Week 6: Electromagnetic energy, spectrum, reflection and refraction

Electromagnetic energy and intensity: Let us begin by recalling that the electric and magnetic field and the direction of propagation constitute a triplet of mutually orthogonal vectors. We now define a quantity, a vector $\vec{\mathbf{S}}$ as follows:

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = c^2\epsilon_0\vec{\mathbf{E}} \times \vec{\mathbf{B}} \tag{1}$$

To understand what this vector is first look at its direction–it is along the direction of propagation $\vec{\mathbf{k}}$ of the wave. What about its dimensions? This is easy to figure out. Given $E = cB$ we get $S = c\epsilon_0 E^2$. Therefore

$$[S] = [c][\epsilon_0][E^2] = (LT^{-1})(Q^2M^{-1}L^{-3}T^2)(M^2L^2T^{-4}Q^{-2}) = MT^{-3} = (ML^2T^{-2})/(L^2T) \tag{2}$$

Thus, S has dimensions of energy per unit area, per unit time which is the flux of the traveling wave. The vector $\vec{\mathbf{S}}$ therefore represents the flux of energy as the wave travels in a certain direction. $\vec{\mathbf{S}}$ is known as the **Poynting vector**.

Given the definition of $\vec{\mathbf{S}}$ we can calculate its divergence. We have

$$\nabla \cdot \vec{\mathbf{S}} = \frac{1}{\mu_0}\nabla \cdot \left(\vec{\mathbf{E}} \times \mathbf{B}\right) \tag{3}$$

We will now use a vector identity given as,

$$\nabla \cdot \left(\vec{\mathbf{P}} \times \vec{\mathbf{Q}}\right) = \vec{\mathbf{Q}} \cdot \vec{\nabla} \times \vec{\mathbf{P}} - \vec{\mathbf{P}} \cdot \vec{\nabla} \times \vec{\mathbf{Q}} \tag{4}$$

Exercise: Proof the above-stated vector identity.

This will give, for the Poynting vector,

$$\nabla \cdot \left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = \vec{\mathbf{B}} \cdot \vec{\nabla} \times \vec{\mathbf{E}} - \vec{\mathbf{E}} \cdot \vec{\nabla} \times \vec{\mathbf{B}} \tag{5}$$

We also have, from the Maxwell equations in vacuum,

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \quad ; \quad \vec{\nabla} \times \vec{\mathbf{B}} = \mu_0\epsilon_0\frac{\partial \vec{\mathbf{E}}}{\partial t} \tag{6}$$

Therefore,

$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{\mu_0}\left[\vec{\mathbf{B}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} - \mu_0\epsilon_0\vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial t}\right] \tag{7}$$

$$= -\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right] \tag{8}$$

The quantity in square brackets on the R. H. S. above (in the last line) is the energy density of the electric and magnetic fields. We write this as

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2 \tag{9}$$

Note that we have $u_E = u_B$ because $E = cB$ (in vacuum). Thus, we may write $u = \epsilon E^2 = \frac{1}{\mu_0}B^2$. The energy flowing through space in the form of an electromagnetic wave is **shared equally** between the constituent electric and magnetic fields. Using the expression for u we get,

$$\vec{\nabla} \cdot \vec{\mathbf{S}} = -\frac{\partial u}{\partial t} \tag{10}$$

which, clearly, is reminiscent of **the continuity equation** discussed earlier. If we use the divergence theorem of Gauss we get, upon integrating over a volume V bounded by a surface A,

$$\int_A \vec{\mathbf{S}} \cdot \hat{\mathbf{n}}\ dA = -\frac{\partial}{\partial t}\int_V u\ dV \tag{11}$$

This simply means the following: **the net energy flux across the surface A enclosing the volume V, is equal to the time rate of change of energy in that volume.**–a result known as **Poynting's theorem.** The theorem expresses an obvious **conservation law**.

