

Please read below information carefully before proceeding with the test

- Take $g = 10 \text{ m/s}^2$ everywhere
- Write final answer in decimal format: 0.6 is fine, $3/5$ is not.
- Answers must be written with at least two places of decimal, whenever appropriate
- Extra rough sheets can be used if needed, however, only this answer script cum question paper will be considered for grading
- Total mark is 55, maximum marks is 50

Questions 1 to 5 carry 1 mark each (No part marks in this section)

Answer the following as briefly as possible, explaining your rationale in one sentence

Q1. Consider a 2-dimensional rigid body. How many equations of equilibrium are needed if the body is subjected to forces parallel to each other?

2 equations

Q2. How does the solution to Q1 change if the parallel forces are also collinear?

1 equation

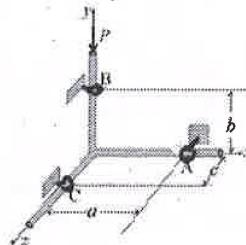
Q3. Consider two coplanar forces A and B. Let the resultant of the two forces be C. The magnitude of the resultant C equals the magnitude of any one of the forces A and B. What is the angle between A and B?

120°

Q4. Consider a member supported by three thin rings and loaded with load P as shown. If the mass of the member is neglected, how many external forces and reactions (incl. moment if any) are there. Just write the total number

3 × 2 + 1 = 7

2 unknowns per ring

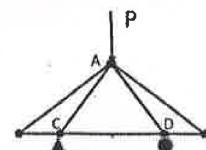


Q5. State true or false with one sentence justification: If a planar rigid body with reaction forces acting at three distinct points on the body is subject to an arbitrary load distribution then the body will always remain in equilibrium.

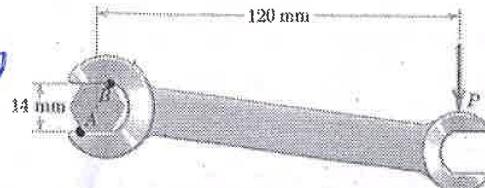
False: 3 roller supports in a line do not ensure equilibrium

Questions 6 to 8 carry 2 mark each (No part marks in this section)

Q6. Consider a truss loaded as shown in the figure, the load in member AC is Comp. (tensile or compressive, choose one option). The load in member CD is TENSILE (tensile or compressive, choose one option)



Q7. The magnitude of the couple forces at the corners A and B when $P = 100N$ is $1484.61N$. Points A and B are part of regular hexagon



$$\text{length } AB = 14/\sin 60^\circ$$

$$\therefore F \times \frac{7}{\sin 60^\circ} = 100 \times 120$$

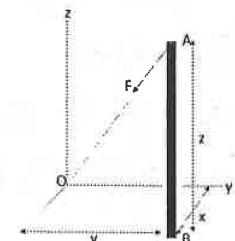
$$F = 1484.61N$$

Q8. The cable AO, exerts a force of $-120i - 90j - 80k$ N at the top of pole AB. If the length of the cable is 34m, the dimension "y" is

-18 m

Unit vector along AO is

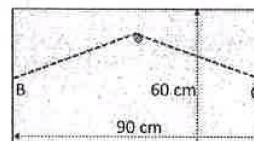
$$\frac{-120i}{170} - \frac{90j}{170} - \frac{80k}{170}$$



$$xi + yj + zk = 34 \left(\frac{-120i}{170} - \frac{90j}{170} - \frac{80k}{170} \right) \Rightarrow y = -18 \text{ mm}$$

Questions 9 to 13 have part marks

Q9. A rectangular uniform picture frame of mass 1 kg with dimensions as shown in the figure is to be hung horizontally (from B to C) as shown with the help of a thread. The maximum tension in the thread can be 15 N. [6 Marks]



$$\sum F_y = 0$$

$$10 - 2T \sin \theta = 0$$

$$T = \frac{5}{\sin \theta}$$

$$15 = \frac{5}{\sin \theta} \Rightarrow \sin \theta = \frac{1}{3}, \theta = 19.47^\circ$$



2 mark

$$\text{length, } l = \frac{90}{\cos 19.47^\circ} = 95.45 \text{ cm}$$

The minimum length of thread needed is 95.45 cm? 3 mark

Q10. The 75-kg mass is suspended by cables attached to three vertical posts having height 2 m each. Point A is at (0, 1.2, 0) m.

