{\frac{C}{L}} = \frac{\omega_0}{2\beta} = \frac{10^5}{2500} = 40$$

Q4B

An ideal LCR series circuit follows the equation, $0.01\ddot{q} + 2\dot{q} + 10^6 q = 0$ (in SI unit). The quality factor of the circuit is

Ans: 50; Range 50±1

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0}{2\beta} = \frac{\sqrt{\frac{10^6}{0.01}}}{2/0.01} = \frac{100}{2} = 50$$

1 Marks

Part A

Q4C

An ideal LCR series circuit follows the equation, $0.01\ddot{q} + 2\dot{q} + 10^8 q = 0$ (in SI unit). The quality factor of the circuit is

Ans: 500; Range 500±1

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0}{2\beta} = \frac{\sqrt{\frac{10^8}{0.01}}}{2/0.01} = \frac{1000}{2} = 500$$

1 Marks

Part A

Q5A

A damped harmonic oscillator follows the equation of motion, $2\ddot{x} + b\dot{x} + 10x = 0$. Initially, it oscillates with an amplitude of 25.0 cm; because of the damping, the amplitude falls to three-fourths of this initial value at the completion of four oscillations.

(a) What is the value of b?

(b) How much energy has been "lost" during these four oscillations?

Ans: (a) 0.10, Range: 0.1±0.01

(b) 44%, Range: 44±2

2 Marks

Part B

The given equation is,

$$2\ddot{x} + b\dot{x} + 10x = 0 \text{ (SI unit)}; A_0 = 25\,cm.$$

$$2\beta = b/2; \omega_0 = \sqrt{5};$$

$$A_4 = A_0 e^{-\beta(4T)};$$

$$\Rightarrow \ln(3/4) = -4 \times \frac{b}{4} \times \frac{2\pi}{\omega_0};$$

$$b = \frac{2.24 \times 0.2877}{6.28} = 0.1024.$$

$$E_4 = E_0 e^{-2\beta(4T)};$$

$$\Rightarrow \frac{E_4}{E_0} = e^{-2\beta(4T)} = e^{-4\beta \times 2 \times (2\pi/\omega_0)};$$

$$\frac{E_4}{E_0} = e^{-0.1024 \times 2 \times 2 \times 3.14/2.24} = e^{-0.57} = 0.56.$$

Energy lost, $\Rightarrow 1 - 0.56 = 44\%$

Q5B

A damped harmonic oscillator follow the equation of motion, $2\ddot{x} + b\dot{x} + 10x = 0$. Initially, it oscillates with an amplitude of 25.0 cm; because of the damping, the amplitude falls to two-third of this initial value at the completion of four oscillations.

(a) What is the value of b?

(b) How much energy has been "lost" during these four oscillations?

Ans: (a) 0.145, Range: 0.145±0.01

(b) 56%, Range: 56±2

2 Marks

Part B

The given equation is,

$$2\ddot{x} + b\dot{x} + 10x = 0 \text{ (SI unit)};\ A_0 = 25\,cm.$$

$$2\beta = b/2;\ \omega_0 = \sqrt{5} = 2.24;$$

$$A_4 = A_0 e^{-\beta(4T)};$$

$$\Rightarrow \ln(2/3) = -4 \times \frac{b}{4} \times \frac{2\pi}{\omega_0};$$

$$b = \frac{2.24 \times 0.4054}{6.28} = 0.145.$$

$$E_4 = E_0 e^{-2\beta(4T)};$$

$$\Rightarrow \frac{E_4}{E_0} = e^{-2\beta(4T)} = e^{-4\beta\times 2\times(2\pi/\omega_0)};$$

$$\frac{E_4}{E_0} = e^{-0.145\times 2\times 2\times 3.14/2.24} = e^{-0.81} = 0.44.$$

$$\text{Energy lost}, \Rightarrow 1 - 0.44 = 56\%$$

Q5C

A damped harmonic oscillator follow the equation of motion, $2\,\ddot{x} + b\,\dot{x} + 10\,x = 0$. Initially, it oscillates with an amplitude of 25.0 cm; because of the damping, the amplitude falls to three-fifth of this initial value at the completion of four oscillations.

(a) What is the value of b?

(b) How much energy has been "lost" during these four oscillations?

