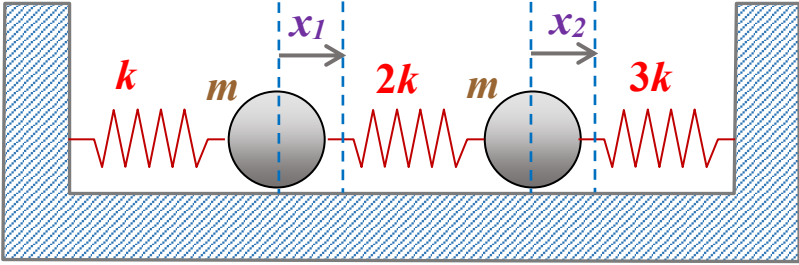
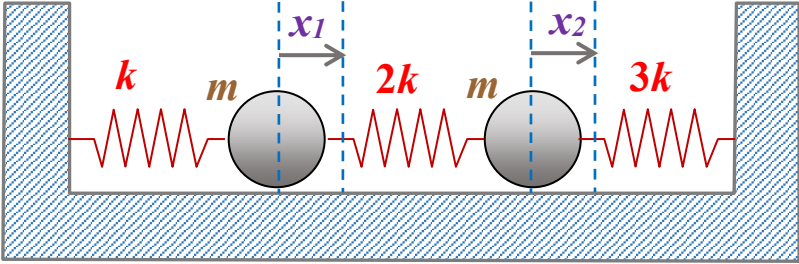
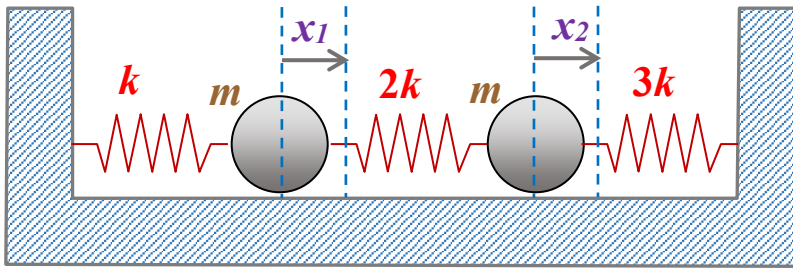


1A	<p>Consider the forced oscillation of a particle described by the equation:</p> <p><b><math>0.5\ddot{x} + 8x = 10 \cos(3t)</math></b> with the initial conditions: <b><math>x(0) = 0, \dot{x}(0) = 0</math></b>. The general solution (complementary function + particular integral) of this equation of motion can be written as: <b><math>x(t) = C_1 \cos(4t) + C_2 \sin(4t) + C_3 \cos(3t)</math></b>. Then the value of <b><math>C_1</math></b> is given by <b><math>C_1 = \text{-----}</math></b></p> <p>Ans. = <b>-2.857</b> (-20/7) (Range: -2.6 to -3.0)</p> <p><b>Solutions (1A), (1B), (1C):</b></p> <p>This equation has the complementary solution</p> <p><math>x_c = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t)</math> where <math>\omega_0 = \sqrt{k/m} = \sqrt{8/0.5} = 4</math></p> <p>And the particular solution <math>x_p = \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos(\omega t)</math></p> <p>The general solution is therefore, <math>x = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) + \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos(\omega t)</math></p> <p>with <math>F_0 = 10</math> and <math>\omega = 3</math> giving <math>\frac{F_0}{m(\omega_0^2 - \omega^2)} = \frac{20}{16-9} = \frac{20}{7}</math></p> <p><math>x = C_1 \cos(4t) + C_2 \sin(4t) + C_3 \cos(3t)</math> where <math>C_3 = \frac{20}{7} = 2.857</math></p> <p>Using initial conditions: <math>x(0) = 0, \dot{x}(0) = 0</math> we see <math>C_1 = -C_3 = -20/7</math> and <math>C_2 = 0</math></p>	1 Marks	Part-A
1B	<p>Consider the forced oscillation of a particle described by the equation:</p> <p><b><math>0.5\ddot{x} + 8x = 10 \cos(3t)</math></b> with the initial conditions: <b><math>x(0) = 0, \dot{x}(0) = 0</math></b>. The general solution (complementary function + particular integral) of this equation of motion can be written as: <b><math>x(t) = A_1 \cos(4t) + A_2 \sin(4t) + A_3 \cos(3t)</math></b>. Then the value of <b><math>A_2</math></b> is given by <b><math>A_2 = \text{-----}</math></b></p> <p>Ans. = <b>0</b> (Range: NA)</p>	1 Marks	Part-A
1C	<p>Consider the forced oscillation of a particle described by the equation:</p> <p><b><math>0.5\ddot{x} + 8x = 10 \cos(3t)</math></b> with the initial conditions: <b><math>x(0) = 0, \dot{x}(0) = 0</math></b>. The general solution (complementary function + particular integral) of this equation of motion can be written as: <b><math>x(t) = A \cos(4t) + B \sin(4t) + C \cos(3t)</math></b>. Then the value of <b><math>C</math></b> is given by <b><math>C = \text{-----}</math></b></p> <p>Ans. = <b>2.857</b> (20/7) (Range: 2.6 to 3.0)</p>	1 Marks	Part-A
2 A	<p>Consider a forced damped oscillator with the equation of motion given by <b><math>m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos(\omega t)</math></b>. The values of <b><math>k = 32, m = 0.5, b = 1</math></b> in MKS units and <b><math>F_0 = 10 \text{ N}</math></b> and <b><math>\omega = 2\omega_0</math></b>, where <b><math>\omega_0^2 = k/m</math></b>. If the steady state solution is given by <b><math>x_p(t) = x_0 \cos(\omega t - \varphi)</math></b>, then the value of <b><math>x_0</math></b> (in meters) is -----</p> <p>Ans. = <b>0.103</b> (Range: 0.08 to 0.2)</p> <p><b>Solutions (2A), (2B), (2C)</b></p> <p><math>\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{32}{0.5}} = 8, \omega = 2\omega_0 = 16</math> and <math>\beta = \frac{b}{2m} = \frac{1}{2 \times 0.5} = 1</math></p> <p>The steady state solution (particular solution) is given by:</p> <p><math>x_p(t) = x_0 \cos(\omega t - \varphi) = \frac{F_0}{m \sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}} \cos(\omega t - \varphi)</math></p> <p><math>x_0 = \frac{F_0}{m \sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}} = \frac{10}{0.5 \sqrt{(64 - 4 \times 64)^2 + (2 \times 2 \times 8)^2}} = 20 \frac{1}{\sqrt{(3 \times 64)^2 + (32)^2}} \approx 0.1027</math></p>	2 Marks	Part-B