For a plane electromagnetic wave with

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{\mathbf{0}} \cos\left(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t\right) \quad ; \quad \vec{\mathbf{B}} = \vec{\mathbf{B}}_{\mathbf{0}} \cos\left(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t\right) \tag{12}$$

we have the instantaneous Poynting vector as,

$$\vec{\mathbf{S}} = c^2\epsilon_0 \left(\vec{\mathbf{E}}_{\mathbf{0}} \times \vec{\mathbf{B}}_{\mathbf{0}}\right) \cos^2\left(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t\right) \tag{13}$$

We now turn to understanding time-averages once again. Recall that we had done this while studying oscillations. Here we give a somewhat more general definition. The main point is we are now defining the time average over an arbitrary time T which is not equal to the periodicity (say τ) of the function $f(t)$. We will see later why we need to do this. Let us define,

$$\langle f(t) \rangle_T = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} f(t)\ dt \tag{14}$$

The periodicity of $f(t)$ is, say, $\tau = \frac{2\pi}{\omega}$. As an example, let us calculate $\langle e^{i\omega t}\rangle_T$ using the above definition. It is easy to check that

$$\langle e^{i\omega t}\rangle_T = \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} e^{i\omega t} = (sinc\ u)\ e^{i\omega t} \tag{15}$$

where we have defined,

$$sinc\ u = \frac{sin u}{u} = \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \tag{16}$$

A graph of this function is shown in Figure 1. Note that it dies down to insignificant values (close to zero) quite rapidly. Isolating the real and imaginary parts, we get

$$\begin{aligned}\langle\cos\omega t\rangle_T &= (sinc\ u\ )\cos\omega t\\ \langle\sin\omega t\rangle_T &= (sinc\ u\ )\sin\omega t\end{aligned} \tag{17}$$

When $T = \tau$ we have the usual result (discussed earlier)

$$\langle\cos\omega t\rangle_\tau = \langle\sin\omega t\rangle_\tau = 0 \tag{18}$$

Note, for an arbitrary $T \neq n\tau$ too, the amplitude of the *sinc* function becomes negligible as long as T is very large.

Exercise: Evalaute $\langle\cos^2\omega t\rangle_T$ and $\langle\sin^2\omega t\rangle_T$.

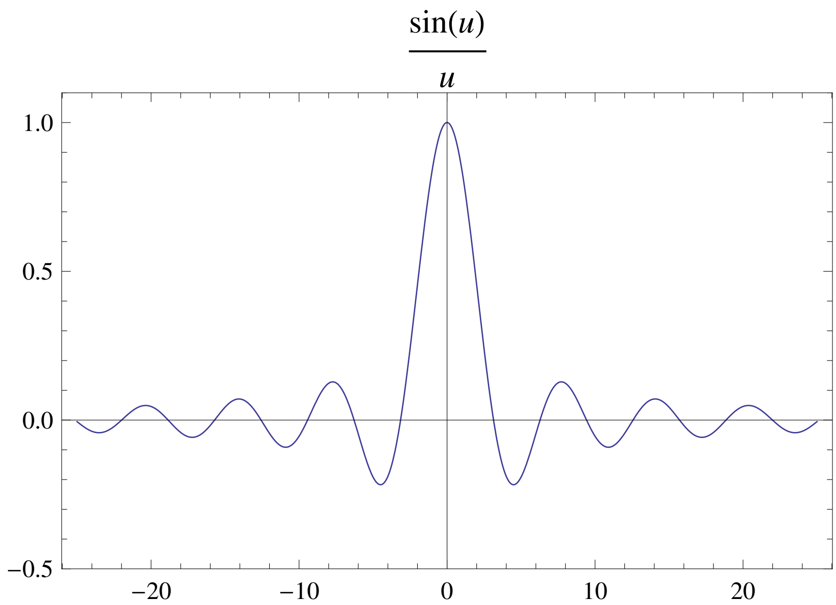

FIG. 1. The Sinc function

Why do we need to do the above? For electromagnetic waves τ is very very small. For example. the frequency of the electric/magnetic field oscillations in light waves is $10^{15}\ s^{-1}$. So the time period will be as small as 10^{-15} s. Therefore, when we do a time average for defining the intensity it is meaningless to do it for one cycle–we never measure one cycle! For instance, if we do an averaging over 10^{-6} seconds, which is possible, we still would require to consider $T = 10^9\tau$. If we do the averaging over a very large T (we may not have $T = n\tau$) the average value will not differ much from that for one cycle (or an integral multiple of one cycle), simply because of the nature of the *sinc* function.

