1 A	If the electric field of a photon is represented by $\vec{E}(in\ SI\ units) = 2\hat{i}\ cos[(3y + 4z) - 2t]$ then the magnitude of the momentum of the photon is ---- $\hbar$ (SI Units). Ans: 5 Range: NA Hints: $p = \hbar\sqrt{3^2 + 4^2} = 5\hbar$	1 Marks	Part-A
1 B	If the electric field of a photon is represented by $\vec{E}(in\ SI\ units) = 4\hat{i}\ cos[(6y + 8z) - 2t]$ then the magnitude of the momentum of the photon is ---- $\hbar$ (SI Units). Ans: 10 Range: NA Hints: $p = \hbar\sqrt{6^2 + 8^2} = 10\hbar$	1 Marks	Part-A
1 C	If the electric field of a photon is represented by $\vec{E}(in\ SI\ units) = 5\hat{i}\ cos[(4y + 3z) - 2t]$ then the magnitude of the momentum of the photon is ---- $\hbar$ (SI Units). Ans: 5 Range: NA Hints: $p = \hbar\sqrt{4^2 + 3^2} = 5\hbar$	1 Marks	Part-A
2A	A beam of 400 nm light is incident on a metal with a work function 2 eV placed in the magnetic field of magnitude ‘B’. The most energetic electrons, emitted perpendicular to the field are bent in circular arcs of radius 10 cm. The value of ‘B’ (in µT) is (Write the answer correct upto two decimal places) (Take hc=1240 nm-eV, 1 eV= 1.6 x 10^{-19} J, m_e=9 x 10^{-31}kg) Ans: 35.2 Range: 35 to 36 Hints: $K = \frac{hc}{\lambda} - \phi = 1.76 \times 10^{-19} J \rightarrow p = \sqrt{2mK} = 5.63 \times 10^{-25}\ Kg\frac{m}{s} \rightarrow B = \frac{p}{re} \rightarrow$ 35.2 µT	2 Marks	Part-B
2B	A beam of 400 nm light is incident on a metal with a work function 2 eV placed in the magnetic field of magnitude ‘B’. The most energetic electrons, emitted perpendicular to the field are bent in circular arcs of radius 20 cm. The value of ‘B’ (in µT) is (Take hc=1240 nm-eV, 1 eV= 1.6 x 10^{-19} J, m_e=9 x 10^{-31}kg) (Write the answer correct upto two decimal places) Ans: 17.60 Range: 17 to 18 $K = \frac{hc}{\lambda} - \phi = 1.76 \times 10^{-19} J \rightarrow p = \sqrt{2mK} = 5.63 \times 10^{-25}\ Kg\frac{m}{s} \rightarrow B = \frac{p}{re}$ $\rightarrow$ 17.6 µT	2 Marks	Part-B
2C	A beam of 400 nm light is incident on a metal with a work function 2 eV placed in the magnetic field of magnitude ‘B’. The most energetic electrons, emitted perpendicular to the field are bent in circular arcs of radius 15 cm. The value of ‘B’ (in µT) is (Take hc=1240 nm-eV, 1 eV= 1.6 x 10^{-19} J, m_e=9 x 10^{-31}kg)	2 Marks	Part-B