	<p>The driven oscillator has its maximum amplitude at a frequency</p> $\omega = \sqrt{\omega_0^2 - \left(\frac{b^2}{2m^2}\right)} = \sqrt{\omega_0^2 - 2\beta^2} = \sqrt{64 - 2} = \sqrt{62} \approx 7.874$ <p>For the velocity amplitude, we differentiate displacement equation yielding</p> $v_0 = \frac{F_0}{m} \frac{\omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\beta\omega)^2}} ; \text{ for maximum velocity amplitude we maximize } v_0 \text{ w.r.t } \omega \text{ this results } \omega = \omega_0 = 8$		
2B	<p>Consider a forced damped oscillator with the equation of motion given by <math>m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos(\omega t)</math>. The values of <math>k = 32, m = 0.5, b = 1</math> in MKS units and <math>F_0 = 10 \text{ N}</math> and <math>\omega = 2\omega_0</math> where <math>\omega_0^2 = k/m</math>. The steady state solution is given by <math>x_p(t) = x_0 \cos(\omega t + \phi)</math>. If <math>\omega_m</math> be the angular frequency of driven oscillator for which the amplitude is maximum, then the value of <math>\omega_m^2</math> in <math>\text{rad}^2/\text{sec}^2</math> is -----</p> <p>Ans. = <b>7.874 (Range: 7 to 8)</b></p>	2 Marks	Part-B
2C	<p>Consider a forced damped oscillator with the equation of motion given by <math>m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos(\omega t)</math>. The values of <math>k = 32, m = 0.5, b = 1</math> in MKS units and <math>F_0 = 10 \text{ N}</math> and <math>\omega = 2\omega_0</math> where <math>\omega_0^2 = k/m</math>. The steady state solution is given by <math>x_p(t) = x_0 \cos(\omega t + \phi)</math>. If <math>\omega_v</math> denote the angular frequency of driven oscillator for which the velocity amplitude is maximum, then the value of <math>\omega_v</math> in <math>\text{rad/sec}</math> is -----</p> <p>Ans. <b>8 (Range: NA)</b></p>	2 Marks	Part-B
3 A	<p>A coupled pendula setup consists of two identical pendula and a light spring. Initially the left mass is at <math>x = 2.0 \text{ cm}</math> and the right mass is at <math>x = 0</math>. The masses are then released. The left mass starts oscillating with a period of <b>1.1 sec</b>. At <math>t = 11 \text{ sec}</math>, both masses oscillate with equal amplitude. At <math>t = 22 \text{ sec}</math>, the right mass is oscillating with maximum amplitude but the left mass is motionless. If <math>\omega_b</math> and <math>\omega_p</math> are the normal mode angular frequencies, then from these data calculate the value of beat frequency, <math>\omega_b - \omega_p</math> in <math>\text{rad/sec}</math> -----</p> <p>Ans. <b>0.1428</b> (1/7) (Range: 0.1 to 0.2)</p> <p><b>Solutions (3A), (3B), (3C):</b></p> <p>The beating is caused by the mixing of two normal modes. Initially left mass is at maximum amplitude and right mass is at equilibrium, after 22 sec., the right mass is at maximum amplitude. Therefore, at 44 sec. The left mass will be at maximum amplitude, hence beat frequency is 1/44 Hz.</p> <p>So, <math>f_b - f_p = 1/44</math> i.e., <math>\frac{\omega_b}{2\pi} - \frac{\omega_p}{2\pi} = 1/44</math> or <math>\omega_b - \omega_p = \frac{2\pi}{44} = \frac{\pi}{22} = \frac{1}{7} = 0.14279</math></p> <p>When So, <math>f_b - f_p = 1/40 = 0.025</math></p> <p>Given the period of fast oscillation is 1.1 sec. So <math>T_{\text{fast}} = \frac{2\pi}{\omega_{\text{average}}} = \frac{4\pi}{\omega_b + \omega_p} = 1.1</math> so <math>\omega_b + \omega_p = \frac{4\pi}{1.1} = \frac{80}{7} = 11.423 \rightarrow \frac{(\omega_b + \omega_p)}{2} = 5.71</math></p>	1 Marks	Part-A
3 B	<p>In the experiment of coupled pendula consisting two identical pendula and a light spring, at <math>t = 0</math>, the right mass is at <math>x = 0</math> and the left mass is at <math>x = 2.2 \text{ cm}</math>. The two masses are then released. The left mass starts oscillating with a period of <b>1.1 sec</b>. At <math>t = 10 \text{ sec}</math>, both masses oscillate with equal amplitude. At <math>t = 20 \text{ sec}</math>, the right mass is oscillating with maximum amplitude but the left mass is motionless. If <math>f_b</math> and <math>f_p</math> are the normal mode frequencies, then calculate the value of beat frequency, <math>f_b - f_p</math> in <math>\text{Hz}</math> -----</p>	1 Marks	Part-A

	Ans. <b>0.025</b> (1/40) (Range: 0.01 to 0.03)		
3 C	<p>The left mass of a coupled pendula apparatus at <math>t = 0</math> is at position <math>x = 2.0 \text{ cm}</math> and the right mass is at <math>x = 0</math>. The masses are then released. The left mass starts oscillating with a period of <math>1.1 \text{ sec}</math>. At <math>t = 10 \text{ sec}</math>, both masses oscillate with equal amplitude. At <math>t = 20 \text{ sec}</math>, the right mass is oscillating with maximum amplitude but the left mass is motionless. If <math>\omega_b</math> and <math>\omega_p</math> are the normal mode angular frequencies, then the average angular frequency, <math>(\omega_b + \omega_p)/2</math> in <math>\text{rad/sec}</math> is -----</p> <p>Ans: <b>5.71</b> (Range: 4.71 to 6.71)</p>	1 Marks	Part-A
4A	 <p>For the spring-mass system shown above, the force on the left mass is calculated as</p> $F_1 = Akx_1 + Bkx_2 \quad (A \text{ and } B \text{ are integers})$ <p>The value of A and B are ----- and -----</p> <p>Ans: A= -3, B= 2 (Range: NA)</p> <p><b>Solutions to (4A), (4B), (4C):</b></p> $F_1 = -kx_1 + 2k(x_2 - x_1) \text{ and } F_2 = -2k(x_2 - x_1) - 3kx_2$ <p>Therefore,</p> $F_1 = -3kx_1 + 2kx_2$ $F_2 = 2kx_1 - 5kx_2 \text{ and } F_2 - F_1 = 5kx_1 - 7kx_2$	2 Marks	Part-B
4B	 <p>For the spring-mass system shown above, the force on the right mass is calculated as</p> $F_2 = Ckx_1 + Dkx_2 \quad (C \text{ and } D \text{ are integers})$ <p>The value of C and D are ----- and -----</p> <p>Ans: C= 2, D= -5 (Range: NA)</p>	2 Marks	Part-B
4C		2 Marks	Part-B



For the spring-mass system shown above, the force on the left mass and that on the right mass are calculated as  $F_1$  and  $F_2$  respectively and the difference is given by

$$F_2 - F_1 = pkx_1 + qkx_2 \quad (p \text{ and } q \text{ are integers})$$

The value of  $p$  and  $q$  are ----- and -----

Ans:  $p=5, q=-7$  (Range: NA)

5A

A coupled pendula setup with identical pendula has  $m = 0.10 \text{ kg}$ ,  $l = 0.10 \text{ m}$  and  $k = 5.0 \text{ N/m}$ . The angular frequencies of the two normal modes (pendulum mode and breathing mode) are respectively given by  $\omega_p$  and  $\omega_b$ . In a place where  $g = 10 \text{ m/s}^2$ , the value of  $\omega_p$  and  $\omega_b$  in **rad/sec** are ----- and -----

Ans:  $\omega_p = 10.0$ ,  $\omega_b = 14.14$

**Solution (5A):**

The angular frequencies of the pendulum mode and breathing mode are respectively

$$\omega_p = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.10}} = 10.0 \text{ and } \omega_b = \sqrt{\frac{g}{l} + \frac{2k}{m}} = \sqrt{\frac{10}{0.10} + \frac{10}{0.10}} \approx 14.14$$

2  
Marks

Part-  
B

5B

A coupled pendula setup with identical pendula has  $m = 0.40 \text{ kg}$ ,  $l = 0.40 \text{ m}$  and  $k = 5.0 \text{ N/m}$ . The angular frequencies of the two normal modes (pendulum mode and breathing mode) are respectively given by  $\omega_p$  and  $\omega_b$ . In a place where  $g = 10 \text{ m/s}^2$ , the value of  $\omega_p$  and  $\omega_b$  in **rad/sec** are ----- and -----

Ans:  $\omega_p = 5$ ,  $\omega_b = 7.07$

**Solution (5B):**

The angular frequencies of the pendulum mode and breathing mode are respectively

$$\omega_p = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.40}} \approx 5 \text{ and } \omega_b = \sqrt{\frac{g}{l} + \frac{2k}{m}} = \sqrt{\frac{10}{0.40} + \frac{10}{0.40}} = 5\sqrt{2} \approx 7.07$$

