Parameter estimation

Sol 1.

Consider a random sample $(X_1, X_2, X_3 - - X_n)$

$\mu = \theta_1$ (mean) $\qquad n^2 = \theta_2$ (variance)

likelihood f^n $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$

to max log

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right]$$

(i) $$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_1} = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}$$

mean

(ii) $$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_2} = \sum_{i=1}^{n} \left[-\frac{1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

↳ Variance

Sol 2 binomial distribution $B(n, \theta)$

$p = \theta \quad , \quad q = 1 - \theta$

Pmf ✓

$$f(x; n, \theta) = {}^nC_x \, \theta^x (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^nC_{x_i} \, \theta^{x_i} (1-\theta)^{n-x_i}$$

taking log

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln {}^nC_{x_i} + x_i \ln\theta + (n-x_i) \ln(1-\theta) \right]$$

diff. w.r.t. θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

find θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{(1-\theta)x_i - \theta(n-x_i)}{\theta(1-\theta)} \right] = 0$$

$$\sum_{i=1}^{n} (1-\theta)x_i - (n-x_i)\theta = 0$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot m$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}}$$