Week 10

Polarisation

Earlier, while introducing the electromagentic wave, we had talked about polarisation. Polarisation was shown to arise because of the **vector** and **transverse** nature of electromagnetic waves. It is related to the orientation of the electric field on the wavefront, as the wave propagates in time. We had discussed the definitions of linear, circular and elliptically polarised light waves in detail.

The first question that we may ask is — is natural light polarised? The light from the Sun, the artificial light that we see around us—is it polarised? In a sense, the answer is yes. Light is usually emitted by atomic emitters Each atom when excited radiates a polarised wavetrain which last for about 10^{-8} seconds. Emissions with the same frequency can combine to form a single wavetrain which also will last for about 10^{-8} seconds. New polarisations are continuously created and the polarisation therefore changes in a rapid and random fashion. Thus natural light is in a state of random and rapidly varying polarisation. It isn't really unpolarised. Natural light can be decomposed into two incoherent, mutually orthogonal, linearly polarised waves of equal amplitude but with a relative phase which varies rapidly and randomly. However, note that a purely monochromatic wave is always completely polarised—it is an infinite wavetrain and is, of course, an idealisation. Natural light or any light as such is never completely polarised however much you try to do so with your devices. We always have partial polarised light.

An important law about polarised light is **Malus' law**. Look at Figure 3. We have natural light which is made to pass through a polariser which has a pass axis along an angle θ (as shown). We generate E_{01} just beyond this polariser. E_{01} sums the components along the pass axis, of the randomly varying fields in natural light. We then make the light pass through an analyser (a second polariser). The analyser has a pass axis along the vertical. Hence the field just after the analyser is $E_{01} \cos \theta$. Thus, the intensity is given as

$$I(\theta) = \frac{c\epsilon_0}{2} E_{01}^2 \cos^2 \theta \tag{1}$$

This is Malus' law. If $\theta = \frac{\pi}{2}$ then the intensity is zero –this is the case of crossed polariser-analyser.

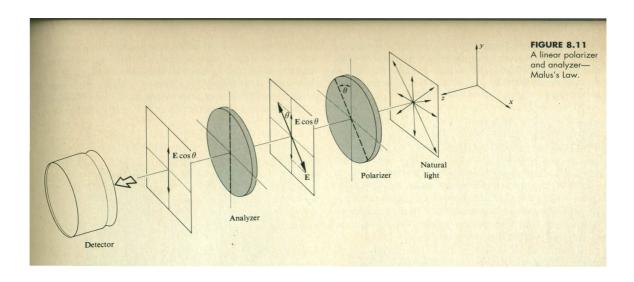


FIG. 1. Malus' law, figure from Hecht, Optics

We now turn to the important issue of how to produce polarised light. There are several ways of doing so. We list below four well-known methods:

- Dichroism or selective absorption
- Polarisation by scattering
- Polarisation by reflection
- Double refraction or birefringence

We will discuss each of them briefly.

Dichroism or selective absorption: Dichroism involves the selective absorption of one of the two mutually orthogonal components of natural light. The standard old device is the wire grid polariser (see Figure 4). In this device, the electric field component $(\vec{\mathbf{E}})_{\parallel}$ is absorbed by the grid wires while the perpendicular component $(\vec{\mathbf{E}})_{\perp}$ is transmitted. Thus, the transmission axis is the direction normal to the grid wires. In 1960, Bird and Parish Jr. constructed a grid with 2160 wires per mm. Each wire was less than a wavelength across. The wire grid is not used much nowadays but it serves as a good classroom example.

There are certain crystals wich have a preferred direction called the **optic axis** and which possess the properties of selective absorption. Among them is **tourmaline** which is in the boron silicate class $(NaFe_3B_3Al_6Si_6O_{27}(OH)_4)$. If light falls on such a crystal the electric field component perpendicular to the optic axis is absorbed while the parallel component is transmitted. In fact the transmitted light is also a bit absorbed. Looking at the crystal from different directions one can see different colours –hence the name dichroic.