2  
Marks

Part-  
B

5C

A coupled pendula setup with identical pendula has  $m = 0.20 \text{ kg}$ ,  $l = 0.20 \text{ m}$  and  $k = 5.0 \text{ N/m}$ . The angular frequencies of the two normal modes (pendulum mode and breathing mode) are respectively given by  $\omega_p$  and  $\omega_b$ . In a place where  $g = 10 \text{ m/s}^2$ , the value of  $\omega_p$  and  $\omega_b$  in **rad/sec** are ----- and -----

Ans:  $\omega_p = 7.07$ ,  $\omega_b = 10$

**Solution (5C):**

The angular frequencies of the pendulum mode and breathing mode are respectively

$$\omega_p = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.2}} \approx 7.07 \text{ and } \omega_b = \sqrt{\frac{g}{l} + \frac{2k}{m}} = \sqrt{\frac{10}{0.2} + \frac{10}{0.2}} \approx 10$$

2  
Marks

Part-  
B

6 A	<p>A particle of mass 0.08 kg in a potential <math>V(x) = V_0 e^{\alpha x^2}</math> exhibit a simple harmonic motion for small displacements from its equilibrium position. If the time period of the small oscillations about the equilibrium position is <math>4\pi</math> s, then the value of <math>\alpha</math> (in <math>\text{m}^{-2}</math>) is (Take <math>V_0 = 1\text{V}</math>)</p> <p>Ans: 0.125 (Range: NA) (if you take it as gravitational potential)</p> <p>0.01 (If you take it as potential energy)</p> <p>Hints: <math>F = m \frac{dV}{dx} \rightarrow \ddot{x} + 2\alpha x = 0 \rightarrow T = 2\pi \sqrt{\frac{1}{2\alpha}}</math></p> <p><math>F = \frac{dV}{dx} \rightarrow m\ddot{x} + 2\alpha x = 0 \rightarrow T = 2\pi \sqrt{\frac{m}{2\alpha}}</math> ( if you take it as energy)</p>	Part-A
6 B	<p>A particle of mass 0.08 kg in a potential <math>V(x) = V_0 e^{\alpha x^2}</math> exhibit a simple harmonic motion for small displacements from its equilibrium position. If the time period of the small oscillations about the equilibrium position is <math>2\pi</math> s, then the value of <math>\alpha</math> (in <math>\text{m}^{-2}</math>) is (Take <math>V_0 = 1\text{V}</math>)</p> <p>Ans: 0.5 (Range: NA)</p> <p>And 0.04 (If you take it as potential energy)</p> <p>Hints: <math>F = -m \frac{dV}{dx} \rightarrow \ddot{x} + 2\alpha x = 0 \rightarrow T = 2\pi \sqrt{\frac{1}{2\alpha}}</math></p> <p><math>F = -\frac{dV}{dx} \rightarrow m\ddot{x} + 2\alpha x = 0 \rightarrow T = 2\pi \sqrt{\frac{m}{2\alpha}}</math> ( if you take it as potential energy)</p>	Part-A
6 C	<p>A particle of mass 0.16 kg in a potential <math>V(x) = V_0 e^{\alpha x^2}</math> exhibit a simple harmonic motion for small displacements from its equilibrium position. If the time period of the small oscillations about the equilibrium position is <math>2\pi</math> s, then the value of <math>\alpha</math> (in <math>\text{m}^{-2}</math>) is (Take <math>V_0 = 1\text{V}</math>)</p> <p>Ans: 0.5 (Range: NA)</p> <p>0.08 (If you take it as potential energy)</p> <p>Hints: <math>F = -m \frac{dV}{dx} \rightarrow \ddot{x} + 2\alpha x = 0 \rightarrow T = 2\pi \sqrt{\frac{1}{2\alpha}}</math></p> <p><math>F = -\frac{dV}{dx} \rightarrow m\ddot{x} + 2\alpha x = 0 \rightarrow T = 2\pi \sqrt{\frac{m}{2\alpha}}</math> ( if you take it as potential energy)</p>	Part-A
7 A	<p>A molecule of DNA is <math>2.17 \mu\text{m}</math> long. The ends of the molecule become singly ionized: negative on one end, positive on the other. If the DNA molecule acts like a spring and compresses 1 % upon becoming charged, then the effective spring constant (in <math>\text{nN/m}</math>) of the molecule is (Take: <math>\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}</math>)</p> <p>Ans: 2.30 (Range: 2.28 to 2.32)</p> <p>Hints: DNA molecule compresses 1 % <math>\rightarrow x_f = (2.17 - 0.0217) \times 10^{-6} = 2.1483 \times 10^{-6} \text{ m}</math></p> <p><math>\Delta x = 0.0217 \mu\text{m}</math></p>	Part-B

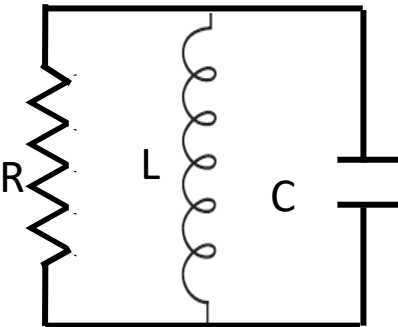
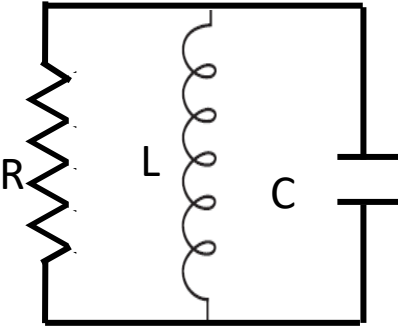
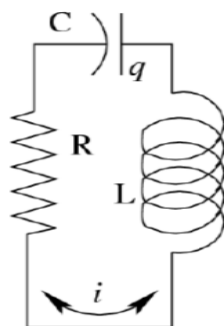
	$F_{coulomb} = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{x_f^2} = k_{eff} \Delta x \rightarrow$ $k_{eff} = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{x_f^2 \Delta x}$		
7 B	<p>A molecule of DNA is 2.2 <math>\mu\text{m}</math> long. The ends of the molecule become singly ionized: negative on one end, positive on the other. If the DNA molecule acts like a spring and compresses 2 % upon becoming charged, then the effective spring constant (in nN/m) of the molecule is (Take: <math>\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}</math>)</p> <p>Ans: 1.13 (Range: 1.11 to 1.15)</p>		Part-B
7 C	<p>A molecule of DNA is 2 <math>\mu\text{m}</math> long. The ends of the molecule become singly ionized: negative on one end, positive on the other. If the DNA molecule acts like a spring and compresses 2 % upon becoming charged, then the effective spring constant (in nN/m) of the molecule is (Take: <math>\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}</math>)</p> <p>Ans: 1.49 (Range: 1.47 to 1.51)</p>		Part-B
8 A	<p>A mass of 0.03 kg rests on a horizontal table and is attached to one end of a spring of spring constant 12 N/m. The other end of the spring is attached to a rigid support. The mass is subjected to a harmonic driving force <math>F = F_0 \cos \omega t</math>, where <math>F_0 = 0.15 \text{ N}</math>, <math>\omega = 20 \text{ rad/s}</math> and a damping force <math>F_d = -rv</math>, where <math>r = 0.06 \text{ kg/s}</math>. The energy dissipated per cycle (in mJ) by the damping force at an angular frequency 20 rad/s is</p> <p>Ans: 58.9 (Range: 55 to 60)</p> <p>Hints: Instantaneous power dissipated <math>P = \text{Force} \times \text{Velocity} \rightarrow</math> The energy dissipated per cycle <math>= \int_0^{2\pi} P dt = \pi r \omega A^2</math> Where A is amplitude of the oscillation.</p> <p>The energy dissipation per cycle:</p> $= \pi r \omega A^2$ $= 3.14 \times 0.06 \times 20 \times \left( \frac{0.15}{0.03 \times \sqrt{[(12/0.03) - 20^2]^2 + 4 \times \left(\frac{0.06}{2 \times 0.03}\right)^2 \times 20^2}} \right)^2$ $= 3.14 \times 0.06 \times 20 \times \frac{0.0225}{0.03 \times ((400 - 400)^2 + 4 \times 1 \times 400)}$ $= 3.14 \times 1.2 \times \frac{0.0225}{0.03 \times 0.03 \times (1600)}$ $= 58.875 \times 10^{-3} \text{ J}.$		Part-B
8 B	<p>A mass of 0.05 kg rests on a horizontal table and is attached to one end of a spring of spring constant 12 N/m. The other end of the spring is attached to a rigid support. The mass is subjected to a harmonic driving force <math>F = F_0 \cos \omega t</math>, where <math>F_0 = 0.2 \text{ N}</math>,</p>		Part-B