Ans: (a) 0.182, Range: 0.182±0.01

(b) 64 %, Range: 64±2

2 Marks

Part B

The given equation is,

$$2\,\ddot{x} + b\,\dot{x} + 10\,x = 0\,(\text{SI unit});\, A_0 = 25\,cm.$$

$$2\beta = b/2; \omega_0 = \sqrt{5} = 2.24;$$

$$A_4 = A_0 e^{-\beta(4T)};$$

$$\Rightarrow \ln(3/5) = -4 \times \frac{b}{4} \times \frac{2\pi}{\omega_0};$$

$$b = \frac{2.24 \times 0.51}{6.28} = 0.182.$$

$$E_4 = E_0 e^{-2\beta(4T)};$$

$$\Rightarrow \frac{E_4}{E_0} = e^{-2\beta(4T)} = e^{-4\beta\times 2\times(2\pi/\omega_0)};$$

$$\frac{E_4}{E_0} = e^{-0.182\times 2\times 2\times 3.14/2.24} = e^{-1.02} = 0.36.$$

$$\text{Energy lost}, \Rightarrow 1 - 0.36 = 64\%$$

Q6A

A damped harmonic oscillator follow the equation of motion, $1.5\,\ddot{x} + 0.23\,\dot{x} + 8\,x = 0$. Initially, it oscillates with an amplitude of 12.0 cm;

(a) Calculate the time (in s) required for the amplitude of the resulting oscillations to fall to one-third of its initial value.

(b) How many full oscillations are made by the block in this time?

2 Marks

Part B

Ans: (a) 14.27, Range: 14.27±1

(b) 5, Range: NA

The given equation is,

$$1.5\,\ddot{x} + 0.23\,\dot{x} + 8\,x = 0;$$

$$2\beta = \frac{0.23}{1.5};\ \beta = 0.077.$$

$$\omega_0 = \sqrt{\frac{8}{1.5}} = 2.3;$$

$$\omega_0 >> \beta : \text{lightly damped.}$$

$$(a).\quad A = A_0 e^{-\beta t};\ \frac{A_0}{3} = A_0 e^{-\beta t};$$

$$\Rightarrow t = \frac{\ln 3}{\beta} = \frac{1.098}{0.077} = 14.27.$$

$$(b).\ t = N\,T,\ N = \frac{t}{T} = \frac{14.27}{2\pi}\omega_0;$$

$$N = \frac{14.27}{2\times 3.14}\sqrt{\frac{8}{1.5}} = 5.26;$$

Q6B

A damped harmonic oscillator follow the equation of motion, $1.5\,\ddot{x} + 0.23\,\dot{x} + 8\,x = 0$. Initially, it oscillates with an amplitude of 12.0 cm;

(a) Calculate the time (in s) required for the amplitude of the resulting oscillations to fall to one-fourth of its initial value.

(b) How many full oscillations are made by the block in this time?

2 Marks

Part B

Ans: (a) 18, Range: 18±1

(b) 6, Range: NA

The given equation is,

$$1.5\,\ddot{x} + 0.23\,\dot{x} + 8\,x = 0;$$

$$2\beta = \frac{0.23}{1.5};\ \beta = 0.077.$$

$$\omega_0 = \sqrt{\frac{8}{1.5}} = 2.3;$$

$$\omega_0 >> \beta : \text{lightly damped.}$$

$$(a).\quad A = A_0 e^{-\beta t};\ \frac{A_0}{4} = A_0 e^{-\beta t};$$

$$\Rightarrow t = \frac{\ln 4}{\beta} = \frac{1.39}{0.077} = 18\,s.$$

$$(b).\ t = N\,T, N = \frac{t}{T} = \frac{18}{2\pi}\omega_0;$$

$$N = \frac{18}{2\times 3.14}\sqrt{\frac{8}{1.5}} = \frac{18\times 2.3}{6.28} = 6.59;$$

Q6C

A damped harmonic oscillator follow the equation of motion, $1.5\,\ddot{x} + 0.23\,\dot{x} + 8\,x = 0$. Initially, it oscillates with an amplitude of 12.0 cm;

(a) Calculate the time (in s) required for the amplitude of the resulting oscillations to fall to two-fifth of its initial value.

(b) How many full oscillations are made by the block in this time?