	(Write the answer correct upto two decimal places) Ans: 23.5 Range: 23 to 24 $K = \frac{hc}{\lambda} - \phi = 1.76 \times 10^{-19} J \rightarrow p = \sqrt{2mK} = 5.63 \times 10^{-25}\ Kg\frac{m}{s} \rightarrow B = \frac{p}{re} \rightarrow$ 23.5 μT		
3A	When an electron is accelerated through a potential difference V, its de Broglie wavelength is $\lambda = \sqrt{\left(\frac{\alpha}{V}\right)}$ for non-relativistic speeds. If λ and V represent numerical values in angstrom and volt respectively, then the magnitude of α is (Write the answer correct up to two decimal places) (Take h = 6.63 x 10^{-34} J-s and m_e = 9 x10^{-31} kg) Ans: 149 Range: 147 to 151 Hints: $eV = \frac{mv^2}{2} = \frac{p^2}{2m} \rightarrow p = \sqrt{(2meV)} \rightarrow \lambda = \frac{h}{p} \rightarrow \lambda = \sqrt{\frac{1.49}{V}}\ nm \rightarrow \sqrt{\frac{149}{V}}\ A$	2 Marks	Part-B
3B	When an electron is accelerated through a potential difference V, its de Broglie wavelength is $\lambda = \frac{\alpha}{\sqrt{V}}$ for non-relativistic speeds. If λ and V represent numerical values in angstrom and volt respectively, then the magnitude of α is (Write the answer correct up to two decimal places) (Take h = 6.63 x 10^{-34} J-s and m_e = 9 x10^{-31} kg) Ans: 12.22 Range: 12 to 13 Hints: $eV = \frac{mv^2}{2} = \frac{p^2}{2m} \rightarrow p = \sqrt{(2meV)} \rightarrow \lambda = \frac{h}{p} \rightarrow \lambda = \sqrt{\frac{1.49}{V}}\ nm \rightarrow \alpha =$ 12.22	2 Marks	Part-B
3C	When an electron is accelerated through a potential difference V, its de Broglie wavelength is $\lambda = \frac{\alpha^2}{\sqrt{V}}$ for non-relativistic speeds. If λ and V represent numerical values in angstrom and volt respectively, then the magnitude of α is (Write the answer correct up to two decimal places) (Take h^2 = 3m_e kg.eV.nm^2 where m_e = 9 x 10^{-31} kg) Ans: 3.5 Range: 3 to 4 Hints: $eV = \frac{mv^2}{2} = \frac{p^2}{2m} \rightarrow p = \sqrt{(2meV)} \rightarrow \lambda = \frac{h}{p} \rightarrow \lambda = \sqrt{\frac{1.49}{V}}\ nm \rightarrow \alpha =$ 3.5	2 Marks	Part-B
4A	Davisson and Germer experiment is repeated with a crystal that has the spacing between Bragg's planes is 0.08 nm. First maximum is seen at θ = 60° for a monochromatic beam of electrons. The wavelength (in nm) associated with the scattered electron is (Write the answer correct up to two decimal places) Ans: 0.14	1 Marks	Part-A

	Range: 0.13 to 0.15 Hints: $2\phi + \theta = \pi; \lambda = \frac{2d}{n}\sin(\phi) = \frac{2d}{n}\cos\left(\frac{\theta}{2}\right) \rightarrow \lambda = 0.138\, nm$		
4B	Davisson and Germer experiment is repeated with a crystal that has the spacing between Bragg's planes is 0.06 nm. First maximum is seen at θ = 60° for a monochromatic beam of electrons. The wavelength (in nm) associated with the scattered electron is (Write the answer correct upto two decimal places) Ans: 0.10 Range: 0.08 to 0.12 Hints: $2\phi + \theta = \pi; \lambda = \frac{2d}{n}\sin(\phi) = \frac{2d}{n}\cos\left(\frac{\theta}{2}\right) \rightarrow \lambda = 0.103\, nm$	1 Marks	Part-A
4C	Davisson and Germer experiment is repeated with a crystal that has the spacing between Bragg's planes is 0.1 nm. One maximum is seen at θ = 60° for a monochromatic beam of electrons. The wavelength (in nm) associated with the scattered electron is (Write the answer correct upto two decimal places) Ans: 0.17 Range: 0.15 to 0.19 Hints: $2\phi + \theta = \pi; \lambda = \frac{2d}{n}\sin(\phi) = \frac{2d}{n}\cos\left(\frac{\theta}{2}\right) \rightarrow \lambda = 0.173\, nm$	1 Marks	Part-A
5A	If the expression $[\hat{x}^5, \hat{p}_x]$ is simplified in terms of $\hat{x}$, then the power of $\hat{x}$ is Ans: 4 Range: NA Hint: $[\hat{x}^n, \hat{p}_x] = i\hbar n\hat{x}^{n-1}$	1 Marks	Part-A
5B	If the expression $[\hat{x}^4, \hat{p}_x]$ is simplified in terms of $\hat{x}$, then the power of $\hat{x}$ is Ans: 3 Range: NA Hint: $[\hat{x}^n, \hat{p}_x] = i\hbar n\hat{x}^{n-1}$	1 Marks	Part-A
5C	If the expression $[\hat{x}^3, \hat{p}_x]$ is simplified in terms of $\hat{x}$, then the power of $\hat{x}$ is Ans: 2 Range: NA Hint: $[\hat{x}^n, \hat{p}_x] = i\hbar n\hat{x}^{n-1}$	1 Marks	Part-A
6A	A particle of mass *m* is confined to a one-dimensional well 0 < x < L. At t = 0 its wave function is $$\psi(x,0) = A\, sin\left(\frac{\pi x}{L}\right)$$ where A is the normalisation constant. What is the probability that the particle is found between L/2 and 3L/4 at a later time t_0 ? Ans: 0.41 Range: 0.38 to 0.44 The normalizat ion constnat, $A^2\int_0^L \sin^2\left(\frac{\pi x}{L}\right)dx = 1 \Rightarrow A = \sqrt{\frac{2}{L}};$ $6(a)\ \ P = \frac{2}{L}\int_{L/2}^{3L/4} \sin^2\left(\frac{\pi x}{L}\right)dx = 0.41$	2 Marks	Part-B

