Tutorial 7: Solutions

1. (a) A point source S is a perpendicular distance R away from the centre of a circular hole of radius a in an opaque screen. If the distance to the periphery is $(R+l)$, show that Fraunhofer diffraction will occur on a very distant screen when

$$\lambda R >> \frac{a^2}{2}$$

What is the smallest satisfactory value of R if the hole has a radius of 1 mm,$l \leq \frac{\lambda}{10}$ and $\lambda = 500nm$?

(b) The angular distance between the centre and the first minimum of a single slit Fraunhofer diffraction pattern is called the half angular width; write and expression for it. Find the corresponding half linear width (i) when no focusing lens is present and the slit-viewing screen distance is L, and (ii) when a lens of focal length f2 is very close to the aperture. Is the half-linear width also the distance between the successive minima?

1.**(a)**

$$(R+l)^2 = R^2 + a^2$$

If l is very small so that wave is almost plane then $R = \frac{a^2}{2l}$ or, $lR = \frac{a^2}{2}$. Therefore, as $\lambda \gg l$ so, $\lambda R \gg \frac{a^2}{2}$. From the relation $lR = \frac{a^2}{2}$, $\frac{a^2}{2R} = l \leq \lambda/10$. So $R_{min} = 5a^2/\lambda = 10\,m$.

(b) Half angular width = angular distance between the 1st minima and central maxima. From the condition for diffraction minima we get

$$\sin\theta = \pm\frac{\lambda}{b} \approx \theta$$

Half linear width = Distance to the screen from aperture $\times$ $\tan\theta \approx L\theta$.

(i) So,

$$L\theta \approx \pm L\frac{\lambda}{b}$$

(ii)

$$L\theta \approx \pm f_2(\frac{\lambda}{b})$$

2. (a)In the case of diffraction from a single slit, what is the ratio of irradiances (intensities) at the central peak maximum to the first of the secondary maxima?

(b) If the irradiance of central Fraunhofer diffraction maximum of a single slit is I_0 , obtain the ratio $\frac{I}{I_0}$ for a point on the screen that is $\frac{3}{4}$ of a wavelength farther from one edge of the slit than the other.

(a)

$$\frac{I}{I_0} = \frac{\sin^2 \beta}{\beta^2}$$

Treating the peaks as midway between two minima, for first of secondary maxima $\beta = \frac{3\pi}{2}$. Therefore

$$\frac{I}{I_0} \approx .045$$

Note: Although this is a crude approximation but it holds good for higher order secondary maxima. The actual values of β for secondary maxima are solutions of the transcendental equation $\beta = \tan\beta$ and these are $\pm 1.43\pi, \pm 2.459\pi, \pm 3.47\pi, ...$ So more accurate answer would be $\frac{\sin^2(1.43\pi)}{(1.43\pi)^2} \approx 0.048$.

(b)

$$\frac{I}{I_0} = \frac{\sin^2 \beta}{\beta^2}$$

Where

$$\beta = \frac{\pi a \sin\theta}{\lambda}$$

Here path difference between the two rays coming from the two edges of the slit is

$$a \sin\theta = \frac{3\lambda}{4}$$

Therefore, $\frac{I}{I_0} = \frac{\sin^2(\frac{3\pi}{4})}{(\frac{3\pi}{4})^2} = \frac{8}{9\pi^2} \approx 0.09$

3. A collimated beam of microwaves impinge on a metal screen that contains a long horizontal slit that is 20 cm wide. A detector moving parallel to the screen in the far-field region locates the first minimum of intensity at an angle of 36.87 degrees above the central axis. Determine the wavelength of the radiation.

The condition for minima is $a\sin\theta = m\lambda$ Here slit-width $a = 20\,cm$, angle $\theta = 36.87^o$, and for the first minimum $m = 1$. Therefore $\lambda = 12\,cm$.

4. (a) (a) A double-slit diffraction pattern is formed using mercury green light at 546.1 nm. Each slit has a width of 0.1 mm. The pattern reveals that the fourth order interference maxima are missing from the pattern.

(i) What is the slit separation?

(ii) What is the irradiance (intensity) of the first three orders of interference fringes, relative to the zeroth order maximum?

