Tutorial 6: Solutions

1. A stationary radiating system consists of a linear chain of parallel oscillators separated by a distance d. The phase of the oscillators varies linearly along the chain, Find the time dependence of the phase difference $\Delta\phi$ between neighbouring oscillators such that the principal maximum of the radiation will be scanning the surroundings with a constant angular velocity ω.

Figure 1 shows the radiating system consisting of a chain of parallel oscillators separated by a distance d.

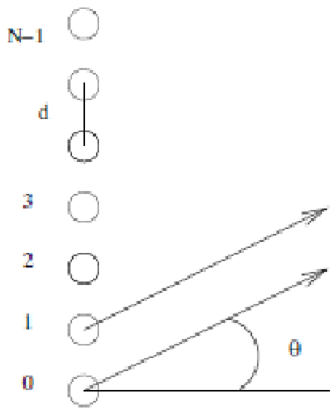

FIG. 1. Linear chain of parallel oscillators

Consecutive sources have a phase difference $\Delta\phi$ at their position. The path differences in the radiation from source $1, 3, ...(N-1)$ with respect to 0^{th} oscillator, are $d\sin\theta, 2d\sin\theta, ...(N-1)d\sin\theta$. Consequently, the phase differences are: $\frac{2\pi}{\lambda}\{d\sin\theta, 2d\sin\theta, ...(N-1)d\sin\theta\}$. Let, $\frac{2\pi}{\lambda}d\sin\theta = \alpha$. So, the resultant phasor at an angle θ,

$$
\begin{aligned}
P &= E\left[1 + e^{i(\alpha+\Delta\phi)} + e^{2i(\alpha+\Delta\phi)} + ... + e^{(N-1)i(\alpha+\Delta\phi)}\right] \\
&= E\left[\frac{1 - e^{iN(\alpha+\Delta\phi)}}{1 - e^{i(\alpha+\Delta\phi)}}\right]
\end{aligned}
$$

So the intensity

$$
\begin{aligned}
I &= PP^* \\
&= E^2\left[\frac{1-\cos N(\alpha+\Delta\phi)}{1-\cos(\alpha+\Delta\phi)}\right] \\
&= E^2\frac{\sin^2\frac{N(\alpha+\Delta\phi)}{2}}{\sin^2\frac{(\alpha+\Delta\phi)}{2}}
\end{aligned}
$$

So, for $(\alpha+\Delta\phi)/2 = k\pi$, $(k = 0, \pm1, \pm2, ...)$, both numerator and denominator of the intensity expression become zero and we have a $\left(\equiv \frac{0}{0}\right)$ form which gives a finite value using the L'Hospital rule. So, the condition for principal maximum to be observed at an angular position θ is

$$\Delta\phi = 2k\pi - \frac{2\pi}{\lambda}d\sin\theta$$

Now, if the principal maximum 'scan' the surrounding with a constant angular velocity ω, then $\theta = \omega t + \alpha$, ($\alpha$ =arbitrary constant). So finally we have,

$$\Delta\phi = 2k\pi - \frac{2\pi}{\lambda}d\sin(\omega t + \alpha)$$

2. (a) An expanded beam of red light from a He-Ne laser ($\lambda = 632.8nm$) is incident on a screen containing two narrow horizontal slits separated by $0.200mm$. A fringe pattern appears on a white screen held $1.00m$ away. (i) How far (in radians and in millimetres) above and below the central axis are the first zeros of intensity? (ii) How far (in mm) from the axis is the fifth bright band? (iii) Compare these two results.

(b) A small aperture of diameter $0.1mm$ at a distance of $1m$ is used to illuminate two slits with light of wavelength $\lambda = 550nm$. The slit separation is $d = 1mm$. What is the fringe spacing and the expected visibility of the fringe pattern?

(a) (i) We know that for $m-th$ order dark fringe,

$$
\begin{aligned}
\theta_m &\approx \frac{(2m+1)\lambda}{2d} \\
\text{or,} \quad y_m &\approx \frac{(2m+1)\lambda D}{2d}
\end{aligned}
$$

The symbols in the above formula have their usual meanings. Here, $\lambda = 632.8nm$, $D = 1.0m$, and $d = 0.200mm$. Therefore the first zeros of intensity will occur at $(m = 0)$

$$y = \frac{D\lambda}{2d} \tag{1}$$

$$y = 1.58\,mm \tag{2}$$

$$\theta = 1.6 \times 10^{-3}\,rad \tag{3}$$

(ii) The fifth bright band ($n = 5$) will be at

$$y = n\frac{D\lambda}{d} \tag{4}$$

So,

$$y = 1.58\,cm \tag{5}$$

(iii) The spectrum is equispaced. Hence, the first bright band from the axis is $1.58 \times 2\,mm$ away. So, the position of the fifth bright band is $(1.58 \times 2) \times 5\,mm = 1.58\,cm$ away.