Finally, let us define the intensity or irradiance which is central to everything we will do later. The time averaged value of the magnitude of $\vec{\mathbf{S}}$ when $T >> \tau$ is the intensity or irradiance. Thus we have

$$I = \langle S \rangle_T \tag{19}$$

For harmonic (cosine or sine) electric/magnetic fields, we get

$$I = \langle S \rangle_T = \frac{c}{\mu_0} \langle B^2 \rangle_T = \epsilon_o c \langle E^2 \rangle_T \tag{20}$$

In a dielectric which is linear, homogeneous and isotropic, we have

$$I = \langle S \rangle_T = \frac{v}{\mu_0} \langle B^2 \rangle_T = \epsilon_o v \langle E^2 \rangle_T \tag{21}$$

where v is the speed of the electromagnetic wave inside the dielectric.

In experiments, we measure this time-averaged value. Not directly though. We convert the light energy into electrical energy using photo-sensitive detectors. So, the intensity is measured in terms of current or voltage. There must be a calibration between the current/voltage and the absolute value of light intensity– usually in your lab experiments this is assumed to be a linear relation. The absolute value of the light energy is not very important in the kind of experiments you perform in the lab class. The variation of the intensity is more important.

Electromagnetic radiation sources and spectrum: Before we begin discussing phenomena associated with light we must know a bit about the sources which are responsible for creating electromagnetic radiation. Obviously, since electric and magnetic fields are involved, the sources must be charges and currents. Since the fields are time-dependent the sources must also be time-dependent. Thus, a static or an uniformly moving charge will not generate electromagnetic radiation. For a uniformly moving charge, we surely have electric and magnetic fields but we can always jump to the frame of the moving charge in which the charge will be static and the magnetic field will disappear. Therefore, we need **non-uniformly moving charges to create a radiation field**. More precisely, it is an

accelerating charge which is capable of generating electromagnetic radiation far far away from the source. As an example, we may consider an oscillating dipole moment,

$$\vec{\mathbf{p}} = \vec{\mathbf{p}}_{\mathbf{0}} \cos \omega t \tag{22}$$

The radiation electric field (magnitude) due to such a dipole is given as

$$E_{rad} = \frac{\mu_0 p_0 \omega^2 \sin\theta}{4\pi r} \frac{\cos(kr - \omega t)}{r} \tag{23}$$

This is the term which dominates far away (i.e. at infinity) from the sources. Note that the radiation electric field goes as $\frac{1}{r}$ (not $\frac{1}{r^2}$). Therefore, the intensity along the direction of propagation can be found and is given as

$$I(\theta) = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta}{r^2} \tag{24}$$

The main feature in the above expression is the ω^4 dependence and the $\sin^2\theta$ dependence. Thus for larger ω (or smaller λ) the radiated intensity is higher. The blue sky we see has a relation to this formula and its ω^4 dependence though there are some subtleties associated with the response of the human eye. Note that **colour** is **not an instrinsic property** of visible electromagnetic waves.

We have not done any derivation of the electric field and intensity above. The purpose was to just give you an idea. The derivation is beyond the scope of this course.

Let us now look at the electromagnetic spectrum in some detail. In principle, there is no limit on the wavelength or frequency of electromagnetic waves. In fact between the variously named types there are no **fundamental** differences since Maxwell's equations do not depend on the wavelength explicitly. Further, there are no gaps in the wavelength/frequency ranges. However, physical properties for the various types do differ and such specific properties are utilised in various applications. Here is a list of the various types:

Radio waves: These were first observed by Heinrich Hertz in 1887 in his famous spark-gap experiment. He had an open loop as the detector. Hertz not only observed the electromagnetic radiation but also studied its properties.

The frequency range for radio waves is a few Hz to 10^9 Hz. λ ranges from 0.3m to many kilometers. There is no upper limit– waves with wavelengths of the order to 18 million miles have been observed. Wavelengths of the order of $10^6 - -10^{11}$m are called **Ultra Low**

Frequency waves (ULF). An example of such ULF waves is known to arise in the small amplitude fluctuations of the Earth's magnetic field.

Microwaves: The frequency range for microwaves is 10^9 Hz to 3×10^{11} Hz and the wavelength range is from 30 cm to 1 mm. There are many examples of microwaves in the universe– the cosmic microwave background (CMB) being the most prominent one with a maximum wavelength of about 1.06 mm. The CMB is a relic radiation from the distant past and is a perfect black body with a temperature close to 3 degree Kelvin. There is also the 21 cm radiation emitted by neutral hydrogen (HI) in galaxies, which is of great importance in astrophysics. Further, most of our communication (mobiles for instance) is done using the microwave band. Another example of the use of microwaves appears, as the name suggests, in microwave ovens.

