Report of Homework of Mattia Sanchioni

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In this homework we have to apply the techniques learned during the lectures in order to complete some tasks.
Given two images of two different cars we have to find some important features such edges, ellipses or symmetric point. With this information, later, we calibrate the camera (zero-screw, but not natural), localize it and find the position of some points.

Let's analyze tasks point-wise.

Extraction of image features

First of all images are uploaded, the photo used in the homework is '*Image1*'.

After that it is transformed in grey scaled:

```
% both transformations are equivalent
rgb2gray(image1);
0.299*images(:,:,1) + 0.587*images(:,:,2) + 0.114*images(:,:,3);
```

Now we can start with the first point in which we have to select the most important features useful for the following points.

The approach, that I have used to extract information from image, is to select a restricted area in image.
In order to do this I wrote the function `selectRegion()` that, given an image, allows to select a section and return the selected region and the coordinates of the rectangle, necessaries to translate the features selected in the region and then visualize in the original image.

After that the selection is derived for having a better visualization of the edges. During the fourth laboratory professor Boracchi showed us the code to make edge detection. I used this code adding the possibility to chose different threshold such that binary or hard threshold.
The edge detection is implemented in the function `findEdges()` in which it is possible to choose *binary, hard* or *Canny* thresholds.

All these functions are used in `findConic()` with which it is possible to detect the conics that represent the wheels. The detection is made with 5 points, because the automatic implemented in `findEllipses()` requires some filters applied to the image as erosion and dilation (not done due to lack of time), because it works well but, because of the noise, a lot of ellipses are detected but not the wheels.

To calculate the conic matrix from 5 points belonging the conic, we used the right null space that is the vector that right multiplied return the null vector. In my code the matrix A represents the system of equations of points belonging the conic.

$$x^T * C * x = 0$$

$$A * N = \underline{0}$$

A is a 5x6 matrix, and *N* is the RNS of A ($N = RNS(A)$), more precisely it is a vector with the coefficients that satisfies this equation:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

$$C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

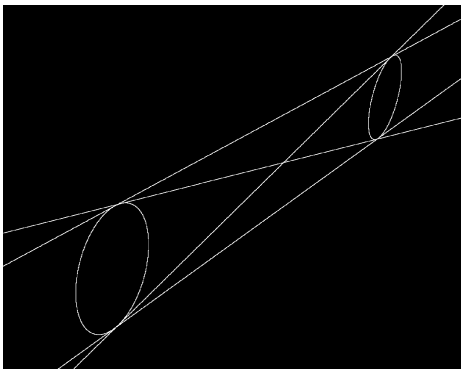
To give the possibility to visualize on the original image the conics just detected I implemented the function `fromConicToProfile()` that return a binary image with all zero except the points of the conic. Moreover `showProfileOpt()` return ad image as the original except in the point of the conic that are white. I optimized this function reducing its computational time of a tenth. Originally I use two *for loops* linked and spent 3/4 seconds, after changing approach I used matrix and logical operations that are more efficient and optimized.

Determine the ratio between diameter and wheel-to-wheel distance

Using the C matrices calculated in the previous point, I calculated the lines tangent both conics. To do this I used the dual conic, the set of line such that

$$\underline{l} \cdot C^* \cdot \underline{l}$$

$$\begin{cases} \underline{l} \cdot C_1^* \cdot \underline{l} \\ \underline{l} \cdot C_2^* \cdot \underline{l} \end{cases} \Longrightarrow 4 \text{ lines}$$



After that I have to calculate the back transformation matrix necessary to calculate the ratio between diameter and distance wheel-to-wheel.

As seen during a lecture we are able to calculate the matrix with the image of two pair of lines and a conic.

First of all I calculated the pair of parallel lines `line1 & line2` and `line3 & line4`. Also from line we find vanishing points that represent the direction of parallel lines, respectively `vpoint1 & vpoint2`.