(b) Here, $\lambda = 550nm$, and $d = 1mm$. We know, the angular fringe spacing is

$$\begin{aligned} \Delta\theta &= \frac{\lambda}{d} \\ &= 5.5 \times 10^{-4}\,rad \end{aligned}$$

Visibility is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

For, Young's double slit arrangement it becomes (for derivation see the coherence chapter of the book by SB and SPK or Week7 lectures of SK this semester)

$$V = \frac{\sin\left(\frac{\pi d\alpha}{\lambda}\right)}{\frac{\pi d\alpha}{\lambda}}$$

where α is the angular extent of the source. Here the diameter of the aperture is $0.1\,mm$ and the distance of the slit position from the aperture position is $1\,m$. Hence $\alpha = 10^{-4}\,rad$. So using all other numerical values we get $V = 0.95$.

3. (a) In a Lloyds mirror experiment (see Figure 2.) a light wave emitted directly by the source S interferes with the wave reflected by the mirror M. As a result an interference pattern is formed on the screen Sc. The source and the screen are separated by a distance $l = 1m$. At a certain position of the source the fringe width is equal to $\Delta x = 0.25mm$. After the source is moved away from the plane of the mirror by $\Delta h = 0.60mm$, the fringe width decreases by a factor $\eta = 1.5$. Find the wavelength of light.

(b) Consider the experiment with Fresnels mirrors. The angle between the mirrors $\alpha = 12$, the distances r and b are $r = 10.0cm$ and $b = 130cm$. The wavelength of light is $\lambda = 0.55mm$. Find:

(i) The fringe width on the screen and the number of possible maxima. (ii) The shift of the interference pattern on the screen when the source slit S is displaced by $\delta l = 1.0mm$ along the arc of radius r about the centre O. (iii) The maximum width δ_{max} of the source slit at which the fringe pattern on the screen can still be observed sufficiently sharp.

(c) A plane wave of light with wavelength $\lambda = 0.70m$ falls normally on the base of a biprism made of glass (n = 1.520) with refracting angle $\theta = 5 degrees$. Behind the biprism there is a plane parallel plate, with the space between them filled up with benzene (n = 1.500). Find the fringe width on a screen placed behind this system.

(a) We know the general formula for fringe spacing

$$\Delta x = \frac{l\lambda}{d} \tag{6}$$

Here, for Lloyd's mirror arrangement, d is the separation between the source S and its image. From the above equation we find that

$$\frac{\Delta x}{\eta} = \frac{l\lambda}{d + 2\Delta h} \tag{7}$$

Since d increases to $d + 2\Delta h$ when the source is moved away from the mirror plane by Δh. Thus,

$$d = \frac{2\Delta h}{\eta - 1}$$
$$\text{and} \quad \lambda = \frac{2\Delta x \Delta h}{(\eta - 1)l}$$

Finally, $\lambda = 0.6\mu m$.

(b) (i)Here $S'S'' = d = 2r\alpha$ and $l = b + r$. Therefore,

$$\Delta x = \frac{(b + r)\lambda}{2\alpha r} \tag{8}$$

putting $b = 1.3m$, $r = 0.1m$, $\lambda = 550nm$, $\alpha = 12'$, we get $\Delta x \approx 1.1mm$. Hence, the number of possible maxima

$$\frac{2b\alpha}{\Delta x} + 1 \approx 9 \tag{9}$$

(ii) When the slit moves by δl along the arc of radius r the incident ray on the mirror rotates by $\frac{\delta l}{r}$. Therefore, the shift of fringe will be of magnitude

$$b\frac{\delta l}{r} = 13\,mm \tag{10}$$

(iii) If the width of the slit is δ then we can imagine the slit to consist of two narrow slits with separation δ. The fringe pattern due to the wide slit is the superposition of the pattern due to these two narrow slits. The full pattern will not be sharp at all if the pattern due to narrow slits are $\frac{1}{2}\Delta x$ apart because then the maxima due to one will fall on the minima due to the other. Thus we can say that the condition for the fringe pattern be observed sufficiently sharp is

$$b\frac{\delta}{r} \leq \frac{\Delta x}{2}$$

So

$$\delta_{max} = (1 + \frac{r}{b})\frac{\lambda}{4\alpha} = 42\mu m \tag{11}$$

(c) For a Fresnel's biprism arrangement, we know that $d = 2a\delta$, where a is the distance of the position of the source (S) behind the biprism and δ is the angle of deviation for each of the refracting halves of the biprism.