2 Marks

Part B

Ans: (a) 18, Range: 18±1

(b) 4, Range: NA

The given equation is,

$$1.5\,\ddot{x} + 0.23\,\dot{x} + 8\,x = 0;$$

$$2\beta = \frac{0.23}{1.5};\ \beta = 0.077.$$

$$\omega_0 = \sqrt{\frac{8}{1.5}} = 2.3;$$

$\omega_0 >> \beta$: lightly damped.

$$(a).\quad A = A_0 e^{-\beta t};\ \frac{2A_0}{5} = A_0 e^{-\beta t};$$

$$\Rightarrow t = \frac{\ln(5/2)}{\beta} = \frac{0.916}{0.077} = 11.9\,s.$$

$$(b).\ t = N\,T,\ N = \frac{t}{T} = \frac{11.9}{2\pi}\omega_0;$$

$$N = \frac{11.9}{2\times 3.14}\sqrt{\frac{8}{1.5}} = \frac{11.9\times 2.3}{6.28} = 4.35;$$

Q7A The suspension system of a 1600 kg automobile "sags" 10 cm when the chassis is placed on it. Also, the oscillation amplitude decreases by 40% each cycle. Estimate the values of the damping constant *b* for the spring and shock absorber system of one wheel, assuming each wheel supports 400 kg.

2 Marks

Part B

Ans: 650 kg/s, Range: 650±10 15

The given equation is,

$$F = k\,x;$$

$$k = \frac{m\,g}{x} = \frac{400 \times 10}{0.1};$$

$$k = 40000\,N\,/\,m.$$

The damping constant (b) is,

$$A = A_0 e^{-\beta t}; \frac{A_2}{A_1} = e^{-\beta T} = 0.6;$$

$$-\frac{b}{2m} \times \frac{2\pi}{\omega} = \ln(0.6);$$

$$\omega_0 = \sqrt{40000\,/\,400} = 10\,\text{Hz}.$$

$$\omega^2 = \omega_0^2 - \beta^2 \approx \omega_0^2$$

$$-\frac{b}{400} \times \frac{3.14}{10} = -b \times \frac{3.14}{4000} = -0.51;$$

$$b = 0.51 \times 4000\,/\,3.14 = 650\,\text{kg/s}.$$

Q7B The suspension system of a 1600 kg automobile "sags" 10 cm when the chassis is placed on it. Also, the oscillation amplitude decreases by 30% each cycle. Estimate the values of the damping constant *b* for the spring and shock absorber system of one wheel, assuming each wheel supports 400 kg.

2 Marks

Part B

Ans:460 kg/s, Range: 460±10 13

The given equation is,

$$F = k\,x;$$

$$k = \frac{m\,g}{x} = \frac{400\times 10}{0.1};$$

$$k = 40000\,N\,/\,m.$$

The damping constant (b) is,

$$A = A_0 e^{-\beta t}; \frac{A_2}{A_1} = e^{-\beta T} = 0.7;$$

$$-\frac{b}{2m}\times\frac{2\pi}{\omega} = \ln(0.7);$$

$$\omega_0 = \sqrt{40000\,/\,400} = 10\,\text{Hz}.$$

$$\omega^2 = \omega_0^2 - \beta^2 \approx \omega_0^2$$

$$-\frac{b}{400}\times\frac{3.14}{10} = -b\times\frac{3.14}{4000} = -0.36;$$

$$b = 0.36\times 4000\,/\,3.14 = 460\,\text{kg/s}.$$

Q7C The suspension system of a 1600 kg automobile "sags" 10 cm when the chassis is placed on it. Also, the oscillation amplitude decreases by 35% each cycle. Estimate the values of the damping constant *b* for the spring and shock absorber system of one wheel, assuming each wheel supports 400 kg.

2 Marks

Part B

Ans:548 kg/s, Range: 548±1012

The given equation is,

$$F = k\,x;$$

$$k = \frac{m\,g}{x} = \frac{400 \times 10}{0.1};$$

$$k = 40000\,N/m.$$

The damping constant (b) is,

$$A = A_0 e^{-\beta t}; \frac{A_2}{A_1} = e^{-\beta T} = 0.65;$$

$$-\frac{b}{2m} \times \frac{2\pi}{\omega} = \ln(0.65);$$

$$\omega_0 = \sqrt{40000/400} = 10\,\text{Hz}.$$

$$\omega^2 = \omega_0^2 - \beta^2 \approx \omega_0^2$$

$$-\frac{b}{400} \times \frac{3.14}{10} = -b \times \frac{3.14}{4000} = -0.43;$$

$$b = 0.43 \times 4000/3.14 = 548\,\text{kg/s}.$$

Q8A

An electrical circuit L=0.25 H, C=1 μF and R=1 Ω connected in a series. (a) How many oscillations will it make before the amplitude of the current is reduced by one-tenth of initial. (b) The percentage of energy lost in first cycle is .

