

Indian Institute of Technology Kharagpur  
Department of Mathematics  
MA11003 - Advanced Calculus  
Problem Sheet - 8  
Autumn 2022

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**1 Solve the following homogeneous differential equations:**

a.  $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$

b.  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

c.  $\frac{d^2y}{dx^2} + a^2y = 0$

d.  $2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$

e.  $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$

f.  $\left(\frac{d^2y}{dx^2} + y\right)^3 \left(\frac{d^2y}{dx^2} + \frac{dy}{dx} + y\right)^2 = 0$

g.  $\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$

h.  $\frac{d^4y}{dx^4} = m^4y$

i.  $\frac{d^4y}{dx^4} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$

**2 Solve the following initial value problems:**

a.  $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0;$   $x(0) = 2, x'(0) = 0$

b.  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0;$   $y(0) = 1, y'(0) = 0$

c.  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0;$   $y(0) = 4, y'(0) = 1$

d.  $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0;$   $y(0) = \alpha, y'(0) = 2\pi$

e.  $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} - 22\frac{dy}{dx} + 56y = 0;$   $y(0) = 1, y'(0) = -2, y''(0) = -4$

f.  $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$   $y(0) = 1, y'(0) = 1$

### 3 Solve the following differential equations:

- a.  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$
- b.  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cosh x$
- c.  $\frac{d^2y}{dx^2} - 4y = e^x + \sin 3x$
- d.  $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$
- e.  $\frac{d^2y}{dx^2} - 4y = x \sin hx$
- f.  $\frac{d^2y}{dx^2} - 4y = x^2$
- g.  $\frac{d^2y}{dx^2} + 4y = e^x + \sin 3x + x^2$
- h.  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$
- i.  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} \sin 2x$
- j.  $(D^2 + 1)y = \operatorname{cosec} x$
- k.  $\frac{d^3y}{dx^3} + y = \sin 3x - \cos^2 \frac{x}{2}$

### 4 Solve the following problems:

- a. Show that the substitution  $z = \sinh^{-1} x$  transforms the equation  $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 4y$  into  $\frac{d^2y}{dz^2} = 4y$ .
- b. Show that all circle of radius  $r$  are represented by the differential equation  $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = r\frac{d^2y}{dx^2}$ .
- c. Construct a linear homogenous second order Differential equation such that the given functions are solution of differential equation
  - (i)  $u(x) = x, v(x) = e^x$ ,
  - (ii)  $u(x) = \frac{1}{x}, v(x) = e^{-x}$ .