Questions 1 to 8 carry one mark each (NO part marks in this section)

1. A simply supported beam is subjected to point load at its mid-point. If the length of the beam is doubled, the maximum bending moment ___increases___ (increases/decreases) by a factor of 2.

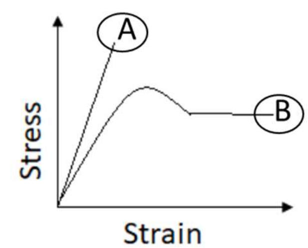
2. A simply supported beam is subjected to point load at its mid-point. If the length of the beam is doubled, the maximum shear force increases (increases/decreases) by a factor of 1.

(decreases by a factor of 1, remains same are also correct answers)

3. A brass structure is subjected to load resulting in stresses, $\sigma_{xx} = 10\ MPa, \sigma_{yy} = -23 MPa\ and\ \tau_{xy} = 0$. What is the shear strain, γ_{xy} developed in the brass structure corresponding to this state of stress?

Shear strain is zero

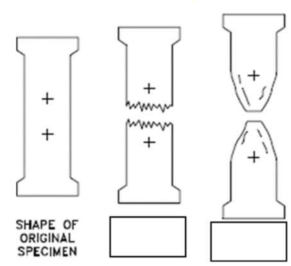
Materials A and B were subjected to tensile tests and the following stress strain curves were obtained (see adjacent figure). ***Study the stress strain curves carefully and answer questions 4, 5 and 6***



4. Which material is brittle (A and/or B)? A

5. Which material has higher Young's/Elastic modulus (A/B/Can't say)? A

6. In the figure, the shape of the original specimen and the fractured specimen is provided. In the box, below the fractured specimen image, mention which image belongs to which material (A / B)



A B

7. How many quantities/components are needed to completely define the state of stress at a point for a general 3-Dimensional loading? The material is homogenous, isotropic (Just write the number, no need to name the quantities/components) 6

8. Fill in the blanks, 1 N/m^2 = __10^{-6}_MPa

Questions 9 to 12 carry three marks each

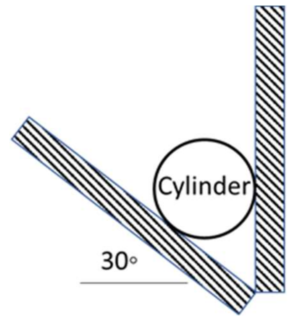
9. Write the approximate values of the following quantities with proper units, if any

Poisson's ratio of an incompressible solid 0.5

Young's modulus of steel any number between 175 -240 GPa

Young's modulus of aluminum any number between 60 -105 GPa

10. A cylinder is supported by a vertical wall and inclined plane as shown in the figure. All surfaces are rough. Enlist all possible impending motions when the cylinder in figure is subject to a counter clockwise torque



-rotation about own axis

- roll without slip towards left

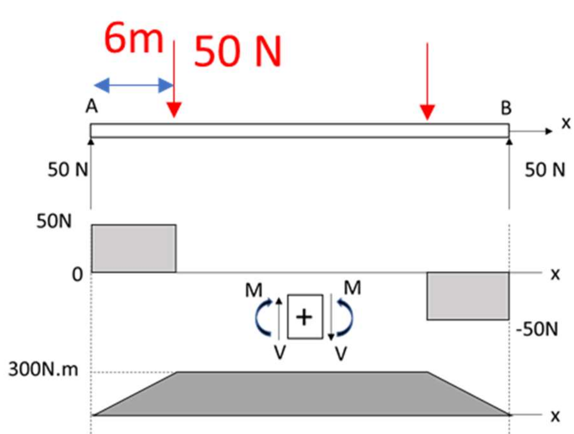
11. As part of a large assembly, a single shear pin (diameter d) connected to plate with thickness, t has reactions equal to X and Y N in the x and y directions. What is the expression for bearing stress developed in the plate with thickness, *t*?