Infra-red: The frequency range for infra-red radiation is 3×10^{11} Hz to 4×10^{14} Hz. The wavelength range is 780 nm to $15,000$ nm. Infra-red radiation is emitted copiously from hot bodies such as electric heaters etc. The human body also emits infra-red radiation in the range of wavelength $3000 - 10,000$ nm.

Visible light: The frequency range is 3.84×10^{14} Hz to 7.69×10^{14} Hz. The wavelength range is $400 - 800$ nm.

Ultraviolet: The frequency range is 8×10^{14} Hz to 3.4×10^{15} Hz. This corresponds to roughly 100 to 400 nm.

X-rays: The frequency range is 2.4×10^{16} Hz to 5×10^{19} Hz. Wavelengths are of the order of $1\mathring{A}$ to $100\mathring{A}$.

Gamma-rays: These are the highest energy waves – their energy lies in the range 10^4 eV to 10^{19} eV. The wavelengths of gamma rays are too small (less than an Angstrom) to observe any optical properties.

From here, we will slowly move on towards understanding phenomena associated with visible light waves. Some of these are reflection, refraction, interference, diffraction, polarisation. We will try to keep and eye on the electromagnetic perspective as we move along. Many of the associated phenomena we will discuss are related to natural occurences. We would like to understand and quantify. Some of the phenomena to be discussed will be directly related to the experiments in the laboratory class associated with this course.

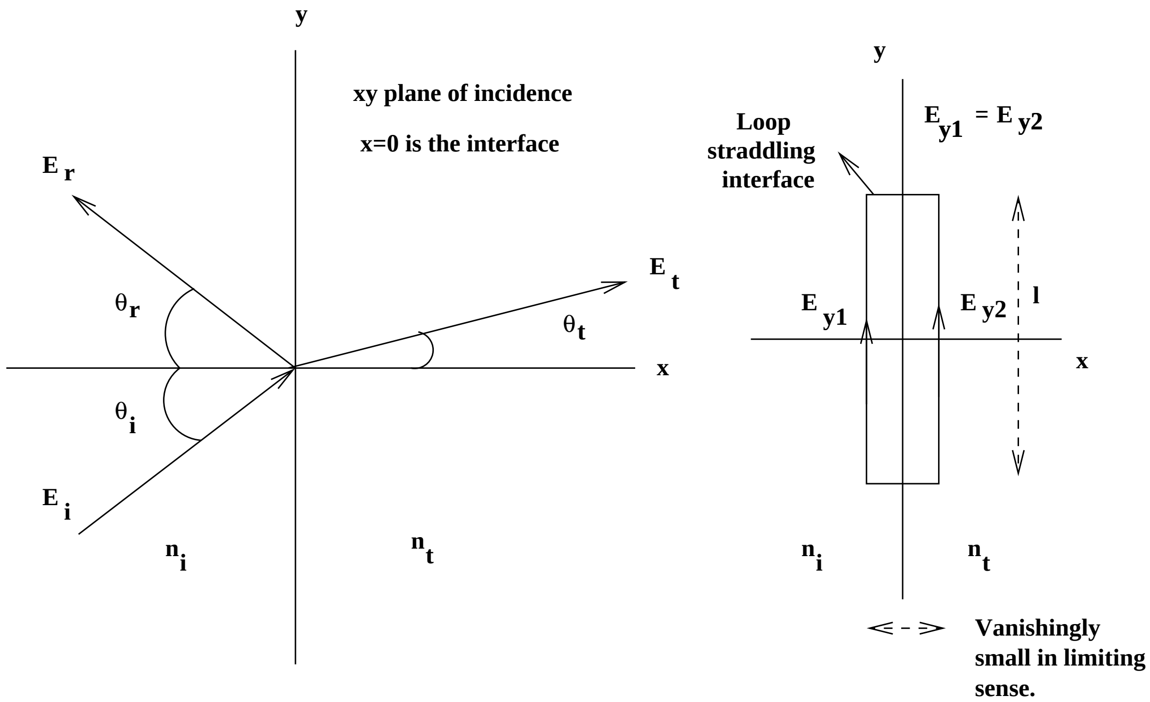

FIG. 2. Reflection and Refraction. In the left figure, the arrows denote the incident, reflected and refracted wave vectors.