6B	A particle of mass *m* is confined to a one-dimensional well 0 < x < L. At t = 0 its wave function is $$\psi(x,0) = A\,sin\left(\frac{\pi x}{L}\right)$$ where A is the normalisation constant. What is the probability that the particle is found between L/4 and L/2 at a later time t_0 ? Ans: 0.41 Range: 0.38 to 0.44 The normalizat ion constnat, $A^2\int_0^L \sin^2\left(\frac{\pi x}{L}\right)dx = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$; $6(b)\ P = \frac{2}{L}\int_{L/4}^{L/2} \sin^2\left(\frac{\pi x}{L}\right)dx = 0.41$	2 Marks	Part-B
6C	A particle of mass *m* is confined to a one-dimensional well 0 < x < L. At t = 0 its wave function is $$\psi(x,0) = A\,sin\left(\frac{\pi x}{L}\right)$$ where A is the normalisation constant. What is the probability that the particle is found between L/4 and 3L/4 at a later time t_0 ? Ans: 0.82 Range: 0.78 to 0.86 The normalizat ion constnat, $A^2\int_0^L \sin^2\left(\frac{\pi x}{L}\right)dx = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$; $6(c)\ P = \frac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\frac{\pi x}{L}\right)dx = 0.82$	2 Marks	Part-B
7A	An electron is confined in the ground state in a one-dimensional box of width $10^{-10}\ meter$. Its energy is 20 eV. The energy of the electron in its first exited state is Ans: 80 eV Range: NA Ans : $E_3 = 2^2 E_1 = 4 X 20$ e V = 80 eV	1 Marks	Part-A
7B	An electron is confined in the ground state in a one-dimensional box of width $10^{-10}\ meter$. Its energy is 20 eV. The energy of the electron in its second exited state is Ans: 180 eV Range: NA Ans : $E_3 = 3^2 E_1 = 9 X 20$ e V = 180 eV	1 Marks	Part-A
7C	An electron is confined in the ground state in a one-dimensional box of width $10^{-10}\ meter$. Its energy is 30 eV. The energy of the electron in its first exited state is Ans: 120 eV	1 Marks	Part-A

<table>
<tr><td></td><td>Range: NA
Ans : $E_3 = 2^2 E_1 = 4 \text{ X } 30 \text{ e V} = 120 \text{ eV}$</td><td></td><td></td></tr>
<tr><td>9A</td><td>A particle of mass m, which moves freely inside an infinite potential well of length a, has the following initial wave function at t =0:
$$\psi(x,0) = \frac{A}{\sqrt{a}} sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} sin\left(\frac{5\pi x}{a}\right)$$
In an experiment to measure the energy of the system, the value of the energy comes out to be $E = \frac{h^2}{8ma^2}$. The corresponding probability associated with the outcome of the measurement is : (Please provide exact value upto 2 decimal places)