Here we have to assume that the space between the biprism and the glass plate, filled with benzene, constitutes two complementary prisms. Each of these prisms are oppsitely placed back to back with each halves of the biprism. Then the two prisms being oppositely placed, the net deviation produced by them is

$$\delta = (n-1)\theta - (n'-1)\theta = (n-n')\theta \tag{12}$$

Hence $d = 2a\theta(n-n')$ So

$$\Delta x = \frac{(a+b)\lambda}{2a\theta(n-n')} \tag{13}$$

for plane incident wave $a \to \infty$. Thus, the above equation becomes

$$\Delta x = \frac{\lambda}{2(n-n')\theta} \tag{14}$$

Given that $\lambda = 0.70\,\mu m$, $n = 1.520$, $n' = 1.500$, $\theta = 5^o = \pi/36\,rad$. So $\delta x \approx 0.2mm$

4. (a) Find the minimum thickness of a film with refractive index 1.33 at which light with wavelength $0.64\mu m$ experiences maximum reflection while light with wavelength $0.40\mu m$ is not reflected at all. The angle of incidence is equal to $30 degrees$.
(b) A thin film of alcohol (n = 1.36) lies on a flat glass plate (n = 1.51). When monochromatic light, whose wavelength can be changed, is incident normally, the reflected light is a minimum for $\lambda = 512nm$ and a maximum for $\lambda = 640nm$. What is the thickness of the film?

(a) For interference of reflected wave from a thin film of thickness d and refractive index n, the condition for maxima and minima in the fringe pattern is

$$\begin{aligned} 2nd\cos\theta_t &= (k+\frac{1}{2})\lambda \quad \text{(for maximum)} \\ &= k'\lambda \quad \text{(for minimum)} \end{aligned}$$

where, θ_t is the angle of transmission. If θ_i is the angle of incidence, then from Snell's law $\cos\theta_t = \sqrt{1-n_0^2\sin^2\theta_i/n^2}$. For air medium $n_0 = 1$ and given that $\theta_i = 30^o$. Hence, for maximum reflection in illumination

$$2d\sqrt{n^2-\frac{1}{4}} = (k+\frac{1}{2})0.64\mu m \tag{15}$$

and for minimum illumination in reflection

$$2d\sqrt{n^2-\frac{1}{4}} = (k')0.40\mu m \tag{16}$$

where $k\,,k'$ are integers. Equatting the above two equations we get an equation $5k' = 4(2k+1)$. For the smallest integer solutions, $k = 2$, $k' = 4$. Given that $n = 1.33$, hence minimum thickness will be $d = 0.65\mu m$
(b) Both of the reflections for interference occur from rarer to denser medium–one reflection from air($n_0 = 1.0$) to alcohol($n_1 = 1.36$) and the other one from alcohol ($n_1 = 1.36$) to glass ($n_2 = 1.51$). So, in both of the reflected waves, additional π phase difference will be introduced. Hence, condition for maxima and minima are

$$\begin{aligned} 2n_1d\cos\theta_t &= p\lambda_1 \quad \text{(for maximum)} \\ &= (q+\frac{1}{2})\lambda_2 \quad \text{(for minimum)} \end{aligned}$$

For normal incidence $\cos\theta_t = 1$. Given that $\lambda_1 = 640\,nm$, $\lambda_2 = 512\,nm$. Hence we equating the condition we get $5p = 2(2q+1)$. For the smallest integer solutions $p = 2, q = 2$. Thus, the minimum thickness is $d = 0.47\,\mu m$.

5. (a) A glass plate of (n = 1.5) is coated with a polymeric film (n = 2). Calculate the coating thickness so as to observe (i) maximum and (ii) minimum reflection using a light of wavelength $\lambda = 500nm$.