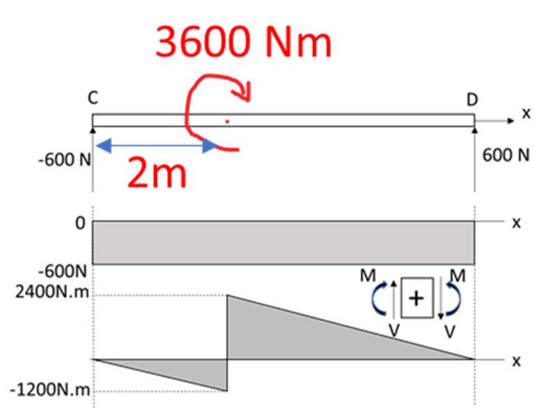
$$\frac{\sqrt{(X^2+Y^2)}}{dt}$$

12. The Young's modulus, E of a material is 85 GPa, and the Poisson's ratio, ν is 0.33. What is the value of the bulk modulus? Give magnitude with proper units.

$$k=\frac{E}{3(1-2\nu)}$$

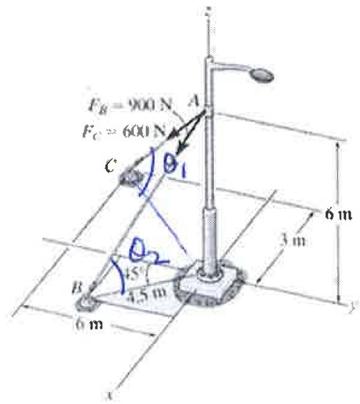
13. The two beams AB and CD are subjected to certain external loadings and the corresponding reaction forces. The applied forces are not shown in the figure. The corresponding shear force and bending moment diagrams for each beam is provided below the beam with the sign convention chosen as shown. In each of the two beams, sketch the location(s), the nature (point load/ couple/uniformly distributed load), direction and magnitude of the of external load(s). **[4×2=8 Marks]**





14. A pole is loaded as shown in the figure, what is the axial reaction force at point 3 meter below the point A?

[6 Marks]



Axial reaction force

$= 900 \sin\theta_1 + 600 \sin\theta_2$

$= 900 \times \frac{4}{5} + 600 \times \frac{2}{3}$

$= 1120 N.$

$\tan\theta_1 = \frac{6}{4.5}$

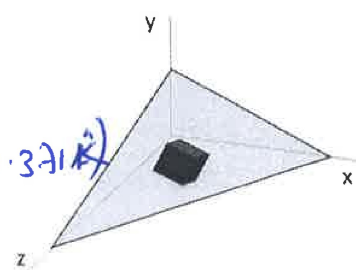
$\sin\theta_1 = 4/5$

$\tan\theta_2 = \frac{6}{\sqrt{6^2+3^2}}$

$\sin\theta_2 = 2/3$

---

15. The 5-kg box is at rest on the sloping surface. The y axis points upward. The unit vector 0.557i + 0.743j +0.371k is perpendicular to the sloping surface. What is the magnitude of the friction force exerted on the box by the surface? **[6 Marks]**



$\vec{N} = N_{mag}(0.557\hat{i} + 0.743\hat{j} + 0.371\hat{k})$

$\vec{w} = -(49.05)\hat{j}$

$\Sigma F = \vec{f} + \vec{N} + \vec{w} = 0$

$\vec{f} = -w - N$

Also, f (friction force) parallel to surface

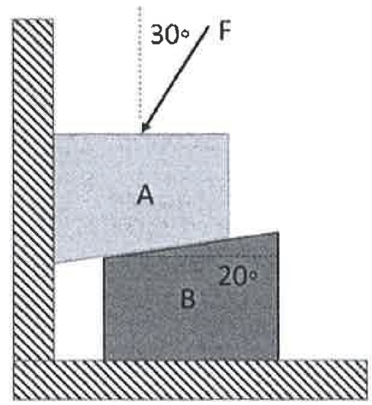
$\vec{f} \cdot (0.557\hat{i} + 0.743\hat{j} + 0.371\hat{k}) = 0$

$\Rightarrow N_{mag} = 36.45 N$

$f = (-20.31\hat{i} + 22\hat{j} - 13.52\hat{k})$

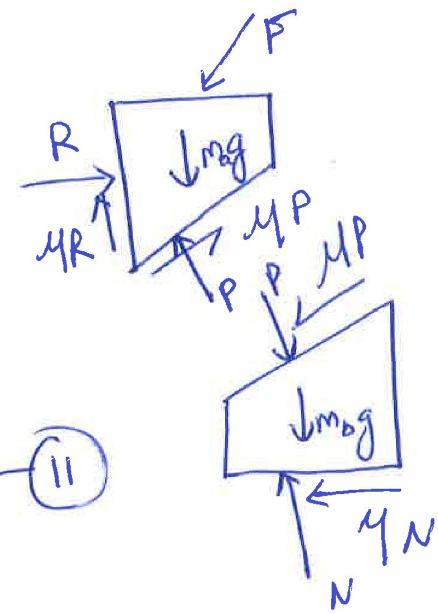
$\|f\| = 32.83 N$

16. The masses of the blocks are m_A=30 kg and m_B=70 kg. Between all of the contacting surfaces, μ_s= 0.1. The force F, is applied at an angle of 30° with the vertical, the angle of the wedge is 20°. What is the largest force F that can be applied without causing the blocks to slip?