	<p><math>\omega = 20 \text{ rad/s}</math> and a damping force <math>F_d = -rv</math>, where <math>r = 0.06 \text{ kg/s}</math>. The energy dissipated per cycle (in mJ) by the damping force at an angular frequency <math>20 \text{ rad/s}</math> is</p> <p>Ans: 2.3 (Range: 2.2 to 2.4)</p> <p>Hints: Instantaneous power dissipated <math>P = \text{Force} \times \text{Velocity} \rightarrow</math> The energy dissipated per cycle <math>= \int_0^{2\pi} P dt = \pi r \omega A^2</math> Where <math>A</math> is amplitude of the oscillation.</p> <p>The energy dissipation per cycle :</p> $= \pi r \omega A^2$ $= 3.14 \times 0.06 \times 20 \times \left( \frac{0.2}{0.05 \times \sqrt{[(12/0.05) - 20^2]^2 + 4 \times \left(\frac{0.06}{2 \times 0.05}\right)^2 \times 20^2}} \right)^2$ $= 3.14 \times 0.06 \times 20 \times \frac{0.04}{0.05 \times 0.05 \times ((240 - 400)^2 + 4 \times 0.36 \times 400)}$ $= 3.0144 \times 20 \times \frac{1}{25600 + 576} = 2.3 \times 10^{-3} \text{ J.}$		
8 C	<p>A mass of <math>0.05 \text{ kg}</math> rests on a horizontal table and is attached to one end of a spring of spring constant <math>12 \text{ N/m}</math>. The other end of the spring is attached to a rigid support. The mass is subjected to a harmonic driving force <math>F = F_0 \cos \omega t</math>, where <math>F_0 = 0.2 \text{ N}</math>, <math>\omega = 2 \text{ rad/s}</math> and a damping force <math>F_d = -rv</math>, where <math>r = 0.06 \text{ kg/s}</math>. The energy dissipated per cycle (in mJ) by the damping force at an angular frequency <math>2 \text{ rad/s}</math> is</p> <p>Ans: 0.108 (Range: 0.09 to 0.13) 0.108</p> <p>Hints: Instantaneous power dissipated <math>P = \text{Force} \times \text{Velocity} \rightarrow</math> The energy dissipated per cycle <math>= \int_0^{2\pi} P dt = \pi r \omega A^2</math> Where <math>A</math> is amplitude of the oscillation.</p> <p>The energy dissipation per cycle:</p> $= \pi r \omega A^2$ $= 3.14 \times 0.06 \times 2 \times \left( \frac{0.2}{0.05 \times \sqrt{[(12/0.05) - 2^2]^2 + 4 \times \left(\frac{0.06}{2 \times 0.05}\right)^2 \times 2^2}} \right)^2$ $= 3.14 \times 0.06 \times 2 \times \frac{0.04}{0.05 \times 0.05 \times ((240 - 4)^2 + 4 \times 0.36 \times 4)}$ $= 3.0144 \times 2 \times \frac{1}{55696 + 5.76} = \frac{60.288}{55701.76} = 1.08 \times 10^{-4} \text{ J.}$		Part-B
9 A	<p>A mass of <math>0.03 \text{ kg}</math> rests on a horizontal table and is attached to one end of a spring of spring constant <math>12 \text{ N/m}</math>. The other end of the spring is attached to a rigid support. The mass is subjected to a harmonic driving force <math>F = F_0 \cos \omega t</math>, where <math>F_0 = 0.15 \text{ N}</math>, <math>\omega = 20 \text{ rad/s}</math> and a damping force <math>F_d = -rv</math>, where <math>r = 0.06 \text{ kg/s}</math>. The Amplitude (in cm) of the steady-state oscillations at frequency <math>20 \text{ rad/s}</math> is</p>		Part-A

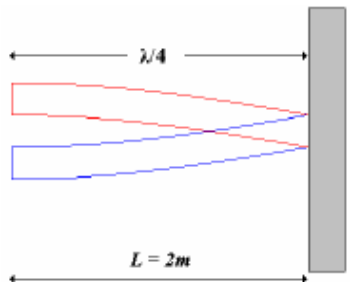
	<p>Ans: <b>12.5 (Range: 10 to 15)</b></p> <p>Hints: <math>A = \frac{F_0}{\left[ m\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2} \right]}</math></p>		
9 B	<p>A mass of 0.03 kg rests on a horizontal table and is attached to one end of a spring of spring constant 12 N/m. The other end of the spring is attached to a rigid support. The mass is subjected to a harmonic driving force <math>F = F_0 \cos \omega t</math>, where <math>F_0 = 0.15</math> N, <math>\omega = 2</math> rad/s and a damping force <math>F_d = -rv</math>, where <math>r = 0.06</math> kg/s. The Amplitude (in cm) of the steady-state oscillations at frequency 2 rad/s is</p> <p>Ans: 1.3 (Range: 1 to 1.5)</p> <p>Hints: <math>A = \frac{F_0}{\left[ m\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2} \right]}</math></p>		Part-A
9 C	<p>A mass of 0.03 kg rests on a horizontal table and is attached to one end of a spring of spring constant 12 N/m. The other end of the spring is attached to a rigid support. The mass is subjected to a harmonic driving force <math>F = F_0 \cos \omega t</math>, where <math>F_0 = 0.15</math> N, <math>\omega = 20</math> rad/s and a damping force <math>F_d = -rv</math>, where <math>r = 0.06</math> kg/s. The magnitude of the phase angle (in radians) between the driving force and the displacement of the mass for steady-state oscillations at a frequency 20 rad/s is</p> <p>Ans: 1.57 (Range: 1.5 to 1.6)</p> <p>Hints: <math>\phi = \tan^{-1} \left( \frac{2\beta\omega}{\omega_0^2 - \omega^2} \right)</math></p>		Part-A
10 A	<p>A mass of 0.5 Kg, attached to a spring, exhibiting under damped oscillations with a time period <math>T = \frac{2\pi}{6}</math> sec in a medium with damping constant (r) 2 N s/m. When the medium is changed then the corresponding change in the time period is <math>\frac{2\pi}{4}</math> sec. Calculate the damping constant (r) of the new medium in N s/m.</p> <p><b>Ans: 5.85 (Range: 3 to 6)</b></p> <p><b>Solution (10A):</b></p> $\text{New Time period} = \frac{2\pi}{6} + \frac{2\pi}{4} = \frac{5\pi}{6}; \omega_{\text{new}} = 2.4;$ $\omega_1^2 = \omega_0^2 - \beta_1^2; \omega_0^2 = \frac{k}{M} = \omega_1^2 + \beta_1^2; k = 0.5(6^2 + 2^2) = 20 \text{ N/m}^2$ $\beta_2^2 = \left( \frac{r_2}{2M} \right)^2 = \omega_0^2 - \omega_{\text{new}}^2 = \frac{20}{0.5} - 2.4^2$ $r_2 = 2M\sqrt{40 - 5.76} = \sqrt{34.24} = 5.85 \text{ N s/m}$	2 Marks	Part-B
10 B	<p>A mass of 0.5 Kg, attached to a spring, exhibiting under damped oscillations with a time period <math>T = \frac{2\pi}{6}</math> sec in a medium with damping constant (r) 2 N s/m. When the medium is changed then the corresponding change in the time period is <math>\frac{2\pi}{5}</math> sec. Calculate the damping constant (r) of the new medium in N s/m.</p> <p><b>Ans: 5.70 (Range: 3 to 6)</b></p> <p><b>Solution (10B):</b></p> $\text{New Time period} = \frac{2\pi}{6} + \frac{2\pi}{5} = \frac{11\pi}{15}; \omega_{\text{new}} = 2.73;$	2 Marks	Part-B

