Solutions for Tutorial # 9

1. Design an interference anti-reflecting coating for the wavelength $\lambda = 550$ nm by depositing a layer of MgF_2 ($n = 1.38$) on crown glass ($n = 1.52$).

   (a)What should be the minimum thickness of the layer ?

   (b) What wavelength will be strongly reflected due to the coating ?

(a) Lets say thickness of the film is t. Then the total distance travelled by the wave inside the film is 2t, assuming light entered normal to the interface

Then the total distance should be equal to half the wavelength of the light in the film

So, $$2t = \frac{\lambda_{film}}{2}$$

Wavelength of the film is .. $$\lambda_{film} = \frac{\lambda_{air}}{n} = \frac{550}{1.38} = 398.5nm$$

Hence the thcikness should be .. $$t = \frac{\lambda_{film}}{4} = 99.64nm$$

Note :: Path or Phase difference between rays is discarded - as both the rays will have pi-phase difference due to reflection they get cancel out.

(b) Wavelength which will be strongly reflected is ..

$$\lambda = 2nt = 2 \times 1.38 \times 99.64nm = 275nm$$

2. A soap film of thickness 5.5×10^{-5} cm is viewed at an angle of 45°. Its index of refraction is 1.33. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light.

A soap film of thickness 5.5x10-5 cm viewed at an angle 45° (angle of incidemce i)

According to Snell's law , if n is refractive index & r is angle of refraction $n = \frac{Sin(i)}{Sin(r)}$

Therefore, $1.33 = \frac{Sin(45°)}{Sin(r)}$ Thus, $Sin(r) = \frac{0.707}{1.33} = 0.5317$ Or, $r = Sin^{-1}(0.5317) = 32.12$

So, $Cos(r) = 0.847$

Condition for darkness is, $$2dnCos(r) = m\lambda$$

For values of m = 1,2,3,4, …. We will get corresponding wavelengths which will remian absent

For m =1 , $$\lambda_1 = 2 \times 5.5 \times 10^{-5} \times 1.33 \times 0.847 \approx 1240nm$$

For m =2 , $$\lambda_2 = 2 \times 5.5 \times 10^{-5} \times 1.33 \times 0.847/2 \approx 620nm$$

For m =3 , $$\lambda_3 = 2 \times 5.5 \times 10^{-5} \times 1.33 \times 0.847/3 \approx 413nm$$

For m =4 , $$\lambda_4 = 2 \times 5.5 \times 10^{-5} \times 1.33 \times 0.847/4 \approx 310nm$$

Wavelengths 620 nm and 413 nm which fall in the visible spectrum will remian absent

3. The convex surface of radius 200 cm of a plano-convex lens rests on a concave spherical surface of radius 400 cm (see figure 1) and Newtons rings are viewed with reflected light of wavelength 600 nm. Calculate the diameter of the 8th bright ring.

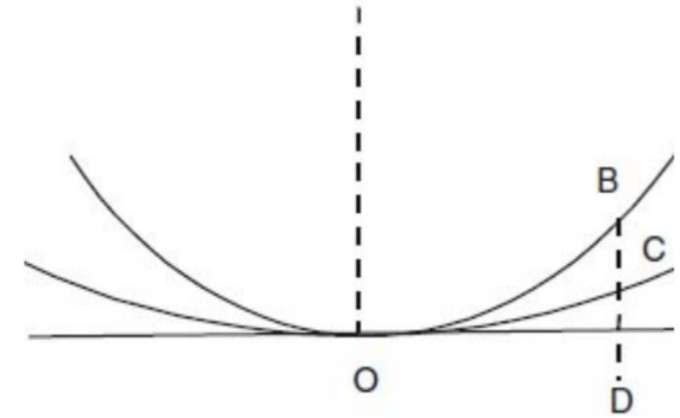


Figure 1:

Convex surface of radius 200 cm (R_1) of a plano convex lens rests on a Concave spherical surface of radius 400 cm (R_2)