(iii)

(b) Show that for a double slit Fraunhofer pattern, if a = mb , the number of bright fringes (or parts thereof) within the central diffraction maximum will be equal to 2m.

4.a(i) Conditions for diffraction minima and interference maxima are $asin\theta = m\lambda$ and $b\sin\theta = p\lambda$ respectively, where a is the slit width and b is the slit separation (i.e. center-to-center separation). Now for a certain value of θ when the above two conditions are satisfied simultaneously the corresponding interference maximum becomes missing. So the condition for missing order is $\frac{b}{a} = \frac{p}{m}$. As $p = 4$, $m = 1$, so $b = 4a = 0.4\,mm$.

(ii) We know that for double slit diffraction intensity distribution is

$$I = I_{max}\frac{\sin^2(\alpha)}{\alpha^2}\cos^2\beta$$

Now,$\alpha = \frac{\pi asin\theta}{\lambda}$, $\beta = \frac{\pi bsin\theta}{\lambda}$, $b = 4a$. Then for first order interference maximum $\beta = \pi$, so $\alpha = \pi/4$; for 2nd order interference maximum $\beta = 2\pi$, so $\alpha = \pi/2$; and for 3rd order interference maximum $\beta = 3\pi$, so $\alpha = 3\pi/4$. So the corresponding ratios of intensities are

$$\frac{I_1}{I_{max}} = \frac{8}{\pi^2} = 0.81$$
$$\frac{I_2}{I_{max}} = \frac{4}{\pi^2} = 0.405$$
$$\frac{I_3}{I_{max}} = \frac{8}{9\pi^2} = 0.09$$

(b). For $a = mb$, if m be an integer then, m^{th} order interference maximum will be missed. So the observed number of peaks within the central diffraction maximum is $2m - 1$ $(0, \pm1, \pm2..... \pm (m - 1))$. Now if we count the half-fringes at the adjacent to the first minimum, then the total count of the bright fringes is $2m$. If m is not an integer then also we get the same count of bright fringes if we count the fractional fringes[See the book Optics by Hecht, 4th edition, p.p-436].

5. Make a rough sketch for the irradiance pattern from seven equally spaced slits having a separation-to-width ratio of 4. Label the points on the x-axis with the corresponding values of β and γ. Note: β and γ are parameters in $I(\theta) = I_0(\frac{sin^2\beta}{\beta^2})(\frac{sin^2 N\gamma}{sin^2\gamma})$.

5. $\frac{b}{a} = 4$

$$\beta = \frac{a\pi}{\lambda}\sin\theta$$
$$\gamma = \frac{b\pi}{\lambda}\sin\theta$$

Using all these equations we get, $\beta = \gamma/4$. Given $N = 7$. The qualitative plot looks like Fig. 3, which is a combination of Fig. 1 and Fig. 2.

6. (a) For oblique incidence, the grating equation is $d(sin\theta_i + sin\theta_m) = m\lambda$, where θ_i and θ_m are, respectively, the angles of incidence and mth order diffraction. What is the angle of minimum deviation for mth order diffraction? If the angle of minimum deviation in the 1st order spectrum is 20^o for mercury blue line of $\lambda = 4358\mathring{A}$, determine the number of lines per cm of the grating.
(b) A grating has 800 lines/cm. Calculate the minimum width of the grating required to resolve the components of Sodium D-lines of wavelengths 5890 $\mathring{A}$ and 5896 $\mathring{A}$ respectively in the 2nd order.

