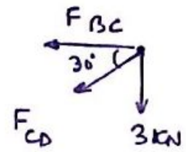
TUTORIAL PROBLEMS

SOLUTION

TUTORIAL - 4

1. Since we need to find force in every member it is profitable to use method of joint for finding the forces.

Usually in the method of joint we first calculate the external reaction forces by considering FBD of the entire truss. This is, however, not mandetary. We can start from any joint that has two unknown forces. From the truss it is seen that joint C is such a joint. From its FBD (figure 1.1) we get



Joint-C

Figure 1.1

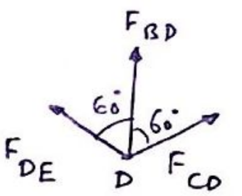
$$F_{CD} \sin 30^\circ + 3 = 0 \qquad \cdots(1)$$

and

$$F_{BC} + F_{CD} \cos 30^\circ = 0 \qquad \cdots(2)$$

ie $F_{CD} = -6$ (kN), $F_{BC} = 3\sqrt{3}$ (kN) $= 5.1962$ kN

Positive sign of a force (which in FBD is drawn with arrow head pointing away from the joint) indicates the member is under tension. Thus $F_{CD} = 6$ kN (C), $F_{BC} = 5.2$ kN (T).



Joint - D

Figure 1.2

We can now proceed to joint D which has two unknowns (see figure 1.2), namely, F_{BD} and F_{DE}. The force balance equations applied to the joint give

$$F_{DE} = F_{CD} = -6 \text{ (kN)} \qquad \cdots(3)$$

and

$$F_{BD} = -2 \times F_{CD} \times \cos 60^\circ = -F_{CD} = 6 \text{ (kN)}. \qquad \cdots(4)$$

Now it is possible to go to joint B whose FBD is shown in figure 1.3. The equilibrium equations can be written as

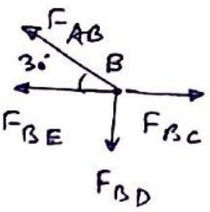


Figure 1.3

$$F_{AB} \cos 30^\circ + F_{BE} = F_{BC} \qquad \cdots(5)$$

and

$$F_{AB} \sin 30^\circ = F_{BD}. \qquad \cdots(6)$$

Thus $F_{AB} = 12$ kN and $F_{BE} = 3\sqrt{3} - 6\sqrt{3} = -3\sqrt{3}$ (kN)

It may be seen that we can now go only to joint E since it has only one external reaction force and another force F_{AE} as shown in figure 1.4. The equilibrium equations yield

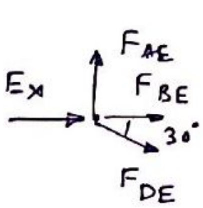


Figure 1.4

$$F_{AE} = F_{DE} \sin 30^\circ = -3 \text{ kN}. \qquad \cdots(7)$$

and

$$E_x = -(F_{BE} + F_{DE} \cos 30^\circ) = 6\sqrt{3} \text{ (kN)}. \qquad \cdots(8)$$

To get the free in the bar CD in two equations we can start at joint E, whose FBD is shown in figure 2.1. Since the net force is zero, the sum of components of all the forces along any direction will also vanish. Thus taking components along n-n we get

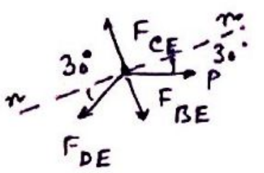
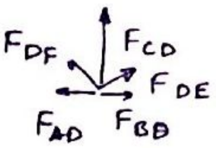


Figure 2.1.

$$F_{DE} \cos 30^\circ - P \cos 30^\circ = 0 \Rightarrow F_{DE} = P. \quad \cdots (1)$$

Next we go to the joint D whose FBD is shown in figure 2.2. The force balance equation along the vertical direction gives



$$F_{CD} + F_{DE} \cos 30^\circ + F_{DF} \cos 30^\circ = 0 \quad \cdot (2)$$

But DF is a zero force member. So

$$F_{CD} = -F_{DF} \cos 30^\circ = -\frac{\sqrt{3}}{2} P.$$

The rod CD is under compressive force of $0.866P$ ~~(kN)~~.

