The mass in a spring mass system performs simple harmonic oscillations along a straight line with a period (T) of 0.5 s and an amplitude (A) of 5 cm. The magnitude of the average velocity of the mass averaged over the time interval during which it travels a distance A/2, starting from (i) the extreme position (ii) the equilibrium position (Answer should be correct up to two decimal places) Ans. (i) 0.3 m/s (ii) 0.6 m/s
A particle of mass m = 0.3 kg in the potential V (x) $=x^3+x^2\exp(\frac{x^2}{L^2})$ (L= 0.9 m) is found to behave like a simple harmonic oscillator for small displacements from equilibrium. The period of the simple harmonic oscillator is (Answer should be correct up to two decimal places) Ans: 2.43 s range: 2.35 to 2.55
A mass of 1.5 kg attached to a spring with spring constant 500 N/m produces critical damping. If the mass at equilibrium position receives an impulse that gives it a velocity of 1.5 m/s at t=0 then the maximum value of the resultant displacement is (Take $e \approx 2.72$) (Answer should be correct up to two decimal places) Ans: 0.03 m 0.025 m to 0.035 m

Solutions:

1. $T = 0.5\,s$; $A = 5\,cm$.

(i) Extreme position:

$$x = A\cos(\omega t + \phi)$$

Amplitude : A to $A/2$ $\Rightarrow$ $t = -\dfrac{\phi}{\omega}$ to $t = \dfrac{\pi}{3\omega} - \phi/\omega$

$$\langle v \rangle = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v\,dt$$

$$\langle v \rangle = -\frac{3A}{T} = -0.3\ m/s$$

$$|\langle v \rangle| = \underline{0.3\ m/s}$$

(ii) Equilibrium position

$$x = A\sin(\omega t + \phi)$$

Amplitude : 0 to $A/2$ $\Rightarrow$ $t = -\phi/\omega$ to $t = \dfrac{\pi}{6\omega} - \phi/\omega$

$$|\langle v \rangle| = \frac{6A}{T} = \underline{0.6\ m/s}.$$

2.

$$V(x) = x^3 + x^2 \exp\left(\frac{x^2}{L^2}\right) \qquad L = 0.9\ m.$$

Equilibrium position: $\left.\frac{dV}{dx}\right|_{x_0} = 0 \Rightarrow x_0 = 0$

$k = \left(\frac{d^2V}{dx^2}\right)_{x_0=0} = 2$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2.43\ s$$

3. $m = 1.5$ kg $\quad k = 500$ N/m.

$\beta = w_0$ (critical damping)

$$\beta = \sqrt{\frac{k}{m}} = \sqrt{\frac{500}{1.5}} = 18.25\ rad\,s^{-1}$$

$$x(t) = (A + Bt)e^{-\beta t}$$

@ $t = 0$, $x = 0$, $\dot{x} = 1.5$

$\Rightarrow A = 0, B = 1.5$

$\left.\frac{dx}{dt}\right|_{t_{max}} = 0 \Rightarrow t_{max} = \frac{1}{\beta}$

$\Rightarrow x_{max} = 0.03\,m$

1. A particle of mass *m* attached to a spring executing a damped oscillatory motion. The spring constant is 41 N/m^2, mass of the particle is 0.5 Kg, and the friction constant (r) is 1 N s/m^2 . (The initial conditions x(0) = 2, v(0) = 0).

i) Find the magnitude of the phase factor α.

ii) Find the amplitude of the oscillations.

Ans: It is an under damped motion. So the solution is $x(t) = A\, e^{-\beta t} \cos(\omega t - \alpha)$
Since *x(0) and v(0)* are given we can obtain the amplitude and phase (in radians).

Phase ~ 0.11 radians and $A = 2/\cos(\alpha)$ ~2.01 meters

Range: 0.10 to 0.12

Range: 2 to 2.1

3. A spring mass system, in absence of external force, exhibiting damped harmonic motion. At $t = 0$, it starts with an initial energy E_0 and after a time t_1 the energy drops to $E_0/5$. Taking mass of the block as 2 Kgs, spring constant 10 N/m, and friction constant (r) 1 N s/m^2 find the time t_1.

Ans: $E = E_0$ Exp(- 2beta t) = E_0 Exp(- r t/(m))

E_0 /5 = E_0 Exp(- 2beta t_1)

ln(5) = 2beta t_1

t1 = ln(5)/2beta = ln(5)*m/r = 3.21 sec

3.1 to 3.3

Questions: Class Test # 1

1. An oscillating LC circuit consists of a 7.5 mH inductor and a 3 μF capacitor. If the maximum charge on the capacitor is 3 μC, the maximum current would be

Ans: $$I_{\max} = \omega q_{\max} = \frac{q_{\max}}{\sqrt{LC}} = \frac{3\mu}{\sqrt{7.5\,mH \times 3\,\mu F}} = 20\,mA$$

2. An electrical circuit L=0.25 H, C=1 μF and R=1 Ω connected in a series. (a) How many oscillations will it make before the amplitude of the current is reduced by a factor of e. (b) Estimate the quality factor for the above LCR circuit.