Newtons rings are viewed with reflected light of wavelength 600 nm (λ)

In the figure optical path difference is 2 X CB, lets say CB=d & OD = r

So, $$d = \frac{r^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - (1)$$

For constructive interferences $$d = (2m-1)\frac{\lambda}{4} - (2)$$

Therefore, $$(2m-1)\frac{\lambda}{4} = \frac{r^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - (3)$$

For 8th bright ring, m=8 $r_8 = 0.425$ By substituting values of R_1, R_2, m & λ in eqn (3)

Therefore diameter of 8th bright ring, $D_8 = 0.85$

4. Consider Michelsons interferometer experiment with Na-light where two very closely spaced wavelength (namely D_1 and D_2) are present. When one mirror of the interferometer is now moved through a distance of 0.15 mm the fringes are found to disappear. If the mean wavelength for the two components of the D lines of sodium light is 589.3 nm, find the difference between their wavelengths.

Fringe disappear when maxima of one wavelength coincide with minima of another wavelength.

Maxima Condition, $$2dSin\theta = m\lambda$$

Minima Condition, $$2dSin\theta = (m + 1/2)\lambda$$

Assuming mirrors are perpendicular to each other and taking Fringe disappearance condition ..

$$2d = m\lambda_1 = (m + 1/2)\lambda_2$$

Therefore, $$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = \frac{1}{2}$$

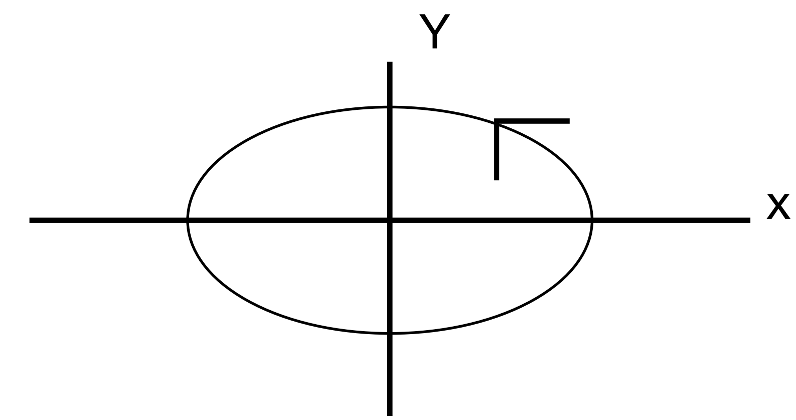
$$\Rightarrow 2d \cdot \frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2} = \frac{1}{2} \qquad \Rightarrow \lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{4d}$$

$$\Rightarrow \lambda_1 - \lambda_2 \simeq \frac{\lambda_{mean}^2}{4d} \; as \; [\lambda_1\lambda_2 = \lambda_{mean}^2]$$

$$\Rightarrow \lambda_1 - \lambda_2 = \frac{(589.3 \times 10^{-9})^2}{4 \times 0.15 \times 10^{-3}} \simeq 0.578nm$$

5. The electric field components of a plane electromagnetic wave are, $E_x = 2E_0\cos(\omega t - kz + \phi)$ and $E_y = 2E_0\sin(\omega t - kz)$. Draw the diagram showing the state of polarization when,