Note that by considering $F_{DF} = 0$; ie DF as a zero free member we have implicitly used one more equation, namely, force balance equation at F along direction perpendicular to AC. Thus, truly speaking, we have ~~Co~~ used three equations. But the fact that DF is a zero force member can be found merely by inspection. This step can be mentally done requiring no explicit writing of any equation.

3. The FBD of the whole truss is shown in figure 3.1. The forces A_x, A_y and D_x are found using the equations of equilibrium

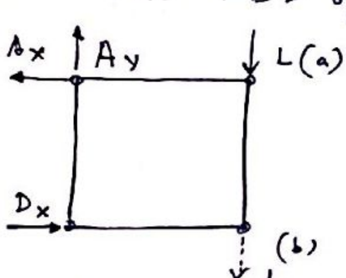


Figure 3.1

$$A_x = D_x = \frac{4}{3} L, \quad \text{and} \quad A_y = L. \quad \cdots (1)$$

It can be seen that at least one of the cables should be operative since otherwise the structure will collapse. The situation when none is operative is shown in figure 3.1 (an inoperative ie zero force member can be removed without any effect).

One can also show that both the cables do not work at the same time. This will be shown for case (a) but the same consideration may be used for the other case. When both are under tension then member CD and AB will both be under compression. This can be proved from the FBD's of joints B and C, shown in figures 3.2 (a) and 3.2 (b), respectively.

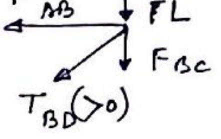


Fig 3.2 (a)

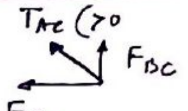


Figure 3.2 (b).

fact, from the FBD's we get

$$F_{CD} = -T_{AC}\cos\theta = -T_{AC}\times\frac{4}{5} \quad \text{and,} \quad F_{BC} = -T_{AC}\sin\theta = -T_{AC}\times\frac{3}{5}$$

and $F_{AB} = -T_{BD}\cos\theta = -\frac{4}{5}T_{BD}$, $T_{BD}\sin\theta + F_{BC} + L = 0$.

From the last expression we get $T_{BD}\times\frac{3}{5} - T_{AC}\times\frac{3}{5} = -L$, ie

$$T_{BD} = T_{AC} - \frac{5}{3}L. \qquad \cdots (2)$$

Considering the horizontal components of total forces at joint D we get

$$D_x + F_{CD} + T_{BD}\cos\theta = 0 \quad \text{ie} \quad T_{BD}\times\frac{4}{5} - T_{AC}\times\frac{4}{5} - \cancel{\tfrac{4}{3}}\frac{4}{3}L = 0$$

which is the same as equation (2). The balance of vertical forces at the same joint gives the same result

$$F_{BD} = -T_{BD}\sin\theta = -\frac{3}{5}T_{BD}. \qquad \cdots (3)$$

The equilibrium equations at joint A yield

$$A_y = T_{AC}\sin\theta + F_{AD} \Rightarrow L = \frac{3}{5}T_{AC} - \frac{3}{5}T_{BD}$$

and $$A_x = F_{AB} + T_{AC}\cos\theta \Rightarrow \frac{4}{3}L = -\frac{4}{5}T_{BD} + T_{AC}\times\frac{4}{5}$$

which are identical.

From the above analysis it is seen that under the given loading all the sides of the rectangle are under compression but the diagonals are under tension. Also from equation (2) we have $T_{BD} < T_{AC}$. However, it is interesting to note that the members remain under compression even if the direction of L is reversed (of course, when the direction of L is reversed $D_x = -\frac{4}{3}L$ which is impossible under the given configuration of the roller. This is not a problem since mechanics does not change even if the roller is replaced by another one having stops at both sides) since in the derivation of forces in members the load does not appear. This is highly improbable since the equations ~~are~~ of equilibrium are linear in terms of forces and consequently the forces in the member vary linearly with the applied load.