Draw the relevant Free body diagram(s), write the relevant equations of equilibrium **[10 Marks]**

For block A,



$\Sigma F_x = 0 \Rightarrow$

$R - F\sin 30^\circ + \mu_s P\cos 20 - P\sin 20 = 0$ — (I)

$\Sigma F_y = 0 \Rightarrow$

$\mu R - F\cos 30 - m_A g + P\cos 20^\circ + \mu_s P\cos 20^\circ = 0$ — (II)

Similarly block B,

$\Sigma F_x = 0 \Rightarrow$

$P\sin 20 - \mu P\cos 20 - \mu N = 0$ — (III)

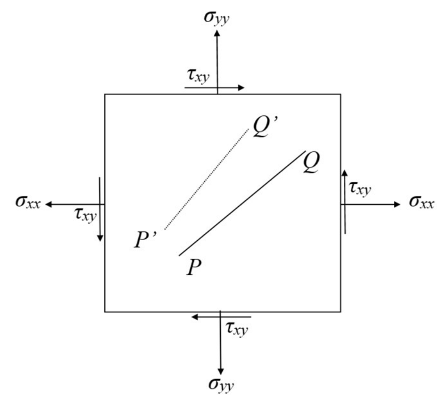
$\Sigma F_y = 0 \Rightarrow$

$-P\cos 20 - \mu P\sin 20 - m_B g + N = 0$ — (IV)

Solving (I), (II), (III) and (IV)

$R = 212N$, $P = 456N$, $N = 1130N$ and $\underline{\underline{F = 197N}}$

17. An object is subjected to external loading in such a way that the stress within a small part within the body is σ_{xx}, σ_{yy}, and τ_{xy}. Under the action of these stresses, the strains developed within the body are: ϵ_{xx}, ϵ_{yy} and ϵ_{xy}, respectively. Consider an imaginary line PQ within the body. The initial coordinates of P are: (x, y) and those of Q are: $(x+\Delta_x, y+\Delta_y)$. Post straining, the points P and Q get displaced to new points P' $(x+u_x, y+u_y)$ and Q' $(x+\Delta_x + u_x+\Delta u_x, y+\Delta_y+u_y+\Delta u_y)$. If the initial direction cosines of the line PQ are n_x, and n_y, then does the following expression hold true (show your derivations):



$$\frac{P'Q'-PQ}{PQ} = \varepsilon_{xx}n_x^2 + \varepsilon_{yy}n_y^2 + \varepsilon_{xy}n_x n_y$$

[15 Marks]

$$
\begin{aligned}
\left(P'Q'\right)^2 &= \left(\Delta x + \Delta u_x\right)^2 + \left(\Delta y + \Delta u_y\right)^2 \\
&= \Delta x^2 + 2\Delta x \Delta u_x + \Delta u_x^2 + \Delta y^2 + 2\Delta y \Delta u_y + \Delta u_y^2 \\
&= \left(\Delta x^2 + \Delta y^2\right) + 2\left(\Delta x \Delta u_x + \Delta y \Delta u_y\right) + \left[\Delta u_x^2 + \Delta u_y^2\right] \\
&\sim \left(\Delta x^2 + \Delta y^2\right) + 2\left(\Delta x \Delta u_x + \Delta y \Delta u_y\right) \\
&= PQ^2 + 2\left(\Delta x \Delta u_x + \Delta y \Delta u_y\right)
\end{aligned}
$$

Here, we have neglected the terms $\left[\Delta u_x^2 + \Delta u_y^2\right]$ under the assumption that $\Delta u_x \ll 1$ and $\Delta u_y \ll 1$. Now, we have to relate Δu_x and Δu_y with the corresponding strains. This is achieved by expanding the terms as follows:

$$
\begin{aligned}
\Delta u_x &= \frac{\partial u_x}{\partial x}\Delta x + \frac{\partial u_x}{\partial y}\Delta y, \\
\Delta u_y &= \frac{\partial u_y}{\partial y}\Delta y + \frac{\partial u_y}{\partial x}\Delta x.
\end{aligned}
$$

