

# Step-Down/ Type-A Chopper - Detailed Analysis

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## Buck/ Step-Down Converter

1. Reduce vltg level at op side.  $V_o \leq V_s$   
@ $T_{on}$ ,  $V_o = V_s$ .  
@ $T_{off}$ ,  $V_o = 0$  as load current flow by FD. So load terminals get shorted.  
 $V_o = \delta V_s \Rightarrow$  Load vltg is indpdt of load current.
2. So load vltg can be varied by varn of  $\delta$ .

## R-Load

When sw is close,  $v_o = V_s$ .

When sw is open,  $v_o = 0$ .

$$i_o = \frac{V_o}{R}$$

Diode is useless here.

## RL Load

When sw is close,  $V_o = V_s$ . FD is inactive. Indt is charged in this state.

When sw is open,  $V_o = 0$ . Indt discharge by diode path.

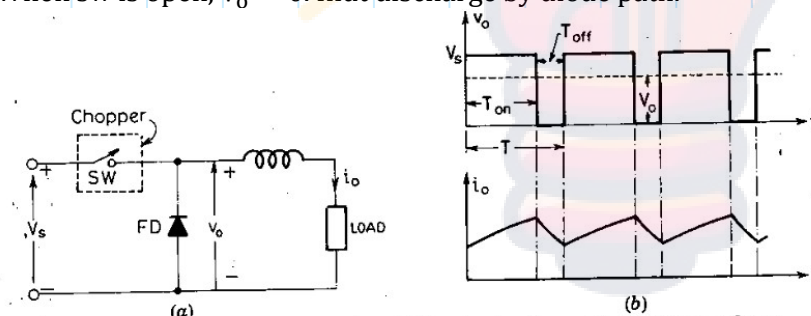


Fig. 7.2 (a) Elementary chopper circuit and (b) output voltage and current waveforms.

[Cont Load Current]

## Avg Load Voltage

$$V_o = \frac{1}{T} \int_0^T v_o dt = \frac{1}{T} \int_0^{T_{on}} V_s dt = \frac{V_s T_{on}}{T} = \delta V_s$$

So varying  $\delta$ , we can step down op vltg across load.

sw is always OFF  $\Rightarrow \delta = 0 \Rightarrow V_o = 0$

sw is always ON  $\Rightarrow \delta = 1 \Rightarrow V_o = V_s$

$\Rightarrow V_o \in [0, V_s]$

## Avg Load Current

$$I_o = \frac{V_o}{R} = \frac{\delta V_s}{R}$$

[R-Load]

## RMS Op Vltg

$$V_{o(rms)} = \sqrt{\frac{1}{T} \int_0^{T_{on}} V_s^2 dt} = \sqrt{\frac{V_s^2 T_{on}}{T}} = V_s \sqrt{\delta}$$

For lossless converter, ip power et op power given by:

$$P_i = \frac{1}{T} \int_0^{\delta T} V_o i dt = \frac{1}{T} \int_0^{\delta T} \frac{V_o^2}{R} dt = \frac{1}{T} \frac{V_s^2}{R} (\delta T) = \frac{\delta V_s^2}{R}$$

Effective Ip Rst seen by power source P

$$R_i = \frac{V_s}{I_o} = \frac{V_s}{\delta V_s / R} = \frac{R}{\alpha}$$

### SS Time-Domain Analysis of Type-A Chopper with RLE-Load

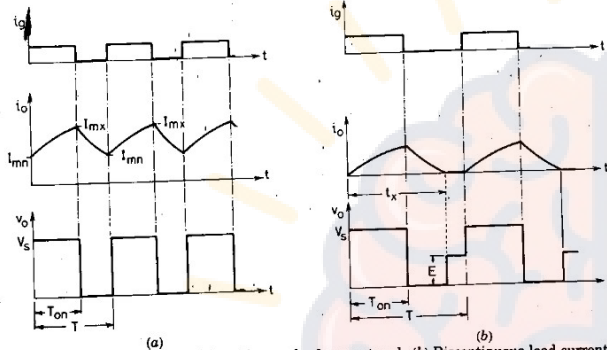


Fig. 7.12. Type-A chopper (a) continuous load current and (b) Discontinuous load current.