(a) Given

$$d(\sin\theta_i + sin\theta_m) = m\lambda$$

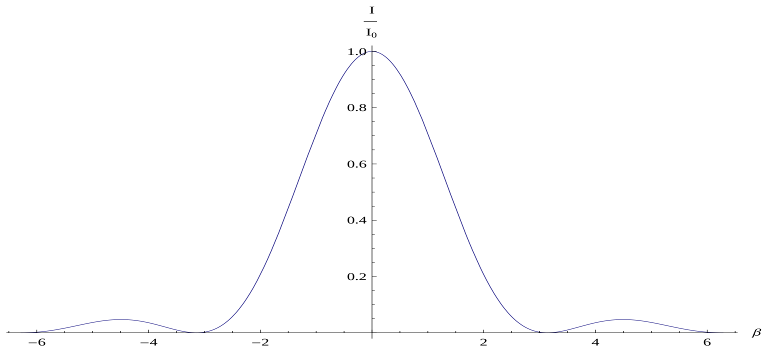

FIG. 1: $\frac{sin^2\beta}{\beta^2}$ vs β

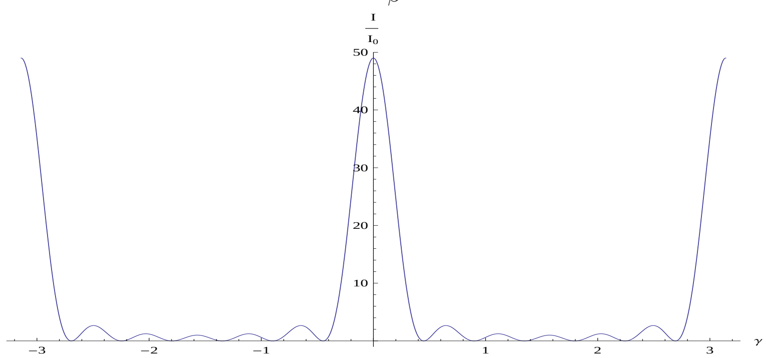

FIG. 2: $\frac{sin^2 N\gamma}{sin^2\gamma}$ vs γ

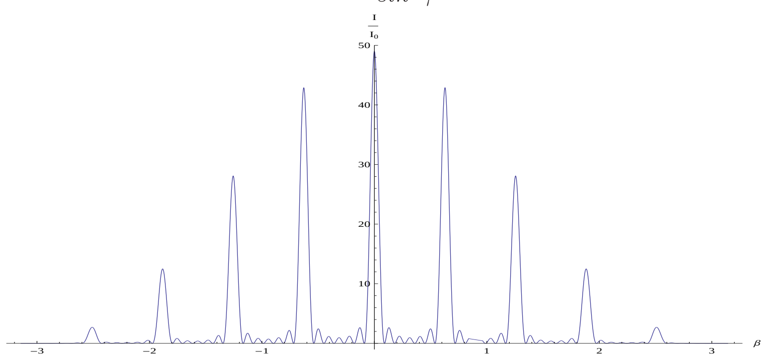

FIG. 3: $\frac{I}{I_0}$ vs β

or,

$$2d\sin\frac{\theta_i+\theta_m}{2}\cos\frac{\theta_i-\theta_m}{2}=m\lambda$$

Deviation suffered by the ray is $\delta=\theta_i+\theta_m$. So, $\sin(\delta/2)$, (hence δ) will have minimum value when $cos\frac{\theta_i-\theta_m}{2}$ is maximum.

So, $\theta_i = \theta_m = \delta/2$. If $\delta = 20^o$, then

$$d = \frac{\lambda}{2\sin\frac{\delta}{2}} = 0.22\,\mu m$$

So the number of lines of the grating is 7969 lines/cm.

(b) Number of lines per cm is given by $\frac{1}{d} = 800$ per cm, where d is the width of the grating. For grating with total number of lines N, its resolving power at the n^{th} order spectrum is $\frac{\lambda}{d\lambda} = nN$. Putting n=2, average wavelength $\lambda = 5893\,\text{Å}$ and $d\lambda = 6\,\text{Å}$, we get N=491. Therefore width is $\frac{491}{800}cm = 0.61cm$

7. (a)A beam of linearly polarized light with its electric field vertical impinges perpendicularly on an ideal linear polarizer with a vertical transmission axis. If the incoming beam has an irradiance of 200 Watts/m^2 , what is the irrradiance of the transmitted beam?
(b)At what angle will the reflection of the sky coming off the surface of a pond (n = 1.33)completely vanish when seen through a Polaroid filter?

(a) Malus law: $I = I_0\cos^2\theta$, where I is the intensity of the light transmitted by the polarizer whose axis of transmission makes an angle θ with the electric field vector of the incident linearly polarized light.Here $\theta = 0°$ So, $I = 200\,watts/m^2$.
(b) If the angle of incidence (θ_i) is the Brewster's angle then the reflected light becomes linearly polarized and hence, can be extinguished completely when seen through a Polaroid filter. Now Brewster's angle θ_p is given by $\theta_i = \theta_p = 90^o - \theta_t$, which leads $\eta = \tan(\theta_p)$. Putting $\eta = 1.33$, we get θ to be 53°.