The above argument suggests that one of the tensions should vanish. Since $T_{BD} < T_{AC}$, then we must have $T_{BD} = 0$ which makes

$$T_{AC} = \frac{5}{3}L, \quad F_{AB} = F_{AD} = 0, \quad F_{CD} = -\frac{4}{3}L, \quad F_{BC} = -L.$$

The forces in the members for case (b) can be similarly found. Note that in this case also equation (2) remains valid. In fact the external reaction forces are the same as in Case (a). The balance of vertical force components at joint A and D gives the required relation. Thus, once again $T_{BD} = 0$

however, this time $F_{BC} = 0$ which can be easily seen from free body diagram of joint B (not drawn). Note that the loads in all the members excluding BC remain the same as the load is slided from point B to C. In the first case the load gets transferred to the cable AC through BC while in the second case it is directly applied.

We have used a rather lengthy discussion to find out which cable member should carry load. This can be avoided if you imagine the truss to be deformable (although only virtually). You can easily see which of the cable gets compressed by noticing whose length is shortened. For example, the deformation leads to the following configuration (see figure 3.4) from which it becomes clear that AC is the only load carrying cable.

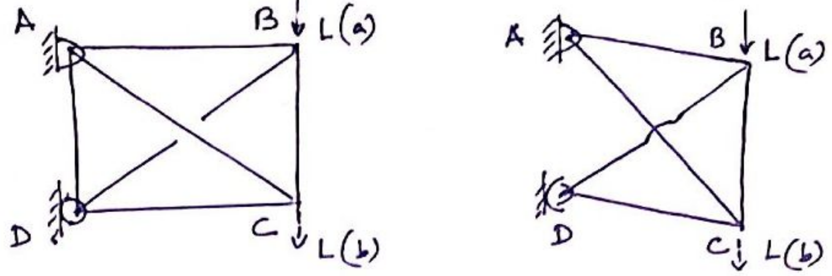


Figure 3.4

4. It can be any cut will lead to four unknown forces in four members. But it can also be seen that the members DE and EF are zero force member. Hence a cut across FD, GM and ML yields a statically determinate system as shown in figure 4.1

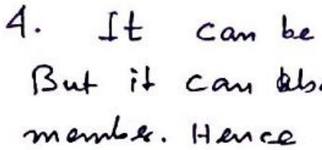


Figure 4.1

Taking moment about point M we get

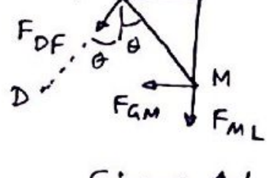
$$1800 \times \frac{1.2}{2} + F_{DF}\left(\sin\theta \times 2.4 + \cos\theta \times \frac{1.2}{2}\right) = 0 \quad \cdots (1)$$

where $\tan\theta = \frac{0.6}{2.4}$.

From (1) we get $F_{DF} = -927.699$ (N).

From force balance along horizontal direction one finds

$$F_{GM} = -F_{DF}\sin\theta = 225 \text{(N)}. \quad \cdots (2)$$



Again, by inspection one can see that FN is also a zero force member since no horizontal force is given at N. In this case we can take a cut across FN, FM, DG and DC as shown in figure 4.2

Now taking moment about point D we get

$$-1800 \times \frac{1.2}{2} - F_{FM}\left(2.4 \times \sin\theta + \frac{1.2}{2}\cos\theta\right) = 0 \quad \cdots (3)$$

and the force balance equation along horizontal direction

$$F_{DG} = -F_{FM} \sin\theta \qquad (4)$$

Looking at the formal similarity between equations (3), (4) and (1), (2) one can write

$$F_{DG} = 225 \text{ (N)}.$$

The last part of the above calculation is not necessary. The fact that $F_{DG} = F_{GM}$ can be established by taking the cut across DC, DG, GM and ML as shown in figure 4.3. The horizontal force balance equation directly yields that $F_{DG} = F_{GM}$.

