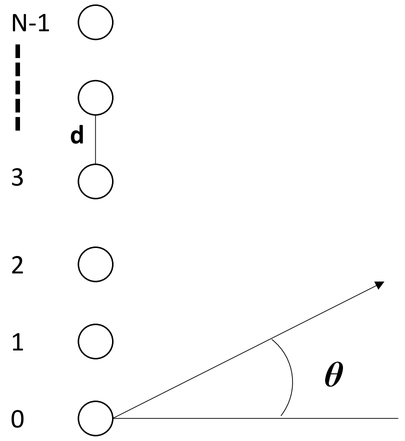
1. **A stationary radiating system consists of a linear chain of parallel oscillators separated by a distance d. The phase of the oscillators varies linearly along the chain. Find out the angular position $\theta$ for $n^{th}$ order maxima.**

**Ans.**

Figure shows that the radiating system which consists of the chain of the parallel oscillator separated by a distance d.




Each consecutive sources have a phase difference of $\delta\phi$ at their position and the path difference is $d\sin\theta$.

Total phase difference ($\varphi$) = $\delta\phi + k_0 d\sin\theta$

The total field at an angle $\theta$ is

$$E_T = E_0[1+e^{i\varphi}+e^{2i\varphi}+----+e^{(N-1)i\varphi}]$$

$$E_T = E_0[\frac{1-e^{iN\varphi}}{1-e^{i\varphi}}]$$

$$I = E_T E_T^*$$

$$I = E^2[\frac{1-\cos N\varphi}{1-\cos\varphi}]$$

$$I = E^2[\frac{\sin^2\frac{N\varphi}{2}}{\sin^2\frac{\varphi}{2}}]$$

For $\frac{\varphi}{2} = n\pi, (n = \pm1, \pm2, \pm3, ---)$ both numerator and denominator of the intensity expression becomes zero and we have ($\equiv\frac{0}{0}$) form and that will give a finite maximum value (using L'Hospital rule). So the condition for maxima at an angular position $\theta$ is

$$\delta\phi + k_0 d \sin\theta_n = 2n\pi$$

$$\theta_n = \sin^{-1}\left(n - \frac{\delta\phi}{2\pi}\right)\frac{\lambda}{d}$$

2. **Consider a two-slit Youngs interference experiment with $\lambda$ = 500 nm where fringes are generated on a screen N placed 1 meter apart from the slits.**
   **(a) The fringe width decreases 1.2 time when the slit width is increased by 0.2 mm. calculate the original fringe width.**
   **(b) When a thin film of a transparent material is put behind one of the slits, the zero order fringe moves to the position previously occupied by the 4th order bright fringe. The index of refraction of the film is n = 1.2. Calculate the thickness of the film.**

**Ans. (a)**
As the standard fringe-width formula is

$$\Delta y = \frac{D}{d}\lambda \quad ......(1)$$

Now the fringe width decreases 1.2 time when the slit width is increased by 0.2 mm, so

$$\frac{\Delta y}{1.2} = \frac{D}{d+0.2}\lambda \quad ......(2)$$

From (1) and (2)

$$d = 1mm$$

So

$$\Delta y = 0.5\,mm$$

**(b)** Intensity maxima occur when the optical path differenceis $\Delta = m\lambda$
Thus

$$\delta\Delta = \delta m\lambda$$

when the film is inserted, the optical path changes by

$$\delta\Delta = t(n-1)$$

wheret is the thickness of the film. As the interference pattern shifts by 4 fringes

$$\delta m = 4$$

Hence

$$t = \frac{4\lambda}{n-1} = 10\,\mu m$$

3. **In a Lloyd's mirror experiment (see Figure 1), a bright wave emitted directly by the source S interferes with the wave reflected by the mirror M. As a result an interference fringe pattern is formed on the screen N. The source and the screen is separated by a distance *l* = 1 m. At a certain position of the source the fringe width on the screen is equals to Δx = 0.25 mm. After the source is moved away from the plan of mirror by Δh = 0.60 mm, the fringe width decreases by a factor η = 1.5. Find the wavelength of the light.**

