TUTORIAL PROBLEMS

SOLUTION

TUTORIAL-3

1 Case-I: When the car is lifted by placing the jack at point C, the FBD of the car consists of the following four forces

(i) $N_A \hat{k}$: normal reaction at point A acting vertically upward
(ii) $N_B \hat{k}$: normal reaction at B acting vertically upward
(iii) $N_C \hat{k}$: ~~normal~~ vertical jacking force
(iv) $-W \hat{k}$: weight of the car

We here make an assumption that the tilt of the car about AB axis is so small that the dimensions given in the plan view of the vehicle are not altered.

To get N_C we balance the moment of all forces about axis A-B. This gives

$$-W \times \frac{1.575}{2} + N_C \times 1.575 = 0 \quad \text{--- (1)}$$

ie $$N_C = \frac{W}{2} = 800 \text{ kg-wt or } 800 \times 9.81 (N) = 7848 (N)$$

Now use moment balance equation about point A. This gives

$$N_B \times (1.400 + 0.28 + 1.12) - W \times (1.40 + 0.28) + N_C \times 1.40 = 0 \quad \text{--- (2)}$$

ie $$N_B = \frac{1600 \times 1.68 - 800 \times 1.4}{2.8} g \ (N)$$

$$= 560 g \ (N)$$
$$= 5493.6 \ (N) \quad [g = 9.81 \ m/s^2]$$

By force balance equation we get

$$N_A = W - N_C - N_B = (800 - 560) g \ (N) \quad \text{--- (3)}$$
$$= 2354.4 \ (N)$$

Case-II: The procedure is similar to the one discussed above. The moment balance about an axis A-B gives the same result for N_D, vertical jacking force required to be applied at point D, ie $N_D = 7848 (N)$.

Moment equation about point A is written as

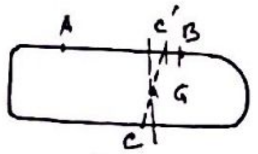
$$N_B \times 2.8 - W \times 1.68 + N_D \times 1.68 = 0 \quad \cdots (4)$$

or $$N_B = \frac{W}{2} \times \frac{1.68}{2.8} = 800 \times \frac{1.68}{2.8}\, g = 4708.8 \text{ (N)}.$$

Force balance along vertical direction gives

$$N_A = W - N_C - N_B = 3139.2 \text{ (N)}.$$

A second method can also be used by resolving the weight of the car in three forces at the three support points. For example when jack is used at point C the weight can be resolved as $\frac{W}{2}$ at C and $\frac{W}{2}$ at point C' lying on AB at a distance (1120 mm − 280) from B (see figure 1.1). This component can be broken again in two parts



$BC' = 1120 - CD$
$= 1120 - 280$ mm

$$N_A = W/2 \times \frac{BC'}{AB} = 800\, g \times \frac{840}{2800} = 2354.4 \text{ (N)}$$

and $$N_B = \frac{W}{2} \times \frac{AC'}{AB} = 800\, g \times \frac{2800 - 840}{2800} = 5493.6 \text{ (N)}$$

These weights are supported by reactions at A and B. Similar process can be followed when the jack is held at point D.

2. The free body diagram of the welded steel beams is shown in figure 2.1. Five forces, namely, three cable tensions T_A, T_B and T_C and two weights of the beams each $W = 100g$ (N), act vertically as shown in FBD.

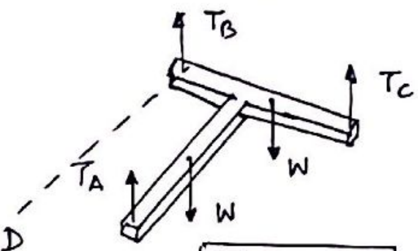


Figure 2.1

Taking moment about BC gives

$$-T_A \times 2.4 + W \times \frac{2.4}{2} = 0 \quad \cdots (1)$$

ie $$T_A = \frac{W}{2} = 50\, g \text{ (N)} = 490 \text{ (N)}$$

(Assuming $g = 9.8\ m/s^2$).