Secondly I found the line at infinity, that is the set of vanishing points.

Moreover, we studied that every conic intersect the line at infinity in two points `I, J`, called circular point, because every conic intersect the line at infinity in this two point.

$$I = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} \qquad J = \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$$

So, I calculate the image of circular points:

$$\begin{cases} \underline{l'_\infty}\,\underline{x} = 0 \\ \underline{x}^T C \underline{x} = 0 \end{cases} \Longrightarrow I', J'$$

The image of absolute conic at infinity is defines as:

$$C^{*'}_\infty = I' J'^T + I'^T J'$$

With the SVD decomposition we are able to have three matrices `U, S, V` such that:

$$svd(C^{*'}_\infty) = U * S * V'$$

In addition we know that we can calculate the image of absolute conic at infinity with the similarity `Hr`. Knowing this, and the transformation rule of conic we can write

$$C^*_\infty = Hr \cdot C^{*'}_\infty \cdot Hr^T$$
$$C^{*'}_\infty = Hr^{-1} \cdot C^*_\infty \cdot Hr^{-T}$$
$$C^*_\infty = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}$$
$$Hr^{-1} = U \qquad C^*_\infty = S$$

In my case the matrix S isn't the matrix of absolute conic at infinity, so I decomposed it into three matrices

$$S = T\, C_{\infty}^{*\prime}\, T$$
$$Hr = (UT)^{-1} e$$

Applying the back transformation to the intersection points `v1, v2` (intersection between tangents and conics, found previously), we can calculate the diameter of wheels and the distance between wheel-to-wheel (from the center of the wheels).

```
>> ratio = diameter/distancewtow

ratio =

    0.1813
```

Camera calibration

In this point I used the previous feature to calibrate the camera, determining the matrix K. To do this, it's necessary the image of absolute conic

$$\Omega_{\infty} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

Since the camera is not natural, and it is zero-skew the matrix has 4 parameters, therefore 4 constraints.
The angle between two directions can be calculate as follows:

$$\cos\theta = \frac{d_1^T d_2}{\sqrt{d_1^T d_1}\sqrt{d_2^T d_2}}$$
$$d_1^T d_2 = v_1^T\, \omega\, v_2$$
$$if\ \theta = 90^{\circ} \qquad v_1^T\, \omega\, v_2 = 0$$

I have already two vanishing points, finding another vanishing point I get three constraints, I need another one.
The image of absolute conic is the set of circular points, since I have already calculate a pair of circular points I used them as other constraint.

$$\begin{cases} vp_1^T\, \omega\, vp_1 & 1 \\ vp_2^T\, \omega\, vp_2 & 2 \\ vp_3^T\, \omega\, vp_3 & 3 \\ I^T\, \omega\, I & 4 \end{cases}$$

Fur equations in four parameter. In this way I obtained ω.