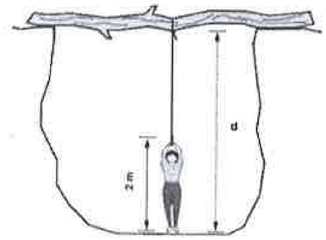
Substituting back in the previous equation we get:

$$
\begin{aligned}
\left(P'Q'\right)^2 &= PQ^2 + 2\left[\Delta x\left(\frac{\partial u_x}{\partial x}\Delta x + \frac{\partial u_x}{\partial y}\Delta y\right) + \Delta y\left(\frac{\partial u_y}{\partial y}\Delta y + \frac{\partial u_y}{\partial x}\Delta x\right)\right] \\
\implies \left(P'Q'\right)^2 - PQ^2 &= 2\frac{\partial u_x}{\partial x}\Delta x^2 + 2\frac{\partial u_y}{\partial y}\Delta y^2 + 2\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\Delta x \Delta y \\
\left(P'Q' + PQ\right)\left(P'Q' - PQ\right) &= 2\frac{\partial u_x}{\partial x}\Delta x^2 + 2\frac{\partial u_y}{\partial y}\Delta y^2 + 2\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\Delta x \Delta y \\
\sim 2PQ\left(P'Q' - PQ\right) &= 2\frac{\partial u_x}{\partial x}\Delta x^2 + 2\frac{\partial u_y}{\partial y}\Delta y^2 + 2\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\Delta x \Delta y \\
\frac{\left(P'Q' - PQ\right)}{PQ} &= \frac{\frac{\partial u_x}{\partial x}\Delta x^2 + \frac{\partial u_y}{\partial y}\Delta y^2 + \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\Delta x \Delta y}{PQ^2}
\end{aligned}
$$

We know that $n_x = \Delta x / PQ$, $n_y = \Delta y / PQ$, $\epsilon_{xx} = \frac{\partial u_x}{\partial x}$, $\epsilon_{yy} = \frac{\partial u_y}{\partial y}$, and $\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$. Substituting everything back, we get:

$$
\frac{\left(P'Q' - PQ\right)}{PQ} = \epsilon_{xx} n_x^2 + \epsilon_{yy} n_y^2 + 2\epsilon_{xy} n_x n_y
$$

18. An explorer lowers herself into a pit using the arrangement shown in Figure, comprising a rigid tree trunk and a massless linearly elastic rope of undeformed hanging length of 5 meter and undeformed diameter of 20 mm. When her hands reach the end of the rope, as shown in Figure, her toes just reach the bottom of the pit (without touching it). If the weight of the explorer is 600 N and her stretched length is 2 meter (as shown in the figure), estimate the depth, d of the pit (see figure), if the Young's modulus of the rope is known to be 10 MPa **[6 Marks]**



Elongation of the rope $= \dfrac{600 \times 5 \times 10^3}{\frac{\pi}{4} \times 20^2 \times 10}$

$= 955\,mm$

$\therefore$ Pit depth $= 5000 + 2000 + 955\,mm$

$= 7955\,mm$

$= 7.955\,m$

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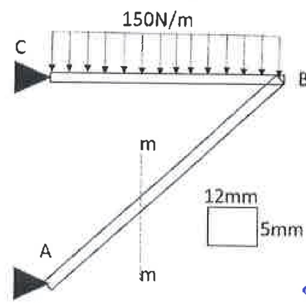
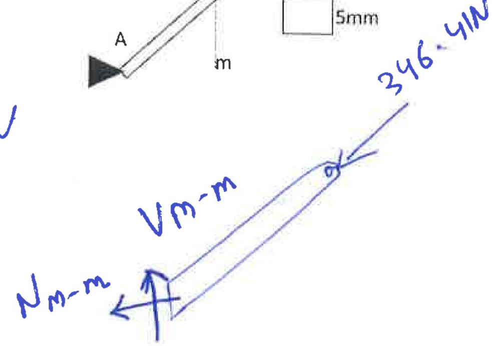
19. Consider a frame with loading as shown in in the figure, all the joints are pin-joints. Member BC is 4 meters long while angle CBA is 60°. The width and thickness of the member BC is 12 and 5 mm respectively . Calculate the normal and shear stress in member AB along vertical section m-m as shown **[6 Marks]**