Moment about ~~point~~ line BD gives (see FBD)

$$T_C \times 2.4 - W \times \frac{2.4}{2} - W \times 0.9 + T_A \times 0.9 = 0 \quad \cdots (2)$$

Thus $$T_C = \frac{1}{2.4}\left[\frac{2.4}{2} + 0.9 - \frac{0.9}{2}\right] W = 673.75 \text{ (N)}.$$

The force balance equation gives

$$T_B = 2W - (T_A + T_C) = (2 - 0.5 - 0.6875)\, W$$
$$= 796.25 \text{ (N)}. \quad \cdots (3)$$

The second method described for problem 1 can be used easily for this problem as well. The weight of rod BC can be resolved as $\frac{W}{2}$ at B and C each. The weight of rod passing through A can be resolved as $\frac{W}{2}$ passing through A and B $\frac{W}{2}$ at the junction which can be further resolved as $\frac{W}{2} \times \frac{1.5}{2.4}$ at B and $\frac{W}{2} \times \frac{0.9}{2.4}$ at C. Thus the total weight of the angle becomes equivalent to three weights

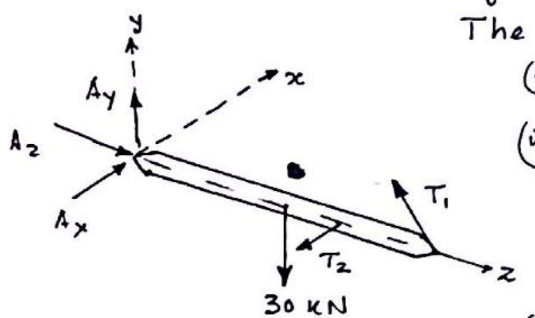
(i) $\frac{W}{2} = 490\,(N)$ at A

(ii) $\frac{W}{2} + \frac{W}{2} \times \frac{1.5}{2.4} = 0.8125\,W = 796.25\,(N)$ at B

and (iii) $\frac{W}{2} + \frac{W}{2} \times \frac{0.9}{2.4} = 0.6875\,W = 673.75\,(N)$ at C.

The cable tensions cancell the forces at respective location.

3. The Free Body Diagram of the boom is shown in figure 3.1.



The forces are

(i) $\vec{F}_A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, at $(0,0,0)$

(ii) $\vec{T}_1 = T_1 \dfrac{(4\hat{i} + 3\hat{j} - 9\hat{k})}{\sqrt{4^2+3^2+9^2}}$ at $(0,0,9)$ m

(iii) $\vec{T}_2 = -T_2\hat{i}$ at $(0,0,6)$ m.

(iv) $\vec{W} = -30\hat{j}$ (KN) at $(0,0,4)$ m.

The equations of equilibrium are given by

(i) Force balance equation:

$$\vec{F}_A + \vec{T}_1 + \vec{T}_2 + \vec{W} = 0 \quad \text{--(1)}$$

or

$$\left.\begin{aligned} A_x + \frac{4T_1}{\sqrt{106}} - T_2 &= 0, \quad \text{--(2a)} \\ A_y + \frac{3T_1}{\sqrt{106}} - 30 &= 0, \quad \text{--(2b)} \\ \text{and} \quad A_z - \frac{9T_1}{\sqrt{106}} &= 0. \quad \text{--(2c)} \end{aligned}\right\}$$

(ii) Moment balance about point A

$$4\hat{k} \times (-30\hat{j}) + 6\hat{k} \times (-T_2\hat{i}) + 9\hat{k} \times \left(\frac{4\hat{i} + 3\hat{j} - 9\hat{k}}{\sqrt{106}}\right) T_1 = 0. \quad (3)$$

or in component form:

$$120 - \frac{27}{\sqrt{106}} T_1 = 0 \qquad \cdots (4a)$$

$$-6T_2 + \frac{36}{\sqrt{106}} T_1 = 0 \qquad \cdots (4b)$$

From equations 2(a) - 2(c) and 4(a) - 4(b) we get

$$T_1 = \frac{120\sqrt{106}}{27} = 45.75 \text{ (kN)}.$$

$$T_2 = \frac{6\,T_1}{\sqrt{106}} = 120 \times \frac{6}{27} = 26.67 \text{ (kN)}$$

$$A_x = 8.89 \text{ (kN)}$$

$$A_y = 16.67 \text{ (kN)}$$

$$A_z = 40.00 \text{ (kN)}$$

The total reaction force at A is $F_A = \sqrt{A_x^2 + A_y^2 + A_z^2} = 44.24$ (kN).