From the figure

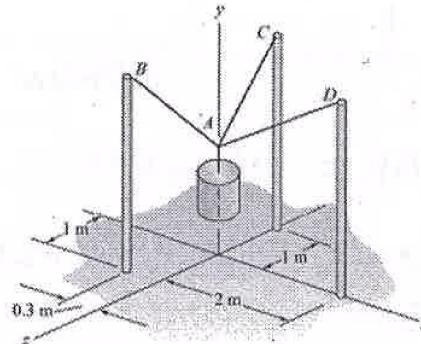
$$A(0, 1.2, 0); C(0, 2, -1)$$

$$B(-0.3, 2, 1); D(2, 2, 0)$$

1 mark

The unit vector along AC is $\underline{0} \hat{i} + \underline{0.625} \hat{j} - \underline{0.781} \hat{k}$

[2 Marks]



Unit vectors are

$$\lambda_{AB} = -0.228 \hat{i} + 0.608 \hat{j} + 0.760 \hat{k}$$

$$\lambda_{AD} = 0.928 \hat{i} + 0.371 \hat{j} + 0 \hat{k}$$

The forces are $F_{AB}\lambda_{AB}$, $F_{AC}\lambda_{AC}$ and $F_{AD}\lambda_{AD}$
 $-750 \hat{j}$

The equations of equilibrium are

$$\left\{ \begin{array}{l} \sum F_x = 0 \Rightarrow -0.228 F_{AB} + 0 + 0.928 F_{AD} = 0 \\ \sum F_y = 0 \Rightarrow 0.608 F_{AB} + 0.625 F_{AC} + 0.371 F_{AD} - 750 = 0 \\ \sum F_z = 0 \Rightarrow 0.760 F_{AB} - 0.781 F_{AC} = 0 \end{array} \right.$$

2 marks

Solving we get.

The tension in cable AB is 573.68 N

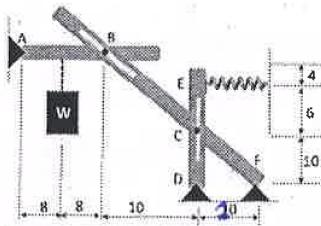
The tension in cable AC is 558.26 N

$1 \times 3 = 3$ mark

The tension in cable AD is 140.95 N

[6 Marks]

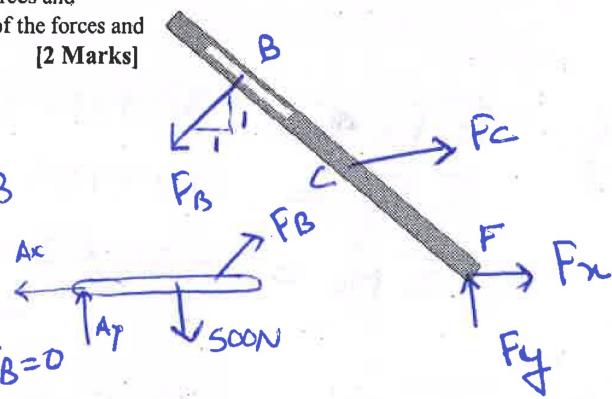
Q11. Study the frame as shown in the figure, all dimensions are in cm. The mass of the suspended object is $W = 50\text{kg}$. The slotted member DE is vertical. The frame is pinned at A, D and F



Outline of the member BEF is given. On it label the forces and reactions, clearly defining the directions and location of the forces and reactions.