(a) $\phi = 0$ (b) $\phi = \frac{\pi}{2}$ (c) $\phi = \frac{\pi}{4}$



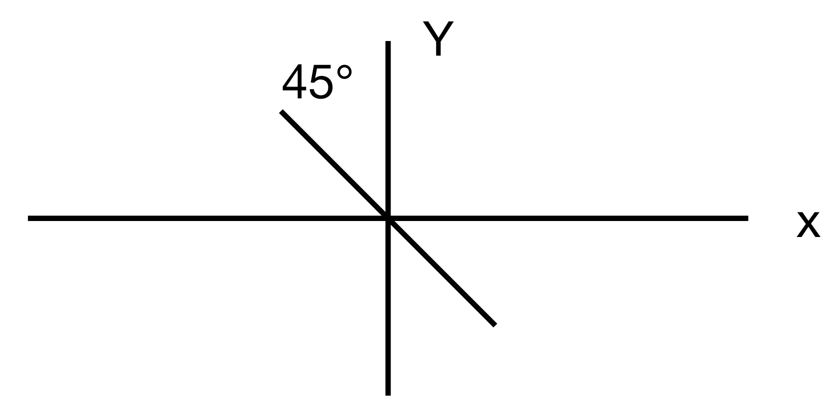
(a) $For\ \phi = 0 \quad E_x = 2E_0cos(\omega t - kz) \qquad E_y = 2E_0sin(\omega t - kz)$

Therefore, $$\frac{E_x^2}{(2E_o)^2} + \frac{E_y^2}{(2E_o)^2} = 1$$

State of Polarization : If we vary ωt from 0 to 2π we will get anti-clockwise Rotation when looking towards incoming light. So it is left elliptically polarized.

(b) $For\ \phi = \frac{\pi}{2} \quad E_x = 2E_0cos(\omega t - kz + \pi/2) = -2E_0sin(\omega t - kz) = -E_y$

State of Polarization : Linearly polarized



(c) $For\ \phi = \pi/4 \qquad E_x = 2E_0cos(\omega t - kz + \pi/4)$

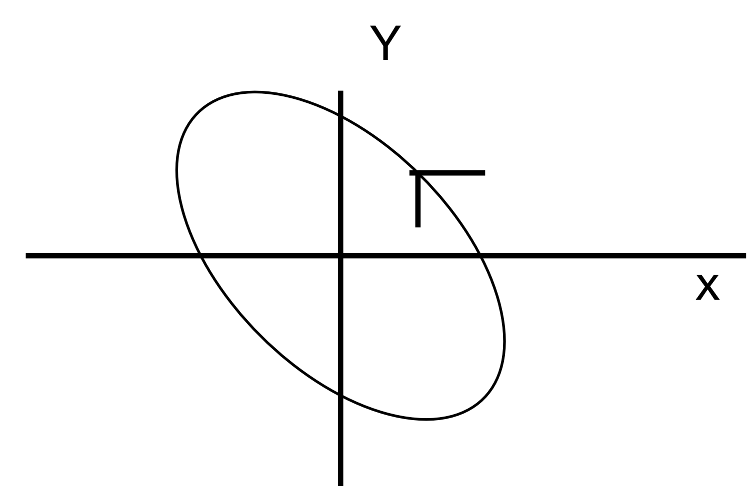
So, $$\frac{E_x}{2E_0} = \frac{1}{\sqrt{2}}cos(\omega t - kz) - \frac{1}{\sqrt{2}}sin(\omega t - kz)$$

Therefore, $$E_x = \frac{2E_0}{\sqrt{2}}[\sqrt{1 - \frac{E_y}{(2E_0)^2}} - \frac{E_y}{2E_0}] \quad \Rightarrow (E_x + \frac{E_y}{\sqrt{2}})^2 = \frac{(2E_0)^2}{2}(1 - \frac{E_y^2}{(2E_0)^2})$$

$$\Rightarrow \frac{E_x^2}{(2E_0)^2} + \frac{E_y^2}{2(2E_0)^2/3} + \sqrt{2}\frac{E_xE_y}{(2E_0)^2} = 1/2 \quad \Rightarrow 2(\frac{E_x}{2E_0})^2 + 3(\frac{E_y}{2E_0})^2 + 2\sqrt{2}(\frac{E_x}{2E_0})(\frac{E_y}{2E_0}) = 1$$

If we assume $\frac{E_x}{2E_0} = X, \frac{E_y}{2E_0} = Y$ We get, $2X^2 + 3Y^2 + 2\sqrt{2}XY = 1$

Equation of an ellipse with tilted major axis.



State of Polarization : If we look against incoming light, we will observe an anti-closkwise rotation over an elliptic path making an angle of 3π/4 between +ve side of major axis and x-axis.