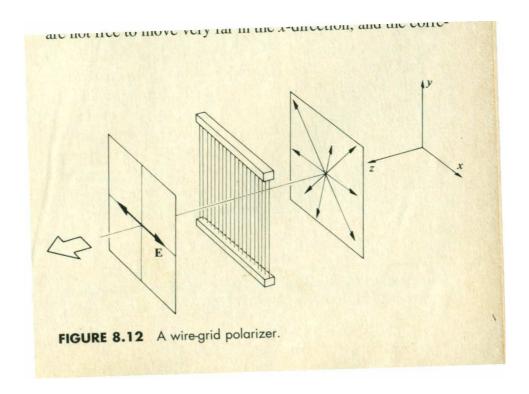


FIG. 2. Wire grid polariser, figure from Hecht, Optics

Another polarizer of the wire-grid type is the H-sheet invented by Herbert Land in 1938. Polyvinyl alcohol is heated and stretched – the long chain hydrocarbons thereby get aligned. This sheet is then dipped in ink rich in iodine and the iodine molecules attach along the hydrocarbon chains forming a wire-grid. The electric field component along the iodine chain is absorbed while the perpendicular component is transmitted. The principle is the same as the wire grid polariser.

Polarisation by scattering: When unpolarised light is incident on a molecular scatterer the electric field induces a dipole moment in the molecule. The dipole oscillates (because the field is oscillatory) and therefore it generates radiation. If the electric field of the incident wave is polarised parallel to this page then the observer standing at a position just below the scatterer and on the line perpendicular to the direction of the wave, does not see any radiation. On the other hand, if the electric field is perpendicular to this page then the same observer does see a radiation. Since incident light will have both components, the observer will eventually see only that light which is polarised perpendicular to the plane of this page. Thus, the process of scattering generates polarised light.

Polarisation by reflection: We have discussed this in some detail while studying reflection and refraction. We had noted that the reflection coefficients $r_{||}$ and r_{\perp} for $n_1 < n_2$ (n_1 being the incidence medium) have the property that for a certain $\theta_i = \theta_p r_{||}$ can become zero but r_{\perp} is never be zero (in fact it is always negative). Thus, on reflection a specific polarisation is produced (the one perpendicular to the plane of incidence). The θ_p is called the polarisation angle or Brewster's angle and is given as

$$\tan \theta_p = \frac{n_2}{n_1} \tag{2}$$

For air to glass $\theta_p = 57$ degrees.

Double refraction or birefringence: This is a phenomenon which occurs in anisotropic crystals. The arrangement of atoms results in the anisotropy. Usually, the dielectric constant (and hence the refractive index given as $n = \sqrt{\frac{\epsilon}{\epsilon_0}}$, taking $\mu = \mu_o$ in the dielectric medium) is assumed as a single number for a given material. In anisotropic crystals the dielectric constant is no longer a single number, but a set of numbers, denoted collectively as ϵ_{ij} where i, j can be x, y, z. Hence, when an electric field enters such a medium, the resulting electric displacement $\vec{\mathbf{D}}$ is given as $D_i = \epsilon_{ij} E_j$ where the j is summed over. For example $D_x = \epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z$. Thus, we have a 3×3 matrix for ϵ_{ij} , which can be diagonalised by a choice of basis. It is this character of the dielectric constant which is responsible for the creation of two different directions of propagation, with different linear polarisations, in an anisotropic crystal. We shall not go into the details of the analysis but only state the facts qualitatively.

Such anisotropic crystals, by virtue of their anisotropy can have preferred directions (one or more). These directions are known as the **optic axes**. A crystal is said to be **uniaxial**, **biaxial** or **triaxial** depending on its having one, two or three different optic axes. Note that the **optic axis** is a **direction**, it is not just a single line. For simplicity, let us assume a uniaxial crystal with its optic axis along the x direction. If the electric field vector of the incident electromagnetic wave $\vec{\mathbf{E}}$ is **along** the optic axis direction, then the wave sees a refractive index denoted by n_e , known as the **extraordinary refractive index**. On the other hand, if the $\vec{\mathbf{E}}$ of the incident electromagnetic wave is **perpendicular** to the optic axis, the wave sees a refractive index n_o (**the ordinary refractive index**). This implies