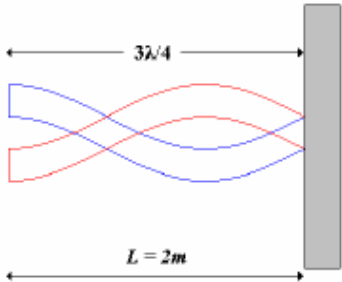
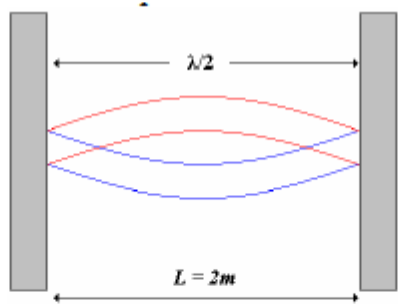
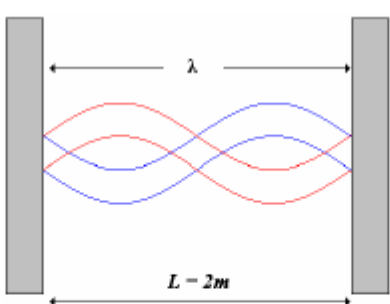


	$\omega_1^2 = \omega_0^2 - \beta_1^2 ; \omega_0^2 = \frac{k}{M} = \omega_1^2 + \beta_1^2 ; k = 0.5(6^2 + 2^2) = 20 \text{ N/m}$ $\beta_2^2 = \left(\frac{r_2}{2M}\right)^2 = \omega_0^2 - \omega_{new}^2 = \frac{20}{0.5} - 2.73^2$ $r_2 = 2M\sqrt{40 - 7.45} = 5.70 \text{ N s/m}$		
10 C	<p>A mass of 0.5 Kg, attached to a spring, exhibiting under damped oscillations with a time period <math>T = \frac{2\pi}{5}</math> sec in a medium with damping constant (r) 1 N s/m. When the medium is changed then the corresponding change in the time period is <math>\frac{2\pi}{3}</math> sec. Calculate the damping constant (r) of the new medium in N s/m.</p> <p><b>Ans: 4.74 (Range: 3 to 6)</b></p> <p><b>Solution (10C):</b> Solution</p> $\text{New Time period} = \frac{2\pi}{3} + \frac{2\pi}{5} = \frac{16\pi}{15}; \omega_{new} = 1.875;$ $\omega_1^2 = \omega_0^2 - \beta_1^2 ; \omega_0^2 = \frac{k}{M} = \omega_1^2 + \beta_1^2 ; k = 0.5(5^2 + 1^2) = 13 \text{ N/m}$ $\beta_2^2 = \left(\frac{r_2}{2M}\right)^2 = \omega_0^2 - \omega_{new}^2 = \frac{13}{0.5} - 1.875^2$ $r_2 = 2M\sqrt{26 - 3.51} = 4.74 \text{ N s/m}$	2 Marks	Part-B
11A	<p>A mass of 0.5 Kg is attached to a spring, with the spring constant 4.5 N/m, exhibiting damped oscillations. The damping constant (r) is 5 N s/m. At the initial time (t = 0) an initial velocity is given to the mass which is 2 m/s and the initial position <math>x = 0</math>. Find the position (in cm) of the mass after 0.1 seconds. Note that <math>\omega_0^2 = 9 ; \beta = 5</math>.</p> <p><b>Ans: 12.45 (Range: 12 to 13)</b></p> <p><b>Solution (11A):</b> Oscillations of overdamped. The solution is <math>x(t) = \frac{v_0}{\omega} e^{-\beta t} \text{Sinh}(\omega t)</math></p> $v_0 = 2 \frac{m}{s} ; \omega = \sqrt{\beta^2 - \omega_0^2} = \sqrt{25 - 9} = 4$ $x(t = 0.1) = 0.124567 \text{ m} \Rightarrow 12.45 \text{ cm}$	1 Marks	Part-A
11 B	<p>A mass of 0.5 Kg is attached to a spring, with the spring constant 4.5 N/m, exhibiting damped oscillations. The damping constant (r) is 5 N s/m. At the initial time (t = 0) an initial velocity is given to the mass which is 2 m/s and the initial position <math>x = 0</math>. Find the position (in cm) of the mass after 0.2 seconds. Note that <math>\omega_0^2 = 9 ; \beta = 5</math>.</p> <p><b>Ans: 16.33 (Range: 15.3 to 17.3)</b></p> <p><b>Solution (11B):</b> Oscillations of overdamped. The solution is <math>x(t) = \frac{v_0}{\omega} e^{-\beta t} \text{Sinh}(\omega t)</math></p> $v_0 = 2 \frac{m}{s} ; \omega = \sqrt{\beta^2 - \omega_0^2} = \sqrt{25 - 9} = 4$ $x(t = 0.2) = 0.1633 \text{ m} = 16.33 \text{ cm}$	1 Marks	Part-A

11 C	<p>A mass of 0.5 Kg is attached to a spring, with the spring constant 4.5 N/m, exhibiting damped oscillations. The damping constant (<math>r</math>) is 5 N s/m. At the initial time (<math>t = 0</math>) an initial velocity is given to the mass which is 2 m/s and the initial position <math>x = 0</math>. Find the position (in cm) of the mass after 0.5 seconds. Note that <math>\omega_0^2 = 9</math> ; <math>\beta = 5</math>.</p> <p>Ans: <b>14.88 (Range: 13.8 to 15.8)</b></p> <p><b>Solution (11C):</b> Oscillations of overdamped. The solution is <math>x(t) = \frac{v_0}{\omega} e^{-\beta t} \sinh(\omega t)</math> <math>v_0 = 2 \frac{m}{s}</math> ; <math>\omega = \sqrt{\beta^2 - \omega_0^2} = \sqrt{25 - 9} = 4</math> <math>x(t = 0.5) = 0.1488 m = 14.88 cm</math></p>	1 Marks	Part-A
12 A	<p>The given LCR (<math>L=0.01</math> H, <math>C=1\mu F</math>, and <math>R=400 \Omega</math>) circuit follows the differential equation for voltage</p> $C \frac{d^2 V}{dt^2} + \frac{1}{R} \frac{dV}{dt} + \frac{1}{L} V = 0$ <p>The quality factor of the circuit is</p> <p>Ans: <b>4 (3 to 5)</b></p> <p>Hints: <math>Q_p = R \sqrt{\frac{C}{L}} = 400 \sqrt{\frac{10^{-6}}{0.01}} = 4,</math></p> 	1 Mark	Part A
12 B	<p>The given LCR (<math>L=0.1</math> mH, <math>C=1\mu F</math>, and <math>R=400 \Omega</math>) circuit follows the differential equation for voltage</p> $C \frac{d^2 V}{dt^2} + \frac{1}{R} \frac{dV}{dt} + \frac{1}{L} V = 0$ <p>The quality factor of the circuit is</p> <p>Ans: <b>40 (39 to 41)</b></p> <p>Hints: <math>Q_p = R \sqrt{\frac{C}{L}} = 400 \sqrt{\frac{10^{-6}}{0.1 \times 10^{-3}}} = 40,</math></p> 	1 Mark	Part A
12 C	<p>The given LCR (<math>L=0.01</math> H, <math>C=1\mu F</math>, and <math>R=2 \Omega</math>) circuit follows the differential equation for charge. The quality factor of the circuit is</p> <p>Ans: <b>50 (49 to 51)</b></p> <p>Hints:</p> $L \ddot{q}(t) + R \dot{q} + \frac{1}{C} q = 0$ $Q_s = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{2} \sqrt{\frac{0.01}{10^{-6}}} = \frac{100}{2} = 50,$ 	1 Mark	Part A