[2 Marks]

Start with member AB



$$\sum M_A = 0$$

$$-500 \times 8 + \frac{1}{\sqrt{2}} \cdot 16, P_B = 0$$

$$P_B = 354\text{N}$$

In Member BCF

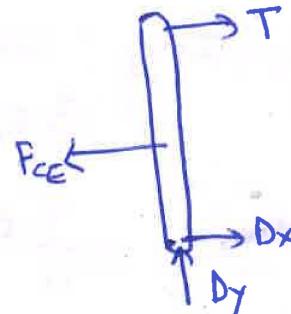
$$\sum M_F = 0 \Rightarrow P_B \cdot (20\sqrt{2}) - F_C \times 10 = 0$$

$$\Rightarrow P_C = 1000\text{N}$$

In member DCE

$$\sum M_D = 0 \Rightarrow -T \times 16 + P_C \times 10 = 0$$

$$T = 625\text{N}$$

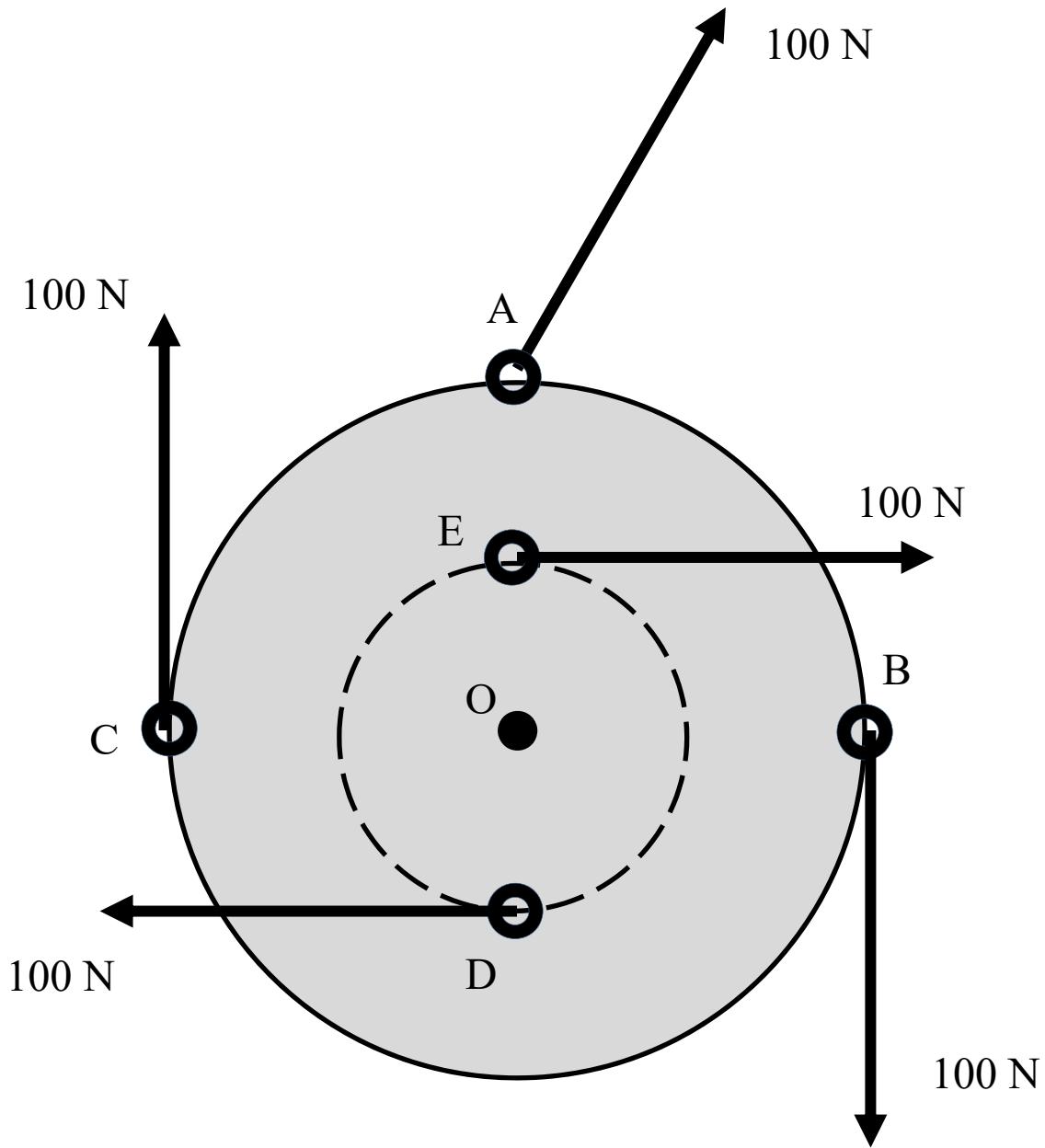


The magnitude of reaction at C is 1000 N

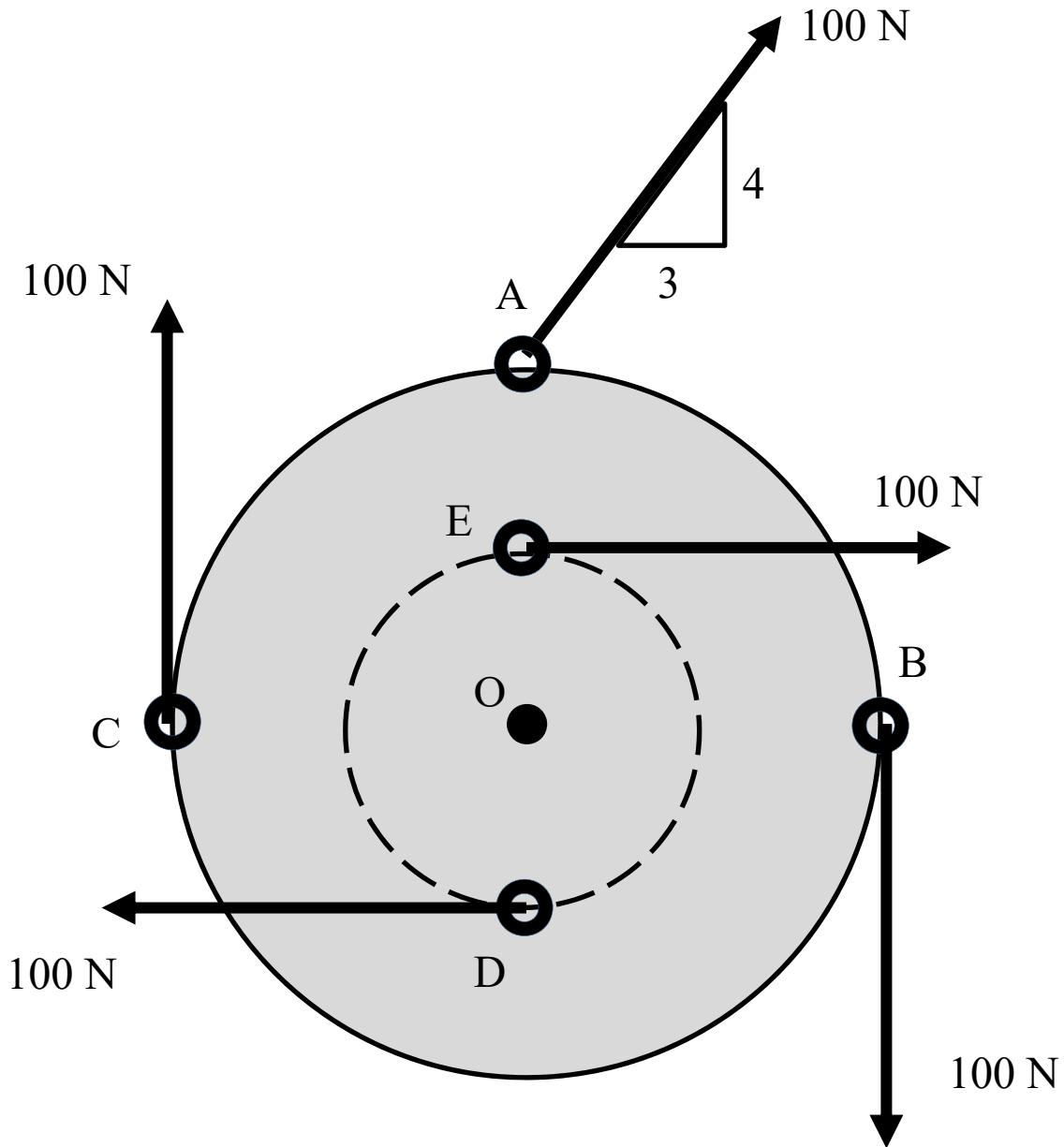
[4 Marks]