that refraction at the interface will be different for these two cases. Consequently, there will be two rays inside the crystal, the one due to n_e is known as the **extraordinary ray** or e-ray and the other due to n_o known as the **ordinary ray** or o-ray. In an arbitrarily polarised incident electromagnetic wave both possibilities exist and therefore, we will see two refracted rays and hence two images of any object. The e-ray does not quite obey Snell's law because the n_e is dependent on the angle of incidence. The o-ray obeys Snell's law, as usual. The polarisations of the electric fields which travel along the e-ray and o-ray are also different. For the e-ray, the polarisation is along the optic axis while for the o-ray it is perpendicular to the optic axis. Hence, double refraction can be used to generate specific linearly polarised states of light. The quantity

$$\Delta n = n_e - n_o \tag{3}$$

is a measure of double refraction or birefringence. If $\Delta n > 0$ we have a positive crystal, while if $\Delta n < 0$ we have a negative crystal. Also, note that the values of n_e and n_o are wavelength dependent. At $\lambda = 589.3$ nm, we know the following numbers for different crystals:

Tourmaline: $n_o = 1.669$, $n_e = 1.638$, Negative. Calcite: $n_o = 1.6584$, $n_e = 1.4864$, Negative.

Quartz: $n_o = 1.5443$, $n_e = 1.5534$, Positive.

Sodium Nitrate: $n_o = 1.5854$, $n_e = 1.3369$, Negative.

Ice: $n_o = 1.309$, $n_e = 1.313$, Positive.

One of the devices which employs double refraction to generate a linearly polarised light wave is the Nicol prism. Two calcite pieces are cut in a special way and cemented. Figure 5 shows the two triangular sections joined along a hypotenuse. The cement used is called Canada Balsam and has a refractive index $n_{cb} = 1.54$. An incident ray is split into an o-ray and an e-ray both being linearly polarised. Since $n_o > n_{cb}$, the o-ray can be made to reflect totally and internally at the cementing layer. It is then absorbed by the edge where black paint is used. The e-ray travels through the cementing layer after some refraction (note $n_{cb} > n_e$) and finally emerges outside, almost parallel to the incident ray, but shifted. Since the e-ray carries a definite polarisation (electric field parallel to optic axis) – we get polarised light at the output of a Nicol prism. In your polarimeter experiment in the lab class you make use of the Nicol prism (placed inside the polarimeter tube) to create linearly polarised light which is then passed through the tube containing sugar solution. There are several other types of polarising devices which make use of double refraction such as the Wollaston

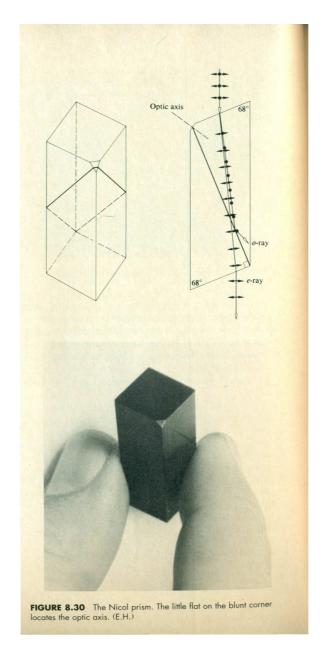


FIG. 3. Nicol prism, figure from Hecht, Optics

prism or the Glan-Foucault prism (for details see the chapter on polarisation in in the book, Optics by Hecht).

The other important device which employs double refraction is the wave plate. This is simply a birefringent crystal of thickness, say h. Assume the optic axis along the y direction and the crystal in the xy plane. A light wave incident along z will have its electric field on the xy plane. This $\vec{\mathbf{E}}$ can be decomposed into x, y components. The y component will respond to n_e and the x component to n_0 . Hence the path lengths traveled will be equal to

 $n_e h$ and $n_o h$. If we can adjust h such that

$$|n_e - n_o|h = \frac{\lambda}{4} \tag{4}$$

then the wave plate is called a quarter wave plate. If

$$|n_e - n_o|h = \frac{\lambda}{2} \tag{5}$$

then we have a half wave plate.