Reflection and refraction: You must have studied reflection and refraction of light in high school. There, by using **Fermat's principle of least time** you may have obtained the laws of reflection and refraction. The basic notion we take from Fermat's principle is that of the **optical path**. We know that for the discrete case this is defined as $\sum_{i=1}^{m} n_i s_i$ where n_i is the refractive index of the ith medium and s_i is the geometric distance traveled by light in that medium. For a continuously varying refractive index we have the optical path as $\int_P^Q n(s)ds$. We will keep these facts in mind.

Let us now try and see if we can prove the laws of reflection and refraction using the concept of light as an electromagnetic wave. In the Figure 2, we have two media with refractive indices n_i and n_t separated by a boundary at, say $x = 0$. We define the **plane of incidence** as the xy plane. Light is incident on the boundary (at $(0, 0)$) –it is then reflected and refracted. The incident wave vector is $\vec{\mathbf{k}}_i$. The reflected wave vector is $\vec{\mathbf{k}}_r$ and the transmitted wave vector is $\vec{\mathbf{k}}_t$. These are shown in Figure 2. All the three vectors lie in the plane of incidence. The angle of incidence, reflection and refraction are, respectively θ_i, θ_r and θ_t. Let us now write down the electric fields of the incident, reflected and refracted waves. Note that the electric fields can be **in the plane of incidence** or **perpendicular to the plane of incidence** or **in an arbitrary direction with components parallel and perpendicular to the plane of incidence** . The electric field must, of course be perpendicular to the propagation vector $\vec{\mathbf{k}}$ and to $\vec{\mathbf{B}}$. These electric fields are:

$$
\begin{aligned}
\vec{\mathbf{E}}_{\mathbf{i}} &= \vec{\mathbf{E}}_{\mathbf{0i}}\ e^{i(k_{ix}x+k_{iy}y-\omega_i t)} \\
\vec{\mathbf{E}}_{\mathbf{r}} &= \vec{\mathbf{E}}_{\mathbf{0r}} e^{i(k_{rx}x+k_{ry}y-\omega_r t)} \\
\vec{\mathbf{E}}_{\mathbf{t}} &= \vec{\mathbf{E}}_{\mathbf{0t}} e^{i(k_{tx}x+k_{ty}y-\omega_r t)}
\end{aligned} \tag{25}
$$

We also have,

$$
k_i^2 = \frac{\omega_i^2 n_i^2}{c^2} \quad ; \quad k_r^2 = \frac{\omega_r^2 n_i^2}{c^2} \quad ; \quad k_t^2 = \frac{\omega_t^2 n_t^2}{c^2} \tag{26}
$$

where

$$
k_i^2 = k_{ix}^2 + k_{iy}^2 \quad ; \quad k_r^2 = k_{rx}^2 + k_{ry}^2 \quad ; \quad k_t^2 = k_{tx}^2 + k_{ty}^2 \tag{27}
$$

We can also write down the $\vec{\mathbf{B}}$ fields but it is not really necessary for our purposes right now.

We note that Maxwell's equations hold in both the regions. They also must hold at the boundary at $x = 0$. How do the fields fit at the boundary (defined by $x = 0$)? To understand how they actually fit, let us go back to Faraday's law in integral form:

$$
\oint \vec{\mathbf{E}} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\ dA \tag{28}
$$

We would like to apply this across a small loop Γ straddling the boundary (see Figure 2). Now, the area of the loop is small and in the limit it goes to zero. So, if the magnetic field is finite (as it is) we have the R. H. S. of the above equation as zero. We assume that the electric fields are in the plane of incidence. Since the path is along y and the length traversed along y is l we have:

$$
E_{1y} \cdot l - E_{2y} \cdot l = 0 \tag{29}
$$

Note that the contribution along the horizontal path segments vanishes when we take the limit $\epsilon \to 0$ (i.e. the surface just straddles the boundary). Thus, we must have,

$$
E_{1y} = E_{2y} \tag{30}
$$

Note that E_{1y} is the total y component of the electric field in the medium n_i while E_{2y} is the same for the medium n_t. Now, if we consider the electric fields in the medium n_i there are reflected and incident fields while in medium n_t, we have only the transmitted field. Therefore, using the fact that the boundary is at $x = 0$, we have

$$E_{0iy}e^{i(k_{iy}y-\omega_i t)} + E_{0ry}e^{i(k_{ry}y-\omega_r t)} = E_{0ty}e^{i(k_{ty}y-\omega_t t)} \tag{31}$$