Note that we had only two equilibrium equations (4(a) - 4(b)) from moment balance. It was seen that $M_z = \vec{M}_A \cdot \hat{k}$ is trivially zero. This implies that the structure (ie boom) is inadequately constrained. Although the boom is in equilibrium under the given loading, the equilibrium can be easily disturbed if the loading is changed. For example, if a couple is applied along AB there is no constraint to prevent the rotation of the boom. The cables are unable to provide restraining torque in that case.

4. The FBD of the rod is shown in figure 4.1. The forces can be vectorially represented as

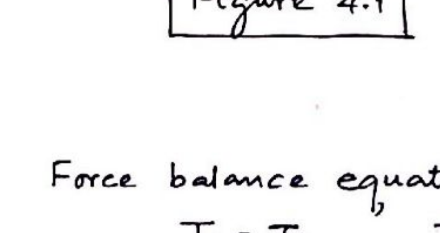


Figure 4.1

$$\vec{T}_1 = T_1 \left[\frac{1.2\hat{j} + 0.8\hat{k}}{\sqrt{(1.2)^2 + (0.8)^2}} \right]$$

$$\vec{T}_2 = T_2 \left[\frac{1.2\hat{j} + 0.8\hat{i} - 0.8\hat{k}}{\sqrt{(1.2)^2 + (0.8)^2 + (0.8)^2}} \right]$$

$$\vec{T}_3 = T_3 \left[\frac{1.2\hat{j} - 0.8\hat{i} - 0.8\hat{k}}{\sqrt{(1.2)^2 + (0.8)^2 + (0.8)^2}} \right]$$

$$\vec{W} = -50g\,\hat{j}.$$

Force balance equation gives the following result immediately

$$T_2 = T_3, \quad T_1 \times \frac{0.8}{\sqrt{(1.2)^2 + (0.8)^2}} - 2T_2 \times \frac{0.8}{\sqrt{(1.2)^2 + (0.8)^2 + (0.8)^2}} = 0 \qquad \cdot\cdot (1), (2)$$

and $$T_1 \times \frac{1.2}{\sqrt{(1.2)^2+(0.8)^2}} + 2\,T_2 \times \frac{1.2}{\sqrt{(1.2)^2+(0.8)^2+(0.8)^2}} - 50\,g = 0 \quad \cdots (3)$$

Solving (1) - (3) one gets

$$T_1 = 294.75 \text{ (N)}$$
$$T_2 = T_3 = 168.53 \text{ (N)}$$

These three tension forces must trivially satisfy the moment equation for the body to be in equilibrium. However taking moment about G we get

$$\vec{M}_G = 1.2\,\hat{k} \times \vec{T}_1 - 2\,\hat{k} \times (\vec{T}_2 + \vec{T}_3)$$
$$= 1.2\,\hat{k} \times T_1 \left[\frac{1.2\hat{j} + 0.8\hat{k}}{\sqrt{(1.2)^2+(0.8)^2}}\right] - 2\,\hat{k} \times T_2 \left[\frac{2\times 1.2\hat{j} - 2\times 0.8\hat{k}}{\sqrt{(1.2)^2+(0.8)^2+(0.8)^2}}\right], \quad \cdots (4)$$

However, for $\vec{M}_G$ to be equal to zero the following condition must be satisfied

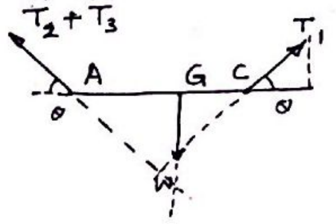
$$\frac{(1.2)^2}{\sqrt{(1.2)^2+(0.8)^2}}\,T_1 = \frac{4\times 1.2}{\sqrt{(1.2)^2+(0.8)^2+(0.8)^2}}\,T_2 \quad \cdots (5)$$

Comparison with equation (2) shows that the two relations are different, implying both force balance equation and moment balance equation can not be satisfied simultaneously. The only conclusion that can be reached is that the body is not in static equilibrium.

It can be noted that if $\frac{CG}{AG}$ becomes equal to one then $\vec{M}_G = 0$. In this case the body may remain in equilibrium although it is under-constraint since $\bar{M}_G \cdot \hat{j} = 0$ and $\bar{M}_G \cdot \hat{k} = 0$ trivially.