Ans: 0.6
Range: NA

$\phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$
They are orthonormal hence $\langle \phi_n | \phi_m \rangle = \int \phi_n^* \phi_m dx = \delta_{nm}$.
Convenient to write $\psi(x,0)$ in terms of $\phi_n(x)$
$\therefore \psi(x,0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} \sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$.
$= \frac{A}{\sqrt{2}} \phi_1(x) + \sqrt{\frac{3}{10}} \phi_3(x) + \frac{1}{\sqrt{10}} \phi_5(x)$.
Since $\psi(x,0)$ must be normalised $\langle \psi | \psi \rangle = 1$
$\therefore \frac{A^2}{2} + \frac{3}{10} + \frac{1}{10} = 1 \rightsquigarrow A = \sqrt{\frac{6}{5}}$.
$\therefore \psi(x,0) = \sqrt{\frac{3}{5}} \phi_1(x) + \sqrt{\frac{3}{10}} \phi_3(x) + \sqrt{\frac{1}{10}} \phi_5(x)$

$\therefore$ For $E_1 \rightsquigarrow P(E_1) = |\langle \phi_1 | \psi \rangle|^2 = 3/5$
$E_3 \rightsquigarrow P(E_3) = |\langle \phi_3 | \psi \rangle|^2 = 3/10$
$E_5 \rightsquigarrow P(E_5) = |\langle \phi_5 | \psi \rangle|^2 = 1/10$</td><td>2 Marks</td><td>Part-B</td></tr>
<tr><td>9B</td><td>A particle of mass m, which moves freely inside an infinite potential well of length a, has the following initial wave function at t =0:
$$\psi(x,0) = \frac{A}{\sqrt{a}} sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} sin\left(\frac{5\pi x}{a}\right)$$
In an experiment to measure the energy of the system, the value of the energy comes out to be $E = \frac{9h^2}{8ma^2}$. The corresponding probability associated with the outcome of the measurement is : (Please provide exact value upto 2 decimal places)

Ans: 0.3
Range: NA</td><td>2 Marks</td><td>Part-B</td></tr>
<tr><td>9C</td><td>A particle of mass m, which moves freely inside an infinite potential well of length a, has the following initial wave function at t =0:</td><td>2 Marks</td><td>Part-B</td></tr>
</table>

	$$\psi(x,0) = \frac{A}{\sqrt{a}} sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} sin\left(\frac{5\pi x}{a}\right)$$ In an experiment to measure the energy of the system, the value of the energy comes out to be $E = \frac{25h^2}{8ma^2}$. The corresponding probability associated with the outcome of the measurement is : (Please provide exact value upto 2 decimal places) Ans: 0.1 Range: NA		
10	The pair is not an orthonormal wavefunction: $$(i).\ \psi_1(x) = e^{-ax^2};\ (ii).\ \psi_2(x) = xe^{-ax^2};$$ $$(iii).\ \psi_3(x) = x^2e^{-ax^2};(iv).\ \psi_4(x) = x^3e^{-ax^2}$$ Choices: (a) (i),(ii) (b) (i),(iii) (c) (ii),(iii) (d) (i),(iv) Ans: (b) Range : NA Ans: The integral of orthonormal functions is zero. Check the pair for even function. The integral of the even function is non-zero. Or You can calculate the overlapping integral of the pairs from -∞ to ∞	1 mark	Part A
11	The values of A and B for the given wavefunctions are $$\psi_1(x) = Ae^{-4x}\ \forall\ x > 0;\ \psi_2(x) = Be^{-4x^2}$$ Ans : A=2.83; Range: 2.80 to 2.85 B=1.26; Range: 1.22 to 1.30 Ans: $\psi_1(x) = Ae^{-4x}\ \forall\ x > 0;$ $\int_0^\infty A^2e^{-8x}dx = 1 \Rightarrow A^2 \left.\frac{e^{-8x}}{-8}\right\vert_0^\infty = 1;$ $A^2 = 8; \Rightarrow A = 2\sqrt{2} = 2.83$ $\psi_1(x) = Be^{-4x^2};$ $\int_0^\infty B^2e^{-8x^2}dx = 1 \Rightarrow B^2\sqrt{\frac{\pi}{8}} = 1;$ $B^2 = \sqrt{\frac{8}{\pi}}; \Rightarrow B = 1.26$	2 marks	Part B