The quarter wave plate has the property that it can convert linearly polarised light into circularly polarised light. Let us see how. Assume that the incident light makes an angle of 45 degrees with the optic axis.

$$\vec{\mathbf{E}} = E_0 \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} \right) \cos \left(\omega t - kz \right) \tag{6}$$

If we introduce a quarter wave plate on its path a phase difference of $\frac{\pi}{2}$ (equal to a path difference of $\frac{\lambda}{4}$) will be introduced. Therefore, the output will be

$$\vec{\mathbf{E}} = E_0 \left[\hat{\mathbf{i}} \cos \left(\omega t - kz \right) + \hat{\mathbf{j}} \cos \left(\omega t - kz + \frac{\pi}{2} \right) \right]$$
 (7)

which is a circularly polarised light wave. If we did not assume the incident wave to have a polarisation of 45 degrees then the output would be elliptically polarised light. It is also possible to do the reverse, i.e. produce linearly polarised light from circularly polarised light using a quarter wave plate.

On the other hand, a half wave plate will just rotate the plane of polarisation since it introduces a phase difference of π .

Optical activity: Any material (liquid or solid, transparent) which causes the $\vec{\mathbf{E}}$ field of an incident plane electromagnetic wave to rotate is known as optically active. The associated phenomenon is optical activity. If the rotation of the plane of polarisation is clockwise when one looks into the direction of the source, we call the material dextro-rotatory (d-rotatory). For anticlockwise rotation the material is levo-rotatory (l-rotatory). Quartz is knowns to be optically active. Sugar solution is also known to be optically active. Dextro and levo rotatory properties are produced by crystal structures which are mirror images of each other—this is known as the handed-ness or chirality property. The structures

are **enantiomorphs** of each other. Among other optically active substances are benzil, $NABrO_3$.

Fresnel gave a very simple explanation of the phenomenon of optical activity. It is based on electromagnetic theory. The incident linearly polarised wave (produced say by a Nicol prism) can be thought of as a superposition of a left-circular and a right circular wave. We write them as

$$\vec{\mathbf{E}}_{\mathbf{R}} = \frac{E_0}{2} \left[\hat{\mathbf{i}} \cos \left(k_R z - \omega t \right) + \hat{\mathbf{j}} \sin \left(k_R z - \omega t \right) \right]$$
 (8)

$$\vec{\mathbf{E}}_{\mathbf{L}} = \frac{E_0}{2} \left[\hat{\mathbf{i}} \cos \left(k_L z - \omega t \right) - \hat{\mathbf{j}} \sin \left(k_L z - \omega t \right) \right]$$
 (9)

where $k_R = k_0 n_R$ and $k_L = k_0 n_L$. Note that we have used k_L and k_R , i.e. the velocities of the right circular and left circularly polarised waves are different inside the optically active medium. We now add $\vec{\mathbf{E}}_{\mathbf{R}}$ and $\vec{\mathbf{E}}_{\mathbf{L}}$ to get the total field. This gives

$$\vec{\mathbf{E}} = E_0 \left[\cos \left(\frac{k_L + k_R}{2} - \omega t \right) \right] \left\{ \hat{\mathbf{i}} \cos \left(k_R - k_L \right) \frac{z}{2} + \hat{\mathbf{j}} \sin \left(k_R - k_L \right) \frac{z}{2} \right\}$$
(10)

At the position where the light enters, z = 0, and we have

$$\vec{\mathbf{E}} = E_0 \cos \omega t \hat{\mathbf{i}} \tag{11}$$

At a distance z = d, i.e. after passing through the optically active medium, we find that there is a rotation of the plane of polarisation given by an amount

$$\beta = -\frac{(k_R - k_L)d}{2} \tag{12}$$

which is equivalent to

$$\beta = \frac{\pi d}{\lambda_0} \left(n_L - n_R \right) \tag{13}$$

If $n_L > n_R$ we have d-rotatory. If $n_L < n_R$ we have l-rotatory. The quantity $\frac{\beta}{d}$ is the specific rotatory power. For quartz it is around 21.7 deg./mm.

In the case of liquids with a given concentration c of an optically active substance, the optical rotation is proportional to the concentration as well as the length. We have $\beta \propto dc$. The proportionally constant is usually referred as the specific rotation. For more details and proper definitions (as per convention) see the discussion in the lab manual on the experiment on polarimetry.