If we now use the requirement that the equation must be valid for $y = 0$, we have

$$E_{0iy}e^{-i\omega_i t} + E_{0ry}e^{-i\omega_r t} = E_{0ty}e^{-i\omega_t t} \tag{32}$$

Thus, two oscillations (of different frequencies) add up to another oscillation (with yet another frequency). When is this possible? The only way it can happen is if

$$\omega_i = \omega_r = \omega_t \tag{33}$$

Exercise: Prove the above result by using the $t = 0$ expressions for the above equation as well as its first and second derivatives (also evaluated at $t = 0$).

This gives us a fundamental result: **the frequencies do not change on reflection and refraction.**

We can employ the same trick by saying that the boundary equation holds for all t. This would give us

$$E_{0iy}e^{ik_{iy}y} + E_{0ry}e^{ik_{ry}y} = E_{0ty}e^{ik_{ty}y} \tag{34}$$

which yields (as for the $y = 0$ case discussed just above),

$$k_{iy} = k_{ry} = k_{ty} \tag{35}$$

From an earlier result, we had $k_i^2 = k_r^2$. Therefore,

$$k_i^2 = k_{ix}^2 + k_{iy}^2 = k_{rx}^2 + k_{ry}^2 = k_r^2 \tag{36}$$

Using the equality of the y components, we get

$$k_{ix}^2 = k_{rx}^2 \tag{37}$$

which eventually leads to

$$k_{ix} = \pm k_{rx} \tag{38}$$

Obviously, it is meaningful to take $k_{ix} = -k_{rx}$, because the other root (i.e $k_{ix} = k_{rx}$) just gives the incident wave vector. The two conditions,

$$k_{ix} = -k_{rx} \quad k_{iy} = k_{ry} \tag{39}$$

finally lead to the reflected wave given as

$$\vec{\mathbf{E}}_r = \vec{\mathbf{E}}_{0r} e^{i(-k_{ix}x + k_{iy}y - \omega_i t)} \tag{40}$$

Consider the facts: (i) the angle θ_i is the angle between the vector $\mathbf{k_i}$ and the positive x axis (i.e. $\hat{\mathbf{i}}$), (ii) the angle θ_r is the angle between the vector $\mathbf{k_r}$ and the negative x axis (i.e. $-\hat{\mathbf{i}}$) Thus, cos $\theta_i = \frac{\mathbf{k_i} \cdot \hat{\mathbf{i}}}{k_i} = \frac{k_{ix}}{k_i}$ and cos $\theta_r = \frac{\mathbf{k_r} \cdot (-\hat{\mathbf{i}})}{k_r} = \frac{-k_{rx}}{k_r} = \frac{k_{ix}}{k_i}$. So, we get $\cos\theta_i = \cos\theta_r$, which implies

$$\theta_i = \theta_r \tag{41}$$

Now, the transmitted wave satisfies

$$k_{ty} = k_{iy} \qquad \frac{k_t^2}{n_t^2} = \frac{k_i^2}{n_i^2} \tag{42}$$

From the relations

$$\frac{k_{iy}}{k_i} = \sin\theta_i \quad \frac{k_{ty}}{k_t} = \sin\theta_t \tag{43}$$

we get

$$k_i \sin\theta_i = k_t \sin\theta_t \tag{44}$$

Since

$$\frac{k_i}{k_t} = \frac{n_i}{n_t} \tag{45}$$

we finally obtain Snell's law,

$$n_i \sin\theta_i = n_t \sin\theta_t \tag{46}$$

Let us now understand briefly (without deriving too much) the **phase change of π on reflection**. This essentially follows from the so-called reflection coefficients. Using the known boundary conditions for the tangential and normal components (w.r.t. the interface) of the electric and magnetic fields one can calculate the reflection coefficients. There are two cases: (i) when the electric field vector is perpendicular to the plane of incidence (ii) when the electric field vector is parallel to the plane of incidence. One can derive the reflection coefficients for these cases to be:

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \tag{47}$$

$$r_{||} = \left(\frac{E_{0r}}{E_{0i}}\right)_{||} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i} \tag{48}$$

There are similar transmission coefficients as well named as $t_{\perp}$ and $t_{||}$ which we do not discuss here. All of these together are called the **Fresnel relations.** As mentioned before, we have **not** derived them here.