12	Which of the following functions would make satisfactory wavefunctions $(a).\ \psi_1(x) = Ae^{a\|x\|};\ (b).\ \psi_2(x) = Ae^{-ax^2}e^{-\frac{i}{\hbar}Et};$ $(c).\ \psi_3(x) = \frac{A}{x-4}e^{-ax^2};(d).\ \psi_4(x) = Ae^{-ax}e^{-\frac{i}{\hbar}Et}$ Ans : (b) Range: NA Sol: $(a).\ \psi_1(x) = Ae^{a\|x\|};$ function diverges at $x = \pm\infty$ $(b).\ \psi_2(x) = Ae^{-ax^2}e^{\frac{i}{\hbar}Et}$;function is well behaved for all x $(c).\ \psi_3(x) = \frac{A}{x-4}e^{-ax^2}$;function diverge at $x = 4$ $(d).\ \psi_4(x) = Ae^{-ax}e^{\frac{i}{\hbar}Et}$;function diverges at $x = -\infty$	1 mark	Part A
13	The wavefunction of a particle moving in the x-direction is $\psi(x) = Ax(7-x)\ \ \forall\ 0 < x < 7$ $= 0;$ elsewhere The value of Δx is Ans: 1.32 Range: 1.25 to 1.40 Sol: $\langle x\rangle = \int_{-\infty}^{\infty} x\|\psi\|^2\,\mathrm{d}x = \int_0^L x\frac{30}{L^5}x^2(L-x)^2\,\mathrm{d}x$ $= \frac{30}{L^5}\int_0^L (L^2x^3 - 2Lx^4 + x^5)\,\mathrm{d}x = 30L\left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right) = \frac{L}{2}$ $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2\|\psi\|^2\,\mathrm{d}x = \int_0^L x^2\frac{30}{L^5}x^2(L-x)^2\,\mathrm{d}x$ $= \frac{30}{L^5}\int_0^L (L^2x^4 - 2Lx^5 + x^6)\,\mathrm{d}x = 30L^2\left(\frac{1}{5} - \frac{2}{6} + \frac{1}{7}\right) = \frac{2L^2}{7}$ Ans: $A = \sqrt{\frac{30}{L^5}}$; $<x> = L/2;\ <x^2> = 2\frac{L^2}{7};$ $\Delta x = \sqrt{<x>^2 - <x^2>} = \sqrt{7}/2 = 1.32$	2 marks	Part B
14	An electron is free between 0 and 7 nm, defined through a given wavefunction $\psi(x) = Ax(7-x)\ \ \forall\ 0 < x < 7$ $= 0;$ elsewhere The average energy (<E>) is Ans: 7.8 meV Range : 7.5 meV to 7.9 meV Sol:	2 marks	Part B

	$= \int_{-\infty}^{\infty} \psi^* \left(\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \right) \mathrm{d}x = \frac{30}{L^5} \frac{-\hbar^2}{2m} \int_0^L x(L-x) \frac{\partial^2}{\partial x^2} (x(L-x))\, \mathrm{d}x$ $= \frac{30}{L^5} \frac{\hbar^2}{m} \int_0^L x(L-x)\, \mathrm{d}x = \frac{30}{L^5} \frac{\hbar^2}{m} L^3 \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{5\hbar^2}{mL^2}$ $< E >= \int_{-\infty}^{+\infty} \psi^*(x) \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \psi(x)\, dx$ $= \frac{30}{7^5} \left(-\frac{\hbar^2}{2m} \right) \int_0^7 x(7-x) \frac{\partial^2}{\partial x^2} x(7-x)\, dx$ $= \frac{5\hbar^2}{m7^2} = \frac{5 \times (1.055 \times 10^{-34})^2}{9.11 \times 10^{-31} \times 49 \times 10^{-18}} \frac{1eV}{1.6 \times 10^{-19}} = 7.8m\,\mathrm{eV}$		