From member BC,

$\Sigma M_C = 0$ (+) $\quad F_{AB} = 346.41\,N$



$N_{m-m} = 173.21\,N$

$V_{m-m} = 300\,N$

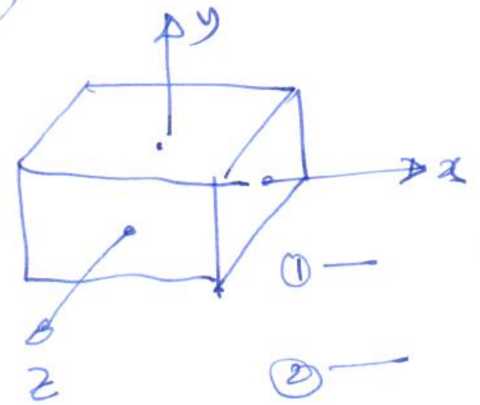
$A_{m-m} = \dfrac{(12 \times 5)}{\cos 60} = 120\,mm^2$

$\sigma_{m-m} = \dfrac{N_{m-m}}{A_{m-m}} = 1.44\,N/mm^2$

$\tau_{m-m} = 1.5\,N/mm^2$

(1)

Prob. (19) Solution



$\sigma_y = \Delta$

$\epsilon_x = \epsilon_z = 0$

(1) — $\sigma_x - \nu(\sigma_y + \sigma_z) = E\epsilon_x = 0$

(2) — $\sigma_y - \nu(\sigma_x + \sigma_z) = E\epsilon_y = E\Delta$

(3) — $\sigma_z - \nu(\sigma_x + \sigma_y) = E\epsilon_z = 0$

from (1) and (3) (subtracting)

$$\sigma_x - \sigma_z - \nu(\sigma_y + \sigma_z) + \nu(\sigma_x + \sigma_y) = 0$$

$$\sigma_x - \sigma_z - \nu\left[\cancel{\sigma_y} + \sigma_z - \sigma_x - \cancel{\sigma_y}\right] = 0$$

$$\sigma_x + \nu\sigma_x - \sigma_z - \nu\sigma_z = 0$$

$$\sigma_x(1+\nu) - \sigma_z(1+\nu) = 0$$

$$\Rightarrow \boxed{\sigma_x = \sigma_z} \quad (4)$$

from (2) and (4)

$$\sigma_y - \nu(\sigma_x + \sigma_x) = E\Delta$$

$$\sigma_y - 2\nu\sigma_x = E\Delta$$

$$2\nu\sigma_x = \sigma_y - E\Delta$$

$$\sigma_x = -\frac{E\Delta}{2\nu} + \frac{\sigma_y}{2\nu} \quad (5)$$

inserting (5) into (3) and using $\sigma_x = \sigma_z$

$$\sigma_x - \nu\sigma_x - \nu\sigma_y = 0 \qquad (1-\nu)\left(-\frac{E\Delta}{2\nu} + \frac{\sigma_y}{2\nu}\right) = \nu\sigma_y$$

$$\Rightarrow \nu\sigma_y - (1-\nu)\frac{\sigma_y}{2\nu} = -\frac{E\Delta}{2\nu}(1-\nu)$$

$$\left[\nu - \frac{1-\nu}{2\nu}\right]\sigma_y = -\frac{E\Delta}{2\nu}(1-\nu)$$

$$\left(\frac{2\nu^2 - 1 + \nu}{\cancel{2\nu}}\right)\sigma_y = -\frac{E\Delta(1-\nu)}{\cancel{2\nu}}$$

$$\sigma_y = -\frac{E\Delta(1-\nu)}{(2\nu^2 - 1 + \nu)} = \frac{E\Delta(1-\nu)}{(1-\nu-2\nu^2)}$$

$$\sigma_y = \frac{E\Delta(1-\nu)}{(1+\nu)(1-2\nu)} \Rightarrow \frac{F_0}{4a^2} = \frac{Ec(1-\nu)}{L_0(1+\nu)(1-2\nu)}$$

$$\boxed{F_0 = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \frac{4Eca^2}{L_0}}$$

now $\Delta = \frac{c}{L_0}$

$\sigma_y = \frac{F}{4a^2}$