Quantum physics

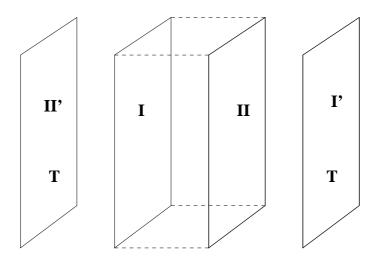
Till now we have tried to understand the behaviour of light as an electromagnetic wave. We have shown that it is a **transverse**, **vector** wave propagating in vacuum at the speed $c = 3 \times 10^8$ m/s. We have studied various aspects of such electromagnetic waves in detail and have tried to understand some of its properties through phenomena like reflection, refraction, interference, diffraction and polarisation. Our theoretical understanding of some of these phenomena have also been demonstrated in several experiments which you have performed in your lab sessions.

Despite the success in understanding the behaviour of light as a wave, there were several unanswered questions which bothered physicists during the end of the nineteenth century. There were experimental results which could not be understood in terms of the wave nature of light. We shall now briefly dwell upon each of these results and try to see how they were eventually understood using the notion of light energy as made up of discrete quanta of energy. This is the photon picture of light and in essence, it brought forth the notion of particle properties of waves. Newton had indeed discussed a corpuscular theory of light many years ago. However, it was Max Planck's light quantum hypothesis in 1900, which laid the foundations of our understanding of the particle-like behaviour of light.

Black body radiation: Among others, one of the results which worried scientists a lot, during the end of the nineteenth century, was the spectrum of **black body radiation**. Let us now try to see where the problem arose.

The ability of a body to radiate is related to its ability to absorb radiation. A body at a constant temperature T, i.e. in thermal equilibrium must absorb and emit at the same rate in order to maintain its constant temperature. A body that absorbs **all radiation** that is incident on it, regardless of the frequency is known as a **black body**.

In the 1860s, Kirchoff studied properties of thermal radiation in quite some detail. One can show that the black body defined above is a better emitter of radiation than anything else. To see this let us look at a simple experiment. Consider two identical pairs of dissimilar surfaces. These are I,I' & II, II' and are placed as shown in the Figure 4. No temperature difference is observed between I' and II'. At a given temperature, I and I' radiate at the rate e_1 while II, II' radiate at the rate e_2 . The surfaces I, I' absorb some fraction a_1 of the radiation falling on them. Similarly II, II' absorb some fraction a_2 . Hence I' absorbs



I and I' are identical and different from identical II, II' I' and II' are at temperature T.

FIG. 4. Kirchoff analysis on e/a= constant.

energy from II at the rate a_1e_2 and II' absorbs energy at the rate a_2e_1 . Since I' and II' remain at the same temperature, we have

$$a_1 e_2 = a_2 e_1 \tag{14}$$

which implies

$$\frac{a_1}{a_2} = \frac{e_1}{e_2} \tag{15}$$

The ability of a body to emit radiation is proportional to its ability to absorb radiation. Now, let us assume I, I' are black bodies. Thus $a_1 = 1$ and therefore

$$e_1 = \frac{e_2}{a_2} \tag{16}$$

Since II, II' are not black bodies, we have $e_1 > e_2$ —thus a black body radiates energy faster than any other body. These results, as mentioned earlier were due to Kirchoff. He also noted all black bodies behave identically, in the sense that the nature of the radiation is dependent only on the temperature and the frequency.

What is the frequency (or wavelength) spectrum of this black body radiation? The experimental curve was known quite well. This is shown in Figure 5. The problem was in the theoretical understanding of this curve/ There were many independent attempts by various

physicists. Wilhelm Wien proposed that the curve could be understood as satisfying the formula

$$u(\nu) = \nu^3 g(\frac{\nu}{T}) \tag{17}$$

where $u(\nu)d\nu$ is the energy density and $u(\nu)$ is the energy density per spectral unit (frequency). He also showed that $\lambda_{max}T = b$ (a constant). The constant b, from experiments turned out to be 0.2898 cm-K. Wien further tried a fit using the formula (C and β are two dimensionful fitting parameters),

$$u(\nu) = C\nu^3 e^{-\beta\nu} \tag{18}$$

and it worked quite well for large ν (i.e. small λ). However, it was not clear why it worked and also it did not work in the low frequency domain.