13 A	<p>An electrical circuit <math>L=0.1</math> H, <math>C=1</math> <math>\mu</math>F and <math>R=200</math> <math>\Omega</math> connected in a series. (a) The ratio of initial current amplitude to the current amplitude after 2 oscillations is (b) The percentage of energy lost in the first cycle is</p> <p>Ans: (a) <b>67 (Range:62 to 72)</b>  (b) 98.5 % (Range: 97.5 to 99)</p> $\omega_0^2 = \frac{1}{LC} = \frac{10^6}{0.1} = 10 \times 10^6, \text{ \&, } \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{200}{0.2}\right)^2 = 1 \times 10^6$ $\Rightarrow \omega_0^2 > \beta^2, \text{ an under damped case}$ $\Rightarrow \omega = \sqrt{\omega_0^2 - \beta^2} = 3 \times 10^3$ <p>The current amplitude varies as, <math>I = e^{-\beta t} A \cos(\omega t + \phi)</math></p> <p>Time taken for 2 oscillation, <math display="block">t = NT = \frac{N 2\pi}{\omega} = \frac{2 \times 2\pi}{3 \times 10^3} \text{ s}</math></p> <p>The current amplitude ratio, <math display="block">\frac{A_2}{A_0} = e^{-\beta t} = e^{-4\pi/3} = 0.015</math></p> $\therefore \frac{A_1}{A_0} = e^{-\beta T} \Rightarrow \frac{E_1}{E_0} = \left(\frac{A_1}{A_0}\right)^2 = e^{-2\beta T} = 0.015,$ <p>Energy lost in first cycle, % loss, <math display="block">\frac{\Delta E}{E} = \frac{E_0 - E_1}{E_0} \times 100 = 98.5\%</math></p>	2 Marks	Part-B
13 B	<p>An electrical circuit <math>L=(100/41)</math> H, <math>C=(1/41)</math> <math>\mu</math>F and <math>R=(180/41)</math> k <math>\Omega</math> connected in a series. (a) The ratio of initial current amplitude to the current amplitude after 2 oscillations is (b) The percentage of energy lost in the first cycle is</p> <p>Ans: (a) <b>16.94 (Range:15.94 to 17.94 )</b>  (b) 94 % (Range: 93 to 96)</p> $\omega_0^2 = \frac{1}{LC} = 41^2 \times 10^4, \text{ \&, } \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{180000/41}{2 \times (100/41)}\right)^2 = 9^2 \times 10^4$ $\Rightarrow \omega_0^2 > \beta^2, \text{ an under damped case}$ $\Rightarrow \omega = \sqrt{\omega_0^2 - \beta^2} = 40 \times 10^2$ <p>The current amplitude varies as, <math>I = e^{-\beta t} A \cos(\omega t + \phi)</math></p> <p>Time taken for 2 oscillation, <math display="block">t = NT = \frac{N 2\pi}{\omega} = \frac{2 \times 2\pi}{40 \times 10^2} \text{ s}</math></p> <p>If the current amplitude decreased by, <math display="block">\frac{A_2}{A_0} = e^{-\beta t} = e^{-(9 \times 10^2) \left(\frac{4\pi}{40 \times 10^2}\right)} = e^{-2.83} = 0.059</math></p> $\therefore \frac{A_1}{A_0} = e^{-\beta T} \Rightarrow \frac{E_1}{E_0} = \left(\frac{A_1}{A_0}\right)^2 = e^{-2\beta T} = 0.059,$ <p>Energy lost in first cycle, % loss, <math display="block">\frac{\Delta E}{E} = \frac{E_0 - E_1}{E_0} \times 100 = 94.1\%</math></p>	2 Marks	Part-B

13 C	<p>An electrical circuit <math>L=1</math> H, <math>C=1</math> <math>\mu</math>F and <math>R=200</math> <math>\Omega</math> connected in a series. (a) The ratio of current amplitude to the initial current amplitude after 2 oscillation is (b) The percentage of energy lost in the first cycle is</p> <p>Ans: (a) <b>0.284 (Range:0.27 to 0.30 )</b>  (b) 71.6 % (Range: 69 to 73)</p> $\omega_0^2 = \frac{1}{LC} = 1 \times 10^6, \text{ \&, } \beta^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{200}{2}\right)^2 = 1 \times 10^4$ $\Rightarrow \omega_0^2 > \beta^2, \text{ an under damped case}$ $\Rightarrow \omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{99} \times 10^2$ <p>The current amplitude varies as, <math>I = e^{-\beta t} A \cos(\omega t + \phi)</math></p> <p>Time taken for 2 oscillation, <math>t = NT = \frac{N 2\pi}{\omega} = \frac{2 \times 2\pi}{\sqrt{99} \times 10^2} \text{ s}</math></p> <p>If the current amplitude decreased by, <math>\frac{A_2}{A_0} = e^{-\beta t} = e^{-1 \times 10^2 \left(\frac{4\pi}{\sqrt{99} \times 10^2}\right)} = 0.284</math></p> $\therefore \frac{A_1}{A_0} = e^{-\beta T} \Rightarrow \frac{E_1}{E_0} = \left(\frac{A_1}{A_0}\right)^2 = e^{-2\beta T} = 0.284,$ <p>Energy lost in first cycle, <math>\% \text{ loss, } \frac{\Delta E}{E} = \frac{E_0 - E_1}{E_0} \times 100 = 71.6\%</math></p>	2 Marks	Part-B
14A	<p>Find the first two of the lowest standing wave frequencies on a rod of length 2m clamped at one end and free at the other. Waves on a metal rod travel at <b>3450m/s</b>.</p> <p><math>f_1 = \text{----- Hz}</math> and <math>f_2 = \text{----- Hz}</math></p> <p>Ans. <math>f_1 = 431.25</math> and <math>f_2 = 1293.75</math></p> <p>Range: <math>f_1 = 425</math> to <math>435</math></p> <p><math>f_2 = 1285</math> to <math>1300</math></p>  <p>To find the frequency of this standing wave, we use Equation (16-1):</p> $v = \lambda f$ <p>From our forced locations of nodes and antinodes in the drawing we also know that:</p> $L = \lambda/4 \text{ or } \lambda = 4L$ <p>Substituting and solving for <math>f</math> we conclude that the frequency of the standing wave is:</p> $f = v/(4L) = (3450\text{m/s}) / (4 \bullet 2\text{m}) = 431.25\text{Hz}$	2 Marks	Part-A

