Course: PH11003: Physics: 2022–2023(II)

*** marked problems are for classwork, the rest for homework**

Tutorial 8: Matter waves and quantum physics

1. * (a) Light from the Sun arrives at the Earth, an average of 1.5×10^{11} m away, at the rate 1.4×10^3 $Watts/m^2$ of area perpendicular to the direction of the light. Assume that sunlight is monochromatic with a frequency 5×10^{14} Hz.

(i) How many photons fall per second on each square metre of the Earth's surface directly facing the Sun?

(ii) What is power output of the Sun, and how many photons per second does it emit?

(b) A 1000 Watt radio transmitter operates at a frequency of 880 kHz. How many photons per second does it emit?

2.* (a) Illuminating the surface of a metal alternately with light of wavelengths $\lambda_1 = 0.35\mu$m and $\lambda_2 = 0.54\mu$m, it was found that the corresponding maximum velocities of the photoelectron differ by a factor of 2. Find the work function of the metal.

(b) The maximum energy of photoelectrons from Alumunium is 2.3 eV for radiation of 2000$\mathring{A}$ and 0.90 eV for radiation of 2580$\mathring{A}$. Use these data to calculate Planck's constant and the work function of Alumunium.

3. * (a) A Compton effect experiment is performed in such a way that the scattered photon and the recoil electron are detected only when their paths are at right angles ($\theta + \phi = \frac{\pi}{2}$).

(i) Show that under these conditions $\lambda' = \frac{\lambda}{\cos\phi}$

(ii) Show that the energy of the scattered photon is m_0c^2.

(iii) Find the energy of the recoil electron.

(iv) What is the minimum energy of the incident photons for which the experiment can be done ?

(b) Show that when a free electron is scattered in a direction making an angle θ with the incident photon in a Compton scattering, the kinetic energy of the electron is:

$$E_k = \frac{h\nu\,(2\alpha\cos^2\theta)}{[(1+\alpha)^2 - \alpha^2\cos^2\theta]} \tag{1}$$

where $\alpha = \frac{h\nu}{m_0 c^2}$.

4. (a) Find the de Broglie wavelength of a 1 mg grain of sand blown by the wind at a speed 20 m/s.

* **(b)** Calculate the de Broglie wavelength of an electron when its energy is 1 eV, 100 eV, 1000 eV. At what value of the kinetic energy is the de Broglie wavelength of an electron equal to its Compton wavelength?

(c) Show that the ratio of the de Broglie wavelength to the Compton wavelength of the same particle is equal to $\sqrt{\left(\frac{c}{v}\right)^2 - 1}$.

5. * **(a)** A parallel beam of electrons accelerated by a potential difference of 25 Volts falls normally on a diaphragm with two narrow slits separated by a distance , 50 μm. Calculate the distance between neighboring maxima of the diffraction pattern on a screen located at a distance, 100 cm from the slits.

(b) A parallel beam of monoenergetic electrons falls normally on a diaphragm with narrow square slit of width 1 micrometre. Find the velocity of the electrons if the width of the central diffraction maximum formed on a screen located 50 cm from the slit is equal to 0.36 mm.

6. * **(a)** The position and momentum of a 1 keV electron are simultaneously determined. If the position is located to within one Angstrom what is the percentage of uncertainty in its momentum?

(b) Verify that the uncertainty principle can be written in the form $\Delta L \Delta\theta \geq \frac{\hbar}{2}$ where ΔL is the uncertainty in the angular momentum of a particle and $\Delta\theta$ is the uncertainty in its angular position.

* **(c)** An atom in an excited state has a lifetime of 12 nanoseconds and in a second excited state the lifetime is 23 nanoseconds. What is the uncertainty in energy of the photon when an electron makes a transition between these two states?

* **7.** (a) A particle is located in a one dimensional square potential with infinitely high walls. The width of the well is a. Find the normalised wave functions of the stationary states of the particle taking the midpoint of the well as the origin of the x coordinate.

(b) A particle is confined in a one dimensional box located between $x = 0$ and $x = L$. The potential is infinity at the walls and zero inside. Determine the probability of finding the particle in a region of width $\Delta x = 0.1L$ at each of the following locations when it is in its ground state: $x = 0, 0.25L, 0.5L.0.75L, L$. Repeat this exercise for the first excited state.

8. * (a) An electron is trapped in a region of width $L = 10$ Angstroms. What are the energies of the ground state and the first two excited state? An electron in the first excited state de-excites to the ground state. What is the wavelength of the emitted radiation?
(b) The wavefunction of a particle confined in a one dimensional box, between $x = 0$ and $x = L$, is given to be

$$\psi(x,t) = \frac{3}{5}\psi_1(x,t) + \frac{4}{5}\psi_2(x,t) \tag{2}$$

where $\psi_1(x,t)$ and $\psi_2(x,t)$ are the ground and first excited state wavefunctions of the particle in a box. What are the possible outcomes and their probabilities if the energy of the particle is measured?