The tension in the spring is 625 N

[4 Marks]



An uniform disc of radius 2 m was supposed to be held in place against the action of external forces (which are not shown) by applying the forces shown at points A, B and C on the circumference as well as forces at the points D and E which are 1 m from the center O. Due to certain problems application of forces at B and D was no longer possible. Given that the line of action as well as the point of application of the remaining forces had to remain unchanged, find the new forces that were required to be applied at the points A, E and C so that the disc would still be kept in place.

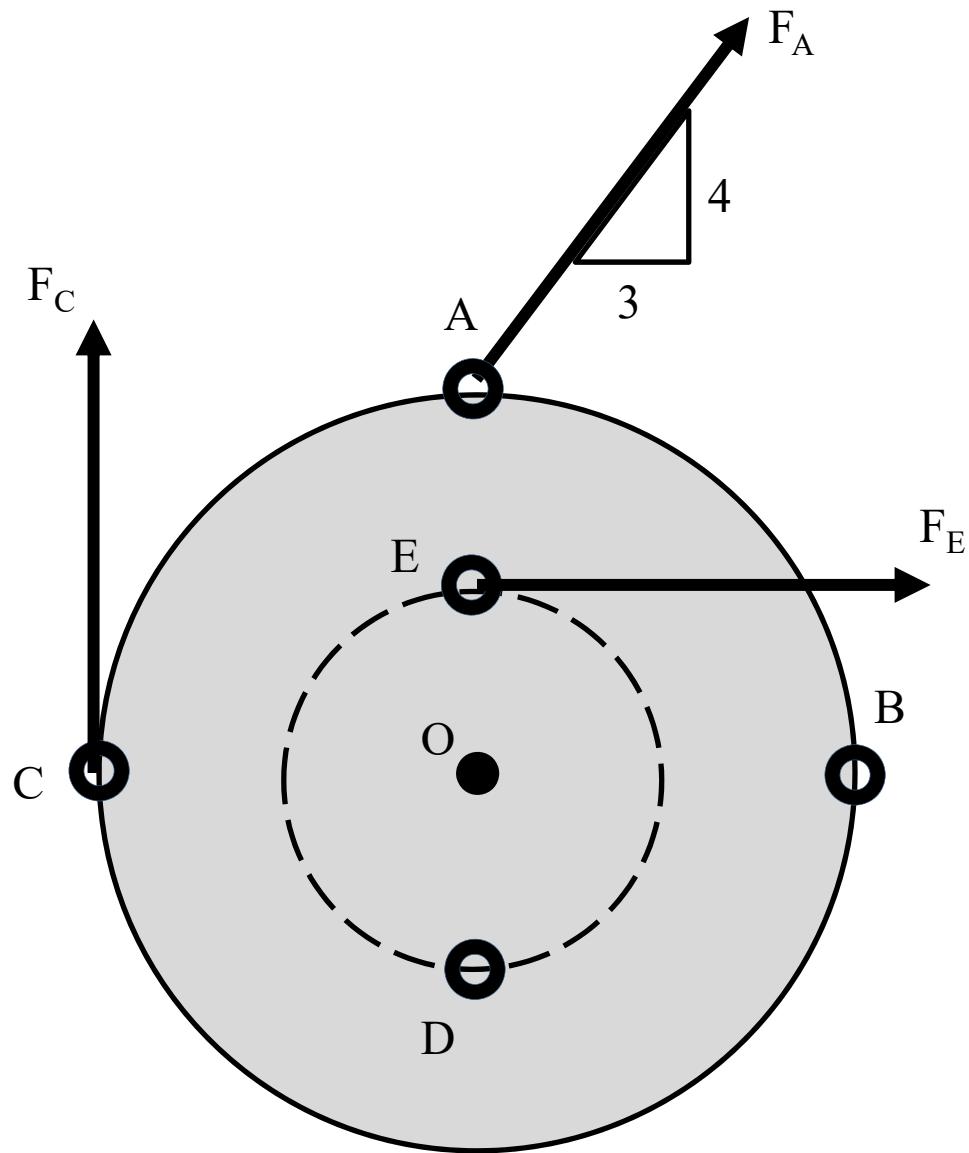


Initially

$$\sum F_x = 60 + 100 - 100 = 60,$$

$$\sum F_y = 80 + 100 - 100 = 80,$$

$$\begin{aligned} \sum M_o &= -60 \times 2 - 100 \times 1 - 100 \times 2 - 100 \times 1 - 100 \times 2 \\ &= -720 \end{aligned}$$