(b) Diffuse monochromatic light with $\lambda = 0.60m$ falls on a thin file of refractive index $n = 1.5$. Determine the film thickness if the angular separation of neighboring maxima observed in reflected light at angles close to 45 degrees to the normal is equal to $\delta\theta = 2.0$ degrees.

(a) A glass plate($n_2 = 1.5$) is coated with a polymeric film of $n_1 = 2$ and the wavelength is $\lambda = 500nm$. Here both sides of coating film are rarer medium. Hence, the conditions for maximum and minimum reflection are those given in the solution of 4(a).

i)Hence the coating thickness of this film to observe maximum reflection is

$$d = \frac{\lambda}{4n_1} = 62.5nm \tag{17}$$

ii) and the thickness for observing minimum reflection is

$$d = \frac{\lambda}{2n_1} = 125nm \tag{18}$$

(b)For maximum intensity,

$$2nd\cos\theta_t = (k + \frac{1}{2})\lambda \tag{19}$$

From Snell's law

$$n_0 \sin\theta_i = n\sin\theta_t \tag{20}$$

For air to film surface reflection, $n_0 = 1$. Now, from Eq. (19), we have

$$-2nd\sin\theta_t\delta\theta_t = \Delta k\lambda \tag{21}$$

For neighboring maxima $\Delta k = -1$. From Eq. (20),

$$\delta\theta_t = \frac{\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}\delta\theta_i \tag{22}$$

So, using Eqs. (20),(21) and (22) we get the finally,

$$d = \frac{\lambda\sqrt{n^2 - \sin^2\theta_i}}{\sin\, 2\theta_i\delta\theta_i} \tag{23}$$

Given that, $n = 1.5$, $\theta_i = 45^o$, $\lambda = 0.6\,\mu m$, $\delta\theta_i = 2^o = \pi/90\,rad$. So, $d = 22.74\,\mu m$.

6. Under the influence of gravity, a wedge- shaped film is formed when a metal is vertically dipped inside a soapy solution (refractive index = 1.34). A coherent and monochromatic light of wavelength $\lambda = 488nm$ falls near-normally on this wedge. The experimentalist observes 12 fringes per cm. Determine the wedge angle of the soap film.

For wedge shaped film $d = \alpha x$, where x is the distance of the point of incidence of light from the apex. Here, $n = 1.34$, $\lambda = 488nm$. The experimentalist observes $12\,fringe/cm$, hence $\Delta x = 0.83mm$. for normal incidence we know,

$$\Delta x = \frac{\lambda}{2n\alpha} \tag{24}$$

where α = wedge angle of soap film. Hence

$$\alpha = \frac{\lambda}{2n\Delta x}\alpha = .00022radian = .013degree \tag{25}$$

7. *(a) A Newtons ring apparatus (comprising of a spherical surface of radius 1 m) is illuminated by light with two wavelength components. One of the wavelengths is 546 nm. If the 11th bright ring of the 546 nm fringe system coincides with the 10th ring of the other. (i) What is the second wavelength? (ii) What is the radius at which overlap takes place and the thickness of the air-film there?
(b) When a Newtons ring apparatus is immersed in a liquid, the diameter of the eighth dark ring decreases from 2.92 cm to 2.60 cm. What is the refractive index of the liquid?

(a) The formula for a bright ring in the fringe pattern of Newton's ring arrangement is

$$r_k^2 = (k + \frac{1}{2})\frac{\lambda R}{n} \tag{26}$$

For air film, $n = 1$.
(i)Here it is given that, radius of curvature $R = 1m$, and $\lambda = 546nm$ If 11 th bright ring($k = 10$) of λ_1 coincide with the 10 th ring ($k = 9$) of λ_2, then

$$21\lambda_1 = 19\lambda_2 \tag{27}$$

Hence $\lambda_2 = 603.5nm$

(ii) Therefore the radius at which overlap takes place is

$$r = 2.4mm \tag{28}$$

thickness of air film at that point will be

$$t = \frac{r^2}{2R} = 2.87\,\mu m \tag{29}$$

(b) The diameter of the eighth dark ring decreases from 2.92 cm to 2.60 cm Since we know the diameter of the m th dark ring is

$$(D_m)^2 = 4m\lambda R \tag{30}$$

when film is air if the intermediate medium is liquid of refractive index of n, then diameter of the m th dark ring will be

$$(D_m)^2 = \frac{4m\lambda R}{n} \tag{31}$$

From the above two equations we get the refractive index of liquid is $n = 1.26$

8.(a) An interference filter is designed for normal incidence of 488 nm light. The refractive index of the spacer is 1.35. If an observer uses a spectrometer and looks at the reflection of white light from the surface of the filter at 30 degrees with respect to surface normal, what wavelength should the observer see?