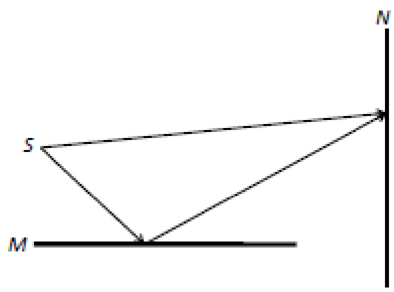



Figure 1: *Lloyd's Mirror*

As the fringe spacing can be express as

$$\Delta x = \frac{l}{2h}\lambda \text{ ......(1)},$$

where$h$ is the distance between mirror to source.
Now, the source is moved away from the plan of mirror by $\Delta$h = 0.60 mm, the fringewidth decreases by a factor $\eta$ = 1.5 then rom equation (1)

$$\frac{\Delta x}{1.5} = \frac{l}{2(h+.6)}\lambda \text{ ......(2)}$$

From (1) and (2)

$$h = 1.2\,mm$$

and from equation (1)

$$\lambda = 0.6 \ \mu m$$

**4. A point source S is located at the origin of a coordinate system (see Figure 2) emits a spherical sinusoidal wave $E_1 = A\frac{D}{r}\cos(\frac{2\pi}{\lambda}r - \omega t)$ where r is the distance from S. In addition there is a plane wave propagating along x-axis. The form of the plane wave is given as $E_2 = A\cos(\frac{2\pi}{\lambda}x - \omega t)$. Both wave are incident on a flat screen perpendicular to the x-axis and at a distance D from the origin, as shown in the figure (Fig. 2). Compute the resultant intensity I at the screen as a function of the distance y from the x-axis when y << D. Express *I* in terms of y, D, $\lambda$ and the intensity $I_0$ at y = 0.**

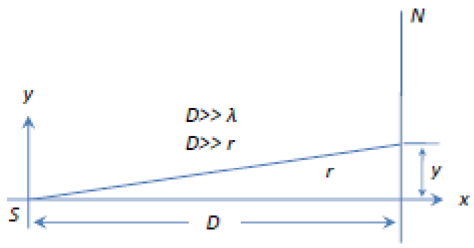



Figure 2:

**Ans**

The electric field E at a point, distance r from the origin, on the screen is given by

$$E \;=\; E_1 + E_2 \;=\; A\,[\frac{D}{r}\cos(\frac{2\pi}{\lambda}r - \omega t) \;+\; \cos(\frac{2\pi}{\lambda}x - \omega t)]$$

$$\approx\; A[\cos(\frac{2\pi}{\lambda}r - \omega t) \;+\; \cos(\frac{2\pi}{\lambda}x - \omega t)]$$

$$We\ have\ used\ the\ approximation\ r \approx D.$$

$$For\ y << D,\ we\ have$$

$$r \;=\; (D^2 + y^2)^{\frac{1}{2}} \;\approx\; D\,(1 + \frac{y^2}{2D^2})$$

$$and\ thus$$

$$E \;=\; A\,[\cos\,(\frac{2\pi D(1 + \frac{y^2}{2D^2})}{\lambda} - \omega t) \;+\; \cos\,(\frac{2\pi D}{\lambda} - \omega t)]$$

$$=\; 2A\cos\frac{\pi y^2}{D\lambda}.\,\cos\left(\frac{2\pi D\left(1 + \frac{y^2}{4D^2}\right)}{\lambda} - \omega t\right)$$

$$Hence$$

$$I\ \alpha\ E^2\ \alpha\ \cos^2\frac{\pi y^2}{D\lambda}.$$

$$Let\quad I_0 = 4A^2 cos^2\,(\frac{2\pi D(1 + \frac{y^2}{4D^2})}{\lambda} - \omega t)$$

$$I \;=\; I_0 \cos^2(\frac{\pi y^2}{D\lambda})$$

5. **In Newton's ring set-up, the diameter of the 10<sup>th</sup> bright ring changes from 1.32 cm to 1.17 cm, when a liquid is introduced between the lens and the glass plate. Calculate the refractive index of the liquid.**

**Ans.**

Diameter of the fringe in the absence of the glass plate

$$d_m^2 = 4(m + \frac{1}{2})R\lambda = (1.32)^2 \;.........(1)$$

And Diameter of the fringe in the presence of the glass plate

$$d_m^2 = 4(m + \frac{1}{2})R\frac{\lambda}{n} = (1.17)^2 \;........(2)$$

From (1) and (2)

$$n \;=\; 1.272$$