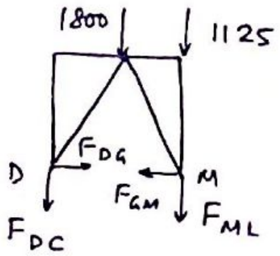


Figure 4.3.

It is interesting to note that although there are four unknowns in this special cut three of the forces are concurrent. This fact helps to find F_{ML} and F_{DC} directly. In fact taking moment about point M we get $F_{DC} \times 1.2 + 1800 \times \frac{1.2}{2} = 0$ ie $F_{DC} = -900$ (N).

By force balance equation we get

$$F_{ML} = -(1800 + 1125 + F_{DC}) = -2025 \text{ (N)}.$$

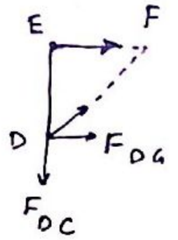


Figure 4.4

Once F_{DC} is known one can take a cut across EF, FD, DG and DC as shown in figure 4.4. Taking moment about point F we get

$$F_{DC} \times \frac{2.4}{2} + F_{DG} \times 2.4 = 0 \qquad (5)$$

ie $\quad F_{DG} = + \frac{F_{DC}}{2.4} \times 0.6 = +\frac{900}{4} = +225 \text{ (N)}.$

Note that in the last process we have not considered the zero force members. The special cut shown in figure 4.3 is normally used while solving a problem involving K-truss.

5. We first take a cut across DE, EL and LK and get FBD as shown in figure 5.1. Taking moment about point L we get

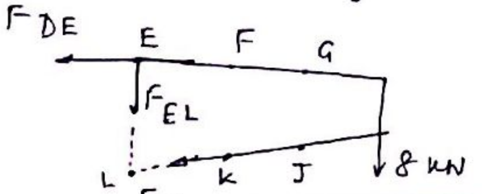


Figure 5.1.

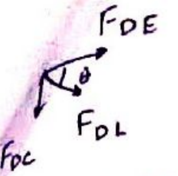
$$F_{DE} \times 2 - 8 \times 6 = 0 \qquad (1)$$

ie $\quad F_{DE} = 24$ (kN).

Since the force in DE is now known we can use equilibrium equations at joint D, shown in figure 5.2

From equilibrium equation we get

$$F_{DL} \cos\theta = -F_{DE} \quad , \left(\cos\theta = \cos 45^\circ = \frac{1}{\sqrt{2}}\right) \quad \cdots (2)$$



ie $F_{DL} = -24\sqrt{2}\,(kN) = -33.94\,(kN)$.

Figure 5.2

Note the power of the method of the section when it is combined with that of joint.

6. Here we take a cut across BC, BE and EF to get the free body as shown in figure 6.1. Taking moment about point E one gets

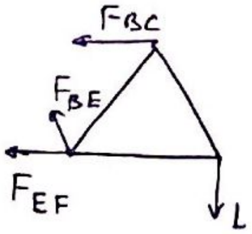


Figure 6.1.

$$F_{BC} \times \frac{\sqrt{3}}{2} a - L \times a = 0 \quad \cdots (1)$$

ie $F_{BC} = \frac{2L}{\sqrt{3}}$.

Similarly balancing the forces along vertical direction we get

$$F_{BE} \cos 30^\circ = L \quad \cdots (2)$$

ie $F_{BE} = \frac{2L}{\sqrt{3}}$.