*(b) One of beams of the interferometer passes through a small glass container containing a cavity 1.30 cm deep. When a gas is allowed to slowly fill the container, a total of 186 dark fringes are counted to move past a reference line. The light used has a wavelength of 610 nm. Calculate the index of refraction of the gas at its final density, assuming that the interferometer is in vacuum.

(a) Interference filter is designed for normal incidence of $\lambda_0 = 488$ nm light. The refractive index of the spacer is $n = 1.35$ If the observer wants to look the reflection at $\alpha = 30$ degrees with respect to surface normal. Then

$$\lambda = \lambda_0 \sqrt{1 - \frac{\sin^2 \alpha}{n^2}} \tag{32}$$

Hence observed wavelength will be $\lambda = 453.2$ nm

(b) Here it is given that $l = 1.30cm$, $\lambda = 610nm$ and order of fringes move are $m = 186$. We know

$$(n-1)2l = m\lambda \tag{33}$$

putting all the given values, we get the value of index of refraction of the gasas $n = 1.0047$.

9.(a) Looking into a Michelson interferometer one sees a dark central disc surrounded by concentric bright and dark rings. One arm of the device is 2 cm longer than the other, and the wavelength of the light is 500 nm. Determine (i) the order of the central disc and (ii) the order of the 6th dark ring from the center.

(b) In an experiment with Michelson interferometer, the distance travelled by the mirror for two successive positions of maximum distinctness is 0.2945 mm. If the mean wavelength for the two components of the source is 548.3 nm, calculate the difference between the two wavelengths.

(a) Here one arm of the device is $d = 2cm$ longer than the other and the operating wavelength is $\lambda = 500nm$. As the central fringe is dark, we can write

$$2d = m_0\lambda \tag{34}$$

where m_0 is the order of the central disc. i) Hence,

$$m_0 = \frac{2d}{\lambda} \approx 80000 \tag{35}$$

ii) The equation for a dark ring is

$$2d\cos\theta_m = m\lambda$$

$m = m_0$ corresponds to the central disc. Hence, the order of sixth dark ring from the center will be $(m_0 - 6) = 79994$.

(b) In this experiment, the distant traveled by the mirror is $d = 0.2945mm$ and the mean wavelength of the two source is $\lambda_{av} = 548.3nm$. Since, we know for concordance,

$$2d = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \approx \frac{(\lambda_{av})^2}{\lambda_1 - \lambda_2} \tag{36}$$

hence

$$\lambda_1 - \lambda_2 \approx 0.5nm \tag{37}$$

10 (b). A Fabry-Perot etalon is designed from a single slab of transparent material having a high refractive index (n = 4.5) and a thickness of 2.5 cm. The uncoated surfaces of the slab have a reflectance (r^2) of 0.90. If the slab is used in the vicinity of wavelength 546 nm, determine: (i) the highest order fringe in the interference pattern, (ii) the ratio (I_{max}/I_{min}), (iii) the chromatic resolving power.

The refractive index of the slab is $n = 4.5$ and thickness is $d = 2.5cm$ and the reflactance of uncoated surface is $r^2 = 0.90$, If the slab is used in the vicinity of wavelength $\lambda = 546nm$ Then, (i) the highest order fringe in the interference pattern is

$$m_0 = \frac{2d}{\lambda} = 91575 \tag{38}$$

(ii)

$$\frac{I_{max}}{I_{min}} = \frac{(1+R)^2}{(1-R)^2} = 361 \tag{39}$$

Where R is reflectance and $R = r^2$.

(iii) The chromatic resolving power is defined as

$$\frac{\lambda}{\Delta\lambda} = 1.515 m_0 \sqrt{F} \tag{40}$$

where coefficient of finesse (F) can be calculated from

$$F = \frac{4R}{(1-R)^2} = 360 \tag{41}$$

Hence resolving power comes out as

$$\frac{\lambda}{\Delta\lambda} = 2632332 \tag{42}$$