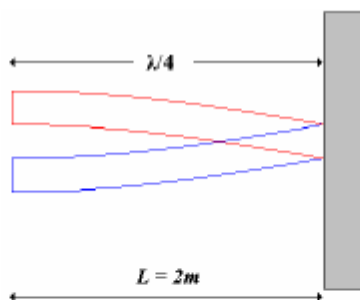
	 <p>Counting the number of nodes and antinodes in the drawing we see that here:  <math>L = 3\lambda/4</math> or <math>\lambda = 4L/3</math>          Substituting and solving for <math>f</math> we conclude that the frequency of the standing wave is:  <math>f = 3v/(4L) = (3 \bullet 3450\text{m/s}) / (4 \bullet 2\text{m}) = 1293.75\text{Hz}</math></p>		
14B	<p>Find the first two of the lowest standing wave frequencies on a rod of length 2m clamped at both ends. Waves on a metal rod travel at <b>3450m/s</b>.</p> <p><math>f_1 = \text{----- Hz}</math> and <math>f_2 = \text{----- Hz}</math></p> <p><b>Ans. <math>f_1 = 862.5</math> and <math>f_2 = 1725</math></b></p> <p><b>Range: <math>f_1 = 855</math> to <math>870</math></b></p> <p><b><math>f_2 = 1720</math> to <math>1730</math></b></p>  <p>This is the rather clearly half of a wave so that we see:  <math>L = \lambda/2</math> or <math>\lambda = 2L</math>          Substituting and solving for <math>f</math> we conclude that the frequency of the standing wave is:  <math>f = v/(2L) = (3450\text{m/s}) / (2 \bullet 2\text{m}) = 862.5\text{Hz}</math></p>  <p>We can see that this is a full wave, so we get:  <math>L = \lambda</math>          Substituting and solving for <math>f</math> we conclude that the frequency of the standing wave is:  <math>f = v/L = (3450\text{m/s}) / (2\text{m}) = 1725\text{Hz}</math></p>	2 Marks	Part- A
14C	<p>Waves on a metal rod of length 2m travel at <b>3450m/s</b>. Find the lowest standing wave frequency (<math>f_1</math>) on the rod if the rod is clamped at one end and free at the other. If the rod is clamped at both ends then what is the lowest standing wave frequency (<math>f_2</math>)</p>	2 Marks	Part- A

$f_1 = \text{----- Hz}$  and  $f_2 = \text{----- Hz}$

Ans.  $f_1 = 431.25$  and  $f_2 = 862.5$

Range:  $f_1 = 425$  to  $435$

$f_2 = 855$  to  $870$



To find the frequency of this standing wave, we use Equation (16-1):

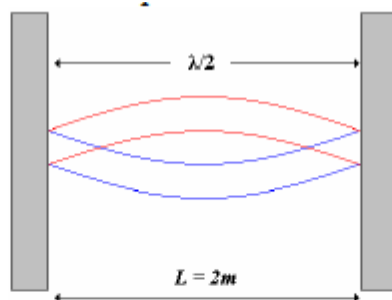
$$v = \lambda f$$

From our forced locations of nodes and antinodes in the drawing we also know that:

$$L = \lambda/4 \text{ or } \lambda = 4L$$

Substituting and solving for  $f$  we conclude that the frequency of the standing wave is:

$$f = v/(4L) = (3450\text{m/s}) / (4 \bullet 2\text{m}) = 431.25\text{Hz}$$



This is the rather clearly half of a wave so that we see:

$$L = \lambda/2 \text{ or } \lambda = 2L$$

Substituting and solving for  $f$  we conclude that the frequency of the standing wave is:

$$f = v/(2L) = (3450\text{m/s}) / (2 \bullet 2\text{m}) = 862.5\text{Hz}$$

15  
A

Consider two waves defined by the wave functions

$y_1(x, t) = 0.50 \sin \left( \frac{2\pi}{3.00} x + \frac{\pi}{2.00} t \right)$  and  $y_2(x, t) = 0.50 \sin \pi \left( \frac{x}{3.00} - \frac{2t}{4.00} \right)$  in SI units. The speed of the waves in SI unit are respectively  $v_1 = \text{-----}$  and  $v_2 = \text{-----}$

Ans:  $v_1 = 0.75$  and  $v_2 = 1.50$

Range:  $v_1 = 0.70$  to  $0.80$ ;  $v_2 = 1.40$  to  $1.60$

Solution:  $v_1 = \frac{\omega}{k} = \frac{\pi}{2.00} / \frac{2\pi}{3.00} = 0.75 \text{ m/sec}$  and  $v_2 = \frac{\omega}{k} = \frac{2\pi}{4.00} / \frac{\pi}{3.00} = 1.5 \text{ m/sec}$

1  
Marks

Part-  
B

15B	<p>Consider two waves defined by the wave functions <math>y_1(x, t) = 0.20 \sin \left( \frac{2\pi}{6.00} x - \frac{\pi}{2.00} t \right)</math> and <math>y_2(x, t) = 0.20 \cos \frac{\pi}{3} (x - 1.5 t)</math> in SI units. The speed of the waves in SI unit are respectively <math>v_1 = \text{-----}</math> and <math>v_2 = \text{-----}</math></p> <p><b>Ans: <math>v_1 = 1.50</math> and <math>v_2 = 1.50</math></b></p> <p><b>Range: 1.40 to 1.60 for both.</b></p> <p>Solution: <math>v_1 = \frac{\omega}{k} = \frac{\pi}{2.00} / \frac{2\pi}{6.00} = 1.50 \text{ m/sec}</math> and <math>v_2 = \frac{\omega}{k} = \frac{1.5\pi}{3} / \frac{2\pi}{6} = 1.50 \text{ m/sec}</math></p>	1 Marks	Part-B
15C	<p>Consider two waves defined by the wave functions <math>y_1(x, t) = 0.50 \sin \pi \left( \frac{x}{4.00} - \frac{2t}{4.00} \right)</math> and <math>y_2(x, t) = 0.20 \cos \frac{\pi}{3} (x + 1.5 t)</math> in SI units. The phase velocity of the waves in SI unit are respectively <math>v_1 = \text{-----}</math> and <math>v_2 = \text{-----}</math></p> <p><b>Ans: <math>v_1 = 2.0</math> and <math>v_2 = 1.50</math> or <math>-1.50</math></b></p> <p><b>Range: <math>v_1 = \text{NA}</math></b></p> <p><b><math>v_2 = 1.40</math> to <math>1.60</math></b></p> <p><b><math>-1.40</math> to <math>-1.60</math></b></p> <p>Solution: <math>v_1 = \frac{\omega}{k} = \frac{2\pi}{4.00} / \frac{\pi}{4.00} = 2.0 \text{ m/sec}</math> and <math>v_2 = \frac{\omega}{k} = \frac{1.5\pi}{3} / \frac{2\pi}{6} = 1.50 \text{ m/sec or } -1.50 \text{ m/sec}</math></p>	1 Marks	Part-B
16 A	<p>A wave is described by the equation <math>Y = 10 \sin 2\pi (2t - x/\lambda)</math>. All quantities are in SI units. If the maximum particle velocity is equal to 6 times the wave velocity, then the wavelength (in m) of the wave is</p> <p>Ans: 10.47</p> <p>Range: 9.5 to 11.5</p> <p>Sol: <math>(dy/dt)_{\max} = 40\pi = 6 \cdot 2 \cdot \lambda</math></p>	1 Marks	Part-A
16 B	<p>A wave is described by the equation <math>Y = 5 \sin 2\pi (4t - x/\lambda)</math>. All quantities are in SI units. If the maximum particle velocity is equal to 6 times the wave velocity, then the wavelength (in m) of the wave is</p> <p>Ans: 5.23</p> <p>Range: 4.5 to 6.5</p> <p>Sol: <math>(dy/dt)_{\max} = 40\pi = 6 \cdot 4 \cdot \lambda</math></p>	1 Marks	Part-A