8. (a) A birefringent crystal of thickness d has its optic axis parallel to the surface of the crystal. What should be the value of d (in μm) if the crystal is to be used as a quarter wave plate for light of wavelength $\lambda = 589.3nm$ ($n_e = 1.5334$, $n_0 = 1.5443$).
(b) A ray of yellow light is incident on a calcite plate at 50 degrees. The plate is cut so that the optic axis is parallel to the front face and perpendicular to the plane of incidence. Find the angular separation between the two emerging rays.
(a) For quarter wave plate $d = \frac{\lambda}{4(n_e - n_0)} = 13.52\,\mu$m.

(b)

$$n_o = 1.6584$$
$$n_e = 1.4864$$

Using Snell's law,

$$\sin\theta_i = n_o \sin\theta_{to} = 0.766$$
$$\sin\theta_i = n_e \sin\theta_{te} = 0.766$$

So,

$$\theta_{to} \approx 27°35'$$
$$\theta_{te} \approx 31°4'$$

So, $\Delta\theta \approx 3°29'$

9. (a) An ideal polarizer is rotated at a rate ω between a similar pair of stationary crossed polarizers. show that the emergent flux density will be modulated at four times the rotational frequency. In other words show that

$$I = I_1(1 - cos4\omega t)$$

where I_1 is the flux density emerging from the first polarizer and I is the final flux density.

(b) A beam of plane polarized light falls on a polarizer which rotates about the axis of the ray with an angular velocity ω= 21 rad/s. Find the energy of light passing through the polarizer per revolution if the flux of the incident light energy is equal to 4.0 mW.

(a) Intensity of the beam after transmission through first polaroid is I_1. If the angle between the pass axes of the first polaroid and the middle polaroid is θ, then from Malus's law,

$$I_2 = I_1 \cos^2\theta$$

Now, the angle between pass axes of the middle and the last polaroid is $90-\theta$. The emergent intensity will be,

$$
\begin{aligned}
I &= I_1 \cos^2\theta \cos^2(90^o - \theta) \\
&= \frac{I_1}{4} \sin^2 2\theta \\
&= \frac{I_1}{8}(1 - \cos 4\omega t)
\end{aligned}
$$

(b) Per revolution the flux will be $\phi_0/2$= 2.0 mw.

The angular velocity = 21 rad/s

Time required for one revolution = $\frac{2\pi}{21}$s.

So, energy passing in $\frac{2\pi}{21}$ s is $0.6mJ$.

10.(a) As we know, substances such as sugar and insulin are optically active, they rotate the plane of polarization in proportion to both the path length and the concentration of the solution . A glass vessel is placed between a pair of crossed linear polarizers, and 50 percent of the natural light incident on the first plarizer is transmitted through the second polarizer. By how much did the sugar solution in the cell rotate the light passed by the first polarizer?

(b) The specific rotation for sucrose dissolved in water at 20 degree Centigrade ($\lambda_0 = 589.3nm$) is +66.45 degree per 10 cm of path traversed through a solution containing 1 gm of active substance per cm^3 of solution. A vertical linearly polarised state of Sodium light enters at one end of a 1m tube containing 1000 cm^3 of solution, of which 10 gm is sucrose. At what orientation will the linearly polarised state of light emerge?

(a) Natural light is a unpolarized light. So after passing through the first polarizer, the intensity becomes $I_1 = I_0/2$, where I_0 is the intensity of the natural light. Since the glass vessel is kept between two crossed linear polarizer, so the final intensity

$$I = I_1 \sin^2\theta$$

where θ is the angle of rotation by the optically active substance. Now, $I = I_0/2$. So, $\sin^2\theta = 1$ or, $\theta = \pi/2$.

(b) We know,$\theta = slm/10$, where s is the specific rotation.

Now,l=100 cm

m=active substance per c.c =0.01g/cm^3

s= specific rotation= 66.45^o per 10 cm per 1 gm of active substance in 1 c.c. solution.

so,$\theta = 66.45 \times 10 \times 0.01 = 6.645^\circ$ and right hand rotation.