That the body can not be in equilibrium can also be seen in the following way. The resultant of $\vec{T}_2$ and $\vec{T}_3$ is a force that lies in the vertical plane since $\vec{T}_2 \cdot \hat{i} = -\vec{T}_3 \cdot \hat{i}$. Thus the given forces can be expressed equivalently as three forces lying in the same plane. For three planar forces the condition of equilibrium is that they must pass through the same point. It may be seen (see figure 4.2) that this is possible only when $CG = AG$.

Figure 4.2



5. The free body diagram of the door is shown in figure 5.1

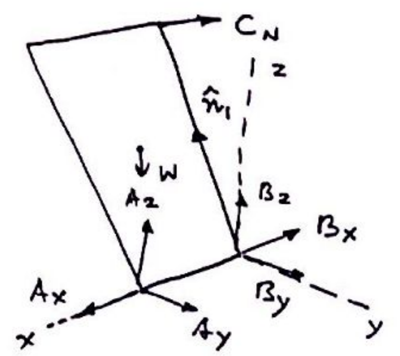


Figure 5.1

The forces on the door can be written in the vector form as

$$\vec{C}_N = C_N \,\hat{n}$$
$$\vec{W} = -mg\,\hat{k}$$
$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$
$$\vec{B} = -B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

where $\hat{n}$ is the unit vector perpendicular to the wall. The unit vector can be expressed in terms of $\hat{i}$ and $\hat{j}$ as (see figure 5.2)

$$\hat{n} = \sin 35^\circ\,\hat{i} + \cos 35^\circ\,\hat{j}.$$

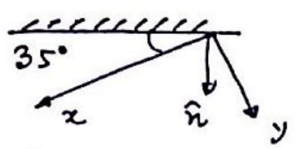


Figure 5.2.

The points of application of $\bar{C}_N$ and $\bar{W}$ are $h\,\hat{n}_1$ and $\frac{\omega}{2}\hat{i} + \frac{h}{2}\hat{n}_1$, respectively, where $\hat{n}_1$ is the unit vector along BC.

Now

$$\hat{n}_1 = -\sin 15^\circ\,\hat{j} + \cos 15^\circ\,\hat{k}.$$

Balance equation of moment about x-axis gives

$$\left(h\left(-\sin 15^\circ\,\hat{j} + \cos 15^\circ\,\hat{k}\right) \times C_N\left(\sin 35^\circ\,\hat{i} + \cos 35^\circ\,\hat{j}\right)\right)\cdot\hat{i}$$
$$+\left(\left(\frac{\omega}{2}\hat{i} + \frac{h}{2}\left(-\sin 15^\circ\,\hat{j} + \cos 15^\circ\,\hat{k}\right)\right)\times(-mg)\,\hat{k}\right)\cdot\hat{i} = 0 \quad \cdots (1)$$

i.e,

$$-C_N\,h\,\cos 15^\circ\,\cos 35^\circ + \frac{h}{2}\,mg\,\sin 15^\circ = 0$$

or

$$C_N = \frac{mg}{2}\,\frac{\tan 15^\circ}{\cos 35^\circ} = 0.164\,mg \quad \cdots (a)$$

The force balance equation yields

$$\bar{C}_N + \vec{W} + \vec{A} + \vec{B} = 0$$

or componentwise

$$A_x = B_x - C_N \sin 35^\circ \quad (2)$$
$$A_y + B_y + C_N \cos 35^\circ = 0 \quad (3)$$
$$A_z + B_z - mg = 0 \quad (4)$$

Also considering moment about z-axis we get

$$\left(h\left(-\sin 15^\circ\,\hat{j} + \cos 15^\circ\,\hat{k}\right) \times C_N\left(\sin 35^\circ\,\hat{i} + \cos 35^\circ\,\hat{j}\right)\right)\cdot\hat{k} + \omega A_y = 0$$

ie

$$C_N\,h\,\sin 35^\circ\,\sin 15^\circ + \omega A_y = 0 \quad \cdots (5)$$

ie

$$A_y = -0.0243\,mg\,\frac{h}{\omega}, \quad \cdots (b)$$

and hence from equation (3)