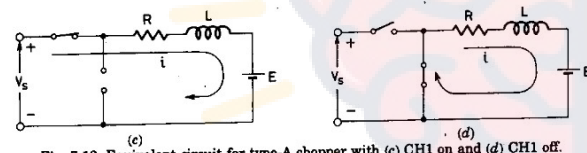


Fig. 7.12. Equivalent circuit for type-A chopper with (c) CH1 on and (d) CH1 off.

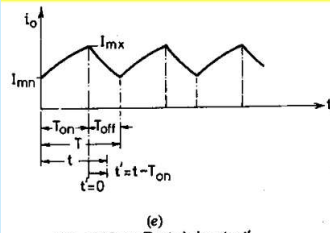


Fig. 7.12 (e) Pertaining to  $t'$ .

For RLE-load,

@  $t \in [0, T_{on}]$ , sw is on

$$\Rightarrow V_s = Ri + \frac{L di}{dt} + E$$

@  $t \in [T_{on}, T]$  or  $t' \in [0, T - T_{on} = T_{off}] \Rightarrow t' = t - T_{on}$   
sw is off, load current flow by FD.

$$\Rightarrow 0 = Ri + \frac{L di}{dt} + E$$

2 boundary values are  $I_{mn}$ ,  $I_{mx}$ .

$$\begin{aligned} RI + L[sI - I_{mn}] &= \frac{V_s - E}{s} \\ I(s) &= \frac{V_s - E}{s(R + Ls)} + \frac{LI_{mn}}{R + Ls} \\ &= \frac{V_s - E}{Ls(s + \frac{R}{L})} + \frac{I_{mn}}{s + \frac{R}{L}} \\ \Rightarrow i(t) &= \frac{V_s - E}{R} (1 - e^{-\frac{Rt}{L}}) + I_{mn} e^{-\frac{Rt}{L}} \end{aligned}$$

$$RI + L[sI - I_{mx}] = -\frac{E}{s}$$

...

$$\Rightarrow i(t') = -\frac{E}{R} (1 - e^{-\frac{Rt'}{L}}) + I_{mx} e^{-\frac{Rt'}{L}}, t \in [T_{on}, T]$$

just replace  $t$  by  $t'$ ,  $V_s = 0$  and  $I_{mn}$  by  $I_{mx}$

$$\text{Let } \left[ e^{-\frac{T_{on}}{\tau}} = m, e^{-\frac{T_{off}}{\tau}} = n, \tau = \frac{L}{R} \Rightarrow mn = e^{-\frac{T}{\tau}} \right]$$

@  $t = T_{on}$ ,  $i(t) = I_{mx}$

$$\Rightarrow I_{mx} = \frac{V_s - E}{R} (1 - e^{-\frac{R}{L} T_{on}}) + I_{mn} e^{-\frac{R}{L} T_{on}}$$

@  $t' = T_{off}$ ,  $i(t) = I_{mn}$

$$\Rightarrow I_{mn} = -\frac{E}{R} (1 - e^{-\frac{R}{L} T_{off}}) + I_{mx} e^{-\frac{R}{L} T_{off}}$$

mn

mx

$$\left[ \begin{aligned} &= \frac{V_s - E}{R}(1 - m) + mI_{mn} \\ &= -\frac{E}{R}(1 - n) + I_{mx}n \end{aligned} \right]$$

Put  $I_{mn}$  in  $I_{mx}$  and solve for  $I_{mx}$ , then find  $I_{mn}$  from obtd eqn:

$$\Rightarrow I_{mx} = \frac{V_s - E}{R}(1 - m) + m \left[ -\frac{E}{R}(1 - n) + I_{mx}n \right]$$

$$\Rightarrow I_{mx} = -\frac{E}{R} + \frac{V_s}{R} \cdot \frac{(1 - m)}{1 - mn} = -\frac{E}{R} + \frac{1 - e^{-\frac{T_{on}}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

and

$$\Rightarrow I_{mn} = -\frac{E}{R}(1 - n) + \left[ -\frac{E}{R} + \frac{V_s}{R} \cdot \frac{(1 - m)}{1 - mn} \right] n$$

$$\Rightarrow I_{mn} = -\frac{E}{R} + \frac{n(1 - m)}{1 - mn} = -\frac{E}{R} + \frac{1 - \frac{1}{m}}{1 - \frac{1}{mn}} = -\frac{E}{R} + \frac{1 - e^{-\frac{T_{on}}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

$$\text{If sw cont conduct, } T_{on} = T \Rightarrow I_{mx} = I_{mn} = \frac{V_s - E}{R}$$