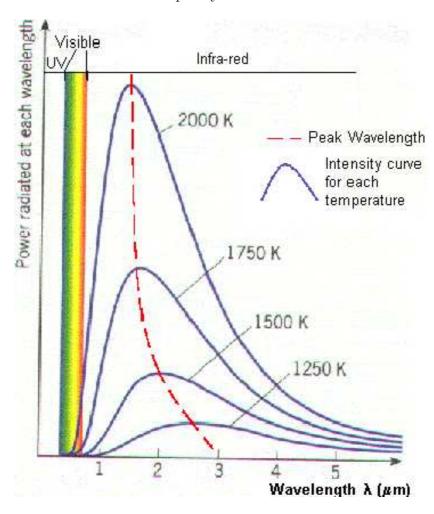


FIG. 5. Black-body spectrum

On the other front, Rayleigh and Jeans used classical electromagnetic theory to explain the

curve. They arrived at a formula,

$$u(\nu) = \frac{8\pi\nu^2}{c^3}kT\tag{19}$$

This seemed to work well in the **large wavelength** (small frequency) region but failed near small wavelengths (large frequencies).

Around 1900, Max Planck came up with the formula

$$u(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{20}$$

Magically, the formula fitted the curves perfectly with $h = 6.63 \times 10^{-34}$ J-s (Planck's constant).

Exercise: Show that Planck's constant h has the dimensions of angular momentum.

Exercise: Write down Planck's law in terms of λ (wavelength). Take $u(\nu)d\nu$ and convert ν to λ using $c=\nu\lambda$.

It is also quite easy to check that in the large wavelength limit (small frequency) one could expand the exponential below in a series and get back the Rayleigh-Jeans result. Similarly, when one takes the small wavelength limit one can ignore the minus one in the denominator and obtain Wien's result. Everything seemed to work and Planck was rather surprised. He then spend a couple of months trying to understand why this was happening. Using thermodynamics, he was able to show that one must **postulate** that the amount of radiation for each frequency came in discrete packets of energy, $h\nu$,— the so-called **light quanta**. If there are n quanta then the energy is $nh\nu$. This is known as the **light quantum hypothesis**. Planck's radiation law for black bodies was put on a firmer footing and correctly understood much later, only through the work of **Satyendra Nath Bose** in 1924 who proposed the famous **Bose-Einstein statistics** and the concept of the **boson**.

Photoelectric effect: Another experimental result which existed towards the end of the 19th century was the photoelectric effect. This was first observed by Hertz while he was doing his experiments to prove the existence of electromagnetic waves. Hertz noticed that if he used a shield to cut-off the ultraviolet wavelengths emitted in the spark, then, at the receiver coil, the length of the spark got reduced. The receiver coil obviously had metallic electrodes and it seemed that the ultraviolet frequency waves facilitated a longer and broader spark at the receiver. This was a mystery which required a solution. Later on,

many others, including Lenard, did experiments with metallic electrodes, by shining light (or ultraviolet radiation) on them and they also observed a similar effect. However, it was not until Einstein came into the scene that the photoelectric effect was understood. Einstein made the bold assertion that one needs to use the **Planck quantum hypothesis in order to understand the photoelectric effect**. Einstein insisted, quite correctly, that it was not possible to explain the results using standard electromagnetic theory. Einstein wrote down the famous photoelectric effect equation as

$$\frac{1}{2}mv^2 = h\nu - W\tag{21}$$

where, as is clear, he used the $E = h\nu$ hypothesis of Planck. W is the work function of the metal and is a material property. There are several important features of the photoelectric effect which we list below.

- Polished metal plates when irradiated emits electrons, not positive ions.
- Plates emit electrons provided the frequency of the incident light is above a certain frequency, called the threshold.
- The magnitude of current is dependent on the intensity of the incident light.
- The energy of the emitted photoelectron is independent of the energy of the source but depends on the frequency of the incident light.
- There is no time-delay between the arrival of the light wave and the emission of the photoelectron. The measured delay was found to be as small as 10^{-9} seconds. This fact could not be explained using electromagnetism.

Thus, we see that the photoelectric effect too (like black body radiation) also made use of Planck's quantum hypothesis and thereby provided further support to the particle nature of light. Later, Millikan did extensive experiments to confirm Einstein's photoelectric equation and the ensuing conclusions.

Compton scattering: Somewhat later, in 1923, Arthur Compton devised a scattering experiment using a thin metallic foil and an incident electromagnetic wave. He noticed that there were peaks at two different wavelengths in the scattered electromagnetic wave intensity and one of them was dependent on the angle of scattering of the e.m. wave. The other wavelength was the same as that of the incident wave. Once again this experiment could not be explained using classical electromagnetism and known facts about the scattering

of e.m. waves.