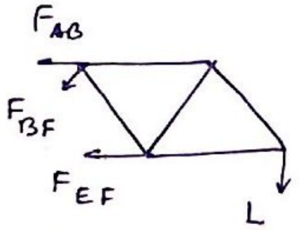


Figure 6.2

In order to get F_{BF} we take section by cutting across AB, BF and EF. The FBD is shown in figure 6.2. Considering vertical force balance equation we get

$$F_{BF} \cos 30^\circ + L = 0 \quad \cdots (3)$$

ie $F_{BF} = -\frac{2L}{3}$.

7. Since the force in member CD is known we can take a cut across CD, BD, BE and EF. The free body diagram of the part of the truss is shown in figure 7.1

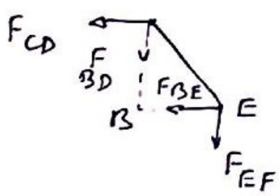


Figure 7.1.

Taking moment balance about point B we get

$$F_{CD} \times 16 - F_{EF} \times 16 = 0 \quad \cdots (1)$$

ie $F_{EF} = F_{CD} = 120\,(kN)$.

From force balance equation we get

$$F_{BE} = -F_{CD} = -120\,(kN) \quad \cdots (2)$$

and $F_{BD} = -F_{EF} = -120\,(kN) \quad \cdots (3)$

order to get the force in member BF consider the equilibrium equations at joint B (FBD is shown in figure 7.2)

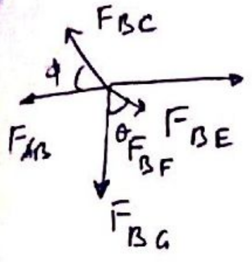


Figure 7.2

From geometry it may be seen that

$$\tan\theta = \frac{4}{3} = \tan\phi$$

Consider the equilibrium equation along horizontal direction. We get

$$F_{BF}\sin\theta - F_{BC}\cos\phi = F_{AB} - F_{BE}$$
$$= -50 + 120 = 70 \qquad \cdots (4)$$

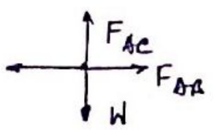


Figure 7.3

Consider next the FBD of joint A (shown in figure 7.3)

We see, at once, that

$$F_{AC} = W = 60 \times g \text{ (kN)}. \qquad \cdots (5)$$

Further, from the symmetry of the structure it may be concluded that

$$F_{CJ} = F_{BC}.$$

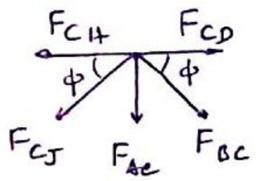


Figure 7.4

Consider now the joint at C. The FBD is shown in figure 7.4. The vertical force balance equation yields

$$F_{AC} + (F_{BC} + F_{CJ})\sin\phi = 0 \qquad \cdots (6)$$

ie
$$F_{BC} = -\frac{F_{AC}}{2\sin\phi} = -\frac{5W}{8}.$$

From equation (4) we get

$$F_{BF} = \left(70 - \frac{5W}{8} \times \frac{3}{5}\right) \times \frac{5}{4}$$
$$= \frac{350}{4} - \frac{15}{32}W.$$

Putting $g = 9.81 \ m/s^2$ we get

$$F_{BF} = -188.41 \text{ (kN)}.$$

The FBD's of the members are shown in figures 8.1 and 8.2

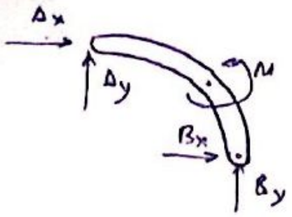


Fig 8.1

Considering moment balance equation about point A we get

$$B_x \times R + B_y \times R + M = 0 \quad \cdots ①$$

Similarly consider moment equation about point C in FBD shown in figure 8.2. We get

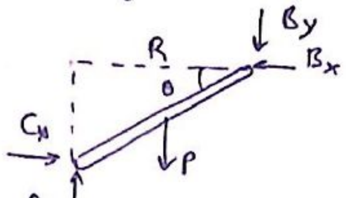


Figure 8.2

$$-P \times \frac{R}{2} - B_y \times R + B_x \times R \tan 30^\circ = 0 \quad \cdots ②$$