$$B_y = mg\left(0.024\,\frac{h}{\omega} - 0.1343\right). \qquad \cdots (c)$$

Similarly, considering moment about y-axis one gets

$$-A_z\,\omega + \left(\left(h\left(-\sin 15^\circ \hat{j} + \cos 15^\circ \hat{k}\right) \times C_N\left(\sin 35^\circ \hat{i} + \cos 35^\circ \hat{j}\right)\right)\cdot \hat{j}\right.$$
$$+ \left(\frac{\omega}{2}\hat{i} + \frac{h}{2}\left(-\sin 15^\circ \hat{j} + \cos 15^\circ \hat{k}\right) \times \left(-mg\,\hat{k}\right)\right)\cdot \hat{j} = 0 \qquad \cdots (6)$$

or $$-A_z\,\omega + h\,C_N\,\sin 35^\circ \cos 15^\circ + \frac{\omega}{2}\,mg = 0$$

which yields

$$A_z = \frac{mg}{2} + 0.0909\,mg\,\frac{h}{\omega} = mg\left(0.5 + 0.091\,h/\omega\right). \qquad \cdots (d)$$

From equations (4) and (d)

$$B_z = mg\left(0.5 - 0.091\,\frac{h}{\omega}\right). \qquad \cdots (e)$$

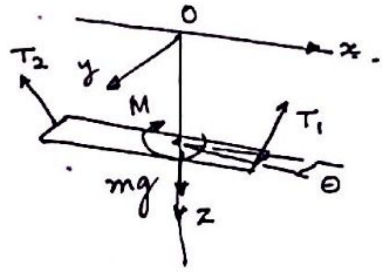
Note that we have exhausted all the equations of equilibrium but could not solve for A_x and B_x individually. The problem is statically indeterminate when A_x and B_x are both present. This happens when the door touches both A and B. In practical situation the door touches only one of the stops since there is always a small gap present. In this case either A_x or B_x should be zero. The stop which is touched by the door may be decided by the following reasoning.

It is seen that the stop can exert normal force only along the direction that secures contact. According to the figure given in the problem stop A can exert a force only along negative x-direction which the stop B exerts in the opposite direction. According to the free body diagram A_x or B_x should be negative. From equation (3) it is seen that this be the case when $B_x = 0$. The reaction force A_x is given by

$$A_x = -0.1514\,g. \qquad \cdots (f)$$

Note that in this problem we have separately considered the moment balance equation about individual axes. This was not essential since equations (1), (5) and (6) could have been obtained by considering the moment balance equation about the origin.

6. It can be seen from the symmetry of the system that in equilibrium position the uniform bar remains in the horizontal position. The configuration of the rod can be described by an angular orientation with respect to AB in horizontal plane and a vertical rise. The free body diagram of the rod is shown in figure 6.1.



The forces on the rod are as following

(i) $mg\,\hat{k}$ passing through origin

(ii) $\bar{T}_1 = T_1\,\hat{n}_1$

(iii) $\bar{T}_2 = T_2\,\hat{n}_2$

and couple of moment $+M\hat{k}$,

where $\hat{n}_1$ and $\hat{n}_2$ are unit vectors along the cords.

It can be easily established from geometry that

$$\hat{n}_1 = +\frac{b}{2}(1-\cos\theta)\,\hat{i} - \frac{b}{2}\sin\theta\,\hat{j} - h\,\hat{k} \Big/ \left[\left(\frac{b}{2}(1-\cos\theta)\right)^2 + \left(\frac{b}{2}\sin\theta\right)^2 + h^2\right]^{1/2}$$

and

$$\hat{n}_2 = -\frac{b}{2}(1-\cos\theta)\,\hat{i} + \frac{b}{2}\sin\theta\,\hat{j} - h\,\hat{k} \Big/ \left[\left(\frac{b}{2}(1-\cos\theta)\right)^2 + \left(\frac{b}{2}\sin\theta\right)^2 + h^2\right]^{1/2}$$

where h' is the vertical distance from the line AB. To get the above expressions for the unit vectors see the projections of the rod in x-y plane as shown in figure 6.2.

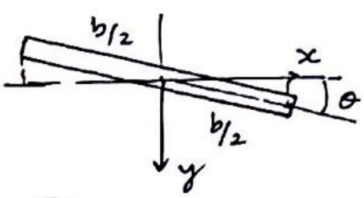


Figure 6.2.

