

**Indian Institute of Technology Kharagpur**  
**Department of Mathematics**  
**MA11003 - Advanced Calculus**  
**Problem Sheet -13A**  
**Autumn 2022**

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1. Evaluate the triple integral

(i)  $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx$  directly.

(ii)  $\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^6 \frac{dz dy dx}{\sqrt{x^2+y^2}}$  using cylindrical coordinates.

(iii)  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{1+x^2+y^2+z^2}$  using spherical coordinates.

2. Compute  $\iiint xy \, dz dy dx$  over the volume enclosed by planes  $z = x + y$  and  $z = 0$ , and between the surfaces  $y = x^2$  and  $x = y^2$ .

3. Compute  $\iiint \frac{z \, dz dy dx}{\sqrt{x^2+y^2}}$  if the region is bounded by paraboloid  $x^2 + y^2 = 2z$  and the cylinder  $x^2 + y^2 = 4$  in the first octant.

4. Find the integral  $\iiint e^{(x^2+y^2+z^2)^{3/2}} dx dy dz$  in the region  $R = x, y, z \geq 0, x^2 + y^2 + z^2 \leq 1$  using spherical coordinates.

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