Ans: $$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{0.25} = 4\times10^6, \quad \&, \; \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{1}{0.5}\right)^2 = 4$$

$$\Rightarrow \omega_0^2 > \beta^2, \text{an under dampped case}$$

The current amplitude varies as, $I = e^{-\beta t}(A\cos\omega t + B\sin\omega t)$

where, $\omega = \sqrt{\omega_0^2 - \beta^2}$

Suppose N complete oscillation occurred for time *t,*

$$t = NT = \frac{N\,2\pi}{\omega} \approx \frac{N\,2\pi}{\omega_0}$$

If the current amplitude decreased by e times,

$$-\beta t = -1,$$

$$-\frac{R}{2L}\frac{N\,2\pi}{\omega_0} = -1, \Rightarrow N = \frac{2L\,\omega_0}{2\pi R} = \frac{0.25\times2\times10^3}{3.14\times1} = 0.16\times10^3 = 160$$

159 to 161

499 to 501

The quality factor, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{0.25}{1}\times10^6} = 0.5\times10^3 = 500$$

3. An AC source with amplitude 200 V and angular frequency 200 rad/s is connected to a series circuit consisting of resistance 150 Ω, a coil with inductance 1 H, and a capacitor of capacitance 100 μF. Estimate the current amplitude in the circuit, the voltage amplitude on the capacitor.

Ans: $$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{100} = 10^4, \quad \&, \ \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{250}{2}\right)^2 = 125\times 125$$

$$\Rightarrow \omega_0^2 < \beta^2, \text{an over dampped case}$$

The current amplitude is,

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} = \frac{200}{\sqrt{150^2 + (200-50)^2}} = \frac{200}{150\sqrt{2}} = 0.943\,\text{A}$$

0.93 to 0.95

The amplitude of the voltage oscillations on the capacitor will be equal to

$$V_c = \frac{q_0}{C} = \frac{I_0}{\omega C} = \frac{0.943}{200\times 100}\times 10^6 = 47.1\,\text{V}$$

46.9 to 47.2

1. The graph shows the power resonance curve of a certain mechanical system which is driven by a force of constant magnitude but variable angular frequency ω. Find the following:

(a) The resonant quality factor (Q) of the system. (1 Mark)

Ans: 50

(b) At some instant of time, the driving force is removed when the energy of the system is E0. Find the time taken for the energy of the system to decrease to E0e−2 (1 Mark)

Ans: 1 s

1 (a) $Q = \dfrac{\omega_0}{2\beta}$ where $2\beta = \omega_2 - \omega_1$

(b) Energy of a damped SHO decreases as $E_0 e^{-2\beta t}$

2. An object of mass 4 kg is attached to a spring of spring constant '(4/3) × 10^3 N/m' and is acted upon by an external force F= F_0 cos(ωt) where F_0 is 8/3 N. The resonant quality factor (Q) of the system is 20. Once steady state has been reached:

(a) What is the amplitude of forced oscillations at $\omega = \omega_0$? (ω_0 is the angular frequency of the free undamped oscillations) **(1 Mark)**

Ans: 4 cm

Range: 3.8 to 4.2

(b) What is the mean power input to maintain the forced oscillations at an angular frequency two percent greater than ω_0. [Take g = 10 ms^{-2}] **(1 Mark)**

Ans: 0.59 Watt

Range: 0.54 to 0.64

2 (a) $$A_{max} = \frac{F_0}{m(2\beta)\,\omega_{res}}$$ where

$$\omega_{res} = \omega_0\sqrt{1 - \frac{1}{2Q^2}} \approx \omega_0$$ as Q is large

ω_0 and 2β can be found since 'k', m and Q are provided.

(b) $$\langle P \rangle = \frac{F_0^2 \beta}{m} \frac{1}{\left(\frac{\omega_0^2}{\omega} - \omega\right)^2 + (2\beta)^2}$$

3. A damped simple harmonic oscillator has a natural frequency ω_0 and is driven by an external force with a variable frequency ω. It is observed that the amplitude has a maximum at a frequency ω_1 while the velocity has a maximum at a frequency ω_2. Which of the following is true:

(3) From the expression for amplitude & velocity as a function of freq. it can be seen that (i) $\omega_0 = \omega_2$ but $\omega_1 \neq \omega_2$

(ii) $\omega_1 \neq \omega_0$ and $\omega_1 \neq \omega_2$