16 C	<p>A wave is described by the equation <math>Y = 4 \sin 2\pi (6t - x/\lambda)</math>. All quantities are in SI units. If the maximum particle velocity is equal to 6 times the wave velocity, then the wavelength (in m) of the wave is</p> <p>Ans: 4.19</p> <p>Range: 3.5 to 5.5</p> <p><math>(dy/dt)_{\max} = 48\pi = 6 \cdot 6 \cdot \lambda</math></p>	1 Marks	Part-A
17 A	<p>A travelling pulse is represented by <math>f(x, t) = \frac{0.6}{(10x+20t)^2+4}</math>, where <math>x</math> and <math>f</math> are in meters and <math>t</math> in seconds.</p> <p>(a) The distance travelled by the pulse in 5 sec is ----- m</p> <p>(b) The maximum displacement is ----- cm</p> <p>Ans: (a) 10; Range: NA</p> <p>(b) 15 cm ; Range: 14 to 16</p> <p>Sol: <math>f(x, t) = \frac{0.6}{(10x+20t)^2+4} = \frac{0.6}{100(x+2t)^2+4}</math></p> <p>(a) The distance travelled by the wave= velocity * time= 2*5= 10 m</p> <p>(b) The displacement is maximum when <math>x+2=0 \rightarrow f_{\max} = 0.6/4 = 15 \text{ cm}</math></p>	2 Marks	Part-B
17 B	<p>A travelling pulse is represented by <math>f(x, t) = \frac{1}{(20x-40t)^2+5}</math>, where <math>x</math> and <math>f</math> are in meters and <math>t</math> in seconds.</p> <p>(a) The distance travelled by the pulse in 3 sec is ----- m</p> <p>(b) The maximum displacement is ----- cm</p> <p>Ans: (a) 6; Range: NA</p> <p>(b) 20; Range: 19 to 21</p>	2 Marks	Part-B
17 C	<p>A travelling pulse is represented by <math>f(x, t) = \frac{10}{(5x+20t)^2+5}</math>, where <math>x</math> and <math>f</math> are in meters and <math>t</math> in seconds.</p> <p>(a) The distance travelled by the pulse in 2 sec is ----- m</p> <p>(b) The maximum displacement is ----- m</p> <p>Ans: (a) 8 ; Range: NA</p> <p>(b) 2 ; Range: NA</p>	2 Marks	Part-B



18A	<p>On a string with a length of 0.75 m and a mass of 150 g formed 1.5 sinusoidal waves with a frequency of 50 Hz. The tension of the rope is</p> <p>Ans: 125 N (Range: NA)</p> <p>Sol: Given, <math>l = 0.75</math> m, <math>m = 0.15</math> kg, <math>\nu = 50</math> Hz.</p> $\mu = \frac{0.15}{0.75} = 0.2 \text{ kg/m.}$ $\lambda = \frac{0.75}{1.5} = 0.5 \text{ m, } v = 50 \times 0.5 = 25 \text{ m/s.}$ $T = v^2 \mu = 625 \times 0.2 = 125 \text{ N.}$	1 Mark	Part-A
18B	<p>On a string with length of 1.05 m and mass of 105 g formed 1.5 sinusoidal waves with frequency of 50 Hz. The tension of the rope is</p> <p>Ans: 122.5 N (Range: 121 to 124)</p> <p>Sol: Given, <math>l = 1.05</math> m, <math>m = 0.105</math> kg, <math>\nu = 50</math> Hz.</p> $\mu = \frac{0.105}{1.05} = 0.1 \text{ kg/m.}$ $\lambda = \frac{1.05}{1.5} = 0.7 \text{ m, } v = 50 \times 0.7 = 35 \text{ m/s.}$ $T = v^2 \mu = 1225 \times 0.1 = 122.5 \text{ N.}$	1 Mark	Part-A
18C	<p>On a string with length of 1.2 m and mass of 150 g formed 1.5 sinusoidal waves with frequency of 40 Hz. The tension of the rope is</p> <p>Ans: 128 N (Range: NA)</p> <p>Sol: Given, <math>l = 1.2</math> m, <math>m = 0.15</math> kg, <math>\nu = 40</math> Hz.</p> $\mu = \frac{0.15}{1.2} = 0.125 \text{ kg/m.}$ $\lambda = \frac{1.2}{1.5} = 0.8 \text{ m, } v = 40 \times 0.8 = 32 \text{ m/s.}$ $T = v^2 \mu = 1024 \times 0.125 = 128 \text{ N.}$	1 Mark	Part-A
19A	<p>A wire of length 1.5 meters and mass 150 grams is fixed between two points under a tension of 90 N. A standing wave has formed which has seven nodes including the end points. What is the frequency (in Hz) of this wave? <b>2 Marks</b></p> <p>Ans: 60 (Range: NA)</p> <p>Sol: <math>Frequency = n \frac{velocity}{2 \text{ length}}</math></p> $velocity = \sqrt{\frac{T}{\mu}} ; \mu = \frac{mass}{length} = \frac{0.15}{1.5} = 0.1 \frac{Kg}{m}; velocity = 30 \text{ m/s}$ $Frequency = 6 \frac{30}{2 * 1.5} = 6 * 10 = 60 \text{ Hz}$	2 Marks	Part-B

19B	<p>A wire of length 2.5 meters and mass 250 grams is fixed between two points under a tension of 40 N. A standing wave has formed which has four nodes including the end points. What is the frequency (in Hz) of this wave?</p> <p>Ans: 12 (Range: NA)</p> <p>Sol: <math>Frequency = n \frac{velocity}{2 length}</math></p> $velocity = \sqrt{\frac{T}{\mu}} ; \mu = \frac{mass}{length} = \frac{0.25}{2.5} = 0.1 \frac{Kg}{m}; velocity = 20 m/s$ $Frequency = 3 \frac{20}{2 * 2.5} = 3 * 4 = 12 Hz$	2 Marks	Part-B
19C	<p>A wire of length 1.5 meters and mass 150 grams is fixed between two points under a tension of 90 N. A standing wave has formed which has six nodes including the end points. What is the frequency (in Hz) of this wave?</p> <p>Ans: 50 (Range: NA)</p> <p>Sol : <math>Frequency = n \frac{velocity}{2 length}</math></p> $velocity = \sqrt{\frac{T}{\mu}} ; \mu = \frac{mass}{length} = \frac{0.15}{1.5} = 0.1 \frac{Kg}{m}; velocity = 30 m/s$ $Frequency = 5 \frac{30}{2 * 1.5} = 5 * 10 = 50 Hz$	2 Marks	Part-B
20A	<p>A string of length 3 meters fixed between two points, A (left) and B (right), under tension 90 N. A standing wave has produced with five nodes including the end points. What is the position of the third antinode from the end B (in m) ?</p> <p>Ans: <b>+1.88 (Range:1.75 to 1.95) or -1.88 (Range:-1.75 to -1.95)</b></p> <p>Sol : Distance between two nodes is <math>3/(5-1) = 3/4</math> meters.  Answer is <math>3/4 + 3/4 + 3/8 = 15/8 = 1.875</math> meters</p>	1 Mark	Part-A
20B	<p>A string of length 4 meters fixed between two points, A (left) and B (right), under tension 100 N. A standing wave has produced with five nodes including the end points. What is the position of the third antinode from the end B (in m) ?</p> <p>Ans: <b>+2.5 (Range: 2.4 to 2.6) or -2.5 (Range: -2.4 to -2.6)</b></p> <p>Sol : Distance between two nodes is <math>4/(5-1) = 1</math> meter.  Answer is <math>4/4 + 4/4 + 4/8 = 2.5</math> meters</p>	1 Mark	Part-A
20C	<p>A string of length 2 meters fixed between two points, A (left) and B (right), under tension 70 N. A standing wave has produced with five nodes including the end points. What is the position of the third antinode from the end B (in m) ?</p> <p>Ans: <b>+1.25 (Range: 1.15 to 1.35) or -1.25 (Range: -1.15 to -1.35)</b></p> <p>Sol : Distance between two nodes is <math>2/(5-1) = 0.5</math> meter.  Answer is <math>0.5 + 0.5 + 0.25 = 1.25</math> meters</p>	1 Mark	Part-A