Physics of Waves

PH11003

Tutorial 10 Solution

Topic : Polarization

28 January 2023

[10.1] Describe the polarization of the following wave and write an expression for its magnetic filed.

$$\vec{E} = E_0(\hat{x} + i\hat{y})e^{i(kz-\omega t)}$$

Solution:

$$Re(E) = \hat{x}E_0 cos(kz-\omega t) - \hat{y}E_0 sin(kz-\omega t)$$

The magnetic field B in vacuum is

$$B = \int (\partial_z E_y \hat{x} - \partial_z E_x \hat{y})dt$$

$$B = \int [-kE_0 cos(kz-\omega t)\hat{x} + kE_0 sin(kz-\omega t)\hat{y}]dt$$

$$B = \frac{E_0}{c} sin(kz-\omega t)\hat{x} + \frac{E_0}{c} cos(kz-\omega t)\hat{y}$$

[10.2] The degree of polarization of partially polarized light is P = 0.25. Find the ratio of intensities of the polarized component of this light and the natural component.

Solution:

Suppose the partially polarized light consists of natural light of intensity I_1 and plane polarized light of intensity I_2 with direction of vibration parallel to, say, x-axis.

Then when a polaroid is used to transmit it, the light transmitted will have a maximum intensity $(1/2)I_1 + I_2$, when the principal direction of the polaroid is parallel to x-axis, and will have a minimum intensity $(1/2)I_1$ when the principal direction is perpendicular to x-axis.

Thus,
$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_2}{I_1 + I_2}$$

So,
$$\frac{I_2}{I_1} = \frac{P}{1-P} = \frac{0.25}{0.75} = \frac{1}{3}$$

[10.3] Initially unpolarized light passes in turn through three linear polarizers with transmission axes at 0°, 30°, and 60°, respectively, relative to the horizontal. What

is the intensity of the product light, expressed as a percentage of the unpolarized light intensity?
Solution:
The first polarizer blocks 1/2 of the unpolarized incident light. Then applying Malus' law for the last two polarizers,

$$I_1 = \frac{1}{2} I_0$$
$$I_2 = I_1 \cos^2(30°) = \frac{1}{2} I_0 \cos^2 30°$$
$$I_3 = I_2 \cos^2(60° - 30°) = \frac{1}{2} I_0 \cos^4 30° = 0.2815\, I_0 \Rightarrow 28.15\% \text{ of } I_0$$

[10.4] An ideal polarizer is rotated at a rate ω between similar pair of stationary crossed polarizers. Show that the emergent flux density will be modulated at four times the rotational frequency. In other words, show that

$$I = \frac{I_1}{8}(1 - cos\ 4\omega t)$$

where I_1 is the flux density emerging from the first polarizer and I is the final flux density.
Solution:
Denoting the angle that the rotating polarizer makes with the first stationary polarizer by θ, we have $\theta = \omega t$. The flux density after the rotating polarizer is

$$I_2 = I_1 cos^2\theta$$

And after the second stationary polarizer it is

$$I = I_3 = I_2 cos^2(90 - \theta) = I_1 cos^2\theta sin^2\theta$$

so

$$I = \frac{I_1}{4} sin^2 2\theta = \frac{I_1}{8}(1 - cos\ 4\theta) = \frac{I_1}{8}(1 - cos\ 4\omega t)$$

[10.5] How thick should a half-wave plate of mica be in an application where laser light of 632.8 nm is being used? Appropriate refractive indices for mica are 1.599 and 1.594.
Solution:

$$\frac{\lambda}{2} = t\,(\Delta n) \text{ or } t = \frac{\lambda}{2\,\Delta n} = \frac{632.8 \times 1 -^{-7}\ \text{cm}}{2\,(1.599 - 1.594)} = 0.063\,\text{mm}$$

[10.6] At what angles will light, externally and internally reflected from a diamond-air interface, be completely linearly polarized? For diamond, n = 2.42.
Solution
The polarizing angle is given by the relation, $\tan\theta_p = \frac{n_2}{n_1}$ .So for $n_{\text{air}} = 1$ and $n_{\text{diam}} = 2.42$

$$\text{Internal reflection:}\ \ \theta_p = \tan^{-1}\left(\frac{n_{\text{air}}}{n_{\text{diam}}}\right) = \tan^{-1}\left(\frac{1}{2.42}\right) = 22.5°$$
$$\text{External reflection:}\ \ \theta_p = \tan^{-1}\left(\frac{n_{\text{diam}}}{n_{\text{air}}}\right) = \tan^{-1}\left(\frac{2.42}{1}\right) = 67.5°$$