From the force balance equations we get

$$(T_1 - T_2)\,\frac{b}{2}\,(1-\cos\theta) = 0 \qquad \text{--- (1)}$$

$$(T_1 - T_2)\,\frac{b}{2}\,\sin\theta = 0 \qquad \text{- (2)}$$

and

$$(T_1 + T_2)\,\frac{h'}{\left[h'^2 + \frac{b^2}{2}(1-\cos\theta)\right]^{1/2}} = mg \qquad \text{- (3)}$$

The moment balance equation about z-axis gives

$$(T_1 + T_2)\,\frac{b}{2} \times \frac{b}{2}\,\frac{\sin\theta}{\left[h'^2 + \frac{b^2}{2}(1-\cos\theta)\right]^{1/2}} = M \qquad \text{.. (4)}$$

From equations (3) and (4) we get

$$\frac{2M}{b\,mg} = \frac{b}{2h'}\sin\theta. \qquad \text{.. (5)}$$

Now, the length of the string is unchanged. This implies

$$\left(\frac{b}{2}(1-\cos\theta)\right)^2 + \left(\frac{b}{2}\sin\theta\right)^2 + h'^2 = L^2 \qquad \text{.. (6)}$$

which yields

$$\frac{b}{2h'}\cos\theta = \frac{h'}{b} - \frac{b}{2h'} \qquad \text{.. (7)}$$

From equations (5) and (6) we get

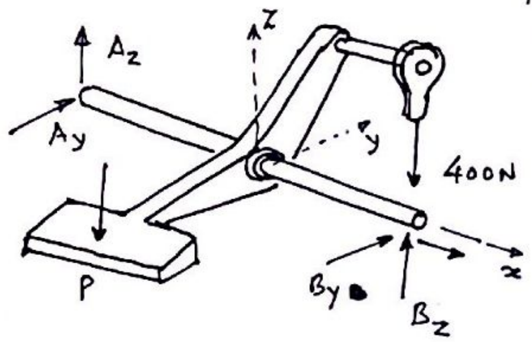
$$\left(\frac{b}{2h'}\right)^2 = \left(\frac{2M}{bmg}\right)^2 + \left(\frac{h'}{b} - \frac{b}{2h'}\right)^2$$

ie $$\left(\frac{h'}{b}\right)^2 = 1 - \left(\frac{2M}{bmg}\right)^2 . \qquad \cdots (8)$$

The vertical rise of the rod is given by

$$\Delta h = b - h' = b\left[1 - \sqrt{1 - \left(\frac{2M}{bmg}\right)^2}\,\right] . \qquad \cdots (9)$$

7. The free body diagram of the foot pedal and the beari shaft is shown in figure 7.1.



Note that only two reaction forces are shown at the bearings. By doing this we assume the bearings to be short in size (length) and incapable of restricting rotation. Further, it is assumed that the bearing can not arrest translational motion along its axis. (The bearing which does this is called thrust bearing)

Considering moment balance equation about x-axis we get

$$P \times 0.2 = T \times 0.12 \cos 30^\circ = 400 \times 0.12 \cos 30^\circ . \qquad \cdots (1)$$

Therefore $P = \dfrac{400 \times 0.12}{0.2} \cos 30^\circ \text{ (N)} = 207.85 \text{ (N)}$

Taking moment of all forces about y-axis and equating it to zero we get

$$(B_z - A_z) \times 0.1 = 400 \times 0.06 \qquad \cdots (2)$$

Similarly about z-axis we get

$$B_y = A_y . \qquad \cdots (3)$$

Force balance equation along y-axis and z-axis give, respectively,

$$A_y + B_y = 0 \qquad \cdots (4)$$

and $$A_z + B_z = P + T = 607.85 . \qquad \cdots (5)$$

From equations (2), (3), (4) and (5) we get

$A_y = 0$, $B_y = 0$,

$$B_z = \frac{607.85 + 240}{2} = 423.93 \text{ (N)},$$

and $$A_z = 607.85 - 423.93 = 183.93 \text{ (N)} .$$

8. We Consider the free body diagram of the drill, sleeve and the mount taken as a single body. The free body diagram is shown in figure 8.1.