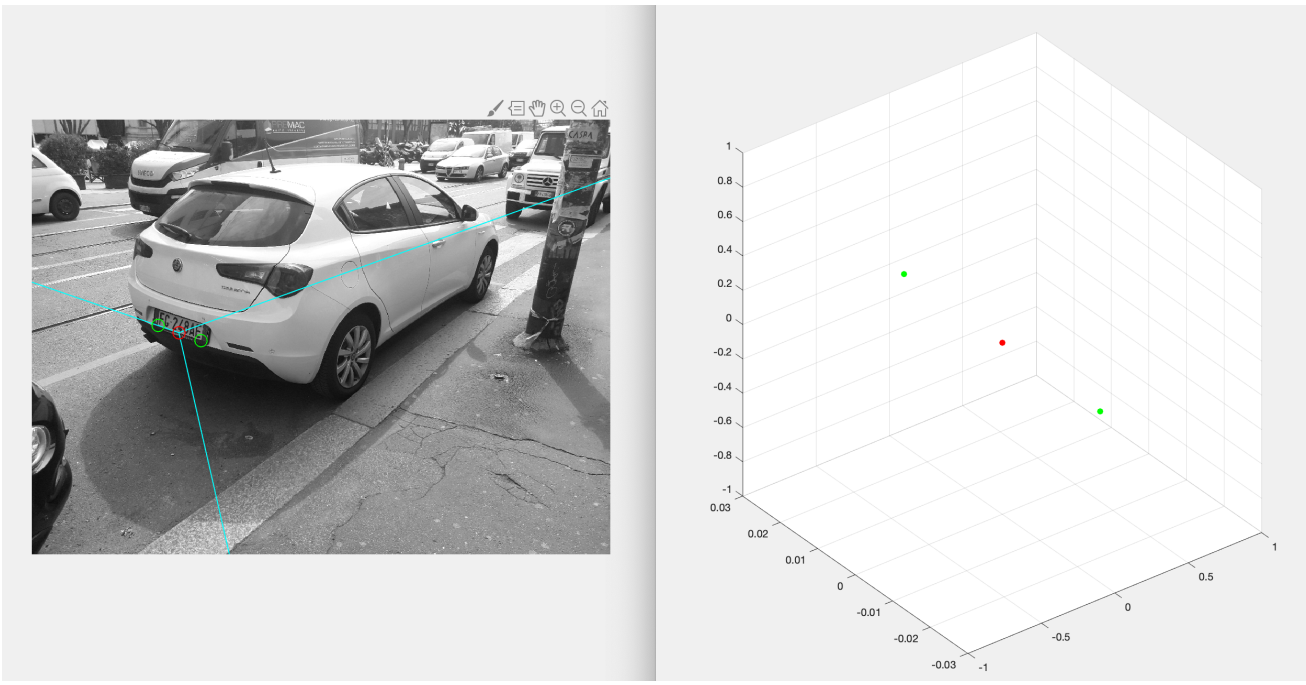
$$\omega^* = \omega^{-1} = KK^T$$

With Cholesky decomposition, return, from *image of absolute conic*, an upper triangular matrix K, the camera calibration.

```
>> K

K =

   1.0e+03 *

    2.9205         0    2.3052
         0    2.9749    2.1227
         0         0    0.0010
```

3D position reconstruction

Now let's fix a reference frame on the midpoint of the lower side of the car license plate.



Using symmetric pairs of relative points, I can map and find their 3D position. In order to do this I computed the back transformations of plane XY, XZ and YZ. They are necessary to calculate the coordinates of symmetric points.

Selected a pair of symmetric points, I can use them to find the midpoint between them using the vanishing point Y (the point on the left of the image). The theory says that the cross ratio remains invariant under a linear projective mapping.

$$CR = \frac{x_1 - y}{x_1 - z} / \frac{x_2 - y}{x_2 - z}$$
$$CR(a, b, c, d) = CR(a', b', c', d')$$

If one of this point goes to infinity the cross ration is equal to -1.

$$\frac{x_1 - y}{x_1 - z} / \frac{x_2 - y}{x_2 - z} = -1$$

$$\begin{aligned} y, z &\quad where\ ever \\ x_1 &\quad midpoint \\ x_2 &\quad \infty \end{aligned}$$

y and z are the symmetric points, x2 is the vanishing point, now I found x1.

$$x_1 = \frac{yx_2 + zx_2 + 2yz}{2x_2 - z - y}$$

Found the midpoint I set it as center of reference axes. The coordinates of p1 and p2 (y and z in the previous formulas) are calculated as the distance from the center to p1 and p2 in YZ plane.

After taht selection manually pair of symmetric points they are 3d positioned. Similar to what done previously. To find y coordinate compute the distances between points P1 and P2 to midpoint in YZ plane. Moreover the z coordinate is the distance between midpoint and the center in YZ plane. Finally x coordinate is the distance between center and midpoint in XY plane.

Camera localization