### Steady State Ripple

$$\text{Ripple Current} = \Delta I = I_{mx} - I_{mn} = \frac{V_s}{R} \cdot \frac{(1 - m)}{1 - mn} - \frac{E}{R} - \left[ \frac{V_s}{R} \cdot \frac{n(1 - m)}{1 - mn} - \frac{E}{R} \right]$$

$$= \frac{V_s}{R} \cdot \frac{(1 - n)(1 - m)}{1 - mn} = \frac{V_s}{R} \left[ \frac{(1 - e^{-\frac{T_{on}}{\tau}})(1 - e^{-\frac{T_{off}}{\tau}})}{1 - e^{-\frac{T}{\tau}}} \right]$$

$$= \frac{V_s}{R} \left[ \frac{(1 - e^{-\frac{\delta T}{\tau}})(1 - e^{-(1-\delta)\frac{T}{\tau}})}{1 - e^{-\frac{T}{\tau}}} \right]$$

$$\text{PU Ripple Crnt} = \frac{I_{mx} - I_{mn}}{V_s/R} = \frac{(1 - e^{-\frac{\delta T}{\tau}})(1 - e^{-(1-\delta)\frac{T}{\tau}})}{1 - e^{-\frac{T}{\tau}}}$$

This ripple current is indepdt of E.

With L inc,  $\tau = \frac{L}{R}$  inc and  $\frac{T}{\tau}$  dec, so pu ripple current dec.

Ripple current has max value  $\Delta I_{mx}$  @  $\delta = 0.5$ . Put  $\frac{T}{\tau} = x$

$$\Delta I_{mx} = \frac{V_s}{R} \left[ \frac{(1 - e^{-0.5x})(1 - e^{-0.5x})}{1 - e^{-x}} \right] = \frac{V_s}{R} \left[ \frac{(1 - e^{-0.5x})(1 - e^{-0.5x})}{(1 - e^{-0.5x})(1 + e^{-0.5x})} \right]$$

$$= \frac{V_s}{R} \left[ \frac{(1 - e^{-0.5x})}{(1 + e^{-0.5x})} \right] = \frac{V_s}{R} \tanh \frac{x}{4} = \frac{V_s}{R} \tanh \frac{T}{4\tau} = \frac{V_s}{R} \tanh \frac{R}{4fL}$$

$$\Rightarrow \Delta I_{mx} = \frac{V_s}{R} \tanh \frac{R}{4fL}$$

$$\text{If } 4fL \gg R \Rightarrow \tanh \frac{R}{4fL} = \frac{R}{4fL} \Rightarrow \Delta I_{mx} = \frac{V_s}{4fL}$$

$\Rightarrow \Delta I_{mx}$  is iprop to chopping freq and L

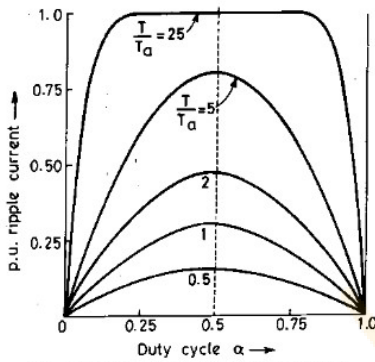


Fig. 7.13. Per unit ripple current as a function of  $\alpha$  and  $T/T_a$ .

