Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 7
Autumn 2022

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1. Find the order and degree of the following differential equations:
   (i) $\left(\frac{d^2x}{dt^2}\right)^2 + \frac{d^2x}{dt^2} + t\frac{dx}{dt} = 0.$
   (ii) $t\frac{d^2y}{dt^2} + t^2\frac{dy}{dt} - \sqrt{y}\cos t = 2t^2 - 3t + 4.$
   (iii) $\sqrt{y + \left(\frac{d^2y}{dx^2}\right)^2} = \left(\frac{dy}{dx}\right)^5.$
   (iv) $\frac{d^2y}{dx^2} = \frac{1}{4}\left(2 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{7}{2}}.$
   (v) $\left(1 + \frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \frac{dy}{dx}.$
2. Form ODE by eliminating the arbitrary constants:
   (i) $y = (A + Bx)e^{kx}$, where A, B are arbitrary constant.
   (ii) $y = \ln(\sin(x + a)) + b.$
   (iii) $y = \alpha x + \alpha - \alpha^3.$
   (iv) $(x - h)^2 + (y - k)^2 = a^2$, where (h, k) is the center as parameter and a is the radius of the circle as constant.
   (v) Obtain the differential equation of all circles each of which touches the x-axis at the origin.
   (vi) Obtain the differential equation of the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, in which λ is the arbitrary parameter, and a, b are given constants.
3. Solve the following Initial Value Problems:
   (i) $x\cos\left(\frac{y}{x}\right)(ydx + xdy) = y\sin\left(\frac{y}{x}\right)(xdy - ydx), \quad y(1) = \pi.$
   (ii) $(xy^2 - e^{\frac{1}{x^3}})dx - x^2ydy = 0, \quad y(1) = 0.$
   (iii) $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1, \quad y(0) = 1.$
4. Check whether the differential equations are homogeneous, otherwise reduce it to homogeneous, and then solve it:
   (i) $x\sin\left(\frac{y}{x}\right)dy = (y\sin\frac{y}{x} - x)dx.$
   (ii) $2xydy = (y^2 - x^2)dx.$
   (iii) $(x + 2y - 3)dy = (2x - y + 1)dx.$
5. Check whether the differential equations are exact, otherwise reduce it to exact differential equation by multiplying with the proper integrating factor, then solve it:
   (i) $x^2ydx - (x^3 + y^3)dy = 0.$

(ii) $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$.

(iii) $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$.

(iv) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

(v) $x\frac{dy}{dx} + (3x + 1)y = xe^{-2x}$.

6. Solve the following ODEs by reducing them to linear differential equations:

(i) $\frac{dy}{dx} + \frac{1-2x}{x^2}y = 1$.

(ii) $\sqrt{a^2 + x^2}\frac{dy}{dx} + y = \sqrt{a^2 + x^2} - x$.

(iii) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)e^x \sec y$.

(iv) $y(2xy + e^x)dx - e^x dy = 0$.

(v) $y^2 + (x - \frac{1}{y})\frac{dy}{dx} = 0$.

(vi) $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$.

(vii) $\cos x\frac{dy}{dx} - y\sin x + y^2 = 0$.

(viii) $ydy + by^2dx = a\cos xdx$.