Physics of Waves

PH11003

Tutorial 5

Waves

08 December 2022

[5.1] A transverse wave on a string is given by $y(x,t) = 2.4cos[\frac{\pi}{20}(0.5x - 40t)]$ where both x and y are in centimeters. Find: (a) the maximum particle velocity; (b) the particle velocity at x = 1.5 cm at t = 0.25 s; (c) the maximum particle acceleration; (d) the acceleration at x = 1.5 cm and t = 0.25 s.

[5.2] The wave function of a standing wave on a string is given by $y(x,t) = 0.02sin(0.3x)cos(25t)$ where x and y are in centimeters and t is in seconds. (a) What is the length of the string if this function represents the third harmonic? (b) At what points is the particle velocity permanently zero?

[5.3] A longitudinal standing wave $\xi = a\ cos(kx)\ cos(\omega t)$ is maintained in a homogeneous medium of density ρ. Find the expressions for : (a) Potential energy density and (b) Kinetic energy density.

[5.4] The phase velocity of a surface wave on a liquid of density ρ and surface tension T is given by

$$v_p = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}\right)^{1/2}$$

where λ is the wavelength of the wave and g is the acceleration due to gravity, (a) Find the group velocity of the surface wave, (b) Find the λ for which v_p is minimum, (c) Evaluate the minimum value of v_p and the corresponding v_g.

[5.5] A linear array of particles with equal masses m are connected by identical springs whose stiffness constant is k. The equilibrium position of the nth particle is $x_n = na$, while s_n is its displacement from equilibrium. (a) Show that

$$m\frac{d^2s_n}{dt^2} = k(s_{n+1} + s_{n-1} - 2s_n)$$

(b) Show that $s_n = Asin(kx_n - \omega t)$ is a solution provided that

$$\omega^2 = \frac{4k}{m} sin^2(\frac{ka}{2})$$

Answers:

[5.1] (a) 15.1 cm/s (b) -15.0 cm/s (c) 94.7 cm/s^2 (d) -11.1 cm/s^2

[5.2] (a) 31.4 cm (b) 10.5 cm, 20.9 cm