

Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 6
Autumn 2022

1. Expand $f(x, y) = e^{(2x+xy+y^2)}$ in powers of x and y upto second order term.
2. Expand $f(x, y) = \sin(xy)$ in powers of $(x - 1)$ and $(y - \pi/2)$ up to second degree term, and then find the remainder term.
3. Expand $f(x, y) = e^y \sin x$ in Taylor's series upto second order term about $(\frac{\pi}{2}, 1)$. Also estimate the value of $f(x, y) = e^y \sin x$ when $x = \frac{51}{100}\pi$, $y = 0.99$.
4. Expand $f(x, y) = x^2y + \sin y + e^x$ in powers of $(x - 1)$ and $(y - \pi)$ upto second order terms using Taylor's theorem and find the remainder term.
5. Show that
$$\sin x \sin y = xy - \frac{1}{6}[(x^3 + 3xy^2) \cos(\theta x) \sin(\theta y) + (y^3 + 3x^2y) \sin(\theta x) \cos(\theta y)],$$
 where $0 < \theta < 1$.
6. Classify the local extremum of the following functions:
 - (a) $f(x, y) = x^2y - 2xy^2 + 3xy + 4$.
 - (b) $f(x, y) = 2(x - y)^2 - x^4 - y^4$.
 - (c) $f(x, y) = x^3 - 12x + y^3 + 3y^2 - 9y$.
 - (d) $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.
 - (e) $f(x, y) = x^2y - xy^2 + 4xy - 4x^2 - 4y^2$.
7. Verify that $x^3y^2(1 - x - y)$ has a maximum at $(\frac{1}{2}, \frac{1}{3})$.
8. Find the absolute maximum and minimum values of $f(x, y) = 4x^2 + 9y^2 - 8x - 12y + 4$ over the rectangle in the first quadrant bounded by the lines $x = 2$, $y = 3$ and the co-ordinate axes.
9. Find the global extremum of $f(x, y) = x^2 + xy + y^2$ over the circular region $R = \{(x, y) / x^2 + y^2 \leq 1\}$.
10. Find the absolute maximum and minimum value of the function $f(x, y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \leq 1$.