Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 10
Autumn 2022

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1. Discuss the convergence of improper integrals using definition:

(i) $\int_{1}^{\infty} \frac{1}{x^4} dx$

(ii) $\int_{10}^{\infty} \frac{1}{x \ln x} dx$

(iii) $\int_{0}^{1} \frac{\ln x}{x^2} dx$

(iv) $\int_{-\infty}^{\infty} \frac{x}{(x^2+1)} dx$

(v) $\int_{1}^{2} \frac{4x}{(x^2-4)^{\frac{1}{3}}} dx$

(vi) $\int_{1}^{\infty} \frac{x+1}{x^{\frac{3}{2}}} dx$

(vii) $\int_{-\infty}^{0} \frac{1}{(3-x)^{\frac{1}{2}}} dx$

(viii) $\int_{-2}^{\infty} \sin(x) dx$

(ix) $\int_{1}^{\infty} \frac{1}{(1+x)\sqrt{x}} dx$

(x) $\int_{1}^{2} \frac{1}{x \ln^2 x} dx$

2. Discuss the convergence of the following integrals :

(i) $\int_{0}^{1} \frac{1}{x^2+\sqrt{x}} dx$

(ii) $\int_{0}^{\infty} \frac{1}{x+e^x} dx$

(iii) $\int_{0}^{\infty} \frac{1}{x^2+xe^x} dx$

(iv) $\int_{0}^{\infty} \frac{1-\cos(x)}{x^2} dx$

(v) $\int_{1}^{\infty} \frac{x}{(1+x)^3} dx$

(vi) $\int_{1}^{\infty} \frac{x}{3x^4+5x^2+1} dx$

(vii) $\int_{-\infty}^{\infty} e^{-|x|} dx$

(viii) $\int_{0}^{\infty} \frac{\cos x}{e^x} dx$

(ix) $\int_{1}^{\infty} e^{x+x^{-1}} dx$

(x) $\int_{0}^{1} \frac{e^x}{x^2} dx$

3. Examine the convergence of the following integrals :

(i) $\int_{0}^{1} \frac{1}{(x+2)\sqrt{x(1-x)}} dx$

(ii) $\int_{0}^{\infty} x^{-\frac{1}{2}} e^{-x} dx$

(iii) $\int_{1}^{\infty} \frac{1}{x^{\frac{1}{2}}(1+x)^{\frac{1}{4}}} dx$

(iv) $\int_{0}^{\infty} \frac{\cos(x)}{\sqrt{x^3+x}} dx$

(v) $\int_{0}^{1} \frac{x^{p-1}}{1-x} dx$

4. Prove that $\int_{0}^{\frac{\pi}{2}} \frac{x^m}{\sin^n x} dx$ is convergent iff $n < m+1$

5. Show that the improper integral $\int\limits_0^1 \frac{\sin(\frac{1}{x})}{\sqrt{x}} dx$ is convergent.

6. Prove that the integral $\int\limits_0^\infty \left(\frac{1}{x+1} - \frac{1}{e^x}\right) \frac{1}{x} dx$ is convergent

7. Test the convergent of $\int\limits_0^\infty e^{-x^2} dx$

8. Explain the convergence of $\int\limits_0^1 \frac{\ln x}{\sqrt{x}} dx$

9. Show that $\int\limits_0^1 x^{m-1}(1-x)^{n-1} dx$ is convergent iff m, n are both positive.

10. Show that $\int\limits_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}(bx)}{x} dx = \frac{\pi}{2} \log(\frac{a}{b}) \quad 0 < b < a$

11. Prove that $\int\limits_0^\infty \frac{\sin(x)(1-\cos(x))}{x^2} dx = \log 2$