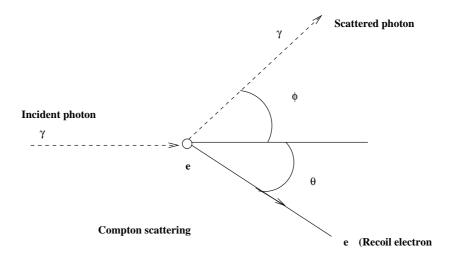


FIG. 6. Compton scattering

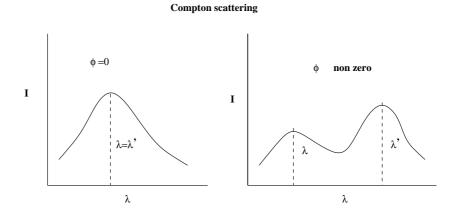


FIG. 7. I vs λ in Compton scattering for $\phi = 0$ and $\phi \neq 0$.

In fact, an understanding of Compton scattering requires the use of both the two new and revolutionary ideas that were born at the beginning of the 20th century – **special relativity** and quantum theory. Let us see how.

Look at the Figure 6. We will try to analyse the scattering event as an elastic scattering of particles—here the particles being the free electron and the incident photon. The first question is what are the energies and momenta of the incident photon and the free electron? To write them down we use the **relativistic relation** between the energy and momentum of a particle given as:

$$E^2 = p^2 c^2 + m_0^2 c^4 (22)$$

where p is magnitude of the momentum of a particle of **rest mass** m_0 . From this formula, we can easily see that the velocity of the particle will be

$$v = \frac{dE}{dp} = \frac{pc^2}{\sqrt{p^2c^2 + m_0^2c^4}} \tag{23}$$

Since v = c for the photon, we note that $m_0 = 0$ (for the photon) and therefore the energy of the photon is E = pc. Assuming the fact that the photon energy is $E = h\nu$ (Planck's hypothesis) we find that the momentum of the photon is $p = \frac{h\nu}{c}$. Also, from the energy formula, we have the rest (p = 0) energy of the electron as m_0c^2 . We make use of the above facts while analysing the kinematics of the scattering event. Using the conservation of energy we have

$$h\nu + m_0 c^2 = h\nu' + \sqrt{p^2 c^2 + m_0^2 c^4}$$
 (24)

where ν' is the energy of the scattered photon of a different wavelength and p is the momentum of the recoiling electron. If ϕ is the angle of scattering of the photon and θ the scattering angle of the recoil electron, we obtain, using momentum conservation along the horizontal and vertical directions,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + p\cos\theta\tag{25}$$

$$\frac{h\nu'}{c}\sin\phi = p\sin\theta\tag{26}$$

If we eliminate θ from these two equations we obtain

$$\left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - \frac{2h\nu\nu'}{c^2}\cos\phi = p^2 \tag{27}$$

On the other hand, from the conservation of energy, we get

$$\left(\frac{h\nu}{c} - \frac{h\nu'}{c} + m_0 c\right)^2 - m_0 c^2 = p^2 \tag{28}$$

Using the expression for p^2 from the energy conservation equation in the θ -eliminated equation from momentum conservation, we obtain,

$$c\frac{\nu - \nu'}{\nu \nu'} = \frac{h}{m_0 c} \left(1 - \cos \phi \right) \tag{29}$$

or, equivalently,

$$\lambda' - \lambda = \lambda_c \left(1 - \cos \phi \right) \tag{30}$$

where $\lambda_c = \frac{h}{m_0 c}$ is the **Compton wavelength** of the scatterer (here, the electron). The above formula tallies well with the experimental observations. Note the dependence of the wavelength λ' on ϕ (shown, qualitatively in Figure 7). The Compton wavelength of the electron is found to be 2.42×10^{-12} m.

The Compton scattering result provides support to two of the major revolutions in physics which took place at the beginning of the 20th century—special relativity and quantum theory. It is also another experimental result (apart from black body radiation and the photoelectric effect) which is understood using the **light quantum hypothesis**, i.e. the particle nature of electromagnetic waves.