### Limit of Continuous Conduction/ Critical Current

$$I_{mn} = 0 \Rightarrow \frac{V_s}{R} \cdot \left[ \frac{e^{\frac{T_{on}}{\tau}} - 1}{e^{\frac{T}{\tau}} - 1} \right] - \frac{E}{R} = 0 \Rightarrow \frac{T_{on}}{\tau} = \ln \left( 1 + \frac{E}{V_s} [e^{\frac{T}{\tau}} - 1] \right)$$

$$\Rightarrow \frac{T_{on}}{T} = \delta = \frac{\tau}{T} \ln \left( 1 + \frac{E}{V_s} [e^{\frac{T}{\tau}} - 1] \right)$$

### Formula List

$$m = e^{-\frac{T_{on}}{\tau}}, n = e^{-\frac{T_{off}}{\tau}}$$

$$mn = e^{-\frac{T}{\tau}}$$

$$I_{mx} = \frac{V_s}{R} \cdot \frac{1 - m}{1 - mn} - \frac{E}{R}$$

$$I_{mn} = \frac{V_s}{R} \cdot \frac{n(1 - m)}{1 - mn} - \frac{E}{R}$$

$$I_{mx} - I_{mn} = \frac{V_s}{R} \cdot \frac{(1 - n)(1 - m)}{1 - mn}$$

### Extinction Time ( $t_x$ )

Above load crnt expressions apply for cont load currents.

Crnt can be discont due to larger Toff.

$$t_x \in [T_{on}, T]$$

$$@t = t_x, I_{mn} = 0$$

$$I_{mx} = \frac{V_s - E}{R} (1 - e^{-\frac{R}{L} T_{on}}) + [0] e^{-\frac{R}{L} T_{on}} = \frac{V_s - E}{R} (1 - e^{-\frac{T_{on}}{\tau}})$$

$$I_{mn} = 0 = -\frac{E}{R} (1 - e^{-\frac{t_x - T_{on}}{\tau}}) + I_{mx} e^{-\frac{t_x - T_{on}}{\tau}}$$

$$= -\frac{E}{R} (1 - e^{-\frac{t_x - T_{on}}{\tau}}) + \left[ \frac{V_s - E}{R} (1 - e^{-\frac{T_{on}}{\tau}}) \right] e^{-\frac{t_x - T_{on}}{\tau}}$$

$$\Rightarrow t_x = T_{on} + \tau \ln \left[ 1 + \frac{V_s - E}{E} (1 - e^{-\frac{T_{on}}{\tau}}) \right]$$

$$\text{Avg Op Vltg for RLE discont mode} = V_o = \frac{\int_0^{T_{on}} V_s dt + 0 + \int_{t_x}^T E dt}{T} = \frac{V_s T_{on} + E(T - t_x)}{T}$$

### Fourier Analysis of Op Voltage

Refer vltg wf for cont load current. Vltg is indpdt of load ckt.

$$v_o = V_o + \sum_{n=1}^{\infty} v_n$$

$$v_n = A_n \sin(n\omega t + \theta_n) = \frac{2V_s}{n\pi} \sin n\pi\delta \sin(n\omega t + \theta_n) \rightarrow \text{value of } n\text{th harmonic vltg.}$$

$$V_o = \delta V_s \rightarrow \text{DC Level of Vltg}$$

$$\theta_n = \tan^{-1} \left( \frac{\sin 2\pi n\delta}{1 - \cos 2\pi n\delta} \right)$$

$A_n \rightarrow$  Ampt of nth harmonic vltg.

$$@ \sin n\pi\delta = 1, A_{n(\text{mx})} = \frac{2V_s}{n\pi} = \frac{0.636V_s}{n} \text{ [Volts]}$$

$$\text{Its rms value} = A_{n(\text{mx})\text{rms}} = \frac{2V_s}{\sqrt{2}n\pi} = \frac{0.45V_s}{n} \text{ [Volts]}$$

$$\text{Harmonic crnt in load} = i_n = \frac{V_n}{Z_n}$$

$$Z_n = \sqrt{R^2 + (n\omega L)^2} \rightarrow \text{Load impd at harmonic freq } n\omega \text{ Hz}$$

$$\text{For nglb load rst } R, i_n = \frac{V_n}{n\omega L} \Rightarrow i_n \propto \frac{V_s}{n^2}$$

$\Rightarrow$  Harmonic vltg dec with n inc.

### **Det of harmonic component without calculating harmonics.**

$$\text{AC ripple vltg} = V_r = \sqrt{V_{\text{rms}}^2 - V_o^2}$$

$$V_{\text{rms}} = \sqrt{\delta} V_s, V_o = \delta V_s \Rightarrow V_r = V_s \sqrt{\delta(1 - \delta)}$$

$$\text{RF is ratio of ac ripple vltg to avg vltg} = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1}$$

**dtbom**