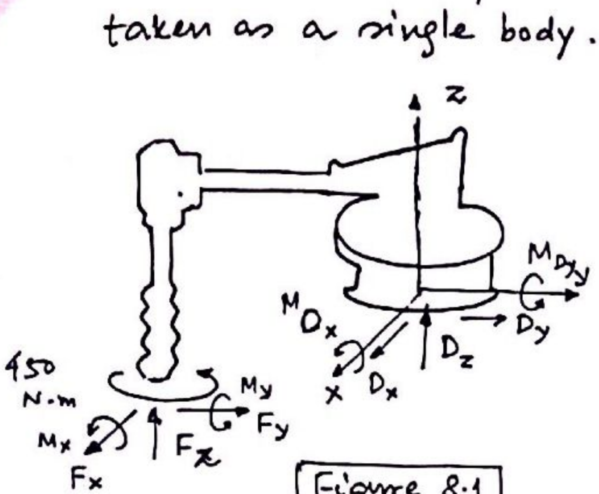


Figure 8.1

Note that in the free body diagram the drill torque is shown in anti'clockwise direction as seen from z-axis. This is because the drill experiences a torque from the ground in the direction opposite to that of the driving torque. At the mount no reaction torque is shown in z-direction since it is free to swivel about the axis.

Considering moment balance equation about z-axis we get

$$450 + F_x \times 2.4 = 0 \qquad \cdots (1)$$

ie, $F_x = -\dfrac{450}{2.4}$ (N).

From force balance equation along x-axis one gets

$$D_x = -F_x = \frac{450}{2.4} \text{ (N)}. \qquad \cdots (2)$$

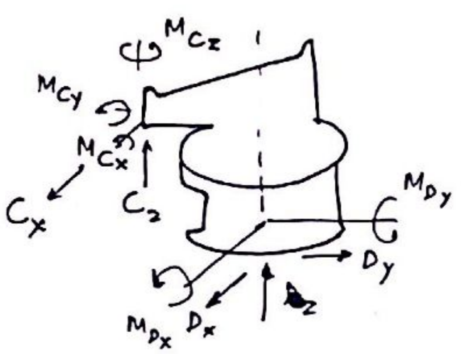
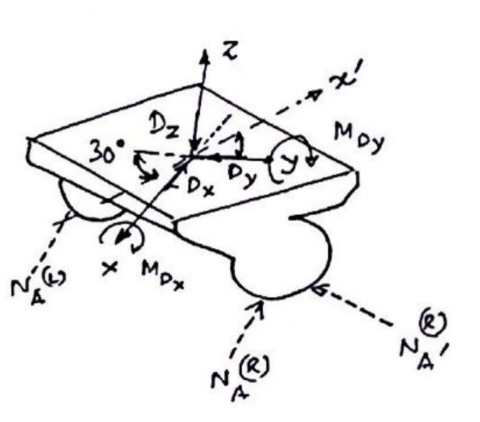


Figure 8.2

To obtain D_y we use the free body diagram of the mount and the sleeve as shown in figure 8.2. Since the arm can slide freely within the sleeve no reaction force is shown along y-axis. Considering the force balance equation along y-axis we get

$$D_y = 0. \qquad \cdots (3)$$



Consider, now, the free body diagram of the cart, shown in figure 8.3. The forces on the wheels exerted by the obstacle are shown by dotted lines since it is not known which of the obstacles A or A' is going to exert the force. The moment balance equation about the z-axis gives the following result

Either $N_A^{(L)} = N_A^{(R)}$ or $N_{A'}^{(L)} = N_{A'}^{(R)}$ $\cdots (4)$

Further, since D_x is positive the force balance equation, along the length-wise direction (x'-axis in FBD) leads to the following conclusion

$$2N_A^{(R)} \sin 30^\circ + D_x \sin 30^\circ = 0 \qquad \cdots (5)$$

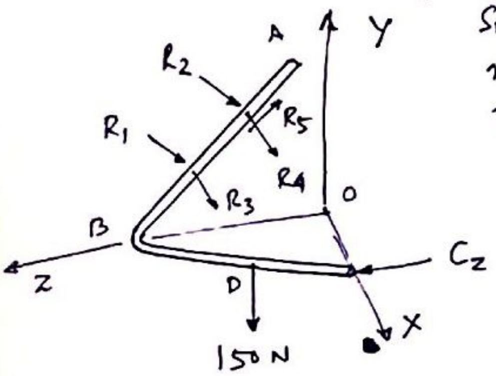
or $$2N_{A'}^{(R)} \sin 30^\circ - D_x \sin 30^\circ = 0 \qquad \cdots (6)$$

However, since N_A or $N_{A'}$ can only assume positive values it becomes apparent that the wheel must touch the obstacle A′. The value of $N_{A'}$ is given by

$$N_{A'}^{(R)} = \frac{D_x}{2} = \frac{450}{2 \times 2.4} = 93.75 \text{ (N)}.$$

Note that in the given problem many reaction forces and moments remain unsolved. These are not needed. In solving the required quantity, namely, $N_{A'}$, we have only used a few of the equilibrium equations.