Ans: (a)366, Range: 366±7

(b) 1.25, Range: 1.25±0.05

2 Marks

Part B

$$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{0.25} = 4\times 10^6, \quad \&, \ \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{1}{0.5}\right)^2 = 4$$

$$\Rightarrow \omega_0^2 > \beta^2, \text{an under dampped case}$$

Suppose N complete oscillation occurred for time *t*,

$$i = i_0 e^{-\beta t}; \frac{i_N}{i_0} = e^{-\beta t} = 0.1;$$

$$-\beta t = \ln(0.1); \Rightarrow t = 2.303/2 = 1.15$$

$$t = NT = \frac{N2\pi}{\omega} \approx \frac{N2\pi}{\omega_0};$$

$$N = \frac{\omega_0}{2\pi} t = \frac{2\times 10^3}{2\times 3.14}\times 1.15 = 366.$$

$$\therefore \frac{i_1}{i_0} = e^{-\beta T};$$

$$\Rightarrow \frac{E_1}{E_0} = \left(\frac{i_1}{i_0}\right)^2 = e^{-2\beta T};$$

$$= e^{-2\times 2\times \frac{2\pi}{2\times 10^3}} = e^{-0.0126} = 0.9875,$$

$$\%\,\text{loss}, \frac{E_0 - E_1}{E_0}\times 100 = 1.25\%$$

Q8B

An electrical circuit L=0.25 H, C=1 μF and R=1 Ω connected in a series. (a) How many oscillations will it make before the amplitude of the current is reduced by one-fourth of initial. (b) The percentage of energy lost after second cycle is .

Ans: (a)221, Range: 221±5

(b) 2.5%, Range: 2.5±0.2

2 Marks

Part B

$$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{0.25} = 4\times10^6, \quad \&, \ \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{1}{0.5}\right)^2 = 4$$

$$\Rightarrow \omega_0^2 > \beta^2, \text{an under dampped case}$$

Suppose N complete oscillation occurred for time *t*,

$$i = i_0 e^{-\beta t}; \frac{i_N}{i_0} = e^{-\beta t} = 0.25;$$

$$-\beta t = \ln(0.25); \Rightarrow t = 1.39/2 = 0.695.$$

$$t = NT = \frac{N 2\pi}{\omega} \approx \frac{N 2\pi}{\omega_0};$$

$$N = \frac{\omega_0}{2\pi} t = \frac{2\times10^3}{2\times3.14}\times0.695 = 221.$$

$$\therefore \frac{i_2}{i_0} = e^{-\beta(2T)};$$

$$\Rightarrow \frac{E_2}{E_0} = \left(\frac{i_2}{i_0}\right)^2 = e^{-4\beta T};$$

$$= e^{-4\times2\times\frac{2\pi}{2\times10^3}} = e^{-0.0251} = 0.975,$$

$$\%\,\text{loss}, \frac{E_0 - E_2}{E_0}\times100 = 2.5\%$$

Q8C

An electrical circuit L=0.25 H, C=1 μF and R=1 Ω connected in a series. (a) How many oscillations will it make before the amplitude of the current is reduced by one-fifth of initial. (b) The percentage of energy lost after four cycle is .

Ans: (a)256, Range: 256±6

(b) 4.9%, Range: 4.9±0.3

2 Marks

Part B

$$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{0.25} = 4\times10^6, \quad \&, \ \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{1}{0.5}\right)^2 = 4$$

$$\Rightarrow \omega_0^2 > \beta^2, \text{an under dampped case}$$

Suppose N complete oscillation occurred for time *t*,

$$i = i_0 e^{-\beta t}; \frac{i_N}{i_0} = e^{-\beta t} = 0.2;$$

$$-\beta t = \ln(0.2); \Rightarrow t = 1.61/2 = 0.805.$$

$$t = NT = \frac{N2\pi}{\omega} \approx \frac{N2\pi}{\omega_0};$$

$$N = \frac{\omega_0}{2\pi} t = \frac{2\times10^3}{2\times3.14}\times0.805 = 256.$$

$$\therefore \frac{i_4}{i_0} = e^{-\beta(4T)};$$

$$\Rightarrow \frac{E_4}{E_0} = \left(\frac{i_4}{i_0}\right)^2 = e^{-8\beta T};$$

$$= e^{-8\times2\times\frac{2\pi}{2\times10^3}} = e^{-0.05024} = 0.951,$$

$$\%\,\text{loss}, \frac{E_0 - E_4}{E_0}\times100 = 4.9\%$$