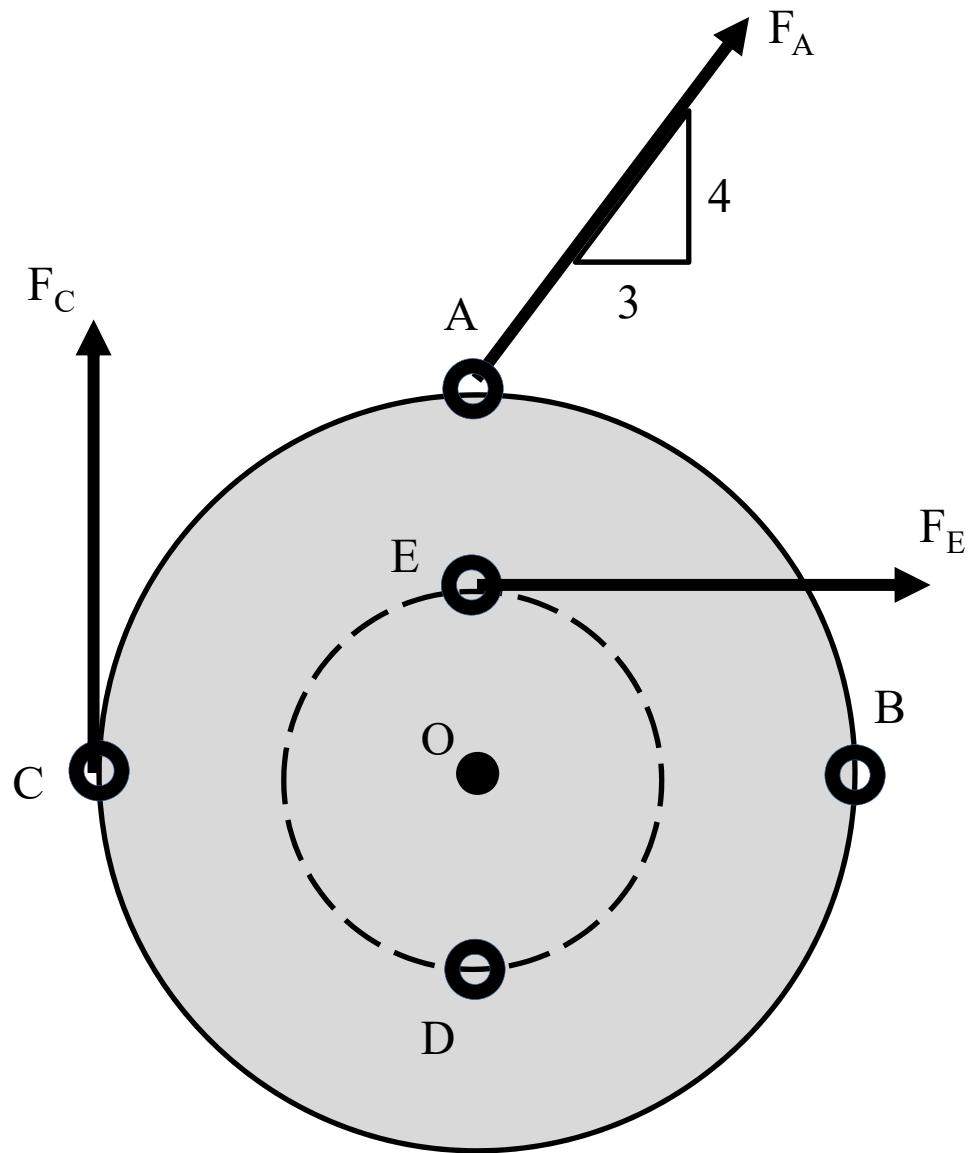


$$\text{Finally } \sum F_x = \frac{3}{5} F_A + F_E, \sum F_y = \frac{4}{5} F_A + F_C,$$

$$\sum M_o = -\frac{3}{5} F_A \times 2 - F_E \times 1 - F_C \times 2$$

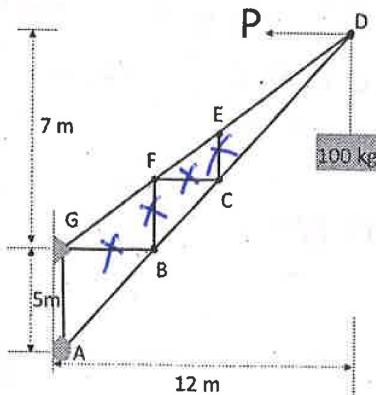
$$\Rightarrow \frac{3}{5} F_A + F_E = 60, \quad (1) \quad \frac{4}{5} F_A + F_C = 80, \quad (1)$$

$$-\frac{3}{5} F_A \times 2 - F_E \times 1 - F_C \times 2 = -720 \quad (2)$$

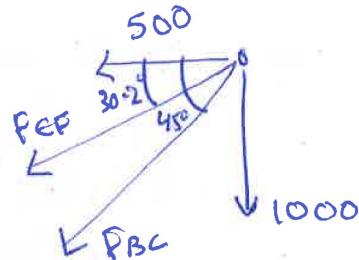


$$\begin{aligned}
 \frac{3}{5}F_A + F_E &= 60, \quad \frac{4}{5}F_A + F_C = 80, \quad -\frac{3}{5}F_A \times 2 - F_E \times 1 - F_C \times 2 = -720 \\
 \Rightarrow 3F_A + 5F_E &= 300, \quad 4F_A + 5F_C = 400, \quad 6F_A + 5F_E + 10F_C = 3600 \\
 \Rightarrow 5F_E &= 300 - 3F_A, \quad 5F_C = 400 - 4F_A \Rightarrow 10F_C = 800 - 8F_A \\
 \Rightarrow 6F_A + 5F_E + 10F_C &= 3600 \Rightarrow 6F_A + 300 - 3F_A + 800 - 8F_A = 3600 \\
 \Rightarrow F_A &= -500, \quad (2)F_C = 480, \quad (2)F_E = 360 \quad (2)
 \end{aligned}$$

Q13. Consider a truss as shown in the figure. At point D, there is a mass of 100 kg and general load P N. With a cross mark, indicate the members which are zero force members. (There will be negative marking for wrong identification of the zero-force members) [2 Marks]



b.) if $P = 500$ N, the load in member BC is -2398.81 N [4 Marks]



$$F_{EF} \cos 30.2 + P_{BC} \sin 45 + 500 = 0$$

$$F_{EF} \sin 30.2 + P_{BC} \cos 45 + 1000 = 0$$

Solving we get, $P_{BC} = -2398.81$ N

c.) Minimum load P to ensure that the member ED is not in a state of tension is 1000N? [4 Marks]

ED is a zero force member if the resultant of two external forces passes through CD.

CD is at 45-degrees with the horizontal.
So, if $P = 100 \times 10$, then ED is a zero force member

$$\underline{P = 1000 N}$$

Space for rough work