9. The free body diagram of the bent rod is shown in figure 9.1.



Since the width of the bracket is small it can not prevent rotation about any of the transverse axes. The reaction forces R_1 and R_2 are on Y-Z plane and perpendicular to AB. R_3 and R_4 are along X-axis while R_5 is along AB. At point C only bearing force C_z exists.

To find out C_z it is most profitable to use moment balance equation about AB, since the rest of the unknown forces do not contribute to this moment.

The forces are represented vectorially as $\bar{C}_z = C_z \hat{k}$, $\vec{F} = -150\hat{j}$ (N). The position vector of their points of application from point B can be represented as

$$\vec{r}_{BC} = (-0.15\hat{k} + 0.20\hat{i}) \text{ m}$$

and $$\vec{r}_{BD} = 0.15 \times \vec{n}_{BC} = 0.15\left(\frac{-0.15\hat{k} + 0.20\hat{i}}{\sqrt{(0.15)^2 + (0.20)^2}}\right) \text{ (m)}.$$

The moment balance equation about AB can be written as

$$\left[\vec{r}_{BC} \times C_z\hat{k} + \vec{r}_{BD} \times (-150\hat{j})\right] \cdot \hat{n}_{AB} = 0 \qquad \cdots (1)$$

where $$\vec{n}_{AB} = \frac{-0.3\hat{j} + 0.15\hat{k}}{\sqrt{(0.3)^2 + (0.15)^2}}. \qquad \cdots (2)$$

From equation (1) we get

$$\frac{0.2 \times 0.3}{\sqrt{(0.3)^2 + (0.15)^2}} C_z - \frac{150 \times (0.15 \times 0.20)}{\sqrt{(0.15)^2 + (0.20)^2}} \times \frac{0.15}{\sqrt{(0.3)^2 + (0.15)^2}} = 0$$

or $C_z = \dfrac{150 \times (0.15 \times 0.2) \times (0.15)}{0.2 \times 0.3 \times \sqrt{(0.15)^2 + (0.2)^2}}$ (N)

$= 45$ (N)

10. In order to get the change in the reaction forces over their nominal values (which are caused by the weight of the aeroplane) we draw the free body diagram of the aircraft as shown in figure 10.1. The weight is not shown and no friction force is shown in wheel A because the braking force is applied only in the remaining two wheels. It should be noted that ΔN_A, ΔN_B and ΔN_C, which are changes in their values can be both positive or negative.

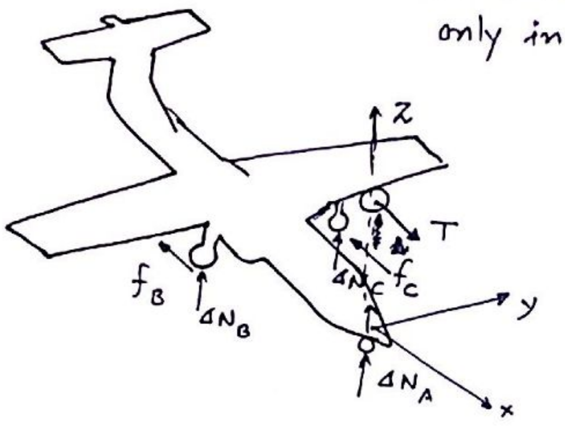


Figure 10.1

Considering the moment balance equation about x-axis we get

$$\Delta N_B = \Delta N_C \quad \cdots (1)$$

Further taking the moment balance equation about y-axis one has

$$(\Delta N_B + \Delta N_C) \times 4 + T \times 2 = 0 \quad \cdots (2)$$

From equations ① and ② we get

$$\Delta N_B = \Delta N_C = -\frac{T}{4} = -500 \text{ (N)}.$$

The force balance equation along z-direction yields

$$\Delta N_A + \Delta N_B + \Delta N_C = 0 \quad \cdots (3)$$

or $\Delta N_A = -2\,\Delta N_B = 1000$ (N).

— END —