Let us assume $n_i < n_t$ (air-glass, air-water). Then, we have $n_i \cos\theta_i < n_t \cos\theta_t$ always. Therefore, $r_{\perp}$ is negative. Hence, we may write,

$$(E_{0r})_{\perp} = -r_{\perp}(E_{0i})_{\perp} = e^{i\pi} r_{\perp}(E_{0i})_{\perp} \tag{49}$$

Thus, there is a **phase shift of** π in the reflected wave, for all angles of incidence. See Figure 3.

The nature of $r_{||}$ is however different because $n_t \cos\theta_i - n_i \cos\theta t$ can be > 0, < 0 or equal to zero. Till $\theta_i = \theta_p$ (where θ_p is defined as $n_t \cos\theta_p = n_i \cos\theta_t$) there is no phase change. Beyond θ_p there is a phase change of π because $r_{||}$ becomes negative (see Figure 3). The angle θ_p is called the **polarisation angle** or **Brewster's angle**. One can show that $n_t \cos\theta_p = n_i \cos\theta_t$ implies $\theta_p + \theta_t = \frac{\pi}{2}$. This also results in the following relation from Snell's law

$$\tan\theta_t = \frac{n_i}{n_t} \tag{50}$$

from which, given the refractive indices one can find θ_t and hence θ_p. For example, if $n_i = 1$ and $n_t = 1.5$, then $\theta_p = 57.57$ degrees and $\theta_t = 33.43$ degrees.

Brewster's angle is called the polarisation angle because $r_{||}$ is zero at this angle of incidence. Therefore, if we have an incident light wave which has components parallel and perpendicular to the plane of incidence, the reflected wave will retain only one polarisation (the perpendicular component). Thus, we have a method of **producing polarised light**. We shall come back to this again at a later stage.

Finally let us briefly mention **total internal reflection**. For this to happen we must have (from Snell's law with $\theta_t = \frac{\pi}{2}$)

$$\sin\theta_i = \frac{n_t}{n_i} \tag{51}$$

If the above equation has to have a solution, one needs $n_t < n_i$. Hence the propagation must be from a denser to a rarer medium. The angle $\theta_i = \theta_c$ for which this happens (given the refractive indices n_t and n_i) is known as the **critical angle**. For a glass-air interface, the critical angle is close to 41.8 degrees.

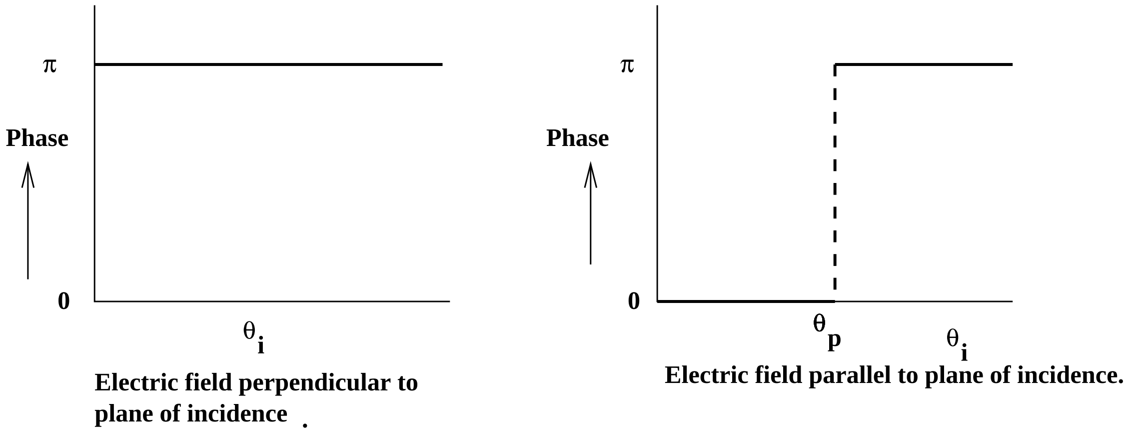

FIG. 3. Phase versus angle of incidence