(a) When $B_x = 0$, we get

$$M = -B_y \times R = +\frac{PR}{2} \quad \text{ie} \quad M = 0.5PR \text{ (CCW)}$$

(b) When $B_y = 0$, we get

$$M = -B_x R = -PR \cdot \frac{\sqrt{3}}{2} = -0.866\,PR.$$

$$\text{ie} \quad M = 0.866\,PR \text{ (CW)}$$

9. It is first noted that BD is a two force member. Keeping this in mind the FBD's of the members are drawn as shown in figures 9.1 and 9.2.

Fig 9.1

Fig 9.2.

Taking moment about point A one gets

$$-F \sin\theta \times 0.5 - 250 \times 0.8 = 0 \quad \cdots ①$$

$$\text{ie} \quad F = -\frac{500 \times 0.8}{\sin 60^\circ} \quad \cdots ⓐ$$

Considering moment about point E we get

$$-R \times 0.2 - F \sin 60^\circ \times 0.4 = 0 \quad \cdots ②$$

$$\text{ie} \quad R = -\frac{F \sin 60^\circ}{0.2} \times 0.4 = -2F \sin 60^\circ \quad \cdots ⓑ$$

From equation ⓐ and ⓑ we get

$$R = 2 \times 500 \times 0.8 = 800 \text{ (N)}.$$

10. The cross piece of the bolt cutter is a two-force member. The frame is also symmetric about horizontal ~~p~~ line passing through the main pin. The free body diagrams of the two members are shown here (in figures 10.1 and 10.2.

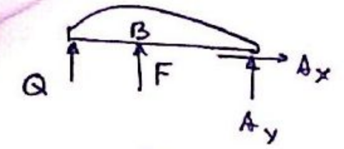


Figure 10.1

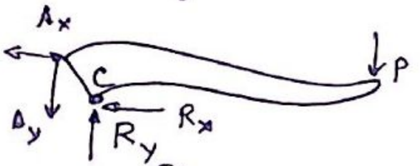


Figure 10.2.

Taking moment about point B, we get

$$A_y \times 60 = Q \times 20 \quad \cdots (1)$$

Also the horizontal force balance equation we get $\quad A_x = 0 \quad \cdots (2)$

Now considering moment equation about point C one gets

$$P \times 180 = A_y \times 30 \quad (\text{since } A_x = 0) \quad \cdots (3)$$

Thus, from equation (1) and (3) we get

$$P = \frac{A_y}{6} = \frac{Q}{18} \quad \cdots (a)$$

The force developed is $Q = 18\,P = 2700$ (N).

11. Here note that the member AB is a two-force member and ~~so is the piston~~. Thus the free body diagram of the handle of the hand pump will be as shown in figure 11.1 The FBD of the piston is shown in figure 11.2. Note that in the piston both side wise reaction force as well as a moment have been shown. The force P_{net} appearing in the FBD is the resultant force of the oil and the atmospheric pressure.

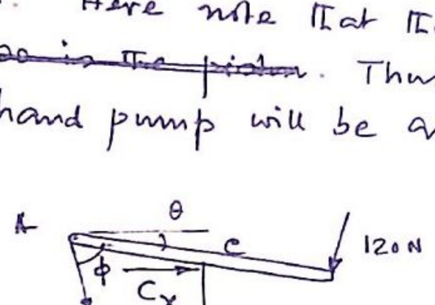


Figure 11-1

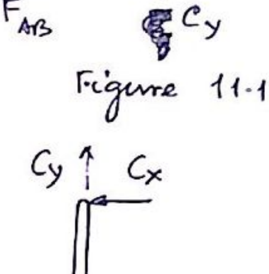
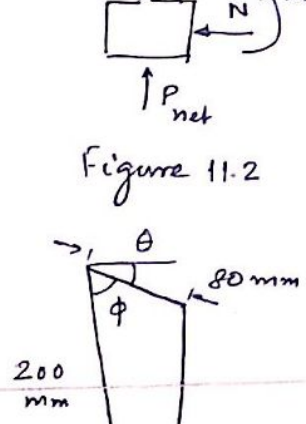


Figure 11.2



From geometry we get (see figure 11.3)

$$\cos(\theta + \phi) = \frac{80 \cos\theta - 40}{200} \quad \cdots (a)$$

ie $\quad \phi = 64.26°$.

Taking moment about point C in FBD of Figure 11.1 we get

$$F_{AB} \times \sin\phi \times 80 = 120 \times 200 \quad \cdots (1)$$

ie $$F_{AB} = \frac{120 \times 200}{80 \times \sin\phi} = 333.05\,(\text{N}).$$

The vertical force balance equation gives

$$C_y = -F_{AB} \sin(\theta + \phi) - 120 \times \cos\theta$$
$$= -443.126\,(\text{N}).$$

$P_{net} = -C_y = 443.126\,(N)$ (as seen from FBD of the piston)
we get

$$p = \frac{P_{net}}{\frac{\pi}{4} d_p^2} = 0.2666 \times 10^6 \frac{N}{m^2} = 0.27 \text{ MPa}.$$

12. Since member BC is a two-force member the free body diagrams of OD and AB ~~are~~ can be drawn as in figures 12.1 and 12.2.

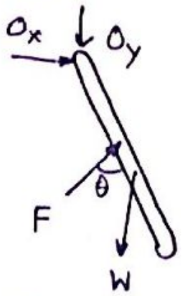


Figure 12.1

Taking moments about points O and A, we get the following two equations

$$F \sin\theta \times 0.4 - 80g \times 0.8 \sin 30^\circ = 0 \quad \text{--- (1)}$$

and, $$M - F \sin\theta \times 0.9 \quad \text{--- (2)}$$

Thus we get

$$M = 0.9 \times F\sin\theta = \frac{0.9}{0.4} \times 80g \times 0.8 \times \sin 30^\circ$$

$$= 72g = 706.32\,(\text{N-m})$$

assuming $g = 9.81 \frac{m}{s^2}$.

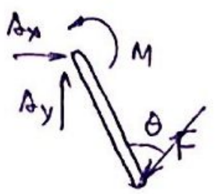


Figure 12.2

13. Since the frame is externally statically determinate, we can consider the entire frame to get reaction forces at A and F. The FBD is shown in figure 13.1.

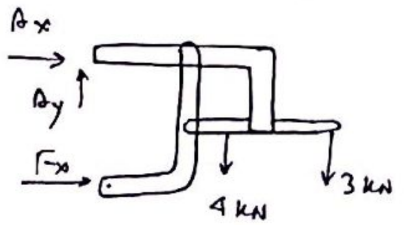


Figure 13.1

The following three equations are resulted from equilibrium

$$A_y = 7\,(kN) \quad (\Sigma F_y = 0) \quad \text{--- (1)}$$

$$A_x = -F_x \quad (\Sigma F_x = 0) \quad \text{--- (2)}$$

$$-(250 + 375) \times A_x - 4 \times (500 + 250) - 3 \times (500 + 500 + 250) = 0 \quad (\Sigma M_F = 0) \quad \text{--- (3)}$$

ie $A_x = -F_x = -10.8\,(kN)$.

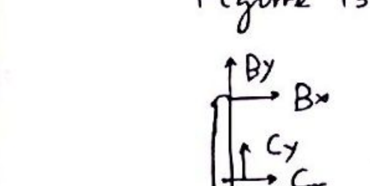


Figure 13.2

Consider, now, free body diagram of the member FCB as shown in figure 13.2. Taking moment about point C we get

$$-B_x \times 250 + F_x \times 375 = 0, \quad \text{--- (4)}$$

ie $B_x = 16.2\,(kN)$.

∴ the force balance equation we further get

$$C_x = -(B_x + F_x) = -(10.8 + 16.2) = -27 \,(\text{kN}).$$

We finally consider the free body diagram of member CDE as shown in figure 13.3

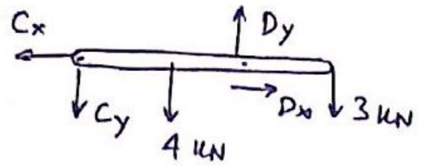


Figure 13.3

Taking moment about point C we get

$$-4 \times 250 + D_y \times 500 - 3 \times 750 = 0 \quad \cdots (5)$$

ie $\quad D_y = 6.5$ (kN).

Again from the force balance equation from the same FBD one gets

$$C_y = D_y - 7 = 6.5\,(\text{kN}) - 7\,(\text{kN}) = -1.5\,(\text{kN})$$

and $\quad C_x = D_x = -27$ (kN).

Consider once again the FBD of member FCB in figure 13.2. The vertical force balance equation gives

$$B_y = -C_y = 1.5 \,(\text{kN}).$$

Thus, the forces on the member ABD, whose FBD is shown in figure 13.4 The reaction forces in different joints are as given below

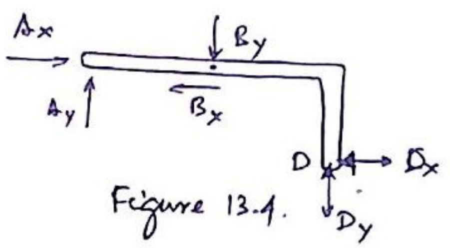


Figure 13.4.

At A : $\quad A_x = 10.8$ kN (←)

$\qquad A_y = 7$ kN (↑)

At B : $\quad B_x = 16.2$ kN (←)

$\qquad B_y = 1.5$ kN (↓)

At D : $\quad D_x = 27$ kN (→)

$\qquad D_y = 6.5$ kN (↓)

14. Consider FBD of AB as shown in figure 14.1. Note that ED is a two force member. The reactions at B are written along two orthogonal directions, not along x- and y- axis. Since the force at B is a single force with unknown magnitude and direction we can resolve it along any two orthogonal directions with unknown components.

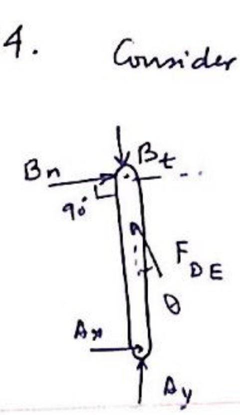


Figure 14.1

Further, B_n is the pressure applied by the piston.

ie $\quad B_n = \frac{\pi}{4} \times (0.05)^2 \times 400$ kN.

...ing moment about point A we get

$$F_{DE} \sin\theta \times (AE) = (AB) \times B_n = 0.2 B_n \quad (1)$$

From geometry (see figure 14.2)

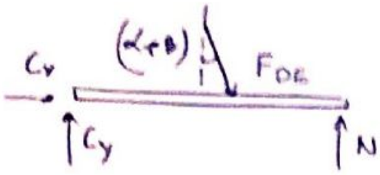
Fig 14.2

$\alpha = \theta$, since $BD = CD = 75$ mm

and $AE = 2 \times (AD) \times \cos\alpha$

Thus
$$F_{DE} = \frac{0.2 B_n}{\sin\alpha \times 2 \times (AD) \times \cos\alpha}$$

Next, consider the free body diagram of member CG as shown in figure 14.3.



Taking moment about point C we get

$$F_{DE} \times \cos(\alpha+\theta) \times (CD) = N \times (CG)$$

ie
$$N = F_{DE} \cos 2\alpha \times \frac{CD}{CG}$$

$$= \frac{0.2 \times B_n}{0.075 \times \sin 2\alpha} \times \cos 2\alpha \times \frac{0.1}{0.25}$$

$$= 2.30 \text{ (kN)}$$

— END —